Dissipative Trapped Electron Modes in the Presence of Impurities

K. T. Tsang

MASTER

OAK RIDGE NATIONAL LABORATORY

OPERATED BY UNION CARBIDE CORPORATION FOR THE ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Printed

Available from
National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road

5285 Springfield, Virginia 22151
Price: Printed Copy 3.50 Microfiche 2.25

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Contract No. W-7405-eng-26

Thermonuclear Division

DISSIPATIVE TRAPPED ELECTRON MODES IN THE PRESENCE OF IMPURITIES

by

K. T. Tsang

(to be submitted to Nuclear Fusion)

MAY 1976

NOTICE This document contains information of a preliminary nature and was prepared primarily for internal use at the Oak Ridge National Laboratory. It is subject to revision or correction and therefore does not represent a final report.

OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37830
operated by
UNION CARBIDE CORPORATION
for the
ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISSIPATIVE TRAPPED ELECTRON MODES IN THE PRESENCE OF IMPURITIES *

K. T. Tsang
Oak Ridge National Laboratory
Oak Ridge, Tennessee 37830

ABSTRACT

The effect of impurities on low frequency drift modes of a toroidally-confined plasma is investigated by the gyro-kinetic equation. It is assumed that electrons are in the banana regime and ions in the plateau regime. Impurity collision damping is found to be significant in the usual trapped electron mode. A new instability due to the impurities can occur for normal profiles and impurities peaked at the center. Quasi-linear considerations show that impurities will be driven outward if such an instability occurs.

I. INTRODUCTION

Extensive studies have recently appeared on the various aspects of dissipative trapped electron modes¹⁻⁶ due to the possibility that they may be responsible for the anomalous electron transport in present or next generation tokamak experiments. Analysis of ORMAK data⁷ tends to indicate that shear is not always strong enough to stabilize these modes in their pure form.³ However, this analysis is inconclusive since there are cases when half of the plasma column does not satisfy the stability criterion and the other half does. For this reason, we want to treat the mode in a more realistic way, i.e., to include the effects of impurities.

*Research sponsored by the U. S. Energy Research and Development Administration under contract with the Union Carbide Corporation.

The obvious effect of impurities is to increase the electron and ion collision frequencies. The main ion collisions can contribute to the damping directly, while electron collisions cannot, since the growth rate is a complicated function of $\epsilon\omega/v_e$. However, the increase of electron collisionality can decrease the trapped electron population and increase the number of electrons that can resonate with the wave. This effect has already been treated in Reference 6.

The more direct way in which impurities enter the dispersion relation is through their contribution, as additional ion species, to the charge neutrality condition. This leads to two important effects. First, the reduction of growth rate due to impurities collisions. This is the same effect found in the trapped ion mode.¹⁰ Second, a new interval in phase velocities of the modes is introduced.

For convenience, we consider the case with only one species of impurity present. Usually we have the relation: $v_I < v_i < v_e$, where v_I , v_i and v_e are the thermal velocities of impurity, main ion and electron respectively. Hence, we can distinguish two modes. The first one is the usual trapped electron mode, which occurs when the parallel phase velocity $\omega/k_{\parallel}$ (where ω is the mode frequency and $k_{\parallel}$ is the parallel wave number) is between v_i and v_e . It is modified only slightly by the presence of impurities. When $\omega/k_{\parallel}$ is between v_I and v_i we obtain the impurity trapped electron mode similar to the impurity drift mode of Reference 8. The impurity trapped electron mode usually has a mode frequency less than the electron diamagnetic drift frequency. Because of this difference, the trapped electron term is more destabilizing because i) the universal unstable term is never small even

when $k_{\perp} \rho_i \ll 1$, where $k_{\perp}$ and ρ_i are respectively the perpendicular wave number and ion gyro-radius, and ii) the electron toroidal magnetic drift frequency, ω_{De} , can no longer be neglected compared with ω , so that the Landau resonance associated with ω_{De} greatly amplifies the trapped electron terms.

Effects of impurities on drift waves, collisionless trapped particle modes and trapped ion instabilities have been previously treated before by various authors.⁸⁻¹⁰ Dissipative trapped electron modes in the presence of impurities have also been discussed briefly by Bhadra.¹¹ However, his treatment is local, and finite Larmor radii effects are neglected. Because of the recent progress in the linear theory of trapped electron modes,⁵⁻⁶ a more complete and improved calculation is needed in order to analyze the experimental data in a more realistic way.

In the following sections, we first discuss the usual trapped electron mode in an impure plasma. Then we will treat the impurity trapped electron mode. The treatment is based on the gyro-kinetic equation recently derived¹² so that we can easily retain the gyro-radius effects of main ion and impurity. This is a handier and more logical way than starting from the Vlasov equation and integrating along the characteristic. Impurities and main ion are assumed to be in higher collisional regimes (plateau or even Pfirsch-Schluter) than the electrons so that no significant part of those with $v_{\parallel} < \sqrt{e} v_{\perp}$ is trapped in the magnetic well. Electrons are assumed to be deep in the banana regime so that all of those with $v_{\parallel} < \sqrt{e} v_{\perp}$ are trapped. In the following, we assume one species of impurity for convenience. The results can readily be generalized to multispecies impurities.

II. USUAL TRAPPED ELECTRON MODE

We assume a low β toroidal plasma with toroidal coordinates r , θ and ζ , where θ and ζ are the angles measured poloidally and toroidally. The electrostatic perturbation is assumed to be $\phi = \hat{\phi}(r, \theta) \exp(-i\omega t - i l \zeta + i m \theta)$, where $\hat{\phi}$ has only slow θ dependence compared with $\exp(i m \theta)$. The toroidal and poloidal mode numbers satisfy $l q(r_0) = m$, where $q(r)$ is the safety factor and r_0 is the radial location of the rational surface of interest.

The equilibrium distribution functions are assumed to be Maxwellians: F_e , F_i and F_I . The subscripts e , i and I denote electron, main ion and impurity respectively.

The perturbed ion and impurity distribution functions f_i and f_I can be shown to satisfy¹²

$$(\omega - \omega_{D\beta} + i\nu_{\parallel} \frac{n \cdot \nabla}{n}) g_{\beta} - (\omega - \omega_{* \beta}^T) \frac{e}{T_{\beta}} F_{\beta} \langle \exp(i L_{\beta}) \rangle = i \nu_{\beta} g_{\beta} \quad (1)$$

$\beta = i, I$

where

$$f_{\beta} \equiv - \frac{Z_{\beta} e \phi}{T_{\beta}} F_{\beta} + g_{\beta} \exp(-i L_{\beta}),$$

$$\langle \phi e^{i L_{\beta}} \rangle \equiv J_0 \left(\frac{k v_{\perp}}{\Omega_{\beta}} \right) \phi + J_1 \left(\frac{k v_{\perp}}{\Omega_{\beta}} \right) \frac{v_{\perp}}{2 k \Omega_{\beta}} \frac{\partial^2 \phi}{\partial r^2},$$

$L_{\beta} = k v_{\perp} / \Omega_{\beta} \cos \phi$, ϕ being the gyrophase angle,

$$\omega_{* \beta}^T = \omega_{* \beta} \left[1 + n_{\beta} \left(\frac{M_{\beta} v^2}{2 T_{\beta}} - \frac{3}{2} \right) \right],$$

$$\omega_{* \beta} = k \frac{c T_{\beta}}{Z_{\beta} e B} \frac{\partial \ln N_{\beta}}{\partial r}, \quad \omega_{D \beta} = - \left(v_{\parallel}^2 + \frac{v_{\perp}^2}{2} \right) \frac{\cos \theta}{\Omega_{\beta} R},$$

$$\eta_{\beta} = \frac{\partial \ln T_{\beta}}{\partial \ln N_{\beta}},$$

$$k = \frac{m}{r}, \quad \Omega_{\beta} = \frac{Z_{\beta} e B}{M_{\beta} C},$$

J_0 and J_1 are the Bessel functions. Note that the form we wrote down for $\langle \phi \exp(i L_{\beta}) \rangle$ is valid only in the so-called prime coordinates.¹² The magnetic drift frequencies $\omega_{D \beta}$ in Eq. (1) will be neglected later. However, $\omega_{D e}$ for the trapped electrons will be retained. There are two reasons to justify this. First, in present experiments T_{β} is much less than T_e . Secondly, since ions and impurities are in plateau regime, the $\cos \theta$ -dependence of $\omega_{D \beta}$ tends to average to zero, due to free-streaming or collisional randomization in the poloidal direction. Only those electrons trapped in the unfavorable curvature of the tokamak field will have nonzero average $\omega_{D e}$ and have significant effect on the mode. A Bhatnagar-Gross-Krook (BGK) collision model is used in Eq. (1). This model is valid in the limit of small $k_{\theta} \rho_i$, but not accurate enough when $k_{\theta} \rho_i$ is of order one and larger.¹² However, to include any realistic collision model in the treatment will lead to immense complication, which would render the attempt worthless.

Eq. (1) contains the finite gyro-radius effects of the ions and impurities. It is straightforward to obtain the perturbed densities $\tilde{N}_{\beta}$ from Eq. (1):

$$\frac{\tilde{N}_\beta}{N_\beta} = -\frac{Z_\beta e \phi}{T_\beta} + \int dV \frac{(\omega - \omega_{*\beta}^T) Z_\beta e F_\beta}{\omega + i\nu_\beta - k_{||} v_{||}} \frac{1}{T_\beta} \left[J_0^2 + J_0 J_1 \frac{v}{k \Omega_\beta} \frac{2}{r^2} \right] \cdot \phi$$

where J_0 and J_1 have the argument $kv_\perp/\Omega_\beta$ and $k_{||} = (m - 1q)/Rq$.

In the limit $b_\beta = (k \rho_\beta)^2 \ll 1$, with $\rho_\beta^2 = T_\beta/M_\beta \Omega_\beta^2$, we have

$$\begin{aligned} \frac{\tilde{N}_\beta}{N_\beta} = & -\frac{Z_\beta e \phi}{T_\beta} - \frac{Z_\beta e}{T_\beta} \left\{ \left[\left[\omega - \omega_{*\beta} (1 + \eta_\beta/2) \right] Z(\xi_\beta) \right. \right. \\ & - \omega_{*\beta} \eta_\beta (\xi_\beta + \xi_\beta^2 Z(\xi_\beta)) \left. \left(1 - b_\beta + \rho_\beta^2 \frac{\partial^2}{\partial r^2} \right) \right. \\ & \left. \left. + \omega_{*\beta} \eta_\beta Z(\xi_\beta) \right\} \phi / k v_\beta, \end{aligned} \quad (2)$$

where $\xi_\beta = (\omega + i\nu_\beta)/k v_\beta$, $Z(\xi_\beta) = 1/\sqrt{\pi} \int_{-\infty}^{+\infty} \exp(-x^2) dx / x - \xi$ is the plasma dispersion function, and $v_\beta = (2T_\beta/M_\beta)^{1/2}$ is the thermal velocity.

Since $\omega \gg \nu_\beta$, we obtain the following when $v_{I_1} < v_{I_1} < \frac{\omega}{k} < v_e$:

$$\begin{aligned} \frac{\tilde{N}_\beta}{N_\beta} = & \frac{Z_\beta e}{T_\beta} \left\{ -\frac{\omega_{*\beta}}{\omega} - \frac{i\nu_\beta}{\omega} \left(1 - \frac{\omega_{*\beta}}{\omega} \right) \right. \\ & \left. + \left[1 - \frac{\omega_{*\beta}}{\omega} (1 + \eta_\beta) \right] \left(\rho_\beta^2 \frac{\partial^2}{\partial r^2} + \frac{1}{2\xi_\beta^2} - b_\beta \right) \right\} \phi \end{aligned} \quad (3)$$

For the electron part, it is a well-known result^{3,4} that if we assume a flute-like perturbed potential, then

$$\begin{aligned} \frac{\tilde{N}_e}{N_e} = & \frac{e\phi}{T_e} + \sqrt{2\epsilon} \frac{e\phi}{T_e} \left[\left(1 - \frac{\omega_{*e}}{\omega} \right) A_2 - \eta_e \frac{\omega_{*e}}{\omega} A_1 \right] \\ & + i \frac{e\phi}{T_e} \left[\left(1 - \frac{\omega_{*e}}{\omega} \right) R_2 + \eta_e \frac{\omega_{*e}}{\omega} R_1 \right], \end{aligned} \quad (4)$$

with

$$\begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \frac{8\pi^{-1/2}}{\alpha_n^2} \int_0^\infty \frac{v^2 dv e^{-v^2}}{1 - \omega_{De}/\omega + i v_e \alpha_n^2 / 8\epsilon\omega} \begin{Bmatrix} v^2 - \frac{3}{2} \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \frac{\pi^{1/2} \omega}{|k_{||}| v_e} \begin{Bmatrix} \frac{1}{2} \\ 1 \end{Bmatrix} \left[\exp\left(-\frac{\omega^2}{k^2 v_e^2}\right) - \exp\left(-\frac{\omega^2}{\epsilon k^2 v_e^2}\right) \right],$$

and α_n being the n^{th} zero of $J_0(x)$. Equations (3) and (4), and the charge neutrality condition leads to the dispersion relation:

$$\begin{aligned} \phi + \sqrt{2\epsilon} \phi \left[\left(1 - \frac{\omega_{*e}}{\omega}\right) A_2 - \eta_e \frac{\omega_{*e}}{\omega} A_1 \right] + i \left[\left(1 - \frac{\omega_{*e}}{\omega}\right) R_2 + \eta_e \frac{\omega_{*e}}{\omega} R_1 \right] \quad (5) \\ = \sum_{\beta = i, I} \left\{ \frac{\omega_{*e}}{\omega} \frac{r_{ne}}{Z_\beta r_{n\beta}} - i \left(\frac{T_e}{T_\beta} + \frac{\omega_{*e}}{\omega} \frac{r_{ne}}{Z_\beta r_{n\beta}} \frac{v_\beta}{\omega} \right) \right. \\ \left. + \left[\frac{T_e}{T_\beta} + \frac{\omega_{*e}}{\omega} \frac{r_{ne}}{Z_\beta} \left(\frac{1}{r_{n\beta}} + \frac{1}{r_{T\beta}} \right) \right] \left(\rho_\beta^2 \frac{\partial^2}{\partial r^2} + \frac{1}{2\xi_\beta^2} - b_\beta \right) \right\} \phi, \end{aligned}$$

where $\alpha_\beta = Z_\beta^2 N_\beta / N_e$, $r_{n\beta} = (\partial \ln N_\beta / \partial r)^{-1}$, and $r_{T\beta} = (\partial \ln T_\beta / \partial r)^{-1}$.

Assuming the terms involving trapped electrons, collisions, and finite gyro-radius are all small, and using the equilibrium neutrality condition, we obtain

$$\omega = \omega_{*e} + \delta\omega, \text{ where } |\delta\omega| \ll |\omega_{*e}|.$$

In the next order equation, after we expand $k_{||}$ around $r = r_0$, Eq. (5) becomes a Weber equation and can be solved by imposing the outgoing wave boundary condition. Then the eigenvalue of the solution is $\delta\omega$, and we obtain

$$\begin{aligned}
\frac{\gamma}{|\omega_{*e}|} &\equiv \frac{\text{Im} \delta \omega}{|\omega_{*e}|} \\
&= \left(\frac{kc}{eB} \right)^2 T_e M' \left[(2\varepsilon)^{1/2} |\text{Im} A_2| + \frac{1}{2} \left(\frac{m_e |L_s|}{M |r_{ne}|} \right)^{1/2} \ln \frac{1}{\varepsilon} \right] \\
&+ \eta_e \left[(2\varepsilon)^{1/2} \text{Im} A_1 \left(\frac{v_e}{|\varepsilon \omega|} \right)^{-1/4} \left(\frac{m_e |L_s|}{M |r_{ne}|} \right)^{1/2} \ln \frac{1}{\varepsilon} \right] \\
&- \sum_{\beta} \alpha_{\beta} \frac{v_{\beta}}{|\omega_{*e}|} \left(\frac{T_e}{T_{\beta}} + \frac{r_{ne}}{Z_{\beta} r_{n\beta}} \right) \\
&- \left(\frac{M'}{M''} \right)^{1/2} \left| \frac{r_{ne}}{L_s} \right|,
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
M' &= \sum_{\beta} \alpha_{\beta} Z_{\beta}^{-2} M_{\beta} \left[1 + \frac{T_{\beta}}{T_e} \frac{r_{ne}}{Z_{\beta}} \left(\frac{1}{r_{n\beta}} + \frac{1}{r_{T\beta}} \right) \right], \\
M'' &= \left\{ \sum_{\beta} \alpha_{\beta} M_{\beta}^{-1} \left[1 + \frac{T_{\beta}}{T_e} \frac{r_{ne}}{Z_{\beta}} \left(\frac{1}{r_{n\beta}} + \frac{1}{r_{T\beta}} \right) \right] \right\}^{-1}, \\
M &= (M' M'')^{1/2}, \quad L_s^{-1} = r q' / R q^2.
\end{aligned}$$

Eq. (6) looks almost the same as the result given in Ref. 5, except for the ion and impurity collision stabilization terms. The temperature gradient term due to the trapped electrons is exactly the same as in Ref. 5, whereas all the other terms are slightly changed due to the presence of the impurities.

In general, M' and M'' are positive even though $r_{ne} (1/r_{n\beta} + 1/r_{T\beta})$ can have either sign, because $T_{\beta}/T_e Z_{\beta}$ is much less than unity. There may exist some remote possibilities that M'/M'' is negative. In that

case, magnetic shear will contribute to the real part of ω instead of the imaginary part and there will be no more shear stabilization.

Eq. (6) in the limit $k\rho_i \ll 1$ has been used to determine the stability of typical ORMAK discharge parameters. It is found⁷ that the impurity collision term overwhelms all other terms, so that the plasma is stable everywhere.

III. IMPURITY TRAPPED ELECTRON MODE

When the parallel phase velocity of the mode is between the ion and impurity thermal velocities, we have the impurity trapped electron mode. Eq. (3) is still valid for the impurities. For the ions, the BGK model in Eq. (1) is no longer as satisfying. Following Ref. 6, we use a pitch angle collision model for the ion, which is justified by the presence of the impurities. The ion equation can be written as

$$\begin{aligned} (\omega + i v_{\parallel} \vec{n} \cdot \vec{\nabla}) g_i - (\omega - \omega_{*i}^T) \frac{e F_i}{T_i} \left[J_0 + J_1 \frac{v_{\perp}}{2 k \Omega_i} \frac{\partial^2}{\partial r^2} \right] \phi \\ = i \frac{v_i}{2} \frac{\partial}{\partial v_{\parallel}} \left[(v^2 - v_{\parallel}^2) \frac{\partial \hat{g}}{\partial v_{\parallel}} \right]. \end{aligned}$$

Treating only the case of small $k\rho_i$, we get

$$\tilde{N}_i = - \frac{e\phi}{T_i} N_i + \tilde{N}_i',$$

where

$$\tilde{N}_i' = - 4\pi \frac{\omega - \omega_{*i}^T}{|k_{\parallel}|} i \frac{e\phi}{T_i} \int_0^{\infty} v dv F_i \int_0^{\infty} dp \frac{\sin(pv)}{p} \exp\left(i \frac{\omega}{|k_{\parallel}|} p - \frac{v v^2 p^3}{6 |k_{\parallel}|}\right), \quad (7)$$

by the same method used in Ref. 6.

The charge neutrality condition then becomes

$$\begin{aligned}
 & \phi + (2\epsilon)^{1/2} \phi \left[\left(1 - \frac{\omega_{*e}}{\omega}\right) A_2 - \eta_e \frac{\omega_{*e}}{\omega} A_1 \right] \\
 & + i \phi \left[\left(1 - \frac{\omega_{*e}}{\omega}\right) R_2 + \eta_e \frac{\omega_{*e}}{\omega} R_1 \right] \\
 & = -\alpha_1 \left[\frac{T_e}{T_I} \phi - \frac{T_e}{e} \frac{\tilde{N}_i'}{N_i} \right] \\
 & + \alpha_I \frac{T_e}{T_I} \left[-\frac{\omega_{*I}}{\omega} - i \frac{v_I}{\omega} \left(1 - \frac{\omega_{*I}}{\omega}\right) \right] \phi \\
 & + \alpha_I \frac{T_e}{T_I} \left[1 - \frac{\omega_{*I}}{\omega} (1 + \eta_I) \right] \left(\rho_I^2 \frac{2}{r^2} + \frac{1}{2\xi_I^2} - b_I \right) \phi .
 \end{aligned} \tag{8}$$

Only the adiabatic responses in Eq. (8) are lowest-order quantities; hence, it implies

$$\omega = \frac{1}{Z_I} \frac{\alpha_I \omega_{*e}}{1 + \alpha_1 T_e/T_i} \frac{r_{ne}}{r_{nI}} + \delta\omega \tag{9}$$

with $\delta\omega$ much smaller than the first right-hand term.

The terms R_1 , R_2 and $\tilde{N}_i'$ are treated as perturbations in the next order equation. In calculating the contribution from $\tilde{N}_i'$, a technique similar to that in Ref. 6 is used. The part of $\delta\omega$ due to $\tilde{N}_i'$ is

$$(1 + \alpha_i \tau) \frac{T_e}{e} \frac{\alpha_i}{N_e} \omega \frac{\int_{-\infty}^{+\infty} dx \tilde{N}_i' \phi^{(0)}}{\int_{-\infty}^{+\infty} dx \phi^{(0)2}},$$

where $x = r - r_0$, $\phi^{(0)}$ is the lowest-order radial eigen solution.

The integral $\int_{-\infty}^{+\infty} dx \tilde{N}_i' \phi^{(0)}$ can be reduced to the form:

$$\begin{aligned}
I &= \int_0^{\infty} \frac{dx}{x} \exp \left[ia - b/x - \sigma x^2 \right] \\
&\approx \int_0^{x_0} \frac{dx}{x} \exp \left[(ia - b)/x \right] \\
&\approx \ln x_0 / (a^2 + b^2)^{1/2},
\end{aligned}$$

where $a = \omega L_S p/k$, $b = v v_p^3 L_S / 6k$, $\sigma = i|k| (T_I/M_I)^{1/2} / |L_S| \rho_I \omega$, and $x_0 = (\sigma/i)^{-1/2}$. Neglecting the slow p -dependence in the logarithm, and inserting the most probable value of p , i.e., v_i^{-1} , we obtain the following growth rate due to the ions:

$$\frac{\gamma_i}{|\omega|} \approx \sqrt{2} \alpha_i \mu^{1/2} (\ln \mu^{-1/2}) \left[\frac{\tau}{1 + \alpha_i \tau} + \frac{Z_I}{\alpha_I} \frac{r_{nI}}{r_{ni}} \left(1 - \frac{\eta_i}{2} \right) \right] \quad (10)$$

where $\mu = M_i |L_S| \tau \alpha_I / 2 M_I |r_{nI}| (1 + \alpha_i \tau)$, and $\tau = T_e / T_i$. Equation (10) is similar to the well-known result of electron-wave Landau resonance in slab geometry. However, we should recognize that it is an asymptotic result for $\mu \ll 1$. For $\mu \gtrsim 1$, we should replace the right-hand side of Eq. (10) by zero. The exact value of γ_i when $\mu \sim 1$ involves very complicated calculation, and it is not clear whether the perturbative method makes sense in this case. Taking into account all the other terms, the total growth rate of this mode is:

$$\begin{aligned}
\frac{\gamma}{|\omega|} &= \left(\frac{Z_I r_{nI}}{\alpha_I r_{ne}} - \frac{1}{1 + \alpha_i \tau + \alpha_I b_I T_e / T_I} \right) \left[(2\epsilon)^{1/2} \left| \text{Im } A_2 \right| + \frac{1}{2} \left(\frac{m_e}{M_I} \left| \frac{L_S}{r_{nI}} \right| \frac{\alpha_I}{1 + \alpha_i} \right)^{1/2} \ln \frac{1}{\epsilon} \right] \\
&+ \eta_e \frac{Z_I r_{nI}}{\alpha_I r_{ne}} \left[(2\epsilon)^{1/2} \text{Im } A_1 \left(\left| \frac{v_e}{\epsilon \omega} \right| \right) - \frac{1}{4} \left(\frac{m_e}{M_I} \left| \frac{L_S}{r_{nI}} \right| \frac{\alpha_I}{1 + \alpha_i \tau} \right)^{1/2} \ln \frac{1}{\epsilon} \right]
\end{aligned} \quad (11)$$

$$\begin{aligned}
& - \alpha_I \frac{T_e}{T_I} \frac{v_I}{|\omega|} \left(\frac{1}{1 + \alpha_i \tau} + \frac{T_I}{\alpha_I T_e} \right) + \frac{\gamma_i}{|\omega|} \\
& - \left[1 + \frac{T_I}{\alpha_I T_e} (1 + \alpha_i \tau) (1 + \eta_I) \right] \left| \frac{r_{nI}}{L_s} \right|
\end{aligned}$$

Eq. (11) tells us that shear is only provided by the impurities.

Although apparently the size of the shear term is not reduced, it is smaller compared with the destabilizing trapped electron terms, since $Z_I r_{nI}/\alpha_I r_{ne}$ is usually greater than one. Ion collision stabilization is missing from our assumed collision model, but it will appear if we use a hybrid pitch angle and BGK collision model. In any case, judging from our experience in last section, it is not as important as the impurity collision. The total effect of the ion pitch angle collisions is to induce an ion-wave collisional resonance which gives rise to γ_i . From Eq. (10), we observe that when $\mu \ll 1$, the ion term is destabilizing if

$$\frac{\tau}{1 + \alpha_i \tau} + \frac{Z_I r_{nI}}{\alpha_I r_{ni}} \left(1 - \frac{\eta_i}{2} \right) < 0.$$

This condition is also given in Ref. 8. The most interesting terms are those due to the trapped electrons. When $Z_I r_{nI}/\alpha_I r_{ne} > 0$ and $\eta_e > 0$, a violent instability will occur. Unlike the usual trapped electron mode, $\text{Im } A_2$ and the circulating electron resonance term is not multiplied by a small number even when kp_i is very small. Hence, these two sources of instability are always present. If we assume r_{nI} is of the same order as r_{ne} , $Z_I r_{nI}/\alpha_I r_{ne}$ is usually much larger than one for the impurity level in present-day tokamaks. Thus, the effect of A_1 , A_2 and the untrapped electron resonance will be magnified.

When r_{nI} is negative, i.e., when impurities are peaked at the center, the resonance associated with the magnetic curvature drift sets in.

For the deeply-trapped electron, we have

$$\omega_{De}/\omega \approx - \frac{m_e v^2}{2T_e} \frac{r_{nI}}{R} \frac{Z_I}{\alpha_I} (1 + \alpha_i \tau).$$

Since the ratio $r_{nI} Z_I (1 + \alpha_i \tau) / R \alpha_I$ is of order unity, this resonance is more pronounced than that in the usual trapped electron mode.

The discussion above implies that the condition $r_{nI}/r_{ne} > 0$, $\eta_e > 0$ is a very unstable situation with respect to the impurity trapped electron mode. Hence, the quasi-linear consequence is that impurities start to diffuse outward until the driving term of the instability vanishes. This is confirmed by the calculation of the quasi-linear diffusion flux for impurities. We have, by integrating the quasi-linear Vlasov equation over velocity space,

$$\frac{\partial N_I}{\partial t} = - \frac{1}{r} \frac{\partial}{\partial r} r \Gamma_I,$$

and

$$\Gamma_I = 2 \sum_k \frac{c}{B} k |\tilde{\phi}_k|^2 \text{Im} \frac{\tilde{N}_{Ik}}{\tilde{\phi}_k}.$$

Substituting $\tilde{N}_{Ik}$ for this mode in, we get

$$\Gamma_I = - \sum_k \left(\frac{Z_I e \phi_k}{T_I} \right)^2 \frac{(v_I k)^2}{|\omega|} \rho_I^2 \frac{N_I}{r} \quad (12)$$

$$\cdot \left\{ \left(\frac{Z_I r_{nI}}{\alpha_I r_{ne}} - \frac{1}{1 + \alpha_i \tau + \alpha_I T_e / T_I b_I} \right) \left[\sqrt{2\epsilon} |\text{Im} A_2| \right. \right.$$

$$\begin{aligned}
& + \frac{1}{2} \left(\frac{m_e}{M_I} \left| \frac{L_s}{r_{nI}} \right| \frac{\alpha_I}{1 + \alpha_i \tau} \right)^{1/2} \ln \frac{1}{\epsilon} \Bigg] \\
& + \eta_e \frac{Z_I r_{nI}}{\alpha_I r_{ne}} \left[\sqrt{2\epsilon} \operatorname{Im} A_1 - \frac{1}{4} \left(\frac{m_e}{M_I} \left| \frac{L_s}{r_{nI}} \right| \frac{\alpha_I}{1 + \alpha_i \tau} \right)^{1/2} \ln \frac{1}{\epsilon} \right] + \frac{\gamma_i}{\omega} \Bigg\}
\end{aligned}$$

Whenever the instability occurs, the sum of the terms inside the curly bracket in Eq. (12) is positive. For the case $\partial N_I / \partial r < 0$ (peaked at the center), the impurity particle flux is positive (outward). This may provide us with an explanation for the experimental fact that so far no impurities peaked at the center as predicted by neoclassical theory are observed.

IV. CONCLUSION

The introduction of impurities in the consideration of trapped electron modes leads us to distinguish between two modes: the usual trapped electron mode and the impurity trapped electron mode, which are basically the usual drift mode and the impurity drift mode, respectively, in toroidal geometry. Both modes are studied in great detail. The growth rate of the usual trapped electron mode is given in Eq. (6). For ORMAK discharges data, the collisional damping term due to impurity overwhelms all other terms and the mode is stabilized. The growth rate of the impurity trapped electron mode is given in Eq. (11). It is found that a strong instability will occur if $r_{nI}/r_{ne} > 0$ and $\eta_e > 0$. Whenever such an instability occurs, the quasi-linear impurity particle flux is found to be positive (outward). This may explain the lack of impurity peaking at the center as predicted by neoclassical theory in most tokamak discharges.

ACKNOWLEDGEMENTS

The author wishes to thank J. D. Callen and P. J. Catto for helpful discussions and valuable comments.

REFERENCES

1. W. Horton, Jr. et al., Plasma Physics and Controlled Nuclear Fusion Research (Proc. Fifth Int. Conf., Tokyo, 1974).
2. C. S. Liu, M. N. Rosenbluth, and W. M. Tang, Phys. Fluids, to be published.
3. J. D. Callen et al., Bull. Am. Phys. Soc. 19, 863 (1975).
4. W. M. Tang et al., Princeton Plasma Physics Laboratory Report MATT-1153, 1975.
5. P. J. Catto, K. T. Tsang, J. D. Callen, and W. M. Tang, ORNL-TM-5158, submitted to Phys. Fluids for publication.
6. K. T. Tsang, J. D. Callen, and P. J. Catto, ORNL-TM-5236, to be published.
7. E. C. Crume and D. L. Shaeffer, private communication.
8. B. Coppi, H. P. Furth, M. N. Rosenbluth, and R. Z. Sagdeev, Phys. Rev. Lett. 17, 377 (1966).
9. B. Coppi, G. Rewolt, and T. Schep, Mass. Institute of Technology Plasma Research Report-754, February, 1975.
10. M. Dobrowolny, Nuclear Fusion 14, 599 (1974).
11. D. K. Bhadra, Phys. Fluids 18, 380 (1975).
12. P. J. Catto and K. T. Tsang, submitted to Phys. Fluids for publication.

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

INTERNAL DISTRIBUTION

ORNL/TM-5287

- 1-3. Lab Records
- 4-5. Central Research Library
- 6. Y-12 Document Reference
- 7. Thermonuclear Library
- 8. ORNL Patent Office
- 9. J. D. Callen
- 10. D. G. McAlees
- 11. R. A. Dory
- 12. G. G. Kelley
- 13. J. F. Clarke
- 14. O. B. Morgan
- 15. Lab Records - RC
- 16. Research & Technical Support Division, (ORO) ERDA
- 17. M. W. Rosenthal
- 18-43. K. T. Tsang

EXTERNAL DISTRIBUTION

- 44. Plasma Physics Library, Plasma Physics Laboratory, Princeton University, Forrestal Campus, P. O. Box 451, Princeton, NJ 08540
- 45. Controlled Thermonuclear Research Library, Lawrence Livermore Laboratory, P. O. Box 808, Livermore, CA 94550
- 46. Q. Division Library, c/o F. L. Ribe, Los Alamos Scientific Laboratory, P. O. Box 1663, Los Alamos, NM 87544
- 47. Controlled Thermonuclear Research Library, c/o Weston M. Stacey, Jr., Argonne National Laboratory, 9700 S. Cass Ave., Argonne, IL 60439
- 48. CTR Computer Center, c/o Dr. John Killeen, Lawrence Livermore Laboratory, P. O. Box 808, Livermore, CA 94550
- 49. Librarian, Culham Laboratory, U. K. Atomic Energy Authority, Abingdon, Oxon, OX14, 3DB, United Kingdom
- 50. Ruth Lengye, Bibliothek, Max Planck Institut fur Plasmaphysik, 8046 Garching bei Munchen, Federal Republic of Germany
- 51. Library, Centre de Recherches en Physique des Plasma, 21 Avenue des Bains, 1007, Lausanne, Switzerland
- 52. A. M. Dupas, Documentation S.I.G.N., Department de La Physique du Plasma, Et de La Fusion Controlee, Association EURATOM-CEA, Sur La Fusion, Centre D'Etudes Nucleaires, BP 85 Centre Du TRI 38041 Grenoble Cedex (France)
- 53. Bibliotheque, Service du Confinement des Plasmas, C.E.A., B.P. No. 6, 92, Fontenay-Aux-Roses (Seine) France

54. Library, International Centre for Theoretical Physics, Trieste, Italy
55. Library, Laboratorio Gas Ionizzati, Frascati, Italy
56. V. E. Ivanov, Physical-Technical Institute of the Ukrainian Academy of Sciences Sukhumi, U.S.S.R.
57. M. S. Rabinovich, Lebedev Institute of Physics, Academy of Sciences of the U.S.S.R., Leninsky Prospect 53, Moscow, U.S.S.R.
58. Thermonuclear Laboratory, Kurchatov Institute of Atomic Energy, 46 Ulitsa Kurchatova, P. O. Box 3402, Moscow, U.S.S.R.
59. Library, Institute for Plasma Physics, Nagoya University, Nagoya, Japan 464
60. Library, FOM-Institut voor Plasma-Fysica, Rijnhuizen, Jutphaas, Netherlands
61. Plasma Physics Group, Department of Engineering Physics, Australian National University, P. O. Box 4, Canberra A.C.T. 2600 Australia
62. Thermonuclear Library, Japan Atomic Energy Research Institute, Tokai, Naka, Ibaraki, Japan
63. CTR Reading Room, c/o Prof. Dieter J. Sigmar, Room 37-391, M.I.T., Cambridge, MA 02139
64. CTR Reading Room, c/o Prof. D. W. Kerst, Dept. of Physics, Sterling Hall, Univ. of Wisconsin, Madison, WI 53706
65. CTR Reading Room, c/o Prof. I. B. Bernstein, Yale University, New Haven, Conn. 06510
66. Center for Plasma Physics and Thermonuclear Research, Univ. of Texas, Physics Building 330, Austin, TX 78712
67. CTR Reading Room, c/o Prof. B. D. Fried, Physics Dept., Univ. of California, Los Angeles, CA 90024
68. CTR Reading Room, c/o Prof. David C. Montgomery, Physics and Astronomy Dept., Univ. of Iowa, Iowa City, Iowa 52240
69. Magneto-Fluid Dynamics Library, Courant Inst. of Math. Sci., New York Univ., 251 Mercer St., New York, NY 10012
70. CTR Reading Room, c/o Prof. Allan N. Kaufman, Physics Dept., Univ. of California, Berkeley, CA 94720

71. CTR Reading Room, c/o Prof. W. B. Thompson, Univ. of California, San Diego, Physics Dept., La Jolla, CA 92037
72. CTR Reading Room, c/o Prof. Alvin W. Trivelpiece, Dept. of Physics and Astronomy, Univ. of Maryland, College Park, MD 20742
73. CTR Reading Room, c/o Prof. Terry Kammash, 103 Research Admin. Bldg., N. Campus, Univ. of Michigan, Ann Arbor, MI 48105
74. CTR Reading Room, c/o Dr. Ravi N. Sudan, Phillips Hall, Cornell Univ., Ithaca, NY 14850
75. Prof. Marshall N. Rosenbluth, Institute for Advanced Study, Princeton, NJ 08540
76. CTR Reading Room, c/o Prof. Roy Gould, California Inst. of Tech., Pasadena, CA 91103
77. CTR Reading Room, c/o Prof. R. Gross, Plasma Research Lab., Columbia Univ., New York, NY 10027
78. Dr. Nicholas A. Krall, Science Applications, Inc., P. O. Box 2354, 1200 Prospect St., La Jolla, CA 92037
79. CTR Reading Room, c/o Dr. Jay P. Boris, Plasma Physics, Naval Research Laboratory, Washington, DC 20390
80. CTR Library, General Atomic Co., P. O. Box 81608, San Diego, CA 92138
81. CTR Library, c/o Dr. Alan F. Haught, United Technologies Research Labs, East Hartford, Conn. 06108
82. Dr. Robert E. Price, Division of Controlled Thermonuclear Research, U. S. Energy Research and Development Administration, Washington, D.C. 20545
83. Dr. Bennett Miller, Division of Controlled Thermonuclear Research, U. S. Energy Research and Development Administration, Washington, D. C. 20545
84. Dr. Arthur Sleeper, Division of Controlled Thermonuclear Research, U. S. Energy Research and Development Administration, Washington, D. C. 20545
85. Dr. Walter Sadowski, Division of Controlled Thermonuclear Research, U. S. Energy Research and Development Administration, Washington, D.C. 20545
86. Dr. D. H. Priester, Division of Controlled Thermonuclear Research, U. S. Energy Research and Development Administration, Washington, D. C. 20545

87. Dr. Paul H. Rutherford, Princeton Plasma Physics Lab., Princeton Univ., Princeton, NJ 08540
88. Dr. L. D. Pearlstein, L-388, Lawrence Livermore Laboratory, P. O. Box 808, Livermore CA 94550
89. Dr. J. P. Friedberg, Los Alamos Scientific Laboratory, Los Alamos, NM 87544
90. Dr. David W. Ross, Center for Plasma Physics and Thermonuclear Research, Dept. of Physics, Univ. of Texas, Austin, TX 78712
91. Dr. Gareth E. Guest, General Atomic Co., P. O. Box 81608, San Diego, CA 92138
92. Dr. Claude Mercier, Service du Theorie des Plasmas, Centre d'Etudes Nucleaires, Fontenay-aux-Roses (Seine) France
93. Dr. J. B. Taylor, Culham Laboratory, UKAEA Abingdon, Oxon, OX14, 3DB, United Kingdom
94. Dr. D. Pfirsch, Institute for Plasma Physics, 8046 Garching bei Munchen, Federal Republic of Germany
95. Dr. V. D. Shafranov, I. V. Kurchatov Inst. of Atomic Energy, 46 Ulitsa Kurchatova, P. O. Box 3402, Moscow, U.S.S.R.
96. Dr. Harold Grad, Courant Inst. of Math. Sci., New York Univ., 251 Mercer St., New York, NY 10012
97. Dr. J. G. Cordey, Culham Laboratory, UKAEA Abingdon, Oxon, OX14, 3DB, United Kingdom
98. Dr. David Baldwin, L-388, Lawrence Livermore Lab., P. O. Box 808, Livermore, CA 94550
99. Prof. Bruno Coppi, Dept. of Physics, M.I.T., Cambridge, MA 02139
100. Dr. Harold P. Furth, Princeton Plasma Physics Laboratory, Princeton Univ., P. O. Box 451, Princeton, NJ 08540
- 101-127. Technical Information Center, ERDA