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TRANSPORT OF A MULTIPLE ION
SPECIES PLASMA IN THE
PFIRSCH-SCHLÜTER REGIME

BY

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**PLASMA PHYSICS
LABORATORY**

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TRANSPORT OF A MULTIPLE ION SPECIES PLASMA
IN THE PFIRSCH-SCHLÜTER REGIME

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ABSTRACT

The classical parallel friction coefficients, which relate the collisional friction forces to the flow of particles and heat along the magnetic field, are calculated for a multiple ion species plasma. In the short mean free path regime, the neoclassical Pfirsch-Schlüter transport coefficients for a toroidally confined multispecies plasma are computed in terms of the classical friction coefficients. The dependence of the neoclassical cross-field transport on the equilibration of the parallel ion temperature profiles is determined.

I. INTRODUCTION

The calculation of the neoclassical cross-field transport coefficients for a multiple ion species plasma in the collisional Pfirsch-Schlüter regime¹ relies on an appropriate set of classical parallel transport relations. Previously, Braginskii² considered classical transport along a magnetic field for a simple electron and single ion species plasma. In Section II of this paper we generalize the method of Ref. 2 to compute the classical parallel transport relations for a multispecies plasma. The presence of multiple ion species considerably complicates the classical transport problem. The mutual collisional friction forces between several ion species of comparable masses produces a "reshaping" of the species' distribution functions, and hence a generally nonadditive change in the transport coefficients, which must be accounted for at a kinetic level. Nevertheless, the multispecies transport coefficients are required to assess the predictions of neoclassical transport theory for tokamak plasmas, which are currently comprised of several high-Z impurity species in addition to hydrogen. Furthermore, the presence of various ionization states of the same impurity precludes the analogy with the Lorentz problem which allowed Rutherford³ to obtain transport coefficients for a single ion and heavy impurity species plasma.

The calculation in Section II yields expressions for the parallel frictional forces in a classical multispecies plasma in terms of the parallel particle and heat flows. Using these results, the neoclassical transport coefficients are computed in

Section III in terms of the classical friction coefficients. The strong equilibration of the parallel temperature profiles in a multiple ion species plasma is shown to modify the standard Pfirsch-Schlüter transport process for an unequilibrated plasma. Finally, in Section IV we evaluate the classical friction coefficients for two specific situations of practical importance, corresponding to the multispecies generalization of the Lorentz problem, and the mutual interaction of various charge states of a single heavy impurity species.

By restricting the present calculation to a collisional multiple ion species plasma, the validity of the results derived here is limited to the relatively cool and contaminated region of the plasma discharge where the mean free path of each constituent is shorter than the magnetic field connection length. Nevertheless, this region, which extends inwards from the plasma periphery, can occupy a significant fraction of the discharge cross section in current tokamak devices.

II. CALCULATION OF CLASSICAL TRANSPORT COEFFICIENTS

We seek to determine the perturbed distribution function f_{a1} of species (a), which satisfies the Fokker-Planck equation² describing classical transport along a magnetic field $\underline{B}$:

$$v_{||} [A_{1a} - L_1^{(3/2)}(x_a^2) A_{2a}] f_{a0} = \sum_b [C_{ab}(f_{a1}, f_{b0}) + C_{ab}(f_{a0}, f_{b1})] \quad (1)$$

Here, f_{a0} is a local Maxwellian, the generalized forces are:

$$A_{1a} = v_{||} \ln p_a - \frac{q_a}{T_a} E_{||} \quad , \quad (2a)$$

$$A_{2a} = v_{||} \ln T_a \quad , \quad (2b)$$

$v_{||} = (\underline{B}/B) \cdot \nabla$, $x_a^2 = (v/v_{Ta})^2$, where $v_{Ta} = (2T_a/m_a)^{1/2}$, and $L_k^{(3/2)}(x_a^2)$ is the Sonine polynomial² of order 3/2 and index k.

Note that the collision operator C_{ab} in Eq. (1) couples the transport of species (a) with all other species (b) through mutual (a) - (b) collisions, which are most effective when $m_a \lesssim m_b$.

The solution of Eq. (1) can be obtained in two equivalent forms. The first is to obtain each distribution function f_{a1} as a linear combination of the generalized forces:

$$f_{a1} = \sum_{b;j} v_{||} \phi_{aj}^b(v) A_{jb} f_{a0} \quad (3)$$

where $\phi_{aj}^b(v)$ are functions of velocity magnitude v which must be determined by solving Eq. (1). Using Eq. (3), all the

moments of f_{a1} can be determined explicitly in terms of the generalized forces.

The alternative solution of Eq. (1), which is pursued here in consideration of our goal of obtaining Pfirsch-Schlüter transport coefficients, is based on a generalized moment equation approach.⁴ Multiplying Eq. (1) by $m_a v_{||} L_k^{(3/2)}(x_a^2)$ and integrating over velocity yields the system of moment equations:

$$p_a A_{1a} \delta_{0k} - \frac{5}{2} p_a A_{2a} \delta_{1k} = \int m_a v_{||} L_k^{(3/2)} C_a d\underline{v} \quad (4)$$

where $C_a \equiv \sum_b C_{ab}$. Defining the parallel friction force on species (a) (mean rate of collisional momentum generation)

$$R_a = \int m_a v_{||} C_a d\underline{v} \quad (5a)$$

and the parallel heat friction

$$H_a = \int m_a v_{||} [(v/v_{Ta})^2 - 5/2] C_a d\underline{v} \quad (5b)$$

we observe that the first two ($k = 0, 1$) inhomogeneous moments in Eq. (4) correspond to classical parallel momentum balance and heat flux balance, respectively:

$$v_{||} p_a - q_a n_a E_{||} = R_a, \quad (6a)$$

$$\frac{5}{2} n_a v_{||} T_a = H_a. \quad (6b)$$

For $k \geq 2$, we obtain the homogeneous relations

$$\int m_a v_{||} L_k^{(3/2)} C_a dv = 0, \quad k \geq 2 \quad (7)$$

whose significance will be presently discussed.

Now we proceed to expand f_{a1} in a Sonine polynomial series²:

$$\begin{aligned} f_{a1} &= \frac{2v_{||}}{v_{Ta}^2} f_{a0} \sum_{k=0}^{\infty} u_{ak} L_k^{(3/2)}(x_a^2) \\ &= \frac{2v_{||}}{v_{Ta}^2} f_{a0} \left[u_{||a} - \frac{2}{5} L_1^{(3/2)}(x_a^2) \frac{q_{||a}}{p_a} + \sum_{k=2}^{\infty} u_{ak} L_k^{(3/2)}(x_a^2) \right] \end{aligned} \quad (8a)$$

where, using the orthogonality of $L_k^{(3/2)}$ we have:

$$n_a u_{ak} = \frac{\int L_k^{(3/2)}(x_a^2) v_{||} f_{a1} dv}{\{ [L_k^{(3/2)}(x_a^2)]^2 \}} \quad (8b)$$

Here, braces denote $\{A(v)\} = [8/3(\pi)^{1/2}] \int_0^{\infty} x^4 \exp(-x^2) A(xv_{Ta}) dx$.

In Eq. (8a) we have identified the coefficients of the first two Sonine polynomials as $u_{||a} = \int v_{||} f_{a1} / n_a dv$, the parallel flow velocity, and $q_{||a} = \int [(1/2)m_a v^2 - (5/2)T_a] v_{||} f_{a1} dv$, the random heat flux. Note that $u_{||a}$ and $q_{||a}$ are hydrodynamic variables, in the sense that they appear in the fluid continuity and heat equations.

The solution of the classical transport Eq. (1) can now be restated in terms of the friction forces R_a and H_a , and the hydrodynamic variables $u_{||a}$ and $q_{||a}$. In particular, the goal of classical transport theory is to provide transport relations of the general form:

$$R_a = \sum_b [\ell_{11}^{ab} u_{||b} + \frac{2}{5} \ell_{12}^{ab} (\frac{q_{||b}}{p_b})] , \quad (9a)$$

$$H_a = \sum_b [\ell_{21}^{ab} u_{||b} + \frac{2}{5} \ell_{22}^{ab} (\frac{q_{||b}}{p_b})] , \quad (9b)$$

thereby expressing the friction in terms of the independent hydrodynamic variables. The quantities ℓ_{mn}^{ab} are the classical friction coefficients. They are inversely related to the classical transport coefficients, which relate the parallel flows $u_{||a}$ and $q_{||a}$ to the forces A_{ja} . Indeed, the two inhomogeneous moment equations appearing in Eq. (4) may be solved, using Eqs. (9), to determine $u_{||a}$ and $q_{||a}$ in terms of the driving forces.

Note that in order to obtain the transport relations in the form of Eqs. (9), we must first determine the expansion coefficients u_{ak} appearing in Eq. (8a), for $k \geq 2$, in terms of the hydrodynamic variables. This is precisely the role of the homogeneous Eq. (7), which may be solved to determine u_{ak} , for $k \geq 2$, in terms of $u_{||a}$ and $q_{||a}$. Then, the distribution function in Eq. (8a) becomes a function of the hydrodynamic quantities alone, and the appropriate moments of the collision operator, Eqs. (5), may be evaluated to yield the desired transport relations in Eqs. (9).

Thus, the present classical transport calculation reduces to an appropriate approximate solution of the homogeneous system, Eq. (7), for u_{ak} ($k \geq 2$). We remark that our solution, Eqs. (9), differs somewhat from that of Ref. 2, corresponding more closely to the "13-moment" approach of Grad.⁴ In

Braginskii's representation of the transport relations, R_a and $q_{||a}$ appear as the dependent quantities expressed in terms of the flows $u_{||a}$ and the generalized forces A_{2a} . Thus, Braginskii solves the moment equation for $k = 1$, Eq. (6b), to determine $q_{||a}$ in terms of $u_{||a}$ and A_{2a} . The present representation is symmetric, expressing the odd velocity moments (R_a and H_a) of the collision operator in terms of the odd moments ($u_{||a}$ and $q_{||a}$) of the distribution function. As we show in Section III, both representations of the transport relations are useful in determining the neoclassical transport coefficients, although the present form, Eqs. (9), is more natural for the classical transport calculation of this section.

We now define the N^{th} order approximation, $f_{a1}^{(N)}$, for the distribution function as follows:

$$f_{a1}^{(N)} = \frac{2v_{||}}{v_{Ta}^2} f_{a0} [u_{||a} - \frac{2}{5} L_1^{(3/2)}(x_a^2) \frac{q_{||a}}{p_a} + \sum_{k=2}^{N+1} u_{ak} L_k^{(3/2)}(x_a^2)] \quad (10a)$$

with the associated constraint equations for the quantities u_{ak} , $k \geq 2$, now reduced to a finite system:

$$\int v_{||} L_k^{(3/2)} C_a d\underline{v} = 0, \quad 2 \leq k \leq N+1. \quad (10b)$$

Consider the two lowest-order approximations, corresponding to $N = 0$ and $N = 1$ respectively.

A. $N = 0$

This order corresponds exactly with Grad's 13-moment approximation [for $f_{a1} \propto v_{||} \phi(v)$]. The distribution function is truncated after its first two moments:

$$f_{a1}^{(0)} = \frac{2v_{||}}{v_{Ta}^2} f_{a0} [u_{||a} - \frac{2}{5} L_1^{(3/2)}(x_a^2) \frac{q_{||a}}{p_a}]$$

and there is no system of constraint equations to be solved corresponding to Eq. (10b). Note that since $f_{a1}^{(0)}$ is already described solely in terms of the hydrodynamic variables, it is straightforward to evaluate the transport relations in the representation of Eqs. (9). However, since no kinetic information was used to obtain $f_{a1}^{(0)}$, it might be expected (and it is readily confirmed²) that the resulting transport relations contain inaccuracies of order unity, thus making the zero-order approximation of limited utility for the calculation of accurate transport relations.

B. $N = 1$

The first-order approximation for f_{a1} can be written:

$$f_{a1}^{(1)} = \frac{2v_{||}}{v_{Ta}^2} f_{a0} [u_{||a} - \frac{2}{5} L_1^{(3/2)}(x_a^2) \frac{q_{||a}}{p_a} + u_{a2} L_2^{(3/2)}(x_a^2)] \quad (11)$$

where $L_1^{(3/2)}(x_a^2) = (5/2) - x_a^2$ and $L_2^{(3/2)}(x_a^2) = (35/8) - (7/2)x_a^2 + (1/2)x_a^4$. As noted in Ref. 2, this approximation yields accuracy of a few percent for the transport coefficients. The

surprising increase in accuracy in going from the $N = 0$ to the $N = 1$ approximation can probably be explained by noting that the ℓ_{mn} are variational.⁵ The first-order constraint, Eq. (10b), from which u_{a2} must be determined in terms of $u_{||a}$ and $q_{||a}$, becomes for each species (a):

$$\int v_{||} L_2^{(3/2)}(x_a^2) C_a [f_{a1}^{(1)}, f_{b1}^{(1)}] dv = 0. \quad (12)$$

Equation (12) yields a linear algebraic equation for the quantities u_{a2} . Explicitly, we obtain:

$$u_{a2} = - \sum_b \alpha_{ab} u_{||b} + \frac{2}{5} \sum_b \beta_{ab} \left(\frac{q_{||b}}{p_b} \right) + \sum_b \Delta_{ab} u_{b2} \quad (13)$$

where

$$\alpha_{ab} = \frac{[M_a^{20} \delta_{ab} + (v_{Ta}/v_{Tb}) N_{ab}^{20}]}{(M_a^{22} + N_{aa}^{22})}, \quad (14a)$$

$$\beta_{ab} = \frac{[M_a^{21} \delta_{ab} + (v_{Ta}/v_{Tb}) N_{ab}^{21}]}{(M_a^{22} + N_{aa}^{22})}, \quad (14b)$$

$$\Delta_{ab} = \frac{-(v_{Ta}/v_{Tb}) N_{ab}^{22} (1 - \delta_{ab})}{(M_a^{22} + N_{aa}^{22})}, \quad (14c)$$

and the matrix coefficients M_{ab}^{jk} and N_{ab}^{jk} are defined as:

$$M_{ab}^{jk} = \int \left(\frac{v_{||}}{v_{Ta}} \right) L_j^{(3/2)}(x_a^2) C_{ab} \left[\frac{2v_{||}}{v_{Ta}} f_{a0} L_k^{(3/2)}(x_a^2), f_{b0} \right] dv, \quad (15a)$$

$$N_{ab}^{jk} = \int \left(\frac{v_{||}}{v_{Ta}} \right) L_j^{(3/2)}(x_a^2) C_{ab} \left[f_{a0}, \frac{2v_{||}}{v_{Tb}} L_k^{(3/2)}(x_b^2) f_{b0} \right] dv, \quad (15b)$$

and

$$M_a^{jk} = \sum_b M_{ab}^{jk}. \quad (15c)$$

The symmetry properties of the matrix coefficients, M_{ab}^{jk} and N_{ab}^{jk} , which follow from the self-adjoint property of the collision operator near thermal equilibrium,⁵ are given by:

$$M_{ab}^{jk} = M_{ab}^{kj}, \quad (16a)$$

$$T_a N_{ab}^{jk} = T_b N_{ba}^{kj}. \quad (16b)$$

In Appendix A, the matrix coefficients are computed explicitly (for $j, k \leq 2$) in terms of v_{Ta}/v_{Tb} for the case when C_{ab} is the linearized Coulomb collision integral.²

The general solution of Eq. (13) is of the form:

$$u_{a2} = - \sum_b \hat{\alpha}_{ab} u_{||b} + \frac{2}{5} \sum_b \hat{\beta}_{ab} \left(\frac{q_{||b}}{p_b} \right). \quad (17)$$

For a small number of plasma constituents, the explicit inversion of Eq. (13) is straightforward. For an arbitrarily large number of species, an approximate inversion of Eq. (13) is desirable.

To this end, we note the relation (Appendix A):

$$\sum_b |\Delta_{ab}| \leq 0.327 \quad (18)$$

which indicates that the coefficients $\hat{\alpha}_{ab}$ and $\hat{\beta}_{ab}$ in Eq. (17) may be obtained by expanding Eq. (13) in the small parameter Δ_{ab} , yielding:

$$\hat{\alpha}_{ab} = \alpha_{ab} + \sum_c \Delta_{ac} \alpha_{cb} + \dots, \quad (19a)$$

$$\hat{\beta}_{ab} = \beta_{ab} + \sum_c \Delta_{ac} \beta_{cb} + \dots. \quad (19b)$$

The smallness of Δ_{ab} follows from Eq. (14c), which shows that Δ_{ab} is the ratio of the $L_2^{(3/2)}$ moment of the integral portion (N_{ab}^{22}) of the collision operator acting on $L_2^{(3/2)}$, which tends to smooth out $L_2^{(3/2)}$, to the differential portion (M_{ab}^{22}) of C_{ab} acting on $L_2^{(3/2)}$, which enhances the structure of $L_2^{(3/2)}$.

The friction forces may now be computed from the defining relations, Eqs. (5), by using $f_{al}^{(1)}$ to evaluate C_a approximately. Thus we obtain for the symmetric friction coefficients ℓ_{mn}^{ab} = ℓ_{nm}^{ba} appearing in Eqs. (9):

$$\ell_{11}^{ab} = m_a [M_a^{00} \delta_{ab} - M_a^{02} \hat{\alpha}_{ab} + (v_{Ta}/v_{Tb}) N_{ab}^{00} - \sum_k (v_{Ta}/v_{Tk}) N_{ak}^{02} \hat{\alpha}_{kb}] , \quad (20a)$$

$$\ell_{12}^{ab} = m_a [-M_a^{01} \delta_{ab} + M_a^{02} \hat{\beta}_{ab} - (v_{Ta}/v_{Tb}) N_{ab}^{01} + \sum_k (v_{Ta}/v_{Tk}) N_{ak}^{02} \hat{\beta}_{kb}] , \quad (20b)$$

$$\ell_{22}^{ab} = m_a [M_a^{11} \delta_{ab} - M_a^{12} \hat{\beta}_{ab} + (v_{Ta}/v_{Tb}) N_{ab}^{11} - \sum_k (v_{Ta}/v_{Tk}) N_{ak}^{12} \hat{\beta}_{kb}] . \quad (20c)$$

Furthermore, by considering momentum conservation in (a)-(b) collisions, $R_{ab} + R_{ba} = 0$, we can derive the relation

$$M_{ab}^{0j} + \left(\frac{m_b v_{Tb}}{m_a v_{Ta}} \right) N_{ba}^{0j} = 0 \quad (21)$$

which can be used to eliminate N_{ak}^{0j} from Eqs. (20). Equation (21) is also sufficient to guarantee the Galilean invariance of the friction forces.

The result Eqs. (20), together with Eqs. (14) and (19) represent the complete solution of the classical multispecies parallel transport problem. Note that Eqs. (20) are valid for arbitrary mass ratios m_a/m_b of colliding species as well as an arbitrary number of collisional ion species. Using the explicit forms for M_{ab}^{jk} and N_{ab}^{jk} given in Appendix A, we may readily derive the inequalities $-\beta_{ab} \leq 0.31 M_{ab}^{22} (M_a^{22} + N_{aa}^{22})^{-1}$ and $(v_{Ta}/v_{Tb}) N_{ab}^{12} \leq 0.49 |M_{ab}^{11}|$ for $a \neq b$. These relations may be used to estimate the last term in Eq. (20c), and the following important result is obtained:

$$\sum_{b \neq a} |\ell_{22}^{ab}| \leq 0.33 |\ell_{22}^{aa}| \quad (22a)$$

Thus, the matrix of friction coefficients ℓ_{22} is nearly diagonal. Equation (22a) may be invoked to obtain the inverse matrix ℓ_{22}^{-1} through first order in $|\ell_{22}^{ab}|/|\ell_{22}^{aa}| \sim \Delta_{ab}$:

$$(\ell_{22}^{-1})^{ab} = (\ell_{22}^{aa})^{-1} [\delta_{ab} - (\ell_{22}^{ba}/\ell_{22}^{bb}) (1 - \delta_{ab})] \quad (22b)$$

Equation (22b) will be required in Section III to relate the neo-classical transport coefficients to the friction coefficients computed in this Section.

III. CALCULATION OF PFIRSCH-SCHLÜTER TRANSPORT COEFFICIENTS

In an axisymmetric tokamak, the neoclassical cross-field fluxes $\Gamma_a^\psi = \langle \int \underline{v} \cdot \nabla \psi f_a \rangle$ (particle flux) and $q_a^\psi/T_a = \langle \int \underline{v} \cdot \nabla \psi [x_a^2 - (5/2)] f_a \rangle$ (random heat flux), may be expressed in terms of the parallel friction forces as follows⁶:

$$\Gamma_a^\psi = -I \left\langle \frac{R_a}{m_a \Omega_a} \right\rangle, \quad (23a)$$

$$\frac{q_a^\psi}{T_a} = -I \left\langle \frac{H_a}{m_a \Omega_a} \right\rangle. \quad (23b)$$

Here $I = RB_T$ is the toroidal flux function, ψ is the poloidal flux function, Ω_a is the gyrofrequency of species (a), and $\langle \dots \rangle$ indicates the flux surface average operator. When all species are in the collisional (Pfirsch-Schlüter) regime, it is well-known³ that the classical parallel friction forces, computed in Section II, completely determine the neoclassical fluxes using Eqs. (23). Thus it is clear that the solution of the neoclassical transport problem requires expressing the parallel flows $u_{||a}$ and $q_{||a}$ in terms of the cross-field gradients.

The continuity equation

$$\nabla \cdot \underline{u}_a = 0$$

together with the perpendicular component of momentum balance determine the lowest-order (in v/Ω) flow within a magnetic surface:

$$q_{||a} = \frac{-IT_a}{e_a B} \frac{\partial \ln p_a}{\partial \psi} \left(1 - \frac{B^2}{\langle B^2 \rangle}\right) + \frac{B \langle B q_{||a} \rangle}{\langle B^2 \rangle} \quad (24)$$

Here, $\langle B q_{||a} \rangle$ is a constant on a magnetic flux surface to be determined below.

To determine $q_{||a}$, we now consider two distinct physical limits corresponding to weak or strong temperature equilibration along B , respectively.³

A. Weak Temperature Equilibration

In this case, the component of the heat flux in the magnetic surface satisfies the continuity equation

$$\nabla \cdot \underline{q}_a = 0 \quad (25)$$

independently for each species (a), i.e., there is no coupling of the particle heat flows within the surface to this order in v/Ω . This situation is most likely to occur in a plasma composed of particles with widely disparate masses. We then readily find:

$$\frac{q_{||a}}{p_a} = - \frac{5}{2} \frac{IT_a}{e_a B} \frac{\partial \ln T_a}{\partial \psi} \left(1 - \frac{B^2}{\langle B^2 \rangle}\right) + \frac{B \langle B q_{||a}/p_a \rangle}{\langle B^2 \rangle} \quad (26)$$

Inserting Eqs. (24) and (26) into Eqs. (9) yields:

$$R_a = - \int_b \frac{IT_b}{e_b B} (\ell_{11}^{ab} \psi_{1b} + \ell_{12}^{ab} \psi_{2b}) \left(1 - \frac{B^2}{\langle B^2 \rangle}\right) + \frac{B \langle B R_a \rangle}{\langle B^2 \rangle} \quad (27a)$$

$$H_a = - \sum_b \frac{IT_b}{e_b B} (\ell_{21}^{ab} A_{1b}^\psi + \ell_{22}^{ab} A_{2b}^\psi) \left(1 - \frac{B^2}{\langle B^2 \rangle}\right) + \frac{B \langle BH_a \rangle}{\langle B^2 \rangle}, \quad (27b)$$

where $A_{1a}^\psi = \partial \ln p_a / \partial \psi$, $A_{2a}^\psi = \partial \ln T_a / \partial \psi$ and we have combined the surface constants $\langle Bu_{||a} \rangle$ and $\langle Bq_{||a} / p_a \rangle$ into the new unknowns $\langle BR_a \rangle$ and $\langle BH_a \rangle$. In the absence of an induced electric field, the parallel momentum balance and heat flux balance equations

yield the constraint relations $\langle BR_a \rangle = \langle BH_a \rangle = 0$, whence Eqs.

(27) become explicit expressions for the parallel friction forces in terms of the cross-field gradients. Writing $\Gamma_a^\psi = \sum_{n,b} L_{ln}^{ab} A_{nb}^\psi$ and $q_a^\psi / T_a = \sum_{n,b} L_{2n}^{ab} A_{nb}^\psi$, the neoclassical cross-field transport coefficients are:

$$T_a L_{mn}^{ab} = T_b L_{nm}^{ba} = \left(\frac{I^2 T_a T_b}{e_a e_b} \right) (\langle B^{-2} \rangle - \langle B^2 \rangle^{-1}) \ell_{mn}^{ab}. \quad (28)$$

Equation (28) reveals the close relationship of neoclassical cross-field transport in the collisional regime to the classical parallel dissipative processes.⁷

We remark that since in general $H_a \neq H_b$ for the situation considered here, it follows from Eq. (6b) that

$$\nabla_{||} (T_a - T_b) \neq 0 \quad (29)$$

which is a statement of the (assumed) weak interspecies temperature coupling within a magnetic surface. Physically, in the absence of any interspecies temperature equilibration, the lighter, more mobile species will tend to have the smoother parallel temperature profile, leading to the nonzero difference in Eq. (29).

B. Strong Temperature Equilibration

The strongly equilibrated plasma satisfies (by definition) $\nabla_{||} (T_a - T_b) = 0$ for all species (a) and (b). Thus $H_a/n_a = (5/2) \nabla_{||} T$, where T is the common temperature of all the species, and it follows from Eq. (23b) that the neoclassical heat fluxes are now strongly coupled as well:

$$\frac{e_a}{n_a} q_a^\psi = \frac{e_b}{n_b} q_b^\psi \quad (30)$$

Physically, strong temperature equilibration reduces the difference in the species' parallel temperature gradients present in Case (A) until the cross-field heat fluxes, driven by $\nabla_{||} T$, satisfy Eq. (30).

To determine the parallel heat flow in this strongly coupled case, we note that now the total random heat flow $\underline{q} = \sum_a \underline{q}_a$ satisfies the homogeneous continuity relation

$$\nabla \cdot \underline{q} = 0 \quad (31)$$

from which it follows [cf. Eq. (26)]:

$$\sum_a \frac{q_{||a}}{T} = - \frac{5}{2} \frac{IT}{B} \frac{\partial \ln T}{\partial \psi} \left(\sum_a \frac{n_a}{e_a} \right) \left(1 - \frac{B^2}{\langle B^2 \rangle} \right) + \frac{B \langle \underline{B} \cdot \underline{q} / T \rangle}{\langle B^2 \rangle} \quad (32)$$

To determine the individual fluxes $q_{||a}$, it is necessary to invert the system of moment equations [cf. Eq. (6b)]:

$$H_a \equiv \sum_b [\ell_{21}^{ab} u_{||b} + \frac{2}{5} \ell_{22}^{ab} (\frac{q_{||b}}{p_b})] = \frac{5}{2} n_a \nabla_{||} T \quad (33)$$

for $q_{||a}$ in terms of $u_{||a}$ and $\nabla_{||} T$ to yield:

$$\frac{q_{||a}}{T} = - \sum_b c_2^{ab} n_b u_{||b} + \frac{5}{2} \hat{\ell}_{22}^a n_a \nabla_{||} T \quad (34)$$

Note that Eq. (34) is the form of the transport relation given by Braginskii² for a simple plasma. Here, the constant c_2^{ab} , and $\hat{\ell}_{22}^a$, are the classical transport coefficients, which are inversely related to the friction coefficients ℓ_{21} and ℓ_{22} . We emphasize that Eqs. (9b) and (34) are different, but equivalent, representations of the transport relations. As such, they are related through the moment Eq. (33), and no further kinetic information is required in transforming between representations. Denoting the inverse of the matrix ℓ_{22} by ℓ_{22}^{-1} , we find the relation between the transport and friction coefficients:

$$c_2^{ab} = \frac{5}{2} \frac{n_a}{n_b} \sum_k (\ell_{22}^{-1})^{ak} \ell_{21}^{kb} \quad (35a)$$

$$\hat{\ell}_{22}^a = \frac{5}{2} \sum_k (\ell_{22}^{-1})^{ak} n_k \quad (35b)$$

The matrix elements $(\ell_{22}^{-1})^{ab}$ are given explicitly in terms of ℓ_{22}^{ab} in Eq. (22b). Summing Eq. (34) and using Eq. (32) yields:

$$\begin{aligned} \frac{5}{2} \nabla_{||} T = & \left[- \frac{5}{2} \frac{IT}{B} \frac{\partial \ln T}{\partial \psi} \left(\sum_a \frac{n_a}{e_a} \right) \left(1 - \frac{D^2}{\langle B^2 \rangle} \right) \right. \\ & \left. + \sum_{a,b} c_2^{ab} n_b \left(u_{||b} - \frac{\langle B u_{||b} \rangle^B}{\langle B^2 \rangle} \right) \right] \left(\sum_a n_a \hat{\ell}_{22}^a \right)^{-1} \quad (36) \end{aligned}$$

Here we have used the relation $\langle B \nabla_{||} T \rangle = 0$ to eliminate the integration constant $\langle \underline{B} \cdot \underline{q}/T \rangle$ appearing in Eq. (32). Using Eq. (23b) we obtain the neoclassical heat flux:

$$\frac{q_a^\psi}{T} = \frac{n_a}{e_a} \left(\sum_b L_{21}^b A_{1b}^\psi + L_{22} A_2^\psi \right) , \quad (37)$$

where

$$L_{21}^b = \frac{I^2 T (n_b/e_b) \left(\sum_k c_2^{kb} \right)}{\sum_k n_k \hat{\ell}_{22}^k} (\langle B^{-2} \rangle - \langle B^2 \rangle^{-1}) , \quad (38a)$$

$$L_{22} = \frac{(5/2) I^2 T \sum_k (n_k/e_k)}{\sum_k n_k \hat{\ell}_{22}^k} (\langle B^{-2} \rangle - \langle B^2 \rangle^{-1}) . \quad (38b)$$

Using Eq. (36) in Eq. (34) to compute $q_{||a}$ in terms of the gradients allows us to evaluate the particle flux from Eq. (9a):

$$\Gamma_a^\psi = \sum_b [L_{11}^{ab} A_{1b}^\psi + L_{21}^a \left(\frac{n_b}{e_b} \right) A_2^\psi] \quad (39)$$

where

$$L_{11}^{ab} = \frac{I^2 T}{e_a e_b} (\langle B^{-2} \rangle - \langle B^2 \rangle^{-1}) \left[\ell_{11}^{ab} + \frac{2}{5} n_b \sum_k \ell_{12}^{ak} \left(\frac{\hat{\ell}_{22}^k \sum_j c_2^{jb}}{\sum_j n_j \hat{\ell}_{22}^j} - \frac{c_2^{kb}}{n_k} \right) \right] . \quad (40)$$

The symmetry properties of the transport coefficients $L_{11}^{ab} = L_{11}^{ba}$ follow directly from the symmetry of the friction coefficients and the inverse matrix $(\ell_{22}^{-1})^{ab}$. The relation $\sum_k \ell_{12}^{ak} \hat{\ell}_{22}^k = n_a \sum_k c_2^{ka}$, used to simplify Eq. (39), is a consequence of these symmetries. Note that now the neoclassical transport coefficients are proportional to the inverse of the classical transport coefficient

$\sum \hat{\ell}_{22}^k n_k$, which is itself inversely proportional to the friction coefficients ℓ_{22}^{ab} . The presence of the explicit summations in Eqs. (38) and (40) indicates that the neoclassical fluxes are affected by the parallel heat transport of all the equilibrated species collectively.

In order to determine the value of the equilibration rate which separates the two limiting regimes computed above, we now investigate analytically the solution of the coupled heat flow equations for a two-ion species plasma. (The generalization to multispecies is straightforward.) By subtracting from the heat equation its flux surface average, and neglecting quantities of second order in the driving forces we find:

$$\nabla_{(\psi)} \cdot \mathbf{q}_A = - \frac{3}{2} \frac{n_A}{\tau_{AB}^E} (\tilde{T}_A - \tilde{T}_B) \quad (41)$$

where $\nabla_{(\psi)}$ denotes the ψ derivatives are deleted from the ∇ operator, $\tilde{T}_A = T_A - \langle T_A \rangle$ is the part of the temperature which varies on a flux surface, and $1/\tau_{AB}^E = (n_B/n_A) (1/\tau_{BA}^E)$ is the energy equilibration rate. The presence of the equilibration term in Eq. (41) accounts for the coupling of the parallel temperature gradients of species A and B.

For simplicity, we solve Eq. (41) in the low- β , large aspect ratio limit. Then, $\nabla_{(\psi)} \cdot \mathbf{q}_A = (1/rh) \partial/\partial \theta (h q_{A\theta})$, where $h = 1 + r/R_0 \cos \theta$. Differentiating Eq. (41) once with respect to θ , and using the heat flux balance $H_A = (5/2) n_A \nabla_{||} \tilde{T}_A$ together with $q_{A\theta} = (B_P/B) q_{||A} + (5/2) (p_A/e_A B) (\partial T_A / \partial r)$, yields the following equation for $q_{||A}$, subject to the constraint $\langle q_{||A}^B \rangle = 0$:

$$\frac{\partial^2}{\partial \theta^2} [h(q_{||A} - q_{||A}^0)] - \frac{\tau^*}{\tau_E^{AB}} [q_{||A} - q_{||A}^\infty] = 0 \quad (42)$$

where

$$\tau^* = - \frac{6}{25} \frac{L_c^2}{p_A} [\ell_{22}^{AA} + \frac{n_A}{n_B} \frac{p_A}{p_B} \ell_{22}^{BB} - \ell_{22}^{AB} (\frac{p_A}{p_B} + \frac{n_A}{n_B})]$$

and $L_c = rB/B_p$ is the connection length. Here, $q_{||A}^0$ is the parallel heat flux computed in Eq. (26) for the unequilibrated plasma $\tau^*/\tau_E^{AB} \rightarrow 0$, and $q_{||A}^\infty$ is the heat flux given in Eq. (34) for the equilibrated case $\tau^*/\tau_E^{AB} \rightarrow \infty$. The appropriate solution of Eq. (42), taking into account the periodicity of $q_{||A}$, is:

$$q_{||A} = \frac{q_{||A}^0 + (\tau^*/\tau_E^{AB}) q_{||A}^\infty}{1 + (\tau^*/\tau_E^{AB})} \quad (43)$$

Here, we have also noted the $\cos \theta$ dependence of $q_{||A}^0$ and $q_{||A}^\infty$. To estimate τ^* , we can use the friction coefficients computed in Section IV to find:

$$\frac{\tau^*}{\tau_E^{AB}} \approx \left(\frac{v_{AA}}{\omega_{TA}} \right)^2 (v_{AA} \tau_E^{AB})^{-1} \quad (44)$$

where species (A) is the heavier species, $\omega_{TA} = v_{TA}/L_c$ is the transit frequency and v_{AA} is the slowing down frequency of species (A). Typically, for species (A) in the Pfirsch-Schlüter regime $v_{AA}/\omega_{TA} > 1$ and $v_{AA} \tau_E^{AB} \sim (m_A/m_B)^{1/2} (n_A e_A^2 / n_B e_B^2)$, the plasma will be strongly equilibrated provided the mass ratio m_B/m_A is not too small. Thus, the solution $q_{||A} = q_{||A}^\infty$ is the one of widest applicability in a multiple ion species plasma.

The variation of the parallel heat flow $q_{||A}$, and hence the transport coefficients, with the parameter τ^*/τ_E^{AB} accounts for the discrepancy between the results of Rutherford³ ($\tau^*/\tau_E^{AB} = 0$) and Tsang and Crume⁸ ($\tau^*/\tau_E^{AB} = \infty$). For situations where τ^*/τ_E^{AB} takes on intermediate values, expressions of the form Eq. (43) may be used to compute the neoclassical transport coefficients as functions of τ^*/τ_E^{AB} . Indeed, for the example of two species considered here, the transport matrix is of the form:

$$L_{mn}^{ab} = \frac{(L_{mn}^{ab})^0 + (\tau^*/\tau_E^{AB}) (L_{mn}^{ab})^\infty}{1 + (\tau^*/\tau_E^{AB})} \quad (45)$$

where the unequilibrated values, denoted by superscript (0), are given in Eq. (28) and the equilibrated values, denoted by superscript (∞), have been computed in Eqs. (38) and (40). A quantitative comparison between $(L_{mn}^{ab})^0$ and $(L_{mn}^{ab})^\infty$ for a two ion species plasma is given at the end of Section IV. There it is shown that, over a wide range of practical impurity strengths, $n_B e_B^2 / n_A e_A^2$, the dominant effect of temperature equilibration is to reduce the heat flow of a heavy high- Z_B impurity by a factor $\sim (m_B/m_A)^{1/2} (e_B/e_A) \gg 1$.

When there are more than two plasma species, the various rates of equilibration between the species are in general different, and the transport coefficients are thus characterized by several equilibration constants of the form $\tau_j^* (\ell_{22}^{AB}) / \tau_E^{AB}$. In particular, the heat flow of the lighter (hydrogenic) component may be uncoupled from that of the heavier equilibrated species.

IV. APPLICATIONS

We now use the results of the preceding sections to explicitly evaluate the classical friction coefficients ℓ_{mn}^{ab} (and hence the neoclassical transport coefficient) for two situations of practical importance. We first extend the results of Rutherford³ to a multispecies "Lorentz" plasma in which the various plasma constituents satisfy the mass ratio ordering $m_H \ll m_1 \ll m_2 \dots$, where H refers to the hydrogenic (lightest) species. The situation is typically realized in tokamak experiments with, for example, the intermediate mass species (1) corresponding to oxygen and the more massive species (2, 3,) to metallic ions. Then we consider the situation of a light hydrogenic species in the presence of a more massive ion species which may be distributed over its various charge states. Experimentally, the massive ion may again be oxygen or a metallic ion species.⁹

It is convenient to define the effective scattering parameters:

$$Z_{ab} = n_b e_b^2 / n_a e_a^2, \quad (46a)$$

$$\bar{Z}_a = \sum_{b \neq a, H} n_b e_b^2 / n_a e_a^2 = \sum_{b \neq a, H} Z_{ab}, \quad (46b)$$

$$Z_a^* = \sum_{b > a} Z_{ab} \quad (b > a \text{ denotes } m_b > m_a). \quad (46c)$$

The parameters Z_a^* and $\bar{Z}_a$ play the role of "effective charges" in scattering species (a).

A. Multispecies "Lorentz" Plasma: $m_H \ll m_1 \ll m_2 \dots$

For collisions of lighter ions (a) on heavier ions (b), the matrix elements $M_{ab}^{jk} = -(n_a/\tau_{aa}) Z_{ab} \mu_1^{jk}$, where $\{4\tau_{ab}/[3(\pi)^{1/2}]\}^{-1} = 4\pi n_b (e_a e_b)^2 \ln \Lambda / (v_{Ta}^3 m_a^2)$ and the matrix μ_1 is:

$$\mu_1 = \begin{pmatrix} 1 & \frac{3}{2} & \frac{15}{8} \\ \frac{3}{2} & \frac{13}{4} & \frac{69}{16} \\ \frac{15}{8} & \frac{69}{16} & \frac{433}{64} \end{pmatrix}. \quad (47a)$$

To lowest order in the small mass ratio m_a/m_b , we find

$(v_{Ta}/v_{Tb}) N_{ab}^{jk} = -M_{ab}^{j0} \delta_{k0}$. For like particle collisions, we find $M_{aa}^{jk} + N_{aa}^{jk} = -(2)^{1/2} (n_a/\tau_{aa}) \mu_2^{jk}$, where

$$\mu_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & \frac{3}{4} \\ 0 & \frac{3}{4} & \frac{45}{16} \end{pmatrix}. \quad (47b)$$

The vanishing entries in μ_2 correspond to momentum conservation in like particle collisions.

Regarding collisions of heavy particles (b) with lighter ones (a), to lowest order in v_{Tb}/v_{Ta} it is sufficient to retain

the terms M_{ba}^{jk} and N_{ba}^{jk} corresponding to $j = 0$ (or $k = 0$) for which $M_{aa}^{j0} + N_{aa}^{j0} = 0$. The remaining terms do not effect momentum balance and result in $O(v_{Tb}/v_{Ta})$ corrections to the transport coefficients. Thus we find:

$$M_{ba}^{j0} = - \frac{n_a z_{ab}}{\tau_{aa}} \left(\frac{m_a}{m_b} \right) \delta_{j0} \quad (48)$$

and N_{ba}^{0j} can be determined using Eq. (21). [Since $M_{ba}^{j0}/M_{bb}^{jk} \sim z_{ba}(v_{Tb}/v_{Ta})$ for $j, k \neq 0$, we neglect in the present analysis the small range of scattering strengths $z_{ba} > v_{Ta}/v_{Tb} \gg 1$ for which the presence of species (b) makes an insignificant contribution to the transport problem. This approximation corresponds to neglecting the influence of the C_{ba} operator in the kinetic equation for f_{b1} .]

Using Eqs. (47) and (48) to evaluate the friction coefficients yields:

$$\ell_{11}^{aa} = -z_a^* C_1^*(z_a^*) \frac{m_a n_a}{\tau_{aa}} - \sum_{k < a} \frac{m_k n_k z_{ka}}{\tau_{kk}} \left[C_1^*(z_k^*) + \left(1 - \frac{z_{ka}}{z_k^*} \right) [1 - C_1^*(z_k^*)] \right], \quad (49a)$$

$$\ell_{11}^{ab} = \begin{cases} C_1^*(z_a^*) \frac{m_a n_a z_{ab}}{\tau_{aa}} - \sum_{k < a} \frac{m_k n_k z_{ka}}{\tau_{kk}} \left(\frac{z_{kb}}{z_k^*} \right) [C_1^*(z_k^*) - 1] , & m_a \ll m_b , \\ C_1^*(z_b^*) \frac{m_b n_b z_{ba}}{\tau_{bb}} - \sum_{k < b} \frac{m_k n_k z_{ka}}{\tau_{kk}} \left(\frac{z_{kb}}{z_k^*} \right) [C_1^*(z_k^*) - 1] , & m_a \gg m_b , \end{cases} \quad (49b)$$

$$\ell_{12}^{aa} = \frac{m_a n_a}{\tau_{aa}} z_a^* C_2^*(z_a^*) , \quad (49c)$$

$$\ell_{12}^{ab} = \begin{cases} -\frac{m_b n_b}{\tau_{bb}} z_{ba}^* C_2^*(z_b^*) , & m_b \ll m_a , \\ 0 & , \quad m_b \gg m_a , \end{cases} \quad (49d)$$

$$\ell_{22}^{ab} = -\frac{m_a n_a}{\tau_{aa}} C_3^*(z_a^*) \delta_{ab} . \quad (49e)$$

Here, the functions $C_j^*(x)$ are defined as follows:

$$\begin{aligned} C_1^*(x) &= 1 - \frac{(15/8)^2 x}{(45/16)(2)^{1/2} + (433/64)x} \\ &\doteq 0.48 + \frac{0.31}{0.59 + x} , \end{aligned} \quad (50a)$$

$$\begin{aligned} C_2^*(x) &= \frac{3}{2} - \frac{(15/8) [(3/4)(2)^{1/2} + (69/16)x]}{(45/16)(2)^{1/2} + (433/64)x} \\ &\doteq 0.30 + \frac{0.41}{0.59 + x} , \end{aligned} \quad (50b)$$

$$C_3^*(x) = (2)^{1/2} + \frac{13}{4} x - \frac{[(3/4)(2)^{1/2} + (69/16)x]^2}{(45/16)(2)^{1/2} + (433/64)x}$$

$$= 1.13 + 0.50x + \frac{0.55x}{0.59 + x} \quad (50c)$$

Note the close agreement of $C_1^* = C_1 + C_2^2/C_3$, $C_2^* = (5/2)C_2/C_3$ and $C_3^* = (25/4C_3)x$ with the functions given by Rutherford.³

Provided the mass ratios m_j/m_{j+1} are finite (but small), there may be interspecies temperature coupling. Thus, according to Eq. (44), well into the Pfirsch-Schlüter regime the equilibrated neo-classical transport coefficients should be used. From Eq. (49e), $(\ell_{22}^{-1})^{ab} = (\ell_{22}^{ab})^{-1}$ so the equilibrated coefficients are readily calculable using Eqs. (49).

An alternative calculation of the above results based on an analogy with the electron-ion problem,¹⁰ is presented in Appendix B.

B. One Hydrogenic Species m_H and Several Different Charge States of a More Massive Impurity of Mass $m_I \gg m_H$.

The various impurity charge states will be identified as (a), (b), For collisions of hydrogen with the impurity state (a), we can use the results of Case A to write $M_{Ha}^{jk} = -(n_H Z_{Ha} / \tau_{HH}) \mu_1^{jk}$, $(v_{TH} / v_{Ta}) N_{Ha}^{jk} = -M_{Ha}^{j0} \delta_{k0}$ and $M_{aH}^{j0} = -(n_H Z_{Ha} / \tau_{HH}) (m_H / m_I) \delta_{j0}$. Here we note that the mass for each state (a) is equal to m_I . Assuming $T_a = T_b$ due to strong temperature equilibration, the matrix elements between the various charge state (a) and (b) are readily evaluated:

$$M_{ab} = - \frac{n_a}{(2)^{1/2} \tau_{ab}} \begin{pmatrix} 1 & \frac{3}{4} & \frac{15}{32} \\ \frac{3}{4} & \frac{59}{16} & \frac{417}{128} \\ \frac{15}{32} & \frac{417}{128} & \frac{8385}{1024} \end{pmatrix}, \quad (51a)$$

$$N_{ab} = \frac{n_a}{(2)^{1/2} \tau_{ab}} \begin{pmatrix} 1 & \frac{3}{4} & \frac{15}{32} \\ \frac{3}{4} & \frac{27}{16} & \frac{225}{128} \\ \frac{15}{32} & \frac{225}{128} & \frac{2625}{1024} \end{pmatrix}. \quad (51b)$$

The hydrogenic friction coefficients ℓ_{mn}^{Ha} can be computed from Eqs. (49). The coefficients for charge state (a) become (for $a \neq b$):

$$\begin{aligned} \ell_{11}^{aa} = & - \frac{m_H n_H}{\tau_{HH}} \frac{Z_{Ha}^2}{Z_H^*} [C_1^*(Z_H^*) + (\frac{Z_H^*}{Z_{Ha}} - 1)] - \frac{m_I n_a}{(2)^{1/2} \tau_{aa}} \\ & \times \{ \bar{Z}_a D_1(\bar{Z}_a) - \frac{15}{32} [d_o - D_o(\bar{Z}_a)] + \Delta D_1(\bar{Z}_a) \}, \end{aligned} \quad (52a)$$

$$\begin{aligned} \ell_{11}^{ab} = & - \frac{m_H n_H}{\tau_{HH}} \frac{Z_{Ha} Z_{Hb}}{Z_H^*} [C_1^*(Z_H^*) - 1] + \frac{m_I n_a}{(2)^{1/2} \tau_{aa}} Z_{ab} \\ & \times \{ D_1(\bar{Z}_a) + D_1(\bar{Z}_b) + \frac{15}{32} [d_o - D_o(\bar{Z}_a) - D_o(\bar{Z}_b)] - 1 + \Delta D_2(\bar{Z}_a, \bar{Z}_b) \}, \end{aligned} \quad (52b)$$

$$\ell_{12}^{aH} = - \frac{m_H n_H}{\tau_{HH}} z_{Ha} C_2^* (z_H^*) , \quad (52c)$$

$$\ell_{12}^{aa} = \frac{m_I n_a}{(2)^{1/2} \tau_{aa}} \{ \bar{z}_a D_2(\bar{z}_a) - (\frac{225}{128}) [d_o - D_o(\bar{z}_a)] + \Delta D_3(\bar{z}_a) \} , \quad (52d)$$

$$\begin{aligned} \ell_{12}^{ab} = \frac{m_I n_a z_{ab}}{(2)^{1/2} \tau_{aa}} & \left(-\frac{15}{4} D_1(\bar{z}_a) - D_2(\bar{z}_b) + \frac{15}{4} \left\{ 1 - \frac{15}{32} [d_o \right. \right. \\ & \left. \left. - D_o(\bar{z}_a) - D_o(\bar{z}_b)] \right\} + \Delta D_4(\bar{z}_a, \bar{z}_b) \right) , \end{aligned} \quad (52e)$$

$$\ell_{22}^{aa} = \frac{m_I n_a}{(2)^{1/2} \tau_{aa}} \{ -D_3(\bar{z}_a) + (\frac{225}{128}) (\frac{15}{4}) [d_o - D_o(\bar{z}_a)] + \Delta D_5(\bar{z}_a) \} , \quad (52f)$$

$$\begin{aligned} \ell_{22}^{ab} = \frac{m_I n_a z_{ab}}{(2)^{1/2} \tau_{aa}} & \left\{ \frac{15}{4} [D_2(\bar{z}_a) + D_2(\bar{z}_b)] + (\frac{225}{128}) (\frac{15}{4}) [d_o - D_o(\bar{z}_a) \right. \\ & \left. - D_o(\bar{z}_b)] - \frac{63}{16} + \Delta D_6(\bar{z}_a, \bar{z}_b) \right\} . \end{aligned} \quad (52g)$$

The functions $D_j(x)$ are given by:

$$D_0(x) = (12 + \frac{1677}{96} x)^{-1} \doteq 0.057(0.69 + x)^{-1} , \quad (53a)$$

$$D_1(x) = 1 - \frac{15}{32} x D_0(x) \doteq 0.97 + \frac{0.019}{0.69+x}, \quad (53b)$$

$$D_2(x) = \frac{3}{4} - \frac{15}{32} \left(\frac{16}{5} + \frac{417}{60} x \right) D_0(x) \doteq 0.56 + \frac{0.043}{0.69+x}, \quad (53c)$$

$$D_3(x) = 2 + \frac{59}{16} x - \frac{15}{32} \left(\frac{16}{5} + \frac{417}{60} x \right)^2 D_0(x) \doteq 1.60 + 2.39x + \frac{0.10x}{0.69+x}, \quad (53d)$$

and the constant $d_0 = \sum_{k \neq H} D_0(\bar{z}_k)$. The functions $\Delta D_j(x)$ and $\Delta D_j(x, y)$, which arise from the small order Δ corrections to the friction coefficients, are given for completeness in Appendix C. However, for values of $\bar{z}_a$ and $\bar{z}_{ab}$ of practical importance, for which $D_j \geq 1$, the quantities ΔD_j (for $j \neq 4, 6$) typically represent corrections of less than ten percent and may be therefore neglected in Eqs. (52) while retaining sufficient accuracy.

Since $D_1 \doteq 1$ and $D_2 \doteq 0.56$ are both clearly of order unity, the frequency characterizing collisions between charge states (a) and (b) is evidently $[(2)^{1/2} \tau_{ab}]^{-1}$. Here, the factor $(2)^{1/2}$ arises from the fact that the reduced mass for (a)-(b) collisions is just $(1/2)m_I$. The presence of $\bar{z}_a$ in the friction coefficients of species (a) results from the collisional interaction of charge state (a) with all other charge states.

Because of the expected strong temperature equilibration among the various charge states, the results Eqs. (38) and (40) for the neoclassical transport coefficients in terms of the ℓ_{mn}^{ab} computed in Eqs. (52) must be used. Since $D_3 > d_0 - D_0$ and $D_2 > (d_0 - D_0)$, Eqs. (52f-g) simplify to $(2)^{1/2} \ell_{22}^{aa} \doteq -(m_I n_a / \tau_{aa}) D_3(\bar{z}_a)$

and $(2^{1/2})\ell_{22}^{ab} = (m_I n_a / \tau_{ab}) [D_4(\bar{z}_a) + D_4(\bar{z}_b)]$, where

$$D_4(x) = 0.34 + \frac{0.067}{0.69 + x} \quad (54)$$

and we have accounted for ΔD_6 in Eq. (54). Note that the explicit forms for ℓ_{22}^{ab} obviously satisfy the inequality, Eq. (22a). The inverse matrix ℓ_{22}^{-1} is given in Eq. (22b) which becomes for the present situation:

$$\begin{aligned} (\ell_{22}^{-1})^{ab} = & - \frac{(2^{1/2})\tau_{aa}}{m_I n_a} D_3^{-1}(\bar{z}_a) \{ \delta_{ab} + z_{ba} [D_4(\bar{z}_a) \\ & + D_4(\bar{z}_b)] D_3^{-1}(\bar{z}_b) (1 - \delta_{ab}) \} . \end{aligned} \quad (55)$$

Finally, we determine quantitatively the effect of parallel temperature profile equilibration, considered in Section III, on the transport coefficients of a two ion species plasma. To facilitate comparison with the results of Refs. 3 and 8, first consider the example of two ions satisfying the small, but finite, mass ratio ordering $m_i \ll m_1$. [Here, species (i) need not be hydrogenic.] According to the criterion Eq. (44), the unequilibrated transport coefficients $(L_{mn}^{ab})^0$, which are related to the friction coefficients in Eq. (28) and were calculated in Ref. 3 for the present situation, should apply at the low-collision frequency boundary of the Pfirsch-Schlüter regime for species (1), $v_{11}/\omega_{T1} \sim 1$. For increasing values of v_{11}/ω_{T1} , the equilibrated transport coefficients $(L_{mn}^{ab})^\infty$, given in Eqs. (38) and (40) and computed

numerically in Ref. 8, become applicable. Since only $\nabla_{||} T_i$ and $\nabla_{||} T_1$ are effected by equilibration, we choose for brevity to compare the coefficient of $\partial T / \partial r$ in the heat flux ($m = n = 2$):

$$\sum_b L_{22}^{ab} = - \frac{T^2}{e_i} (\langle B^{-2} \rangle - \langle B^2 \rangle^{-1}) \frac{m_i n_i}{\tau_{i1}} \hat{L}_{22}^a \quad (56a)$$

where the unequilibrated value of $\hat{L}_{22}^a$ is:

$$(\hat{L}_{22}^a)^0 = \left(\frac{m_a n_a \tau_{ii}}{m_i n_i \tau_{aa}} \right) \left(\frac{e_i}{e_a} \right)^2 \left[\frac{C_3^*(Z_a^*)}{\alpha} \right] \quad (56b)$$

and the equilibrated value is:

$$(\hat{L}_{22}^a)^\infty = \frac{n_a e_i}{n_i e_a} \frac{[1 + \alpha (e_i/e_1)^3] [C_3^*(\alpha)/\alpha]}{\{1 + [C_3^*(\alpha)/C_3^*(0)] (n_1 m_i \tau_{11} / n_i m_1 \tau_{ii})\}} \quad (56c)$$

Here, $\alpha \equiv Z_i^* = n_1 e_1^2 / n_i e_i^2$ is the scattering strength of species (i) relative to species (1). In Eq. (56a) we have assumed (for simplicity) that the radial temperature gradients are equal for both species.

In Fig. 1 the normalized quantities $(\hat{L}_{22}^i)^0$, $(n_i e_1 / n_1 e_i) (\hat{L}_{22}^1)^0$ and $(\hat{L}_{22}^1)^\infty$ are graphed versus α for various charge ratios e_1/e_i and for fixed mass ratio $m_1/m_i = 16$. The deviation of the value of the lighter ion (i) transport coefficient from its unequilibrated value becomes appreciable for scattering strengths $\alpha \sim (e_1/e_i)^3$, or $n_1/n_i \sim e_1/e_i$. Thus, temperature equilibration has an important effect on $\hat{L}_{22}^i$ only if species (i) is not the main

hydrogenic population. In contrast, Fig. 1 clearly indicates a range of α , $0 < \alpha \lesssim 2(m_z/m_i)^{1/2}(e_z/e_i)^4$ over which there is a substantial (order of magnitude) reduction in the heavy ion heat transport due to equilibration. These effects are in agreement with the qualitative physical influence expected of strong profile coupling.

For the interaction of two equal mass particles of charge states (a) and (b), we can write, assuming equal radial temperature profiles and using Eqs. (52) and (55):

$$L_{22}^a = - \frac{I^2 T}{e_a^2} (\langle B^{-2} \rangle - \langle B^2 \rangle^{-1}) \frac{m_i n_a}{(2)^{1/2} \tau_{ab}} \hat{L}_{22}^a \quad (57)$$

where the unequilibrated value is:

$$(\hat{L}_{22}^a)^0 = \frac{D_3(z_{ab})}{z_{ab}} - 0.78 \left(\frac{e_a}{e_b} \right) \quad (58a)$$

and the equilibrated value is:

$$(\hat{L}_{22}^a)^\infty = \frac{1 + z_{ab} (e_a/e_b)^3}{z_{ab} [D_3^{-1}(z_{ab}) + (e_a/e_b)^4 D_3^{-1}(z_{ab}^{-1}) + 1.56 (e_a/e_b)^2 D_3^{-1}(z_{ab}) D_3^{-1}(z_{ab}^{-1})]} \quad (58b)$$

Here, we have noted the relation $\bar{z}_{ab} = \bar{z}_{ba}^{-1}$ for the two charge states considered.

The transport coefficients in Eqs. (58) have been graphed in Fig. 2 versus z_{ab} for several values of the charge ratio e_a/e_b . The discrepancy between the two limiting values of $\hat{L}_{22}^a$ increases as e_a/e_b departs from unity. For the case $e_a/e_b = 4$,

the equilibrated coefficient exceeds the unequilibrated value over the range of effective charge $z_{ab} \geq 1$ and, for $z_{ab} \geq 2$ is of opposite sign. Thus, a substantial error would be incurred if the coefficients $(L_{mn}^{ab})^0$ were used here instead of the proper equilibrated values $(L_{mn}^{ab})^\infty$.

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APPENDIX A

When C_{ab} is the linearized Fokker Planck operator describing Coulomb collisions, the matrix elements defined in Section II are readily evaluated using the generating function for the Sonine polynomials² $L_k^{(3/2)}(x_a^2)$:

$$\sum_{j,k=0} \xi^j \eta^k M_{ab}^{jk} = - \frac{n_a}{\tau_{ab}} \bar{m}_{ab}(\xi, \eta) , \quad (A1a)$$

$$\sum_{j,k=0} \xi^j \eta^k N_{ab}^{jk} = \frac{T_a}{T_b} \frac{v_{Tb}}{v_{Ta}} \frac{n_a}{\tau_{ab}} \bar{n}_{ab}(\xi, \eta) . \quad (A1b)$$

Here, $1/\tau_{ab} = [4/3(\pi)^{1/2}] \tau_{ab}$, where $v_{ab} = 4\pi n_b (e_a e_b)^2 \ln \Lambda / (v_{Ta}^3 m_a^2)$, and

$$\begin{aligned} \bar{m}_{ab} = & (1 - \xi\eta)^{-3/2} (1 - \xi)^{-1} (1 - \eta)^{-1} \left(\frac{1+x+y}{B} \right)^{1/2} \\ & \times \left[1 - x_{ab}^{-2} \frac{(x+y)}{B} + 5x_{ab}^{-4} \frac{xy}{B^2} + \frac{2x_{ab}^{-2} xy}{B^2 (1+x+y)} \right] , \end{aligned} \quad (A2a)$$

$$\bar{n}_{ab} = (1 - \xi)^{-1} (1 - \eta)^{-1} \left(\frac{(1+x)(1+y)}{A} \right)^{1/2} \left(1 + \frac{x_{ab}^{-2} x + y}{A} + \frac{3x_{ab}^{-2} xy}{A^2} \right) \quad (A2b)$$

where $x_{ab} = v_{Ta}/v_{Tb}$ and

$$x = \xi/(1 - \xi) , \quad y = \eta/(1 - \eta) , \quad (A2c)$$

$$A = x_{ab}^{-2}(1+x) + (1+y) , \quad (A2d)$$

$$B = x_{ab}^{-2}(1+x+y) + 1 . \quad (A2e)$$

Expanding $\bar{m}_{ab}$ and $\bar{n}_{ab}$ around $\xi, \eta = 0$ yields (taking account of $T_a \neq T_b$ for $m_a \neq m_b$ in $j = 0$ momentum moments only):

$$M_{ab}^{00} = - \frac{n_a}{\tau_{ab}} \left(1 + \frac{m_a}{m_b}\right) \frac{x_{ab}^3}{(1+x_{ab}^2)^{3/2}} = - \frac{m_b}{m_a} x_{ba} N_{ba}^{00} , \quad (A3a)$$

$$M_{ab}^{01} = M_{ab}^{10} = - \frac{3}{2} \frac{n_a}{\tau_{ab}} \left(1 + \frac{m_a}{m_b}\right) \frac{x_{ab}^5}{(1+x_{ab}^2)^{5/2}} = - \frac{m_b}{m_a} x_{ba} N_{ba}^{01} , \quad (A3b)$$

$$M_{ab}^{02} = M_{ab}^{20} = - \frac{15}{8} \frac{n_a}{\tau_{ab}} \left(1 + \frac{m_a}{m_b}\right) \frac{x_{ab}^7}{(1+x_{ab}^2)^{7/2}} = - \frac{m_b}{m_a} x_{ba} N_{ba}^{02} , \quad (A3c)$$

$$M_{ab}^{11} = - \frac{n_a}{\tau_{ab}} (1+x_{ab}^2)^{-5/2} x_{ab} \left(\frac{15}{2} + 4x_{ab}^2 + \frac{13}{4} x_{ab}^4 \right) , \quad (A3d)$$

$$M_{ab}^{12} = M_{ab}^{21} = - \frac{n_a}{\tau_{ab}} (1+x_{ab}^2)^{-7/2} x_{ab}^3 \left(\frac{63}{4} + 6x_{ab}^2 + \frac{69}{16} x_{ab}^4 \right) , \quad (A3e)$$

$$M_{ab}^{22} = - \frac{n_a}{\tau_{ab}} (1 + x_{ab}^2)^{-9/2} x_{ab} \left(\frac{175}{8} + 28x_{ab}^2 + \frac{459}{8} x_{ab}^4 + 17x_{ab}^6 + \frac{433}{64} x_{ab}^8 \right), \quad (A3f)$$

$$N_{ab}^{11} = \frac{27}{4} \frac{n_a}{\tau_{ab}} \frac{T_a}{T_b} \frac{x_{ab}^2}{(1 + x_{ab}^2)^{5/2}}, \quad (A3g)$$

$$N_{ab}^{12} = \frac{225}{16} \frac{T_a}{T_b} \frac{n_a}{\tau_{ab}} \frac{x_{ab}^2}{(1 + x_{ab}^2)^{7/2}}, \quad (A3h)$$

$$N_{ab}^{22} = \frac{2625}{64} \frac{T_a}{T_b} \frac{n_a}{\tau_{ab}} \frac{x_{ab}^4}{(1 + x_{ab}^2)^{9/2}}. \quad (A3i)$$

Now consider $\Delta_{ab}^*(x_{ab}) = -x_{ab} N_{ab}^{22} / M_{ab}^{22}$. Using Eqs. (A3f,i) yields:

$$\Delta_{ab}^* = \frac{T_a}{T_b} \frac{6.06x_{ab}^4}{3.23 + 4.14x_{ab}^2 + 8.48x_{ab}^4 + 2.51x_{ab}^6 + x_{ab}^8}. \quad (A4)$$

For $T_a/T_b \approx 1$, $\Delta_{ab}^*(x_{ab}^2)$ has a maximum near $m_b/m_a \approx 4/3$. Note that Δ_{ab}^* goes to zero in either of the two mass ratio limits:

$$\Delta_{ab}^* \rightarrow 1.88x_{ab}^4, \quad x_{ab} \ll 1, \quad (A5a)$$

$$\rightarrow 6.06x_{ab}^{-4}, \quad x_{ab} \gg 1. \quad (A5b)$$

A plot of $\Delta_{ab}^*(x_{ab}^2)$ is presented in Fig. 3. Using the fact that $M_{ab}^{22} < 0$ and $\Delta_{ab}^* \leq 0.327$ leads to the inequality, Eq. (18).

APPENDIX B

We now compute R_{ab} and H_a for a multispecies plasma satisfying the Lorentz mass ratio ordering

$$m_e \ll m_1 \ll m_2 \ll \dots \ll m_N \quad (B1)$$

In this limit, the coupling of heavier species (b) to lighter ones (a) is weak and can be neglected in the kinetic equation for species (b) to lowest order in $(m_a/m_b)^{1/2} (n_a z_a^2 / n_b z_b^2)$. Thus, the kinetic Eq. (1) simplifies to

$$\begin{aligned} v_{||} [A_{1a} - L_1^{(3/2)} (x_a^2) A_{2a}] f_{a0} \\ = \left(\sum_{b>a} v_{ab} \right) x_a^{-3} \mathcal{L} f_{a1} + \left(\sum_{b>a} 2v_{ab} u_{||b} / v_{Ta}^2 \right) f_{a0} + C_{aa}(f_{a1}) \end{aligned} \quad (B2)$$

where $\mathcal{L} = (1/2) v^2 \partial / \partial v_{||} [1 - (v_{||}/v)^2] \partial / \partial v_{||}$ and v_{ab} is the constant collision frequency defined in Appendix A. With the substitution $f_{a1} = (2v_{||} u_a / v_{Ta}^2) f_{a0} + f_{a1}^*$, where

$$u_a = \left(\sum_{b>a} v_{ab} u_{||b} \right) \left(\sum_{b>a} v_{ab} \right)^{-1}$$

we note (since C_{aa} annihilates a shifted Maxwellian) that f_{a1}^* satisfies the equation

$$v_{||} [A_{1a} - L_1^{(3/2)} (x_a^2) A_{2a}] f_{a0} = \left(\sum_{b>a} v_{ab} \right) x_a^{-3} \mathcal{L} f_{a1}^* + C_{aa}(f_{a1}^*) \quad (B4)$$

which is equivalent to the electron-ion kinetic equation, with species (a,b) playing the role of (e,i) and $\sum_{b>a} v_{ab}$ the role of v_{ei} . Hence, the solution of Eq. (B4) can be written immediately in analogy with the electron-ion problem,^{2,10} yielding:

$$f_{a1} = \frac{2v_{||} f_{a0}}{v_{Ta}^2} [u_a + \phi_{a1}(v, z_a^*) (u_{||a} - u_a) + \phi_{a2}(v, z_a^*) \frac{q_{||a}}{p_a}] \quad (B5)$$

where $z_a^* = \sum_{b>a} v_{ab}/v_{aa}$ is the effective charge scattering species (a). Although the functions ϕ_{a1} , ϕ_{a2} can be related to $L_j^{(3/2)}(x_a^2)$, we only need their moments to compute the friction forces:

$$R_{ab} = -C_1^*(z_a^*) \frac{m_a n_a}{\tau_{ab}} (u_{||a} - u_a) + \frac{2}{5} \frac{m_a n_a}{\tau_{ab}} C_2^*(z_a^*) \frac{q_{||a}}{p_a} + \frac{m_a n_a}{\tau_{ab}} (u_{||b} - u_a), \quad m_a < m_b, \quad (B6a)$$

$$H_a = \sum_{b>a} H_{ab} = \bar{z}_a C_2^*(z_a^*) \frac{m_a n_a}{\tau_{aa}} (u_{||a} - u_a) - \frac{2}{5} \frac{m_a n_a}{\tau_{aa}} C_3^*(z_a^*) \frac{q_{||a}}{p_a}. \quad (B6b)$$

Thus using the relation $R_a = \sum_{b>a} R_{ab} + \sum_{b<a} (-R_{ba})$, we readily obtain the friction coefficients in the Lorentz limit, which are in complete agreement with the results obtained in Eqs. (49).

APPENDIX C

The corrections ΔD_j to the friction coefficients in Eqs. (52) may be expressed in terms of the functions D_j given in Eqs. (53) and the constants d_0 and

$$d_1 = \sum_{k \neq H} [D_0(\bar{z}_k)]^2 \quad (C1)$$

as follows:

$$\begin{aligned} \Delta D_1(x) = 2.56 \{ [d_0 - D_0(x)] [(2x+1)D_0(x) - d_0] \\ + d_1 - [D_0(x)]^2 \} , \end{aligned} \quad (C2a)$$

$$\begin{aligned} \Delta D_2(x, y) = 2.56 \{ (1 + xy) D_0(x) D_0(y) + [d_0 - D_0(x) - D_0(y)] \\ \times [d_0 - xD_0(x) - yD_0(y)] - d_1 + [D_0(x)]^2 + [D_0(y)]^2 \} , \end{aligned} \quad (C2b)$$

$$\begin{aligned} \Delta D_3(x) = 9.61 \left([d_0 - D_0(x)] \{ (1+x) D_0(x) - d_0 \right. \\ \left. - 0.57 [D_2(x) - 0.75] \} + d_1 - [D_0(x)]^2 \right) , \end{aligned} \quad (C2c)$$

$$\begin{aligned} \Delta D_4(x, y) = 9.61 \left(-\{ D_0(y) - 0.57x [D_2(y) - 0.75] \} D_0(x) \right. \\ \left. + [d_0 - D_0(x) - D_0(y)] \{ xD_0(x) - 0.57 [D_2(y) - 0.75] - d_0 \} \right. \\ \left. + d_1 - [D_0(x)]^2 - [D_0(y)]^2 \right) , \end{aligned} \quad (C2d)$$

$$\Delta D_5(x) = 36.05 \left([d_o - D_o(x)] \{d_o - D_o(x) + 1.14 [D_2(x) - 0.75]\} - d_1 + [D_o(x)]^2 \right) , \quad (C2e)$$

$$\begin{aligned} \Delta D_6(x,y) = & 36.05 \left(D_o(x) D_o(y) + 0.32 [D_2(x) - 0.75] [D_2(y) - 0.75] \right. \\ & + [d_o - D_o(x) - D_o(y)] \{d_o + 0.57 [D_2(x) - 0.75] \\ & \left. + 0.57 [D_2(y) - 0.75]\} - d_1 + [D_o(x)]^2 + [D_o(y)]^2 \right) \quad (C2f) \end{aligned}$$

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762312

Figure 1. Graph of the normalized ion heat fluxes in response to a radial temperature gradient versus α for $m_1/m_i = 16$, comparing the unequilibrated (0) and equilibrated (∞) values. (PPPL 762312)

762311

Figure 2. Graph of the normalized ion heat fluxes in response to a radial temperature gradient versus Z_{ab} for two different charge states of a single plasma species. The dashed lines are unequilibrated values. The solid lines are equilibrated values. (PPPL 762311)

762259

Figure 3. Graph of the small coupling parameter Δ_{ab}^* versus the mass ratio m_b/m_a for $T_a/T_b = 1$, revealing its relative peakedness around $m_b/m_a = 1.33$, where $\Delta_{ab}^* = (\Delta_{ab}^*)_{\max} = 0.327$. (PPPL 762259)