STUDIES OF THE STABILITY OF A
FUSION BURN IN A REVERSED FIELD PINCH DEVICE

BY

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THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Nuclear Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 1977

Urbana, Illinois

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ACKNOWLEDGMENT

The author wishes to express sincere gratitude to Professor George H. Miley, her advisor, without whose guidance and invaluable suggestions this work would not have been possible. Many of the author's colleagues, and in particular Professor G. A. Gerdin and Mr. R. A. Stark, provided beneficial criticism and helpful remarks which expediated completion of the work. The author also thanks all those members of the secretarial and drafting office staff of the Nuclear Engineering Program who became involved with this thesis for their excellent work.

The author appreciates especially the patience of her husband, Brian, who always supports and even encourages this kind of activity.

This research was supported by the United States Energy Research and Development Association under contract EY-76-S-02-2218.

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Section I

INTRODUCTION

The reversed field pinch is an axisymmetric toroidal device. It differs from a tokamak primarily in that it operates with poloidal and toroidal magnetic fields of comparable magnitude and with a safety factor, q , less than unity. Although the detailed theory is incomplete, field reversal has been demonstrated to provide stability under these conditions. Further, it is thought that the configuration can be stable at relatively high values of β , the ratio of plasma to magnetic pressure. (All symbols used in this discussion are defined in Appendix I.)

This configuration was recognized in experiments at Culham Laboratory, UK¹. Because of its potential advantages, more extensive experimental investigations and theoretical studies have been subsequently carried out both there²⁻⁵, at Los Alamos Scientific Laboratory⁶ and in Japan⁷.

The purpose of the present study is to simulate, in a global computer model, the burn of a plasma contained in a reversed field toroidal configuration and to investigate various prospective methods of prolonging the duration of a stable burn. Extension of the stable period is desirable since a longer burn leads to greater energy multiplication, Q . As discussed in Section III, results from the present studies show that variations in normal operating parameters have only small effects on the length of the burn. Consequently, in order to achieve longer operating cycles with attractive Q values ($Q \geq 5$), a positive control system such as cold fuel injection appears to be necessary.

Figure 1 Field-reversing helical current configuration.

Figure 2 Reversed field pinch: magnetic field configuration.

Figure 3. Stability of a reversed field pinch (from reference 2).

Field reversal is achieved in a toroidal device when the current induced in the plasma relaxes to what is thought to be a helical configuration (Figure 1). Studies by Taylor^{8,9} have shown that this configuration is a minimum energy state for the plasma. The poloidal component of current thus created gives rise to a toroidal magnetic field which opposes that externally imposed on the plasma. This results in the total toroidal magnetic fields inside and outside the plasma being opposite in sign with a zero toroidal field at the plasma edge. The plasma passes through a highly turbulent phase during the setting up of the configuration and remains turbulent even after it is achieved. (Note that the pitch and radius of the toroidal helix followed by the current can vary throughout the operating cycle.) The turbulence of the plasma leads to anomalies in some of the plasma parameters; the resistivity remains approximately constant at high ($kT \geq 0.5$ keV) plasma temperatures¹⁰, and also the energy and particle confinement times are enhanced significantly.

The reversed field pinch has not been found to be susceptible to the Kruskal-Shafranov limitation¹¹ ($q > 1$). This pinch can be stable with toroidal and poloidal magnetic fields of similar magnitude and does not require the very large toroidal fields at present thought to be necessary for conventional tokamak confinement. Thus the configuration has the potential both to relieve the technological problems associated with achieving these high fields and to reduce the amount of external energy stored in the imposed magnetic field during operation. Inevitable inefficiencies cause part of any external energy stored in the system to be lost. The configuration allows a freer choice of aspect ratio

(major radius/minor radius) which can also facilitate construction and maintenance of the device.

Although the reversed field configuration is not susceptible to the Kruskal-Shafranov limitation, its stability is subject to the Suydam criterion¹² and to the necessity of avoiding magnetohydrodynamic kink instabilities¹³. A more fundamental requirement is that field reversal, once achieved, be maintained. Together these conditions for stability restrict^{2,14} the plasma to a bounded region of β - θ space as shown in Figure 3. The parameter, θ , a measure of the ratio of poloidal to toroidal magnetic field strength is a natural parameter for describing this type of device. (The remaining, broken line in Figure 3 indicates the criterion for avoiding current reversal at the plasma surface. Such current reversal may give rise to surface instabilities². If the plasma passes into this or any of the other unstable regimes the computer code simply presents a message to that effect and continues computation. No further work on this particular potential instability has been carried out in this global model.) A preliminary study indicates that for reactors of the class discussed here, the dependence, if any, of the stability limitations on machine parameters is weak. This deserves further study however, especially if grossly different situations are to be considered.

A number of prospective methods of extending the duration of the stable burn have been investigated. Techniques examined have included careful choice of the different factors affecting the voltage profile (Figure 5) and of the imposed magnetic field; the effects of different

plasma dimensions and first wall materials have also been studied. It is important to remember, however, that few materials are suitable for the first wall. Reversed field toroidal devices are expected to rely on wall stabilization¹⁵ which uses image currents to reduce magnetohydrodynamic motion such as kinking and eliminates the need for a vertical field to prevent radial expansion. This requires either a highly conducting first wall, introducing many difficulties of maintenance, or a, probably complex, system of windings similar to that proposed by Lawson¹⁵. In either case it is unlikely that there will be a wide range of suitable materials available to satisfy these requirements. Also considered is the possibility of controlling plasma stability by cold refuelling or additional heating at times when an impending violation of the stability limitations is anticipated during a burn. Continuous monitoring of β and θ by the code makes possible such anticipation and also investigation of these control methods.

Section II

THE CODE

A global computer model has been developed which describes the burn of a plasma contained in a reversed field toroidal device. The code uses a Sarafyan embedding form of the Runge-Kutta technique¹⁶ to solve the coupled first order differential equations for the various particle number densities and temperatures as functions of time. The equations used are shown in Appendix II.

The model considers the plasma to be in one of five regimes depending on the approximate validity or otherwise of the following assumptions:

ion and electron temperatures are approximately equal,
slowing down of charged fusion products is instantaneous,
DT reaction rates dominate all others.

It is necessary to specify the initial regime of the plasma. The code then monitors suitable parameters and changes regime when appropriate. The conditions governing the various regimes are shown in Figure 4. Parameters m_1 and m_2 are input to the code. For all the runs described here $m_1 = 0.5$ and $m_2 = 20$. In regimes 1, 2 and 3, only DT reactions are considered whereas in regimes 4 and 5, the effects of DD reactions are taken into account. In addition the reactions of fusion product tritium and helium-3 are included; this is normally a small correction in the present work however. Similarly in regimes 1, 2 and 4, slowing down of charged fusion products is assumed to be instantaneous whereas in regimes 3 and 5, the hot (birth energy) product particles are treated as a separate species from the same type of particles at

Figure 4 Assumptions governing different regimes of computer code.

Figure 5 Parameters describing imposed voltage profile.

background temperatures. (No provision is made in the code for a direct change from regime 3 to regime 4 since this would require instantaneous dumping of the energy of the hot product alpha particles remaining at the time of the change.)

The basic parameters input to the code are those which describe the size of the device and the material of the first wall, the imposed toroidal magnetic field and voltage profile, and the rate and duration of any refuelling and additional heating. The voltage profile is used to determine the plasma current and resulting magnetic field. The form of this calculation is described in detail below. The code monitors the values of β and θ which are used to assess the potential stability or otherwise of the plasma. This process is discussed at the end of this section. The code finally determines, for each increment of time, the average fusion power and the resulting average wall loading.

The voltage, current, magnetic field model

The program is designed to accept as input the voltage profile imposed upon the plasma. (Note that this will be significantly smaller than the external voltage applied to the device.) It is assumed to start at some non-zero value, rise linearly to a maximum and then fall linearly to zero. The initial and maximum values along with the rise time and total 'on-time' thus define the imposed voltage profile. The initial voltage is not taken to be zero since the code is presently unable to describe the cold and very turbulent regime encountered during start-up. The plasma is assumed to have achieved, at least instantaneously, a reversed field configuration and possible developments of the burn from that time are considered.

As discussed earlier the current caused by the imposed voltage is thought to relax a helical configuration around the torus (Figure 1). The helix becomes more tightly wound as the configuration is set up. However once field reversal is established the pitch remains more nearly constant so long as the plasma remains stable⁹. The code includes the approximation that this is so. Note also that the resulting field line pitch varies with field line radius and consequently a spatially average value must be used in this global model. The pitch of the current helix is defined here by α , the ratio of the path length, L , around the helix to the major circumference of the torus, that is $L = 2\pi R\alpha$ where R is the major toroidal radius. The value of α used to that needed just to achieve field reversal (defined as $B_{TC} = 1.5 B_{IMP}$) with the initial conditions. (An investigation has been made of the sensitivity of the results to variations in this somewhat arbitrary figure of 1.5. This is discussed in the next section.) The helix is assumed to have an average radius equal to half the plasma radius.

During the comparatively rapid voltage rise, inductive effects are small and the relationship between voltage and current is approximately that given by Ohm's law:

$$\underline{j} = \underline{E}/\eta \quad (1)$$

With the definitions and assumptions described above this reduces to

$$I / \pi(a/2)^2 = V / \eta 2\pi R\alpha \quad (2)$$

or $I = KV$ where $K = a^2 / 8R\alpha\eta$. As the voltage falls, however, the inductive effects associated with the electrical energy stored in the magnetic field become significant. This causes the plasma current to fall much more slowly than would otherwise be the case. The appropriate

expressions for the plasma current are thus:

during the voltage fall,

$$I(t) = K \left\{ V(t) + \frac{V_{\max} \tau}{(t_2 - t_1)} \left[1 - \exp\left(\frac{t_1 - t}{\tau}\right) \right] \right\} \quad (3)$$

and after the imposed voltage has fallen to zero,

$$I(t) = I(t_2) \exp\left(\frac{t_2 - t}{\tau}\right). \quad (4)$$

where t_1 is the voltage rise time and t_2 is the total 'on-time'.

τ (compare with $\tau \sim L/R$ for an inductive circuit) is the decay time for the current and is given¹⁷ by

$$\tau = 4\pi \times 10^{-7} a^2 / \eta \quad (5)$$

where η is the plasma resistivity. (All equations are presented in SI units.) At low ($kT_e \lesssim 0.5$ keV) temperatures, the value of resistivity used is that due to Spitzer and Härm¹⁸. As discussed earlier, the turbulence of a plasma in reversed field confinement leads to anomalies in a number of plasma parameters; among these anomalies is that the resistivity is approximately constant for higher temperatures. Thus the decay time, τ , is in general a function of time. However for hot ($kT_e \gtrsim 0.5$ keV) plasmas, η and hence τ remain approximately constant. The plasma radius, a , is approximately constant in all the cases considered.

In order to estimate the poloidal and toroidal components of the magnetic field created by the plasma current, the helix is considered as a series of connected loops. The toroidal field so created is calculated as that caused by n current loops of radius $a/2$. The number

of loops, n , equivalent to the helix is thus given by

$$2\pi R\alpha \approx 2\pi R + n2\pi a/2 \quad (6)$$

from which

$$n \approx 2R(\alpha-1)/a \quad (7)$$

Thus the toroidal field created by the plasma current is

$$B_{TC} \approx 4IR (\alpha-1) \times 10^{-7}/a^2 \quad (8)$$

Similarly the poloidal field is given by

$$B_{PC} \approx 2I \times 10^{-7}/a \quad (9)$$

Since the direction of the toroidal field created in this way opposes that imposed externally on the plasma, the total magnetic fields acting are

$$B_T(t) \approx [4R(\alpha-1) \times 10^{-7}/a^2]I(t) - B_{IMP} \quad (10)$$

$$B_P(t) \approx (2 \times 10^{-7}/a) I(t) \quad (11)$$

where R , a and α are all approximately constant during the stable period of interest.

The stability criteria

As discussed earlier, the constraints imposed on the plasma by the necessity of avoiding kink instabilities, loss of field reversal or violation of the Suydam criterion can be expressed as restrictions in the region of β - θ space which the plasma may occupy. The constraints² are shown in Figure 3. The code monitors the values of β and θ and presents an appropriate message if any of the constraints is violated.

The durations of the stable burn for different combinations of the external conditions can be compared. The code indicates the exact location of the plasma in β - θ space at the time when it loses stability;

this facilitates appropriate adjustment of the conditions to give a longer burn. Moreover, since β and θ are monitored throughout, potential violations of their stability limitations can be foreseen. Loss of stability, if anticipated sufficiently early, may sometimes be able to be avoided by corrective action such as cold refuelling or extra heating. The stable burn period may thus be prolonged. It is intended to develop the code further so that such corrective action can be taken automatically.

Section III

RESULTS, CONCLUSIONS AND FUTURE WORK

As already discussed, the computer code indicates violation of any of the plasma stability criteria by printing a suitable message. Such messages continue throughout any period during which the plasma is operating in an unstable regime. This apart, the code continues to operate with the assumptions applicable to a stable plasma. Thus an extended run can be made even though the results obtained are probably not physically realizable. In this way a comparison, illustrated in Figure 6, could be made with the unpublished results¹¹ of Long et al. (whose work did not incorporate these constraints of stability). As discussed earlier, it is inappropriate to model a very cold plasma with this code. It was therefore not possible to match their initial temperature ($kT \sim 1\text{eV}$) and for this reason the temperature rise and peak power output occurred earlier in the run in this study than in the comparative one. Also in that work the poloidal plasma β was held constant once peak power output had been reached. In view of these differences agreement between the two cases seems good. The base reactor parameters used in most of these studies were also taken from suggestions in the work¹¹ of Long et al. The values used are shown in Table 1.

Many of the results of these studies are illustrated by plotting operating curves in β - θ space (Figures 8-13). In all cases presented, the end of the curve corresponding to high values of θ represents the start of the run and both β and θ initially fall. The discontinuity in the curve corresponds to the end of the voltage rise period. After

this time the plasma current falls only slowly, as discussed in Section II, and hence θ rises slowly while β increases rapidly as the plasma temperature rises. It should be noted that the plasma does not follow the curve at constant speed; this is indicated on curve C of Figure 10 by way of example. The broken lines on Figures 9-13 correspond to the stability limitations shown in Figure 3.

Sensitivity studies

Investigations were made of the sensitivity of the results to each of the following assumptions or approximations:

particle confinement is neoclassical,

current decay time has the value due to Adlam and Tayler¹⁷,

α is calculated from an initial reversal condition, $B_{TC} = 1.5 B_{IMP}$.

Figures 7 and 8 illustrate the results of these studies.

Particle confinement

Two runs were compared, one of which used neoclassical confinement times and the other, the expressions for confinement time appropriate to the trapped ion and trapped electron modes¹⁹. As expected, in the latter case loss of fuel density was much more rapid and the burn was quenched quickly because of this effect. Before significant loss of fuel occurred however, there was little difference between the two cases either in power output or in behavior with respect to the stability criteria.

Decay time

Since this expression for current decay time¹⁷ previously has been applied to cylindrical or to tokamak configurations, it

Figure 6 Temperature and wall loading during plasma burn. Broken lines indicate the results¹¹ of Long et al. Wall loading is that due to fusion power only.

Figure 7 Wall loading due to fusion power for different values of various model parameters. (Radiation wall loading introduces a correction of less than a few percent in burns of this length.)

Figure 8. Operating lines for different values of various model parameters.

was thought advisable to investigate the effects of small changes in this parameter. However scaling of the decay time by an order of magnitude in either direction produced no significant effects either on the power produced or on the duration of stability. Only when the decay time was forced to an unrealistically small value (between two and three orders of magnitude smaller than that described by Adlam) did any appreciable effects become apparent.

Calculation of α

The two runs compared here calculated α from the initial reversal conditions $B_{TC} = 1.5 B_{IMP}$ and $B_{TC} = 2.0 B_{IMP}$. Necessarily, the different values of α obtained ($\alpha \sim 1.10$ and $\alpha \sim 1.14$) lead to different operating curves β - θ space. However, provided that the same form of calculation of α is used in all cases, comparison of the results for different values of input parameters is in no way negated by this. Thus the effectiveness of different possible control techniques can be compared even though quantitative statements about the exact position of a set of operating lines cannot be made without more detailed study. There was no significant difference in calculated power output between these two runs.

The aim of this study was to suggest which of the controllable parameters are most likely to be useful in prolonging a stable plasma burn. A longer burn, and hence a longer cycle time, can result in simpler operation and, in particular, gives enhanced energy multiplication, Q . Energy multiplication will be greater for a longer burn since the input energy is added to the plasma early in the cycle whereas energy is obtained from the plasma throughout the later stages of the burn.

For example, for the runs indicated by lines A and B on Figure 10, the stable burn periods, τ_s , were approximately 6s and 5s respectively. If, in this time dependent model, energy multiplication is defined by

$$Q \equiv \frac{\int_0^{\tau_s} P_o(t) dt}{\int_0^{\tau_s} P_I(t) dt}$$

where P_o and P_I are the fusion and input powers respectively, then the values of Q obtained for the examples mentioned above are

τ_s/s	Q	kT_e/keV	kT_i/keV
5	0.8	18	16
6	1.4	26	22

The temperatures given are those at the time when stability is lost.

It is necessary to extend the burn by several more seconds in order to include within the stable period the regime where the ion temperature exceeds that of the electrons. Calculations indicate that if the longer of the above two burns could be extended to twelve seconds then the energy multiplication would be increased to five.

Investigations were made of the potential effects on plasma stability of changes in plasma radius, imposed magnetic field and any of the four parameters defining the voltage profile. The input parameters for these runs are shown in Table 1 and the resulting operating curves in Figures 9-13. In each of these cases the relative impurity concentration, λ , was deliberately held low ($n_{\text{imp}} = \lambda n_\alpha$ where λ is an input parameter) so that any consequences of the presence of impurities would not obscure the other effects under consideration.

However since the fusion product density remains low during the stable period, the effects of impurities are not expected to be significant. As indicated in the last row of Table 1, a study was also made of the effects of higher impurity concentration and of impurities of higher atomic number; wall stabilization will require wall materials which are highly conducting and hence have comparatively high atomic number. While the plasma in these runs had a slightly increased effective atomic number, Z_e , ($\Delta Z_e \sim 5 \times 10^{-4}$ using values at loss of stability), the build up of fusion products and impurities at this stage in the burn was too low to have any significant effect on the temperature, stability or power output of the plasma. Similarly, until some method is found to extend the stable period appreciably, the burn-up fraction will remain very small.

Different values of plasma radius or imposed magnetic field (Figures 9 and 10) cause widely separated positions of the plasma operating line in β - θ space, although it should be noted that changing the plasma radius results only in a change in θ and not in β . It is of course to be expected that a complementary set of field and device sizes must be chosen if any stable confinement is to be established. However judicious choice of either of these two parameters does not allow fine control of the stable burn time since the relationship between β and θ after peak voltage has been achieved is approximately the same in all cases: the straight line portions of the operating curves are roughly parallel.

Unlike the other parameters studied, the initial value of voltage is not a matter of free choice but is determined by available methods of

Table 1 Computer code input parameters for comparative studies.

INITIAL VOLTAGE/V	MAXIMUM VOLTAGE/V	PLASMA RADIUS/m	RISE TIME/s	FALL TIME/s	IMPOSED MAGNETIC FIELD/T	RELATIVE IMPURITY CONCENTRATION (n_{imp}/n_{α})	IMPURITY ATOMIC NUMBER
20							
17	VI + 10	3	1.0	2.0	1.0	10^{-8}	6
15							
	40						
20	35	3	1.0	2.0	1.0	10^{-8}	6
	30						
		5					
18	33	4	1.0	2.0	1.0	10^{-8}	6
		3					
			1.5				
18	33	3	1.0	2.0	1.0	10^{-8}	6
			0.5				
				3.0			
19	29	3	1.0	2.0	1.0	10^{-8}	6
					1.5		
19	29	3	1.0	2.0	1.0	10^{-8}	6
					0.5		
						10^{-8}	6
19	29	3	1.0	2.0	1.0	10^{-1}	6
						10^{-1}	29

In all cases: $R = 15m$; initially $kT_e = kT_i = 1keV$; $n_D = n_T = 3.5 \times 10^{19} m^{-3}$; no additional heating or refuelling

Figure 9 Operating lines for different values of imposed magnetic field, B .

Figure 10 Operating lines for different values of plasma radius, a .

Figure 11 Operating lines for different values of initial voltage, V_I .

Figure 12 Operating lines for different values of maximum voltage, V_M .

Figure 13 Operating lines for different values of voltage rise time, t .

start-up. Figure 11 shows operating lines for three possible values of initial voltage. All of these runs used the same rise time and voltage increment ($V_M - V_I = 10V$). The maximum voltage achieved was therefore different in each case. This gives rise to the variation in the minimum values of β . Figure 12 illustrates the effects of rising to different voltage maxima from the same base voltage and in the same rise time. The position of the operating curve changes more rapidly for the same change in V_M as the value of V_M falls. Also the gradient of the linear part of the curve is lower for the lower values of maximum voltage. Both of these effects may be useful in helping to contain the plasma longer within the stable region. In the same way Figure 13 indicates that a faster rise time may help to prolong the plasma stability by reducing the rate of increase of β with θ during the burn. Investigations show that increasing the voltage fall time has a similar, but smaller, effect.

Changes in the values of various parameters, which should be reasonably simple to control, have been shown to have significant effects on the stability of the plasma during the period of voltage rise. Some effects have been found which are helpful during the later part of the burn period. In particular the voltage should have a rapid rise time and a comparatively slow fall. Also the peak voltage should be reasonably low in so far as this is consistent with achieving and maintaining field reversal. Even so these effects on the later portion of the burn are all comparatively small and some further method of control is needed. (It should be recalled, however, that only a small increase in burn time leads to substantial enhancement of energy

multiplication.) Some control of plasma temperature should be possible using suitable rates of cold refuelling. On the other hand, if the plasma temperature falls too far additional heating may be necessary. However the investigations described here suggest that any burn which remains stable for a few seconds is more likely to lose stability as its temperature rises and Suydam's criterion is violated than by either of the other routes considered here. Thus cold refuelling appears to be the most promising control method for the later part of the burn.

This computer code in its present form can model such refuelling or additional heating in a global manner. Some preliminary investigations of these control techniques can therefore be made reasonably simply. There are, however, many potential advantages in developing the code from a global to a one dimensional model. As well as giving a more realistic description of a number of plasma parameters, this would enable some modeling of injection to be made. Injection may be able to assist in maintaining the helical current as well as in temperature control. (The question of current maintenance is not explicitly addressed here but could be a serious difficulty.) Clearly it would also be most helpful if the code could describe the initiation of the reversed field pinch configuration. However an adequate model of this cold and turbulent regime will be difficult to determine. While this may seem discouraging, it must at least be remembered that conclusions drawn about the burn regime are generally independent of the method of start-up of the reactor.

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Appendix I

SYMBOLS USED

a	plasma radius
B_{IMP}	imposed magnetic field
B_p	poloidal magnetic field
B_{PC}	poloidal magnetic field caused by plasma current
B_T	toroidal magnetic field
B_{TC}	toroidal magnetic field caused by plasma current
E	electric field
I	plasma current
I_p	poloidal component of plasma current
I_t	toroidal component of plasma current
j	current density
K	$\equiv a^2 / 8R\alpha\eta$
k	Boltzmann's constant
L	pathlength around helix
n	number of loops in helix
n_D	deuterium number density
n_e	electron number density
n_i	ion number density
n_{imp}	impurity number density
n_T	tritium number density
n_α	α particle number density
P_I	input power to plasma
P_O	output fusion power from plasma
P_w	average wall loading

Q	$\equiv \left[\int_0^{\tau_s} P_o(t) dt \right] / \left[\int_0^{\tau_s} P_I(t) dt \right]$
q	$\equiv aB_T / RB_p$ (safety factor)
R	major toroidal radius
T	temperature
T _e	electron temperature
T _i	ion temperature
t	time
t ₁	voltage rise time
t ₂	voltage on-time
V	voltage
V _I	initial voltage
V _M	maximum voltage
Z	atomic number
Z _e	effective atomic number
α	$\equiv L/2\pi R$
β	(plasma pressure) / (magnetic pressure)
ΔZ_e	difference in Z _e between two runs of the code
η	plasma resistivity
θ	(poloidal magnetic field at plasma radius) / (average toroidal magnetic field)
λ	$\equiv n_{imp} / n_\alpha$
τ	current decay time
τ_S	stable burn time
τ_{SD}	slowing down time

Appendix II

EQUATIONS USED

Throughout, a superscript p on a particle number density indicates the density of birth energy, 'product' ions of that species. In each regime the electron number density is obtained by satisfying charge neutrality ($n_e = \sum_{\beta} n_{i\beta} z_{\beta}$) and the impurity number density, n_{imp} , is taken to be proportional to the reaction rate. The power from ohmic heating is divided in the ratio 4:6 between ions and electrons respectively²⁰.

$\tau_p \equiv$ particle confinement time (average value for all ion species)

$\tau_E \equiv$ energy confinement time

$\tau_{ie} \equiv$ energy equilibration time between ions and electrons

$\tau_{SD \cdot i} \equiv$ time for species i to slow down from birth to thermal energies

$W_{ie} \equiv$ rate of energy exchange from electrons to ions

$U_{ia} \equiv$ fraction of energy lost by species a which is given to ions

$U_{ea} \equiv$ fraction of energy lost by species a which is given to electrons

(Note $U_{ia} + U_{ea} = 1$)

Regime 1 assumptions: $T_e \approx T_i$

$$\tau_{SD \cdot \alpha} \lesssim 0.5 \text{ s}$$

$$\frac{n_T}{n_D} \gg \frac{\langle \sigma v \rangle_{DD}}{\langle \sigma v \rangle_{DT}}$$

Particle balance equations

$$\text{deuterium:} \quad \frac{dn_D}{dt} = S_D - n_D n_T \langle \sigma v \rangle_{DT} - \frac{n_D}{\tau_p}$$

$$\text{tritium:} \quad \frac{dn_T}{dt} = S_T - n_D n_T \langle \sigma v \rangle_{DT} - \frac{n_T}{\tau_p}$$

$$\alpha\text{-particles:} \quad \frac{dn_\alpha}{dt} = n_D n_T \langle \sigma v \rangle_{DT} - \frac{n_\alpha}{\tau_p}$$

Energy balance equation

$$\frac{dE}{dt} = n_D n_T \langle \sigma v \rangle_{DT} Q_\alpha - \frac{E}{\tau_E}$$

+ power from injected particles and
additional heating

+ power from ohmic heating

- power lost in bremsstrahlung and
cyclotron radiation

Regime 2 assumptions: $\tau_{SD \cdot \alpha} \lesssim 0.5 \text{ s}$

$$\frac{n_T}{n_D} \gg \frac{\langle \sigma v \rangle_{DD}}{\langle \sigma v \rangle_{DT}}$$

Particle balance equations

$$\text{deuterium:} \quad \frac{dn_D}{dt} = S_D - n_D n_T \langle \sigma v \rangle_{DT} - \frac{n_D}{\tau_p}$$

$$\text{tritium:} \quad \frac{dn_T}{dt} = S_T - n_D n_T \langle \sigma v \rangle_{DT} - \frac{n_T}{\tau_p}$$

$$\text{helium-3:} \quad \frac{dn_H}{dt} = S_H - \frac{n_H}{\tau_p}$$

$$\alpha\text{-particles: } \frac{dn_{\alpha}}{dt} = n_D n_T \langle \sigma v \rangle_{DT} - \frac{n_{\alpha}}{\tau_p}$$

Energy balance equations

$$\text{ions: } \frac{dE_i}{dt} = n_D n_T \langle \sigma v \rangle_{DT} Q_{\alpha} U_{i\alpha} - \frac{E_i}{\tau_E} + W_{ie}$$

+ power to ions from injected particles
and additional heating
+ power to ions from ohmic heating

$$\text{electrons: } \frac{dE_e}{dt} = n_D n_T \langle \sigma v \rangle_{DT} Q_{\alpha} U_{ea} - \frac{E_e}{\tau_E} - W_{ie}$$

+ power to electrons from injected
particles and additional heating
+ power to electrons from ohmic heating
- power lost in bremsstrahlung and
cyclotron radiation

Regime 3 assumptions: $\tau_{SD\alpha} \gtrsim 0.5 \text{ s}$

$$\frac{n_T}{n_D} \gg \frac{\langle \sigma v \rangle_{DD}}{\langle \sigma v \rangle_{DT}}$$

Particle balance equations

$$\text{deuterium: } \frac{dn_D}{dt} = S_D - n_D n_T \langle \sigma v \rangle_{DT} - \frac{n_D}{\tau_p}$$

$$\text{tritium: } \frac{dn_T}{dt} = S_T - n_D n_T \langle \sigma v \rangle_{DT} - \frac{n_T}{\tau_p}$$

helium-3:
$$\frac{dn_H}{dt} = S_H - \frac{n_H}{\tau_p}$$

product α -particles:
$$\frac{dn_\alpha^p}{dt} = n_D n_T \langle \sigma v \rangle_{DT} - \frac{n_\alpha^p}{\tau_{SD \cdot \alpha}} - \frac{n_\alpha^p}{\tau_p}$$

thermal α -particles:
$$\frac{dn_\alpha}{dt} = \frac{n_\alpha^p}{\tau_{SD \cdot \alpha}} - \frac{n_\alpha}{\tau_p}$$

Energy balance equations

ions:
$$\frac{dE_i}{dt} = \frac{n_\alpha^p}{\tau_{SD \cdot \alpha}} Q_\alpha U_{i\alpha} - \frac{E_i}{\tau_E} + W_{ie}$$

- + power to ions from injected particles and additional heating
- + power to ions from ohmic heating

electrons:
$$\frac{dE_e}{dt} = \frac{n_\alpha^p}{\tau_{SD \cdot \alpha}} Q_\alpha U_{e\alpha} - \frac{E_e}{\tau_E} - W_{ie}$$

- + power to electrons from injected particles and additional heating
- + power to electrons from ohmic heating
- power lost in bremsstrahlung and cyclotron radiation

Regime 4 assumptions:

$$\tau_{SD \cdot \alpha} \lesssim 0.5 \text{ s}$$

$$\frac{n_T}{n_D} \lesssim \frac{20 \langle \sigma v \rangle_{DD}}{\langle \sigma v \rangle_{DT}}$$

Particle balance equations

$$\text{deuterium: } \frac{dn_D}{dt} = S_D - n_D^2 \langle \sigma v \rangle_{DD \cdot \text{total}} - n_D n_T \langle \sigma v \rangle_{DT} \\ - n_D n_H \langle \sigma v \rangle_{DH} - \frac{n_D}{\tau_p}$$

$$\text{tritium: } \frac{dn_T}{dt} = S_T - n_D n_T \langle \sigma v \rangle_{DT} + \frac{n_D^2}{2} \langle \sigma v \rangle_{DD_p} - \frac{n_T}{\tau_p}$$

$$\text{helium-3: } \frac{dn_H}{dt} = S_H - n_D n_H \langle \sigma v \rangle_{DH} + \frac{n_D^2}{2} \langle \sigma v \rangle_{DD_n} - \frac{n_H}{\tau_p}$$

$$\alpha\text{-particles: } \frac{dn_\alpha}{dt} = n_D n_T \langle \sigma v \rangle_{DT} + n_D n_H \langle \sigma v \rangle_{DH} - \frac{n_\alpha}{\tau_p}$$

$$\text{protons: } \frac{dn_p}{dt} = \frac{n_D^2}{2} \langle \sigma v \rangle_{DD_p} + n_D n_H \langle \sigma v \rangle_{DH} - \frac{n_p}{\tau_p}$$

Energy balance equations

$$\text{ions: } \frac{dE_i}{dt} = \frac{n_D^2}{2} \langle \sigma v \rangle_{DD_n} Q_H U_{iH} + n_D n_T \langle \sigma v \rangle_{DT} Q_\alpha U_{i\alpha} \\ + \frac{n_D^2}{2} \langle \sigma v \rangle_{DD_p} (Q_T U_{iT} + Q_{p1} U_{ip1}) \\ + n_D n_H \langle \sigma v \rangle_{DH} (Q_{\alpha 2} U_{i\alpha 2} + Q_{p2} U_{ip2}) \\ + W_{ie} - \frac{E_i}{\tau_E}$$

+ power to ions from injected particles
and additional heating

+ power to ions from ohmic heating

electrons:

$$\begin{aligned} \frac{dE_e}{dt} = & \frac{n_D^2}{2} \langle \sigma v \rangle_{DD_n} Q_H U_{eH} + n_D n_T \langle \sigma v \rangle_{DT} Q_\alpha U_{e\alpha} \\ & + \frac{n_D^2}{2} \langle \sigma v \rangle_{DD_p} (Q_T U_{eT} + Q_{p1} U_{ep1}) \\ & + n_D n_H \langle \sigma v \rangle_{DH} (Q_{\alpha 2} U_{e\alpha 2} + Q_{p2} U_{ep2}) \\ & - W_{ie} - \frac{E_e}{\tau_E} \end{aligned}$$

+ power to electrons from injected particles and additional heating

+ power to electrons from ohmic heating

- power lost in bremsstrahlung and cyclotron radiation

Regime 5 assumptions: $\tau_{SD} \cdot \alpha \geq 0.5 \text{ s}$

$$\frac{n_T}{n_D} \lesssim \frac{20 \langle \sigma v \rangle_{DD}}{\langle \sigma v \rangle_{DT}}$$

Particle balance equations

deuterium:

$$\begin{aligned} \frac{dn_D}{dt} = & S_D - n_D^2 \langle \sigma v \rangle_{DD \cdot \text{total}} - n_D n_T \langle \sigma v \rangle_{DT} \\ & - n_D n_T^p \langle \sigma v \rangle_{DT}^p - n_D n_H \langle \sigma v \rangle_{DH} - n_D n_H^p \langle \sigma v \rangle_{DH}^p - \frac{n_D}{\tau_p} \end{aligned}$$

product tritium:
$$\frac{dn_T^p}{dt} = \frac{n_D^2}{2} \langle \sigma v \rangle_{DD_p} - n_D n_T^p \langle \sigma v \rangle_{DT}^p - \frac{n_T^p}{\tau_{SD \cdot T}} - \frac{n_T^p}{\tau_p}$$

thermal tritium:
$$\frac{dn_T}{dt} = S_T - n_D n_T \langle \sigma v \rangle_{DT} + \frac{n_T^p}{\tau_{SD \cdot T}} - \frac{n_T}{\tau_p}$$

product helium-3:
$$\frac{dn_H^p}{dt} = \frac{n_D^2}{2} \langle \sigma v \rangle_{DD_n} - n_D n_H^p \langle \sigma v \rangle_{DH}^p - \frac{n_H^p}{\tau_{SD \cdot H}} - \frac{n_H^p}{\tau_p}$$

thermal helium-3:
$$\frac{dn_H}{dt} = S_H - n_D n_H \langle \sigma v \rangle_{DH} + \frac{n_H^p}{\tau_{SD \cdot H}} - \frac{n_H}{\tau_p}$$

product α -particles:
$$\begin{aligned} \frac{dn_\alpha^p}{dt} = & n_D n_T \langle \sigma v \rangle_{DT} + n_D n_T^p \langle \sigma v \rangle_{DT}^p \\ & + n_D n_H \langle \sigma v \rangle_{DH} + n_D n_H^p \langle \sigma v \rangle_{DH}^p \\ & - \frac{n_\alpha^p}{\tau_{SD \cdot \alpha}} - \frac{n_\alpha^p}{\tau_p} \end{aligned}$$

thermal α -particles:
$$\frac{dn_\alpha}{dt} = \frac{n_\alpha^p}{\tau_{SD \cdot \alpha}} - \frac{n_\alpha}{\tau_p}$$

product protons 1:
$$\frac{dn_{p1}^p}{dt} = \frac{n_D^2}{2} \langle \sigma v \rangle_{DD_p} - \frac{n_{p1}^p}{\tau_{SD \cdot p1}} - \frac{n_{p1}^p}{\tau_p}$$

product protons 2:
$$\begin{aligned} \frac{dn_{p2}^p}{dt} = & n_D n_H \langle \sigma v \rangle_{DH} + n_D n_H^p \langle \sigma v \rangle_{DH}^p \\ & - \frac{n_{p2}^p}{\tau_{SD \cdot p2}} - \frac{n_{p2}^p}{\tau_p} \end{aligned}$$

thermal protons:

$$\frac{dn_p}{dt} = \frac{n_{p1}^p}{\tau_{SD \cdot p1}} + \frac{n_{p2}^p}{\tau_{SD \cdot p2}} - \frac{n_p}{\tau_p}$$

Energy balance equations

ions:

$$\begin{aligned} \frac{dE_i}{dt} = & \frac{n_{\alpha}^p}{\tau_{SD \cdot \alpha}} Q_{\alpha} U_{i\alpha} + \frac{n_H^p}{\tau_{SD \cdot H}} Q_H U_{iH} \\ & + \frac{n_T^p}{\tau_{SD \cdot T}} Q_T U_{iT} + \frac{n_{p1}^p}{\tau_{SD \cdot p1}} Q_{p1} U_{ip1} \\ & + \frac{n_{p2}^p}{\tau_{SD \cdot p2}} Q_{p2} U_{ip2} + W_{ie} - \frac{E_i}{\tau_E} \end{aligned}$$

+ power to ions from injected

particles and additional heating

+ power to ions from ohmic heating

electrons:

$$\begin{aligned} \frac{dE_e}{dt} = & \frac{n_{\alpha}^p}{\tau_{SD \cdot \alpha}} Q_{\alpha} U_{e\alpha} + \frac{n_H^p}{\tau_{SD \cdot H}} Q_H U_{eH} \\ & + \frac{n_T^p}{\tau_{SD \cdot T}} Q_T U_{eT} + \frac{n_{p1}^p}{\tau_{SD \cdot p1}} Q_{p1} U_{ep1} \\ & + \frac{n_{p2}^p}{\tau_{SD \cdot p2}} Q_{p2} U_{ep2} - W_{ie} - \frac{E_e}{\tau_E} \end{aligned}$$

+ power to electrons from injected

particles and additional heating

+ power to electrons from ohmic heating

- power lost in bremsstrahlung and

cyclotron radiation