DEVELOPMENT OF A HIGH RESOLUTION, HIGH SENSITIVITY
CYLINDRICAL CRYSTAL SPECTROMETER FOR LINE SHAPE DIAGNOSTICS OF
X-RAYS EMITTED FROM HOT PLASMAS

Progress Report
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ABSTRACT

This report outlines progress towards development of a high resolution, high throughput, curved crystal spectrometer suitable for line shape diagnostics of X-rays emitted from hot plasmas. The instrument is designed to interface with the MIT Tokamak (Alcator) with the initial aim of studying the prominent MoL lines which occur in the X-ray spectrum. However, it will have the versatility to function over an energy range of at least 1.5 keV to 7 keV allowing determination of temperature, charge state and density distributions for important impurity ions. The spectrometer employs a large, cylindrically bent crystal which focuses the dispersed X-rays along the cylinder axis where they are recorded by a position sensitive high resistance anode proportional counter. Thus a wide energy range of the spectrum can be recorded simultaneously and sensitively from a short duration plasma. Computer control of data acquisition and analysis will allow real-time diagnostics. The report includes an analysis and optimization of spectrometer parameters for interface with Alcator and an analysis of monolithic and individual silicon diode arrays as alternative position sensitive detectors. A Si(Li) detector with computer-controlled ADC system has been on line at Alcator providing several time sequenced spectra during each shot. Bremsstrahlung and impurity ion spectra are discussed and results for electron temperatures as a function of time and position in the plasma are presented.

1.0 INTRODUCTION

Among the most important parameters of fusion-type plasmas are the ion temperature and impurity ion charge state and density distributions. These parameters can be obtained reliably from a detailed analysis of the X-rays emitted by the plasma provided the spectrum is obtained with sufficient resolution and sensitivity.

This progress report provides details on a spectrometer specifically designed to make accurate measurements of X-ray line profiles and ionic charge state distribution in a time dependent (pulsed) hot plasma. Design of the spectrometer was carried out under Contract EY-76-S-02-4021, and progress under that contract is reported herein. The spectrometer will employ a large, cylindrically bent crystal to disperse and focus the X-rays along the cylinder axis where they will be recorded by a position sensitive detector. After fabrication, testing and calibration, the spectrometer will be used initially as a plasma diagnostic on the MIT Tokamak, Alcator. It will have versatility to function over a wide energy range and will provide a particularly unique diagnostic on the next generation of higher temperature Tokamaks now under construction. The report includes a discussion of a new type of photon counting silicon diode array detector which will be necessary to extend the instrument's capability to energies of 6 keV and above. The spectrometer's capability of sensitively recording a wide range of X-ray energies simultaneously will encourage applications at other installations where the plasma is transient (e.g., laser-induced plasma devices) or beam time limitations exist (e.g., synchrotron radiation or high energy heavy ion facilities).

2.0 SCIENTIFIC OBJECTIVES

Current Tokamak discharges are characterized ⁽¹⁾ by electron densities of $0.3 - 1 \times 10^{14} \text{ cm}^{-3}$ and central temperatures from 1 - 3 keV, while proposed next generation machines scheduled for completion in the early 1980's will exhibit temperatures above 5 keV. In these discharges, light ions will be completely stripped but heavy impurity ions, such as Fe, Mo and W from the walls and limiters, will be highly but incompletely stripped. It is anticipated ^(2, 3, 4) that the density, charge state, and distribution of these ions will be of crucial importance in affecting radiation losses, reactivity, and stability, and therefore the feasibility, of future Tokamak reactor plasmas.

Among the diagnostic data most urgently required, then, are not only the central ion temperature, but also the charge state, concentration and spatial distribution of the high Z impurities. An X-ray spectrometer capable of recording time and space resolved spectra, with sufficient resolution to measure the Doppler broadened line widths, appears to be a crucial diagnostic on both current and future Tokamaks. The advantages of this diagnostic are now considered.

2.1 Ion Temperature Measurement

Temperature diagnostics such as laser scattering and neutron spectroscopy need supporting confirmation by other techniques. Neutral atom charge-exchange-energy-measurements depend on uncertain atomic cross-sections. All these methods involve deconvoluting the effects of outer cooler layers of the plasma. Determination of temperature from Doppler broadened spectral line widths, on the other hand, has found successful application to many plasma conditions. The principle is suitable for Tokamak conditions since the temperature relaxation times are short enough that the temperatures measured

are representative of the hydrogen temperature. By choosing the appropriate transitions temperatures characteristic of the center of the plasma can be measured without interference from outer layers. Other sources of spectral line broadening such as Stark, Zeeman and collision broadening are not significant.

For a Maxwellian energy distribution, the Doppler broadened full width at half maximum is given by

$$\frac{\Delta E}{E} = \frac{\Delta \lambda}{\lambda} = 2 (2 \log 2)^{1/2} \left(\frac{kT_i}{Mc^2} \right)^{1/2} \quad (1)$$

where T_i and M are the temperature and mass, respectively, of the impurity ions, and E is the transition energy. Thus, for a heavy ion such as Mo ($M = 96$) at 1 keV, $\Delta E/E = 2.5 \times 10^{-4}$, while for Fe ($M = 56$) at 5 keV, $\Delta E/E = 7.3 \times 10^{-4}$. Since all impurity ions will be highly stripped (e.g., Fe^{23+}), strong transitions in these ions will correspond to energies of several 100 eV to several 1000 eV or $\sim 10 \text{ \AA}$ and less. These lines will be the most easily identified and will provide the most sensitive diagnostic (since the transition energy is of the same order as the electron temperature). In this region the spectrum can be easily dispersed by Bragg reflection from crystal planes with a crystal limited resolution of 0.5 to 2×10^4 , sufficient to deconvolute the Doppler line width.

Another alternative, that of Doppler measurements at vacuum ultraviolet wavelengths which are long enough that the Doppler wavelength profile can be measured, is less attractive. This is because lines in this region correspond to small energy differences (6 to 14 eV) between high principal quantum number states and will therefore be obscure, unidentified, weak lines.

2.2 Ion Charge State and Density Profiles

Analysis of high resolution X-ray spectra corresponding to simple easily identifiable core transitions, which will be the most amenable to modeling and calculation, will provide reliable information of ion charge state distribution and density profiles. These parameters evolve rapidly during the typical 100 ns plasma pulse and therefore time resolved studies are necessary to verify the modeling.

2.3 Helium-like Ion Density Measurements

Absolute electron densities can be determined from spectral line ratios. Capability of measuring line ratios in the He like $n = 2$ to $n = 1$ triplet of highly stripped ions offers a unique opportunity to measure plasma electron density. These levels are relatively closely spaced and therefore the temperature dependence of the excitation rate ratios can be ignored.

The instrument we will describe is ideally suited for these measurements since it will possess high resolution (necessary to discriminate against satellite and other spurious lines) and high sensitivity, and will also record the entire triplet at once eliminating problems associated with shot-to-shot variations.

2.4 Electron Temperature Measurements

Temperature measurements are also obtainable from measures of relative line strengths since the population of the upper state depends upon the collisional excitation rate coefficient C_i . Generally,

$$C_i \propto F(T) e^{-E_i/kT}$$

where E_i is the excitation energy of the i^{th} state and $F(T)$ is some slowly varying function

of temperature. Thus, apart from slowly varying terms, the principal temperature dependence of the ratio of emission lines from states i and j is,

$$e^{-(E_i - E_j)/kT}$$

Line ratio measurements are a good temperature measurement when $(E_i - E_j) > kT$ and when both transitions are strongly allowed. Traditionally, the lithium like line ratios $(2s - 2p)/(2s - 3p)$ have been used for cooler temperature plasmas. This choice is not appropriate for the light ions and high temperatures under consideration for devices like Alcator. For hot plasmas, the states which are available usually involve transitions to the $1s$ level, i.e., the ratio $(1s - 2p)/(1s - 3p)$ could be measured. In most cases $E_{3p} - E_{2p} < kT$ and good statistics are required if the data are to be interpreted properly. As an example, consider hydrogen like oxygen and iron where $\Delta E = E_{3p} - E_{2p} \approx 121$ eV and 1277 eV respectively. Table 1 lists the expected line ratios for several values of plasma temperature. Very accurate line ratio measurements are required before temperatures can be inferred from the data. Good resolution is also required to separate out spurious contributions in the region of the spectral line. Simultaneous measurements and a high sensitivity are required to achieve good statistical accuracy in the intensity measurements.

Table 1 $(1s - 2p)/(1s - 3p)$ Line Ratios

<u>kT (keV)</u>	<u>Oxygen</u>	<u>Iron</u>
0.5	0.79	.078
1.0	0.89	.28
2.0	0.94	.53
5.0	0.98	.77

2.5 Spectrometer Requirements

Taken together, the above scientific objectives require an X-ray spectrometer with the following capabilities:

1. Ability to record spectra up to 7 keV (for transitions with $E \approx kT_i$).
2. Ability to record a segment of spectrum simultaneously (to obtain statistically reliable line ratios as well as to aid in line identification and charge state analysis since the lines shift as the plasma evolves).
3. Energy resolution of $\Delta E/E = 10^{-4}$ (to measure Doppler widths and hence ion temperature).
4. Time resolution of about 10 ms in order to obtain several time sequenced spectra per shot (thus enabling measurement of the time evolution of plasma parameters which is important as a diagnostic and to verify modeling).
5. High sensitivity (to aid in gathering time resolved and statistically significant data).

All these capabilities are realized in the spectrometer described in the next section.

3.0 APPROACH

The concept of the spectrometer is illustrated in Fig. 1. At the heart of the instrument is a large, cylindrically bent crystal which subtends a range of Bragg angles at the entrance slit, located on the cylinder axis. The crystal focuses the reflected and dispersed rays back onto the cylinder axis where they are recorded by a linear, position-sensitive proportional counter. The focusing properties of the cylindrical crystal, used in this way, allow acceptance of a large solid angle from a long narrow slit thus providing maximum throughput without loss of resolution. In addition, the crystal disperses a wide wavelength range onto the position sensitive detector allowing a significant range of spectrum to be recorded simultaneously. To take full advantage of the instrument's potential and to provide a real-time diagnostic, data acquisition will be controlled by a minicomputer.

The instrument is being designed specifically to interface with an X-ray diagnostic port on Alcator with the initial aim of studying the prominent MoL lines which occur at 2.5 keV in the X-ray spectrum. A beryllium windowed, high resistance anode, proportional counter (HRAPC) has been designed to detect the dispersed X-rays. This detector can be fabricated with the required active length (20 cm) and the minimum necessary spatial resolution (0.05 cm).

Disadvantages of the HRAPC are associated with the thin window support structure which interrupts the active length at several places and with the glancing angle of incidence. The glancing incidence, about 60° from normal, increases window opacity and also increases the X-ray conversion path length parallel to the wire, which limits resolution. These severely restrict the application of a HRAPC at both lower (because of window loss) and higher (because of conversion length) energies. Therefore we have made a feasibility study of fabricating a linear array of individual Si chips, with single photon counting capability, for use as a future detector. We have also considered monolithic Si diode arrays, with self-scanning readout, as high resolution detectors. However,

as commercially available, the latter are characterized by high noise and are no more sensitive than photographic film.

In parallel with the spectrometer design effort, a Si(Li) detector system with a fast computer controlled ADC system has been in constant use at Alcator. The capability of this X-ray diagnostic has been extended to allow several time sequenced spectra to be obtained during each shot. Thus, from the Bremsstrahlung spectra between 1 and 6 keV, we have obtained electron temperatures as a function of time and position in the plasma. The ratio of the molybdenum line intensities to Bremsstrahlung is providing some crude estimates of time and spatial density distributions of this impurity.

These investigations have helped define needed rate and spectral information necessary to evaluate the performance of the spectrometer, and have also provided major portions of the required computer-controlled data acquisition and analysis systems. An existing flat crystal scanning spectrometer has been modified and calibrated for installation on Alcator. It will be used with a proportional counter detector to add to the limited data on ion temperature and impurity ion distribution in the Alcator plasma.

4.0 PROGRESS

ERDA support for the design phase of the spectrometer development began on June 1, 1976. A protracted search for a suitable investigator resulted in appointment of a scientist in November, 1976, who did not start full-time employment until January 17, 1977. Thus, full-time work on the project did not start until seven months after the beginning of the contract and for this reason a two month, no cost extension was requested and granted. Work is proceeding now with the full-time attention of the co-investigator, and the principal investigator is continuing to contribute 25% of his time. It is anticipated that all planned milestones of the design phase will be substantially attained by July 31. These were:

1. Preliminary data to be obtained with a simple flat crystal spectrometer/photographic film detector system.
2. Evaluate the efficiency and resolution of the position sensitive proportional counter as a function of energy.
3. Design the curved crystal spectrometer.

Fabrication and testing of the curved crystal spectrometer were reserved for the renewal phase. Because of the late start on the project, laboratory testing of the proportional counter will be behind schedule.

Intensive engineering support has been enlisted to aid in achieving the following tasks by June 30.

1. Design, specify, and let contract for high resistance anode proportional counter (HRAPC).
2. Design and/or specify signal processing electronics for HRAPC.
3. Analyze diode array sensitivity with respect to detector noise and identify realistic performance in the context of specified electronic processing.
4. Design vacuum housing, crystal support system, HRAPC transport system.
5. Construct signal processing electronics for HRAPC.

Contracts for the proportional counter, crystal, and also an X-ray calibration/ test source and control unit will be let shortly. These programs will exhaust the materials and salaries budgets by the end of the contract term (July 31).

A self-scanned monolithic diode array (Reticon) was investigated as a suitable high-resolution, fast readout detector. It may be used for specific high resolution studies in the completed spectrometer, but the signal to noise and severe size limitations of the device encouraged us to evaluate the feasibility of fabricating a diode array of individual low noise elements, each with independent readout. Thus, Task 3 is an addition to the contract requirements.

The preliminary design of the spectrometer is completed. This has included an analysis of aberrations and optimization of the spectrometer parameters for interface with Alcator while providing maximum throughput and resolution. Necessary slit, crystal, and detector sizes and scanning motions have been defined and appropriate mechanisms planned.

The Si(Li) detector system on Alcator has generated electron temperatures and also crude estimates of Mo impurity densities as a function of time and position in the plasma. These results were displayed at the APS Topical Conference on Atomic Processes in High Temperature Plasmas at Knoxville on February 16-18. Work on Alcator will continue and installment of the flat crystal spectrometer, though delayed by contingencies in the plasma group at MIT, will occur during May. The flat crystal spectrometer is of the scanning type and will use a proportional counter as detector (rather than photographic film as specified in the contract) for higher sensitivity and digital readout.

5.0 SPECTROMETER PARAMETERS

A cylindrical spectrometer of the type in which the crystal is bent to a cylindrical surface and the spectra are observed along the cylinder axis has been rarely used. However, the design allows observation of a wide range of the spectrum simultaneously while additionally providing a high throughput and high resolving power, and is thus very suitable for our application. These aspects of the cylindrical spectrometer have been investigated in an early paper.⁽⁶⁾ The geometry is illustrated in Fig. 2.

The crystal radius of curvature is determined by

$$\rho = L \sin \theta_B \quad (2)$$

where L is the distance along the axial ray from the slit to the crystal, and θ_B is the Bragg angle subtended by that ray at the crystal (Fig. 2).

The reflected rays satisfy Bragg's Law:

$$n\lambda = 2d \sin \theta \quad (3)$$

where the range of Bragg angle is determined either by an aperture before the crystal or by the crystal length. In the latter case, with a crystal of length C , the range of Bragg angles is:

$$\Delta \theta_C = \frac{C}{L} \sin \theta_B = \frac{C}{\rho} \sin^2 \theta_B. \quad (4)$$

Since

$$\frac{\Delta E}{E} = \frac{\Delta \lambda}{\lambda} = \Delta \theta \cot \theta, \quad (5)$$

the crystal determined energy range is

$$\left(\frac{\Delta E}{E} \right)_C = \frac{C}{\rho} \sin \theta_B \cos \theta_B. \quad (6)$$

The dispersed rays are refocused onto the cylinder axis at a distance along the axis from the slit given by:

$$x = 2\rho \cot \theta. \quad (7)$$

Hence, if the angular resolution element is $d\theta$, then the spatial resolution at the detector is:

$$dx = \frac{2\rho}{\sin^2 \theta} d\theta \quad (8)$$

For a zero height slit, the angular resolution is determined by a combination of crystal diffraction width, $d\theta_C = W_C$, and slit width, w , resulting in an overall resolution (Fig. 2)

$$d\theta_T = \sqrt{W_C^2 + (w/2L)^2} \quad (9)$$

The resolution-throughput product is optimized when these contributions are equal.

Thus, when

$$w = 2L W_C = \frac{2L \tan \theta}{(E/dE)_C} = \frac{2\rho}{\cos \theta_B (E/dE)_C} \quad (10)$$

where $(E/dE)_C = \tan \theta / W_C$ is the crystal resolving power. The actual angular resolution is then

$$d\theta_T = \sqrt{2} W_C = \sqrt{2} w/2L \quad (11)$$

corresponding to a spatial resolution on the detector of

$$dx_T = \frac{\sqrt{2} w}{\sin \theta_B} = 2 \sqrt{2} \frac{\rho}{\sin \theta_B \cos \theta_B (E/dE)_C} \quad (12)$$

If the slit has a finite height, H , then the resolution is also affected by focusing aberrations of the cylindrical surface. These are such that a point $z = h$ in the slit (at $x = 0$) is imaged to a point at⁽⁶⁾

$$x = 2\rho \cot \theta \frac{(1 - \frac{h}{\rho} \sin \phi)}{1 - \frac{2h}{\rho} \sin \phi} \sqrt{(1 - \frac{h}{\rho} \sin \phi)^2 - \frac{h^2}{\rho^2} \frac{\cos^2 \phi}{\cot^2 \theta}} \quad (13)$$

$$z = \frac{h\rho}{2h \sin \phi - \rho} \quad (14)$$

where ϕ varies from $-\phi_{\max}$ to $\phi_{\max}$ and is the angle which the plane containing the incident ray makes to the central plane of the crystal (both planes containing the cylinder axis).

For $\frac{h}{\rho} \ll 10^{-2}$ and $\phi \ll 10^{-1}$, the x position of the image can be simplified to:

$$x = 2 \rho \cot \theta \left[1 - \frac{h^2}{\rho^2} \left(\frac{\cos^2 \phi}{2 \cot^2 \theta} - \sin^2 \phi \right) \right]. \quad (15)$$

Thus, the maximum range of $x(h, \phi)$ is:

$$\begin{aligned} \Delta x_H &= x(0, 0) - x\left(\frac{H}{2}, 0\right) \\ &= \frac{H^2}{4 \rho \cot \theta} \end{aligned} \quad (16)$$

where H is the total slit height. This is an independent broadening mechanism and must be combined in quadrature with the previous spatial resolution to get the total spatial resolution element at the detector. In practice, it is a small second order effect.

The actual energy resolution can now be calculated from

$$\begin{aligned} \frac{dE}{E} &= \cot \theta \frac{d\theta}{dx} dx \\ &= \frac{\sin \theta \cos \theta}{2 \rho} dx_T. \end{aligned} \quad (17)$$

$$= \sqrt{2} / (E/dE)_C \quad (18)$$

if Δx_H is insignificant.

Finally, we can calculate the expected signal from

$$N = I_0 b \cdot wH \cdot \Omega \cdot P \epsilon \cdot \tau \quad (19)$$

where I_0 is flux from cm^3 of plasma, b is the plasma thickness along the field of view, wH is the entrance slit area, P is the crystal peak reflectivity, ϵ is the detector efficiency and τ is the plasma duration. The solid angle, Ω , is determined by the crystal diffraction width, W_C , and the crystal height, H_C :

$$\Omega = \frac{L W_C \cdot H_C}{L^2} = \frac{w H_C}{2 L^2} \quad (20)$$

assuming the optimum slit width, $w = 2 L W_C$. If the detector pixel length, d , is less than a resolution element at the detector, then Eq. (19) will give the expected signal per pixel with

$$\Omega = \frac{d H_C}{2 L^2} \quad (21)$$

6.0 PERFORMANCE

An important impurity ion in the Alcator plasma is Mo arising from the limiter. Its radiation occurs in the Mo L lines, the most prominent of which cover a wide energy range of about 350 eV centered around 2.5 keV. Another important emission is the Fe K α radiation which covers a range of 300 eV centered at 6.55 keV.

Limitations of the diagnostic access to Alcator require that the slit to crystal distance be at least $L = 120$ cm. In addition, if we consider a reasonable crystal length of $C = 10$ cm and a slit height of $H = 1$ cm, we can tabulate the spectrometer parameters for a choice of crystals (Table 2).

We note that with the crystal length of 10 cm, we obtain only half the desired energy range at the Mo L lines. On the other hand, the diagnostic access to Alcator limits the total divergence of the X-rays to about 0.1 radians, which range of Bragg angles corresponds to a larger energy range as shown in the table. To make full use of the Alcator exit aperture, the crystal can be scanned along its length, across the exit beam. Since the detector size is also limited, it must be scanned parallel to the crystal.

It is interesting to also calculate the relative energy range which can be simultaneously recorded on a detector 2.54 cm long (such as a commercially available self-scanned silicon diode array). In all cases, this is $\frac{\Delta E_D}{E} \approx 0.01$ (note crystal length is half detector length). This is enough to record the He like triplet of Fe²⁴⁺ ($\frac{\Delta E}{E} = .008$), but we could not also simultaneously record the 1s-2p transition of Fe²⁵⁺ (Total $\frac{\Delta E}{E} = .043$) with a detector of length less than 11 cm. A detector length of 5 cm could record the He triplet of all He-like ions from Mass 12 (Mg) and higher.

We note from the table that the required detector resolution is ~ 0.5 mm, and that with this we can achieve the resolution design goal of approximately 10^{-4} , limited by crystal resolution. Higher resolution crystals than those chosen for illustration in Table 2 are available.

Table 2
Spectrometer Performance

Radiation			Mo L	Mo L	Fe K α
Energy	E	eV	2525	2525	6550
Crystal Type			PET	Si 111	LiF
Plane Spacing	2d	cm	8.742	6.284	4.027
Crystal Resolving Power	$(E/\Delta E)_c$	$\times 10^4$	0.7	0.7	1
Slit to Crystal Distance	L	cm		120	
Crystal Length	C	cm		10	
Slit Height	H	cm		1	
Field Aperture	α	rad		0.1	
Bragg Angle	θ_B	deg	34.0	51.1	27.9
Crystal Curvature Radius	ρ	cm	67.1	93.4	56.2
Crystal Limited Energy Range	ΔE_c	eV	174	132	482
Aperture Limited Energy Range	ΔE_A	eV	374	204	1237
Energy Range per 2.54 cm of Detector	$\Delta E_D/E$	$\times 10^{-4}$	88	66	93
Optimum Slit Width	w	mm	0.231	0.425	0.127
Slit Height Broadening	Δx_H	mm	0.025	0.033	0.024
Resolution Element at Detector	dx_T	mm	0.586	0.772	0.385
Relative Energy Resolution	dE/E	$\times 10^{-4}$	2.02	202	1.42
Absolute Energy Resolution	dE	eV	0.51	0.51	0.93
Crystal Height	H_c	cm		10	
Peak Reflectivity	P			0.5	
Detector Efficiency	ϵ			1	
Plasma Flux	I	$\text{cm}^{-3} \text{s}^{-1}$		10^{10}	
Plasma Thickness	b	cm		5	
Plasma Duration	τ	s^{-1}		0.1	
Expected Count Rate	N	photons	4600	15,700	1400

For a Maxwellian energy distribution, the Doppler broadened width is (from Eq. (1)):

$$\frac{\Delta E}{E} = 7.70 \times 10^{-5} (k T_i / M)^{1/2} \quad (22)$$

where M is now the atomic weight and $k T_i$ is in eV. For Mo ($M=96$), and $k T_i = 0.5$ and 5 keV, $\frac{\Delta E}{E} = 1.76$ and 5.56×10^{-4} , respectively. Thus, with presently achieved estimated ion temperatures in Alcator, the instrument resolution will, to some extent, limit the accuracy of the line width (and hence ion temperature) measurements. Planned ion temperature in the next generation of Tokamaks now under construction is in the 5 keV range (in particular, Alcator C should achieve 2 - 3 keV), and the associated Doppler width should be accurately measurable with our instrument. Other plasma contributions to the line width are insignificant except for spin orbit coupling which can be readily calculated.

Estimation of the radiation flux from the plasma is difficult because the necessary rate coefficients are unknown and also the impurity ion diffusion has to be calculated. At ion temperatures of less than 1 keV, the Mo ion will be only partially stripped into the M shell. Thus, Mo L radiation will arise both from recombination and from excitation. A lower limit bound on the flux may be obtained by considering recombination only. The flux per cm^3 of plasma is then, roughly,

$$I = 0.1 n_i n_e \langle \sigma_R v_e \rangle \quad (23)$$

where n_i and n_e are the densities of impurity ions and electrons respectively, $\langle \sigma_R v_e \rangle$ is the total rate coefficient for recombination and we assume that 90% of the recombination occurs to the ground state leaving only 10% to higher, radiative states. Assuming a rate coefficient $10^{-11} \text{ cm}^3 \text{ sec}^{-1}$, $n_e = 3 \times 10^{13} \text{ cm}^{-3}$ and $n_i = 3 \times 10^8 \text{ cm}^{-3}$ (only 0.001% of n_e), we find $I = 9 \times 10^9 \text{ photons cm}^{-3} \text{ s}^{-1}$. Using this flux in Eq. (19), we find the expected signal per resolution element at line center to be (Table 2) about 10^3 or 10^4 photons.

7.0 DETECTOR SYSTEMS

Work to date has been directed towards the evaluation of candidate position sensitive photon detectors whose spatial resolution is compatible with the spectrometer performance objectives. For the geometries under consideration, we require a spatial resolution of at least 0.5 mm with incident photons impacting the detector at an average angle about 56° from the normal to the counter. In fact, a resolution of about 0.1 to 0.2 mm is optimum. This would improve the instrument resolution by allowing determination of the center of gravity of a resolution element and also the instrument line broadening profile.

We have evaluated CCD arrays, high resistance anode proportional counters and linear silicon diode arrays in the context of our application.

We find that the CCD array suffers from poor signal to noise ratios below a few keV, cannot be readily adapted to single photon counting, and is of limited physical size. These issues are elaborated in Section 7.3.

The high resistance anode proportional counter (HRAPC) would be useful for energies up to 20 keV if it were not for problems associated with variations in absorption length which because of the extremely oblique angle of entrance translate almost directly into position uncertainties. We discuss these issues in Section 7.2.

Because the HRAPC requires comparatively simple processing electronics which produce a directly usable output, it constitutes a useful test instrument. Additionally, it requires no cooling. We are obtaining a unit suitable for use at 2.5 keV under the present contract which will be used in the checkout and test of the spectrometer elements and in initial application at Alcator. Final test of the proportional counter system will be carried out in the next phase.

The linear diode array is clearly the only candidate among those considered to offer all of the required performance for the spectrometer application. It has these desirable characteristics:

1. wide photon energy response <1 keV to >25 keV
2. high pulse rate capability $>10^5$ /element
3. single photon detection capability
4. good energy resolution
5. optimum element size (0.1 to 0.2 mm wide).

The disadvantages of the linear diode array are:

1. requires cooling (at least at lower energies)
2. complex electronics
3. requires computer processing of output.

Consequently, we are proposing to specify, acquire, and test a 512 element linear diode array and a portion of the associated electronics for use as the principle photon detector in an upgraded spectrometer system.

7.1 Linear Diode Array System

The plan for implementation of the linear diode array system is shown in Fig. 3. An array of 512 separate diode elements and appropriate biasing is connected to 512 processing chains. The output of each chain is a discriminator pulse which drives an output decoder. This generates a nine-bit event address and event flag which can be processed by computer, fast microprocessor, or special purpose logic. Due to budgetary limitations on our renewal proposal, we plan to implement 16 of the 512 processing chains without the output decoder and digital processor.

7.1.1 Diode Array

Work is currently underway to determine the detailed specification of the diode elements. The objective is to optimize geometric, doping and interconnection factors in such a way as to provide good high energy absorption while maintaining the low noise levels necessary for low energy detection. At present, noise models of the array have

been generated and are being evaluated as part of the total contemplated electronic system (see Appendix A).

7.1.2 Processing Chain

A single processing chain is shown in block diagram form in Fig. 4. The input preamplifier is a charge sensing preamp capable of handling a wide dynamic range of pulse amplitude at rates up to 10^5 /sec. It is implemented in hybrid form and is mounted in close proximity to the array. The preamp employs a low noise FET that may be mounted separately as part of the array substrate which will allow the FET to be cooled as well as the array.

Present activities are directed towards determining the detailed specifications of the pulse shaping amplifier and in determining the suitability of commercially available preamplifiers that could be used in hybrid form. We have identified at least one unit which has four preamps in a standard "flatpack". Emphasis on the preamp relates to packaging considerations. We hope to use existing hybrid designs or at worst direct extensions of them. There are sufficient units involved to permit economical repackaging of designs.

The post amplifiers may be rack mounted discrete/IC units or somewhat miniaturized assemblies mounted on the spectrometer detector mechanism. The exact configuration will be determined in the next term.

The output decoder will have an "acceptance aperture" of 1 μ sec and a processing time of 3 μ sec. This is more than adequate for the rates presently anticipated.

7.1.3 Test of Diffused Silicon Junction

As an indication of the potential of thin diffused silicon junction as X-ray detectors, we have obtained the results of a test conducted by Pehl at Lawrence Berkeley Laboratory. The spectrum obtained from an Fe^{55} K-capture source is shown in Fig. 5 together with the relevant test parameters. These results compare favorably with those obtained

from the best Si(Li) detectors, the FWHM being only a factor of two larger. The important point is that our analysis indicates that this device will operate uncooled with a FWHM considerably smaller than could be obtained from a conventional linear position sensitive proportional counter to be used to detect 6 keV X-rays. Thus, a one-dimensional array of diffused silicon diode junctions appears to be the only way to proceed in adapting our instrument to X-ray energies greater than about a few keV. We plan to purchase an array of 512 diodes, but to instrument only 16 elements with discrete element measurement chains as a feasibility and test program. Additional funding will be required to design and fabricate the 512 hybridized data channels and the computer interface which will be required eventually.

7.2 High Resistance Anode Proportional Counter System

There are two conflicting requirements that limit the use of the high resistance anode proportional counter in our spectrometer application. First, the aperture windows must transmit the lowest energy X-rays of interest with reasonable efficiency. We selected .001" beryllium as the minimum useful thickness which at the angle of 56° from the normal will pass 80% of 2.5 keV photons. The second requirement is that the absorption path have a projected length on the anode of less than 0.5 mm.

If the gas is 1 ATM xenon, this condition is satisfied.

It turns out that a .001" Be window $3/8'' \times 3/8''$ can support 1 ATM (in vacuum there is 1 ATM differential pressure across the window) so that counter having a row of $3/8''$ square windows is feasible. The "blind spots" due to the supporting structure will mask part of the spectrum. However, the counter can be translated and the results combined.

The problem comes when the higher energy response is considered. At 7 keV, 10 ATM of xenon is required to provide a sufficiently short absorption length. However, a window .010" of beryllium is now required to withstand the pressure. Such a window

will not pass the 2.5 keV X-rays to any appreciable extent. Consequently, it is clear that different counters are required for different energy ranges. Pressures above 10 ATM are not feasible because of reduced gas gain.

It should be noted that this situation exists only because of the oblique entrance angle which is dictated by the focusing properties of the spectrometer. The 1 ATM counter could, in fact, be used at higher energies if the photons entered perpendicular to the anode.

7.3 Reticon

We initially purchased and evaluated a Reticon Corporation 1024 element silicon diode array. This is a monolithic linear array of 1024 photodiodes each with associated charge storage capacitor for integrating photocurrent and a multiplex switch for periodic readout via an integrated shift register scanning circuit. Such devices have been used for X-ray detection in the range 1 - 10 keV.⁽⁷⁻¹⁰⁾ However they suffer several drawbacks for our application:

1. The available active length of only 2.54 cm means that only a small range of the spectrum, namely about 0.01 E where E is the central energy, can be recorded in one setting.
2. The available width of only 0.4 mm means that the large available entrance slit height (which could be as large as 2 cm without degrading resolution) cannot be used. Thus 1 and 2 conspire against exploiting two important advantages of the cylindral crystal spectrometer.
3. The resolution elements (diodes) are on 0.025 mm centers whereas the spatial resolution of the spectrometer is like 0.5 mm. Thus the device reads out more information per unit length than is necessary.
4. The dark leakage current of the diode is increased by exposure to radiation to about 10 pA. For an integration period of 0.1 s, leakage current would

then contribute a charge of 1 pCoul compared to the saturation charge of 3.2 pCoul. Thus it is necessary to cool the array to obtain sufficient dynamic range.

5. With a cooled array, the dynamic range is limited by fixed pattern noise due to switching transients. The part of this due to drifts in gains and offsets can be reduced by improved circuit design, and can be subtracted out by making dark measurements. The ultimate limitation is the thermodynamic noise on each individual element which is typically 0.1% of the saturation level. The amplitudes of these fluctuations correspond to 3.2×10^{-15} couls or 2×10^4 electrons. Since 3.66 eV of photon energy is needed to produce an electron in Si the minimum detectable signal is 30 photons of energy 2.5 keV (in an array element of 0.025 mm x 0.4 mm). This is no more than the sensitivity of photographic film at the same energy.
6. The solid angle subtended by an individual resolution element associated with a single pixel of the detector is $\Omega = 0.025 \text{ cm} \times H_c / 2L^2 = 8.7 \times 10^{-7} \text{ s. r.}$ Also the slit height can be only 0.04 cm so the expected signal in 0.1 sec, from Eq. (19), is only $N = 2 \times 10^{-10} I_0$ photons which is undetectable unless I is more than 10 times greater than the lower bound estimated in Eq. (23).

8.0 INSTRUMENT DESCRIPTION

Bragg angle calculations for a number of crystals suitable for bending and for several lines of Tokamak interest (e.g., Si $K\alpha$, Mo L, Ar $K\alpha$, Xe L, Cr $K\alpha$, Fe $K\alpha$) showed that for each line a suitable crystal could be found with θ_B in the range 27° to 40° . Thus, in the interest of future versatility, we elected to design our spectrometer around a Bragg angle of 33° , but with sufficient flexibility to accommodate angles from 25° to 41° .

Use of a larger Bragg angle (above 50°) has the advantage of allowing a larger input aperture (hence more throughput), but the range of energies subtended by the crystal is reduced. At large θ_B , the X-rays are incident more normally on the detector which is an advantage with some detector types (see Section 7.2). Since access to these larger Bragg angles may also be needed, the crystal chamber (Fig. 6) has two entrance ports to accommodate central Bragg angles of 33° and 50° .

The crystal is cemented to a block with a cylindrical surface machined in it of appropriate radius of curvature. The block is mounted on a carriage which allows remote translation parallel to the crystal length (cylinder axis). The carriage also allows adjustment of the Bragg angle, crystal vertical position, and tilt. A similar remotely translatable mount is provided for the detector. The detector chamber itself is mounted so that it can rotate up to $\pm 16^\circ$ in the horizontal plane about the crystal vertical rotation axis. Vacuum coupling is by a convoluted bellows of 10" I.D., which diameter is needed to accommodate the limiting Bragg reflected rays from the crystal to the detector.

The spectrometer interfaces to the 10" conflat flanged viewing port of Alcator through a 4" or 6" aperture, pneumatically operated, gate valve. This valve is opened to the spectrometer for about 1 sec only during a shot. The entrance slit is placed close to the plasma torus and a linkage provided to remotely change the slit size. Since the slit is at least 12" from the plasma center, a small slit size combined with a vertical translating aperture in front of the crystal (not shown in Fig. 6) can be used for vertical spatial scans of the plasma.

For laboratory test purposes, an auxiliary adapter tube which supports the entrance slit and test X-ray source is provided for the spectrometer. Polishing and passivating of all internal vacuum vessel surfaces combined with 6" I.D. apertured turbo-pumping will allow fast non-bake cycling to the base pressure required by Alcator (5×10^{-8} Torr). The spectrometer is mounted on a stand convenient for laboratory testing, but an additional platform must be provided to coalign the spectrometer with the Alcator viewing axis which is 8' from the floor.

9.0 PLAN OF WORK

9.1 Current Phase

Before the end of the current term (July 31, 1977) we will carry to completion the following tasks:

1. Specify and let contract for an X-ray diffracting crystal
2. Design, specify, and let contract for high resistance anode proportional counter (HRAPC)
3. Design and/or specify signal processing electronics for HRAPC
4. Analyze diode array sensitivity with respect to detector noise and identify realistic performance in the context of specified electronic processing
5. Design vacuum housing, crystal support system, HRAPC transport system
6. Construct signal processing electronics for HRAPC

9.2 Renewal Phase

During the renewal term (Aug. 1, 1977 to July 31, 1978) we will have the following objectives:

1. Specification, purchase, and test of 512 element linear diode array and associated cooling system.
2. Specification, purchase, or fabrication, and test of 16 electronic signal chains.
3. Assembly and test of diode array system.
4. Completion of tests of high resistance anode proportional counter.
5. Completion of spectrometer mechanisms design.
6. Completion of vacuum chamber and system design and procurement of parts and materials.
7. Fabrication of spectrometer mechanisms and vacuum housing.
8. Assembly, test and calibration of spectrometer.
9. Interface of spectrometer with Alcator and further testing as an X-ray diagnostic tool.

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FIGURE CAPTIONS

- Fig. 1 Cylindrical crystal spectrometer geometry.
- Fig. 2 Cylindrical crystal spectrometer parameters. Upper figure illustrates crystal diffraction limited resolution; lower figure illustrates slit width limited resolution.
- Fig. 3 Program for development and testing of a 512 element linear diode array.
- Fig. 4 A single processing chain attached to an individual diode in the diode array.
- Fig. 5 X-ray spectrum of an Fe^{55} K-capture source obtained with a thin, diffused silicon junction.
- Fig. 6 Schematic of spectrometer design.

1. Crystal	17. Adaptor
2. Crystal Chamber	18. X-ray Source
3. Detector	19. Vacuum Gate Valve
4. Detector Chamber	20. Alcator Adaptor
5. Bellows	21. Mechanical Feed Through
6. Rotary Mechanical Feed Through	22. Slit Movement Linkage
7. Isolation Bellows	23. Slit Plate
8. Vacuum Pump Adaptor	24. Detector Movement
9. Turbo-pump	25. Crystal Movement
10. Vent and TC Gauge	26. Detector Chamber Carriage
11. Vacuum Valve	27. Electrical Feed Through
12. Vent and TC Gauge	28. Stand
13. Trap	29. Bellows Support
14. Mechanical Vacuum Pump	30. Alcator Source
15. X-ray Source Adaptor Tube	31. Alcator Interface
16. Slit Plate	

CYLINDRICAL CRYSTAL SPECTROMETER GEOMETRY

FIGURE 1

FIGURE 2

EROA-DIODE ARRAY

FIGURE 3

FIGURE 4

SOURCE: Fe^{55}

DETECTOR: SI DIFFUSED JUNCTION

THICKNESS: 100 μm

DIAMETER: 3 mm

TEMPERATURE: LIQUID NITROGEN

CAPACITANCE: 7pF

PULSE SHAPING: GAUSSIAN

PEAKING TIME: 17 μs

FIGURE 5

FIGURE 6

SIGNAL AND NOISE IN A PHOTODIODE-AMPLIFIER SYSTEM

1.0 Scope

In the following sections, methods are developed to predict signal and noise performance of a photodiode and preamplifier/shaping amplifier system. The models are developed in a general sense in order to help define which diode parameters can be adjusted for best performance. As a starting point, a charge sensitive preamplifier followed by pseudo-gaussian pulse shaping is used in the development of the model. Noise sources in both the photodiode and preamplifier are considered, and methods for calculating both signal and noise in the context of this processing is presented.

2.0 Method

Signal and noise performance predictions are developed separately in this analysis. For signal, a unit impulse of charge from the photodiode, resulting from input radiation, is processed through the diode, preamplifier, and shaping amplifier networks to determine the normalized peak of the output signal, which, after considering the input flux, can be compared with the peak value of noise calculated. Two methods of determining the peak are considered; one which calculates the actual time response of the network, and one which approximates the peak of the response through the use of curves. The development of the noise response is performed by considering the various noise sources as stochastic signals, using their power density spectra through the system weighting function to determine the mean square value of the output response.

3.0 Signal

3.1 Signal Processing Elements

The signal processing elements are as shown in Figure 3-1, consisting of a photodiode, a charge sensitive preamplifier, and a shaping post amplifier. This system is evaluated here since this combination has been shown by others to be reasonably effective.

The shaping characteristics to be considered are of pseudo-gaussian form, consisting of one differentiation and two integrations. Depending on signal to noise performance and counting rate limitations, the number of differentiations and integrations required in a final system can be analyzed as part of an optimization procedure in a similar manner to what will be developed here.

3.2 Signal Model

3.2.1 Photodiode

The signal model for the photodiode¹ is shown in Fig. 3-2.

R_j is usually in the order of 10^8 ohms and is highly temperature dependent; C_j , which is bias voltage dependent, is in the order of 0.5 pF to 10.0 pF, and R_s is in the order of 500 ohms. When operated with a charge sensing preamplifier with low input impedance, R_j may be neglected.

3.2.2 Preamplifier

The preamplifier signal model is shown in Figure 3-3. For purposes of these calculations, the following assumptions are made:

- 1) the amplifier element bandwidth may be considered infinite.
- 2) the amplifier element input impedance may be considered infinite.

Since all of the input current flows through the feedback network by virtue of the amplifier element's infinite input impedance, this circuit may be unilateralized as shown in Figure 3-4, and may be combined with the photodiode model as shown in Figure 3-5, assuming $A \gg 1$.

3.2.3 Shaping Amplifier

The shaping amplifier analyzed is a pseudo-gaussian type, which has one differentiation and two integrations.^{2,3} In practice, all time constants are usually the same, but to allow for possible future flexibility, the general case of three different time constants will be analyzed. This model is represented in Figure 3-6.

3.2.4 Preamp-Diode Impulse Response

For $R_s = 0$, the model of Figure 3-5 results in

$$\frac{E_o(s)}{I_{in}(s)} = \frac{-A}{(C_c + C_j + AC_f) \left(s + \frac{A}{R_f(C_c + C_j + AC_f)} \right)}$$

FIGURE 3-1

- R_j = junction resistance of the active area
- C_j = junction capacitance of the active area
- R_s = resistance of the bulk silicon material summed with contact resistance
- C_c = diode output terminal and output cable capacitance

FIGURE 3-2

FIGURE 3-3

FIGURE 3-4

FIGURE 3-5

FIGURE 3-6

3.2.4 contd.

Note: For purposes of calculation, R_g will be considered zero. The complete expression, with non-zero R_g is

$$\frac{E_o(s)}{I_{in}(s)} = \frac{-A}{R_5 C_1 (C_C + A C_F) \left(s^2 + \frac{R_F (C_C + A C_F) + A R_5 C_1 + R_F C_1}{R_5 R_F C_1 (C_C + A C_F)} s + \frac{A}{R_5 C_1 (C_C + A C_F)} \right)}$$

3.2.5 Post Amplifier Impulse Response

Referring to Figure 3-6, the impulse response for the shaping amplifier is

$$\frac{E_o(s)}{E_{in}(s)} = \frac{s}{R_2 R_3 C_2 C_3 \left(s + \frac{1}{R_1 C_1} \right) \left(s + \frac{1}{R_2 C_2} \right) \left(s + \frac{1}{R_3 C_3} \right)}$$

3.2.6 System Impulse Response

Combining the above, the system transfer function is

$$\frac{E_o(s)}{I_{in}(s)} = \frac{-A}{(C_C + C_1 + C_F) R_2 R_3 C_2 C_3} \left(\frac{s}{\left(s + \frac{A}{R_F (C_C + C_1 + A C_F)} \right) \left(s + \frac{1}{R_1 C_1} \right) \left(s + \frac{1}{R_2 C_2} \right) \left(s + \frac{1}{R_3 C_3} \right)} \right)$$

3.3 Output Amplitude

3.3.1 Time Response Calculation

The system impulse response of paragraph 3.2.6 can be stated as

$$\frac{E_o(s)}{I_{in}(s)} = K \frac{s}{(s+a)(s+b)(s+c)(s+d)}$$

Assuming a , b , c , and d are different values. The time response associated with this function is of the form

$$\frac{e_o(t)}{i_{in}(t)} = K (a_1 e^{-at} + a_2 e^{-bt} + a_3 e^{-ct} + a_4 e^{-dt})$$

Where the coefficients a_n are determined by partial fraction expansion. The coefficients are

3.3.1 contd.

$$a_1 = \frac{-a}{(b-a)(c-a)(d-a)}$$

$$a_2 = \frac{-b}{(a-b)(c-b)(d-b)}$$

$$a_3 = \frac{-c}{(a-c)(b-c)(d-c)}$$

$$a_4 = \frac{-d}{(a-d)(b-d)(c-d)}$$

With this information, the time response of the network may be calculated and the waveform inspected for pulse shape and amplitude

3.3.2 Approximation of Output

A relatively fast method for approximating the output amplitude and pulse shape exists, provided that the preamplifier fall time constant is much larger than that of the shaping networks, and that all shaping networks have about the same time constants,

$$\text{or } \frac{R_f (C_f + C_1 + A C_f)}{A} \gg R_1 C_1$$

AND

$$R_1 C_1 \approx R_2 C_2 \approx R_3 C_3$$

With these conditions, the curves of Figure 3-7² may be consulted, and for $n = 2$ integrators, $f(t)_{\text{peak}} \sim 0.27$ times the peak output of the preamplifier or $E_o (\text{peak}) = \frac{0.27 Q_{in}}{C_f}$

Where Q_{in} is the charge input and C_f is the value of the preamplifier feedback capacitor.

FIGURE 3-7

4.0 Noise

4.1 Noise Sources

4.1.1 Photodiode Noise Sources

The photodiode noise model¹ is shown in Figure 4-1, and contains three noise sources; i_{s1} , which represents shot noise due to dark current; i_{s2} , which represents the diodes excess (1/f) noise; and e_s , which represents the thermal noise due to the bulk silicon material in the diode. These sources are described as follows:

$$i_{s1} = \sqrt{2qI_0 \Delta f}$$

$$i_{s2} = i_{s1}, \text{ INCREASING AS } 1/f \text{ BELOW } \sim 1 \text{ KHz}$$

$$e_s = \sqrt{4kTR_s \Delta f}$$

WHERE

I_0 = DIODE DARK CURRENT

Δf = NOISE BANDWIDTH

R_j has no thermal noise source associated with it, since it is an energy barrier rather than an actual resistor. Since its value is $\sim 10^8 \Omega$, it is omitted from analysis.

4.1.2 Preamplifier Noise Sources

The preamplifier noise sources are shown in Figure 4-2 for a typical FET input device. This may be represented as an equivalent input noise current and noise voltage source. Typical plots of e_n and i_n for a low noise FET are shown in Figure 4-3. In the model, the four noise generators are described as follows:

e_{n1} - THERMAL NOISE GIVEN AS

$$e_{n1} = \sqrt{4kTR_n \Delta f}$$

WHERE $R_n \approx \frac{2}{3g_m}$ (g_m IS TRANSCONDUCTANCE OF FET)

e_{n2} - EXCESS (1/f) NOISE

i_{n1} - SHOT NOISE DUE TO GATE LEAKAGE CURRENT

GIVEN AS $i_{n1} = \sqrt{2qI_L \Delta f}$ ($I_L = I_{GSS}$ OF FET)

i_{n2} - THERMAL NOISE DUE TO REAL PART OF INPUT ADMITTANCE

FIGURE 4-1

FIGURE 4-2

FIGURE 4-3

$$i_s = \sqrt{2qI_0 \Delta f}$$

$$e_s = \sqrt{4kTR_s \Delta f}$$

$$i_n = \sqrt{2qI_L \Delta f}$$

$$e_n = \sqrt{4kTR_n \Delta f}$$

OR

Where all noise sources are white noise generators.

FIGURE 4-4

4.1.2 contd.

For the best noise performance, the noise bandwidth of the total system should be adjusted to minimize the contributions of e_{n2} and i_{n2} . For the typical curves of Figure 4-3, bandwidth should exclude frequencies below about 1 KHz, making $e_n \simeq e_{n1}$ and above about 100 KHz, making $i_n \simeq i_{n1}$. These values represent shaping time constants between 1.6 μ s and 160 μ s, which are typical for pulse amplifiers. Since the photodiode 1/f noise is similar, the remaining analysis will consider all 1/f noise sources out of the region of interest.

4.1.3 Noise Model

Based on the considerations of 4.1.1 and 4.1.2, the noise model to be analyzed becomes as shown in Figure 4-4.

4.2 Output Noise Calculation

4.2.1 Method

A weighting function, $W(s)$ will be developed for each noise source in the system, to which the power density spectrum for the source will be applied to arrive at the mean square value of the output noise. The total output noise will then be described by

$$e_{no} = \sqrt{e_{o_{i_s}}^2 + e_{o_{i_n}}^2 + e_{o_{e_s}}^2 + e_{o_{e_n}}^2}$$

4.2.2 Power Density Spectra of Input Noise Sources

The power density spectrum of a white noise generator is given by ⁴

$$\frac{1}{2} \text{ WATTS/HZ}$$

4.2.2 contd.

For the noise generators considered here, the power density spectra are given as

<u>SOURCE</u>	<u>POWER DENSITY SPECTRUM</u>
i_s	$q I_0$
i_n	$q I_{GSS}$
e_s	$2 k T R_s$
e_n	$\frac{4 k T}{3 g_m}$

4.2.3 Weighting Functions

Referring to the model of 4.1.3, the weighting functions for each source may be determined as follows:

FOR i_s, i_n

$$W(s) = \left(\frac{A}{(A C_f + C_c + C_j) R_2 R_3 C_2 C_3} \right) \left(\frac{S}{\left(s + \frac{A}{R_f (A C_f + C_1)} \right) \left(s + \frac{1}{R_1 C_1} \right) \left(s + \frac{1}{R_2 C_2} \right) \left(s + \frac{1}{R_3 C_3} \right)} \right)$$

FOR e_s, e_n

$$W(s) = \left(\frac{C_j + C_c + C_f}{(A C_f + C_c + C_j) (R_2 R_3 C_2 C_3)} \right) \left(\frac{S \left(s + \frac{1}{R_f (C_j + C_c + C_f)} \right)}{\left(s + \frac{A}{R_f (C_j + C_c + A C_f)} \right) \left(s + \frac{1}{R_1 C_1} \right) \left(s + \frac{1}{R_2 C_2} \right) \left(s + \frac{1}{R_3 C_3} \right)} \right)$$

4.2.4 Mean Square Value of Output

The power density spectrum of the output is defined⁵ as

$$\Phi_{gg}(j\omega) = \Phi_{vv}(j\omega) |W(j\omega)|^2$$

WHERE $\Phi_{vv}(j\omega)$ IS THE INPUT POWER DENSITY SPECTRUM

The mean square value of the output is given by

$$\Psi_{gg}(0) = \int_{-\infty}^{\infty} \Phi_{gg}(j\omega) d\omega$$

4.2.4 contd.

Since the white noise power density spectra being evaluated here are constants, they can be taken outside the integral sign, giving

$$\varphi_{gg}(0) = \Phi_{vv} \int_{-\infty}^{\infty} |W(j\omega)|^2 d\omega$$

Which may be evaluated using the Cauchy Residue Theorem.

4.2.5 Evaluation of Integrals

$\int_{-\infty}^{\infty} |W(j\omega)|^2$ FOR THE NOISE CURRENT SOURCES
MAY BE WRITTEN AS

$$\int_1 = \int_{-\infty}^{\infty} |W(j\omega)|^2 = K_1 \int_{-\infty}^{\infty} \frac{\omega^2}{(\omega^2 + a^2)(\omega^2 + d^2)(\omega^2 + e^2)(\omega^2 + f^2)} d\omega$$

WHERE

$$K_1 = \frac{A}{(AC_f + C_1) R_2 R_3 C_2 C_3}$$

$$a = \frac{A}{R_4 (AC_f + C_1)}$$

$$d = \frac{1}{R_1 C_1}$$

$$e = \frac{1}{R_2 C_2}$$

$$f = \frac{1}{R_3 C_3}$$

WHICH WILL BE EVALUATED BY THE CAUCHY RESIDUE THEOREM

$$\int_c = 2\pi j \sum \text{RESIDUES}$$

4.2.5 contd.

This function has upper half plan poles at ja , jd , je , and jf . The residues at these poles are evaluated as follows:

$$R_{ja} = \lim_{j \rightarrow a} \frac{(w - ja) \omega^2}{(w - ja)(w + ja)(\omega^2 + d^2)(\omega^2 + e^2)(\omega^2 + f^2)}$$

$$= \frac{-a}{2j(d^2 - a^2)(e^2 - a^2)(f^2 - a^2)}$$

Similarly,

$$R_{jd} = \frac{-d}{2j(a^2 - d^2)(e^2 - d^2)(f^2 - d^2)}$$

$$R_{je} = \frac{-e}{2j(a^2 - e^2)(d^2 - e^2)(f^2 - e^2)}$$

$$R_{jf} = \frac{-f}{2j(a^2 - f^2)(d^2 - f^2)(e^2 - f^2)}$$

And the total integral becomes

$$\int_{-\infty}^{\infty} |W(j\omega)|^2 = 2\pi j (R_{ja} + R_{jd} + R_{je} + R_{jf})$$

The integral for the voltage sources may be found in a similar manner. For these, the integral is written as

$$\int_{-\infty}^{\infty} |W(j\omega)|^2 = K_2 \int_{-\infty}^{\infty} \frac{\omega^2(\omega^2 + g^2)}{(\omega^2 + h^2)(\omega^2 + d^2)(\omega^2 + e^2)(\omega^2 + f^2)}$$

Where

$$K_2 = \frac{C_J + C_C + C_F}{R_L R_3 C_2 C_3 (C_J + C_C + A C_F)}$$

$$g = \frac{1}{R_F (C_J + C_C + C_F)}$$

$$h = \frac{A}{R_F (C_J + C_C + C_F)}$$

4.2.5 contd.

$$d = \frac{1}{R_1 C_1}$$

$$e = \frac{1}{R_2 C_2}$$

$$f = \frac{1}{R_3 C_3}$$

Which will be evaluated the same way.

This function has upper half plane poles at jh , jd , je and jf . These residues are:

$$R_{gh} = \lim_{\omega \rightarrow jh} \frac{(\omega - jh) \omega^2 (\omega^2 + g^2)}{(\omega - jh)(\omega + jh)(\omega^2 + d^2)(\omega^2 + e^2)(\omega^2 + f^2)}$$

$$= \frac{-h(g^2 - h^2)}{2j(d^2 - h^2)(e^2 - h^2)(f^2 - h^2)}$$

$$R_{gd} = \frac{-d(g^2 - d^2)}{2j(h^2 - d^2)(e^2 - d^2)(f^2 - d^2)}$$

$$R_{ge} = \frac{-e(g^2 - e^2)}{2j(h^2 - e^2)(d^2 - e^2)(f^2 - e^2)}$$

$$R_{gf} = \frac{-f(g^2 - f^2)}{2j(h^2 - f^2)(d^2 - f^2)(e^2 - f^2)}$$

4.2.5 contd.

And as before,

$$\int_2 = \int_{-\infty}^{\infty} |W(j\omega)|^2 = 2\pi j (R_{jd} + R_{jd} + R_{je} + R_{jf})$$

Note: Residues at jd , je , jf are different from those developed for the current sources.

4.2.6 RMS Output Noise

For this system, the RMS value of output noise is

$$e_{o\text{rms}} = \sqrt{K_1^2(P_1 + P_2) \int_1 + K_2^2(P_3 + P_4) \int_2}$$

Where P_n are the power density spectra of inputs,

$$P_1 = q I_o$$

$$P_2 = q I_{gss}$$

$$P_3 = 2kTR_s$$

$$P_4 = \frac{4kT}{3g_m}$$

5.0 Sample Problem

5.1 Statement of Problem

Evaluate the signal to noise performance of the following system, operating at 200° K

Diode : 3.5 eV / ion pair

C_j : 5 pf I_o : 10^{-8} A at 300° K

R_s : 500 Ω Input : 7 kev

C_c : 5 pf T : 150° K

Preamp :

A : 10^4 I_{gss} : 10^{-4} at 300° K

R_f : 100 Meg. g_m : .002 mho

C_f : 1 pf T : 250° K

5.1 contd.

Shaping Amp :

$$R_1 C_1 : 8\mu s$$

$$R_2 C_2 : 10\mu s$$

$$R_3 C_3 : 12\mu s$$

5.2 Analysis

5.2.1 Peak Signal Output

From Section 3.3.2, the approximate peak output is

$$E_{o(PEAK)} = \frac{(0.27)(1.6 \times 10^{-12})(7 \times 10^3)}{(1 \times 10^{-12})(3.5)} = 8.6 \times 10^{-5}$$

5.2.2 Noise Output

$$\text{FOR } \int_1, K_1 = 8.3 \times 10^{21}$$

$$R_{ja} = -4.8 \times 10^{-27}$$

$$R_{jb} = 8.3 \times 10^{-26}$$

$$R_{jc} = -2.4 \times 10^{-25}$$

$$R_{jd} = 2.26 \times 10^{-25}$$

$$\int_1 = -1.34 \times 10^{-26}$$

$$\text{FOR } \int_2, K_2 = 9.2 \times 10^6$$

$$R_{jh} = -6 \times 10^{-22}$$

$$R_{jd} = +2.3 \times 10^{-19}$$

$$R_{je} = -3.3 \times 10^{-19}$$

$$R_{jf} = 1.2 \times 10^{-19}$$

$$\int_2 = 2 \times 10^{-20}$$

5.2.2 contd.

The leakage currents of both the diode and preamp input FET double every 10°C . At the operating temperatures given,

$$I_o = 3 \times 10^{-13} \text{ A} \quad \text{AND} \quad I_{\text{GSS}} = 3 \times 10^{-13} \text{ A}$$

$$\therefore P_1 = 4.8 \times 10^{-32}$$

$$P_2 = 4.8 \times 10^{-32}$$

$$P_3 = 2.0 \times 10^{-18}$$

$$P_4 = 2.2 \times 10^{-18}$$

$$K_1^2 (P_1 + P_2) \int_1 = 3.6 \times 10^{-12}$$

$$K_2^2 (P_3 + P_4) \int_2 = 6.9 \times 10^{-24}$$

$$e_{0 \text{ RMS}} = 1.9 \times 10^{-6}$$

$$e_{0(\text{O-PK})} = 5.7 \times 10^{-6}$$

$$\frac{\text{SIG (PK)}}{\text{NOISE (PK)}} = 15.11$$

Since 7 kev X-rays were used as input for this example, the limit of detectability is 463 ev.

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APPENDIX B

1.0 High Resistance Anode Proportional Counter System

1.1 Summary

We have considered the high resistance anode proportional counter (HRAPC) as a position sensitive X-ray detector suitable for use in the spectrometer. We find that the use of the HRAPC is entirely feasible for photons having an average energy of 2.5 kev. The approach offers the advantages of

- a. well developed counter technology
- b. comparatively good availability
- c. simple output processing

The disadvantages of the approach are

- a. some translation of the detector is required to cover the entire "image line"
- b. because of geometric constraints set by the spectrometer, the use of the HRAPC is probably not feasible at higher photon energies than about 4 kev.

Since much of the important work and initial testing relates to the energy region around 2.5 kev and the HRAPC represents the only feasible detector available within present budget constraints we are in the process of instrumenting such a system.

1.2 System Concept

The HRAPC has been implemented in a number of configurations with a variety of electronic signal processing techniques. We have generally utilized the concepts of Ref. 1.

The proportional counter is similar to any other counter except that instead of a low resistance anode (typically fine tungsten wire), its anode has a high resistivity. The state of the art is a pyrolytic-carbon-coated quartz fiber having a diameter of .001" and a resistivity of about 8.5 kilohms/mm. (The exact resistivity is not critical, but it should be uniform over the anode length. In practice, uniform resistivity is obtained.)

1.2 contd.

The counter can be considered to be a distributed RC delay line whose "lumped circuit" equivalent is shown in Fig. 1-1. It can be shown that if the charge pulses arriving at each end are shaped by identical processing to produce bipolar pulses, the relative difference in output voltage zero crossing times is directly related to the position on the anode nearest which the secondary gain producing avalanche occurred. Borkowski and Kopp state that for a high input impedance voltage preamplifier followed by bi-polar pulse shaping, the zero crossing time differential is given by:

$$|\Delta t_x| \approx Sx$$

where x is the distance along the anode measured from one end and

$$S = \rho (C_L L + 2C_{IN})$$

where ρ = resistivity in ohms/mm
 L = total anode length in mm
 C_L = capacitance per unit length
 C_{IN} = input capacitance

It turns out that a wide variety of filters can be used to generate the bipolar signals whose relative zero crossing times are analyzed.

Our concept entails the measurement of $|\Delta t_x|$ in terms of an output voltage pulse whose amplitude is proportional to $|\Delta t_x|$ and therefore to event position x . The basic processing concept is shown in Fig. 1-2. Here we consider only events whose "energy" lie within specified limits. This permits operation using low intensity inputs in the presence of background and is useful for test. It also permits rough energy evaluation independent of spectral analysis. The delay in the "stop" chain is arbitrary and simply ensures that a "start" zero crossing always occurs in time before a "stop" zero crossing as displayed to the time to height converter.

1.2.1 Zero Crossing Sensitivity

Our counter is approximately 200 mm long. The resistivity is 8.5×10^3 ohms/mm. The capacitance from anode to body (C_L) is calculated by treating the counter as a cylinder of diameter 25 mm. The capacitance is ²

$$C_L L = \frac{55.6 \times 10^{-3} \times 200 \text{ pf}}{\ln \frac{25}{.025}} \quad E_1 (1)$$

The input capacitance is comprised of the anode insulator capacitance and any amplifier input capacitance. It is apparent from Eq 1 that the addition of input capacitance increases the preamplifier noise referred to the input. An optimization is necessary here. However, a realistic value is 3.0 pf. By Eq 1 this gives

$$|\Delta t_x| = 8.5 \times 10^3 (1.6 + 2 \times 3) 10^{-12} \approx 65 \text{ nsec/mm} \quad E_2 (2)$$

We require a resolution better than 0.5 mm. We set the desired resolution at 0.2 mm and obtain

$$|\Delta t_x|_{\text{MIN}} \approx 65 \times .2 = 13 \text{ nsec}$$

The maximum value of $|\Delta t_x|$ is $65 \times 200 \times 2 = 26,000$ nsec. The factor 2 is due to the artificial delay in the "stop" chain which is equal to the $|\Delta t_x|$ for events occurring at one end i. e. $|\Delta t_x| = 65 \times 200$ nsec. Since the minimum resolution time is 13 nanosecond, the electronics must have a range of $26,000/13$ or $2000/1$. We take output noise at the time to height converter at .005 volts. A full scale signal of 10 volts is seen to be adequate.

1.2.2 Input Preamplifier

An input voltage sensitive preamplifier has been developed. The circuit is shown in Fig. 1-3. The primary component of input capacitance is gate to drain capacitance C_{GD} which is increased by the Miller effect. Because of the compara-

1.2.2 contd.

tively low input impedance of the base of Q_2 due to the bypass capacitor C_3 and the base to collector Miller capacitance of Q_2 , there is little voltage gain at the drain of Q_1 . The typical value of C_{GD} is about 3.0 pf which is, to a close approximation, equal to the input capacitance.

It should be noted that the gate to source capacitance C_{CS} which is about 7 pf in a common source configuration is effectively neutralized by the feedback.

1.2.3 Position Determination Circuits (PDS)

The Position Determination Circuit can be implemented in a number of ways and a number of proven circuits exist. For initial evaluation and test, laboratory equipment will be used. The principal elements were shown in Fig. 1-2. A linear bipolar amplifier such as the ORTEC 450 will be used.

We are also evaluating the use of delay line clipping which while offering better (faster) zero crossing characteristics is inherently more noisy. Since we are considering what are effectively monochromatic inputs and can operate the proportional counter at comparatively high charge gains, there may be no significant noise problem.

The consideration of interest is the "zero crossing" characteristics of the bipolar pulse. We are concerned with two aspects as shown in Fig. 1-4. There is, in general, a point which is insensitive to the amplitude of the bipolar pulse. Due to non-ideal conditions this point may have some offset V_{TH} which can be positive or negative. This offset point is the effective zero crossing point which defines an event in time. The slope is of concern in that it generates a timing uncertainty.

1.2.3 contd.

The zero crossing time or event is detected by an amplitude discriminator with hysteresis as shown in Fig. 1-5. In the normal state the output is high having a value of V_{HI} which establishes a reference level V_T at the input. When the input exceeds V_T , the discriminator output swings negative to its low value V_{LO} . R_1 , R_2 and the reference voltage are adjusted to drive V_{REF} to the zero crossing point V_{TH} . As the input starts negative it passes through V_{TH} causing the output to change states back to its high value. This negative to positive transition defines the time of the zero crossing. The effect of slope can be seen by considering an example. Let the minimum slope be 2 volts/ μ sec and the threshold or transition uncertainty of the discriminator be 2 mv. The equivalent time uncertainty is

$$\Delta t = \frac{2 \times 10^{-3} \text{ volt} - \text{sec}}{2 \times 10^6 \text{ volt}} = 10^{-9} \text{ sec} \quad \epsilon_3$$

This is well within the objective of 13 nsec.

1.3 Proportional Counter Considerations

There are three principal constraints on the proportional counter that determine the design parameters. First, the aperture windows must be sufficiently thin and of sufficiently low Z material to admit the lowest energy photons of interest.

The window must be vacuum tight since the counter will be operated in vacuum connected to the TOKAMAK. The "tightness" is to preclude contamination. Because the counter is operated in vacuum the window must support a pressure differential equal to its internal gas pressure.

Finally, we require the variations in absorption depth in the counter gas when projected on the anode to be less than the required resolution. This dictates gas composition and pressure.

1.3 contd.

Summarized we have:

1. windows must be thin to transmit X-rays
2. gas must be dense to absorb X-rays in a short distance, i.e. high pressure and/or high density, but
3. window must be capable of supporting the internal counter gas pressure.

The situation is aggravated by the geometry of the application. As shown in Fig. 1-6, the angle of incidence with respect to the window has an average value of 34° . This increases both the effective window thickness and the effect of variations in absorption depth.

1.3.1 Window Thickness

The state of the art is such that small beryllium windows of thickness .0025 cm are feasible. The transmission through the window is given by

$$\frac{I}{I_0} = e^{-\mu x}$$

where I = exit intensity

I_0 = entrance intensity

μ = absorption constant

x = window thickness in cm.

The absorption constant " μ " is a function of the window material and the energy of the incident X-ray. For an energy of 2.4 kev and a window of beryllium, $\mu = 40 \text{ cm}^{-1}$. The effective window thickness is

$$x = \frac{.0025}{\sin 34^\circ} = .0045 \text{ cm.}$$

The resultant transmission ratio I/I_0 is $e^{-40 \times .0045} = .84$. About 84% of the incidence X-rays will be transmitted through the .0025 cm Be window.

1.3.2 Gas Considerations

Because we wish to minimize the internal gas pressure we have selected Xenon 90% CO₂ 10% as the counter gas mixture. This will give minimum absorption length or depth for a given pressure. We define the max. absorption length as that value of length x for which $e^{-\mu x} = e^{-3} = .05$. Here μ is the value for Xenon and varies with gas density and photon energy. For 1 ATM Xenon at standard conditions, and 2.4 kev, $\mu = 45.4 \text{ cm}^{-1}$. For the condition $x = 3$ we obtain

$$x = \frac{3}{\mu} = \frac{3}{45.4} = .066 \text{ cm} = 0.66 \text{ mm}$$

The projected length on the anode is

$$X_p = 0.66 \cos 34 = 0.55 \text{ mm}$$

This is approximately the desired resolution. We are saying that 95% of the incident X-rays from a single input beam of infinitesimal cross section will produce a "spread" along the anode of 0.55 mm. When the output distribution is analyzed, the resolution will be better. For example, 85% of the X-rays will project to a length of .37 mm.

1.3.3 Window Size

Using a requirement that the windows support 1 ATM outward differential pressure, stress analysis shows that a conservative sizing is .9 x .9 cm for individual windows supported by an appropriate rib structure. To minimize shadowing, the support apertures are cut at the 34° incident angle. In operation, the counter must be translated to view the entire image. Since the spectrometer requires a detector translation capability anyway there is no added complexity to the mechanism.

1.3.4 High Energy Considerations

We have considered the use of the proportional counter at energies around 7 kev. The conclusion is that because of the pressure required to control absorption depth variations it may be difficult to obtain satisfactory performance. A suitable counter has a .025 cm Beryllium window and 10 ATM of Xenon. Our concern is whether or not adequate gas gain can be obtained at such a pressure. It should be noted that this situation is due to the oblique angle of incidence which is a consequence of our application. A 10 ATM unit should be obtained and evaluated.

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PROPORTIONAL COUNTER

FIGURE 1-1a

$$R = \rho \Delta l = \frac{R_t}{L}$$

$$C = C \Delta l = \frac{C_t}{L}$$

R_t = ANODE RESISTANCE
 C_t = ANODE TO HOUSING CAPACITANCE
 R_{IN} = TERMINATING RESISTANCE
 C_{IN} = TERMINATING CAPACITANCE

LUMPED CIRCUIT EQUIVALENT

FIGURE 1-1b

HRAPC PROCESSING ELECTRONICS

FIGURE 1-2

FIGURE 1-3 Voltage Preamplifier

ZERO CROSSING CONSIDERATIONS

FIGURE 1-4

ZERO CROSSING DISCRIMINATOR

FIGURE 1-5

APPENDIX C

X-RAY SPECTRA FROM ALCATOR

J. E. Rice

H. I. Helava

This work was performed at MIT in collaboration with J. E. Rice and H. I. Helava.

The study of X-ray spectra in the energy region from 500 eV to 50 keV yields information concerning the electron distribution function (through the continuum) and the concentration of impurities (from line radiation) as a function of time. Since X-ray energy is of order T of the electrons encountered in tokamaks, they provide a particularly good temperature measurement.

The cross-section for production of a bremsstrahlung photon of energy $h\nu$ from an electron of energy ϵ_0 during a collision with a stationary proton, integrated over the angles of the outgoing electron and photon, is well approximated in the region of interest by Greene [1959].

$$\frac{d\sigma_E}{dh\nu} = \frac{5.73}{h\nu} \textcircled{-22} \frac{n_o n_f}{\pi^2} \frac{1-e^{-2\pi n_o}}{1-e^{-2\pi n_f}} \ln \left\{ \frac{n_f + n_o}{n_f - n_o} \right\} \frac{\text{cm}^2}{h\nu} \quad (1)$$

where $n_o^2 = \frac{13.6 \text{ eV}}{\epsilon_o}$ and $n_f^2 = \frac{13.6 \text{ eV}}{\epsilon_o - h\nu}$. The energy spectrum of X-rays is found by convolving this cross section with the electron distribution function $f(\epsilon_o)$

$$I(h\nu) = n_e n_i \int_{h\nu}^{\infty} \sqrt{\frac{2\epsilon_o}{m_e}} h\nu \frac{d\sigma_E}{dh\nu} f(\epsilon_o) d\epsilon_o \frac{\text{energy}}{\text{unit energy-sec-cm}^3} \quad (2)$$

where n_i is the density of protons. If the electron distribution function is maxwellian, this integral is equal to

$$I(h\nu) = 2.6 \textcircled{-14} n_e n_i \sqrt{\frac{13.6 \text{ eV}}{T_e}} e^{-h\nu/T_e} \bar{g} \quad (3)$$

where $\bar{g}$ is an average gaunt factor roughly equal to $\sqrt{\frac{3 T_e}{h\nu}}$. What is actually observed with the detector is

$$F(h\nu) = \frac{\Delta N}{\Delta t \Delta V \Delta \Omega \Delta h\nu} = 2.4 \textcircled{-16} \frac{\bar{g} n_e^2}{\sqrt{T_e}} \frac{e^{-h\nu/T_e}}{h\nu} \quad (4)$$

number of photons
area-cm²-rad-sec-ml angle-unit energy

(T_e is in keV) and is related to $I(h\nu)$ by

$$I(h\nu) = h\nu \int F(h\nu) d\Omega. \quad (5)$$

The geometry of the system is shown in Fig. 1 and a description of the detector is given in the Appendix. Spatial resolution at the plasma is about 1/2 cm and the detector is able to scan from 1 cm above center in the vacuum vessel to 4 cm below. Since photons originate along a given chord, Eq. (4) must be modified to include profile effects,

$$\frac{h\nu}{\Delta t \Delta A \Delta \Omega} \frac{\Delta N}{\Delta h\nu} = 2.4 \quad (-16) \int_L dl \frac{n_e^2(l)}{\sqrt{T_e(l)}} \bar{g}(l) e^{-h\nu/T_e(l)} \frac{\text{photons}}{\text{cm}^2\text{-sec-ster}} \quad (6)$$

where L is the chord length in the plasma, $\Delta V = \Delta A \Delta l$, and $\Delta h\nu$ is the channel width in the MCA system. Parabolic density profiles are usually assumed,

$$n_e(l) = n_0 \left(1 - \frac{r^2}{a_l^2}\right) = n_0 \left(1 - \frac{d^2}{a_l^2} - \frac{l^2}{a_l^2}\right) \quad (7a)$$

where a_l is the limiter radius, and n_0 is the central density, as are gaussian temperature profiles

$$T_e(l) = T_0 e^{-r^2/a^2} = T_0 e^{-d^2/a^2} e^{-l^2/a^2} \quad (7b)$$

where T_0 is the central temperature, a is the 1/e width of the gaussian (both are parameters to be determined) and d is the distance of the chord from the center of the chamber. The width, a , can be determined by viewing along different chords and Eq. (6) can be solved for T_0 by comparison with the observed spectrum following an iterative process described in the Appendix. As an example, Eq. (6) is plotted in Fig. 2 for a width of 5 cm.

So, by measuring the bremsstrahlung continuum, it is possible to obtain the electron distribution function via Eq. (2) or, if the distribution is maxwellian, to obtain the electron temperature (and profile) through Eq. (6).

System Geometry

Figure 2

An example of a non-thermal X-ray spectrum is shown in Fig. 3, which was obtained during a low density ($5 \times 10^{12}/\text{cm}^3$) discharge. There are two distinct regions of continuum: between 8 and 15 keV the slope could arise from a distribution function approximated by a maxwellian with a temperature of ~ 30 keV, while the spectrum above 20 keV can be attributed to a flat electron tail. The presence of these high-energy electrons during low-density shots allows for the non-thermal excitation of the Mo K_α and K_β lines around 18 keV, which are absent at higher densities. In fact, at high densities ($> 10^{14}/\text{cm}^3$), the X-ray spectra are predominantly thermal as shown in Fig. 4 and most of the radiation comes in below an energy a few times T_e . The temperature in this case is around 800 eV and the Mo L line at 2.6 keV is probably thermally excited.

It is interesting to note the time evolution of the electron temperature and the dependences on other plasma parameters, such as current and density, that change in time. Spectra have been collected on time scales of 15 msec at various positions in the plasma and the time evolution of the temperature profile has been determined. Representative data are shown in Fig. 5. The shot is steady state as regards plasma current, and the injection of gas at 30 msec causes the density to peak by 50 msec--which corresponds to the time that the temperature profile peaks on center. The temperature profile has a marked dependence on density. These profiles can be roughly described by gaussians with $a = 10$ cm during the low-density phase and $a = 5$ cm after the density has peaked. The continuum flux is entirely attributable to bremsstrahlung of electrons colliding with protons--recombination radiation is not important, at least at the center of the plasma, and one obtains a $Z_{\text{eff}} = \frac{\sum_i n_i Z_i^2}{\sum_i n_i Z_i}$ near unity.

Figure 3

55 KA/div

H_a

20ms/div

Figure 5

Even with a low Z_{eff} , a large number of X-rays arise from impurity line radiation because the cross sections for producing lines are very strong. The emission of an X-ray line by an atom must be preceded by the creation of a vacancy, either by electron impact ionization or excitation, and the subsequent (nearly spontaneous) filling of the vacancy by an outer shell electron. It is, therefore, necessary to know the appropriate ionization or excitation cross sections. The number of ionized impurity atoms per $\text{cm}^3\text{-sec}$ produced by a maxwellian electron distribution is given approximately by Griem [1967]

$$\left. \frac{dn}{dt} \right|_I = I_Z = n_e n_Z \langle \sigma v \rangle = n_e n_Z 2.5 \left(-6 \right) \frac{n}{\chi^2} \frac{\sqrt{T_e}}{(1 + \frac{T_e}{\chi})} e^{-\chi/T_e} \text{ sec}^{-1} \text{ cm}^{-3} \quad (8)$$

where T_e is in eV, χ is the ionization potential (in eV) for the shell in question and n is the number of equivalent electrons in the shell. A particular line of interest is Mo L near 2.5 keV; the ionization potential from the L shell is about 4 keV and n is 8 (neglecting any fine structure). This expression can be applied to nonthermal ionization as long as the "temperature" of the distribution function in the vicinity of the line is used. A similar formula for excitations is

$$\left. \frac{dn}{dt} \right|_E = n_e n_Z 1.6 \left(-5 \right) \frac{f_{nn^*} \langle g \rangle}{\Delta E \sqrt{T_e}} e^{-\Delta E/T_e} \text{ sec}^{-1} \text{ cm}^{-3} \quad (9)$$

where ΔE is the energy difference between the initial and final shells, f_{nn^*} is the absorption oscillator strength for allowed transitions (not well known for Molybdenum X-ray transitions) and $\langle g \rangle$ is an average gaunt factor equal to .2 in the region of interest. The ratio of Eqs. (8) and (9) for Mo L, assuming $f_{nn^*} = .01$ and $\Delta E = 2300$ eV, is ~ 7 . Thus, since the oscillator strengths are not well known, excitation will be ignored in the subsequent

analysis of the Mo line. For the temperatures encountered in Alcator (< 1 keV), Molybdenum near the center of the plasma is stripped somewhere into the M shell. That is, the charge states are probably between Mo^{32+} and Mo^{24+} , and excitation of L shell electrons is not a necessity for seeing the L line as it would be for a higher temperature plasma, that is, with $\text{Mo}^{1+} > \text{Mo}^{32+}$.

An average Molybdenum density along a given chord can be obtained by measuring the ratio of the L line to the continuum and comparing it to the ratio of Eq. (8) to $\Delta h\nu F(h\nu)$, c.f.

$$\frac{I_{\text{mo}}}{\Delta h\nu F(h\nu)} = \frac{2.5 \text{ } (-6) \int_L \frac{n}{x^2} \frac{\sqrt{\text{Te}(l)}}{(1 + \frac{\text{Te}(l)}{x})} e^{-x/\text{Te}(l)} n_e(l) n_{\text{mo}}(l) dl}{2.4 \text{ } (-16) \int_L \frac{\Delta h\nu}{h\nu} \frac{n_e^2(l)}{\sqrt{\text{Te}(l)}} \bar{g}(l) e^{-h\nu/\text{Te}(l)} dl} \quad (10)$$

or

$$\begin{aligned} \langle n_{\text{mo}} \rangle &= \frac{\int_L n_e(l) n_{\text{mo}}(l) \sqrt{\text{Te}(l)} e^{-x/\text{Te}(l)} dl}{\int_L n_e(l) \sqrt{\text{Te}(l)} e^{-x/\text{Te}(l)} dl} \\ &= \left[\frac{I_{\text{mo}}}{\Delta h\nu F(h\nu)} \right] \frac{\Delta h\nu}{h\nu} 2 \text{ } (-4) \frac{\int_L \frac{n_e^2(l)}{\sqrt{\text{Te}(l)}} \bar{g}(l) e^{-h\nu/\text{Te}(l)} dl}{\int_L \frac{n_e(l)}{\sqrt{\text{Te}(l)}} e^{-x/\text{Te}(l)} dl} \text{ cm}^{-3} \end{aligned} \quad (11)$$

for T_e in eV and n_e in cm^{-3} .

The time development of the density of Mo can be determined quite easily via Eq. (11) and information about the Mo profile can be obtained by measuring $\langle n_{\text{mo}} \rangle$ along different chords. Fig. 6 is a plot of the residual Mo line (with the continuum subtracted out) for the shot shown in Fig. 7. $\langle n_{\text{mo}} \rangle$ for each point is approximately $2 \text{ } (8) \text{ cm}^{-3}$ and does not change in time even though the electron density increases an order of magnitude. The contribution of Mo (assuming Mo^{30+}) to Z_{eff} , even at the beginning of the shot

Figure 7

is less than 1% and it may be concluded that heavy metals do not contribute significantly to Z_{eff} , at least at the center of the plasma. Two interesting points demonstrated in Fig. 6 are that as the temperature increases, the position of the line shifts to higher energy and that its width decreases. It must be remembered that this feature is the sum of contributions from different L lines of different charge states viewed along the chord, so the width is expected to be greater than the width for one charge of Mo which would be about 400 eV. The line shifts as the temperature increases because more outer shell electrons are stripped off and less shielding of the nucleus occurs increasing the energy of the transition. The decrease in width can be interpreted as a tendency of the Mo to occupy only one or two charge states during the quiescent period late in the shot.

Information concerning the spatial distribution of Mo can be obtained by studying spectra taken along different chords, such as in Fig. 8. A casual glance at the two spectra would lead one to conclude that the Mo is concentrated at the center of the plasma, but the actual distribution is shown in the Figure. The point at 9 cm is from a UV estimate. This peaking of the Molybdenum at the edge of the plasma has been noted in Pulsator by Sesnic [1976].

Other metallic impurities have been seen, most notably Cr and Fe, through their non-thermally excited K lines--as in Fig. 4--during low-density discharges. When Alcator was operated with a partial Tungsten limiter, W M radiation was sometimes detected as shown in Fig. 9.

In conclusion, the tendency of the electron distribution function towards thermal as the density is increased has been observed (Fig. 4); time development of the temperature profile has been measured (Fig. 5) and estimates of the absolute Mo concentration have been obtained.

Figure 8

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APPENDIX

A. Properties of the Solid State Si(Li) X-Ray Detector.

In general, the observed spectrum $F'(hv')$ from the X-ray detector is determined by the convolution of the instrumental response $R(hv, hv')$ with the product of the incident X-ray spectrum $F(hv)$ and the detector efficiency $\epsilon(hv)$,

$$F'(hv') = \int_0^{\infty} \epsilon(hv) F(hv) R(hv, hv') dhv.$$

The instrumental response function is a gaussian:

$$R(hv, hv') = \frac{1}{\sqrt{2\pi} \sigma_0(hv)} \exp \left(-\frac{1}{2} \frac{(hv - hv')^2}{\sigma_0^2(hv)} \right)$$

where the width $\sigma_0(hv)$ is a slowly varying function of the statistical fluctuations in the number of charge pairs produced by a photon of energy hv and the characteristic noise σ_n in the preamp electronics:

$$\sigma_0^2(hv) = \sigma_n^2 + Khv.$$

The FWHM of an X-ray line ($= 2.35\sigma_0$) is typically $\sim .230$ eV at 5900 eV.

($\sigma_n = 78$ eV, $K = .70$ eV.)

For $hv < 16$ keV, the efficiency $\epsilon(hv)$ of the Kevex detector is determined by the transmission through a .0025 cm Be entrance window, a nominal 0.3 μ Si dead layer, and a 200 $\text{\AA}$ Au conducting layer. The silicon and gold layers are situated on the front surface of the detecting element, which is located 0.6 cm behind the Be window. A sharp discontinuity in $\epsilon(hv)$ at 1.84 keV and smaller ones at 2.26, 2.74, 3.15, and 3.43 keV are due to K-shell and M-shell X-ray absorption edges in silicon and gold, respectively. $\epsilon(hv)$ is shown in the Figure.

B. The Width of Temperature Profiles Determined by Viewing Along Different Chords.

The iterative process by which profiles are determined proceeds as follows: width a of the gaussian profile is assumed, and inserted into Eq. (7). T_0 is determined for the different values of d by comparison of Eq. (6) with the observed spectra. A gaussian is fit to the various values of T_0 as a function of d and the new width a is reinserted into Eq. (7) and new T_0 s are generated. This process converges after one or two iterations. Since the electron density was so low, the ripple during the beginning of the shot shown in Fig. 5 is well within the statistical errors in the spectra from which the temperatures were determined.