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LARGE ENERGY TRANSFER REACTIONS
IN ION SLOWING-DOWN PROBLEMS:
THEIR EFFECT ON PLASMA PROPERTIES
AND A MULTIGROUP METHOD TO TAKE
THEM INTO ACCOUNT

BY

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Large Energy Transfer Reactions in Ion Slowing-Down
Problems; Their Effect on Plasma Properties and
A Multigroup Method To Take Them into Account*

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ABSTRACT

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Effects of nuclear elastic scattering and large angle ion-ion Coulomb interactions causing large energy transfers from energetic ions slowing down in plasmas to the plasma ions are examined. The effects examined are the fusion probability, the slowing down time, and the fraction of the ion energy imparted to the plasma ions and electrons.

A new multigroup method for the solution of the ion slowing-down equation that can accurately take into account large energy transfer reactions is described.

*Talk given at the Princeton University Plasma Physics Laboratory on July 17, 1975.

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1. INTRODUCTION

The majority of slowing-down problems of ions in plasmas are formulated and solved under the assumptions that (1) Coulomb interactions are the sole mechanism for the transfer of the energy from the test particle (namely, the energetic ion that slows down) to the plasma ions and electrons, and (2) the effect of these (ion-ion and ion-electron) Coulomb interactions on the energy loss of the test particle is equivalent to the effect of friction. The particle is assumed to slow down due to small energy transfer (SET) reactions as if it were subjected to a "Coulomb drag".

In reality there are two mechanisms that can cause the test ion to lose a significant fraction of its energy in a single collision event: These large energy transfer (LET) mechanisms are (1) Large angle ion-ion Coulomb interactions, and (2) Nuclear elastic scattering reactions between the nuclei of the test and the plasma ions.

In this talk I shall discuss the conditions in which LET reactions can have a non-negligible contribution to the slowing-down process (Sec. 2), describe a new method developed to take into account these LET reactions (Sec. 3), and describe the effects the LET reactions might have on the plasma properties (Sec. 4). The data presented in this

report pertain to plasmas of different compositions, densities and temperatures and to different slowing-down ions. No conclusions about the effect of LET reactions on the properties of specific fusion devices are attempted.

The development of the multigroup method and all the calculations reported in this work were done jointly with D. Shvarts.¹⁻⁴

2. MECHANISMS FOR SLOWING DOWN

In order to develop some insight as to under what conditions LET reactions can have non-negligible effects in plasma slowing-down problems, it is useful to compare cross sections and slowing-down rates corresponding to the different slowing-down mechanisms.

2.1 LARGE ANGLE ION-ION COULOMB SCATTERING

Rutherford cross section for Coulomb scattering is

$$\sigma(\Omega, u) = \left(\frac{Z_t Z_p}{2\mu u^2} \right)^2 \frac{1}{\sin^4 \left(\frac{\theta}{2} \right)}, \quad 0 \leq \theta \leq \pi \quad (1)$$

where θ is the scattering angle in the center of mass coordinate system, u is the relative velocity of the two ions, μ is their reduced mass, Z is the charge and p and t

denote, respectively, the bulk plasma ions and the test (i.e. slowing-down) particle. For the sake of simplicity let us assume that the struck particle is initially at rest. Then the kinematics of elastic collisions gives the following relation between the initial (E) and final (E') energies of the incident particle and the scattering angle θ :

$$E' = \frac{1}{2} E \left[(1+\alpha) + (1-\alpha) \cos \theta \right] \quad (2)$$

where

$$\alpha \equiv \left(\frac{M_P - M_t}{M_P + M_t} \right)^2, \quad (3)$$

and M is the mass. Equation (2) tells us that the minimal energy loss per collision is zero and is obtained for $\theta = 0$ whereas the maximal energy loss is $(1-\alpha)E$ and is obtained for $\theta = \pi$. For equal mass ions $\alpha = 0$ so that the incident ion can lose all of its energy in a single 180° collision.

The probability for a large angle Coulomb scattering is, however, very small compared with the probability for a small angle scattering, as seen from Eq. (1). In other words, the Rutherford cross section is extremely anisotropic.

A convenient way for assessing the relative probability for a large energy transfer in ion-ion Coulomb scattering is to examine the Rutherford cross section expressed in terms of the energy $T=E-E'$ transferred from the incident ion to the struck ion. When the latter is initially at rest,

$$\sigma(T, u) = \frac{2\pi}{M_p} \left(\frac{Z_t Z_p}{u} \right)^2 \cdot \frac{1}{T^2}, \quad 0 \leq T \leq (1-\alpha)E. \quad (4)$$

This expression shows clearly that the probability for LET ion-ion Coulomb scattering is very small relative to the probability for small energy transfer.

Equations (1)-(4) are valid also for ion-electrons Coulomb interactions. For such an interaction, however, $(1-\alpha)$ is of the order of 10^{-6} so that the maximal ion energy loss per collision is negligible even for large (in the center of mass) angle scattering.

2.2 NUCLEAR ELASTIC SCATTERING

The (total) cross section for several charged particle nuclear elastic scattering reactions is shown in Fig. 1. The graphs of Fig. 1 are approximate fits to some experimental data found in the literature; they are presented to give a rough idea of the magnitude of nuclear scattering

cross sections and are not to be considered as accurate. It is seen that these cross sections are, in the MeV energy range, of the order of a few barns. This is about an order of magnitude larger than the value of the cross section for the D-T fusion reaction (also shown, for comparison, in Fig. 1).

As a result of a nuclear elastic scattering the incident ion can lose energy anywhere between 0 and $(1-\alpha)E$, depending on the scattering angle (as described by Eq. (2)). Compared with Coulomb scattering, nuclear elastic scattering can be taken approximately to be isotropic (in the center of mass). Consequently, the average energy loss per nuclear elastic scattering event is $\frac{(1-\alpha)}{2} E$.

2.3 RELATIVE RATE OF ENERGY LOSS

The relative rate of loss of energy of an energetic ion slowing down in a plasma as a result, respectively, of ion-ion, ion-electron and nuclear elastic scattering is given, approximately, by ,

$$-\left(\frac{1}{E} \frac{dE}{dt}\right)_i = C_i E^{-3/2} , \quad (5)$$

$$-\left(\frac{1}{E} \frac{dE}{dt}\right)_e = C_e , \quad (6)$$

and

$$-\left(\frac{1}{E} \frac{dE}{dt}\right)_n = C_n E^{+1/2} \sigma_n(E) \quad (7)$$

where

$$C_i \equiv n_i \left(\frac{M_t}{M_p}\right) \sqrt{\frac{2}{M_t}} 2\pi(Z_p Z_t)^2 \ln \Lambda_i \quad (8)$$

$$C_e \equiv N_e \frac{8}{3} \left[\frac{2\pi M_e}{KT_e}\right]^{1/2} \frac{(Z_t)^2 \ln \Lambda_e}{M_t KT_e} \quad (9)$$

and

$$C_n \equiv n_i \left(\frac{1-\alpha}{2}\right) \sqrt{\frac{2}{M_t}} \quad (10)$$

Figure 2 compares the rate of energy loss caused by the three slowing-down mechanisms for the following problem: an alpha particle that slows down in a low density ($n=10^{14} \text{ cm}^{-3}$) D:T (1:1) plasma. The cross section for nuclear scattering was taken to be one barn over the entire energy range. Following are several observations: (a) The higher the energy of the test ion, the relative effectiveness of LET reactions (namely, nuclear elastic scattering) for slowing down the ions increases. (b) We can not expect significant contribution to the slowing-down

process, in the problem examined, when the energy of the alpha particle is below several hundred keV. (c) The relative contribution of nuclear scattering increases (for a given energy of the test particle) with the temperature of the electrons. This relative contribution will also increase with the density of the plasma. The increase in the plasma density causes a reduction in the value of the Coulomb logarithms,

$$\ln \Lambda = (\ln \Lambda)_0 - \frac{1}{2} \ln n ,$$

which is reflected by a decrease in the coefficients C_i and C_e (i.e., in the rate of energy loss owing to ion-ion and ion-electron Coulomb interactions). (d) At extremely high densities the electrons behave as a degenerate gas of fermions and are not effective in slowing down the ions.⁵

In view of the last two points we expect the effects of LET reactions to be more pronounced in laser-fusion (or electron-beam fusion) plasmas as compared with magnetically confined plasmas.

3. A MULTIGROUP METHOD FOR THE SOLUTION OF THE SLOWING DOWN EQUATION WITH LARGE ENERGY TRANSFERS

3.1 BACKGROUND

Most of the slowing-down calculations performed to date (see, for example, references 6-10) used a test particle (or a probability distribution) approach and took Coulomb friction as the only mechanism for energy transfer. We shall refer to this formulation as the Continuous Slowing Down (CSD) model.

Several methods have been developed in recent years for the calculation of the slowing down of energetic ions in plasmas taking into account LET reactions. The first work that considered nuclear elastic collisions in plasma slowing-down problems is, perhaps, the work of Gryzinski.¹¹ He discussed the conditions under which nuclear elastic scattering reactions can play a significant role in the slowing-down process, but, in his calculations, he considered only neutron (originating from fusion reactions) - ion elastic collisions; the slowing down of the recoil ions thus produced he calculated using a CSD model. About a decade later, Wyatt et al.¹² considered nuclear scattering reactions in plasma slowing-down problems and have developed a Fokker-Planck code to take the LET reactions into account. A similar approach was adopted by Lee, Zimmerman and Wood of

LLL.¹³ A completely different approach was taken by Evans¹⁴ who has developed a sophisticated Monte Carlo code that can accurately describe the discrete nature of LET nuclear and Coulomb ion-ion interactions. Bruckner and Brysk¹⁵ have modified the CSD to include nuclear scattering reactions. The energy transfer caused by nuclear elastic scattering is assumed to be continuous: its rate equals, on the average, that of the discrete energy transfer. We shall refer to this model as the modified continuous slowing-down (MCSD) model. Later on Bruckner et al.¹⁶ have improved this model allowing for the discrete nature of the LET reactions. Recently Peres¹⁷ has developed another formulation for the solution of slowing-down problems including LET reactions. His formulation is similar to a Fokker-Planck equation in which the second energy derivative in the expansion of the scattering integral is ignored. Peres' approximation is, in fact, similar to that used by Bartine et al.¹⁸ for electron transport problems.

We have developed⁴ a new method for the calculation of the slowing down of energetic ion in plasmas. The basic equation is similar to that of Peres¹⁷; the difference comes in the treatment of the energy variable. A unique feature of our method is that the energy variable is described in the multigroup formulation. The transformation of the continuous energy variable to the group formulation

becomes possible by expressing the Coulomb friction in terms of effective removal cross sections. The derivation of this multigroup formulation will now be outlined. In so doing, I shall point out the approximations made in several of the methods for the solution of the slowing-down equation mentioned above.

3.2 THE MULTIGROUP FORMULATION

The slowing-down equation for a given species of ions can be written in the form of the following balance equation

$$\Sigma_t(E) \phi(E) = S(E) + \int_{E_c}^{\infty} dE' \Sigma_s(E' \rightarrow E) \phi(E') \quad (11)$$

where S is the source of the energetic ions, ϕ is their flux, Σ_t is the total cross section (accounting for fusion, Σ_f , and scattering, Σ_s , reactions) and $\Sigma_s(E' \rightarrow E)$ is the scattering kernel. Ions slowing below the cutoff energy, E_c , are assumed to be transferred from the slowing-down ion density to the bulk ions population.

Let us divide the scattering integral of Eq. (11) into three parts covering the energy ranges $E_c \leq E' < E - \Delta$, $E - \Delta \leq E' < E + \Delta$ and $E + \Delta \leq E' < \infty$ where Δ is an arbitrary energy width. It is convenient to select Δ so that the first integral, $\int_{E_c}^{E - \Delta}$, will be negligible (that there will be no up scattering). For the present derivation we shall take Δ to

be greater than the maximal energy transfer in ion-electron and ion-ion collisions. The third integral $\int_{E+\Delta}^{\infty}$ accounts for contributions from scattering reactions characterized by LET. These contributions can be taken into account with the aid of a transfer matrix, as is usually done in codes designed to solve neutron slowing-down problems.¹⁹

Most problematic is the second integral, $\int_{E-\Delta}^{E+\Delta}$, which accounts for small energy-transfer collisions owing to, mainly, Coulomb interactions. The cross section for these collisions is proportional to the inverse of the square of the energy transfer (see Eq. (4)). Thus, this cross section is very large for small energy transfers which are not effective in removing the ions from the energy group they belong to. A large cross section for a small energy transfer will lead, in a conventional multigroup formulation, to a large in-group scattering cross section. The magnitude of such an in-group scattering cross section in a plasma slowing-down problem will be much larger than that of the out-of-group scattering cross section. Such a situation is undesirable from a practical point of view; it raises difficulties in the numerical solution of the slowing-down equation. To avoid this difficulty, it is useful to express the second integral in a different form.²⁰ Expressing the scattering integral in terms of the energy transfer, $T \equiv E' - E$, the second integral becomes

$$\int_{-\Delta}^{\Delta} dT \Sigma_S(E+T; T) \phi(E+T)$$

where $\Sigma_s(E';T)$ stands for $\Sigma_s(E' \rightarrow E' - T)$. Assuming that within the small energy range $(-\Delta, \Delta)$ the integrand varies slowly, we may expand it in a Taylor series around E and retain only the first three terms:

$$\int_{-\Delta}^{\Delta} dT \Sigma_s(E+T;T) \phi(E+T) \approx \Sigma_{\text{SET}}(E) \phi(E) + \frac{d}{dE} \left[\left\langle \frac{dE}{dx} \right\rangle_{\text{SET}} \phi(E) \right] + \frac{d^2}{d^2 E} \left[D_{\text{SET}}(E) \phi(E) \right] \quad (12)$$

where

$$\Sigma_{\text{SET}}(E) = \int_{-\Delta}^{\Delta} dT \Sigma_s(E;T) \quad (13)$$

is the removal cross section due to small energy transfer (SET) collisions;

$$\left\langle \frac{dE}{dx} \right\rangle_{\text{SET}} = \int_{-\Delta}^{\Delta} dT T \Sigma_s(E;T) \quad (14)$$

is the average energy loss per unit path-length due to these collisions; and

$$D_{\text{SET}}(E) = \frac{1}{2} \int_{-\Delta}^{\Delta} dT T^2 \Sigma_s(E;T) \quad (15)$$

The resulting slowing-down equation is

$$\left[\Sigma_a(E) + \Sigma_{\text{LET}}(E) \right] \phi(E) = S(E) + \int_{E+\Delta}^{\infty} dE' \Sigma_s(E' \rightarrow E) \phi(E') + \quad (16)$$

$$\frac{d}{dE} \left[\left\langle \frac{dE}{dx} \right\rangle_{\text{SET}} \phi(E) \right] + \frac{d^2}{dE^2} \left[D_{\text{SET}}(E) \phi(E) \right]$$

where

$$\Sigma_a(E) = \Sigma_t(E) - \Sigma_s(E) \quad (17)$$

and

$$\Sigma_{LET}(E) \equiv \Sigma_s(E) - \Sigma_{SET} = \int_0^{E-\Delta} dE' \Sigma_s(E+E') \quad (18)$$

Before proceeding to derive the multigroup equation, let us compare the approximations incorporated in several of the slowing down formulations.

The CSD model ignores LET reactions and neglects the second energy derivative term, but its $\langle dE/dx \rangle$ takes into account all energy transfers possible by Coulomb interactions (i.e., the limits of integration in Eq. (14) cover the entire energy range). The resulting equation is

$$\Sigma_a(E) \phi(E) = S(E) + \frac{d}{dE} \left[\left\langle \frac{dE}{dx} \right\rangle \phi(E) \right] \quad (19)$$

The MCSD uses a similar equation:

$$\Sigma_a(E) \phi(E) = S(E) + \frac{d}{dE} \left[\left\langle \frac{dE}{dx} \right\rangle_{eff} \phi(E) \right] \quad (20)$$

where $\left\langle \frac{dE}{dx} \right\rangle_{eff}$ is an effective $\langle dE/dx \rangle$ term obtained by adding to the Coulombic $\langle dE/dx \rangle$ another term that takes into account, in an average sense, the effect of nuclear

scattering reactions on the energy loss, per unit path length, of the test ion.

In the Fokker-Planck equation one retains both first and second energy derivatives of the Taylor series expansion with $\langle \frac{dE}{dx} \rangle$ and $D(E)$ taking into account LET as well as SET Coulomb interactions (by letting $\Delta \rightarrow \infty$). The two terms of Eq. (16) describing, explicitly, LET reactions are, however, ignored:

$$\Sigma_a(E) \phi(E) = S(E) + \frac{d}{dE} \left[\langle \frac{dE}{dx} \rangle \phi(E) \right] + \frac{d^2}{dE^2} \left[D(E) \phi(E) \right] \quad (21)$$

The modified Fokker-Planck equation is, actually, Eq. (16).

Both Peres¹⁷ and our multigroup methods are based on the following approximation to Eq. (16):

$$\left[\Sigma_a(E) + \Sigma_{LET}(E) \right] \phi(E) = S(E) + \int_{E+\Delta}^{\infty} dE' \Sigma_s(E' \rightarrow E) \phi(E') + \frac{d}{dE} \left[\langle \frac{dE}{dx} \rangle_{SET} \phi(E) \right] \quad (22)$$

Here the LET reactions are expressed explicitly but only one term, accounting only for SET reactions, is retained in the Taylor series expansion. Peres solves Eq. (22) using a finite difference scheme (i.e., he solves a differential equation) whereas we transform Eq. (22) into a multigroup form, which makes it a simple algebraic equation.

To transform Eq. (22) into the multigroup form, we integrate it between E_i^+ and E_i^- which are, respectively, the lower and upper limits of energy group i , obtaining,

$$\left(\Sigma_a^i + \Sigma_{LET}^i \right) \phi_i = S_i + \sum_{j \leq i} \Sigma_{LET}^{j \rightarrow i} \phi_j + \left[\langle \frac{dE}{dx} \rangle_i^- \phi_i^- - \langle \frac{dE}{dx} \rangle_i^+ \phi_i^+ \right] \quad (23)$$

where

$$\phi_i \equiv \int_{E_i^+}^{E_i^-} dE \phi(E) \quad (24)$$

is the total flux in the i^{th} group; S_i is, similarly, the total source in this group; the group constants, Σ_a^i and Σ_{LET}^i , are defined as follows:

$$\Sigma_x^i \equiv \int_{E_i^+}^{E_i^-} dE \Sigma_x(E) \phi(E) / \phi_i \quad (25)$$

and the scattering matrix:

$$\Sigma_{LET}^{j \rightarrow i} \equiv \int_{E_j^+}^{E_j^-} dE' \int_{E_i^+}^{E_i^-} dE \Sigma_L(E' \rightarrow E) \phi(E') / \phi_j \quad (26)$$

where $\Sigma_L(E' \rightarrow E) = \Sigma_S(E' \rightarrow E)$ when $\Delta \leq E' - E$ and zero otherwise. The notation X_i^+ stands for $X(E_i^+)$.

For Eq. (23) to become an equation for the group fluxes, we have to express the fluxes at the group boundaries in terms of the group fluxes. This we achieve with the aid of the transformation:

$$\left\langle \frac{dE}{dx} \right\rangle_i^- \phi_i^- - \left\langle \frac{dE}{dx} \right\rangle_i^+ \phi_i^+ = \sum_{fr}^{i \rightarrow i+1} \phi_{i-1} - \sum_{fr}^{i \rightarrow i+1} \phi_i, \quad (27)$$

where $\sum_{fr}^{i \rightarrow i+1}$ is an effective "friction" cross section; it accounts for the removal of ions, owing to SET interactions, from group i into the adjacent group $i+1$. It is defined as follows:

$$\sum_{fr}^{i \rightarrow i+1} = \left\langle \frac{dE}{dx} \right\rangle_i^+ \phi_i^+ / \phi_i. \quad (28)$$

Hence, the value of this friction cross section depends on the shape of the flux in the i^{th} group. (This is also true for the other group constants, as seen from Eqs. (25) and (26).

To calculate $\sum_{fr}^{i \rightarrow i+1}$ from Eq. (28) it is necessary to know the flux shape, but only in the energy range of group i . The smaller the group width, the less sensitive $\sum_{fr}$ is to the flux shape. Approximate shapes for averaging spectra can easily be obtained from simplified models. Following are two examples: (1) $\phi(E)=C$. (2) $\phi(E)=C/\langle \frac{dE}{dx} \rangle$. This shape is the solution of the CSD model in the absence of absorptions. The friction cross sections associated with the flux shapes described above are

$$\sum_{fr}^{i \rightarrow i+1} = \begin{cases} \left\langle \frac{dE}{dx} \right\rangle_i^+ / \Delta E_i & ; \quad \phi(E) = C \\ E_i^- \left[\int_{E_i^+} \frac{dE}{\left\langle \frac{dE}{dx} \right\rangle} \right]^{-1} & ; \quad \phi(E) = \frac{C}{\left\langle \frac{dE}{dx} \right\rangle} \end{cases} \quad (29)$$

With the transformation of Eq. (27), Eq. (23) becomes

$$\left(\Sigma_a^i + \Sigma_{LET}^i + \Sigma_{fr}^{i \rightarrow i+1} \right) \phi_i = S_i + \sum_{j \leq i} \Sigma_{LET}^{j \rightarrow i} \phi_j + \Sigma_{fr}^{i-1 \rightarrow i} \phi_{i-1} \quad (30)$$

This is an algebraic equation for the group fluxes of the suprathermal ions. It is in the form of the multigroup equation for neutron slowing-down problems.

3.3 SOLUTION STRATEGY

Equation (30) can be solved, for plasma slowing-down problems, in two basic ways: a generation-by-generation approach and a coupled solution.

A schematic representation of a generation-by-generation approach is shown in Fig. 3. The test particle (can be an externally injected particle or one of the fusion reaction products, C or D) can produce recoils by LET collisions with the plasma ions species A or B. The highest energy the recoils can get is $(1-\alpha)E_0$ where E_0 is the initial energy of the test ion. This first generation of recoils can produce, while slowing down, another generation of recoils and so on and so forth.

In the generation-by-generation solution strategy, one first solves Eq. (30) for the flux of the test ions starting with the external source distribution, $S(E)$. Knowing the flux, one can calculate all different kinds of

reaction rates such as the total rate of fusion events induced directly by the test particles,

$$F = \int_{E_c}^{\infty} dE' \Sigma_f(E') \phi(E') , \quad (31)$$

and the source of first generation of recoil ions,

$$S_1(E) = \int_{E_c}^{\infty} dE' \Sigma_s \left[E' \rightarrow (E' - E) \right] \phi(E') . \quad (32)$$

With this source distribution, Eq. (30) can now be solved for the flux of first generation of recoils, and so on and so forth. This procedure has to be applied to each of the plasma ion species. In problems where the slowing-down ions can undergo fusion reactions, the contribution of the fusion reaction products to the suprathermal ion population should also be calculated (following the same generation-by-generation approach).

It turns out that for CTR plasmas, the number of recoil generations that have a meaningful contribution to the suprathermal ion population seldom exceeds two. Consequently, the generation-by-generation approach converges very rapidly. Moreover, it provides a useful insight into the physics of the slowing-down process. It should also be recognized that the solution of Eq. (30) for a given generation can proceed in a straightforward recursive manner: the flux in the highest energy group (index 1) is simply

$$\phi_1 = S_1 / \left[\Sigma_a^1 + \Sigma_{LET}^1 + \Sigma_{fr}^{1 \rightarrow 2} - \Sigma_{LET}^{1 \rightarrow 1} \right] \quad (33)$$

from which we get the flux in group 2,

$$\phi_2 = \frac{S_2 + \Sigma_{LET}^{1 \rightarrow 2} \phi_1 + \Sigma_{fr}^{1 \rightarrow 2} \phi_1}{\Sigma_a^2 + \Sigma_{LET}^2 - \Sigma_{LET}^{2 \rightarrow 2} + \Sigma_{fr}^{2 \rightarrow 3}} \quad (34)$$

and so on.

Another approach for the treatment of the recoil ions is to take their contribution into account in the transfer-matrix term. For problems in which the slowing-down and the plasma ions are of the same species, this is accomplished by replacing the transfer matrix of Eq. (26) by

$$\Sigma_{LET}^{j \rightarrow i} = \int_{E_j^+}^{E_j^-} dE' \int_{E_i^+}^{E_i^-} dE \left[\Sigma_S(E' \rightarrow E) + \Sigma_S(E' \rightarrow (E' - E)) \right] \phi(E') / \phi_j. \quad (35)$$

The formulation becomes more complicated in multi-species problems. The results to be presented in this talk were obtained using the generation-by-generation approach.

3.4 ACCURACY OF THE METHOD

A multigroup code entitled MUGLET was developed⁴ for the solution of Eq. (30). All numerical results to be reported in this talk have been obtained with this code.

The accuracy of the multigroup model was tested⁴ in two different ways: (1) By comparing the solution of Eq. (30) with an exact solution. A simple problem amenable to analytic solution was used for this comparison. (2) By investigating the sensitivity of MUGLET results to the group-width and to the flux shape used for averaging the cross sections. All these comparisons are done for the fusion probability (FP) of a deuteron test particle that slows down in a tritium plasma of cold ions and 20 keV electrons. The density of the plasma is $3 \times 10^{13} \text{ cm}^{-3}$.

The analytic solution was obtained under the following assumptions:

- (1) The energy loss is caused only by Coulomb interaction, for which

$$\left\langle \frac{dE}{dx} \right\rangle = \sqrt{\frac{2}{M_t}} \left[C_i E^{-1} + C_e E^{1/2} \right]. \quad (36)$$

- (2) The fusion cross section, σ_f , is constant. Under these assumptions we find that the probability that a test ion will cause fusion while slowing down from E_0 to E is

$$F(E) = 1 - \exp \left\{ - \frac{2n_i \sigma_f}{C_e \sqrt{\frac{2}{M_t}}} \left[x - \frac{a}{6} \ln \frac{(a+x)^2}{(a^2 - ax + x^2)} - \frac{a}{\sqrt{3}} \tan^{-1} \frac{(2x-a)}{\sqrt{3} a} \right] \right\}_{x=\sqrt{E}}^{x=\sqrt{E_0}}$$

where $a^3 \equiv C_i / C_e$.

(37)

MUGLET's solution to the same problem was obtained by setting, in Eq. (30), $\Sigma_{LET}^i = \Sigma_{LET}^{j \rightarrow i} = 0$; $\Sigma_a(F) = n_i \sigma_f$; and taking Δ to be the maximal energy transfer in elastic collisions. The friction group cross sections, $\Sigma_{fr}^{i \rightarrow i+1}$, were prepared by averaging $\langle dE/dx \rangle$ of Eq. (36) with a $1/\langle dE/dx \rangle$ flux shape (see Eq. (29)). One hundred energy groups having an equal lethargy width of 0.075 covering the energy range from 10 keV to 15 MeV were used.

Excellent agreement was found between the results of MUGLET and the analytic solution; the maximal difference between these results is one percent for initial test-particle energies in the range 100 keV - 15 MeV.

To test the sensitivity of the results of the multigroup model to the group width and averaging spectrum, we have calculated the FP for the test problem described at the beginning of this section using different group widths and averaging spectra. These numerical experimentations take into account LET reactions. The cross sections used for nuclear scattering reactions have been obtained from approximate fits to experimental data (see Fig. 1).

Nuclear scattering is assumed, for simplicity, to be isotropic in the center of mass system. The D-T and D-D fusion cross sections are those of Duane²¹ whereas the T-T fusion reaction has been neglected. The continuous energy loss rate is assumed to result only from Coulomb

interactions. The expression used for the energy loss per unit path length is similar to Eq. (36) except that (1) the Coulomb logarithms ($\ln\Lambda$) are taken to be energy dependent, and (2) $\ln\Lambda_i$ have been changed to account only for small energy transfers. That is, we take $\ln\Lambda_i = \frac{1}{2} \ln(T_{\max}/T_{\min})$ where $T_{\min}$ is the minimal energy transfer (corresponding to the smallest impact parameter, which is Debye length) and $T_{\max}$ is the maximal energy transfer that we want to take into account by the $\langle dE/dX \rangle$ term. For the present calculations we take $T_{\max}$ to be the width of the energy group (group-dependent).

The test problem has been solved using five different energy group structures having equal lethargy widths of 0.6, 0.3, 0.15, 0.075 and 0.05 unit. Three different sets of group constants, generated with the two averaging spectra of Eq. (29), as well as with $\phi(E) = C/E$, are used for each group structure. The latter averaging spectrum is the solution of the slowing-down equation when nuclear elastic scattering is the only mechanism for slowing down, and its cross section is assumed constant and isotropic.

The sensitivity of the fusion probability to the group lethargy width and to the averaging flux shape is shown in Fig. 4. It is observed that:

(1) The results obtained with all three averaging spectra converge as the group width decreases. The maximal spread in the results for a group structure of $\Delta u = 0.075$ is 5%.

(2) The FP is least sensitive to the averaging spectrum $\phi(E) = \text{constant}$. With this spectrum the FP already reaches its asymptotic value when the group width is 0.3 lethargy unit.

Additional investigation is necessary for developing a more complete view of the suitability of the different averaging spectra for different kinds of slowing-down problems.

3.5 DISCUSSION

The transformation of the slowing-down equation from a differential (in energy) to an algebraic equation for the flux of the slowing-down ions opens the way for developing slowing-down models that are more general than the model described above (and incorporated in MUGLET). For example, the multigroup model can be extended directly to space and time-dependent problems, as well as for the description of the directional flux of ions and to the treatment of anisotropic scattering. The resulting space and time-dependent methods will be similar to the corresponding

neutron transport methods. In fact, it is expected that the multigroup model for plasma slowing-down problems could be cast in a form directly amenable for solution by existing neutron transport codes. In addition, the multigroup model can be extended to describe a plasma slowing-down problem involving several ion species.

In certain classes of plasma slowing-down problems, such as space and time-dependent problems, it is desirable (from economic considerations) to reduce the number of energy groups. This can be done by "collapsing" a number of groups into a single broader group, the group constant for which is obtained by an appropriate flux averaging of the multigroup cross sections. This group collapsing method is in routine use for neutron transport problems for which it has proved to be very satisfactory. The common practice is to solve a space-independent multigroup problem with a fine-group structure, use the calculated flux distribution to generate average cross section for a broad-group structure, and use them for space (and also time) dependent calculations.

Owing to the treatment of the energy variable, the multigroup method presented in this section is expected to be more efficient than methods, such as the Fokker-Planck and Peres', which solve the differential (in energy) slowing-down equations. The group treatment of the energy variable will be particularly useful for space and time dependent problems.

Our multigroup model uses the same physical approximations as does the improved model of Brueckner et al.¹⁶ The multigroup model is directly extendable to space and time dependent problems whereas Bruckner's model is not.

A disadvantage of the multigroup model compared to the Fokker-Planck method is that it cannot account for non-linear effects; that is, it is applicable only to problems in which the distribution of the bulk plasma ions (and electrons) is assumed to be known. Additional disadvantage of the multigroup method in its present form is that it does not allow for magnetic field effects.

Another difference between the multigroup and the Fokker-Planck methods is the approximation they use to describe the energy transfer. In the Fokker-Planck formulation the integration in Eq. (15) takes into account small, as well as large energy transfers; that is, it takes $\Delta \rightarrow \infty$. The dominant contribution to $D(E)$ comes from large energy transfer collisions. But these LET collisions are treated only in an approximate manner in the Fokker-Planck method. The multigroup method, on the other hand, treats the LET collisions accurately; the treatment it uses is equivalent to an infinite Taylor series in Eq. (12). For small energy transfers, the multigroup model is less accurate than the Fokker-Planck; it uses only the first term of the Taylor series expansion versus two terms used by the

Fokker-Planck method. We expect, however, that the contribution of the SET collisions to the dispersion of the slowing-down beam is small relative to the contribution of the LET collisions. The relative accuracy of the multigroup and Fokker-Planck methods is yet to be investigated.

4. EFFECTS OF LET REACTIONS ON PLASMA PROPERTIES

Large energy transfer reactions can affect the distribution of the energetic (or suprathermal) ions in the plasma and, hence, many integral parameters such as the number of fusions per test ion, the slowing-down time, the average energy of the suprathermal ions, and the fraction of the energy of the test ion imparted to the plasma electrons and ions.

In this section, I shall bring examples illustrating the magnitude of these effects of the LET reactions and its dependence on the plasma composition, density, temperature and on the kind and initial energy of the test particles. The calculations were performed with MUGLET using the nuclear elastic scattering reactions and the D-T and D-D fusion cross sections of Abulaffia and Peres²² and Duane's²¹ cross section for the T-T fusion reaction. The cross sections used for the Coulomb interactions are the same as described in the previous section (in relation to the

numerical experimentation). The results presented were obtained when MUGLET was in its debugging phase and may not be very accurate in the absolute scale. The relative magnitude of the LET effects, on the other hand, is expected to be accurate.

Subsections 4.2 - 4.4 present results from slowing-down calculations pertaining to dilute plasmas having densities that correspond to the densities of magnetically confined plasmas. LET effects on the properties of dense plasmas, of a density typical to laser or electron-beam pellet compression devices, are discussed in Section 4.5.

4.1 WHAT TO EXPECT

LET reactions can affect a given characteristic of the plasma via two different, albeit related, mechanisms: (1) As a result of a LET reaction, the slowing-down ion might skip over a finite energy region. (2) The energy lost by the test particle in the LET reaction appears as the initial kinetic energy of a recoil ion. Table I summarizes the expected trends of the effect of these two phenomena on the plasma characteristics under consideration.

The discrete jumps over part of the slowing down energy range will tend to reduce the contribution of the test particle itself to the fusion probability, to reduce

the average time it takes the test particle to slow down and to reduce the average energy of the test particle. The reason for the latter phenomenon is that the higher the energy of the ion, the more effective become LET reactions (owing to nuclear elastic scattering) in slowing down the ion (see Fig. 2). The enhanced slowing down, at high energies will also result in an increase in the fraction of the energy of the test ion itself imparted directly to the bulk plasma ions. This is because the ratio $\langle \frac{dE}{dt} \rangle_{\text{ions}} / \langle \frac{dE}{dt} \rangle_{\text{electrons}}$ increases as the energy of the slowing down ion decreases (see Fig. 2).

The recoils produced by the LET reactions of the test ion will have a positive contribution to fusion probability as well as to the fraction of the test ion energy imparted by friction to the plasma ions. The latter phenomenon is attributed to the fact that the average energy the recoils are born with is, usually, smaller than the energy of the test ion that enters the recoil producing reaction; and the lower the ion energy, the larger is $\langle \frac{dE}{dt} \rangle_{\text{ions}} / \langle \frac{dE}{dt} \rangle_{\text{electrons}}$. For the same reason of a lower initial energy, the presence of recoils will tend to reduce the average energy of the suprathermal ion population in the plasma.

4.2 FUSION PROBABILITY

Figure 5 illustrates the effect of LET reactions on the fusion probability (FP) of a fast deuteron that slows down in a dilute ($n_i = 10^{14} \text{ cm}^{-3}$) D:T (1:1) plasma with $T_e = 20 \text{ keV}$, by comparing the results of the multigroup model with the results of the CSD and the MCSD models (see Sec. 3.2). Following are several observations concerning the results presented in Fig. 5:

(a) $(FP)_0 < (FP^B)_0 < (FP^C)_0$, that is, the direct contribution of the test ion to the FP, is overestimated by the CSD model and, to a lesser extent, by the MCSD model. This comparison reflects the (negative) effect of the discrete energy jumps of the test particle caused by LET reactions (see Table I).

(b) For deuteron initial energies below about 0.6 MeV, $(FP^C)_0 > \sum_i (FP)_i$. Above that energy, on the other hand, $(FP^C)_0 < \sum_i (FP)_i$. In the high energy range the (positive) contribution of the recoils to the fusion probability more than compensates for the (negative) contribution of the discrete energy jumps to the FP of the test particle itself.

(c) The MCSD model significantly overestimates, in the high energy range, the contribution of the recoils to the FP. This overestimation is the result of, mainly, the large overestimate, by the MCSD model, of the existence probability of the test ion, as shown in Fig. 6. The "existence probability" $P_{ex}(E)dE$ is the probability that the test ion will reach the energy interval dE around E while it slows down. The minima in P_{ex} calculated with MUGLET is a direct reflection of the effect of the discrete energy jumps caused by LET reactions.

The fusion probability that can be obtained in low density ($n_e = 10^{14} \text{ cm}^{-3}$) D, T and D:T (1:1) plasmas at different electron temperatures ($T_e = 5, 20$ and 100 keV) and for different beams (test ions) is presented in Figs. 7-18. The notation $(A-B)_C$ used in these figures denotes a fusion reaction of type (A-B) that is caused, directly or via the recoil ions, by a test ion of type C. The results obtained with LET reactions taken into account are shown in solid lines. The corresponding results obtained from the CSD model (dashed lines) are also shown for comparison. The latter were also obtained with MUGLET after adjusting it to solve Eq. (19). Following are several observations concerning the results presented in Figs. 7-18:

(a) LET reactions have a very small effect on the FP when the test ion energy is below about 0.5 MeV.

(b) A deuteron injected into a T-only plasma can cause, if energetic enough, T-T fusion reactions in addition to the D-T reactions (see, for example, Fig. 9). Similarly, a triton injected into a D-only plasma can cause, via recoil production, D-D fusion reactions.

(c) In D-only and T-only plasmas, LET reactions reduce the overall D-T fusion probability in the entire range of test particle energies considered. The CSD model also overpredicts the $(D-D)_D$ and the $(T-T)_T$ fusion probabilities over most of the energy range except for the highest energies considered.

(d) The contribution of the recoils to the FP in the D:T (1:1) plasmas is more pronounced than for the single species plasma. This recoil contribution more than compensates for the reduction in the FP owing to the discrete energy jumps when the beam energies are about 1 MeV and higher.

The observations described in (c) and (d) reflect two phenomena:

(1) The dependence of the energy distribution of the recoils on the scattering reaction producing them. The maximal energy of a recoil born in a D-T collision is 4% ($\alpha = 1/25$) of the maximal energy of a recoil born in a D-D or a T-T collision ($\alpha = 0$).

(2) The relative contribution of a recoil born with a given energy to the different fusion probabilities. To evaluate the dependence of this contribution on the initial energy of the ions, it is useful to examine the differential fusion probabilities (DFP) for several reactions shown in Fig. 19. The differential fusion probability $F(E)$ is defined so that $F(E)dE$ is the probability that the ion will fuse while at dE around E . It is calculated from the expression $\sigma v(E)/\langle dE/dt \rangle$ obtained assuming that the slowing down is caused by Coulomb drag only. This is a good assumption as long as (a) LET reactions are much less effective than the Coulomb drag in slowing down the ions, and (b) the probability for fusion while at dE is small. In order of the maximal value of the differential fusion probability and in order of the minimal ion energy for which this maximum occurs, the three fusion reactions are D-T, D-D, and T-T. Incidentally, the peak in the curve of the differential fusion probability for the D-T reaction occurs at energies for which the D-T fusion reaction is close to its maximum

and the rate of energy loss owing to Coulomb drag is close to its minimum.

Consider, for example, the D-T fusion reactions. For a deuteron beam to produce D recoils that are effective in causing a D-T reaction (more than half of the maximal probability that a very energetic deuteron will cause a D-T fusion reaction), it has to have an initial energy exceeding (considerably) 300 keV. This is twice the energy E at which $\int_0^E dE F(E) / \int_0^\infty dE F(E) = 1/2$. Similarly, for a deuteron beam to produce T recoils that are effective in causing a D-T fusion reaction, it has to have an initial energy exceeding about 4 MeV. Figure 16 shows that the contribution of a D produced recoils to the D:T fusion probability in a D:T plasma compensates for the loss of FP caused by the discrete energy jumps of the beam ions for deuterons beam energies of about 1 MeV. At this energy the energetic ($E > 150$ keV) deuterium recoils are numerous enough to have a significant contribution to the FP.

As another example consider the T-T fusion probability of Fig. 9. The T-T differential fusion probability of Fig. 19 implies that for a tritium beam to produce recoils of large enough energy, so as to compensate for the loss in the direct FP of the test tritons, the beam energy should exceed about 6 MeV. Figure 9 shows that the contribution of the recoils overcompensates the effect of

the discrete energy jumps on the T-T FP for beam energies exceeding about 7 MeV, which is in accord with what we ought to expect based on the (differential FP) analysis.

(e) The T-T fusion probability tends to saturate for beam energies exceeding about 3 MeV; it is caused by the steep decline in the T-T fusion cross section above this energy, as given by Duane's²¹ fit to the T-T fusion cross section.

(f) A peculiar behavior of the T-T fusion probability is observed in Fig. 18: with LET reactions taken into account, the FP reaches a maximum for beam energies of about 3.5 MeV and then falls off and has a local minimum at beam energies of about 7 MeV. A possible explanation for this phenomenon is that for tritons of energy below about 7 MeV, a T-T nuclear elastic scattering reaction will bring the average energy of the recoil and slowing-down tritons to below 3.5 MeV, where the summed probability for T-T fusion reaction is smaller than the FP of a single 7 MeV triton. As the beam energy increases, the summed FP increases but the probability for a LET reaction also increases. Up to about 7 MeV, the combined effect of the two phenomena described above is a reduction in the overall FP. As the beam energy increases above 7 MeV, the increase in the summed FP of the recoil and slowing down triton exceeds the

FP of the initial triton, thus bringing about an increase in the FP. It should be noticed that the detailed behavior of the FP of a given type with beam energy depends also on the plasma composition. For example, the local minima behavior of the T-T FP described above does not appear in a T only plasma (Fig. 9).

(g) The neutron production probability in problems of a deuteron beam injected into a D plasma (about half the FP), and in problems of a triton beam injected into a T plasma (twice the fusion probability) is similar for the same plasma electrons temperature and beam energies (up to about 3 MeV).

4.3 SLOWING-DOWN TIME

As a result of a LET reaction, the slowing-down particle can lose, instantaneously, a large fraction of its energy. Consequently, we expect that LET reactions will tend to reduce the slowing-down time (SLDT) of the test particle. But LET reactions produce recoils and the question is how do these recoils affect the SLDT. The initial energy of the recoil can not be larger than the energy the test particle had when entering the recoil producing collision: when the mass of the two colliding ions is different, it will be significantly lower. Consequently

we expect that the overall time it takes for the test ion and its recoils to slow down (referring to the test particle injection time as the initial time) will be smaller as compared with no recoil production.

Figure 20 illustrates the effect of LET reactions on the SLDT of a deuteron test particle in a low density ($n=10^{14} \text{ cm}^{-3}$) D:T plasma at two different electron temperatures. The SLDT calculated with the CSD model is compared with two MUGLET results that allow for LET reactions; one refers to the SLDT of the test particle itself while the other refers to the average SLDT of the test particle and its recoils. It is observed that this average SLDT is smaller than the SLDT of the test particle, as expected from the above discussion.

The method we used for calculating the SLDT with LET reactions taken into account is different, markedly, from the conventional method of the CSD model. In the latter, the SLDT from initial energy E_0 to E is defined as follows

$$\tau(E_0 \rightarrow E) = \int_{E_0}^E \frac{dE}{\langle dE/dt \rangle} \quad (38)$$

This definition is not applicable to problems with discrete energy losses. A different approach that allows for the effect of LET (as well as fusion and other reactions) has been incorporated in the MUGLET code. This is the time-moments formulation developed originally for neutron transport problems.

Consider the time-dependent form of Eq. (11) subjected to a pulse of test ions:

$$-\frac{1}{v} \frac{\partial \phi(E, t)}{\partial t} + \underline{H}(E) \phi(E, t) = S(E) \delta(t) \quad (39)$$

where $\underline{H}$ represents the Boltzmann operator for the steady-state problem:

$$\underline{H}(E) \phi(E) \equiv \Sigma_t(E) \phi(E) - \int_{E_c}^{\infty} dE' \Sigma_s(E' \rightarrow E) \phi(E') \quad (40)$$

The average time it takes the flux of ions to reach energy E is the first time moment:

$$\langle t \rangle (E) \equiv \int dt \, t \phi(E, t) / \int dt \phi(E, t) \quad (41)$$

Similarly, the n^{th} time moment is defined as

$$\langle t^n \rangle (E) \equiv \phi^{(n)}(E) / \phi^{(0)}(E) \quad , \quad (42)$$

where the flux moment, $\phi^{(n)}(E)$, is defined as follows

$$\phi^{(n)}(E) \equiv \int dt t^n \phi(E, t) . \quad (43)$$

It turns out that the flux moments (and therefore also the time moments) can be obtained directly from the recursive solution of a sequence of steady-state equations:

$$\tilde{H}(E) \phi^{(0)}(E) = S(E) \quad (44)$$

$$\tilde{H}(E) \phi^{(n)}(E) = \frac{n}{V} \phi^{(n-1)}(E), \quad n = 1, 2, \dots \quad (45)$$

This formulation is incorporated in MUGLET. In addition to the slowing-down time, MUGLET calculates also the mean deviation,

$$\sigma = \sqrt{|\langle t \rangle^2 - \langle t^2 \rangle|} . \quad (46)$$

4.4 HIGH DENSITY PLASMAS

As mentioned in Section 2, as the density of the plasma increases the smaller become the cross-sections for ion-ion and ion-electron Coulomb interactions, and therefore the effectiveness of the LET reactions increases. An order of magnitude increase in the plasma density results in the reduction of the Coulomb logarithms by about a unity. Referring to the low density plasma considered above, this change in the Coulomb logarithms will result in a 5% reduction in the energy loss rate caused by Coulomb interactions.

Table II illustrates the magnitude of the effects of LET reactions on the properties of dense plasmas subjected to energetic test particles. The results in the Table refer to a 12.2 MeV deuteron (the most energetic recoil ion that can result from the collision of a 14.1 MeV neutron) that slows down in a D:T plasma of a density $n = 5 \times 10^{26} \text{ cm}^{-3}$ and a temperature of $T_e = T_i = 50 \text{ keV}$. The Coulomb logarithms for this problem are, approximately, 5 (as compared with about 20 for the low density plasmas considered above). For this particular example we find from (Table II) that:

(a) With the LET reactions taken into account, the bulk plasma ions get directly (from the slowing-down ion and its recoils) about twice as much energy as predicted by the CSD

model. Most of the energy imparted to the electrons comes from the energetic ion itself whereas the major part of the energy imparted to the plasma ions comes from the recoils. This stems from the fact that the recoils are, on the average, of a lower energy than the test particle, and the ratio $\langle dE/dt \rangle_{\text{ions}} / \langle dE/dt \rangle_{\text{electrons}}$ becomes larger as the energy of the slowing-down ions decreases (see Fig. 2). The difference between the 12.2 MeV initial energy and the total friction energy delivered to the plasma ions and electrons (between 8 and 9 MeV) takes part in the fusion reactions (and appears as an additional kinetic energy of the reaction products).

(b) The total fusion probability in this dense plasma is, for the same deuteron energy, significantly higher (by, approximately, an order of magnitude) as compared with the FP in the dilute plasmas shown in Figs. 14 ($T_e = 20$ keV) and 15 ($T_e = 100$ keV). The effect of the LET reactions is particularly noticeable for the D-T fusion probability; this D-T FP calculated with MUGLET (LET reactions taken into account) is about 3.5 times larger than the FP calculated with the CSD model. The major contribution to D-T fusions comes from the recoils. LET reactions do not affect very much, in this example, the D-D fusion probability.

(c) The slowing-down time of the 12.2 MeV deuteron calculated with the LET reactions taken into account is only half of its value obtained from the CSD model. The average SLDT of the injected deuteron plus its recoils is even shorter.

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Table I. Qualitative Effects of Large Energy Transfer
Reactions on Plasma Properties.

Plasma Property	Phenomenon		
	Discrete Energy Jumps	Recoils Production	Total
Fusion Probability	-	+	- or +
Slowing-Down Time	-	-	-
Average Beam Energy	-	-	-
Fraction of Energy Imparted to Ions	+	+	+

Table II. Effects of Large Energy Transfer Reactions
in Dense Plasmas.

PROPERTY	LET Model		CSD Model
	1st Generation	Total	
Friction Energy (MeV)			
Ions	0.84	2.67	1.45
Electrons	4.77	5.70	7.38
Total	5.61	8.37	8.83
Fusion Probability			
D-T	0.19	0.95	0.27
D-D	0.24	0.33	0.39
Total	0.43	1.28	0.66
SLDT (sec.)	3.66×10^{-12}	2.85×10^{-12}	7.21×10^{-12}

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Fig. 1. Cross Sections for Nuclear Elastic Scattering Reactions of Several Light Nuclei. (Obtained from approximate fits to experimental data.)

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 Fig. 2. Relative Rate of Energy Loss Caused by Nuclear Elastic Scattering and by Ion-Ion and Ion-Electron Coulomb Interaction for Alpha Particle Slowing Down in a D:T (1:1) Plasma with $n_e = 10^{14} \text{ cm}^{-3}$.

GENERATION BY GENERATION APPROACH

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Fig. 3. A Schematic Representation of a Generation by Generation Approach to the Solution of the Slowing-Down Equation.

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Fig. 4. The Dependence of the Fusion Probability on the Group Width and on the Cross-Section Averaging Spectra.

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 Fig. 5. The Effect of Large Energy Transfer Reactions on the Fusion Probability of a Deuteron in a D:T (1:1) Plasma with $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = 20 \text{ keV}$.

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 Fig. 6. The Existence Probability of a 12.5 MeV Deuteron Slowing Down in a D:T (1:1) Plasma with $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = 100 \text{ keV}$.

Fig. 7. Fusion Probabilities in Tritium Plasma of $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = 5 \text{ keV}$. 754423

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Fig. 8. Fusion Probabilities in Tritium Plasma of $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = 20 \text{ keV}$.

754413
 Fig. 9. Fusion Probabilities in Tritium Plasma of
 $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = 100 \text{ keV}$.

Fig. 10. Fusion Probability in D plasma of $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = 5 \text{ keV}$. 754422

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 Fig. 11. Fusion Probability in D plasma of $n_e = 10^{14} \text{ cm}^{-3}$
 and $T_e = 20 \text{ keV}$.

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Fig. 12. Fusion Probability in D plasma of $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = 100 \text{ keV}$.

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Fig. 13. Fusion Probability of a D Beam in a D:T (1:1) Plasma of $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = 5 \text{ keV}$.

754416
Fig. 14. Fusion Probability of a D Beam in a D:T (1:1) Plasma of $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = 20 \text{ keV}$.

Fig. 15. Fusion Probability of a D Beam in a D:T (1:1) Plasma of $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = 100 \text{ keV}$.

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Fig. 16. Fusion Probability of a T Beam in a D:T (1:1) Plasma of $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = 5 \text{ keV}$.

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Fig. 17. Fusion Probability of a T Beam in a D:T (1:1) Plasma of $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = 20 \text{ keV}$.

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 Fig. 18. Fusion Probability of a T Beam in a D:T (1:1) Plasma of $n_e = 10^{14} \text{ cm}^{-3}$ and $T_e = 100 \text{ keV}$.

Fig. 19. Differential Fusion Probability for D-T, D-D and T-T Reactions.

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Fig. 20. The Effect of LET Reactions on the Slowing-Down Time.

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