ORO 5126-14¹⁵

BOUND STATE MOMENTUM DISTRIBUTIONS

Y. Alexander, E. F. Redish and N. S. Wall

Department of Physics and Astronomy
University of Maryland, College Park, Maryland 20742 U.S.A.

March 1977

U. of Md. TR #77-049

U. of Md. PP #77-161

UNIVERSITY OF MARYLAND
DEPARTMENT OF PHYSICS AND ASTRONOMY
COLLEGE PARK, MARYLAND

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

BOUND STATE MOMENTUM DISTRIBUTIONS*

Y. Alexander, E. F. Redish and N. S. Wall**

Department of Physics and Astronomy
University of Maryland, College Park, Maryland 20742 U.S.A.

March 1977

ABSTRACT

Proposed forms for nuclear momentum distributions are investigated. Calculations of (p,p') reactions using those forms are done in a PWIA at angles where the quasielastic peak is seen and also at back angles. The parameters used are derived from (e,e') data, where the nuclear momenta probed overlap with those of the low angle (p,p') experiment. Although there is reasonable agreement for the (p,p') data at 180° , the inclusion of distortion necessitates a different parameter set to obtain agreement for the quasi-free process. We conclude that the (p,p') reaction cannot be readily understood with a simple momentum distribution.

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

*Supported by the U. S. Energy Research and Development Administration.

**Supported by the National Science Foundation.

EP
DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Recently an experimental study of the inclusive reaction $A(p, p')X$ by Frankel, et al.¹ was analyzed by Amado and Woloshyn² in terms of newly proposed nuclear momentum distributions of the forms:

$$n(k) = N_s k \gamma_s / \sinh(\gamma_s k) \quad (1a)$$

$$= N_c / (\cosh(\gamma_c k))^2 \quad (1b)$$

These forms are derived in a study of a one-dimensional many-body problem with a δ -function interaction.³ Such forms have substantial momentum components out to much larger momenta than even a finite temperature Fermi gas. Using this distribution and the assumption of a single scattering mechanism in the (p, p') reaction, there remain two parameters to be determined; namely γ_s (or γ_c) and $\bar{\epsilon}$ (the average binding energy of the struck nucleon). When these values are fixed in one experiment, their applicability to another set of data is a test of the form of the distribution. Amado and Woloshyn use quasielastic electron scattering⁴ to set these parameters; the momentum components probed ($k \lesssim 2 \text{ fm}^{-1}$) are in a region where there also exist data and analyses of quasielastic proton scattering. These experiments can be used to test the plane wave approximation as well as the momentum distributions for consistency between the high momentum and low momentum situations. In particular at 1 GeV, a (p, p') experiment was performed by Corley, et al.^{5,6} in the region $9^\circ \lesssim \theta \lesssim 20^\circ$.

In all the analyses considered it is assumed that single step knockout dominates in $p + A \rightarrow p' + N + (A-1)$ where A and $A-1$ are the initial and final nuclei, respectively. Furthermore if the energy dependence of the $(A-1)$ final state is neglected, the sum over all its final states can be

approximated by closure. The final expression in the plane wave impulse approximation (PWIA) is

$$\frac{d^3\sigma}{dp_1^3} = (2\pi)^4 \frac{E(p_0)}{P_0} \sum_{i=1}^A \int d^3p \, n_i(p) |M|^2 \delta(E(p_1) + E(p_2) + T_{A-1} + \bar{\epsilon} - m - E(p_0)) \quad (2)$$

where $E(p) = (p^2 + m^2)^{1/2}$, T_{A-1} is the kinetic energy of the final nucleus, and M is the off shell nucleon-nucleon scattering amplitude.

The process can be represented diagrammatically as shown in Fig. 1. For (p, p') at low angles, $\vec{p}_1$ is taken to represent the knocked-out particle, and the momentum transfer $\vec{q}$ is $\vec{p}_1 - \vec{p}_0$. For the case of (p, p') at $\theta_{\text{LAB}} = 180^\circ$, small momentum transfer is still possible by taking $\vec{p}_2$ as the observed particle. In the latter case only a small momentum transfer $\vec{q}$ is involved, while $\vec{p}$ must be large and moving towards the projectile.

In either case the elementary cross section is taken to be on-shell at the relative energy for the outgoing pair. Integrating over $\cos \theta$ removes the δ function and leaves an integral over p with limits

$$p_{\pm} = \frac{|\vec{p}_0 - \vec{p}_1| \mp [|\vec{p}_0 - \vec{p}_1|^2 - (1 + \alpha)(|\vec{p}_0 - \vec{p}_1|^2 - b)]^{1/2}}{1 + \alpha} \quad (3)$$

$$\alpha = \frac{m + a}{m_{A-1}}, \quad a = E(p_0) - E(p_1) - E, \quad b = a^2 + 2ma \quad (4)$$

For the Corley^{5,6} and Frankel¹ experiments, there is no overlap of the ranges of integration, but each alone has considerable overlap with the momentum probed in the electron scattering data⁴.

The low angle data are analyzed by Corley⁶ in the PWIA and by Kroll and Wall⁷ in both the PWIA and distorted wave impulse approximations (DWIA). In those calculations it is shown that it is essential to consider the effect of distortion, particularly the attenuation coming from the presence of the imaginary potential⁸. This potential has a depth such that the mean free path, λ , of a 1 GeV proton in nuclear matter is of the order 1 fm. In the calculations of Refs. 6 and 7 the momentum distribution is obtained by Fourier transforming the single particle wave functions obtained in Woods-Saxon binding potentials⁹. These potentials are adjusted to have reasonable shapes and to give the correct binding energies.

We have repeated these PWIA calculations for carbon with some minor modifications. The earlier calculations use an elementary cross section $\left. \frac{d\sigma}{d\Omega} \right|_{NN}$ that is only a function of $|t| = q^2$. However, the center of mass energy of the interacting pair varies by several hundred MeV. When this energy variation is included in the evaluation of $\left. \frac{d\sigma}{d\Omega} \right|_{NN}$ there is a resulting shift of ≈ 10 MeV in the location of the quasi free peak, and a slight improvement of the shape of the spectrum. Here E_{cm}^{NN} is calculated with the final, rather than the initial, pair of interacting nucleons, though this change has little effect on the location or size of the maximum.

The calculation is then performed with a momentum distribution of the form given in Eq. (1). Fig. 2 shows a comparison of several calculations with the experimental data of Ref. 5. Again it should be pointed out that the nuclear momenta making significant contributions to these calculations overlap strongly with those used in the analysis of the quasielastic electron scattering⁴. The particular value of $\gamma_s = 3.18$ fm used by Amado and Woloshyn¹⁰

in a PWIA calculation to fit the data of Frankel¹ is seen to predict a differential cross section $d^2\sigma/d\Omega dE$ which is too high. A value of $\gamma_s \sim 5.0$ fm is seen to reproduce, fairly well, the results of the PWIA calculation using a single particle wave function. By arbitrarily changing γ_s to 2.5 fm one can obtain the correct magnitude for the differential cross section at the maximum. However, on the basis of the Kroll and Wall⁷ calculations, we expect distortion to reduce the magnitude by a factor ~ 2 but not to improve the predicted width. Figure 3 shows these calculations arbitrarily normalized and shifted to a common maximum. It can easily be seen that the width of the curve, adjusted to give the correct maximum cross section, is too large compared to experiment. Table 1 gives the full width at half maximum of the quasi-free scattering spectrum at 15.1° , for 1 GeV protons scattered from ^{12}C . It is clear that both of the distributions, with $\gamma_s = 3.18$ and 2.5 fm, lead to peaks which are too wide.

The various parametrizations of $n(k)$ investigated above are then used for calculating differential cross sections at 180° (Table 2). The value of 3.18 used in Ref. 2 in fact falls below the data; this is especially troublesome since distortion is likely to lower this result even further. The $n(k)$ used in the Corley and Kroll analyses has insufficient high momentum components for $k \sim 5 \text{ fm}^{-1}$ and is useless for comparison at this region. If $\gamma = 5 \text{ fm}$ is used, the cross section is, as expected, many orders of magnitude too low. Finally, the value of $\gamma = 2.5 \text{ fm}$, which in a PWIA yields the correct magnitude of the differential cross section at small angles, results in too large a cross section at 180° .

From the results in Tables 1 and 2 we conclude that a single value for γ is insufficient for expressing the momentum distribution of carbon over such a large momentum range. When distortion effects are included perhaps

even the extrapolation from electron scattering to the Frankel experiment is too much for the model considered. However, a model involving $\gamma = \gamma(k)$ is completely general and loses the simplicity and elegance of the Amado-Woloshyn analyses².

A recent calculation by L. D. Miller¹¹ using relativistic Hartree-Fock wave functions yields a momentum distribution that at large k falls approximately like Eq. (1) with $\gamma \sim 5$. Since there is considerable structure in this $n(k)$, the resulting cross section may be somewhat different, but are still likely to be much too low.

Our calculations using previous results⁷ to estimate the effects of distortion therefore suggest that the inclusive (p,p') reaction on nuclei cannot be explained in terms of a PWIA calculation with a simple momentum distribution of the Amado-Woloshyn type. Multiple scattering approaches to this sort of process are suggested by the success of DWIA calculations⁷ and the earlier work of Blokhintsev¹².

We wish to thank Prof. R. D. Amado and Dr. R. M. Woloshyn for useful discussions. We also thank Dr. L. D. Miller for communicating his results prior to publication. Computations supported by University of Maryland Computer Science Center.

REFERENCES

1. S. Frankel et al., Phys. Rev. Lett. 36, 642 (1976).
2. R. D. Amado and R. M. Woloshyn, Phys. Rev. Lett. 36, 1435 (1976).
3. R. D. Amado, Phys. Rev. C 14, 1264 (1976).
4. R. R. Whitney et al., Phys. Rev. C 9, 2230 (1974).
5. D. M. Corley et al., Nucl. Phys. A184, 437 (1972).
6. D. M. Corley, Ph.D. Thesis, University of Maryland, 1968.
7. F. R. Kroll and N. S. Wall, Phys. Rev. C 1, 138 (1970).
8. H. Palevsky et al., Phys. Rev. Lett. 18, 1200 (1967).
9. N. S. Wall and P. G. Roos, Phys. Rev. 150, 811 (1969).
10. R. M. Woloshyn, private communication.
11. L. D. Miller, private communication.
12. D. I. Blokhintsev, Zh. Eksp. Teor. Fiz. 33, 1295 (1957) [Sov. Phys. JETP 6, 995 (1958)].

FIGURE CAPTIONS

Figure 1. Mechanism for single scattering for (p,p') .

Figure 2. Inclusive proton spectra for 1.05 GeV protons on ^{12}C at 15.10.

..... PWIA calculation with shell model momentum distribution (Ref. 6,7)

----- PWIA calculation with $n(k)$ of Eq. 1a; $\gamma = 5$ fm

- - - - - PWIA calculation with $\gamma = 3.18$ fm

- - - - - PWIA calculation with $\gamma = 2.5$ fm

————— Experimental data (Ref. 5,6)

Figure 3. Inclusive proton spectra of Fig. 2, arbitrarily normalized and shifted. Notation as in Fig. 2, but with data and PWIA calculation (—) essentially coincident.

TABLE I. Full width at half maximum of the quasi free proton scattering spectrum for 1 GeV protons scattered through 15.1° from ^{12}C . The shell model and DWIA results are from Ref. 7, the experimental data are from Ref. 5 and 6.

	Shell model	$\gamma_s = 3.18 \text{ fm}$	$\gamma_s = 2.5 \text{ fm}$	$\gamma_s = 5.0 \text{ fm}$	DWIA	Exp
Width (MeV)	125	155	185	111	145	130

TABLE II. Differential cross section in units of $\text{mb}/(\text{GeV}/c)^3\text{-sr}$ for the scattering of 600 MeV protons from ^{12}C at $\theta_{\text{LAB}} = 180^\circ$

E	$\gamma_s = 2.5 \text{ fm}$	$\gamma_s = 3.18 \text{ fm}$	$\gamma_s = 5.0 \text{ fm}$	Exp
200 MeV	.6	8×10^{-2}	1.0×10^{-5}	$\sim .1$
300 MeV	8×10^{-3}	1.0×10^{-4}	8×10^{-9}	$\sim 1.5 \times 10^{-3}$

Fig. 1

Fig 2.

$^{12}\text{C}(p,p') 15.1^\circ$

$T_{\text{LAB}} = 1.05 \text{ GeV}$

Arbitrary Normalization

Curves Shifted to Data

Fig. 3