STABILITY AND MAGNETIC TEARING
OF FINITE- β
MODIFIED DRIFT WAVES

BY

LIU CHEN, J. HSU, P. K. KAW,
AND
P. H. RUTHERFORD

**PLASMA PHYSICS
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Printed in the United States of America.

Available from
National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151

Price: Printed Copy \$ * ; Microfiche \$3.00

<u>*Pages</u>	<u>NTIS Selling Price</u>
1-50	\$ 4.00
51-150	5.45
151-325	7.60
326-500	10.60
501-1000	13.60

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Liu Chen, J. Hsu, P. K. Kaw, and P. H. Rutherford

Princeton Plasma Physics Laboratory
Princeton University, Princeton, N. J. 08540

PPPL-1376

October 1977

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Stability and Magnetic Tearing of Finite- β
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Liu Chen, J. Hsu,* P. K. Kaw, and P. H. Rutherford
Princeton University Plasma Physics Laboratory
Princeton, New Jersey 08540 USA

ABSTRACT

We present a new simplified approach to the analysis of radial eigenmodes of finite- β modified drift waves in a sheared magnetic field. Applying this approach to the universal drift mode, we recover, for the lowest ($n = 0$) radial eigenmode, the previous result [1] that finite- β effects are stabilizing. For the next ($n = 1$) radial eigenmode, however, we find that finite- β effects further destabilize the mode. Moreover, the corresponding mode structure exhibits nonzero radial (tearing) magnetic perturbations around the mode-rational surface. The consequences of a structure of microscopic magnetic islands, created in this way, for plasma transport are also briefly discussed.

* Present address: General Atomic, San Diego, California USA

1. INTRODUCTION

Low-frequency electrostatic microinstabilities of the drift-wave type are important because they may significantly influence transport processes in magnetically-confined plasmas such as tokamaks. For typical current and future tokamaks with parameters such that $1 \gg \beta \gg m_e/m_i$, it is well known that electrostatic drift waves are coupled to shear Alfvén waves [2]. Here, $\beta = 8\pi n_0 (T_e + T_i)/B^2$ is the ratio between plasma pressure and magnetic pressure. The effects of coupling to shear Alfvén waves modify not only the stability conditions but also the magnetic surface topology, due to the associated magnetic perturbations. In the presence of magnetic shear, radially non-local analyses are necessary to examine these finite- β effects. Previous radial eigenmode analyses [1] of the finite- β modified universal drift waves have concentrated on the lowest ($n = 0$) radial eigenmode, which is least shear stabilized. The conclusion was that finite- β effects play a stabilizing role. Furthermore, because the $n = 0$ eigenmode has odd radial (tearing) magnetic perturbations with respect to the mode-rational surface, the instability does not lead to the reconnection of magnetic field lines. The above analyses, however, are rather involved. In this work, we present a simplified approach to the radial eigenmode analyses. Applying this approach to the universal drift waves, we recover, for the $n = 0$ eigenmode, the

above-mentioned results. For the $n = 1$ eigenmode, however, we find the new result that finite- β effects further destabilize the mode. Furthermore, we find the eigenmode structure exhibits finite radial (tearing) magnetic perturbations around the mode-rational surface. Thus, the instabilities will cause the magnetic field lines to reconnect and produce microscopic magnetic islands around the mode-rational surfaces. Such magnetic surface topologies present the possibility of new and interesting transport mechanisms in tokamak discharges via the process known as magnetic braiding [3]. While in this work, we limit our considerations to the collisionless universal drift waves, the theoretical approach presented here can be readily extended to other types of drift waves; e.g., trapped-electron modes [4].

Section 2 describes the theoretical approach used in this work. The radial eigenmode problem is analyzed in Sec. 3. In Sec. 4, we present the dispersion relation and discuss the stability condition. The eigenmode structures, particularly the radial (tearing) magnetic perturbations, are discussed in Sec. 5. Section 6 contains the final conclusions and a brief discussion of the implications of the results for plasma transport.

2. THEORETICAL APPROACH

We adopt a slab plasma with inhomogeneity in x and a sheared magnetic field $\underline{B} = B(\underline{e}_z + x\underline{e}_y/L_s)$. As to the field variables, we use electric potential ϕ and vector potential A ; hence, $\underline{E} = -\nabla\phi - \partial A/\partial t \underline{e}_z$ and $\underline{b} = \nabla \times A$. With $\beta \ll 1$, we may ignore the compressional Alfvén mode; i.e., we take $A = A_{||}\underline{e}_{||}$ and $b_{||} = 0$. The radial wave equation can then be derived following the standard procedures outlined in Ref. 1. The perturbed electron distribution function is obtained from the drift kinetic equation:

$$f_e = \frac{e f_{em}}{T_e} \left[\phi - \frac{\omega - \omega_{*e}}{\omega - k_{||} v_{||}} \left(\phi - \frac{v_{||} A_{||}}{c} \right) \right]. \quad (1)$$

Here, f_{em} is the electron Maxwellian distribution, $\omega_{*e} = k_y c T_e / e B L_n$ is the electron diamagnetic drift frequency, $L_n = 1/|d \ln n / dx|$, $k_{||} = k_{||}' x$, $k_{||}' = k_y / L_s$, and we have ignored the electron temperature gradients. For the ions, we have

$$f_i = \frac{e f_{im}}{T_e} \left[\frac{\omega_{*e}}{\omega - k_{||} v_{||}} \phi + \frac{\tau + \omega_{*e}/\omega}{\omega - k_{||} v_{||}} k_{||} v_{||} \left(\phi - \frac{v_{||} A_{||}}{c} \right) + \rho_s^2 \left(1 + \frac{\omega_{*e}}{\omega \tau} \right) \nabla_{\perp}^2 \phi \right]. \quad (2)$$

Here, $\tau = T_e / T_i$, $\rho_s = c_s / \Omega_i$, $c_s^2 = T_e / m_i$ and we have taken the small ion-Larmor radius limit; i.e., $|\rho_i^2 \nabla_{\perp}^2 \phi| \ll |\phi|$. From f_e

and f_i , we obtain density and current perturbations; which, after being substituted into the quasineutrality condition and Ampere's law, yield respectively

$$\left(1 + \frac{\omega_{*e}}{\omega\tau}\right) \rho_s^2 \left(\frac{d^2}{dx^2} - k_y^2\right) \phi = F(x) \left(\phi - \frac{\omega A_{||}}{ck_{||}}\right), \quad (3)$$

and

$$\rho_s^2 \left(\frac{d^2}{dx^2} - k_y^2\right) A_{||} = \frac{\omega c}{v_A^2 k_{||}} F(x) \left(\phi - \frac{\omega A_{||}}{ck_{||}}\right), \quad (4)$$

where

$$F(x) = \left(1 - \frac{\omega_{*e}}{\omega}\right) [1 + \xi Z(\xi)] - \frac{k_{||}^2 c_s^2}{\omega^2} \left(1 + \frac{\omega_{*e}}{\omega\tau}\right). \quad (5)$$

Here, $\xi = \omega/|k_{||} v_e| \equiv x_e/|x|$, $x_e = \omega/|k_{||} v_e|$ and Z is the plasma dispersion function. In deriving Eqs. (3) and (4), we have also assumed $|\omega| \gg |k_{||} v_{||}|$ for the ions and ignored ion Landau damping. Equations (3) and (4) constitute the basic set of equations for the radial eigenvalue problem.

To proceed with the radial eigenmode analysis, we adopt the following theoretical approach:

(i) Neglecting k_y^2 terms and resonant-electron effects, Eqs. (3) and (4) are solved by a matching procedure to obtain finite- β modifications to the eigenvalues and mode structures.

(ii) Contributions of finite- k_y^2 terms and resonant electrons to the eigenvalues are then treated by perturbation theory [1, 5]. We note that the major difference between the

present approach and that in Ref. 1 lies in (i). By using a matching procedure, the analyses of finite- β effects become much simplified and physically more transparent. Another minor difference (which does not affect the stability condition) is that we do not take the small argument limit of the Z function and, hence, avoid the singular behavior of the parallel current density at the mode-rational surface.

3. FINITE- β MODIFIED RADIAL EIGENMODE ANALYSES

Ignoring k_y^2 terms and resonant-electron effects, Eqs. (3) and (4) can be combined, integrated once and we obtain

$$\frac{d}{dx} \left(\frac{1}{f(x)} \frac{d\epsilon}{dx} \right) + \Lambda \left(\frac{1}{x^2} - \frac{1}{x_A^2} \right) \epsilon(x) = - \frac{c_1}{x^2}, \quad (6)$$

where

$$\epsilon(x) \equiv d\phi/dx, \quad (7)$$

$$f(x) = 1 + \xi Z_r - x^2/x_T^2, \quad (8)$$

$$x_A^2 = \omega^2 (1 + \omega_{*e}/\omega\tau) / k_{||}^2 v_A^2, \quad (9)$$

$$x_T^2 = \omega^2 (1 - \omega_{*e}/\omega) / k_{||}^2 c_s^2 (1 + \omega_{*e}/\omega\tau), \quad (10)$$

and

$$\Lambda = x_A^2 (1 - \omega_{*e}/\omega) / \rho_s^2 (1 + \omega_{*e}/\omega\tau). \quad (11)$$

$z_r = \text{Re } Z(\xi)$ and c_1 is an integration constant. As $|x| \rightarrow \infty$, we note from Eq. (6) that $\varepsilon(x)$ exhibits two types of perturbations; one is of short-wavelength drift-wave type and the other is of long-wavelength MHD kink or tearing-mode type (i.e., $\varepsilon \propto 1/x^2$). Since we are interested in drift-wave perturbations which do not match to external MHD perturbations (i.e., drift waves which do not need external MHD support), we impose the boundary condition that $|x^2 \varepsilon(x)| \rightarrow 0$ as $|x| \rightarrow \infty$ and, thus, $c_1 = 0$. Equation (6) then reduces to

$$\frac{d}{dx} \left(\frac{1}{f(x)} \frac{d\varepsilon}{dx} \right) + \Lambda \left(\frac{1}{x^2} - \frac{1}{x_A^2} \right) \varepsilon(x) = 0. \quad (12)$$

To solve Eq. (12), we note that there are three spatial scales in this problem; $|x_T| \sim \rho_s (L_s/L_n)^{1/2}$, $|x_A| \sim \rho_s \beta^{1/2} L_s/L_n$ and $|x_e| \sim \rho_s (m_e/m_i)^{1/2} L_s/L_n$. For typical tokamak parameters such that

$$m_e/m_i < \beta < L_n/L_s, \quad (13)$$

we have

$$|x_e| < |x_A| < |x_T|. \quad (14)$$

Equation (12) can then be solved separately in the following three distinctive regions: the outer region $|x| \sim |x_T|$, the middle region $|x| \sim |x_A|$, and the inner region $|x| \sim |x_e|$. The eigenvalue problem as posed by Eq. (12) is then completely determined by imposing the boundary conditions at $|x| = \infty$, the parity

requirements at the mode-rational surface ($x = 0$), and matching the solutions in the three regions.

In the outer region, $|x| \sim |x_T| > |x_A|$, $|x_e|$ and Eq. (12) becomes

$$\frac{d}{dx} \left(\frac{d\epsilon_0}{dx} \frac{1}{f_0(x)} \right) - \frac{\Lambda}{x_A^2} \epsilon_0 = 0, \quad (15)$$

where

$$f_0(x) = 1 - x^2/x_T^2. \quad (16)$$

Integrating Eq. (15) once and recalling $\epsilon_0(x) = d\phi_0/dx$, we have

$$\frac{d^2\phi_0}{dx^2} - \frac{\Lambda}{x_A^2} \left(1 - \frac{x^2}{x_T^2} \right) \phi_0 = c_2 \left(1 - \frac{x^2}{x_T^2} \right). \quad (17)$$

Again, noting that as $|x| \rightarrow \infty$, we are interested in short-wavelength drift-wave perturbations which do not match the external long-wavelength MHD perturbations ($\phi = \text{constant}$; i.e., perturbations of the $m = 1$ internal kink mode type), we, therefore, impose the condition that $c_2 = 0$. Equation (17) then becomes

$$\frac{d^2\phi_0}{dx^2} - \frac{\Lambda}{x_A^2} \left(1 - \frac{x^2}{x_T^2} \right) \phi_0 = 0. \quad (18)$$

Equation (18) admits solutions of parabolic cylinder functions. Imposing the outwardly propagating boundary condition or, equivalently, $|\phi_0| \rightarrow 0$ as $|x| \rightarrow \infty$ for $\text{Im } \omega > 0$, we obtain

$$\phi_0(x) = A_0 U(-\gamma, x/x_0) , \quad (19)$$

where

$$x_0 = (x_T^2 x_A^2 / 4\Lambda)^{1/4} \exp(-i\pi/4) , \quad (20)$$

and

$$\gamma = -x_0^2 \Lambda / x_A^2 . \quad (21)$$

In the middle region, we have $|x_e| < |x| \sim |x_A| < |x_T|$ and Eq. (12) becomes

$$\frac{d^2 \epsilon_M}{dx^2} + \Lambda \left(\frac{1}{x^2} - \frac{1}{x_A^2} \right) \epsilon_M(x) = 0 . \quad (22)$$

The solutions are Whittaker's functions; i.e.

$$\epsilon_M(x) = A_M \left[M_{0,\mu} \left(\frac{x}{x_M} \right) + S W_{0,\mu} \left(\frac{x}{x_M} \right) \right] , \quad (23)$$

where

$$x_M^2 = x_A^2 / 4\Lambda , \quad (24)$$

and, because $|\Lambda| \sim \beta L_s / L_n < 1$,

$$\mu = \left(\frac{1}{4} - \Lambda \right)^{1/2} \approx \frac{1}{2} - \Lambda . \quad (25)$$

In the inner region, $|x| \sim |x_e| < |x_A|, |x_T|$ and Eq. (12) reduces to

$$\frac{d}{dx} \left(\frac{1}{f_I(x)} \frac{d\epsilon_I}{dx} \right) + \frac{\Lambda}{x^2} \epsilon_I(x) = 0, \quad (26)$$

where

$$f_I(x) = 1 + \xi Z_r(\xi). \quad (27)$$

Noting, again, that $|\Lambda| \sim \beta L_s/L_n < 1$, we can solve Eq. (26) by treating the second term as a perturbation. Imposing the parity requirement that $\epsilon_I(x)$ be odd about $x = 0$, we obtain

$$\epsilon_{Io}^{(0)}(x) = A_{Io} \int_0^x f_I(x') dx' \left(1 - \Lambda \int_0^{x'} t^{-2} \epsilon_{Io}^{(0)}(t) dt \right), \quad (28)$$

where

$$\epsilon_{Io}^{(0)}(x) = \int_0^x f_I(x') dx'. \quad (29)$$

For even $\epsilon_I(x)$, we have

$$\epsilon_{Ie}(x) = A_{Ie} \left(1 + \Lambda \int_0^x t^{-1} f_I(t) dt \right). \quad (30)$$

We now take the various limits of ϵ_I , ϵ_M , and ϵ_O to match the solutions. The calculations are straightforward: for $|x| \gg |x_e|$ or $|\xi| \ll 1$, we obtain from Eqs. (28) and (30),

$$\epsilon_{Io}^{>}(x) \approx A_{Io} x (1 - \Lambda \ln x) \approx A_{Io} x^{1-\Lambda}, \quad (31)$$

and

$$\epsilon_{Ie}^{>}(x) \approx A_{Ie} (1 + \Lambda \ln x) \approx A_{Ie} x^{\Lambda}. \quad (32)$$

As to ε_M , we have for $|x/x_M| \ll 1$

$$\varepsilon_M^{<}(x) \approx A_M \left[S \left(\frac{x}{x_M} \right)^\Lambda \frac{\pi^{1/2}}{\cos \Lambda \pi} \cdot \frac{2^{-2\Lambda}}{\Gamma(1/2 + \Lambda)} + \left(\frac{x}{x_M} \right)^{1-\Lambda} \right. \\ \left. \times \left(1 - \frac{S \pi^{1/2} 2^{2\Lambda}}{4 \Gamma(3/2 - \Lambda) \cos \Lambda \pi} \right) \right], \quad (33)$$

and for $|x/x_M| \gg 1$

$$\varepsilon_M^{>}(x) \approx A_M \pi^{-1/2} \left\{ \left[\Gamma\left(\frac{3}{2} - \Lambda\right) 2^{1-2\Lambda} e^{-i\pi(1-\Lambda)} + S \pi^{1/2} \right] \exp\left(-\frac{x}{2x_M}\right) \right. \\ \left. + \Gamma\left(\frac{3}{2} - \Lambda\right) 2^{1-2\Lambda} \exp\left(\frac{x}{2x_M}\right) \right\}. \quad (34)$$

In the outer region, we obtain, from Eq. (19) and $\varepsilon_O(x) = d\phi_O/dx$, that for $|x/x_O| \ll 1$ and $|x/x_M| \gg 1$

$$\varepsilon_O^{<}(x) \approx -\tilde{A}_O \cos(\theta + \gamma^{1/2} x/x_O), \quad (35)$$

where

$$\tilde{A}_O = \frac{A_O}{x_O} \frac{[2\Gamma(3/2 + \gamma)]^{1/2}}{[2\pi(\gamma + 1)]^{1/4}}, \quad (36)$$

and

$$\theta = -\frac{\pi}{4} (2\gamma + 1). \quad (37)$$

Matching $\varepsilon_M^{>}(x)$ and $\varepsilon_O^{<}(x)$, we obtain

$$S = \Gamma(3/2 - \Lambda) 2^{1-2\Lambda} \pi^{-1/2} \left(e^{2i\theta} + e^{i\Lambda\pi} \right), \quad (38)$$

and

$$A_M = - \tilde{A}_O \pi^{1/2} 2^{-2(1-\Lambda)} e^{-i\theta} / \Gamma(3/2 - \Lambda) . \quad (39)$$

Matching $\epsilon_M^<(x)$ and $\epsilon_I^>(x)$, we obtain for $\epsilon_{IO}(x)$

$$S_O = 0 , \quad (40)$$

and

$$A_{IO} \approx A_M / x_M . \quad (41)$$

While, for $\epsilon_{Ie}(x)$, we obtain

$$S_e \approx \pi^{-1/2} 2^{2(1-\Lambda)} \Gamma(3/2 - \Lambda) \cos \Lambda \pi \quad (42)$$

and

$$A_{Ie} \approx 2 A_M . \quad (43)$$

Substituting Eqs. (40) and (42) into Eq. (38), and noting Eq. (37), we derive the following eigenvalue conditions: for odd- ϵ or even- ϕ modes,

$$\gamma_O = 2m + 1/2 - \Lambda ; \quad m = 0, 1, \dots \quad (44)$$

for even- ϵ or odd- ϕ modes,

$$\gamma_e = 2\ell + 3/2 + \Lambda ; \quad \ell = 0, 1, \dots . \quad (45)$$

Equations (44) and (45) can be combined into

$$\gamma = n + 1/2 + (-1)^{n+1} \Lambda , \quad n = 0, 1, \dots . \quad (46)$$

Thus, $n =$ non-negative even integers correspond to odd- ε (even- ϕ) eigenmodes, and $n =$ positive odd integers correspond to even- ε (odd- ϕ) eigenmodes. The Λ term in Eq. (46) represents the finite- β corrections to the zeroth-order eigenvalues $\gamma = n + 1/2$. The corresponding values of $\tilde{A}_O$, A_M and A_I will be calculated in Sec. 5 when we examine the structure of the eigenmodes.

4. DISPERSION RELATIONS

As mentioned in Sec. 1, we include the finite $-k_y^2$ and resonant-electron effects by perturbative theory. Since $\beta \ll 1$ and $|\omega_{A||}/ck_{||}\phi| \sim 0(\beta)$, perturbations in the eigenvalues $(\delta\gamma)$ due to these effects can be calculated assuming the modes are electrostatic. The results are standard, and we shall limit our considerations to the $n = 0, 1$ ($m = 0, \ell = 0$) eigenmodes. Extensions to other eigenmodes are straightforward. We have, for the $n = 0$ ($m = 0$) eigenmode,

$$\delta\gamma(0) = k_y^2 x_0^2 - (2\pi)^{1/2} (i - 1) \gamma(0) \frac{x_e}{|x_0|} \ln \left| \frac{x_0}{x_e} \right|, \quad (47)$$

and for the $n = 1$ ($\ell = 0$) eigenmode

$$\delta\gamma(1) = k_y^2 x_0^2 - \frac{(i - 1)}{2} \gamma(1) \frac{x_e}{|x_0|}. \quad (48)$$

Combining Eqs. (47) and (48) with Eq. (46) gives the final eigenvalue conditions

$$\gamma(0) = \frac{1}{2} - \Lambda + k_Y^2 x_0^2 - (2\pi)^{1/2} (i - 1) \gamma(0) \frac{x_e}{|x_0|} \ln \left| \frac{x_0}{x_e} \right|, \quad (49)$$

and

$$\gamma(1) = \frac{3}{2} + \Lambda + k_Y^2 x_0^2 - \frac{(i - 1)}{2} \gamma(1) \frac{x_e}{|x_0|}. \quad (50)$$

The dispersion relations are then given as follows: for the $n = 0$ radial eigenmode,

$$1 - \frac{\omega_{*e}}{\omega} = - \left(\frac{\tau + 1}{\tau} \right) \left(k_Y^2 \rho_s^2 - \beta + i \left| \frac{L_n}{L_s} \right| + i\beta k_Y^2 \rho_s^2 \left| \frac{L_s}{L_n} \right| \right) \cdot \left[1 - (i - 1) \left(\frac{\pi}{2} \right)^{1/2} \left| \frac{m_e L_s}{m_i L_n} \right|^{1/2} \ln \left(\left| \frac{m_i L_n}{m_e L_s} \right| \right) \right], \quad (51)$$

for the $n = 1$ radial eigenmode,

$$1 - \frac{\omega_{*e}}{\omega} = - \left(\frac{\tau + 1}{\tau} \right) \left(k_Y^2 \rho_s^2 + 3\beta + 3i \left| \frac{L_n}{L_s} \right| - i\beta k_Y^2 \rho_s^2 \left| \frac{L_s}{L_n} \right| \right) \cdot \left(1 - \frac{1}{2} \left| \frac{m_e L_s}{m_i L_n} \right|^{1/2} \right). \quad (52)$$

Equation (51) agrees with that derived in Ref. 1. It shows that, for the $n = 0$ eigenmode, finite- β effects are stabilizing.

While for the $n = 1$ eigenmode, Eq. (52) shows that finite- β effects further destabilize the mode. We note that the condition $|\Lambda| < 1$ requires, in addition to Eq. (13),

$$k_Y^2 \rho_s^2 \beta < 2L_n^2 / L_s^2, \quad (53)$$

which, for typical parameters $\beta \sim 1\%$ and $L_s/L_n \sim 15$, becomes $k_y^2 \rho_s^2 < 1$.

5. EIGENMODE STRUCTURE

Again, we limit our discussion to the $n = 0, 1$ eigenmodes. Furthermore, since we are particularly interested in the magnetic perturbations due to finite- β effects, we shall ignore finite- k_y^2 and resonant-electron effects to facilitate the discussion.

From Eqs. (36), (40), (41), and (44), we have for the $n = 0$ eigenmode

$$\begin{aligned}\tilde{A}_0(0) &= 2^{-1/2} A_0(0)/x_0, \\ A_M(0) &= -\tilde{A}_0(0)/2 = -i 2^{-3/2} A_0(0)/x_0, \\ S_0(0) &= 0,\end{aligned}\tag{54}$$

and

$$A_{IO}(0) = A_M(0)/x_M(0) = -i A_0(0)/2x_0^2.$$

Equation (54) along with Eqs. (19), (23), and (28) specifies the mode structures of the electric field $\epsilon(x)$. The mode structures of $A_{||}$ can be most easily obtained from Eqs. (3) and (4); i.e., with k_y^2 terms neglected,

$$\frac{d^2 A}{dx^2} = \frac{ck_{||}}{\omega} \frac{x_A^2}{x} \frac{d^2 \phi}{dx^2},\tag{55}$$

which becomes, after integrating twice,

$$A_{||}(x) = -x_A^2 \frac{ck_{||}x}{\omega} \int_x^\infty \frac{\varepsilon(x')}{x'^2} dx' \quad (56)$$

For the $n = 0$ eigenmode, with $\varepsilon(x')$ approximated by its value in the electrostatic limit, we obtain

$$A_{||}(x) = A_0(0) \frac{ck_{||}x_A^2}{\omega x_0} \begin{cases} \frac{x}{2x_0} \ln \frac{x}{x_0} , & |x_e| \ll |x| \ll |x_0| \\ \frac{x_0}{x} \exp\left(-\frac{x^2}{4x_0^2}\right) , & |x| \gg |x_0| . \end{cases} \quad (57)$$

For $|x| \ll |x_e|$, $\varepsilon_I(x) \propto x^3$ and $A_{||}(x) \propto x^3$. Thus, in this case, the radial (tearing) magnetic perturbations $b_x(x) = ik_y A_{||}(x)$ vanish at the mode-rational surface ($x = 0$), and the instabilities will not lead to the reconnection of field lines.

As to the $n = 1$ eigenmode, we obtain from Eqs. (36), (39), (42), and (43)

$$\begin{aligned} \tilde{A}_0(1) &= A_0(1)/x_0 , \\ A_M(1) &= \tilde{A}_0(1)/2 = A_0(1)/2x_0 , \\ S_e(1) &\approx 2 , \end{aligned} \quad (58)$$

and

$$A_{Ie}(1) \approx 2A_M(1) = A_0(1)/x_0 .$$

The mode structure of $A_{||}(x)$ can then be readily calculated from Eq. (56),

$$A_{||}(x) = A_O(1) \frac{x_A^2 c k_{||}}{\omega x_O} \begin{cases} -1, & |x| \ll |x_O| \\ \exp\left(-\frac{x^2}{4x_O^2}\right), & |x| \gg |x_O| \end{cases} \quad (59)$$

Thus, $A_{||}(x) \neq 0$ at $x = 0$ and we have a nonvanishing radial (tearing) magnetic perturbations at the mode-rational surface. The magnitude of $b_x(x=0) = ik_y A_{||}(x=0)$ is given by

$$\frac{b_x(x=0)}{B} = \frac{(i-1)}{2} k_y \rho_s \beta \left| \frac{L_s}{L_n} \right|^{1/2} \frac{e A_O(1)}{T_e} \quad (60)$$

In deriving Eq. (60), we have noted that $x_O^2 = -i \rho_s^2 |L_s/2L_n|$. Also, by comparing $A_{||}(x)$, Eq. (59), and

$$\phi(x) \approx A_O(1) (x/x_O) \exp(-x^2/4x_O^2),$$

we note that $|\phi| > |\omega A_{||}(x)/ck_{||}|$ for $|x| > |x_A|$. Thus, the $n=1$ eigenmode is predominantly electrostatic, except in the small region near the mode-rational surface ($|x| \leq |x_A| \sim \rho_s \beta^{1/2} |L_s/L_n|$) where electromagnetic perturbations dominate.

6. CONCLUSIONS AND DISCUSSIONS

We have presented in this work a new simplified approach to radial eigenmode analyses of finite- β modified drift-waves in a sheared magnetic field. This approach is based on separating finite- β effects from electrostatic effects due, for example, to finite- k_y^2 or resonant-electron terms. Observing the existence of

three spatial scales, a matching procedure is employed to obtain the finite- β effects. Electrostatic effects are then included via a perturbation analysis. Applying this approach to the universal drift mode, we recover, for the lowest ($n = 0$) radial eigenmode, the previously obtained results that finite- β effects are stabilizing. For the next ($n = 1$) radial eigenmode, however, we find the new result that finite- β effects are further destabilizing. We have also examined the corresponding eigenmode structure, particularly with respect to the question of finite radial (tearing) magnetic perturbations, b_x . We find for the $n = 0$ eigenmode that b_x vanishes at the mode-rational surface. For the $n = 1$ eigenmode, however, we find b_x has a finite value across the mode-rational surface. Thus, instabilities of the $n = 1$ eigenmodes (in fact, all n -values that are positive odd integers) will lead to the reconnection of field lines, and formation of microscopic magnetic islands.

Finally, let us briefly discuss the implications of such microscopic magnetic islands for anomalous transport in tokamaks. From Eq. (60), the widths of magnetic islands w are given by

$$\frac{w}{L_n} = 2^{3/2} \left| \frac{b_x(0)L_s}{B k_y} \right|^{1/2} = 2\beta^{1/2} \left(\frac{\rho_s}{L_n} \right)^{1/2} \left| \frac{L_s}{L_n} \right|^{3/4} \left| \frac{eA_0(1)}{T_e} \right|^{1/2} .$$

(61)

Meanwhile, the distance between two adjacent mode-rational surfaces in tokamaks is

$$\frac{\Delta r_s}{L_n} = (k_y \rho_s)^{-1} \left(\frac{\rho_s}{L_n} \right) \left(\frac{qR}{L_s} \right) . \quad (62)$$

Here, q is the usual safety factor. Taking typical parameters, $\rho_s/L_n \sim 10^{-2}$, $L_s/L_n \sim 15$, $qR/L_s \sim 1$, $\beta \sim 10^{-2}$, and the usual estimate of the drift waves saturation level $|eA_0(1)/T_e| \sim |e\phi/T_e| \sim \rho_s/L_n$, we have

$$\frac{w}{L_n} \sim 1.5 \times 10^{-2} , \quad (63)$$

and

$$\frac{w}{\Delta r_s} \approx 1.5 k_y \rho_s . \quad (64)$$

As noted in Eq. (53), the validity of our analyses is limited to $k_y^2 \rho_s^2 \lesssim 2^{1/2} L_n/L_s \beta^{1/2} \sim 1$. Thus, for short-wavelength modes, w can be comparable to Δr_s and there exists the interesting possibility of having these microscopic magnetic islands overlapping across several mode-rational surfaces. The overlap of magnetic islands can destroy magnetic surfaces, and produce magnetic field lines stochastically wandering (diffusing) in the radial direction; i.e., magnetic braiding [3]. With electron thermal conduction being mainly along $\underline{B}$, the occurrence of magnetic braiding will lead to enhanced radial heat transport. Assuming the magnetic field lines are stochastic, we may heuristically estimate the

electron energy confinement time τ_E by the following arguments. Taking heat conduction only along $\tilde{B}$, $\tau_E = L^2 \nu_e / v_e^2$. Here, L is the distance along $\tilde{B}$ that electrons must diffuse to reach the plasma boundary, ν_e is the electron-electron collisional frequency, and we have assumed $\nu_e L / v_e > 1$. Since the magnetic island width is w and the magnetic field lines are stochastically wandering in the radial direction, L is approximately given by $L = (a/w)^2 qR$. Here, a is the plasma minor radius. Thus, we estimate τ_E as

$$\tau_E = \left(\frac{a}{w} \right)^4 q^2 R^2 \nu_e / v_e^2. \quad (65)$$

For the parameters $\nu_e \sim 10^5$, $v_e \sim 1.5 \times 10^9$, $qR \sim 10^2$ and, from Eq. (63), $a/w \sim 10^2$, we have $\tau_E \sim 50$ msec; which agrees, in order of magnitude, with experimental values. Again, we stress that the above arguments are heuristic at best, and that much more detailed investigation is needed on this topic.

ACKNOWLEDGMENTS

The authors acknowledge useful discussions with Dr. A. H. Boozer, Dr. T. H. Stix, and Dr. W. M. Tang.

This work was supported by U. S. Energy Research and Development Administration Contract EY-76-C-02-3073.

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