RELIABILITY OF CRBR PRIMARY PIPING:
CRITIQUE OF STRESS-STRENGTH OVERLAP METHOD FOR
COLD-LEG INLET DOWNCOMER

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TECHNICAL NOTE:

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I. INTRODUCTION

A. CRBR Primary Piping Integrity

The integrity of the primary coolant piping of a liquid metal fast breeder reactor (LMFBR) is of fundamental concern for the transport of thermal energy from the reactor over the entire lifetime of the reactor plant. Due to the lack of operating plant experience for large LMFBR conditions, the reliability of the LMFBR primary piping must be inferred from indirect analysis. This analysis includes materials stress analysis (with support from experimental data) and structural reliability methods. The integrity of the Clinch River Breeder Reactor (CRBR) primary piping has been addressed by the applicant in the PSAR⁽¹⁾ and, more recently, in a topical report⁽²⁾ (herein referred to as WARD-D-0127). Particular concern is given to the possibility of a rupture of the cold-leg downcomer section of the piping, near the reactor vessel inlet. A guillotine (double-ended) pipe rupture followed by reactor scram (assumed to be operative per design) leads to boiling of sodium in the hot channel of the core.⁽³⁾ Independent analysis at BNL has confirmed this prediction and furthermore, it was also found that sodium boiled in the hot channel for pipe-break areas (2.2 ft^2) less than that for a double-ended (2.89 ft^2) pipe break⁽⁴⁾.

WARD-D-0127 attempts to establish that loss of primary piping integrity (and its implications for loss of intact, coolable geometry of the core as a result of a pipe rupture) is an extremely rare event and of sufficiently low probability that it need not be included as part of the design basis for the CRBR.

In WARD-D-0127, probabilities of failure of the inlet downcomer are based on three failure phenomena: failure by worst case loading, failure by general fatigue and failure by an initial crack propagating through the piping wall. In the present report, no attempt is made to determine the adequacy of these failure mechanisms to completely describe the failure modes of the primary piping. Nor does this report attempt to establish the adequacy of the stress analysis and fracture mechanics techniques that are employed in WARD-D-0127. Certain generic problems associated with the integrity of LMFBR primary piping have been discussed

in an earlier BNL report.⁽⁵⁾ Rather, this note focuses on the assessment of piping reliability that is presented in Section 6 of WARD-D-0127.

It should also be noted that in its licensing evaluation of the CRBR, the Nuclear Regulatory Commission and, in particular, its Materials Engineering Branch, is not using numerical probabilistic analyses in its assessment of the integrity of the primary coolant piping.

B. Stress-Strength Overlap and Reliability

Central to the reliability assessment is the stress-strength overlap method.⁽⁶⁾ According to this method, the parameters that characterize the applied stress and inherent strength are regarded as random variables with associated probability density functions. The probability of failure is then the probability that the stress will exceed the strength (and the reliability is the probability that the strength will exceed the stress). Given these definitions, the mathematical formulation of the reliability is, in general terms, straightforward.

Let S denote the strength and $P(S)$ denote its probability density function. Similarly, let s denote the stress and $Q(s)$ denote its probability density function. In principle, the variables S and s should be defined only for the physically realizable values of strength and stress. Then

$$\int_{R_S} P(S) dS = \int_{R_s} Q(s) ds = 1 \quad (I.1)$$

where R_S and R_s are the physically allowable ranges of S and s respectively. The probability density function of the random variable $Z = S - s$ is denoted by $L(Z)$ and related to $P(S)$ and $Q(s)$ by

$$L(Z) = \int_{R_S} dS \int_{R_s} ds P(S) Q(s) \delta (Z - (S - s)); \quad (I.2)$$

here $\delta(X)$ is the Dirac delta function⁽⁷⁾. The reliability, R , is then the probability that Z is positive and, from Eq. (1.2), is given by

$$R \equiv \int_{(Z \geq 0)} L(Z) dZ = \int_{R_S} dS \int_{R_s} ds P(S) Q(s) \theta(S-s) \quad (I.3)$$

In Eq. (I.3), $\theta(X) = 1$ for $X \geq 0$ and $\theta(X) = 0$ for $X < 0$. The failure probability, $\bar{R}$, is given by $\bar{R} = 1 - R$.

In order for Eq. (I.3) to meaningfully be regarded as a probability that can, at least in principle, be validated in the realm of experience, $P(S)$ and $Q(s)$ must themselves have probabilistic bases associated with their formulations.

In Section II of this report, a critical review of the reliability assessment of WARD-D-0127 is presented. In Section III, a sensitivity analysis is performed for the reliability assessment based on the stress-strength overlap method. This analysis is based on the population parameters (mean and standard deviation) presented in WARD-D-0127. Section IV summarizes this report and presents its conclusions.

II. REVIEW OF WARD-D-0127 RELIABILITY ASSESSMENT

The assessment of the reliability of the CRBR cold-leg inlet downcomer section of the primary piping is presented in Section 6 of WARD-D-0127. The conclusion of this work is that the failure probability of the cold-leg inlet downcomer piping section is less than 5×10^{-12} per reactor-year in the early years and approaches 1×10^{-10} per reactor-year in the latter years.

In Section 6, the reliability is obtained by evaluating Eq. (I.3), above. The probability density functions for the stress and strength are chosen to be represented by the normal distribution function. The normal distribution is, of course, characterized by two population parameters, the mean and the standard deviation. The means (for the worst case loading, general fatigue and initial crack propagation mechanisms) were obtained from analyses presented in the earlier sections of WARD-D-0127. The standard deviations, however, were subjectively chosen (see p. 6.1-2) due to the lack of a sufficient amount of applicable experimental data to establish actual standard deviations. Thus the arbitrary choice of the normal distribution and the selection of the standard deviation are not well justified in WARD-D-0127.

In connection with the use of the normal distribution, it should be noted that Eq. 5 on p. 6.2-2 of WARD-D-0127 is valid only if the normal distributions for the stress and strength are assumed to extend from minus infinity to plus infinity. This implies a finite probability for negative, and therefore unphysical, stresses and strengths as well as a finite probability for arbitrarily large positive strengths.

For certain values of the population parameters, the negative tail of the normal distribution contributes a negligible relative amount to the reliability ⁽⁸⁾ and therefore its use would be justified. However, the inclusion of a negative tail can yield misleading results. Specifically, it was stated that, for situations where the probability of failure is very small, the probability is quite insensitive to the stress level and is for practical purposes, a function of strength and its uncertainty. This cannot be correct. One can see this by considering

the case where the spread in values of stress is so small it can be treated as a fixed parameter equal to its mean value. Denote the probability that strength, S , is exceeded by stress, s , by $\text{Pr}[S < s]$. If this probability is independent of stress, its value is that given by $s = 0$, or

$$\text{Pr}[S < s] = \text{Pr}[S < 0]. \quad (\text{II.1})$$

But, $\text{Pr}[S < 0]$ is physically zero. A nonzero result is obtained for the normal distribution only because of the nonphysical negative tail of the distribution. In the actual calculations performed for Table 6.2-2, the mean of s was not assumed to be zero, but still the nonphysical negative tail contributed to the failure probability. For example, in the case corresponding to line 1 of Table 6.2-2, the failure probability of 2×10^{-13} includes a contribution from the area under the negative tail of the normal distribution of 7×10^{-14} .

III. SENSITIVITY ANALYSIS FOR THE STRESS-STRENGTH OVERLAP METHOD

In this section, the results of sensitivity studies for the reliability formulation of WARD-D-0127 are presented. This analysis takes the numerical values for the population parameters given in Table 6.2-2 as a basis for study. According to the table, the mean and standard deviations of the strength are approximately 10^{-1} and 10^{-2} , respectively. The mean and standard deviation of the stress are both on the order of 10^{-3} . For the purpose of this sensitivity study, the stress is taken to be a fixed number and not a random variable (or, equivalently, its density function is taken to be a Dirac delta function centered at the mean value of the distribution). This is reasonable in view of the relative narrowness of the stress distribution and the separation of its mean from the mean of the strength distribution. This assumption is made for computational convenience in order to illustrate the sensitivity of the reliability to the distribution for the strength.

A. Estimate of Failure Probabilities from Chebyshev's Inequality

The shape of the distribution function for the strength, treated as a random variable, is not known definitely, either from theoretical considerations or experiment. It is then appropriate to consider what can be said about the probability that the strength (here denoted by S) is less than the stress (denoted by s), given only a knowledge of the mean and standard deviation of the strength variable. Since, in this analysis, it is assumed that the stress s is a fixed number, and not a random variable, it is possible to use the Chebyshev inequality to estimate the failure probability.⁽⁹⁾

According to this inequality

$$\Pr [|S - \bar{S}| \geq k\sigma] \leq \frac{1}{k^2}, \quad (\text{III. A.1})$$

where σ is the standard deviation of S , $\bar{S}$ is the mean of S , and $k > 0$. If the probability density function for S is symmetric about its mean $\bar{S}$, then Eq. (III. A.1) becomes

$$\Pr [S \leq \bar{S} - k\sigma] \leq \frac{1}{2k^2} \quad (\text{III. A.2})$$

Set

$$k = \frac{\bar{S} - s}{\sigma} \quad (\text{III. A.3})$$

Then

$$\text{Pr } [S \leq s] \leq \frac{1}{2} \frac{\sigma^2}{(\bar{S} - s)^2} \quad (\text{III. A.4})$$

The quantity $\text{Pr } [S \leq s]$ is the failure probability, or unreliability. If the distribution is not symmetric, one can still say that

$$\text{Pr } [S \leq s] \leq \frac{\sigma^2}{(\bar{S} - s)^2} \quad (\text{III. A.5})$$

Consider values of $\bar{S}$, σ , and s corresponding to the worst case loading of line 1 of Table 6.2-2 of WARD-D-0127:

$$\bar{S} = .0918$$

$$\sigma = .0124 \quad (\text{III. A.6})$$

$$s = .001448$$

Then, from Eq. (III. A.4), assuming the distribution of S is symmetric,

$$\text{Pr } [S \leq s] \leq 9.4 \times 10^{-3} \quad (\text{III. A.7})$$

This result, Eq. (III. A.7), represents an upper bound on the failure probability, given that the distribution is symmetric, with $\bar{S}$, σ , and s given by Eq. (III. A.6). If the assumption that the distribution is symmetric is removed, then the upper bound on the failure probability is twice as great. Depending on the actual shape of the distribution, the actual failure probability can be much lower. However, the significance of the result is that one cannot be sure that the probability of failure is less than 10^{-2} , if one is not certain about the distribution shape. With the use of historical data it would be possible to justify a failure probability less than 10^{-2} , but it is difficult to see how historical data could justify a

failure probability on the order of 10^{-13} , as given in Table 6.2-2.

B. Normal Probability Distribution

In this subsection, the sensitivity of the reliability to variations of the standard deviation of the strength, σ_S , is presented. The following assumptions were made in the present analysis:

1. The stress s is deterministically known to have the value $s_o = 1.5 \times 10^{-3}$ (see Table 6.2-2 of WARD-D-0127).

2. The strength S is distributed according to a normal (Gaussian) probability density function truncated so that $\Pr [S < 0] = 0$, with mean value $\bar{S}$ and standard deviation σ .

Based on these two assumptions, the unreliability ($\bar{R}$) is given by (see also Figure III.1.).

$$\bar{R} = \Pr [0 \leq S \leq s_o] , \quad (\text{III. B.1})$$

or, in terms of the standardized random variable u (see Figure III.2),

$$\bar{R} = \Pr [-x \leq u \leq -\alpha] , \quad (\text{III. B.2})$$

where

$$u = \frac{S - \bar{S}}{\sigma} , \quad x = \frac{\bar{S}}{\sigma} , \quad \alpha = \frac{\bar{S} - s_o}{\sigma} \quad (\text{III. B.3})$$

By virtue of Eq. (III.B.1) and assumption 2 it follows that

$$\bar{R} = \frac{1}{c} \int_{-x}^{-\alpha} e^{-u^2/2} du \quad (\text{III.B.4})$$

where c is a normalization constant such that

$$\frac{1}{c} \int_{-x}^{\infty} e^{-u^2/2} du = 1 \quad (\text{III.B.5})$$

It can be easily verified that

$$1/c = \frac{1}{\sqrt{2\pi} - I(x)} \quad (\text{III.B.6})$$

where by definition

$$I(x) \equiv \int_x^{\infty} e^{-u^2/2} du \quad (\text{III.B.7})$$

By virtue of Eqs. III.B.4 and III.B.7 it follows that

$$\bar{R} = \frac{1}{c} [I(\alpha) - I(x)] \quad (\text{III.B.8})$$

If s_0 and $\bar{S}$ are assumed constant then x and α become functions of σ (see Eq. (III.B.3)). The unreliability, $\bar{R}$, has been evaluated for various values of σ and the results are tabulated in Table III.1 and shown in Figure III.3. The results indicate an extremely high sensitivity of the unreliability to the value of σ .

For values of x (or α) less than 3.5, the integrals $I(x)$, $I(\alpha)$ in Eq. (III.B.8) have been taken from Tables for the normal distribution. For values of x (or α) greater than 3.5 the integrals have been evaluated from their asymptotic expansion

$$\begin{aligned} I(x) &= \sqrt{2} I(t) \equiv \sqrt{2} \int_t^{\infty} e^{-u^2} du \\ t &= x/\sqrt{2} \\ &= \frac{\sqrt{2} e^{-t^2}}{2t} \left[1 - \frac{1}{2t^2} + \frac{1 \cdot 3}{(2t^2)^2} - \dots + \frac{(-1)^{\eta-1} 1 \cdot 3 \dots (2\eta-3)}{(2t^2)^{\eta-1}} \right] + r_{\eta} \end{aligned} \quad (\text{III.B.9})$$

where

$$|r_{\eta}| < \frac{\sqrt{2} 1 \cdot 3 \cdot 5 \dots (2\eta-1) e^{-t^2}}{2^{\eta} + 1 \quad t^{2\eta} + 1}$$

TABLE III.1

Unreliability ($\bar{R}$) of the cold-leg piping inlet downcomer section as a function of the standard deviation of the strength σ .

$\sigma/\bar{S}$	$x = \bar{S}/\sigma$	$\bar{R}$
1.00	1.00	5.71×10^{-3}
.900	1.11	5.06×10^{-3}
.800	1.25	4.14×10^{-3}
.700	1.43	3.13×10^{-3}
.600	1.66	2.10×10^{-3}
.500	2.00	1.63×10^{-3}
.400	2.50	7.04×10^{-4}
.300	3.33	7.29×10^{-5}
.250	4.00	1.08×10^{-5}
.200	5.00	1.25×10^{-7}
.180	5.55	1.03×10^{-8}
.150	6.66	1.29×10^{-11}
.140	7.14	5.80×10^{-13}
.135	7.40	1.34×10^{-13}
.130	7.69	1.21×10^{-14}
.120	8.33	7.73×10^{-17}
.100	10.00	2.90×10^{-23}

C. Sensitivity to the Shape of the Distribution

Section (III.A) showed that the upper-bound Chebyshev inequality estimate for the failure probability was, for the case examined, on the order of 10^{-2} . On the other hand, the applicant ⁽²⁾ obtained a result of 2×10^{-13} for the failure probability, on the assumption of a normal distribution. It is therefore appropriate to consider the sensitivity of the failure probability to distribution shape. In the present section distribution shapes such that the variable

$$\frac{S - \alpha}{\beta}$$

has a (truncated) Student's t distribution, ⁽¹⁰⁾ with various degrees of freedom, will be examined. The parameters α and β will be chosen in a manner discussed below.

The frequency function for the Student's t distribution with ν degrees of freedom is given by

$$f_{\nu}(t) = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\left(1 + \frac{t^2}{\nu}\right)^{\frac{\nu+1}{2}}} \quad (\text{III.C.1})$$

The variance, $\text{var}(t)$, of t is given by

$$\text{var}(t) = \frac{\nu}{\nu-2}, \quad (\text{III.C.2})$$

and the mean, $E(t)$, of t is zero.

If the probability distribution of the variable

$$y = \frac{S - \alpha}{\beta} \quad (\text{III.C.3})$$

is to be the Student's t distribution with ν degrees of freedom, then the variance of y must be given by Eq. (III.C.2), and the mean, $E(y)$, of

y must be zero. But

$$E(y) = (E(S) - \alpha)/\beta \quad (\text{III.C.4})$$

and

$$\text{var}(y) = \text{var}(S)/\beta^2 \quad (\text{III.C.5})$$

Hence, if y is to have a t distribution with ν degrees of freedom, it is necessary that

$$\alpha = E(S) \quad (\text{III.C.6})$$

and

$$\beta^2 = \frac{\text{var}(S)}{\text{var}(y)} = \frac{\nu-2}{\nu} \text{var}(S) \quad (\text{III.C.7})$$

It is, however, nonphysical to assume that the variable y has a t distribution, since this implies that the strength S has finite probability of attaining negative values.

Accordingly, the sensitivity analysis will be performed, assuming that the variable y has the frequency function

$$g_{\nu}(y) = \begin{cases} Cf_{\nu}(y), & y \geq \frac{-\alpha}{\beta} \\ 0, & y < \frac{-\alpha}{\beta} \end{cases} \quad (\text{III.C.8})$$

where $f_{\nu}(y)$ is given by Eq. (III.C.1), and C is a normalization constant. Then the strength S, related to y by Eq. (III.C.3), will have zero probability of attaining negative values. The distribution $g_{\nu}(y)$ of Eq. (III.C.8) will be referred to as a truncated t distribution.

With the use of the frequency function $g_{\nu}(y)$ (instead of $f_{\nu}(y)$), the variance of y is no longer given by Eq. (III.C.2), and the mean of y is

no longer zero. Hence Eqs. (III.C.6) and (III.C.7) are, strictly speaking, no longer valid. However, if the area

$$\int_{-\infty}^{\frac{-\alpha}{\beta}} f_v(y) dy$$

in the truncated tail of $f_v(y)$ is sufficiently small, then the errors in assuming the validity of Eqs. (III.C.6) and (III.C.7) will be negligible, for the purposes of the sensitivity analysis. The validity of Eqs. (III.C.6) and (III.C.7) will therefore be assumed. However, in one particular case, the error in the use of Eqs. (III.C.6) and (III.C.7) will be evaluated in order to check the assumption that Eqs. (III.C.6) and (III.C.7) hold.

In the sensitivity analysis, the parameters $\bar{S}$, σ , and s chosen corresponded to line 1 of the Worst Case Loading table, Table 6.2-2, of WARD-D-0127. The parameters are

$$\begin{aligned} E(S) = \bar{S} &= .0918 & (III.C.9) \\ \sigma &= .0124, \quad \sigma = \sqrt{\text{var}(S)} \\ s &= .001448 \end{aligned}$$

where $\bar{S}$ is the mean value of the "strength", σ is the standard deviation of the "strength", and s is the "stress", treated as a fixed number, and not a random variable. The words "strength" and "stress" are used here to identify the variables with those in the stress-strength overlap theory; in actuality, the quantity here called "strength" is the strain limit, and the quantity called "stress" is the true effective plastic strain.

Using the values of $\bar{S}$, σ and s given in Eq. (III.C.9), the failure probability was calculated using the frequency function $g_v(y)$ of Eq. (III.C.8), and using Eqs. (III.C.6) and (III.C.7) to define the parameters α and β . (Then $\alpha = .0918$ and $\beta = .0124 \sqrt{(v-2)/v}$). The calculation was performed for a number of degrees of freedom v equal to 3, 13, and 21. The results are given in Table III.2.

TABLE III.2

ν	Failure Probability
3	2.5×10^{-5}
13	2.0×10^{-7}
21	1.8×10^{-8}

It is seen that values of the failure probability very different from the applicant's value of 2×10^{-13} were obtained. It is useful to compare the frequency function for S implied by Eq. (III.C.8) with $\nu = 21$ to the normal frequency function assumed by the applicant. If $h_\nu(S)$ denotes the frequency function for S implied by Eq. (III.C.8), then

$$h_\nu(S) = g_\nu(y) \frac{dy}{dS} = \frac{1}{\beta} g_\nu\left(\frac{S-\alpha}{\beta}\right)$$

For $S \geq 0$, from Eqs. (III.C.8) (III.C.1), and (III.C.7) one obtains

$$h_\nu(S) = \frac{1}{\sigma\sqrt{\nu-2} \sqrt{\pi}} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\left(1 + \frac{(S-\alpha)^2}{(\nu-2)\sigma^2}\right)^{\frac{\nu+1}{2}}}, \quad (III.C.10)$$

where the constant C has been replaced by unity. (For $\nu = 21$ the computed value of C for the given values of α and β was $C = 1 + 6.4 \times 10^{-8}$) For $\nu = 21$, Eq. (III.C.10) becomes

$$h_\nu(S) = \frac{1}{\sigma} \frac{.4145}{\left(1 + \frac{(S-\alpha)^2}{19\sigma^2}\right)^{11}}. \quad (III.C.11)$$

The normal frequency function $N(S)$ is

$$N(S) = \frac{1}{\sigma} \frac{1}{\sqrt{2\pi}} e^{-(S-\alpha)^2/2\sigma^2} \quad (III.C.12)$$

A comparison of values of $\sigma h_\nu(S)$ to $\sigma N(S)$ for various values of $(S-\alpha)/\sigma$ is given in Table III.3. One sees that, for values of S within 2.4 standard deviations of the mean, $h_\nu(S)$ and $N(S)$ are within a few percent of each other. Nevertheless, at very large deviations, there is a considerable difference between $h_\nu(S)$ and $N(S)$.

TABLE III.3

Comparison of predictions of Student's t and normal
frequency functions for various deviations from the mean

$\frac{S - \alpha}{\sigma}$	$\sigma h_v(S), v = 21$	$\sigma N(S)$
0	.4145	.3989
.4	.3780	.3683
.8	.2879	.2897
1.2	.1856	.1942
1.6	.1032	.1109
2.0	.0507	.0540
2.4	.0225	.0224
3.0	5.82×10^{-3}	4.43×10^{-3}
4.0	5.00×10^{-4}	1.34×10^{-4}

The significance of this result is that a distribution which is very similar to a normal distribution for moderately large deviations from the mean can be very different from a normal distribution for the extreme, rarely occurring deviations. It would be very difficult, on the basis of empirical evidence alone, to choose between the distributions, since one could not hope to collect enough data to distinguish between the distributions (differing appreciably only for very rarely occurring events). The truncated t distribution with 21 degrees of freedom yields a failure probability of 2×10^{-8} , and a normal distribution, virtually indistinguishable from it on the basis of empirical evidence, yields a failure probability on the order of 10^{-13} .

The sensitivity analysis has, up to this point, assumed the validity of Eqs. (III.C.6) and (III.C.7), which are, strictly speaking, not valid for the truncated t distribution of Eq. (III.C.8). As was indicated earlier, the effect of this error was analyzed for one particular case. This case corresponded to $\nu = 3$. In this case, an exact analysis led to the result that $\alpha = .09172$ and $\beta = .012991$ for $\bar{S} = .0918$ and $\sigma = .0124$, when the truncated t distribution was used. The approximate method, based on Eqs. (III.C.6) and (III.C.7), assumed, for $\nu = 3$, $\alpha = \bar{S}$ and $\beta = \sigma$. In the exact analysis, the values of α and β were obtained by determining the values of α and β such that

$$\bar{S} = \int_0^{\infty} S g_{\nu} \left(\frac{S - \alpha}{\beta} \right) \frac{1}{\beta} dS \quad (\text{III.C.13})$$

$$\sigma^2 = \int_0^{\infty} (S - \bar{S})^2 g_{\nu} \left(\frac{S - \alpha}{\beta} \right) \frac{1}{\beta} dS, \quad (\text{III.C.14})$$

for the given values of $\bar{S}$ and σ . Equations (III.C.13) and (III.C.14) lead to

$$\frac{\bar{S}}{\beta} = \frac{\gamma \left(\frac{\pi}{2} + \text{Arctan } \gamma \right) + 1}{\frac{\pi}{2} + \text{Arctan } \gamma + \frac{\gamma}{1 + \gamma^2}} \quad (\text{III.C.15})$$

$$\sigma^2 + \bar{S}^2 = \alpha^2 + \beta^2, \quad (\text{III.C.16})$$

where $\gamma = \frac{\alpha}{\beta}$. Equations (III.C.15) and (III.C.16) were solved simultaneously for α and β , with the result $\alpha = .09172$, $\beta = .012991$. The failure probability, using the exact method, was 2.8×10^{-5} , while the approximate method yielded 2.5×10^{-5} . This agreement is considered adequate.

IV. SUMMARY AND CONCLUSION

In this report, a critique of the strength-stress overlap method for the reliability of the CRBR primary heat transport system piping was presented. The report addressed, in particular, the reliability assessment of WARD-D-0127 (Piping Integrity Status Report), which is part of the CRBR PSAR docket. It was found that the reliability assessment is extremely sensitive to the assumed shape for the probability density function for the strength (regarded as a random variable) of the cold-leg inlet downcomer section of the primary piping.

Based on the rigorous Chebyshev inequality, it was shown that the piping failure probability is less than 10^{-2} . On the other hand, it was shown that the failure probability can be much larger than $\sim 10^{-13}$, the typical value put forth in WARD-D-0127.

This second conclusion is based on two sensitivity studies. In the first study, it was shown that the failure probability is a rapidly varying function of the standard deviation of the normal distribution (that was chosen in WARD-D-0127 as the basis for the quantitative analysis). For example, if the standard deviation had been taken to be 18 percent of the mean (as opposed to the WARD-D-0127 value of 13.5 percent), then the failure probability would be greater than 10^{-8} .

In the second sensitivity study, the actual functional form of the probability density function for the strength was varied. The principal conclusion of this study was that a density function that is very similar (within a few percent) to a normal distribution for moderately large deviations from the mean, can be different for the extreme, rarely occurring deviations, and can yield a very different failure probability. As a corollary to this conclusion, it can be said that, since the failure probability is strongly dependent on the probability density function for large deviations from the mean, the accuracy of the reliability prediction is crucially dependent on the mathematical knowledge of the probability density function, as it characterizes extremely rare events.

In conclusion, it was shown that the failure probability for the cold-leg inlet downcomer section of the primary piping is extremely sensitive to assumptions that are inherent in the formalism associated with the quantitative prediction.

REFERENCES

1. Preliminary Safety Analysis Report, Clinch River Breeder Reactor Plant, Project Management Corporation.
2. Piping Integrity Status Report, WARD-D-0127, Westinghouse Electric Corporation, December 1975.
3. Amendment 5 (October 1975) of Ref 1.
4. Monthly Progress Report for February 1976, Fast Reactor Safety Division, Brookhaven National Laboratory.
5. Integrity of LMFBR Primary Piping: A Preliminary Evaluation, J. G. Y. Chow et al., BNL 19855, Brookhaven National Laboratory, September, 1974.
6. E. B. Haugen, Probabilistic Approaches to Design, (John Wiley and Sons, New York, 1968) pp. 145-150.
7. The Dirac delta is extensively used in certain areas of physics and engineering. It has the following properties:

$$\delta(x - x_0) = \begin{cases} 0, & \text{if } x \neq x_0 \\ +\infty, & \text{if } x = x_0 \end{cases}$$

and
$$\int_{-\infty}^{\infty} \delta(x - x_0) dx = 1$$

Furthermore, if $f(x)$ is well-defined at x_0 , then

$$\int_{-\infty}^{\infty} f(x) \delta(x - x_0) dx = f(x_0).$$

Also see P.A.M. Dirac, The Principles of Quantum Mechanics, (Oxford, Clarendon Press) 4th ed (1958); and L. Schwartz, Théorie des distributions (Paris, Hermann, 1950-1951).

8. See Ref 6, p. 46.
9. See H. Cramer, *Mathematical Methods of Statistics* (Princeton University Press, Princeton, 1946) p. 83.
10. See Ref 9, p. 237.

$$f(S) = \frac{1}{C} \exp \left[-\frac{1}{2} \left(\frac{S - \bar{S}}{\sigma} \right)^2 \right]$$

Figure 1. Probability density function for the random variable S

Figure 2. Probability density function for the standardized random variable $u = \frac{s - \bar{s}}{\sigma}$

$$x = \frac{\bar{s}}{\sigma}$$

$$\alpha = \frac{\bar{s} - s_0}{\sigma} = x \left(1 - \frac{s_0}{\sigma} \right)$$

Figure 3. Unreliability $\bar{R}$ versus the Standard Deviation σ