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SUBJECT: Steady State Heat Transfer in ORR Irradiation Capsules

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ABSTRACT

The differential equation and boundary conditions that describe steady state heat transfer in an Oak Ridge Research Reactor (ORR) irradiation capsule have been determined. The finite difference equations needed to obtain a numerical solution to the heat transfer problem were developed and an algorithm for the solution of these equations was specified. The effectiveness and accuracy of the method was not evaluated, since the computer program required for the solution was not written. Analytical solutions for the second order partial differential equation with non-linear boundary conditions describing heat transfer in the problem were attempted. Various simplifying assumptions were not successful.

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1. SUMMARY

Within a High Temperature Gas-Cooled Reactor (HTGR) core, fissile fuel may diffuse out of the fuel microspheres into the surrounding coating under the influence of a temperature gradient. This is called the "amoeba" effect. To provide temperature profiles across the fuel capsule and to help elucidate the mechanism for this phenomenon, both analytical and numerical methods were investigated. Only the numerical technique of finite differences was investigated to provide solution.

The irradiation capsule was divided into regions corresponding to the different materials of construction. The general equation describing steady state heat transfer in all regions is

$$\nabla[k(T)\nabla \cdot T(r,\theta)] = -S \quad (1)$$

This partial differential equation, together with the boundary conditions in each region, were converted to finite-difference form by means of Taylor series expansions and the method of undetermined coefficients.

An algorithm has been developed to compute the temperature profiles from the finite difference equations. Computer programming is still required to obtain the desired results. This new method of solution should provide more accurate results than those previously available since it accounts for the time dependence of material dimensions, heat transfer coefficients, and heat generation terms.

2. INTRODUCTION

2.1 Background

Irradiation tests are being conducted on prototype High Temperature Gas-Cooled Reactor (HTGR) fuel rods with simulated HTR conditions in the Oak Ridge Research Reactor (ORR). The fuel rods are made by placing silicon carbide coated microspheres containing fuel in a cylindrical mold. Pitch is then injected and the rod is heat treated to graphitize the pitch into a carbon matrix.

During irradiation, migration of the fuel into the fuel microsphere coating has been noted. Migration is believed to be due to a thermal diffusion mechanism induced by temperature gradients in the fuel rod. Hence, knowledge of the temperature profile in the fuel rod and surrounding graphite moderator assembly is important in evaluating the results of in-core testing of prototype fuels.

The heat transfer equations which must be solved to determine the temperature profile in an ORR irradiation capsule are complex even when the transient terms are neglected. Complexities are introduced by internal heat generation in both the fuel and moderator caused by nuclear fission of the fuel and gamma radiation heating of the fuel and moderator, both of which depend on capsule position because of axial reactor neutron and gamma flux profiles. Gaps between the fuel rod and moderator generate non-linear boundary conditions dependent on radiation and conduction. In addition, thermal conductivity and material dimensions are a function of both temperature and fluence, the time-integrated neutron flux.

2.2 Previous Work

A numerical solution to the heat transfer problem has been obtained previously, utilizing an available computer program, HEATING3 (14). However, this program requires a considerable number of manual calculations to determine the input parameters required to obtain a solution. Difficulties arise in using HEATING3 because the program was written for fixed boundary problems only. Thus, the temperature profile obtained from HEATING3 may not correspond to the physical dimensions of the fuel capsule at the calculated temperature. HEATING3 does not allow for calculation of these dimensions as they change with temperature from one iteration to the next.

To solve for a temperature profile with HEATING3, one must rely on the previous steady state solution to calculate the new input dimensions, which again will remain fixed for the subsequent solution. The new dimensions are manually calculated external to the program. The time required to obtain output from HEATING3 is about one to two days. The time required to obtain temperature profiles at various axial positions and times is long. For example, temperature profiles at three axial positions and three times could take 15 days.

HEATING3 also requires that all boundaries be parallel to the coordinate system. The geometry of a peripheral fuel rod within the capsule does not conform to this requirement. The shape of this fuel rod must be approximated or distorted to meet the requirements of HEATING3, thereby affecting the accuracy of the solution.

2.3 Objectives and Method of Attack

To obtain an accurate temperature profile in the ORR capsules in a reasonable amount of time, both analytical and numerical techniques were attempted. The analytical solution was attempted by making justifiable simplifications to the complex heat transfer equations that must be solved. The non-linear boundary conditions were examined for any simplifying assumptions that would render them linear. The non-constant coefficients of the equations were also examined to determine the extent of their dependence on temperature.

Standard techniques for the solution of partial differential equations were examined for application to the problem. A numerical approach was developed utilizing a finite difference approximation to the differential equation and the boundary conditions applicable in each section. The Liebman method was employed to solve the finite difference equations algebraically. This approach, unlike that of HEATING3, allowed use of the exact capsule geometry at any temperature and time.

To describe mathematically the physical model, an array of lattice points subdividing a one-eighth section into incremental units was imposed. This lattice was established by placing lines from the origin along radials at fixed angular intervals, $\Delta\theta$, on the one-eighth section and curves of constant radius at fixed radial increments, Δr . Finite difference equations were written for all lattice points.

3. THEORETICAL DISCUSSION

3.1 Steady State Model Derivation

Due to symmetry, only one-eighth of the cross-sectional area of the capsule need be considered. Figure 1 shows the area of interest. The section contains portions of two fuel rods, the graphite moderator, gaps containing helium-neon mixtures, and two stainless steel containment shells.

Cylindrical coordinates with the origin at the center of the capsule seemed best suited for analysis of heat transfer within the selected control volume. The temperature is both r and θ dependent. Actually, there is also an axial dependence, but this gradient is negligible in comparison. Simulation of heat transfer was performed at different axial planes to avoid consideration of the axial dependence.

The one-eighth section may be divided into regions according to the physical boundaries separating the various materials. Gaps filled with sweep gases are not considered as regions. Thus, there are five regions, as shown in Fig. 2.

The differential equation for steady state heat transfer by conduction is derived by making an energy balance on a differential control volume. Thermal conductivity is assumed to be constant within the differential element. An energy balance on a differential volume element, shown in Fig. 3, of dimensions ΔZ , Δr , and $r\Delta\theta$ yields the heat transfer equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = - \frac{S}{k(T)} \quad (2)$$

Equation 2 is applicable within a homogeneous region and is subject to the boundary conditions that apply to that region.

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CIRCULAR CROSS SECTION OF CAPSULE

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1. Center Fuel Rod
2. Peripheral Fuel Rod
3. Graphite Moderator
4. Inner Stainless Steel Shell
5. Outer Stainless Steel Shell

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REGIONS WITHIN ONE-EIGHTH SECTION

DATE 3-24-76	DRAWN BY RCM	FILE NO. CEPS-X-227	FIG. 2
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Differential equation for conduction heat transfer:

$$\frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = - \frac{S}{k(T)}$$

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DIFFERENTIAL VOLUME ELEMENT

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3.2 Boundary Conditions

There are two types of boundary conditions for the regions. Both specify flux at a surface. Due to symmetry the heat flux in the θ -direction is equal to zero or

$$-\frac{\partial T}{\partial \theta} = 0 \quad (3)$$

for $\theta = 0$ or $\pi/4$. At gaps, when the boundary surfaces are parallel to a curve of constant radius, the boundary condition is equal to the heat transfer between the two regions,

$$-k \frac{\partial T}{\partial r} = \frac{k_g}{\delta} (T_B - T_S) + \sigma e F (T_B^4 - T_S^4) \quad (4)$$

When the boundary surfaces are not parallel to a line of constant radius, the boundary condition is

$$-k \frac{\partial T}{\partial n} = \frac{k_g}{\delta} (T_B - T_S) + \sigma e F (T_B^4 - T_S^4) \quad (5)$$

where n is perpendicular to the surface.

The surface temperature, T_S , and the temperature across the gap, T_B , are position-dependent along the boundary surface. An analytical expression for T_S and T_B as functions of r and θ is not known.

Figure 4 presents the nine types of region boundaries possible for the capsule geometry and the boundary condition equations which must be satisfied at each. Note that at corners, two boundary conditions must be satisfied simultaneously. At the center of the central fuel rod, the boundary condition is

$$\frac{\partial T}{\partial r} = 0 \quad (6)$$

by symmetry rather than the boundary condition indicated in Fig. 4 for boundary type three.

Boundary

1, 2
3, 4
5, 6, 7, 8
9

Applicable Boundary
Condition Equation

3
4
3 and 4
5

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REGION BOUNDARIES

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4. ANALYTICAL SOLUTION

The first obstacle in obtaining an analytical solution is the non-linearity of the partial differential equation. Standard methods for solving partial differential equations such as separation of variables, Laplace transform, and series expansion apply to linear equations only. Various techniques for solving non-linear partial differential equations (1, 12) such as the Kirchhoff transformation of the dependent variable attempt to linearize the non-linear equation. For example, with equation

$$\nabla \cdot [k(T) \nabla T] = -S$$

the desired form is

$$\nabla \cdot [\nabla \psi] = -S$$

Hence,

$$\nabla \psi = k(T) \nabla T$$

Let

$$k(T) = d\psi/dT$$

and

$$\psi = \int_0^T k(T') dT'$$

Assume that

$$k(T) = a_0 + a_1 T + a_2 T^2$$

Then

$$\psi = \int_0^T (a_0 + a_1 T' + a_2 T'^2) dT' = a_0 T + \frac{a_1}{2} T^2 + \frac{a_2}{3} T^3$$

Then,

$$v_{\psi}(T) = \frac{d\psi}{dT} vT$$

and the linearized equation is

$$v_{\psi}^2 + S = 0$$

To generate the temperature profile, boundary conditions need to be evaluated. Because the boundary conditions in Eqs. (4) and (5) are not linear, an analytical solution is not possible.

5. NUMERICAL SOLUTION

5.1 Definition of Lattice Point Types

A solution to the heat transfer equation can be obtained numerically. The most powerful numerical technique is that of finite differences which transforms partial differential equations to algebraic equations (4, 6, 11, 12).

To describe heat transfer in the capsule, various combinations of boundary conditions must also be taken into account in the derivation of the finite difference equation, the result being that a number of equations are obtained. Figures 5 and 6 show the locations that a lattice point may have relative to the boundaries. These boundaries can either be parallel or skew to the coordinate system.

Regions with boundaries parallel to the coordinate system are termed normal regions, e.g., the center fuel rod and stainless steel containment shells. If any boundary in a region is not parallel to the coordinate system - e.g., the peripheral fuel rod and graphite moderator, the region is termed special. Figure 5 shows the possible positions of a point in a normal region and Fig. 6 shows those in a special region. Lattice points are either inside a region, in which case they are called interior, or on a boundary surface, in which case they are called boundary points. An interior simple point is one located at a distance greater than Δr or $r_{\Delta\theta}$ away from any boundary and is found in both normal and special regions.

An interior non-simple point is one located at a distance less than Δr or $r_{\Delta\theta}$ from a boundary. This situation may occur in either a normal or a special region. When a boundary surface in a normal region coincides with a lattice line, the lattice points formed by the intersections of lattice lines, in either the r - or θ -direction, and the boundary surface are called regular boundary points. A regular boundary point on a surface not parallel to the coordinate system can only occur when the lattice lines

Interior Non-Simple Point

Regular Boundary Point

Irregular Boundary Point

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LATTICE POINTS IN SPECIAL REGION

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in both the $-r$ and $-\theta$ direction intersect at a point on a boundary. In this case the point is simply called a regular boundary point in a special region.

If a lattice line crosses a boundary which does not coincide with a lattice line, the point formed by the intersection is called an irregular boundary point. In a normal region, irregular boundary points can be formed by the intersection of a boundary with a lattice line in the $-r$ direction only, while in a special region this can occur with either the r - or θ -lattice line.

5.2 Finite Difference Equation for an Interior Simple Point

To transform the differential equation into finite difference form, Taylor series expansions to the second order derivative are formulated to approximate the temperatures of four lattice points surrounding the point of interest (i, j) .

$$T_{i+1,j} = T_{i,j} + \Delta r \left. \frac{\partial T}{\partial r} \right|_{i,j} + \frac{(\Delta r)^2}{2} \left. \frac{\partial^2 T}{\partial r^2} \right|_{i,j} \quad (7)$$

$$T_{i-1,j} = T_{i,j} - \Delta r \left. \frac{\partial T}{\partial r} \right|_{i,j} + \frac{(\Delta r)^2}{2} \left. \frac{\partial^2 T}{\partial r^2} \right|_{i,j} \quad (8)$$

$$T_{i,j+1} = T_{i,j} + (r\Delta\theta) \left. \frac{\partial T}{\partial \theta} \right|_{i,j} + \frac{(r\Delta\theta)^2}{2} \left. \frac{\partial^2 T}{\partial \theta^2} \right|_{i,j} \quad (9)$$

$$T_{i,j-1} = T_{i,j} - (r\Delta\theta) \left. \frac{\partial T}{\partial \theta} \right|_{i,j} + \frac{(r\Delta\theta)^2}{2} \left. \frac{\partial^2 T}{\partial \theta^2} \right|_{i,j} \quad (10)$$

The partial derivatives of the Taylor series expansions are eliminated by the method of undetermined coefficients. Each equation, (7) through (10), is multiplied by a constant, A, B, C, and D respectively. The four equations are then added and all terms involving derivatives are collected on one side.

$$\begin{aligned}
& (A - B)\Delta r \left. \frac{\partial T}{\partial r} \right|_{i,j} + (C - D)(r\Delta\theta) \left. \frac{\partial T}{r\partial\theta} \right|_{i,j} + (A + B) \frac{(\Delta r)^2}{2} \left. \frac{\partial^2 T}{\partial r^2} \right|_{i,j} \\
& + (C + D) \frac{(r\Delta\theta)^2}{2} \left. \frac{\partial^2 T}{r^2 \partial \theta^2} \right|_{i,j} = A(T_{i+1,j} - T_{i,j}) + B(T_{i-1,j} - T_{i,j}) \\
& + C(T_{i,j+1} - T_{i,j}) + D(T_{i,j-1} - T_{i,j}) \quad (11)
\end{aligned}$$

Equating the coefficients in Eq. (11) with those of Eq. (2) gives

$$(A - B) \Delta r = \frac{1}{r}$$

$$(C - D)r\Delta\theta = 0$$

$$(A + B) \frac{(\Delta r)^2}{2} = 1$$

$$(C + D) \frac{(r\Delta\theta)^2}{2} = 1$$

If the system of four equations and four unknowns are solved, the values of the constants A, B, C, and D are obtained.

$$A = \frac{1}{(\Delta r)^2} + \frac{1}{2r\Delta r} = \frac{1}{(\Delta r)^2} \left(1 + \frac{1}{2r}\right)$$

$$B = \frac{1}{(\Delta r)^2} - \frac{1}{2r\Delta r} = \frac{1}{(\Delta r)^2} \left(1 - \frac{1}{2r}\right)$$

$$C = D = \frac{1}{(r\Delta\theta)^2} = \frac{1}{i^2} \frac{1}{(\Delta r)^2 (\Delta\theta)^2}$$

(Note r is equal to $i\Delta r$.) These coefficients are then substituted into the righthand side of Eq. (11).

$$\frac{-S}{k(T_{i,j})} = \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right]_{i,j} = \frac{1}{(\Delta r)^2} \left[\left(1 - \frac{1}{2i}\right) T_{i-1,j} + \left(1 + \frac{1}{2i}\right) T_{i+1,j} - 2T_{i,j} \right] + \frac{1}{i^2 (\Delta r \Delta \theta)^2} (T_{i,j-1} + T_{i,j+1} - 2T_{i,j})$$

where:

$$i = 1, 2, 3, 4, \dots$$

$$j = 1, 2, 3, 4, \dots$$

If $T_{i,j}$ is solved, the finite difference equation is

$$T_{i,j} = \frac{\frac{1}{(\Delta r)^2} \left[\left(1 - \frac{1}{2i}\right) T_{i-1,j} + \left(1 + \frac{1}{2i}\right) T_{i+1,j} \right] + \frac{1}{i^2 (\Delta r \Delta \theta)^2} (T_{i,j-1} + T_{i,j+1}) + \frac{S}{k(T_{i,j})}}{\frac{2}{(\Delta r)^2} + \frac{2}{i^2 (\Delta r \Delta \theta)^2}} \quad (12)$$

5.3 Finite Difference Equation for Interior Non-Simple Point

The distance from a non-simple interior point in a normal region to a boundary is less than Δr . Thus a fraction of this distance must be employed in the derivation of the finite difference equation. Let $a\Delta r$ represent this distance between the point (i,j) and the boundary. The procedure is the same for an interior simple point. For example, assume that point (i,j) is near the boundary of the peripheral fuel rod. In this case the temperature, $T_{i+a,j}$, of the point formed by the intersection of the curve of constant radius and the boundary must be used, rather than $T_{i+1,j}$. Thus,

$$T_{i+a,j} = T_{i,j} + a\Delta r \left. \frac{\partial T}{\partial r} \right|_{i,j} + \frac{(a\Delta r)^2}{2} \left. \frac{\partial^2 T}{\partial r^2} \right|_{i,j} \quad (13)$$

Equations (8), (9), and (10) are again used for the expansion in the other directions. Multiplying Eqs. (13), (8), (9), and (10) through by A, B, C, and D respectively, adding the equations, and collecting the like terms, one obtains

$$\begin{aligned}
& (Aa - B)\Delta r \left. \frac{\partial T}{\partial r} \right|_{i,j} + (C - D)(r\Delta\theta) \left. \frac{\partial T}{\partial\theta} \right|_{i,j} + (Aa^2 + B)\frac{(\Delta r)^2}{2} \left. \frac{\partial^2 T}{\partial r^2} \right|_{i,j} \\
& + (C + D)\frac{(r\Delta\theta)^2}{2} \left. \frac{\partial^2 T}{\partial\theta^2} \right|_{i,j} = A(T_{i+a,j} - T_{i,j}) + B(T_{i-1,j} - T_{i,j}) \\
& + C(T_{i,j+1} - T_{i,j}) + D(T_{i,j-1} - T_{i,j}) \quad (14)
\end{aligned}$$

Again equating the coefficients with those of Eq. (2) gives

$$(Aa - B)\Delta r = \frac{1}{r}$$

$$(C - D)r\Delta\theta = 0$$

$$(Aa^2 + B)\frac{(\Delta r)^2}{2} = 1$$

$$(C + D)\frac{(r\Delta\theta)^2}{2} = 1$$

Solving these equations for the values of the constants A, B, C, and D gives

$$A = \frac{1}{ai(\Delta r)^2} + \frac{1 - \frac{a}{2r}}{\frac{a(\Delta r)^2}{2}(a + 1)}$$

$$B = \frac{1 - \frac{a}{2r}}{\frac{(\Delta r)^2}{2}(a + 1)}$$

$$C = D = \frac{1}{i^2} \frac{1}{(\Delta r\Delta\theta)^2}$$

Substitute these coefficients in the righthand side of Eq. (14) and solve for $T_{i,j}$:

$$T_{i,j} = \frac{T_{i+a,j} \left[\frac{1}{ia(\Delta r)^2} + \frac{1 - \frac{a}{2i}}{\frac{a(\Delta r)^2}{2}(a+1)} \right] + T_{i-1,j} \left[\frac{1 - \frac{a}{2i}}{(\Delta r)^2(a+1)} \right] + [T_{i,j+1} + T_{i,j-1}] \left[\frac{1}{(i\Delta r\Delta\theta)^2} \right] + \frac{S}{k}}{\frac{1}{ia(\Delta r)^2} + \frac{1 - \frac{a}{2i}}{\frac{a(\Delta r)^2}{2}(a+1)} + \frac{1 - \frac{a}{2i}}{(\Delta r)^2(a+1)} + \frac{2}{(i\Delta r\Delta\theta)^2}}$$

By a similar procedure the finite difference equations for the remaining types of non-simple points may be developed.

5.4 Table of Equations

Regular boundary, irregular boundary, and interior non-simple lattice points require geometrical considerations in developing the finite-difference equations. The boundaries of the peripheral fuel rod and the surrounding graphite moderator are treated differently. To circumvent the difficulty of relating the heat flux leaving normal to the fuel rod surface to the system coordinates, the usual procedure of calculating the temperature at a point from the temperature of points in the four surrounding directions has been modified. The conductive heat flux between lattice points is evaluated by numerically approximating the differential boundary conditions and resolving the heat flux by conduction normal to the surface. Thus, input and output fluxes described by the appropriate boundary conditions or heat conduction terms are equated and the temperature at the boundary point is found. Complete procedures are found in Appendix 10.4.2.

The equations for each of the different lattice types are presented in Table 1 together with the boundary type number (Fig. 4) and quadrant numbers (Fig. 7).

6. PROPOSED PROGRAM LOGIC

With the finite difference equations presented in Sect. 5 and the required input information, outlined in Appendix 10.6, it is possible to write a computer program to solve for the temperature at any point in the fuel capsule. The following outline of the necessary program logic includes the evaluation of functions describing the various parameters, the method of differentiation, and the treatment of each type of lattice point. The logic is summarized in the flow chart shown in Fig. 8.

Table 1. Finite Difference Equations

Point Type	Boundary No. (see Fig. 4)	Equation	
Interior Simple Point	-	$T_{i,j} = \frac{\left(\frac{1}{\Delta r}\right)^2 \left[\left(1 - \frac{1}{2i}\right) T_{i-1,j} + \left(1 + \frac{1}{2i}\right) T_{i+1,j} \right] + \frac{1}{(\Delta r \Delta \theta)^2} (T_{i,j-1} + T_{i,j+1}) + \frac{S}{k}}{\frac{2}{\Delta r^2} + \frac{2}{(\Delta r \Delta \theta)^2}} \quad (15)$	
General Equation Interior Non-Simple	-	$T_{i,j} = \frac{AT_i + a_{p,j} + BT_{i-a_n,j} + CT_{i,j+b_p} + DT_{i,j-b_n} + \frac{S}{k}}{A + B + C + D} \quad (16)$ $A = \frac{a_n^2}{a_p a_n + a_p^2} \left[\frac{2}{(a_n \Delta r)^2} - \frac{a_p}{a_n^2 \Delta r^2} \right] + \frac{1}{a_p \Delta r^2}$ $B = \left[\frac{2}{(a_n \Delta r)^2} - \frac{a_p}{a_n^2 \Delta r^2} \right] \frac{a_n}{a_p + a_p}$ $C = \frac{2b_n/b_p}{b_n/b_p (\Delta r \Delta \theta)^2 + (1 - b_n \Delta \theta)^2}$ $D = \frac{2}{b_n/b_p (\Delta r \Delta \theta)^2 + (1 + b_n \Delta \theta)^2}$	
Regular Boundary Normal Region	1	$T_{i,j} = \frac{\frac{2T_{i,j+1}}{(\Delta r \Delta \theta)^2} + \frac{2i-1}{2(\Delta r)^2} T_{i-1,j} + \frac{2i+1}{2(\Delta r)^2} T_{i+1,j} + \frac{S}{k}}{\frac{2}{(\Delta r \Delta \theta)^2} + \frac{2}{(\Delta r)^2}} \quad (17)$	
Regular Boundary Normal Region	2	$T_{i,j} = \frac{\frac{1}{(\Delta r)^2} \left[\left(1 - \frac{1}{2i}\right) T_{i-1,j} + \left(1 + \frac{1}{2i}\right) T_{i+1,j} \right] + \frac{2T_{i,j+1}}{(\Delta r \Delta \theta)^2} + \frac{S}{k}}{\frac{2}{(\Delta r)^2} + \frac{2}{(\Delta r \Delta \theta)^2}} \quad (18)$	
Regular Boundary Normal Region	3	$T_{i,j} = \frac{\frac{1}{(\Delta r \Delta \theta)^2} T_{i,j+1} + \frac{2T_{i+1,j}}{(\Delta r)^2} + \frac{T_{i,j-1}}{(\Delta r \Delta \theta)^2} - \left(\frac{1-2i}{\Delta r}\right) \left[\frac{k_g}{\Delta r} T_\theta + \frac{\pi F}{k} (T_\theta^4 - T_{i,j}^4) \right] + \frac{S}{k}}{\frac{2}{(\Delta r \Delta \theta)^2} + \frac{2}{(\Delta r)^2} - \left[\frac{1-2i}{\Delta r} \right] \frac{k_g}{\Delta r}} \quad (19)$	
Regular Boundary Normal Region	4	$T_{i,j} = \frac{\frac{T_{i,j+1} + T_{i,j-1}}{(\Delta r \Delta \theta)^2} + \frac{2T_{i+1,j}}{(\Delta r)^2} + \frac{S}{k} + \left[\frac{1+2i}{\Delta r} \right] \left[\frac{k_g}{\Delta r} T_\theta + \frac{\pi F}{k} (T_\theta^4 - T_{i,j}^4) \right]}{\frac{2}{(\Delta r \Delta \theta)^2} + \frac{2}{(\Delta r)^2} + \left[\frac{1+2i}{\Delta r} \right] \frac{k_g}{\Delta r}} \quad (20)$	
Regular Boundary Normal Region	5	$T_{i,j} = \frac{\frac{2T_{i,j+1}}{(\Delta r \Delta \theta)^2} + \frac{2T_{i+1,j}}{(\Delta r)^2} + \left(\frac{1+2i}{\Delta r}\right) \left[\frac{k_g}{\Delta r} T_\theta + \frac{\pi F}{k} (T_\theta^4 - T_{i,j}^4) \right] + \frac{S}{k}}{\frac{2}{(\Delta r \Delta \theta)^2} + \frac{2}{(\Delta r)^2} + \left[\frac{1+2i}{\Delta r} \right] \frac{k_g}{\Delta r}} \quad (21)$	
Regular Boundary Normal or Special Region	6	$T_{i,j} = \frac{\left[\frac{2}{(\Delta r)^2} \right] T_{i-1,j} + \frac{2T_{i,j+1}}{(\Delta r \Delta \theta)^2} - \left[\frac{1+2i}{\Delta r} \right] \left[\frac{k_g}{\Delta r} T_\theta + \frac{\pi F}{k} (T_\theta^4 - T_{i,j}^4) \right] + \frac{S}{k}}{\frac{2}{(\Delta r)^2} + \frac{2}{(\Delta r \Delta \theta)^2} + \left[\frac{1+2i}{\Delta r} \right] \frac{k_g}{\Delta r}} \quad (22)$	

Table 1. Finite Difference Equations (continued)

Point Type	Boundary No. (see Fig. 4)	Equation	
Regular Boundary Normal Region	7	$T_{i,j} = \frac{\frac{2T_{i+1,j}}{(i-r)^2} + \frac{2T_{i,j-1}}{(i-r-9)^2} - \left(\frac{1-2i}{i-r}\right) \left[\frac{k_g}{k} T_B + \frac{-ef}{k} (T_B^4 - T_{i,j}^4)\right] + \frac{5}{k}}{\frac{2}{(i-r)^2} + \frac{2}{(i-r-9)^2} + \left(\frac{1-2i}{i-r}\right) \frac{k_g}{k}}$	(23)
Regular Boundary Normal Region	8	$T_{i,j} = \frac{\frac{2T_{i-1,j}}{(i-r)^2} + \frac{2T_{i,j-1}}{(i-r-9)^2} + \left(\frac{i+2i}{i-r}\right) \left[\frac{k_g}{k} T_B + \frac{-ef}{k} (T_B^4 - T_{i,j}^4)\right]}{\frac{2}{(i-r)^2} + \frac{2}{(i-r-9)^2} + \left(\frac{i+2i}{i-r}\right) \frac{k_g}{k}}$	(24)
Regular Boundary Special Region Quadrant 1	9	$T_{i,j} = \frac{\frac{kT_{i,j}}{\Delta r \cos \theta} + \frac{k_g}{G} T_B + \frac{-ef}{k} (T_B^4 - T_{i,j}^4)}{\frac{k}{\Delta r \cos \theta} + \frac{k_g}{G}} \quad \theta = (-, -\theta)$	(25)
Regular Boundary Special Region Quadrant 2	9	$T_{i,j} = \frac{\frac{k_g}{G} T_B + \frac{-ef}{k} (T_B^4 - T_{i,j}^4) - \frac{kT_{i,j}}{\Delta r \cos(-\theta)}}{\frac{-k}{\Delta r \cos(-\theta)} + \frac{k_g}{G}}$	(26)
Regular Boundary Special Region Quadrant 3	9	$T_{i,j} = \frac{\frac{k_g}{G} T_B + \frac{-ef}{k} (T_B^4 - T_{i,j}^4) - \frac{kT_{i,j}}{\Delta r \cos(\gamma-\theta)}}{\frac{k_g}{G} - \Delta r \cos(\gamma-\theta)}$	(27)
Regular Boundary Special Region Quadrant 4	9	$T_{i,j} = \frac{\frac{k_g}{G} T_B + \frac{-ef}{k} (T_B^4 - T_{i,j}^4) + \frac{kT_{i,j}}{\Delta r \cos(-\theta)}}{\frac{-k}{\Delta r \cos(-\theta)} + \frac{k_g}{G}}$	(28)
Irregular Boundary Normal Region	3	$T_{i-a,j} = \frac{\frac{1}{(i-r-9)^2} T_{i-a,j+1} + \frac{2T_{i,j}}{(a-r)^2} + \left(\frac{1}{i-r} + \frac{2}{a-r}\right) \left[\frac{k_g}{k} T_B + \frac{-ef}{k} (T_B^4 - T_{i-a,j}^4)\right]}{\frac{2}{(i-r-9)^2} + \frac{2}{(a-r)^2} + \left(\frac{1}{i-r} + \frac{2}{a-r}\right) \frac{k_g}{k}}$	(29)
Irregular Boundary Point Normal Region	4	$T_{i+a,j} = \frac{\frac{T_{i+a,j+1} + T_{i+a,j-1}}{(i-r-9)^2} + \frac{2T_{i,j}}{(a-r)^2} + \left(\frac{1}{i-r} + \frac{2}{a-r}\right) \left[\frac{k_g}{k} T_B + \frac{-ef}{k} (T_B^4 - T_{i+a,j}^4)\right]}{\frac{2}{(i-r-9)^2} + \frac{2}{(a-r)^2} + \left(\frac{1}{i-r} + \frac{2}{a-r}\right) \frac{k_g}{k}}$	(30)
Irregular Boundary Point Normal Region	5	$T_{i-a,j} = \frac{\frac{2T_{i-a,j+1}}{(i-r-9)^2} + \frac{2T_{i,j}}{(a_p-r)^2} + \left(\frac{1}{i-r} + \frac{2}{(a_p-r)^2}\right) \left[\frac{k_g}{k} T_B + \frac{-ef}{k} (T_B^4 - T_{i-a,j}^4)\right]}{\frac{2}{(i-r-9)^2} + \frac{2}{(a_p-r)^2} + \left(\frac{1}{i-r} + \frac{2}{(a_p-r)^2}\right) \frac{k_g}{k}}$	(31)
Irregular Boundary Point Normal Region	6	$T_{i+a,j} = \frac{\frac{-2}{(a-r)^2} T_{i,j} + \frac{2T_{i+a,j+1}}{(i-r-9)^2} + \left(\frac{1}{i-r} + \frac{2}{a-r}\right) \left[\frac{k_g}{k} T_B + \frac{-ef}{k} (T_B^4 - T_{i+a,j}^4)\right]}{\frac{2}{(a-r)^2} + \frac{2}{(i-r-9)^2} + \frac{k_g}{k} \left(\frac{1}{i-r} + \frac{2}{a-r}\right)}$	(32)
Irregular Boundary Special Region	7	$T_{i-a,j} = \frac{\frac{2T_{i,j}}{(a-r)^2} + \frac{2T_{i-a,j+1}}{(i-r-9)^2} - \left(\frac{1}{i-r} + \frac{2}{a-r}\right) \left[\frac{k_g}{k} T_B + \frac{-ef}{k} (T_B^4 - T_{i-a,j}^4)\right]}{\frac{2}{(a-r)^2} + \frac{2}{(i-r-9)^2} - \left(\frac{1}{i-r} + \frac{2}{a-r}\right) \frac{k_g}{k}}$	(33)

* For quadrant specifications, see Fig. 7.

Table 1. Finite Difference Equations (continued)

Point Type	Boundary No. (see Fig. 3)	Equation	
Irregular Boundary Special Region	8	$T_{i+a,j} = \frac{\frac{2T_{i,j}}{(a\Delta r)^2} + \frac{2T_{i+a,j-1}}{(i\Delta r\Delta\theta)^2} + \left(\frac{1}{i\Delta r} + \frac{2}{a\Delta r}\right)\left[\frac{k_B}{6k}T_B + \frac{\rho c F}{k}(T_B^A - T_{i+a,j}^A)\right]}{\frac{2}{(a\Delta r)^2} + \frac{2}{(i\Delta r\Delta\theta)^2} + \left(\frac{1}{i\Delta r} + \frac{2}{a\Delta r}\right)\frac{k_B}{6k}} \quad (34)$	(34)
Irregular Boundary Point Quadrant 1	9	$T_{i-a,j} = \frac{\frac{kT_{i,j}}{a\Delta r \cos(\gamma-\theta)} + \frac{k_B}{6}T_B + \rho c F(T_B^A - T_{i-a,j}^A)}{a\Delta r \cos(\gamma-\theta) + \frac{k_B}{6}} \quad (35)$	(35)
Irregular Boundary Point Quadrant 2	9	$T_{i-a,j} = \frac{\frac{k_B}{6}T_B + \rho c F(T_B^A - T_{i-a,j}^A) - \frac{kT_{i,j}}{a\Delta r \cos(\gamma-\theta)}}{\frac{k_B}{6} - a\Delta r \cos(\gamma-\theta)} \quad (36)$	(36)
Irregular Boundary Point Quadrant 3	9	$T_{i+a,j} = \frac{\frac{k_B}{6}T_B + \rho c F(T_B^A - T_{i+a,j}^A) - \frac{kT_{i,j}}{a\Delta r \cos(\gamma-\theta)}}{\frac{k_B}{6} - a\Delta r \cos(\gamma-\theta)} \quad (37)$	(37)
Irregular Boundary Point Quadrant 4	9	$T_{i+a,j} = \frac{\frac{k_B}{6}T_B + \rho c F(T_B^A - T_{i+a,j}^A) + \frac{kT_{i,j}}{a\Delta r \cos(\gamma-\theta)}}{a\Delta r \cos(\gamma-\theta) + \frac{k_B}{6}} \quad (38)$	(38)
Irregular Boundary Quadrant 1	9	$T_{i,j-b} = \frac{\frac{kT_{i,j}}{rb\Delta\theta \sin(\gamma-\theta)} + \frac{k_B}{6}T_B + \rho c F(T_B^A - T_{i,j-b}^A)}{rb\Delta\theta \sin(\gamma-\theta) + \frac{k_B}{6}} \quad (39)$	(39)
Irregular Boundary Quadrant 2	9	$T_{i,j-b} = \frac{\frac{k_B}{6}T_B + \rho c F(T_B^A - T_{i,j-b}^A) + \frac{kT_{i,j}}{rb\Delta\theta \sin(\gamma-\theta)}}{\frac{k_B}{6} + rb\Delta\theta \sin(\gamma-\theta)} \quad (40)$	(40)
Irregular Boundary Quadrant 3	9	$T_{i,j+b} = \frac{\frac{k_B}{6}T_B + \rho c F(T_B^A - T_{i,j+b}^A) + \frac{kT_{i,j}}{rb\Delta\theta \sin(\gamma-\theta)}}{\frac{k_B}{6} + rb\Delta\theta \sin(\gamma-\theta)} \quad (41)$	(41)
Irregular Boundary Quadrant 4	9	$T_{i,j+b} = \frac{\frac{k_B}{6}T_B + \rho c F(T_B^A - T_{i,j+b}^A) + \frac{kT_{i,j}}{rb\Delta\theta \sin(\gamma-\theta)}}{\frac{k_B}{6} + rb\Delta\theta \sin(\gamma-\theta)} \quad (42)$	(42)
Irregular Boundary Special Region	6	$T_{i+a,j} = \frac{\frac{2}{(a\Delta r)^2}T_{i+a,j} + \frac{2T_{i+a,j+1}}{(i\Delta r\Delta\theta)^2} + \left(\frac{1}{i\Delta r} + \frac{2}{a\Delta r}\right)\left[\frac{k_B}{6k}T_B + \frac{\rho c F}{k}(T_B^A - T_{i+a,j}^A)\right]}{\frac{2}{(a\Delta r)^2} + \frac{2}{(i\Delta r\Delta\theta)^2} + \frac{k_B}{6k}\left(\frac{1}{i\Delta r} + \frac{2}{a\Delta r}\right)} \quad (43)$	(43)
Irregular Boundary Special Region	5	$T_{i-a,j} = \frac{\frac{2T_{i,j+1}}{(i\Delta r\Delta\theta)^2} + \frac{2T_{i-a,j}}{(a\Delta r)^2} + \left(\frac{1}{i\Delta r} + \frac{2}{a\Delta r}\right)\left[\frac{k_B}{6k}T_B + \frac{\rho c F}{k}(T_B^A - T_{i-a,j}^A)\right]}{\frac{2}{(i\Delta r\Delta\theta)^2} + \frac{2}{(a\Delta r)^2} + \left(\frac{1}{i\Delta r} + \frac{2}{a\Delta r}\right)\frac{k_B}{6k}} \quad (44)$	(44)

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SPECIAL REGION QUADRANT NUMBERS

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SIMPLIFIED LOGIC FLOW DIAGRAM

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8.2

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SIMPLIFIED LOGIC FLOW DIAGRAM

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6.1 Preliminary Function Calculation

The heat generation term for each material is evaluated as a function of fluence. The reactor flux is determined at the point of interest from the reactor flux profile which is given as a function of axial position by a polynomial of the form

$$f = C_1 + C_2Z + C_3Z^2 + C_4Z^4$$

The coefficients C_1 through C_4 are empirical. The flux at each point is considered constant and the fluence is simply,

$$\Phi = f \cdot t$$

The heat generation term for each material can be calculated with another polynomial of the form

$$q = C_1 + C_2\Phi + C_3\Phi^2 + C_4\Phi^4$$

where C_1 through C_4 must again be determined from experimental data.

6.2 Dimensional Changes

The dimensions of each region are calculated from the original dimensions at 25°C, the thermal expansion coefficient of each material, and a fluence dependent function. The arithmetic average temperature for each region must be determined. For the first iteration the temperatures for each region are estimated. In subsequent iterations the temperatures at each point and the average temperature are calculated. The average temperature is calculated as the sum of the temperatures divided by the number of points, or

$$T_{ave} = \frac{\sum_{i=1}^n T_i}{n}$$

The thermal expansion coefficient for each material is determined from a polynomial,

$$b = C_1 + C_2T_{ave} + C_3T_{ave}^2 + C_4T_{ave}^3$$

The coefficients must be obtained from available experimental data (5,8,9).

After the thermal expansion coefficient is determined, the dimensions of the new boundaries can be specified. The radial surface of each region is specified by a radial distance R and two angular dimensions. If R_0 is the distance to a boundary from a fixed point at 25°C , then the radius of the same material at temperature T is

$$R_T = (1 + b)R_0$$

The dimensions of a material also change with fluence and temperature. They were assumed to vary according to the equation

$$R = R_T e^{-K\theta}$$

in which case $R = (1 + b)R_0 e^{-K\theta}$ where K must be determined from the experimental data (9).

To define the special regions, the distance between the center of the fuel rods, R_p , the gap distance, G , between the peripheral fuel rod and the graphite, and the fuel rod radius, R_f , must be specified. The gap distance can be calculated by subtracting the radial distances corresponding to the two surfaces on either side of the gap.

6.3 Iteration Method

A lattice system must be specified by defining the finite difference increments in the R and θ directions, Δr and $\Delta \theta$. The temperature at each lattice point is then calculated by the Liebman method. In this iteration technique the temperature at each lattice point (i, j) is calculated in a specific order. The temperature is first calculated for all odd values of i for a fixed value of j , then even values of i for $j+1$, odd values of i for $j+2$, and so on. This procedure is repeated for all values of j . When the first pass through the entire lattice system is completed, the temperatures at the points omitted during the first pass through the lattice are calculated in the second pass. The order of calculation is accomplished with the use of integer arithmetic and DO-loops for the i and j counters as shown in Fig. 8.

The reason for this alternating point iteration method is to insure that the calculation converges stably since the temperature at any point (i, j) is based on the surrounding temperatures which are calculated during the same iteration.

6.4 Region Determination

To calculate the temperature at a point, the equation which applies in the particular situation must be selected. To find the correct equation, the region containing the point and the type of point must be determined. The coordinates R_i and θ_j for a point are compared with the coordinates specifying the boundaries of the various regions. For example, with one IF-statement, the location of a point can be determined with respect to one boundary. If R_i minus R_{boundary} is equal to zero, the point lies on a boundary. If R_i minus R_{boundary} is less than zero, the point lies to the right of the boundary; and if greater than zero, to the left of the boundary. By comparing R_i with the minimum and maximum coordinates of a region, one can determine whether the point lies in that region or to the left or right of its boundaries.

This process must be repeated for all normal regions. If the point lies outside all of the normal regions, it must be in special regions 2 or 3. Region 3 has approximately the same boundaries as a normal region, so the logic that applies to a normal region must first be applied to find any regular boundary points. However, when a point does fall inside these outer bounds, it still might be a special boundary point on the peripheral fuel rod surface, region 2, or on the graphite moderator surface, region 3. All other points are interior points in regions 3 or 4.

As shown in Fig. 9, the irregular boundary points 1, 2, 3, and 4 fall on a line in the radial direction. The location of these points can be calculated in coordinates with respect to both the center of the capsule and the center of the fuel rod. These coordinates can be converted from one coordinate system to the other as shown in Appendix 10.5. As a result, the radial distances of these points from the center of the capsule can be determined:

$$R_1 = \frac{2R_p \cos(\theta_j) - \sqrt{[2R_p \cos(\theta_j)]^2 - 4(R_p^2 - R_F^2)}}{2} \quad (45)$$

$$R_2 = \frac{2R_p \cos(\theta_j) + \sqrt{[2R_p \cos(\theta_j)]^2 - 4(R_p^2 - R_F^2)}}{2} \quad (46)$$

$$R_3 = \frac{2R_p \cos(\theta_j) - \sqrt{[2R_p \cos(\theta_j)]^2 - 4[R_p^2 - (R_F + G)^2]}}{2} \quad (47)$$

$$R_4 = \frac{2R_p \cos(\theta_j) + \sqrt{[2R_p \cos(\theta_j)]^2 - 4[R_p^2 - (R_F + G)^2]}}{2} \quad (48)$$

<u>Region</u>	<u>Point No.</u>	<u>Point Type</u>
3	45, 25	Interior points, simple or non-simple
2	35	Interior points, simple or non-simple
3	15, 55, 85	Regular boundary points
2	65, 75, 95	Regular boundary points

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LOGIC FOR LOCATING
POINTS IN REGIONS 2 AND 3

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FIG.
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These distances can then be compared with that of the point in question, R_i , at θ_j . If $R_1 = R_2 = R_i$, the point is the regular boundary point at the point where the line in the radial direction is just tangent to the boundary surface of the graphite moderator in region 3 as indicated by point 15. Likewise, if $R_i = R_3 = R_4$, the point R_i is the point where the line in the radial direction is tangent to the fuel rod surface, point 95. Another special situation occurs when θ_j is equal to zero and R_i is equal to either R_1 , R_2 , R_3 , or R_4 . Since the flux at these corner points is in the R direction only, these points are regular boundary points. If R_i is equal to either R_1 , R_2 , R_3 , or R_4 for any θ , the point is a regular boundary point, as indicated by points 55, 65, 75, and 85. If this is not the case, R_i is compared with each of the four distances to determine which region and quadrant the point falls in.

6.5 Lattice Type Determination

After the region containing the point of interest is known, the type of lattice point is determined. Once the point type, region, and associated boundaries are all known, the applicable equation from Table 1 can be selected and solved. Regular boundary points and corner points are located by the region determination logic. The specific boundary is determined when the R or θ coordinates of the point match those of the boundary. All other points are interior, simple, or non-simple.

To determine whether an interior point is simple or non-simple, at a fixed θ the R coordinate of the point must be compared to each of the R coordinates of the region's boundaries to find whether the boundary falls within one incremental distance, ΔR , from R_i . Thus, R_i must be compared to $R_{\min}$ and $R_{\max}$ for the normal regions and $R_{\min}$, $R_{\max}$, R_1 , R_2 , R_3 , and R_4 for the special regions.

A point is an interior simple one if the criterion $R_i - R_b \geq |\Delta r|$ is met. The distance between the point (i, j) and the fuel rod and graphite moderator boundaries in the θ -direction must also be determined. For this comparison the angle corresponding to a point on the boundary surface, θ_B , on the same θ -lattice line must be calculated. For the fuel rod surface, this angle is

$$\theta_B = \sin^{-1}\left(\frac{R_i \sin(\theta_j)}{R_F}\right) \quad (49)$$

and for the graphite surface

$$\theta_B = \sin^{-1}\left[\frac{R_i \sin(\theta_j)}{R_F + G}\right] \quad (50)$$

If $\theta_j - \theta_B \geq |\Delta\theta|$, point (i,j) is clearly an interior simple one, and the finite difference equation (15) can be directly solved for $T_{i,j}$.

Interior non-simple points are easily identifiable since $R_i - R_B < |\Delta r|$ or $\theta_j - \theta_B < |\Delta\theta|$. To calculate the temperature at an interior non-simple point, (i,j) , the distance from (i,j) to the boundary and the temperature of the boundary point, $T_{i+a,j}$ or $T_{i,j+b}$, must be known as discussed in Sect. 5.3.

As shown in Fig. 10, an interior point in a normal region can be irregular only in the r -direction. In the special regions an interior point may be irregular in the r -, θ -, or both directions. The distance between the point of interest, (i,j) , and a boundary is defined as $a\Delta r$ in the r -direction and $b\Delta\theta$ in the θ -direction where $a = (R_B - R_i)/\Delta r$ and $b = (\theta_B - \theta_j)/\Delta\theta$.

The temperatures at the boundary points are calculated next, with one of Eqs. (25) through (44). To calculate the flux between point $(i+a,j)$ or $(i,j+b)$ and a point along a line normal to the surface and on the opposite boundary, the gap distance, G , and the temperature across the gap, T_b , must be known. The calculation to determine G has already been presented in Sect. 6.2. In a normal region, T_b is always on the same lattice line as $T_{i+a,j}$ and the flux between the two points is perpendicular to the surface. Thus, the temperature at T_b , once calculated, is stored and used in subsequent iterations.

Based on an empirical equation derived from data available from previous solutions, Eq. (51) was derived. The surface temperature is obtained as a function of angle γ . For the special regions this is not the case. The temperature across the boundary, T_b , from point $T_{i+a,j}$ or $T_{i,j+b}$ is defined in the direction normal to the surface, but this direction is no longer the same as a lattice line in the r -direction as shown in Fig. 11.

$$T_b = \left(\frac{T_1 + T_2}{2}\right) + \left(\frac{T_1 - T_2}{2}\right) \cos \gamma + A\gamma \quad (51)$$

when T_b is on the fuel rod surface and by

$$T_b = \left(\frac{T_3 + T_4}{2}\right) + \left(\frac{T_3 - T_4}{2}\right) \cos \gamma + A\gamma \quad (52)$$

on the graphite surface. Figure 11 defines T_1 , T_2 , T_3 , T_4 , and γ in the above equations. The equations are derived in Appendix 10.4.3. The angle γ for the fuel rod surface is

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REGULAR AND IRREGULAR POINTS

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FIG.
 10

$$\gamma = \sin^{-1} \left(\frac{R_B \sin(\theta_B)}{R_F} \right) \quad (53)$$

where R_B and θ_B are the coordinates of point $(i+a, j)$ or $(i, j+b)$. When T_b is located on the graphite surface, γ is defined by

$$\gamma = \sin^{-1} \left(\frac{R_B \sin(\theta_B)}{R_F + G} \right) \quad (54)$$

The constant, A , is calculated on the basis of the last known boundary temperatures after every iteration through the lattice from the equation

$$A = \frac{T_b - \left(\frac{T_1 + T_2}{2} \right) - \left(\frac{T_1 - T_2}{2} \right) \cos \gamma}{\gamma} \quad (55)$$

for the fuel rod surface and

$$A = \frac{T_b - \left(\frac{T_3 + T_4}{2} \right) - \left(\frac{T_3 - T_4}{2} \right) \cos \gamma}{\gamma} \quad (56)$$

for the graphite surface.

A least squares fit of the data results in the values of A for Eqs. (51) and (52).

All the information necessary to calculate $T_{i+a, j}$ or $T_{i, j+b}$, the irregular boundary point temperature, is now available. Once these temperatures are computed, the non-simple interior point temperature, $T_{i, j}$, can be computed from the proper finite difference equation, Eq. (16).

6.6 Convergence Criteria

As the temperature at each lattice point $T_{i, j}$ is computed, the difference from the temperature calculated in the previous iteration is also computed as

$$\Delta T = T_{i, j}(n) - T_{i, j}(n-1)$$

where n is the number of the iteration. The largest difference encountered can be stored by checking each difference against those of the other temperatures for that iteration.

After the temperatures for the entire lattice have been calculated, the largest difference can be compared with a predetermined convergence criterion. If this criterion is met, the temperature profile may be printed or punched in the desired form. If not, the entire solution process may be repeated. Ideally, the coordinates of the point corresponding to the largest temperature difference should be made known to discover where the calculated profile represents the poorest approximation.

A second convergence criterion is also advisable. The sum of all the temperature differences can be divided by the total number of lattice points, providing the average temperature variation.

7. CONCLUSIONS

1. The steady-state differential equation describing heat transfer in the irradiation capsule was derived. The boundary conditions applicable to each section boundary were determined. Finite difference equations that will enable calculation of the temperature at each lattice point have been developed for each type of point encountered in the finite difference lattice.

2. The logic required to write a computer program to solve the finite difference equations has been discussed.

3. A computer program based on the equations and logic should give the temperature profile at any axial position in the fuel rod over the irradiation period without manual calculations. Moreover, the temperature profile obtained should correspond to the exact capsule geometry.

4. An analytical solution to the second order partial differential equation with variable coefficients and non-linear boundary conditions was not obtained. Undefined values for boundary conditions make such a solution impossible.

8. RECOMMENDATIONS

1. Write a computer program based on the solution algorithm.

2. Select lattice grid spacing for optimum accuracy and computational efficiency. This will require preliminary runs to determine reasonable increment values.

3. Develop a convergence criterion for the desired accuracy of output. It is recommended that two convergence criteria be used. To check stability

for the entire control volume, use an average error convergence criterion. Also, check point solution stability by storing and comparing temperature differences at each point to find the largest error.

4. Compare computing time and accuracy of the program with HEATING 3 to essay advantages and disadvantages.

9. ACKNOWLEDGMENT

We thank our consultant, M.J. Kania, for his guidance.

10. APPENDIX

10.1 Derivation of Differential Equation

To perform a heat balance on a differential volume, the steady state equation is,

$$\text{heat input} - \text{heat output} + \text{heat generation} = 0$$

or

$$\begin{aligned} - \left(k \frac{\partial T}{\partial r} \right)_r r \Delta \theta \Delta z &+ \left(k \frac{\partial T}{\partial r} \right)_{r+\Delta r} (r + \Delta r) \Delta \theta \Delta z - \left(k \frac{\partial T}{\partial \theta} \right)_\theta \Delta r \Delta z \\ &+ \left(k \frac{\partial T}{\partial \theta} \right)_{\theta+\Delta \theta} \Delta r \Delta z + (S) \Delta r \Delta z r \Delta \theta = 0 \end{aligned}$$

Dividing by $\Delta r \Delta z \Delta \theta$ and letting Δr and $\Delta \theta$ approach zero, one obtains

$$kr \frac{\partial^2 T}{\partial r^2} + k \frac{\partial T}{\partial r} + rk \frac{\partial^2 T}{r^2 \partial \theta^2} + Sr = 0$$

If the generation term, Sr , is brought to the right of the equation and divided by kr , the final form of the equation is,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = - \frac{S}{k}$$

Because a numerical method is used to evaluate the partial differential equation within differential volumes, thermal conductivity is selected at the temperature of the point of interest (i,j) from previous iteration. Hence, thermal conductivity is not written in the position-derivative form, and heat transfer equation follows Poisson's equation.

$$\frac{\partial^2 T_{i,j}(r,\theta)}{\partial r^2} + \frac{S}{k(T_{i,j})} = 0$$

or

$$\frac{\partial^2 T_{i,j}}{\partial r^2} + \frac{1}{r} \frac{\partial T_{i,j}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T_{i,j}}{\partial \theta^2} = - \frac{S}{k(T_{i,j})} \quad (2)$$

10.2 Finite Difference Equation for a Boundary Point

The partial differential equation is applicable to all points within a homogeneous section of the capsule. However, the finite difference representation is different depending on the geometry and/or boundary conditions. For points on a boundary, boundary conditions must be satisfied. The development of equations for boundary points located in the regular regions and the special regions is not identical and therefore is discussed in two subsections which follow.

10.2.1 Equations for Normal Regions

There are eight boundary equations for a region circumscribed by lines parallel to the coordinate axis r and θ as shown in Fig. 4. Four equations apply to two lateral sides plus upper and lower bounds and four equations apply to the corner points.

For boundary 3, the boundary condition is

$$-k \frac{\partial T}{\partial r} = \frac{k_g}{G}(T_B - T_{i,j}) + \sigma e F(T_B^4 - T_{i,j}^4) \quad (4)$$

Figure 12 shows the geometry for the derivation.

From Taylor's formula, the temperature at the three surrounding points, and the finite difference equation is obtained together with boundary conditions,

$$T_{i+1,j} = T_{i,j} + \Delta r \left. \frac{\partial T}{\partial r} \right|_{i,j} + \frac{(\Delta r)^2}{2} \left. \frac{\partial^2 T}{\partial r^2} \right|_{i,j} \quad (7)$$

$$T_{i,j+1} = T_{i,j} + (r\Delta\theta) \left. \frac{\partial T}{\partial \theta} \right|_{i,j} + \frac{(r\Delta\theta)^2}{2} \left. \frac{\partial^2 T}{\partial \theta^2} \right|_{i,j} \quad (9)$$

$$T_{i,j-1} = T_{i,j} - (r\Delta\theta) \left. \frac{\partial T}{\partial \theta} \right|_{i,j} + \frac{(r\Delta\theta)^2}{2} \left. \frac{\partial^2 T}{\partial \theta^2} \right|_{i,j} \quad (10)$$

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FIG.
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$$-k \frac{\partial T}{\partial r} = \frac{k_g}{G}(T_B - T_{i,j}) + \sigma e F(T_B^4 - T_{i,j}^4) \quad (4)$$

Multiply by constants and solve by method of undetermined coefficients.

$$A r \Delta \theta - C \Delta \theta = 0 \quad A = C = \frac{1}{(r \Delta \theta)^2}$$

$$\frac{A(r \Delta \theta)^2}{2} + \frac{C(r \Delta \theta)^2}{2} = 1 \quad A(r \Delta \theta)^2 = 1$$

$$B \Delta r - D k = \frac{1}{r} \quad -D k = \frac{1}{r} - B \Delta r = \frac{1}{r} - \frac{2}{\Delta r}$$

$$\frac{B(\Delta r)^2}{2} = 1 \quad B = \frac{2}{(\Delta r)^2}$$

Thus, the Laplacian operator at i, j is

$$\begin{aligned} \nabla^2 T(r, \theta) \Big|_{i,j} &= \frac{1}{i^2 (\Delta r \Delta \theta)^2} (T_{i,j+1} - T_{i,j}) + \frac{2}{(\Delta r)^2} (T_{i+1,j} - T_{i,j}) \\ &+ \frac{1}{i^2 (\Delta r \Delta \theta)^2} (T_{i,j-1} - T_{i,j}) - \left(\frac{1}{i \Delta r} - \frac{2}{\Delta r} \right) \left[\frac{k_g}{k G} (T_B - T_{i,j}) + \frac{\sigma e F}{k} (T_B^4 - T_{i,j}^4) \right] \end{aligned}$$

Then we can solve for $T_{i,j}$.

$$T_{i,j} = \frac{-\left(\frac{1-2i}{i \Delta r}\right) \frac{\sigma e F}{k} T_{i,j}^4 - \frac{T_{i,j+1}}{(i \Delta r \Delta \theta)^2} - \frac{2 T_{i+1,j}}{\Delta r^2} - \frac{T_{i,j-1}}{(i \Delta r \Delta \theta)^2} + \left(\frac{1-2i}{i \Delta r}\right) \left[\frac{k_g T_B}{k G} + \frac{\sigma e F}{k} T_B^4 \right] - \frac{S}{k}}{-\frac{1}{(i \Delta r \Delta \theta)^2} - \frac{2}{\Delta r^2} - \frac{1}{(i \Delta r \Delta \theta)^2} + \left(\frac{1-2i}{i \Delta r}\right) \frac{k_g}{k G}} \quad (19)$$

Equation for Boundary 1

Because of symmetry, this surface is adiabatic, and the boundary condition is

$$-k \frac{\partial T}{r \partial \theta} = 0$$

Based on the geometry shown in Fig. 13, Taylor's series expansions and the method of undetermined coefficients, $T_{i,j}$ is found to be:

$$T_{i,j} = \frac{\frac{2T_{i,j+1}}{(i\Delta r\Delta\theta)^2} + \frac{2i-1}{2i(\Delta r)^2}(T_{i-1,j}) + \frac{2i+1}{2i(\Delta r)^2}(T_{i+1,j}) + \frac{S}{K}}{\frac{2}{(i\Delta r\Delta\theta)^2} + \frac{2}{(\Delta r)^2}} \quad (17)$$

Equation for Boundary 4

Except for the geometry shown in Fig. 14, the method for this case follows that of Boundary 3. Taylor expansion on point $i-1,j$ is carried out instead of the point $i+1,j$.

$$T_{i,j} = \frac{\left(\frac{1+2i}{\Delta r}\right)\frac{\sigma Fe}{k} T_{i,j}^4 + \frac{T_{i,j+1} + T_{i,j-1}}{(i\Delta r\Delta\theta)^2} + \frac{2T_{i-1,j}}{\Delta r^2} - \left(\frac{1+2i}{i\Delta r}\right)\frac{k_g}{kG} T_B - \frac{\sigma Fe}{k} T_B^4 + \frac{S}{K}}{\frac{2}{(i\Delta r\Delta\theta)^2} + \frac{2}{(\Delta r)^2} + \left(\frac{1+2i}{i\Delta r}\right)\left(\frac{k_g}{Gk}\right)} \quad (10,19) \quad (20)$$

Equation for Boundary 2

The same boundary conditions as those for Boundary 1 apply. After accounting for the geometrical difference shown in Fig. 15, the solution is

$$T_{i,j} = \frac{\frac{1}{(\Delta r)^2} \left[\left(1 - \frac{1}{2i}\right) T_{i-1,j} + \left(1 - \frac{1}{2i}\right) T_{i+1,j} \right] + \frac{2T_{i,j-1}}{(i\Delta r\Delta\theta)^2} + \frac{S}{K}}{\frac{2}{(\Delta r)^2} + \frac{2}{(i\Delta r\Delta\theta)^2}} \quad (18)$$

Equation for Boundary 5

Boundary conditions at this point are

$$k \frac{\partial T}{\partial r} = \frac{k_g}{G} (T_B - T_{i,j}) + \sigma ef (T_B^4 - T_{i,j}^4)$$

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GEOMETRY FOR EQ. (17)

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GEOMETRY FOR EQ. (20)

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GEOMETRY FOR EQ. (18)

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$$\frac{\partial T}{\partial \theta} = 0$$

Using these boundary equations and the geometry shown in Fig. 16 together with Taylor expansions on two points, one finds:

$$T_{i,j} = \frac{-\left(\frac{1+2i}{i\Delta r}\right)\frac{\sigma e F}{k} T_{i,j}^4 + \frac{2T_{i,j+1}}{(i\Delta r\Delta\theta)^2} + \frac{2}{\Delta r^2} T_{i+1,j} + \left(\frac{1+2i}{i\Delta r}\right)\left(\frac{k_g}{Gk} T_B + \frac{\sigma e F}{k} T_B^4\right)}{\frac{2}{(i\Delta r\Delta\theta)^2} + \frac{2}{\Delta r^2} + \left(\frac{1+2i}{i\Delta r}\right)\frac{k_g}{Gk}} \quad (21)$$

All corner points have the same set of boundary conditions. Hence, manipulations on the equation just derived are made only for differences in geometry. These equations are presented below.

The equation for Boundary 6, whose geometry is shown in Fig. 17, is,

$$T_{i,j} = \frac{\left[\frac{2}{(\Delta r)^2}\right]T_{i-1,j} + \frac{2T_{i,j+1}}{(i\Delta r\Delta\theta)^2} - \left(\frac{1+2i}{i\Delta r}\right)\left[\frac{k_g}{Gk} T_B + \frac{\sigma e F}{k}(T_B^4 - T_{i,j}^4)\right] + \frac{S}{K}}{\frac{2}{(\Delta r)^2} + \frac{2}{(i\Delta r\Delta\theta)^2} + \frac{k_g(1+2i)}{Gk i\Delta r}} \quad (22)$$

The equation for Boundary 7, whose geometry is shown in Fig. 18, is,

$$T_{i,j} = \frac{\frac{\sigma e F}{k}\left(\frac{1-2i}{i\Delta r}\right)T_{i,j}^4 + \frac{2}{\Delta r^2} T_{i+1,j} + \frac{2T_{i,j-1}}{(i\Delta r\Delta\theta)^2} - \left(\frac{1-2i}{i\Delta r}\right)\left[\frac{k_g}{Gk} T_B + \frac{\sigma e F}{k} T_B^4\right] + \frac{S}{K}}{\frac{2}{(\Delta r)^2} + \frac{2}{(i\Delta r\Delta\theta)^2} + \left(\frac{1-2i}{i\Delta r}\right)\frac{k_g}{Gk}} \quad (23)$$

The geometry for the equation for Boundary 8 is shown in Fig. 19.

$$T_{i,j} = \frac{\frac{2T_{i-1,j}}{(\Delta r)^2} + \frac{2T_{i,j-1}}{(i\Delta r\Delta\theta)^2} - \left(\frac{1+2i}{i\Delta r}\right)\left[\frac{k_g}{Gk} T_B + \frac{\sigma e F}{k}(T_B^4 - T_{i,j}^4)\right] + \frac{S}{K}}{\frac{2}{(\Delta r)^2} + \frac{2}{(i\Delta r\Delta\theta)^2} + \left(\frac{1+2i}{i\Delta r}\right)\frac{k_g}{Gk}} \quad (24)$$

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GEOMETRY FOR EQ. (21)

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GEOMETRY FOR EQ. (22)

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FIG.
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GEOMETRY FOR EQ. (23)

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GEOMETRY FOR EQ. (24)

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FIG.
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10.2.2 Equations for Special Regions

Regions such as the peripheral fuel rod and graphite moderator, whose boundaries are not parallel to the cylindrical coordinates, are designated as special regions. The different boundaries found in these regions are shown in Fig. 4. Boundary equations (1) through (8) of special region 3, the graphite moderator, are identical with those for the normal regions.

10.3 Interior Non-Simple Point Finite Difference Equation

An interior non-simple point occurs when a lattice point lies close to the boundary at a distance smaller than the selected lattice spatial increment. The procedure to derive the equation for an interior simple point is used again for this case with $a\Delta r$ instead of Δr . In the θ -direction $b\Delta\theta$ is used rather than $r\Delta\theta$. However, temperatures of the boundary points adjacent to the interior non-simple point of interest must also be calculated. At this point the similarity between the regular and special region treatments breaks down.

10.3.1 Equations in Normal Regions

The finite-difference equation for a non-simple interior point located in a normal region has a fractional "a" value, but "b" is still equal to one. Thus, for a point situated close to the right side boundary, with the geometry shown in Fig. 20, the equation is

$$T_{i,j} = \frac{\left[\frac{1}{(ia\Delta r)^2} + \frac{1 - \frac{a}{2i}}{\frac{(a\Delta r)^2}{2} + \frac{(a\Delta r)^2}{2}} \right] T_{i+a,j} + \left[\frac{1 - \frac{a}{2i}}{\frac{(a\Delta r)^2}{2} + \frac{(\Delta r)^2}{2}} \right] T_{i-1,j} + \frac{(T_{i,j+1} + T_{i,j-1}))}{(i\Delta r\Delta\theta)^2} + \frac{S}{K}}{\frac{1}{(ia\Delta r)^2} + \frac{2[1 - (a/2i)]}{(a\Delta r)^2 + (a\Delta r)^2} + \frac{2[1 - (a/2i)]}{(a\Delta r)^2 + (\Delta r)^2} + \frac{2}{(i\Delta r\Delta\theta)^2}} \quad (57)$$

The temperature at the boundary, which constitutes an irregular point, is calculated from the equation derived for regular boundary points with a fractional increment employed in the Taylor series expansion. This is further discussed in Sect. 10.4.1.

10.3.2 Equations in Special Regions

Due to the shape of the peripheral fuel rod and of the graphite moderator surrounding it, both factors "a" and "b" may be less than one. The equation formulated for this situation, based on the geometry shown in Fig. 21, is

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GEOMETRY FOR EQ. (57)

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GEOMETRY FOR EQ. (58)

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$$T_{i,j} = \frac{\frac{1}{(a\Delta r)^2}[(1 - \frac{1}{2i})T_{i-1,j} + (1 - \frac{1}{2i})T_{i+a,j}] + \frac{1}{i^2(a\Delta r b \Delta \theta)^2}(T_{i,j-b} + T_{i,j+1}) + \frac{S}{K}}{\frac{2}{(a\Delta r)^2} + \frac{2}{i^2(a\Delta r b \Delta \theta)^2}} \quad (58)$$

The boundary temperatures are obtained from equations developed for boundary 9, after specifying the boundary point location.

Non-simple interior points close to the outside boundaries of region 3 are treated in the same manner as those in the normal region.

10.4 Irregular Boundary Points

10.4.1 Equations in Normal Regions

Whether in normal or special regions, all irregular boundary points are generated by interior points. An interior point is surrounded by four points, two in the r -direction and two in the θ direction. When a boundary surface is in the vicinity of an interior point, the surface crosses the lattice and intersects either one or two of the lines in the θ - or r -direction. This makes one of the surrounding points an irregular boundary point where the temperature must be found.

In the normal region, a boundary surface only intersects lines in the r -direction. These irregular boundary point temperatures are parameterized as $T_{i+a,j}$. This permits boundary temperatures to be stored relative to an interior point. Since a boundary location may not be fixed, this technique seems to be the only way to refer to boundary temperatures, T_B . The boundary condition between gaps can be thus evaluated.

The irregular boundary point temperature is determined by introducing the boundary condition

$$-k \frac{\partial T}{\partial r} = \frac{k_g}{G}(T_B - T_{i+a,j}) + \sigma \epsilon F(T_E^4 - T_{i+a,j}^4)$$

where $T_B \equiv T_{i-1+a,j}$ or $T_{i+1+a,j}$ depending on the direction of the neighboring region. The treatment is similar to that of a regular boundary point in a normal region, only the lattice distance is smaller in the r -direction as shown in Fig. 10.

Equation (19) is employed for a point near the $R_{\min}$ boundary. Substitute for the appropriate "a" value, and change the subscripts of the temperatures used for the irregular boundary case to the regular boundary case as follows:

$$T_{i+a,j+1} = T_{i+a,j}$$

$$T_{i+a,j} = T_{i,j}$$

$$T_{i+a,j-1} = T_{i,j-1}$$

$$T_{i,j} = T_{i+a,j}$$

For a point near the $R_{\max}$ boundary, Eq. (20) is used to take into account the subscript changes in the equation.

10.4.2 Equations for Regular and Irregular Boundary Points in Special Regions

Different forms of the boundary equation are required to evaluate the temperature at some points in the peripheral or center fuel rod depending on the location of the lattice point. Therefore, there is a need to classify the boundary surfaces into four quadrants as shown in Fig. 7.

There are two cases for each quadrant. In one case, the heat flux in the desired direction requires an approximation of $\partial T/\partial r$ and in the other an approximation of $r(\partial T/\partial \theta)$. Derivation of the equations for the special boundary are classified according to their locations in quadrants 1 through 4. Quadrant 1 is part of the graphite moderator. The geometry employed to calculate the boundary temperature is shown in Fig. 22.

The thermal conductivity of the graphite between the two points can be considered constant and the conduction heat fluxes are vector quantities. If we solve for q ,

$$-k \frac{\partial T}{r \partial \theta} = q \sin \phi$$

or

$$q = -k \frac{\partial T}{r \partial \theta} \frac{1}{\sin \theta} = -k \frac{\partial T}{\partial r} \frac{1}{\cos \theta} \quad (59)$$

The flux at any boundary point is obtained by numerical approximation. For the boundary point $(i, j+b)$ at the intersection of a curve of constant radius passing through the interior point (i, j) and intersecting the boundary surface, we have

$$k \left(\frac{T_{i,j+b} - T_{i,j}}{r b \Delta \theta} \right) \left(\frac{1}{\sin \theta} \right) = \frac{k_g}{G} (T_B - T_{i,j+b}) + \alpha \epsilon F (T_B^4 - T_{i,j+b}^4)$$

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FIG.
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The geometry for this case is shown in Fig. 23. If we solve for $T_{i,j+b}$,

$$T_{i,j+b} \left[\frac{k}{rb_n \Delta \theta \cos(\frac{\pi}{2} - \theta)} + \frac{k_g}{G} \right] = \frac{kT_{i,j}}{rb_n \Delta \theta \cos(\frac{\pi}{2} - \theta)} + \frac{k_g}{G} T_B + \sigma \epsilon F (T_B^4 - T_{i,j+b}^4)$$

$$T_{i,j+b} = \frac{\frac{kT_{i,j}}{rb_n \Delta \theta \cos(\frac{\pi}{2} - \theta)} + \frac{k_g}{G} T_B + \sigma \epsilon F (T_B^4 - T_{i,j+b}^4)}{\frac{k}{rb_n \Delta \theta \cos(\frac{\pi}{2} - \theta)} + \frac{k_g}{G}} \quad (60)$$

Calculation of T_B at the special boundary surface is discussed in Sect. 10.4.3.

However, if point $(i+a,j)$ generated by the line of constant angular direction θ intersecting the boundary in the r -direction as shown in Fig. 24, then

$$k \left(\frac{T_{i+a,j} - T_{i,j}}{a_n \Delta r} \right) \left(\frac{1}{\cos \theta} \right) = \frac{k_g}{G} (T_B - T_{i+a,j}) + \sigma \epsilon F (T_B^4 - T_{i+a,j}^4)$$

If we solve for $T_{i+a,j}$, then

$$T_{i+a,j} = \frac{\frac{kT_{i,j}}{a_n \Delta r \cos \theta} + \frac{k_g}{G} T_B + \sigma \epsilon F (T_B^4 - T_{i+a,j}^4)}{\frac{k}{a_n \Delta r \cos \theta} + \frac{k_g}{G}} \quad (61)$$

Conditions requiring the use of these equations are

If $0 < a \leq 1$, then Eq. (61)

If $0 < b \leq 1$, then Eq. (60)

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GEOMETRY FOR EQ. (61)

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With this suggested program logic, there is no need to interpolate temperature gradients. There are eight equations in all, two for each quadrant, Eqs. (25) through (32).

The same equations apply for regular and irregular boundary points, the distinction being the value of "a" or "b". Since the regular point has lines connected to interior points in both r- and θ -directions, either the numerical gradient $\Delta T/\Delta r$ or $\Delta T/r\Delta\theta$ can be employed. However, $\Delta T/\Delta r$ approximations are assigned to regular points because they are simpler to obtain.

10.4.3 Calculation of T_B on Special Region Boundary

To determine whether there was a relation between angular position and boundary temperature, data available from HEATING 3 was plotted as shown on Fig. 25. The shape of the curve suggested that the surface temperature could be related to the cosine of angle γ or

$$T_B = \frac{T_1 + T_2}{2} - \frac{T_1 - T_2}{2} \cos \gamma + A\gamma \quad (62)$$

where A is a correction factor. To determine the value of A, Eq. (62) may be rearranged

$$T_B - \frac{T_1 + T_2}{2} + \frac{T_1 - T_2}{2} \cos \gamma = A\gamma' \quad (63)$$

where γ' is the smallest angle between the radial line connecting T_B to the center of the fuel rod in region 2 and the zero θ -line; thus $0 < \gamma' < \pi/2$. T_1 and T_2 are the temperatures corresponding to $\gamma = 0$ and $\gamma = \pi$ on the fuel rod surface and T_3 and T_4 on the graphite surface. When the lefthand side of Eq. (63) is plotted as a function of γ , a straight line, Fig. 26, is obtained. The value of A may be calculated for both the fuel rod and graphite surfaces.

10.5 Change of Coordinate System

For the geometrical situation shown in Fig. 27, R_B can be calculated from

$$R_F^2 = R_B^2 + R_P^2 - 2R_B R_P \cos \theta$$

and

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FUEL ROD TEMPERATURE AS A FUNCTION
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FIG.
25

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DETERMINATION OF CORRECTION FACTOR

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FIG.
26

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CHANGE OF COORDINATE SYSTEM

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$$R_B^2 - R_B(2R_p \cos \theta) + R_p^2 - R_F^2 = 0 \quad (64)$$

If we solve for R_B ,

$$R_{B1} = \frac{2R_p \cos \theta + \sqrt{(2R_p \cos \theta)^2 - 4(R_p^2 - R_F^2)}}{2} \quad (46)$$

and

$$R_{B2} = \frac{2R_p \cos \theta - \sqrt{(2R_p \cos \theta)^2 - 4(R_p^2 - R_F^2)}}{2} \quad (45)$$

The angles γ_1 and γ_2 must be calculated to determine T_B . From the geometry shown in Fig. 15, $R_{B1} \sin \theta = R_F \sin \gamma_1$ and $R_{B2} \sin \theta = R_F \sin \gamma_2$. Thus,

$$\gamma_1 = \sin^{-1}\left(\frac{R_{B1} \sin \theta}{R_F}\right) \quad (53)$$

and

$$\gamma_2 = \sin^{-1}\left(\frac{R_{B2} \sin \theta}{R_F}\right) \quad (54)$$

To define sections 1, 2, 3, and 4 of the special regions the line that separates sections 1 and 3 from regions 2 and 4 must be defined. This line connects the center of the fuel rod with the point where $R_B = R_{B1} = R_{B2} = R_p \cos \theta$.

10.6 Nomenclature

- a fractional position of irregular boundary point in R-direction, dimensionless
- b fractional position of irregular boundary point in θ -direction, dimensionless
- e emissivity, dimensionless
- F view factor = 1

G	gap width, cm
h	heat transfer coefficient, $\text{W}/\text{cm}^2\text{-}^\circ\text{C}$
k	thermal conductivity, $\text{W}/\text{cm-}^\circ\text{C}$
r	radial coordinate distance, cm
R	radial coordinate of material, cm
RDL	radial lattice increment
RLMIN	minimum radial lattice dimension
RLMAX	maximum radial lattice dimension
S	heat generation, W/cm^3
t	time, sec
T	temperature, $^\circ\text{C}$
THDL	angular lattice increment
THMAX	maximum angular lattice dimension
THMIN	minimum angular lattice dimension
Z	axial position, cm
γ	fuel rod angular coordinate, radians
ϕ	reactor fluence, $\text{joules}/\text{cm}^2$ or angle from radius, radians
σ	Stephan-Boltzman constant, $5.67 \times 10^{-12} \text{ W}/\text{cm}^2\text{-}^\circ\text{K}^4$

Subscripts

B	boundary property
S	surface property
g	gas property
o	initial value

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