MICROINSTABILITIES IN COMPLEX
MAGNETIC FIELD GEOMETRIES
AND
HIGH- β SHEARED SHEATH STRUCTURE
II

MASTER

Progress Report
for Period March 1, 1976 - February 28, 1977

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ABSTRACT

The new approach for the solution of the Vlasov equation for complex magnetic field geometries and for the determination of ensuing stability criteria for microinstabilities has been developed further by estimation of various parameters in the contexts of Tokamak and Tormac regimes, by carrying out analytically the averaging over the fast cyclotron motion to obtain a compact expression for the dispersion equation, and by considering the ensuing results for stability of the drift wave in various parameter domains. The possibility of extending this formulation, without much modification, to other instabilities such as lower hybrid drift, drift cyclotron, drift cone, has been noted and corresponding studies have been initiated.

The study of high- β , high shear plasma sheath structures has continued both analytically and through computation work. Results in the Vlasov-fluid model for simple ion distribution functions indicate a relation for maximum shear the sheath can support for a given β . Further computer work is aimed at the study of sheath structures with more realistic ion distribution functions. The equations containing finite electron pressure and finite larmor radius viscosity effects have been cast in more tractable and physically suggestive non-Euclidean form. The effect of the rotation of the main plasma on the sheath structure, the coupling of the main plasma to the sheath and its rotational damping, and the formation of a viscous boundary layer between the sheath and the plasma, have been investigated.

INTRODUCTORY REMARKS

This progress report consists of two parts. The studies on "Microinstabilities in Complex Magnetic Field Geometries" are described in Part I by Dr. Pradip Bakshi. The studies on "High- β Sheared Sheath Structures" are described in Part II by Dr. Gabor Kalman. These studies are in compliance with the originally stipulated contract requirements.

Dr. Bakshi has so far devoted approximately 75% of his stipulated time on this contract for the current year. The remaining 25% of his stipulated time will be devoted during the remainder of the current term. Dr. Kalman has so far devoted approximately 92% of his stipulated time on this contract for the current year. The remaining 8% of his stipulated time will be devoted during the remainder of the current term.

PART I

MICROINSTABILITIES IN COMPLEX MAGNETIC
FIELD GEOMETRIES

Pradip Bakshi

Microinstabilities in Complex Magnetic Field Geometries

The long range objective of this program is to develop a new formalism for the study of microinstabilities in various inhomogeneous magnetic field geometries, involving (possibly strong) variations in density and other plasma parameters. The formalism should be capable of describing not only the slow variation situations such as the tokamaks and mirror machines, but also the sharp variation problems such as the tormac sheath or the $2 \times$ IIB radial variation problem.

The basic approach has been described earlier¹⁻³ in the context of shear stabilization of drift waves; we concentrate here on the current program and recent developments. Two significant advances³⁻⁴ during this report period were 1) a careful examination of the parameter domains in the context of the tokamak, and the tormac sheath regimes, and 2) obtaining analytically a generally valid expression for the dispersion operator equation, requiring only that the probe frequency be much smaller than the cyclotron frequency. With this development, it is now possible to apply this formalism (with only minor modifications) to various other instabilities. We have already initiated investigations of the lower hybrid drift and the drift cyclotron instabilities. We have also begun a systematic study of the effects of using different (unperturbed) distribution functions: such a study is a pre-requisite for investigating the loss-cone and drift-cone type instabilities, and also for a

proper analysis of various instabilities in the tormac sheath regime, which has its own characteristic distribution with a loss-zone.

1. Parameter Domains

As described in detail elsewhere³, the key parameters which distinguish our results from previous theories are $p = (\rho s)(\rho k)$, where ρ = ion gyro radius, k =transverse wave number, s is the inverse shear length, and $q = p(\Omega/\omega)$, where Ω is the ion gyro-frequency and ω is the probe frequency. It is easy to show that $q = (s/\epsilon)(\omega^*/\omega)$, where ϵ is the inverse length characterizing the density gradient and $\omega^* = kv_d$ is the drift frequency. The effects of the orbit corrections due to the shear are fully retained in our results (Ref. 3, eqs (6) and (8)); we recover earlier theories⁵⁻⁷ as limiting cases by setting $p = 0$, $q = 0$ at various stages. Typical values for these parameters for the Tokamaks and the Tormac Sheath are as follows:

	Tokamak	Tormac
p	$3 \times 10^{-4} \rho k$	$3 \times 10^{-1} \rho k$
q	$3 \times 10^{-2} \rho k$	$7 \rho k$
Ω/ω	100	20

Thus for Tokamaks, $p \ll 1$ and $q \ll 1$ even for $\rho k \sim 5$ to 10. But for the Tormac sheath, we have $p \ll 1$ and $q \gg 1$ for $\rho k \sim 1$, and $p \gg 1$ and $q \gg 1$ for larger ρk . The corrections for the Tokamak regime can be evaluated as an expansion in q . The Tormac sheath would require a non perturbative treatment, even for moderate ρk . In both cases, $(\Omega/\omega) \gg 1$ and this enables us to average over the fast (cyclotron motion) scale, leading to an operator dispersion

equation as described below.

We also obtained the estimates for the differential operator D (Ref. 3 eq. (11) and (13)) and the physical parameter $u = (k_{\parallel} v_{th,i} / \omega)$ in terms of the $q=0$ eigenmodes. For $pk \leq 1$, $\rho^2 D^2 \sim s/\epsilon$ remains small and the higher derivatives have to be retained for $pk \geq 2$. The parameter $u \ll 1$ for $pk \ll 1$, and remains less than unity up to $pk \geq 10$.

2. Dispersion Equation

A significant step was taken during this report period by analytically carrying out the proper-time integrations. Assuming only that $(\Omega/\omega) \gg 1$, the time integral can be essentially carried out in two stages: averaging over a period of the fast scale Ωt , without affecting the slow scale ωt , and then integrating over the slow scale. Ignoring terms of order (ω/Ω) , we find that integrating eq. (8), Ref. 3 leads to eq. (16) of Ref. 3, thus providing a compact expression for the ion contribution for the drift mode. The electron term can be well approximated by the usual expression, eq. (15), Ref. 3. The ensuing dispersion equation is analyzed as follows depending on the magnitudes of the various parameters:

a) $\rho^2 D^2 < 1$, $u^2 < 1$, $q^2 < 1$.

Expanding to order D^2 and u^2 provides the harmonic oscillator equation. The limit $q=0$ leads to the Pearlstein-Berk description⁶ of the problem; keeping terms to order q^2 provides the lowest order corrections. For $pk \leq 1$, the corrections are small, requiring a slightly higher shear for stability compared to the $q=0$ results. For a value of pk between 1 and 2, the correction changes sign and a slightly smaller shear is required

for stability. As pk increases, so do $(S/\epsilon)_{\text{critical}}$ and q , until for some pk between 2 and 3, the coefficient of the D^2 term passes through zero, indicating a complete breakdown of this level of description. Clearly, order D^4 terms have to be maintained to avoid the singular behavior as the coefficient of D^2 passes through zero.

b) $u^2 < 1$, $\rho^2 D^2$ arbitrary.

This case is being analyzed by taking Fourier transform which interchanges the role of u^2 and D^2 . Then expansion to order u^2 provides a second order differential equation, to be treated as a WKB equation. We can retain q^2 to all orders, since a numerical treatment will be required.

c) $u^2 \approx 1$, $\rho^2 D^2 \geq 1$.

This is the most difficult domain and will have to be analyzed as an integral equation by keeping all the terms.

Full details of case (a), and the methods of attack for cases (b) and (c) are being described in a forthcoming report.⁸

3. Lower Hybrid Drift Instability.

The formalism developed above is applicable with minor modifications to the study of the lower hybrid drift instability. The essential feature of the formulation, that of transverse propagation remains valid here. The ions, with $\omega \approx (\Omega_e \Omega_i)^{1/2} \gg \Omega_i$ are essentially unmagnetized, while the electrons have their cyclotron frequency $\Omega_e \gg \omega$, the probe frequency so that the averaging over the fast scale is a valid approximation. The effects of various other gradients and drifts would have to be incorporated but the basic structure of our

formalism is applicable. An estimate for the shear stabilization criterion, using the usual (small D^2 , u^2 and $q=0$) assumptions has been given recently.⁹ We have initiated a study¹⁰ to determine whether substantial departure from those results can be expected.

4. Drift Cyclotron and Drift Cone Instabilities.

We have also initiated a study¹¹ of the drift cyclotron instability¹² and the possible modifications of its shear stabilization criterion. We have also begun a systematic study¹² of the sensitivity of our results to the details of the distribution functions. This will also be useful in handling the loss-cone, drift-cone type instabilities, and help achieve a reasonable description of instabilities in the Tormac sheath regime, which has its own particular velocity space distribution.

In summary, we have found that the largest departures from conventional results occur for large pk , a domain for which the presently available theories are generally inadequate. We have maintained the philosophy of not making any approximations of convenience. This has often imposed the need to develop additional techniques, in order to further extend the analytical formulation without arbitrary approximations. This has also slowed down the progress at times. The present formulation, however, does provide a manageable framework for studying the various instabilities over a very wide range of parameters without making invalid approximations. This was to be the practical goal of our proposed program and we do see the possibility of accomplishing this successfully in the next year's program.

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Bulletin Subject (Plasma Heading)
in which paper should be placed
19, Linear Instabilities.

A New Formalism for Plasma Microinstabilities in Sheared Magnetic Fields.* W. BELLEW and P. BAKSHI, Boston College.—A new approach for the solution of the Vlasov equation with an arbitrarily sheared magnetic field has been developed using operator inversion and other non-perturbative operator techniques based on similarity transforms, Lie algebras and Zassenhaus expansions. The formalism is illustrated by determining the perturbed distribution functions and density operator for transverse and arbitrary direction drift-waves. (1) The ensuing dispersion equation is essentially an integral equation for the perturbed potential function. The conventional second order differential equation approach is a valid approximation only for small S/ϵ , (S = inverse shear length, ϵ = inverse density variation scale length). (2) Even in that approximation, the earlier results on this problem¹ appear here as the $q \rightarrow 0$ limit in our formalism, ($q = (S/\epsilon)(\omega^*/\omega)$, ω^* = drift frequency); the modifications for $q \neq 0$ are significant for the high shear, high gradient regimes like the Toramak sheath. The approach is applicable to other microinstabilities as well.

*Work supported by U.S.E.R.D.A.

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PART II

HIGH- β SHEARED SHEATH STRUCTURE

Gabor Kalman

High- β Sheared Sheath Structures

In the report period we continued to study and analyze the structure of sheared high- β sheath structures along the lines already indicated in the previously submitted Reports. We also performed computer analysis of simple models. In addition, the problem of plasma rotation in the Tormac geometry and the accompanying formation of boundary layer was investigated.

1. Sheath Structure Analysis

The basic features of the sheath model we have been using are the following: a) The ions are treated through their full dynamical description, either by using Newtonian dynamics or, equivalently, Vlasov equation. b) The electrons are treated in an approximation which is basically equivalent to an expansion in the small parameter $\epsilon = r_e/L$ (r_e and r_i being the electron and ion Larmor radii and L the scale length).^(1,2) To lowest order (ϵ^0) the electrons constitute a pressureless, possibly drifting fluid (similarly to the Freidberg⁽¹⁾ model); to next order (ϵ^1) the electron fluid acquires viscosity and other transport properties, similarly to the D'Ippolito-Davidson⁽²⁾ model. An intermediate stage (say, $O(\epsilon^{1/2})$), distinguished from either of the above models, is obtained and has been considered: in this model the electrons constitute a hot, pressure providing but inviscid fluid with isotropic pressure. c) The plasma is strictly charge neutral and there are no electric fields in the sheath.

The general framework for the lowest order study was worked out earlier. The algebra becomes isomorphic to that

of a (pseudo) particle moving in a given two-dimensional (pseudo) potential. This formalism we now generalized to more complex situations by introducing a non-Euclidean coordinate system following the local sheared field lines. The metric $g_{\mu\nu}$ and the affine connections $\Gamma_{\mu\nu}^{\lambda}$ related to this coordinate system contain important physical information: the former depends on θ' while the latter on θ' and θ'' (θ being the shear angle); this allows one to separate effects related to the shear and to $\text{grad } B^2$. The conservation of pseudo angular momentum which has been established for the lowest order model, but is broken by the electron pressure, has been found to be a special case of a more general conservation law, valid up to ϵ^n , $n < 1$: this allows integration of the sheath structure equations with non-vanishing electron pressure.

The hybrid-kinetic description of high- β plasmas of D'Ippolito and Davidson⁽²⁾ provided the appropriate starting point for the sheath structure equations. However, we have introduced further simplifying assumptions in order to arrive at a fully (viscous) hydrodynamical description of the electrons. We have studied the implications of these equations for the coupling between the locally parallel and perpendicular components of the electron current, $j_{\parallel}$ and $j_{\perp}$. Such coupling is "weak" (of the order $(r_e/r_i)^2$), but it removes, to a great extent, the arbitrariness in the electron current distribution. In the Tormac type geometries if the sheath is "plugged", $j_{\parallel}$ and $j_{\perp}$ are already coupled. Since we have not been able to obtain a full solution of the equations, it is not clear, how this is compatible with the restriction resulting from the coupling. One possibility is that one has to populate the sheath with trapped electrons.

Another, that the sheath has to expand to a width d such that the coupling $(r_e/d)^2$ becomes tolerable (such a behavior would be, of course, rather undesirable).

Computer analysis of the relations governing the Vlasov-fluid model started with simplified ion distribution functions: first monoenergetic ions and secondly ions with step-function distribution have been analyzed. Both of these analyses seem to confirm that for a given β there is a maximum shear that the sheath can support. An analytic approximation for small β gives $\text{Max } (\theta) = A\sqrt{\beta}$ where the constant A depends on the details of the distribution function - it is especially sensitive to any "skewness" of the distribution function in the plasma - sheath interface. The computer results for finite β appear to be in fair agreement with the above formula. The extension of the computer analysis to more realistic Maxwellian and quasi-Maxwellian distributions turned out to be more time consuming than expected. We hope to complete work on such distributions in the near future. Should the above results be corroborated by this work, the establishment of the existence of a maximum shear would be of obvious significance.

2. Boundary Layer for Rotating Plasma

The rotation of the main plasma⁽³⁾ in the Tormac has been suggested^(4,5) as a desirable alternative to rotational transform. The rotating plasma is coupled to the non-rotating sheath by a viscous boundary layer. Using conventional wisdom, the width of the boundary layer can be estimated to be $\delta \approx r/\sqrt{\text{Re}}$ (with r being the minor radius and Re the Reynolds number). Expressing the Reynolds number in terms of the transverse viscosity⁽⁶⁾,

rotational velocity and scale length, one is led to the relation $\delta/r_i = 1/\sqrt{\Omega\tau_i}$. Boozer⁽⁷⁾ estimates Ω to be of the order of the electron collision frequency, which gives a rather undesirable thin boundary layer.

We have done preliminary work on calculating the non-collisional coupling of the rotating plasma through the effect of a current entering the sheath and interacting with its magnetic field. This problem has been approached by considering the force exerted by the main plasma on the sheath, which, in turn, reacts on the main plasma. A further aspect of the problem is the possible evolution of hydrodynamic turbulence due to the non-circular cross section of the torus. Approximating the cross section by a triangle, the hydrodynamic flow pattern can be worked out by complex mapping technique. The study of the flow pattern then leads to the estimation of the possible breakdown of the laminar flow.

Further study of these problems is in progress. Publication of the sheath structure results has been delayed, mostly because of computational problems. A publication on these results is now planned for the near future.

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