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STRUCTURAL DESIGN ASPECTS OF
MAGNETIC COILS FOR
A LINEAR THETA-PINCH HYBRID REACTOR

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R. J. Bartholomew

ABSTRACT

The structural design aspects of a linear theta-pinch hybrid reactor (LTPHR) are centered in the solenoidal adiabatic compression coil (ACC) which must support the high magnetic pulse forces that tend to expand the coil and separate the leads. The structural model is represented by the theory of elasticity solution to a thick-walled cylinder. Dynamic amplification (or attenuation) is considered by a shock spectrum technique. A composite material is postulated, where the conductor material for each strand is clad with a high-strength stainless steel with insulation considered. Yield strength (for isolated-pulse operation) and endurance limit (for repetitive-pulse operation) for the high-strength steel impose magnetic field strength constraints on the coil design. These constraints are combined in an overall energy balance calculation that includes neutronic considerations to determine an optimum ACC design. The computer code ENBAL was used to incorporate neutronic, electrical, and structural constraints into the overall energy balance of the LTPHR. The lead separation problem is solved by designing spaced clamps to hold the leads together over great distances.

I. INTRODUCTION

The adiabatic compression coil (ACC) for a practical linear theta-pinch hybrid reactor (LTPHR)^{1,2} was designed as a multiturn solenoid to which high currents are pulsed through long leads from a relatively remote power supply. The requirement as dictated by energy balance considerations implies the need to generate peak magnetic fields on the order of 20 T, pulsed roughly as half-sine wave forms on the order of 1 to 2 ms as often as four pulses per second. The coil thickness was required to be kept at a minimum in order to maximize the flow of neutrons outward as generated in the plasma located inside of, and concentric with, the axis of the solenoid coil. Hemicylindrical blankets (~1-m-radius and 1-m-long) are situated outside of, and concentric with, the compression coil, each hemicylinder being a self-contained reactor core. An implosion

heating coil (IHC) of low magnetic field (between 1 and 2 T, end-fed) is concentric with and inside the ACC.

This report contains the structural analysis aspects of the design of the ACC and its feed lines. The structural design considerations include static strength for single-pulse operation, dynamic amplification, and endurance limit constraints for repetitive-pulse operation. Design curves are presented in parametric form for use in design iterations. These design curves were generated by a computer code ENBAL⁵ that incorporates neutronic, electrical and structural constraints into the overall energy balance of the LTPHR.

II. PROBLEM FORMULATION AND ANALYSIS METHOD

A. Adiabatic Compression Coil

Because of the high magnetic field required by

the ACC, the structural design problem is rather like that of designing a gun barrel since the pressure field generated by a 25-T coil is 250 MPa. The problem was analyzed by utilizing the theory of elasticity solution for a thick-walled cylinder under internal pressure as described in Ref. 3.

$$\sigma_r = \frac{P_i}{\left(\frac{\Delta b}{b} + 1\right)^2 - 1} \left\{ 1 - \left(\frac{b}{r}\right)^2 \left(\frac{\Delta b}{b} + 1\right)^2 \right\} \quad (1)$$

$$\sigma_\theta = \frac{P_i}{\left(\frac{\Delta b}{b} + 1\right)^2 - 1} \left\{ 1 + \left(\frac{b}{r}\right)^2 \left(\frac{\Delta b}{b} + 1\right)^2 \right\}.$$

The ACC and feed line configuration for one linear module of the plant is as shown in Fig. 1. The reactor core blanket is not included in the figure but is described as two hemicylinders not attached to each other but surrounding the ACC and supported from concrete ledges at the inner wall of the linear theta-pinch hybrid reactor (LTPHR) enclosure.

The reactor core blanket was assumed rigid in two halves, and the dynamic model consisted of these two masses displacing outward with the coil as it expanded under internal pressure for conditions such that the blanket was coupled with the coil. It will

be seen later that the coupling is easily prevented, which can be an advantage in the design of the coil. The strain energy for a given length of coil (L) was determined, and the equivalent stiffness (k_r) for motion at the outward coil radius ($b + \Delta b$) was derived (see Appendix A) and found to be of the form:

$$k_r = \frac{\pi L E \left[\left(\frac{\Delta b}{b} + 1\right)^2 - 1 \right]}{2 \frac{b}{b + 1}^2} [(1 - \nu) + (1 + \nu) \left(\frac{\Delta b}{b} + 1\right)^2] \quad (2)$$

Using the principle of virtual work, the effective force acting at $r = b + \Delta b$ due to the action of internal pressure was found and the dynamic model in the absence of damping was found to be of the type

$$\frac{M_t}{k_r} \ddot{u} + u = \xi(t) \quad (3)$$

Figure 2 shows a schematic of the model. Equation (3) is the governing equation for an undamped linear oscillator. The natural period is given by

$$T = 2\pi \sqrt{\frac{M_t}{k_r}} \quad (4)$$

The solution to Eq. (3) for $\xi(t)$, a half-sine pulse or a versed-sine ($\sin^2$) pulse of duration τ (sec), is well known and is given, for instance, in Ref. 4 and in Appendix B of this report. The shock spectrum technique from Ref. 4 was used to obtain the maximum dynamic amplification or attenuation $\alpha = \alpha(\tau/T)$ for the dynamic model Eq. (3). Figure 3 shows the so-called shock spectrum of α for a

Fig. 1. LTPHR two-coil system.

Fig. 2. Schematic model of LTPHR ACC dynamic system.

half-sine pulse of duration τ . Figure 4 shows a versed-sine shock spectrum of α . The versed-sine spectrum is appropriate if the magnetic field occurs as a half-sine pulse since the internal pressure varies as the magnetic field flux density squared (B^2). Appendix B presents details of how the α are obtained for both the half-sine and versed-sine pulses. The effective pressure (P_{eff}) was then determined from the theory of elasticity displacement solution for the thick cylinder under uniform pressure. In terms of (P_{eff}) this solution was

Fig. 3. Shock spectrum for half-sine pulse.

Fig. 4. Shock spectrum for versed-sine pulse.

derived in Appendix A to be

$$P_{eff} = \frac{\left[\left(\frac{\Delta b}{b} + 1 \right)^2 - 1 \right] E u_{max}}{2b \left(\frac{\Delta b}{b} + 1 \right)} \quad (5)$$

The stresses σ_r , σ_θ were found using P_{eff} for P_i in Eq. (1). The critical stress is the hoop stress σ_θ and is a maximum at $r = b$. Using this value of r in the second part of Eq. (1) and solving for

$$\frac{\sigma_{\theta max}}{P_{eff}}, \text{ we obtain:}$$

$$\frac{\sigma_{\theta max}}{P_{eff}} = \frac{1 + \left(\frac{\Delta b}{b} + 1 \right)^2}{\left(\frac{\Delta b}{b} + 1 \right)^2 - 1} \quad (6)$$

The magnetic field flux density B in tesla is given as a function of P_i in N/m^2 as

$$B = \sqrt{2 \mu_0 P_i} \quad (7)$$

where $\mu_0 = 4 \pi \times 10^{-7}$.

Remembering that

$$P_1 = \frac{1}{\alpha} P_{\text{eff}} \quad (8)$$

where $\alpha = \alpha(\tau/T)$, the dynamic amplification (attenuation) factor.

The stress $\sigma_{\theta\text{max}}$ may be taken to be the endurance limit of a single material. For a composite material of cross-section A, and assuming that only one component carries all the fatigue strength whereas the other component carries the current and is constrained not to deform beyond the elongation allowed by the strength component, we have the relationship

$$\sigma'_{\theta\text{max}} = \sigma_{\theta\text{max}} \frac{A}{A_s} \quad (9)$$

Combining Eqs. (6), (7), (8), and (9), parametric curves with $\Delta b/b$ as a parameter were generated with the structural constraint magnetic field parameter $B_c/\sqrt{A_s/A}$ as an ordinate and the ratio of pulse duration to structural period τ/T as abscissa

$$B_c/\sqrt{A_s/A} = \left\{ 2\mu_0 \frac{\sigma'_{\theta\text{max}}}{\alpha} \frac{\left(\frac{\Delta b}{b} + 1\right)^2 - 1}{1 + \left(\frac{\Delta b}{b} + 1\right)^2} \right\}^{1/2} \quad (10)$$

The load-bearing component stress $\sigma'_{\theta\text{max}}$ may be taken as the endurance limit of the composite material. Curves were constructed from Eq. (10) which show the limits or constraints on the design of pulsed coil systems of the above type. These curves are discussed in Sec. III.

B. ACC Leads

The coil leads are seen from Fig. 1 to be bundled together on either side of the ACC, spaced at the turn intervals along the length of the coil. Since the current is flowing in opposite directions in the two leads of each bundled pair, there will be a magnetic force generated that will try to separate the leads. This force is given by the equation

$$\frac{dF}{d\ell} = \mu_0 \frac{I^2}{2\pi a} \quad (11)$$

The force per unit length ($dF/d\ell$) may be considered as a distributed force between leads tending to separate the leads. Clamps must be designed to prevent undue lead separation. The design problem is to determine the maximum length (ℓ) between clamps and the size of clamps needed.

To determine the length (ℓ) between clamps, each span of leads may be considered a beam of length (ℓ) fixed at each end. The beam bends between clamps, and the maximum bending moment is given by

$$M_{\text{Bmax}} = \frac{dF}{d\ell} \frac{\ell^2}{12} \quad (12)$$

and occurs at either fixed end. Each lead has a rectangular cross section of width (w) and depth (d). The maximum bending stress occurs at the outer fibers and is given by

$$\sigma_{\text{bmax}} = \frac{M_{\text{Bmax}} d/2}{I_{xx}} \quad (13)$$

where

$$I_{xx} = \frac{wd^3}{12} \quad (14)$$

Given w and d , and selecting an allowable σ_{bmax} for the leads, combining Eqs. (12) and (13), and solving for ℓ gives

$$\ell = d \sqrt{\frac{2w \sigma_{\text{bmax}}}{\frac{dF}{d\ell}}} \quad (15)$$

The maximum deflection is at $\ell/2$ and is given by

$$\delta_{\text{max}} = \frac{\frac{dF}{d\ell} \ell^4}{384 E I_{xx}} \quad (16)$$

For the design of the clamps themselves, we consider a single span consisting of the leads of length (ℓ) and two clamps of length (ℓ_c). For this configuration we have the relationship between the reactions (R) at each clamp and the magnetic force per unit length ($dF/d\ell$).

$$2R = \frac{dF}{d\ell}(2\ell_c + \ell) \quad (17)$$

We also have the relationship for the tensile stress at each clamp, which is given by

$$\sigma_{c_t} = \frac{R}{2\ell_c t} \quad (18)$$

where (t) is the thickness of each clamp.

Combining Eqs. (17) and (18) gives a relationship between ℓ_c and t

$$\ell_c = \frac{\ell}{4\sigma_{c_t} t / \frac{dF}{d\ell} - 2} \quad (19)$$

Selecting an allowable value of σ_{c_t} we may solve Eq. (19) for combinations of ℓ_c and t that give a reasonable geometric design.

C. ACC Design Details

The ACC basic design consists of a helix of conductors made up of rectangular strands that are transposed along the helix to insure that each strand will be the same length even, though the conductors are curved to form the helix. For strength considerations, each strand is designed, in general, of a composite of two materials-- one that carries the electric current, designated (m_c), and the other that provides the strength requirement, designated (m_s).

The configuration is such that the m_c material is on the inside of the strand, and the strength material m_s is clad with a certain thickness (t_s) around the m_c material. In this way, with the conductors in helix form, internal pressure will cause equal strains in each of the two materials. With total cross-section area (A) composed of (A_s) for (m_s) and (A_c) for (m_c), we have the following relations for the composite elastic moduli.

$$A = A_s + A_c$$

$$E = \frac{A_s E_s + A_c E_c}{A}$$

$$G = \frac{A_s G_s + A_c G_c}{A} \quad (a)$$

$$\nu = \frac{E}{2G} - 1$$

$$G_s = \frac{E_s}{2(1 + \nu_s)}$$

$$G_c = \frac{E_c}{2(1 + \nu_c)}$$

The maximum hoop load (F_θ) carried by the inner strand is given by

$$F_\theta = \sigma_{\theta\max} A \quad (20)$$

The subsequent hoop stresses in each of the two materials may be shown to be given by:

$$\sigma_{\theta\max} = \frac{E_s}{E} \sigma_{\max} \quad (21)$$

$$\sigma_{\theta c\max} = \frac{E_c}{E} \sigma_{\max}$$

If the conductor material has stress that exceeds its ultimate strength, we have the relationship

$$F_\theta = \sigma_{\theta\max} A = \sigma'_{\theta\max} A_s$$

which gives the important formula (9) previously stated. The ratio A_s/A will clearly remain a constant for any number of strands or coil thickness required by the design.

With the cross-sectional dimensions of a strand given as in the following sketch, we have

$$A_s = 2t_s (w_c + d_c + 2t_s) \quad (22)$$

$$A_c = w_c d_c$$

For radial stress, we must consider the area of a one-unit circumference of a strand as one edge, w_c as the other edge of the conductor material, and $2t_s$ as the total edge length of the strength material. Thus we have

$$A_{cr} = w_c \quad (23)$$

$$A_{sr} = 2t_s$$

The maximum radial stress σ_{rmax} is at the inner radius $r = b$ of the shell [see the first part of Eq. (1)] and is equal to the internal pressure P_i . The maximum radial load is given by

$$F_r = P_i A_r \quad (24)$$

where

$$A_r = A_{cr} + A_{sr}$$

Because of the requirement for equal strain of each material in the radial direction, we have

$$\frac{F_{rc}}{E_c A_{cr}} = \frac{F_{rs}}{E_s A_{sr}} \quad (25)$$

and since

$$F_r = F_{rc} + F_{rs} \quad (26)$$

we can solve for F_{rc} and F_{rs} from Eqs. (25) and (26). Then we can obtain the radial stress maxima from

$$\sigma_{rmax} = \frac{E_s A_r}{E_s A_{sr} + E_c A_{cr}} \sigma_{rmax} \quad (27)$$

$$\sigma_{rcmax} = \frac{E_c A_r}{E_s A_{sr} + E_c A_{cr}} \sigma_{rmax}$$

III. RESULTS

A. Adiabatic Compression Coil

Figure 5 shows the physical dimensions of the cross section of a single strand of the ACC design selected for the LTPHR. The ACC was designed to be composed of 10 turns comprising each helix, with each turn consisting of two tandem sets of 50 strands each, making an overall thickness (Δb) of 10 cm, an inside radius (b) of 14 cm, and turn width of 19 mm (adding 1 mm between the two tandem sets for insulation). A gap of 3.5 mm is provided between each turn of the helix for insulation and cooling purposes. Thus, the total length of each coil module is 225 mm.

The conductor material selected is beryllium-copper, and the strength material is an age-hardenable stainless steel (Custom 455). Table I gives the mechanical properties for these two materials.

Using the composite material described by Table I and Fig. 5 in equation (a), we obtain the following "equivalent" elastic moduli

$$E = 1.62 \times 10^5 \text{ MPa}$$

$$G = 0.614 \times 10^5 \text{ MPa} \quad (b)$$

$$\nu = 0.318$$

Fig. 5. Physical dimensions of cross-section of each conductor strand.

TABLE I
MECHANICAL PROPERTIES OF ACC MATERIALS

Property	Custom 455	Beryllium- Copper
E	2×10^5 MPa	1.3×10^5 MPa
ν	0.29	0.355
σ yield	1634 MPa	172.4-241.3 MPa (annealed) 896.3-1034.2 MPa (hard)
σ ultimate	1689.2 MPa	413.7-551.6 MPa (annealed) 1137.6-1275.5 MPa (hard)
σ_e	758.4 MPa (tension- compression) 965.3 MPa (tension- tension)	-----
Elec. resistance	75 $\mu\Omega$ -cm	4.82-5.82 $\mu\Omega$ -cm

Also, for the configuration used,

$$\begin{aligned} L &= 2 \text{ m} \\ b &= 14 \text{ cm} \\ \Delta b &= 10 \text{ cm} \end{aligned} \quad (c)$$

With these data it is found that

$$k_r = 1.5304 \times 10^{12} \text{ N/m} \quad (28)$$

The approximate weight of the coil plus two blankets is

$$W_T = 0.1868 \times 10^6 \text{ N} \quad (29)$$

which corresponds to a mass M_T of

$$M_T = 1.905 \times 10^4 \text{ kg} \quad (30)$$

Using Eq. (4) we obtain the natural period

$$T = 0.70 \text{ ms} \quad (31)$$

For a half-sine shock pulse, the duration τ was taken to be 2 ms. Thus, we have

$$\frac{\tau}{T} = 2.86 \quad (32)$$

Using a versed-sine shock pulse, which is probably more representative of the actual magnetic pressure pulse, and with the blanket uncoupled from the coil, the mass is about 0.1 of M_T as given by Eq. (30). Thus, the natural period is given as

$$T = 0.23 \text{ ms} \quad (33)$$

Thus, we have with $\tau = 2 \text{ ms}$

$$\frac{\tau}{T} = 9.0$$

Using the dynamic model illustrated in Fig. 2, we obtain the amplification factors from Fig. 3 or Fig. 4, or more precisely, as indicated in Appendix B.

Maximax

$$\alpha = \begin{cases} 1.15, & \text{from Fig. 3} \\ 1.05, & \text{from Fig. 4} \end{cases} \quad (34)$$

Residual

$$\alpha = \begin{cases} 0.3263, & \text{from Eq. B-7} \\ 0.0593, & \text{from Eq. B-12} \end{cases} \quad (35)$$

As derived in Appendix A, we use Eq. (8) to find P_{eff} for both during the pulse and the residual vibration after the pulse using the versed-sine values from Fig. 4 and Eq. B-12).

Maximax

$$P_{\text{eff}} = 1.05 \times 250 = 262.5 \text{ MPa} \quad (36)$$

Residual

$$P_{\text{eff}} = 0.0593 \times 250 = 14.8 \text{ MPa} \quad (37)$$

Using Eq. (1) with P_{eff} substituted for P_1 , we obtain the maximum stresses at $r = b$ of

Maximax

$$\sigma_{rmax} = -262.5 \text{ MPa} \quad (38)$$

$$\sigma_{\theta max} = 533.3 \text{ MPa} \quad (39)$$

Residual

$$\sigma_{rmax} = -14.8 \text{ MPa} \quad (40)$$

$$\sigma_{\theta max} = 30.07 \text{ MPa} \quad (41)$$

Using the physical dimension data of Fig. 4 and Eqs. (22) and (23), we obtain

$$\begin{aligned} A_s &= 8.8 \text{ mm}^2 \\ A_C &= 10.8 \text{ mm}^2 \\ A &= 19.6 \text{ mm}^2 \\ A_{sr} &= 0.8 \text{ mm}^2 \\ A_{cr} &= 9 \text{ mm}^2 \\ A_r &= 9.8 \text{ mm}^2 \end{aligned} \quad (42)$$

Using Eqs. (21), (25), (26), and (27) with the data of Table I and the data of Eqs. (b) and (42), we obtain the stresses in the component materials.

Maximax

$$\begin{aligned} \sigma_{rsmax} &= -386.8 \text{ MPa} \\ \sigma_{\theta smax} &= 658.4 \text{ MPa} \\ \sigma_{rcmax} &= -251.4 \text{ MPa} \\ \sigma_{\theta cmax} &= 428 \text{ MPa} \end{aligned} \quad (43)$$

Residual

$$\begin{aligned} \sigma_{rsmax} &= -21.8 \text{ MPa} \\ \sigma_{\theta smax} &= 37.12 \text{ MPa} \\ \sigma_{rcmax} &= -14.18 \text{ MPa} \\ \sigma_{\theta cmax} &= 24.13 \text{ MPa} \end{aligned} \quad (44)$$

Comparing the results of Eqs. (43) and (44) with data of Table I indicates that for the maximax condition, it is likely that the beryllium-copper component will exceed its ultimate strength (conservatively using the minimum end of the annealed range). Thus, we must assume that the Custom 455 steel component will take the entire maximum load. Thus, we use Eq. (9) to obtain for the maximax condition

$$\sigma'_{\theta smax} = 1187.8 \text{ MPa} \quad (45)$$

The factor of safety for hoop stress based on ultimate strength is therefore

$$(F.S.)_{s\theta} = 1.42 \quad (46)$$

The factors of safety for radial stress based on ultimate strength are computed:

$$(F.S.)_{sr} = \frac{1689.2}{386.8} = 4.37 \quad (47)$$

$$(F.S.)_{cr} = \frac{413.7}{251.4} = 1.65$$

The above results indicate marginal-to-satisfactory factors of safety for the ACC as designed for single-shot (or a few shots) application. However, for repetitive pulses, the endurance limit stresses (σ_e) as shown in Table I indicate that the design will be inadequate. The residual response stresses [Eq. (44)] present no problem because they are less than one-tenth of the tension-compression endurance limits of the steel. However, from Eq. (45) it is seen that the maximax hoop stress of 1187.8 MPa exceeds the steel tension-tension endurance limit of 965.4 MPa. No endurance limit figures are available for the beryllium-copper.

The magnetic field that the design could sustain for, essentially, unlimited life is that field that would produce a stress in the steel not greater than 965.3 MPa. This field may be calculated by the simple ratio

$$\frac{B_1}{B_2} = \sqrt{\frac{\sigma_2}{\sigma_1}} \quad (48)$$

Using $\sigma_e = 965.3 \text{ MPa}$ for σ_2 and $\sigma'_{\theta smax} = 1187.8 \text{ MPa}$ for σ_1 , and $B_2 = 25 \text{ T}$, we find that

$$B_1 = 22.5 \text{ T} \quad (49)$$

Using Eq. (10), the parametric curves of Figs. 6 and 7 were generated. The half-sine values of α were used. Figure 6 shows the ACC geometric constraint curves with $\sigma_e = 965.3 \text{ MPa}$, tension-tension and $\sigma_e = 758.4 \text{ MPa}$, tension-compression; whereas Fig. 7 shows the single-shot curves based on Custom 455 yield strength ($\sigma_{y.s.} = 1634 \text{ MPa}$).

These latter curves should not be used for repetitive-pulse designs, and therefore, are not pertinent constraint curves for the LTPHR. They are presented primarily to show the difference between design curves based on yield strength and those based on fatigue considerations.

Fig. 6. Structural endurance limit magnetic field constraint.

Figure 8 shows the variation in required vs structural limit magnetic fields as a function of τ/T . The structural limit curves are shown for the LTPHR dimensions and for an A_s/A equal to 0.4 structural fraction and period $\tau/T = 0.23$ ms, the coil having the endurance and yield strength properties of

Fig. 7. Structural yield strength limit magnetic field constraint.

Fig. 8. Structural constraint curves for an adiabatic compression coil on a linear theta-pinch hybrid reactor (LTPHR). (Q_E) is the ratio of total (real plus virtual electrical) output to total electrical input, $1, 2$ (E_E) is the implosion field used to preheat the DT plasma, (A_s/A) is the structural volume fraction in the ACC, (T) is the mechanical period of the ACC, (Δb) is the coil thickness (0.1 m), (b) is the inner radius of the ACC (0.14 m), and (L) is the length of the LTPHR required for a given burn time (τ) (end-loss time or half-period of the magnetic field pulse).

Custom 455 stainless steel. The significance of these curves is that the requirements are met for an LTPHR design for all pulse durations greater than about $\tau = 0.25$ ms for unity energy multiplication Q_E , $\tau = 0.62$ ms for doubling energy multiplication, $\tau = 1.09$ ms for tripling energy multiplication, and $\tau = 1.84$ ms for quadrupling energy multiplication. The required total lengths of these LTPHR designs would be 110 m, 270 m, 475 m, and 800 m, respectively. The economic break-even point is exceeded for all designs operating above a τ of 0.53 ms. The yield limit constraint curve is also shown for comparison purposes.

Figure 9 shows an optimization of structural fraction A_s/A for Custom 455 in terms of maximizing Q_E for three different values of LTPHR length. These curves show that A_s/A in the range 0.4-0.5 is optimum.

Fig. 9. Dependency of maximum value of Q_E as determined by endurance constraint on A_s/A for three lengths of theta pinch.

B. ACC Leads

As shown in Fig. 1, the leads are twisted so that the incoming lead and outgoing lead for each helical coil lie flat and tandem to each other as they leave the coil and approach the power supply. Also the 19-mm thickness is doubled for the approach section to the power supply. The design current for the approach section was determined to be

$$I = 0.448 \times 10^6 \text{ A}.$$

The distance (a) between the incoming and outgoing leads, allowing 15 mm between the leads for insulation, was determined from geometry to be

$$a = 100 + 15 = 115 \text{ mm}.$$

With $\mu_0 = 0.4\pi \times 10^{-6}$, using Eq. (10) gives

$$\frac{dF}{dx} = 0.35 \times 10^6 \text{ N/m}. \quad (50)$$

The total lead bundle forms a rectangle of dimensions

$$w = 38 \text{ mm}$$

$$D = 2d + 15 = 2(100) + 15 = 215 \text{ mm}.$$

If we limit the bending stresses to

$$\sigma_{bmax} = 82.74 \text{ MPa},$$

Equation (4) gives a minimum length between bands of

$$l = 424 \text{ mm}. \quad (51)$$

Using the beryllium-copper modulus of elasticity ($E = 1.3 \times 10^5 \text{ MPa}$), the maximum deflection of the leads between clamps is found from Eq. (15) to be

$$\delta_{max} = 0.0716 \text{ mm}. \quad (52)$$

We determine clamp dimensions as follows:

- 1) Limit tensile stress in clamp to

$$\sigma_{ct} = 82.74 \text{ MPa},$$

- 2) Estimate thickness to be 8 mm, and

- 3) Using Eq. (19), calculate

$$l_c = 7.62 \text{ cm} \quad (53)$$

Thus we may design the clamps to have a thickness of 8 mm and a length each of 8 cm. The inside dimensions will be

$$w = 3.8 \text{ cm}$$

$$D = 21.5 \text{ cm}. \quad (54)$$

REFERENCES

1. K. I. Thomassen, R. A. Krakowski, T. A. Oliphant, G. E. Bosler, and D. J. Dudziak, "Prospects for Converting $^{232}\text{Th}^{233}\text{U}$ in a Linear Theta-Pinch," Trans. Am. Nucl. Soc. **19**, 6 (1974).
2. R. A. Krakowski, D. J. Dudziak, T. A. Oliphant, and K. I. Thomassen, "An Engineering Design of a Linear Theta-Pinch Hybrid Reactor (LTPHR)," Trans. Am. Nucl. Soc. **21**, 61 (1975).
3. S. Timoshenko and J. N. Goodier, Theory of Elasticity, 2nd Ed., Chap. 4, p. 60, and Chap. 6, p. 178 (prob. 3) (McGraw-Hill Book Co., New York, 1951).
4. C. M. Harris and C. E. Crede, Eds., Shock and Vibration Handbook, Chap. 8, pp. 8-3, 8-21, and 8-24 (McGraw-Hill Book Co., New York, 1961).
5. R. A. Krakowski, unpublished work, Los Alamos Scientific Laboratory (1975).

NOMENCLATURE

σ_r	Radial stress of thick-walled cylinder.
σ_θ	Circumferential (hoop) stress of thick-walled cylinder.
$\sigma_x, \sigma_y, \sigma_z$	Normal stresses in rectangular xyz cartesian system.
$\tau_{xy}, \tau_{yz}, \tau_{zx}$	Shear stresses in xy, yz, zx planes of rectangular xyz cartesian system.
$\tau_{r\theta}, \tau_{rz}, \tau_{\theta z}$	Shear stresses in $r\theta$, rz , θz , planes of cylindrical coordinate system.
P_i	Internal pressure in thick-walled cylinder.
Δb	Wall thickness of thick-walled cylinder.
b	Inner radius of thick-walled cylinder.
r	Radial coordinate to a material point in thick-walled cylinder.
B	Magnetic flux density in solenoid.
B_c	Magnetic flux density due to structural constraint.
μ_0	Magnetic permeability of free space equal to $4\pi \times 10^{-7}$.
E	Young's modulus of elasticity (general or composite material).
G	Shear modulus (general or composite material).
ν	Poisson's ratio (general or composite material).
E_s, E_c	Young's modulus of elasticity for structural or conductor materials, respectively.
G_s, G_c	Shear modulus for structural or conductor materials, respectively.
ν_s, ν_c	Poisson's ratio for structural or conductor materials, respectively.
A_s, A_c	Cross-sectional area of structural or conductor materials, respectively
A	Total cross-sectional area of composite material.
t_s	Thickness of structural material in composite structure strand.
w_c	Width of conductor material strand.
d_c	Depth of conductor material strand.
A_{sr}, A_{cr}	Bearing area in radial direction of structural material and conductor material, respectively.
F_θ	Maximum hoop load carried by inner strand of composite structure and conductor.
$\sigma_{\theta max}$	Maximum hoop stress of composite material.
$\sigma_{\theta s max}, \sigma_{\theta c max}$	Maximum hoop stress of structural and conductor material, respectively.
$\sigma'_{\theta s max}$	Maximum hoop stress of structural material when conductor ultimate stress is exceeded.
F_r	Maximum radial load of composite material.
F_{rs}, F_{rc}	Maximum radial load of structural material and conductor material, respectively.
$\sigma_{r max}$	Maximum radial stress of composite material.
$\sigma_{rs max}, \sigma_{rc max}$	Maximum radial stress of structural material and conductor material, respectively.
k_r	Equivalent spring constant of coil acting at outer radius $(b + \Delta b)$ in uniform expansion of thick cylinder.

L	Length of adiabatic compression coil.
M_t	Total mass of coil plus blanket or of coil itself.
$\xi(t)$	Displacement of spring k_r at end $b + Ab$.
$u(t)$	Radial displacement of coil as a function of time.
T	Natural period of coil dynamic system.
$u_{\max}$	Maximum radial displacement of coil at $b + Ab$.
$\alpha\left(\frac{\tau}{T}\right)$	Dynamic amplification (attenuation) of undamped oscillator excited by shock pulse.
P_{eff}	Effective internal pressure on coil due to shock amplification (attenuation).
A_s/A	Structural volume fraction of composite coil.
$\frac{dF}{d\ell}$	Magnetic force per unit length between two electrical conductors.
I	Current in electrical conductors.
a	Mean distance between two electrical conductors.
ℓ	Length between clamps of solenoid leads.
$M_{B\max}$	Maximum bending moment between clamps of solenoid lead electrical conductors.
$\sigma_{b\max}$	Maximum allowable bending stress in solenoid leads.
w	Width of rectangular cross section of one lead.
d	Depth of rectangular cross section of one lead.
$\delta_{\max}$	Maximum deflection due to bending of a solenoid lead.
I_{xx}	Cross-sectional area moment of inertia about transverse axis of a solenoid lead.
R	Reaction force at clamps due to distributed magnetic force between clamps of solenoid lead.
ℓ_c	Length of clamp.
t	Thickness of clamp.
D	Inside depth dimension of clamp.
σ_{ct}	Allowable tensile stress tending to separate clamp.
$\sigma_{\text{yield}}, \sigma_{y.s.}$	Yield strength of material.
$\sigma_{\text{ultimate}}, \sigma_{u.s.}$	Ultimate strength of material.
σ_e	Endurance limit of material.
W_T	Total weight of coil plus two blankets, or of the coil alone, as applicable.
τ	Shock pulse duration.
$\frac{\tau}{T}$	Ratio of pulse duration to dynamic system natural period.
B_1, B_2	Magnetic flux density for condition 1 and condition 2.
σ_1, σ_2	Material stress for condition 1 and condition 2.
v_o	Strain energy per unit volume of thick-walled cylinder.
v	Strain energy per unit length of thick-walled cylinder.
U	Strain energy of thick-walled cylinder of length L .
$\epsilon_r, \epsilon_\theta, \epsilon_{r\theta}$	Strains in radial, hoop, and in $r\theta$ plane in cylindrical coordinates.
$u(r, \theta), v(r, \theta)$	Radial and circumferential displacements of material point in cylindrical coordinates.

$f(t), g(r)$	Undetermined functions due to partial integration.
λ	Eigenvalue constant.
A, B, C	Constants of integration.
δW	Virtual work.
$\delta u(b + \Delta b), \delta u(b)$	Virtual displacements at $(b + \Delta b)$ and (b) , respectively.
F_p	Effective force doing virtual work δW through virtual displacement $\delta u(b + \Delta b)$.
$f(t)$	General form of shock pulse.
ξ_p	Peak displacement of spring k_r at end $b + \Delta b$.
$G(\frac{T}{T})$	Form of time variation part of response to half-sine pulse.
$\alpha_R(\frac{T}{T})$	Residual response amplification (attenuation) factor.

APPENDIX A

DERIVATION OF STRUCTURAL DYNAMIC MODEL OF LTPHR ACC WITH REACTOR CORE BLANKET

Structurally, the adiabatic compression coil may be treated as a cylindrical, thick-walled pressure vessel under internal pressure (P_i). The value of P_i is determined by the magnetic flux density B in the following relationship.

$$P_i = \frac{1}{2} \frac{B^2}{\mu_0} \quad , \quad (A-1)$$

where $\mu_0 = 4\pi \times 10^{-7}$,

B = Magnetic flux density (tesla), and

P_i = Internal pressure (N/m^2) .

From the theory of elasticity, the stress components in cylindrical coordinates are given by

$$\sigma_r = \frac{P_i}{(\frac{\Delta b}{b} + 1)^2 - 1} \left[1 - \left(\frac{b}{r}\right)^2 \left(\frac{\Delta b}{b} + 1\right)^2 \right]$$

$$\sigma_\theta = \frac{P_i}{(\frac{\Delta b}{b} + 1)^2 - 1} \left[1 + \left(\frac{b}{r}\right)^2 \left(\frac{\Delta b}{b} + 1\right)^2 \right] \quad (A-2)$$

$$\sigma_z = \tau_{r\theta} = \tau_{rz} = \tau_{\theta z} = 0 \quad .$$

From Ref. 3, p. 148, the general strain energy per unit volume in rectangular coordinates expressed as a function of stress components is given by

$$v_o = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - \frac{\nu}{E}(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x) + \frac{1}{2G}(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2). \quad (A-3)$$

The coordinate transformation law for the stress tensor to transform to cylindrical coordinates from rectangular coordinates to matrix notation is as follows

$$\begin{bmatrix} \sigma_r & \tau_{r\theta} & \tau_{rz} \\ \tau_{r\theta} & \sigma_\theta & \tau_{\theta z} \\ \tau_{rz} & \tau_{\theta z} & \sigma_z \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (A-4)$$

Since the transformation is orthogonal, the inverse of the transformation matrix is equal to its transpose. Therefore, we may write Eq. (A-4) solving for the xyz components of stress in terms of the rθz components as follows:

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma_r & \tau_{r\theta} & \tau_{rz} \\ \tau_{r\theta} & \sigma_\theta & \tau_{\theta z} \\ \tau_{rz} & \tau_{\theta z} & \sigma_z \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (A-5)$$

Expanding Eq. (A-5) in view of the last of Eqs. (A-2) gives

$$\begin{aligned} \sigma_x &= \sigma_r \cos^2\theta + \sigma_\theta \sin^2\theta \\ \sigma_y &= \sigma_r \sin^2\theta + \sigma_\theta \cos^2\theta \\ \tau_{xy} &= (\tau_r - \sigma_\theta) \sin\theta \cos\theta \\ \sigma_z &= \tau_{yz} = \tau_{xz} = 0 \end{aligned} \quad (A-6)$$

The shear modulus G is given by the well known relationship

$$G = \frac{E}{2(1 + \nu)} \quad (A-7)$$

Substituting Eq. (A-6) into (A-3) gives

$$v_o = \frac{1}{2E}(\sigma_r^2 + \sigma_\theta^2 - 2\nu \sigma_r \sigma_\theta) \quad (A-8)$$

Using Eq. (A-2) we obtain

$$v_o = \frac{b^2 P_i^2}{E \left[\left(\frac{\Delta b}{b} + 1 \right)^2 - 1 \right]} \left[(1 - \nu) + (1 + \nu) \left(\frac{b}{r} \right)^4 \left(\frac{\Delta b}{b} + 1 \right)^4 \right]. \quad (A-9)$$

The strain energy per unit length of the tube is given by

$$V = \iint_A v_o \, dA, \quad (A-10)$$

where $dA = r \, dr \, d\theta$.

Thus

$$V = \int_0^{2\pi} \int_b^{b+\Delta b} \left\{ \frac{b^2 P_i^2}{E \left[\left(\frac{\Delta b}{b} + 1 \right)^2 - 1 \right]} \left[(1 - \nu) + (1 + \nu) \left(\frac{b}{r} \right)^4 \left(\frac{\Delta b}{b} + 1 \right)^4 \right] \right\} r \, dr \, d\theta \quad (A-11)$$

Equation (A-11) integrates to give

$$v = \frac{\pi b^2 P_i^2}{E \left[\left(\frac{\Delta b}{b} + 1 \right)^2 - 1 \right]} \left[(1 - \nu) + (1 + \nu) \left(\frac{\Delta b}{b} + 1 \right)^2 \right] \quad (A-12)$$

For a given cylinder of length (L) and uniform cross section, the total strain energy becomes

$$U = VL = \frac{\pi b^2 P_i^2 L}{E \left[\left(\frac{\Delta b}{b} + 1 \right)^2 - 1 \right]} \left[(1 - \nu) + (1 + \nu) \left(\frac{\Delta b}{b} + 1 \right)^2 \right] \quad (A-13)$$

We also need the displacements (u, v) in order to develop the equivalent spring constant k_r at $r = b + \Delta b$. From Ref. 1, the stress-strain and strain-displacement relations are

$$\epsilon_r = \frac{\partial u}{\partial r} = \frac{1}{E}(\sigma_r - \nu \sigma_\theta) \quad , \quad (A-14)$$

$$\epsilon_\theta = \frac{u}{r} + \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{1}{E}(\sigma_\theta - \nu \sigma_r) \quad , \text{ and}$$

$$\gamma_{r\theta} = \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} = 0 \quad .$$

From the first part of Eq. (A-14) with Eq. (A-2) substituted for the stresses and integrating with respect to r, we obtain

$$u(r, \theta) = \frac{r P_i}{E \left[\left(\frac{\Delta b}{b} + 1 \right)^2 - 1 \right]} \left[(1 - \nu) + (1 + \nu) \left(\frac{b}{r} \right)^2 \left(\frac{\Delta b}{b} + 1 \right)^2 \right] + f(\theta) \quad . \quad (A-15)$$

From Eq. (A-15) and the second of Eq. (A-14), we obtain

$$\frac{\partial v}{\partial \theta} = -f(\theta) \quad , \quad (A-16)$$

which gives

$$v(r, \theta) = - \int f(\theta) d\theta + g(r) \quad . \quad (A-17)$$

Using the third part of Eq. (A-14) and the above, we obtain the eigenvalue problem

$$\frac{\partial f(\theta)}{\partial \theta} + \int f(\theta) d\theta = -r \frac{\partial g(r)}{\partial r} + g(r) = \lambda \quad , \quad (A-18)$$

where λ is a constant (eigenvalue). Solving

$$r \frac{dg}{dr} - g(r) = -\lambda \quad , \quad (A-19)$$

we obtain

$$g(r) = Cr + \lambda \quad , \quad (A-20)$$

where C is an arbitrary constant. Solving

$$\frac{d^2 f}{d\theta^2} + f = 0 \quad , \quad (A-21)$$

we obtain

$$f(\theta) = A \sin \theta + B \cos \theta \quad , \quad (A-22)$$

where A and B are arbitrary constants. The displacement general solutions become

$$u(r, \theta) = \frac{r P_i}{E \left[\left(\frac{\Delta b}{b} + 1 \right)^2 - 1 \right]} \left[(1 - \nu) + (1 + \nu) \left(\frac{b}{r} \right)^2 \left(\frac{\Delta b}{b} + 1 \right)^2 \right] + A \sin \theta + B \cos \theta$$

$$v(r, \theta) = A \cos \theta - B \sin \theta + Cr + \lambda \quad . \quad (A-23)$$

The uniform loading of internal pressure P_i requires that u does not vary with θ . Thus,

$$A = B = 0 \quad .$$

As a consequence of uniform internal pressure P_i , there is no tangential displacement. Thus,

$$v(r, \theta) = 0 \quad , \quad (A-24)$$

which requires that

$$C = \lambda = 0 \quad .$$

Finally, we may write the radial displacement u as

$$u(r) = \frac{r P_i}{E \left[\left(\frac{\Delta b}{b} + 1 \right)^2 - 1 \right]} \left[(1 - \nu) + (1 + \nu) \left(\frac{b}{r} \right)^2 \left(\frac{\Delta b}{b} + 1 \right)^2 \right] \quad . \quad (A-25)$$

At the inner surface $r = b$, we have

$$u(b) = \frac{b P_i}{E \left[\left(\frac{\Delta b}{b} + 1 \right)^2 - 1 \right]} \left[(1 - \nu) + (1 + \nu) \left(\frac{\Delta b}{b} + 1 \right)^2 \right] \quad . \quad (A-26)$$

At the outer surface $r = b + \Delta b$, we have

$$u(b + \Delta b) = \frac{2b\left(\frac{\Delta b}{b} + 1\right)P_1}{E\left[\left(\frac{\Delta b}{b} + 1\right)^2 - 1\right]} \quad (A-27)$$

To find the equivalent spring constant acting through the displacement $u(b + \Delta b)$, we use the relation

$$U = \frac{1}{2} k_r u^2(b + \Delta b) \quad (A-28)$$

Using Eqs. (A-13) and A-27), we obtain

$$k_r = \frac{\pi L E \left[\left(\frac{\Delta b}{b} + 1 \right)^2 - 1 \right]}{2 \left(\frac{\Delta b}{b} + 1 \right)^2} \left[(1 - \nu) + (1 + \nu) \left(\frac{\Delta b}{b} + 1 \right)^2 \right] \quad (A-29)$$

The virtual work δW due to the effective force F_p , acting at $r = b + \Delta b$ through a virtual displacement $\delta u(b + \Delta b)$, is given by

$$\delta W = F_p \delta u(b + \Delta b) = 2 \pi b L P_1 \delta u(b) \quad (A-30)$$

The effective force F_p is thus given by

$$F_p = 2 \pi b L P_1 \frac{\delta u(b)}{\delta u(b + \Delta b)} \quad (A-31)$$

$$\frac{\delta u(b)}{\delta u(b + \Delta b)} = \frac{u(b)}{u(b + \Delta b)}, \text{ and therefore}$$

$$F_p = \frac{\pi b L P_1}{\frac{\Delta b}{b} + 1} \left[(1 - \nu) + (1 + \nu) \left(\frac{\Delta b}{b} + 1 \right)^2 \right] \quad (A-32)$$

The dynamic equation of motion with no damping can be placed in the form

$$\frac{M_T}{k_r} \ddot{u} + u = \xi(t) \quad (A-33)$$

where M_T = total mass of coil plus blanket,

$$\xi(t) = \frac{F(t)}{k_r}, \text{ and}$$

u = absolute displacement of M_T radially.

If the dynamic excitation is of the form

$$F(t) = F_p f(t) \quad (A-34)$$

the displacement $\xi(t)$ will be of the form

$$\xi(t) = \xi_p f(t) = \frac{2b\left(\frac{\Delta b}{b} + 1\right)P_1}{E\left[\left(\frac{\Delta b}{b} + 1\right)^2 - 1\right]} f(t) \quad (A-35)$$

as obtained from Eqs. (A-29) and (A-32).

If $f(t)$ is a shock pulse of any form, Ref. 2 gives the relationship of the maximum of the maxima (maximax response and the residual response) as

$$\frac{u}{\xi_p} \max = \alpha \quad (A-36)$$

Thus the maximum radial displacement $u_{\max}(b + \Delta b)$ can be computed from

$$u_{\max}(b + \Delta b) = \alpha \xi_p \quad (A-37)$$

Using Eq. (A-27), we may solve for the effective pressure P_{eff} that will produce $u_{\max}(b + \Delta b)$. From a comparison of Eqs. (A-27) and (A-35), it is evident that

$$P_{\text{eff}} = \alpha P_1 \quad (A-38)$$

The shock spectrum for a half-sine pulse is shown in Fig. 3 and is derived in Appendix B.

APPENDIX B

DERIVATION OF SHOCK SPECTRA FOR UNDAMPED DYNAMIC RESPONSE OF AN OSCILLATOR TO A MAGNETIC FIELD PULSE

For the differential equation of an undamped oscillator of the form

$$\frac{m}{k} \ddot{u} + u = \xi(t) \quad , \quad (B-1)$$

where m = mass of mechanical system,

k = spring constant of mechanical system,

u = absolute displacement of mass, and

$\xi(t)$ = absolute displacement of remote end of spring.

Half-Sine Pulse

Let $\xi(t)$ be represented as a half-cycle sine pulse as

$$\xi(t) = \begin{cases} \xi_p \sin \frac{\pi t}{\tau} & , \quad 0 \leq t \leq \tau \\ 0 & , \quad t > \tau \end{cases} \quad (B-2)$$

The response $u(t)$ is the well-known solution (see Ref. (4) of the form

$$u\left(\frac{t}{T}\right) = \begin{cases} \frac{\xi_p}{1 - \left(\frac{1}{2\tau/T}\right)^2} \left(\sin \left[\frac{\pi}{\tau/T} \left(\frac{t}{T} \right) \right] - \frac{1}{2\tau/T} \sin \left[2\pi \left(\frac{t}{T} \right) \right] \right), & 0 \leq \frac{t}{T} \leq \frac{\tau}{T} \\ \xi_p \left[\frac{\cos \pi \frac{\tau}{T}}{\frac{\tau}{T} \left[\left(\frac{1}{2\tau/T} \right)^2 - 1 \right]} \right] \sin 2\pi \left(\frac{t}{T} - \frac{\tau}{2T} \right), & \frac{t}{T} > \frac{\tau}{T} \end{cases} \quad (B-3)$$

The ratio $(u/\xi_p)_{\max} = \alpha(\tau/T)$ is defined as the shock spectrum, or response spectrum for the dynamic system Eq. (B-1). There will be two regions of maxima to investigate:

- (1) Maxima in the range $0 \leq \frac{t}{T} \leq \frac{\tau}{T}$, and
- (2) Residual maxima in the range $\frac{t}{T} > \frac{\tau}{T}$.

For the region (2), the largest maximum is considered the residual response. The very largest maximum (occurring in either region (1) or region (2)) is termed the maximax response.

The maxima of Eq. (B-3) are found in the usual way. For region (1) we must find the maxima of the function

$$G\left(\frac{t}{T}\right) = \sin \left[\frac{\pi}{\tau/T} \left(\frac{t}{T} \right) \right] - \frac{1}{2\tau/T} \sin \left[2\pi \left(\frac{t}{T} \right) \right] \quad (B-4)$$

Differentiating and setting to zero,

$$\frac{dG}{d\left(\frac{t}{T}\right)} = \frac{\pi}{\tau/T} \left[\cos \left[\frac{\pi}{\tau/T} \left(\frac{t}{T} \right) \right] - \cos \left[2\pi \left(\frac{t}{T} \right) \right] \right] = 0 \quad ,$$

which gives

$$\sin \frac{1}{2} \left(\frac{\pi}{\tau/T} + 2\pi \right) \frac{t}{T} \sin \frac{1}{2} \left(\frac{\pi}{\tau/T} - 2\pi \right) \frac{t}{T} = 0 \quad (B-5)$$

The critical values of time are obtained from Eq. (B-5) as

$$\begin{aligned} \frac{t}{T} &= \frac{2n\tau/T}{1-2\tau/T} \quad , \quad n = 0, 1, 2, \dots \dots \dots \\ \frac{t}{T} &= \frac{2n\tau/T}{1+2\tau/T} \quad , \quad n = 0, 1, 2, \dots \dots \dots \end{aligned} \quad (B-6)$$

The method of calculating the $\alpha(\tau/T)$ is to pick a value of τ/T and calculate those values of t/T from Eq. (B-6) that lie in the region $0 \leq t/T \leq \tau/T$. Those values of t/T are then substituted in Eq. (B-4) and the one that gives the largest $G(t/T)$ is the maximax for region (1). The largest maxima of region (2) are obvious from Eq. (B-3). The maximax residual response is given by

$$\alpha_R\left(\frac{\tau}{T}\right) = \left| \frac{\cos \pi \frac{\tau}{T}}{\frac{\tau}{T} \left[\left(\frac{1}{2 \frac{\tau}{T}} \right)^2 - 1 \right]} \right| \quad (B-7)$$

The two maximax responses from region (1) and region (2) are then compared. The largest of those is then termed the maximax response, and the values of Eq. (B-7) are termed the maximax residual response. This method was used to generate the values of α vs τ/T shown in Fig. 3.

Versed-Sine Pulse

If the magnetic field flux density B is generated in the form of a half-sine pulse, the resulting equivalent internal pressure acting on the coil will be in the form of a sine-squared pulse or what is usually termed in the literature (Ref. 4) as a versed-sine pulse. This is due to the relationship between the equivalent internal pressure of the coil expressed as a function of magnetic flux density B .

$$P_i = \frac{B^2}{2\mu_0} \quad (B-8)$$

Thus, we let $\xi(t)$ be represented by the versed-sine pulse as

$$\xi(t) = \begin{cases} \xi_p \sin^2 \frac{\pi t}{\tau}, & 0 \leq t \leq \tau \\ 0, & t < \tau \end{cases} \quad (B-9)$$

The response $u(t)$ is the well-known solution (see Ref. 4) of the form

$$u\left(\frac{t}{T}\right) = \begin{cases} \frac{\xi_p}{2[1 - (\tau/T)^2]} \left[1 - \left(\frac{\tau}{T}\right)^2 + \left(\frac{\tau}{T}\right)^2 \cos \left[\frac{2\pi}{\tau/T} \left(\frac{t}{T}\right) \right] - \cos 2\pi \left(\frac{t}{T}\right) \right], & 0 \leq \frac{t}{T} \leq \frac{\tau}{T} \\ \xi_p \left[\frac{\sin \pi \frac{\tau}{T}}{1 - \left(\frac{\tau}{T}\right)^2} \right] \sin 2\pi \left(\frac{t}{T} - \frac{1}{2} \frac{\tau}{T} \right), & \frac{t}{T} > \frac{\tau}{T} \end{cases} \quad (B-10)$$

To obtain the shock spectrum, we again find the maxima of (u/ξ_p) from Eq. (B-10). Again there are two regions as defined previously. However, finding the critical values in the usual way leads to a transcendental equation for region (1) values.

$$\sin 2\pi \left(\frac{t}{T}\right) - \frac{\tau}{T} \sin \left[\frac{2\pi}{\tau/T} \left(\frac{t}{T}\right) \right] = 0 \quad (B-11)$$

Equation (B-11) was solved by a trial and error procedure, and the resulting maxima were then computed from Eq. (B-10) and compared to obtain the maximax.

The largest maxima for region (2) are obvious from the second part of Eq. (B-10). The maximax residual response is thus given by

$$\alpha_R\left(\frac{\tau}{T}\right) = \left| \left[\frac{\sin \pi \frac{\tau}{T}}{1 - (\tau/T)^2} \right] \right| \quad (B-12)$$

The two maximax responses from region (1) and region (2) are then compared as before for the half-sine pulse. The largest of these is then termed the maximax response, and the values of Eq. (B-12) are termed the maximax residual response. This method was used to generate the values of α vs τ/T shown in Fig. 4.