HTGR Steam Generator Modeling

T. W. Kerlin

OAK RIDGE NATIONAL LABORATORY

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HTGR STEAM GENERATOR MODELING

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ABSTRACT

Research activities at The University of Tennessee on gas cooled reactor dynamics are described. The main activity is on steam generator modeling, using approaches ranging from a relatively simple linear representation to a detailed nonlinear representation. Model comparisons will involve simulations of the Fort St. Vrain reactor steam generator, with emphasis on the evaluation of accuracy vs computation costs. A smaller effort is also in progress for modeling the reactor core, the main turbine and the blower turbines. Preparations are described for using test data from Fort St. Vrain for validating the dynamic models and identifying important design parameters in the plant.

INTRODUCTION

The research activities at The University of Tennessee on gas cooled reactor dynamics during 1975 are discussed in this progress report.

The main purposes of this work are:

1. component modeling. The emphasis is on steam generator modeling. Several different modeling approaches ranging from a relatively simple few node, linear representation to a detailed multi-node, nonlinear representation are being developed and compared. The comparison will involve simulation of the Fort St. Vrain steam generator with each different model and an evaluation of accuracy (relative to the most detailed model) vs. computation costs will be determined. A smaller effort involving modeling of other plant components is also in progress. This includes the reactor core, the main turbine and the blower turbines.
2. model validation. Preparations are being made for using test data from Fort St. Vrain (to be obtained in a separate cooperative program between General Atomic Company and The University of Tennessee) for checking the dynamic models and identifying important design parameters in the plant. This involves methods development for performing the identification and preparation of system models for use in the model-reference identification.

CHAPTER I

FORT ST. VRAIN STEAM GENERATOR MODELING

S. I. Chang

I.1 A Description of the System

The Fort St. Vrain plant has two identical once-through type steam generator loops arranged in parallel, each of which consists of six individual steam generator modules, located within the cavity of the prestressed concrete reactor vessel (PCRVR) and beneath the reactor core (see Figure I.1). Each steam-generator module contains a heat-transfer section with a main steam production region and an integral reheater, as well as a PCRVR penetration section with suitable inlet and outlet connections and helium-pressure barriers.

The steam generator is used to transfer the heat from the helium coolant to the water/steam. The helium flow that cools the reactor core enters the steam generator at high temperature and gives up its heat, first to the reheat steam section, then to the once-through main steam/water section. The sections produce main steam and reheat steam as demanded by the plant load and main turbine requirements.

The main design parameters of the system are shown in Table I.1.

The modules (Figure I.2) are divided into three bundles (heating surface sections) arranged one above the other, made up of helically wound tubes supported by perforated plates attached to the central support structure.

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fig. 1.1. Fort St. Vrain 330-MW(e) HTGR.

TABLE I.1
FORT ST. VRAIN NUCLEAR POWER PLANT
STEAM GENERATOR DESIGN CONDITIONS

	100% Steam Flow	25% Steam Flow
<u>Load</u>		
Generator Output (MW)	342.0	81.2
Station Output (MW)	330.2	67.4
<u>Helium</u>		
Flow (lb/hr)	3,452,400	1,026,000
Outlet Pressure (psia)	686	589
Inlet Temperature (F)	1263	1085
Outlet Temperature (F)	632	501
Pressure Drop (psi)	3.47	0.36
<u>Feedwater/Main Steam</u>		
Flow (lb/hr)	2,305,326	574,776
Outlet Pressure (psia)	2512	2419
Inlet Temperature (F)	403	299
Outlet Temperature (F)	1005	1000
Pressure Drop (psi)	590	45
<u>Reheat Steam</u>		
Flow (lb/hr)	2,245,366	556,928
Outlet Pressure (psia)	600	151
Inlet Temperature (F)	673	570
Outlet Temperature (F)	1002	1000
Pressure Drop (psi)	42	11

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Fig. 1.2. Fort St. Vrain Nuclear Power Plant Steam Generator Module.

I.1.1 Helium Circuitry

Helium leaves the core and travels downward to the steam generator. The gas then passes over the reheater, finishing superheater, pre-superheater, evaporator, and economizer surfaces consecutively. Cold helium exits at the bottom of the steam generator modules and flows into the four suction chambers of the helium circulators.

I.1.2 Main Steam Circuitry

Feedwater is distributed to the 12 modules through flow-control valves and enters each module through a ringheader. The heat-transfer surfaces of the economizer-evaporator-presuperheater (EES) and finishing superheater (SHII) bundles are composed of helically wound tubes originating at the feedwater subheaders. In the bottom EES bundle, the water is preheated, evaporated, and presuperheated. The steam flow is counter current to the helium flow passing over these tubes. Then the crossover tubes lead the presuperheated steam to the finishing superheater (SHII) which is arranged for co-current flow above the EES bundle. This arrangement is required to minimize tube metal temperatures and enhances stability in this section. Superheated steam leaves the SHII bundle via tubes and is collected in the main steam ringheader, which is connected to the main steam piping.

I.1.3 Integral Reheater Circuitry

Cold reheat steam enters each steam-generator module through an annulus. The steam is discharged from the annulus to the reheater (RH) bundle. The RH bundle, also composed of helically wound tubes, is

the uppermost heating surface in the steam generator, and is arranged for counter current flow. Hot reheat steam is carried via the central reheat pipe through the primary and secondary closures and returns to the intermediate pressure (IP) turbine.

I.2 Lumped Parameter Analysis

The dynamic analysis of a subsystem such as a once through steam generator (OTSG) is a distributed parameter problem. However, a lumped parameter approach is commonly used for its simplicity and ease of solution by either analog or digital techniques. The lumped parameter modeling technique generally results in the physical system being represented in the following form:

$$G(x, \dot{x}, t) = F_1(x, t) + F_2(u, t) \quad 1.2.1$$

$$Y(t) = F_3(x, u, t) . \quad 1.2.2$$

For dynamic analysis, it is often desirable to have a linear set of coupled differential equations. Equation (1.2.1) can be linearized by expanding each term in a Taylor Series about an operating point. If the terms above first order are neglected, the result is valid as long as the variables stay close to the operating point.

The form of the linearized set of equations is:

$$A\delta\dot{x} = B\delta x + C\delta u \quad 1.2.3$$

$$\delta y = D\delta x + E\delta u. \quad 1.2.4$$

A set of equations of this form was obtained by the System Analysis

Branch of the Reactor Engineering Department of General Atomic in the mathematical model LAP (Linear Analysis Program).

I.3 Model Formulation

The linear model of LAP includes the primary coolant system, secondary coolant system, and reactor system. Each of these subsystem models was derived from first principles (conservation of mass, momentum, energy, and neutrons) except for an approximation used for the main steam section in the OTSG in early studies. An empirical or transfer function approach was adopted for the main steam section. This empirical model was obtained by tuning arbitrary constants in LAP's⁽¹⁾ transfer function type model to produce agreement between LAP and TAP.⁽²⁾

The development and implementation of a linear steam generator model derived from first principles was undertaken at GA (work performed by W. D. Leech⁽³⁾) because it was believed that an OTSG linear model based on first principles would be superior to the original model for two reasons. First, a model based on first principles yields a better physical understanding. Second, the tuning of the arbitrary constants which appear in the transfer function involved a large degree of qualitative judgement as to when satisfactory agreement was reached between TAP and LAP.

In order to account for movement of the beginning and end of the boiling zone, the main steam section was divided into three regions with moving boundaries corresponding to the economizer, evaporator, and superheater. The first node upper boundary was defined as the

length (from the inlet) required for the water to be heated to saturation. The second node upper boundary, was defined by the additional length required to heat the water from saturated liquid to saturated vapor. The third node was defined as the remaining length by the steam generator. This formulation allows movement of the points where boiling begins (0% quality) and ends (100% quality) and yields a low order representation of the physical system that retains all of the physical processes usually considered important.

I.4. Mathematical Model

I.4.1 Assumptions

In this modeling, the assumptions can be grouped into two sets: those relating to equation developments for each system component and those made in obtaining a simplified conceptual system from the real system. More details of system assumptions or approximations are made in the development following this section.

Assumptions listed in the following five points refer specifically to equation development for the main steam section bundle.

1. Fluid properties are uniform at any given cross section.
2. Fluid properties for each node are weighted by the properties at boundaries.
3. No heat conduction occurs in the flow direction.
4. Dynamic effects of gas-pressure changes are negligible, so the gas flow rate is assumed to be time dependent only.
5. Balanced flow and uniform heat flux exist at any node in the main steam sections.

I.4.2 Basic Equations

Figure I.3 shows a counter flow heat exchanger. The locations X_2 and X_3 are free to move but always remain at fixed quality. For the steam generator model, X_2 is the location of saturated liquid (0% quality) and X_3 is the location of saturated vapor (100% quality).

a. Conservation of Mass.

The basic relationship is:

Flow in - Flow out = rate of change of mass stored.

Fig. I.3. Schematic Representation of a Moving Boundary Model for a Counter Flow Heat Exchanger.

Gas side

$$WG_i = WG_{in} \quad (i=1,2,3) \quad I.4.1$$

where WG = gas flow rate (lb/sec)

WG_{in} = inlet gas flow rate.

Here we assume the gas flow rate is a function of time only and the same in every section.

Water/steam side

$$\begin{aligned} (WS_i - \frac{1}{\bar{V}_i} \cdot A \cdot \dot{X}_i) - (WS_{i+1} - \frac{1}{\bar{V}_{i+1}} \cdot A \cdot \dot{X}_{i+1}) = \\ A \cdot \frac{d}{dt} [\frac{1}{\bar{V}_i} (X_{i+1} - X_i)] \end{aligned} \quad I.4.2$$

where

$\dot{}$ = indicated derivative with respect to time

$-$ = indicates average quantity in the node

WS = water/steam flow rate (lb/sec)

V = specific volume (ft³/lb)

$\bar{V}$ = average specific volume in the node (ft³/lb)

A = water/steam flow cross section (ft²)

X = location along the flow path (ft)

$i = 1, 2, 3$. Note that the X_1 and X_4 are fixed, so that $\dot{X}_1 = \dot{X}_4 = 0$

Rewriting Equation (I.4.2)

$$\begin{aligned} WS_i - WS_{i+1} = A \cdot \frac{d}{dt} [\frac{1}{\bar{V}_i} \cdot (X_{i+1} - X_i)] \\ + \frac{1}{\bar{V}_i} \cdot A \cdot \dot{X}_i - \frac{1}{\bar{V}_{i+1}} \cdot A \cdot \dot{X}_{i+1} \end{aligned} \quad I.4.3$$

Conservation of Energy

The basic relationship is:

Energy flow in - Energy flow out - Rate of energy transferred
out = rate of energy stored.

a. Gas

$$(WG_{i+1} \cdot HG_{i+1} + A_g \cdot \rho_{i+1} \cdot HG_{i+1} \cdot \dot{X}_{i+1}) - (WG_i \cdot HG_i + A_g \cdot \rho_i \cdot HG_i \cdot \dot{X}_i) - Q_{gm} = A_g \cdot C_v \cdot \frac{d}{dt} [\bar{\rho}_i \cdot \overline{TG}_i (X_{i+1} - X_i)] \quad 1.4.4$$

where

HG = enthalpy of gas (Btu/lb)

ρ_i = gas density (lb/ft³)

Q_{gm} = heat transfer rate from gas to metal (Btu/sec)

C_v = constant volume specific heat for the gas (Btu/lb°F)

TG = gas temperature (°F)

We assume that the helium is an ideal gas under the conditions that exist in the steam generator. Then

$$HG = C_p (TG + 459.6) \quad 1.4.5$$

where

C_p = constant pressure specific heat for the gas (Btu/lb°F).

The gas density can also be related to the gas pressure and gas temperature.

$$\rho_i = PG_i \cdot 0.373343 / (TG_i + 459.6) \quad (i = 1, 2, 3, 4). \quad 1.4.6$$

Equation (I.4.4) can be rearranged to yield

$$\begin{aligned}
 & WG_{i+1} \cdot HG_{i+1} - WG_i \cdot HG_i - Q_{gm} = A_g \cdot C_v \cdot \frac{d}{dt} [\bar{\rho}_i \cdot \bar{TG}_i \cdot \\
 & \quad (x_{i+1} - x_i)] \\
 & - A_g \cdot \rho_{i+1} \cdot HG_{i+1} \cdot \dot{x}_{i+1} + A_g \cdot \rho_i \cdot HG_i \cdot \dot{x}_i
 \end{aligned} \quad \text{I.4.7}$$

b. Metal

The basic relationship is:

Heat flow in - Heat flow out = rate of heat stored in the metal.

$$\begin{aligned}
 Q_{gm} - TM_i \cdot M \cdot C_{pm} \cdot \dot{x}_i - Q_{ms} + TM_{i+1} \cdot M \cdot C_{pm} \cdot \dot{x}_{i+1} = \\
 M \cdot C_{pm} \cdot \frac{d}{dt} [\bar{TM}_i \cdot (x_{i+1} - x_i)]
 \end{aligned} \quad \text{I.4.8}$$

where

$\bar{TM}$ = average metal temperature for the node ($^{\circ}\text{F}$)

C_{pm} = metal heat capacity (Btu/lb- $^{\circ}\text{F}$)

Q_{ms} = heat transfer rate from metal to steam (Btu/lb)

M = metal mass per unit length (lb/ft).

We assume the metal boundary temperature to be given by the arithmetic average of adjacent node temperatures

$$TM_i = \frac{1}{2} \cdot (\bar{TM}_{i-1} + \bar{TM}_i) \text{ for } i = 2, 3. \quad \text{I.4.9}$$

Substitute Equation (I.4.9) into (I.4.8) and rearrange to yield

$$\begin{aligned}
 Q_{gm} - Q_{ms} = M \cdot C_{pm} \cdot [(x_{i+1} - x_i) \cdot \bar{TM}_i + \frac{1}{2} (\bar{TM}_i - \bar{TM}_{i+1}) \cdot \\
 \dot{x}_{i+1} - \frac{1}{2} \cdot (\bar{TM}_i - \bar{TM}_{i-1}) \cdot \dot{x}_i].
 \end{aligned} \quad \text{I.4.10}$$

The heat transfer rates are evaluated as follows:

$$Q_{gm} = [\text{Heat transfer area} \cdot (\overline{TG}_i - \overline{TM}_i)] / [K_{gi} \cdot (\overline{WG}_i)^{-C_{gi}} + R_i \cdot 0.5] \quad \text{I.4.11}$$

$$Q_{ms} = [\text{Heat transfer area} \cdot (\overline{TM}_i - \overline{TS}_i)] / [K_{si} \cdot (\overline{WS}_i)^{-C_{si}} + R_i \cdot 0.5] \quad \text{I.4.12}$$

where

K_g = constant for gas to metal heat transfer

C_g = flow exponent for gas to metal heat transfer

K_s = constant for metal to water heat transfer

C_s = flow exponent for metal to water heat transfer

R = thermal resistivity of the metal times the tube thickness

TS = water/steam temperature.

C. Water/Steam

$$(\overline{WS}_i \cdot \overline{HS}_i - \frac{1}{V_i} \cdot \overline{HS}_i \cdot A \cdot \dot{x}_i) - (\overline{WS}_{i+1} \cdot \overline{HS}_{i+1} - \frac{1}{V_{i+1}} \cdot \overline{HS}_{i+1} \cdot A \cdot \dot{x}_{i+1}) + Q_{ms} = A \cdot \frac{d}{dt} \left[\frac{1}{V_i} \cdot \bar{u}_i \cdot (x_{i+1} - x_i) \right] \quad \text{I.4.13}$$

where

HS = enthalpy of water/steam (Btu/lb)

u = internal energy of water/steam (Btu/lb).

Rearranging Equation (I.4.13) yields:

$$\overline{WS}_i \cdot \overline{HS}_i - \overline{WS}_{i+1} \cdot \overline{HS}_{i+1} + Q_{ms} = A \cdot \frac{d}{dt} \left[\frac{1}{V_i} \cdot \bar{u}_i \cdot (x_{i+1} - x_i) \right] + A \cdot \left[-\frac{1}{V_{i+1}} \cdot \overline{HS}_{i+1} \cdot \dot{x}_{i+1} + \frac{1}{V_i} \cdot \overline{HS}_i \cdot \dot{x}_i \right] \quad \text{I.4.14}$$

Conservation of Momentum

Since it is assumed the flow on the gas side of the steam generator is time dependent only, there is no need for a momentum equation in that region.

In the water/steam, the basic principle is:

[forces = Rate of change of momentum.

$$\begin{aligned}
 P_i - P_{i+1} - K_f \overline{WS}_i^2 - \left(\frac{1}{144 \cdot \bar{V}_i} \right) \cdot \text{PITCH} \cdot (X_{i+1} - X_i) \\
 = \left(\frac{1}{144 \cdot g \cdot A} \right) \frac{d}{dt} [\overline{WS}_i \cdot (X_{i+1} - X_i)]
 \end{aligned}
 \tag{I.4.15}$$

where

K_f = friction drop coefficient (PSIA/(lbm/sec)²)

g = gravitational constant (ft/sec²)

PITCH = height increase per unit length.

In Equation (I.4.15) the first two terms are the pressure force. The third term is the friction loss. The fourth term is pressure change due to changes in elevation. Due to the coiled nature of the steam generator the PITCH = 0.05, so we can cancel the fourth term. The coefficient $\left(\frac{1}{144 \cdot g \cdot A} \right)$ of the differential term in the momentum equation is quite small. Furthermore, the sign of $\dot{X}_i$ and $\dot{X}_{i+1}$ are the same all the time. So, it can be cancelled without introducing significant error in the model. After cancelling these terms in Equation (I.4.15), we have the general form of the flow pressure drop relation:

$$\overline{WS}_i = \sqrt{(P_i - P_{i+1})/K_i}$$

where K_i is a head loss coefficient. These algebraic relationships are

$$WS_1 = \sqrt{(\bar{P}_0 - \bar{P}_1)/K_1} \quad 1.4.16$$

$$WS_2 = \sqrt{(\bar{P}_1 - \bar{P}_2)/K_2} \quad 1.4.17$$

$$WS_3 = \sqrt{(\bar{P}_2 - \bar{P}_3)/K_3} \quad 1.4.18$$

$$WS_4 = \sqrt{(\bar{P}_3 - \bar{P}_4)/K_4} \quad 1.4.19$$

d. The weighting factors

The equations which were derived in the above include both the average quantities and end point quantities for the nodes. Additional relationships will be necessary in order to have as many equations as unknowns.

For each node, a weighting factor is defined and the average value for any quantity, $\bar{Y}$, is calculated as follows:

$$\bar{Y}_i = Y_i \cdot F_i + Y_{i+1} \cdot (1 - F_i) \quad F_i \leq 1.$$

Note that if F equals 0.5, $\bar{Y}$ is the linear average. This averaging method is used to calculate $\bar{TG}$, $\bar{TH}$, $\bar{WS}$, $\bar{P}$, and $\bar{HS}$. The average quantities $\bar{U}$, $\bar{V}$, and $\bar{TS}$ are then calculated from steam tables using $\bar{P}$ and $\bar{HS}$ as the independent variables. The values of $F_1 = 0.75$, $F_2 = 0.67$ and $F_3 = 0.30$ were used in all the calculations for Section I of this report.

This concludes the derivation of the lumped nodes equations for a once-through steam generator. Before further development, the assumptions behind the derivation are summarized as follows:

1. Gas flow rate is a function only of time.
2. The enthalpy at X_2 and X_3 are functions of pressure with fixed quality.

3. No heat conduction in the flow direction.
4. Balanced flow and uniform heat flux exists at any node.
5. In the economizer, the flow in equals flow out.
6. The average water/steam flow rate used in the equation is assumed to be a linear average of the flow in and out of each node.
7. The water/steam flow rate is proportional to the square root of pressure difference.

1.4.3 Summary of Equations

Now for the purpose of review, the following equations are a summary of those that must be solved for each node of a moving boundary heat exchanger.

The twelve differential equations are:

$$\begin{aligned}
 WS_i - WS_{i+1} = A \left\{ \frac{d}{dt} \left[\frac{1}{V_i} \cdot (X_{i+1} - X_i) - \frac{1}{V_{i+1}} \cdot \dot{X}_{i+1} \right. \right. \\
 \left. \left. + \frac{1}{V_i} \cdot \dot{X}_i \right\} \quad (i = 1, 2, 3)
 \end{aligned}
 \tag{1.4.20}$$

Note that $\dot{X}_1 = \dot{X}_4 = 0$

$$\begin{aligned}
 WG_{i+1} \cdot HG_{i+1} - WG_i \cdot HG_i - Q_{gm} = A_g \cdot C_v \cdot \frac{d}{dt} \cdot \\
 [\bar{\rho}_i \cdot \bar{TG}_i \cdot (X_{i+1} - X_i)] - A_g \cdot \rho_{i+1} \cdot HG_{i+1} \cdot X_{i+1} \\
 + A_g \cdot \rho_i \cdot HG_i \cdot \dot{X}_i \quad (i = 1, 2, 3)
 \end{aligned}
 \tag{1.4.21}$$

$$\begin{aligned}
Q_{gm} - Q_{ms} &= M \cdot C_{pm} \cdot [(X_{i+1} - X_i) \cdot \frac{1}{\overline{Tm}_i} \\
&+ \frac{1}{2} (\overline{Tm}_i - \overline{Tm}_{i+1}) \cdot \dot{X}_{i+1} - \frac{1}{2} \cdot (\overline{Tm}_i - \overline{Tm}_{i-1}) \cdot \dot{X}_i] \\
(i &= 1, 2, 3)
\end{aligned}
\tag{I.4.22}$$

$$\begin{aligned}
WS_i \cdot HS_i - WS_{i+1} \cdot HS_{i+1} + Q_{ms} &= A \cdot \frac{d}{dt} \left[\frac{1}{V_i} \cdot \bar{u}_i \cdot \right. \\
(X_{i+1} - X_i)] &+ A \cdot \left[\frac{1}{V_i} \cdot HS_i \cdot \dot{X}_i - \frac{1}{V_{i+1}} \cdot HS_{i+1} \cdot \dot{X}_{i+1} \right] \\
(i &= 1, 2, 3).
\end{aligned}
\tag{I.4.23}$$

where HS_2 and HS_3 are functions of pressure with fixed quality only.

The six algebraic equations are

$$WG_i = WG_{in} \quad (i = 1, 2, 3) \tag{I.4.24}$$

$$WS_i = \sqrt{(\bar{P}_{i-1} - \bar{P}_i)/K_i} \quad (i = 2, 3, 4). \tag{I.4.25}$$

The eighteen unknowns for the model are:

1. Three average gas temperature ($\overline{TG}_1, \overline{TG}_2, \overline{TG}_3$) described by differential Equation (I.4.21).
2. Three metal temperature ($\overline{Tm}_1, \overline{Tm}_2, \overline{Tm}_3$) described by differential Equation (I.4.22).
3. Three water/steam pressures ($\bar{P}_1, \bar{P}_2, \bar{P}_3$) described by differential Equation (I.4.20).
4. Two locations (X_2, X_3) and one enthalpy (HS_4) described by differential Equation (I.4.23).

5. Three gas flows (WG_1, WG_2, WG_3) described by algebraic Equation (I.4.24).
6. Three water/steam flow (WS_2, WS_3, WS_4) described by algebraic Equation (I.4.25).

The assumed knowns, or inputs to the problem are:

1. Inlet water pressure (P_1)
2. Inlet water flow and enthalpy (WS_1, HS_1).
3. Fixed locations (X_1, X_4)
4. Inlet gas temperature and flow (TG_4, WG_{in}).

I.5 Linearization of Equations

The previously derived equations are linearized in the following sections.

I.5.1 Equation (I.4.20) Linearization

Equation (I.4.20) is repeated as follows:

$$WS_i - WS_{i+1} = A \left\{ \frac{d}{dt} \left[\frac{1}{\bar{V}_i} \cdot (X_{i+1} - X_i) - \frac{1}{\bar{V}_{i+1}} \cdot \dot{X}_{i+1} + \frac{1}{\bar{V}_i} \cdot \dot{X}_i \right] \right\} \quad I.5.1$$

We assume $\bar{V}_i$ is a function of $\bar{HS}_i$ and $\bar{P}_i$. The average quantities $\bar{HS}_i$ and $\bar{P}_i$ are obtained by weighting the end point values for each node as follows:

$$\bar{HS}_i = W_i \cdot HS_i + (1 - W_i) \cdot HS_{i+1} \quad I.5.2$$

$$\bar{P}_i = W_i \cdot P_i + (1 - W_i) \cdot P_{i+1} \quad I.5.3$$

The linearized form of Equation (I.4.29) is

$$\delta WS_i - \delta WS_{i+1} = A_{1,i} \delta \dot{X}_{i+1} + A_{2,i} \delta \dot{HS}_i + A_{3,i} \delta \dot{HS}_{i+1} \\ + (A_{4,i} + A_{5,i}) \delta \dot{P}_i + A_{6,i} \delta \dot{X}_i$$

where

$$A_{1,i} = A \cdot \left(\frac{1}{V_i} - \frac{1}{V_{i+1}} \right)$$

(Note: All the values for each variable in the coefficients represent the steady state in the system.)

$$L_i = X_{i+1} - X_i$$

$$A_{2,i} = -A \cdot L_i \cdot \left(\frac{\partial \bar{V}_i}{\partial HS_i} \right)_{P_i} / \bar{V}_i^2 \quad i = 1$$

$$= 0 \quad i = 2, 3$$

($\bar{V}$ is only a function of pressure when $i = 2, 3$).

$$A_{3,i} = -A \cdot L_i \cdot \left(\frac{\partial \bar{V}_i}{\partial HS_{i+1}} \right)_{P_{i+1}} / \bar{V}_i^2 \quad i = 3$$

$$= 0 \quad i = 1, 2$$

$$A_{4,i} = -A \cdot L_i \cdot \left(\frac{\partial \bar{V}_i}{\partial P_i} \right)_{HS_i} / \bar{V}_i^2$$

$$A_{5,i} = -A \cdot L_i \cdot \left(\frac{\partial \bar{V}_i}{\partial P_{i+1}} \right)_{HS_{i+1}} / \bar{V}_i^2$$

$$A_{6,i} = -A \cdot \left(\frac{1}{V_i} - \frac{1}{V_i} \right)$$

δHS_i can be related to the temperature TS_i as follows:

$$\delta HS_i = \left(\frac{\partial HS_i}{\partial P_i} \right)_{TS_i} \delta P_i + \left(\frac{\partial HS_i}{\partial TS_i} \right)_{P_i} \delta TS_i$$

Since enthalpy is almost independent of pressure for subcooled water, i.e.,

$$\left(\frac{\partial HS_1}{\partial P_1}\right)_{TS_1} \approx 0.$$

For $i = 1$, we have

$$\delta HS_1 = \left(\frac{\partial HS_1}{\partial TS_1}\right)_{P_1} \delta TS_1.$$

The final form of the linearized Equation (I.4.20) is:

for $i = 1$.

$$\begin{aligned} \delta WS_1 - \delta WS_2 = & A_{1,1} \delta \dot{X}_2 + A_{2,1} \cdot \left(\frac{\partial HS_1}{\partial TS_1}\right)_{P_1} \cdot \delta \dot{TS}_1 \\ & + (A_{4,1} + A_{5,1}) \delta \dot{P}_1 \end{aligned} \quad \text{I.5.4}$$

$i = 2$

$$\delta WS_2 - \delta WS_3 = A_{6,2} \delta \dot{X}_2 + (A_{4,2} + A_{5,2}) \delta \dot{P}_2 + A_{1,2} \delta \dot{X}_3. \quad \text{I.5.5}$$

$i = 3$.

The term HS_4 can be related to the main steam temperature (TS_4) and pressure ($\bar{P}_3$) as follows:

$$\delta HS_4 = \left(\frac{\partial HS_4}{\partial \bar{P}_3}\right)_{TS_4} \delta \bar{P}_3 + \left(\frac{\partial HS_4}{\partial TS_4}\right)_{\bar{P}_3} \delta TS_4$$

$$= (1 - w_3) \left(\frac{\partial HS_4}{\partial P_4}\right)_{TS_4} \delta \bar{P}_3 + \left(\frac{\partial HS_4}{\partial TS_4}\right)_{P_4} \delta TS_4.$$

After δHS_4 is replaced by $\delta \bar{P}_3$ and δTS_4 , we have

$$\begin{aligned} \delta WS_3 - \delta WS_4 = & A_{3,3} \left(\frac{\partial HS_4}{\partial TS_4} \right)_{P_4} \delta TS_4 + G_1 \delta \bar{P}_3 \\ & + A_{6,3} \delta \dot{X}_3 \end{aligned} \quad \text{I.5.6}$$

where

$$G_1 = (A_{4,3} + A_{5,3}) + A_{3,3} \cdot (1 - W_3) \cdot \left(\frac{\partial HS_4}{\partial P_4} \right)_{TS_4}.$$

I.5.2 Equation (I.4.21) Linearization

Equation (I.4.21) is repeated as follows:

$$\begin{aligned} WG_{i+1} \cdot HG_{i+1} - WG_i \cdot HG_i - Q_{gm} = & A_g \cdot C_v \cdot \frac{d}{dt} [\bar{\rho}_i \cdot \overline{TG}_i \cdot (X_{i+1} - X_i)] \\ & - A_g \cdot \rho_{i+1} \cdot HG_{i+1} \cdot \dot{X}_{i+1} + A_g \cdot \rho_i \cdot HG_i \cdot \dot{X}_i \quad (i = 1, 2, 3). \end{aligned} \quad \text{I.5.7}$$

We assume HG is a function of TG only and ρ is constant during a transient. Let

$$\begin{aligned} HG_i &= C_{pg} \cdot (TG_i + 459.6) \\ \overline{TG}_i &= TG_i \cdot F_i + TG_{i+1} \cdot (1 - F_i) \\ \overline{WG}_i &= WG_i \cdot F_i + WG_{i+1} \cdot (1 - F_i). \end{aligned} \quad \text{I.5.8}$$

The linearized form of Equation (I.4.21) is as follows

$$\begin{aligned}
 & B_{1,i} \delta WG_{i+1} + B_{2,i} \delta WG_i + B_{3,i} \delta TG_{i+1} + C_{4,i} \delta \overline{TG}_i \\
 & + B_{4,i} \delta TG_i + B_{5,i} \delta X_{i+1} + B_{6,i} \delta X_i + B_{7,i} \delta \overline{TM}_i = \\
 & A_{7,i} \delta \dot{X}_{i+1} + A_{8,i} \delta \dot{X}_i + A_{9,i} \delta \dot{\overline{TG}}_i
 \end{aligned}$$

where

$$E_{1,i} = \pi \cdot OD \cdot L_i \cdot (\overline{TG}_i - \overline{TM}_i) \cdot K_{gi} \cdot C_{gi} \cdot WG^{-(C_{gi} + 1)} / \\
 (K_{gi} \cdot WG^{-C_{gi}} + 0.5 R_i)^2$$

$$E_{2,i} = \pi \cdot OD \cdot (K_{gi} \cdot WG^{-C_{fi}} + 0.5 R_i)^{-1}$$

$$B_{1,i} = HG_{i+1} - E_{1,i} \cdot (1 - F_i)$$

$$B_{2,i} = -HG_i - E_{1,i} \cdot F_i$$

$$B_{3,i} = WG_{i+1} \cdot C_{pg}$$

$$C_{4,i} = -E_{2,i} \cdot L_i$$

$$B_{4,i} = -WG_i \cdot C_{pg}$$

$$B_{5,i} = -E_{2,i} \cdot (\overline{TG}_i - \overline{TM}_i)$$

$$B_{6,i} = -B_{5,i}$$

$$B_{7,i} = E_{2,i} \cdot L_i$$

I.5.9
(cont.)

$$A_{7,i} = A_g \cdot (C_{vg} \cdot \bar{\rho}_i \cdot \overline{TG}_i - \rho_{i+1} \cdot HG_{i+1})$$

$$A_{8,i} = -A_g \cdot (C_{vg} \cdot \bar{\rho}_i \cdot \overline{TG}_i - \rho_i \cdot HG_i)$$

$$A_{9,i} = A_g \cdot C_{vg} \cdot L_i \cdot \bar{\rho}_i$$

For $i = 1$

$$\begin{aligned} B_{1,1} \delta WG_2 + B_{2,1} \delta WG_1 + B_{3,1} \delta TG_2 + C_{4,1} \delta \overline{TG}_1 + B_{4,1} \delta TG_1 \\ + B_{5,1} \delta X_2 + B_{7,1} \delta \overline{TM}_1 = \\ A_{7,1} \delta \dot{X}_2 + A_{9,1} \delta \dot{\overline{TG}}_1 \end{aligned} \quad 1.5.9$$

$i = 2$

$$\begin{aligned} B_{1,2} \delta WG_3 + B_{2,2} \delta WG_2 + B_{3,2} \delta TG_3 + C_{4,2} \delta \overline{TG}_2 + B_{4,2} \delta TG_2 \\ + B_{5,2} \delta X_3 + B_{6,2} \delta X_2 + B_{7,2} \delta \overline{TM}_2 = \\ A_{7,2} \delta \dot{X}_3 + A_{8,2} \delta \dot{X}_2 + A_{9,2} \delta \dot{\overline{TG}}_2 \end{aligned} \quad 1.5.10$$

$i = 3$

$$\begin{aligned} B_{1,3} \delta WG_4 + B_{2,3} \delta WG_3 + B_{3,3} \delta TG_4 + C_{4,3} \delta \overline{TG}_3 + B_{4,3} \delta TG_3 \\ + B_{6,3} \delta X_3 + B_{7,3} \delta \overline{TM}_3 = \\ A_{8,3} \delta \dot{X}_3 + A_{9,3} \delta \dot{\overline{TG}}_3 \end{aligned} \quad 1.5.11$$

Note that either the variable $\delta \overline{TG}$ or the variable δTG can be eliminated by using Equation (I.5.8). In order to make changes of the weight factor (F_i) in the system dynamic response study convenient, we keep both variables in the equation.

I.5.3 Equation (I.4.22) Linearization

Equation (I.4.22) is repeated as follows:

$$Q_{gm} - Q_{ms} = M \cdot C_{pm} \cdot [(X_{i+1} - X_i) \cdot \dot{\overline{TM}}_i + \frac{1}{2} \cdot (\overline{TM}_i - \overline{TM}_{i+1}) \cdot \dot{X}_{i+1} - \frac{1}{2} \cdot (\overline{TM}_i - \overline{TM}_{i-1}) \cdot \dot{X}_i] \quad (i = 1, 2, 3). \quad 1.5.12$$

We assume the term $\overline{TS}_i$ in Q_{ms} is a function of $\overline{P}_i$ and $\overline{HS}_i$.

And, let

$$\overline{P}_i = W_i \cdot P_i + (1 - W_i) \cdot P_{i+1}$$

$$\overline{HS}_i = W_i \cdot HS_i + (1 - W_i) \cdot HS_{i+1}.$$

The linearized form of Equation (I.4.22) as follows:

$$\begin{aligned} & B_{8,i} \delta X_{i+1} - B_{8,i} \delta X_i + B_{9,i} \delta \overline{TM}_i + B_{10,i} \delta \overline{TG}_i \\ & + B_{12,i} \delta WG_{i+1} + B_{13,i} \delta WG_i + (B_{14,i} + B_{15,i}) \delta \overline{P}_i \\ & + B_{16,i} \delta HS_i + B_{17,i} \delta HS_{i+1} + B_{18,i} \delta WS_i + B_{19,i} \delta HS_{i+1} \\ & = A_{11,i} \delta \dot{\overline{TM}}_i + A_{12,i} \delta \dot{X}_{i+1} + A_{13,i} \delta \dot{X}_i \end{aligned}$$

where

$$E_{4,i} = \infty \cdot (KS_i \cdot WS^{-CS_i} + 0.5R_i)^{-1}$$

$$E_{5,i} = \infty \cdot L_i \cdot KS_i \cdot CS_i \cdot WS^{-(CS_i+1)} \cdot (TH_i - TS_i) \\ \cdot (KS_i \cdot WS^{-CS_i} + 0.5R_i)^{-2}$$

$$u_{8,i} = E_{2,i} \cdot (TG_i - TH_i) - E_{4,i} \cdot (TH_i - TS_i) = 0.$$

(For at the steady state, the heat transferred in equals the heat transferred out.)

$$B_{9,i} = -E_{2,i} \cdot L_i - E_{4,i} \cdot L_i$$

$$B_{10,i} = E_{2,i} \cdot L_i$$

$$B_{12,i} = E_{1,i} \cdot (1 - F_i)$$

$$B_{13,i} = E_{1,i} \cdot F_i$$

$$B_{14,i} = E_{4,i} \cdot \left(\frac{\partial TS_i}{\partial P_i} \right)_{HS_i} \cdot L_i$$

$$B_{15,i} = E_{4,i} \cdot \left(\frac{\partial TS_i}{\partial P_{i+1}} \right)_{HS_{i+1}} \cdot L_i$$

$$B_{16,i} = E_{4,i} \cdot \left(\frac{\partial TS_i}{\partial HS_i} \right)_{P_i} \cdot L_i \quad i = 1$$

$$= 0$$

$$i = 2, 3$$

(For $i = 2, 3$, TS_i is function of P_i only with fixed quality.)

$$B_{17,i} = E_{4,i} \cdot \left(\frac{\partial \overline{TS}_i}{\partial HS_{i+1}} \right) P_{i+1} \cdot L_i \quad i = 3$$

$$= 0 \quad i = 1, 2$$

$$B_{18,i} = -E_{5,i} \cdot W_i$$

$$B_{19,i} = -E_{5,i} \cdot (1 - W_i)$$

$$A_{11,i} = M \cdot C_{pm} \cdot L_i$$

$$A_{12,i} = \frac{1}{2} M \cdot C_{pm} \cdot (\overline{TM}_i - \overline{TM}_{i+1})$$

$$A_{13,i} = -\frac{1}{2} M \cdot C_{pm} \cdot (\overline{TM}_i - \overline{TM}_{i-1}).$$

The final linearized equations for each node are:

For $i = 1$

$$\begin{aligned} & B_{9,1} \delta \overline{TM}_1 + B_{10,1} \delta \overline{TG}_1 + B_{12,1} \delta WG_2 + B_{13,1} \delta WG_1 \\ & + (B_{14,1} + B_{15,1}) \delta \overline{P}_1 + B_{16,1} \cdot \left(\frac{\partial HS_1}{\partial TS_1} \right) P_1 \cdot \delta TS_1 + B_{18,1} \delta WS_1 \\ & + B_{19,1} \delta WS_2 = A_{11,1} \delta \overline{TM}_1 + A_{12,1} \delta \overline{TM}_2 \end{aligned} \quad I.5.12$$

$$i = 2$$

$$\begin{aligned} & B_{9,2} \delta \overline{TM}_2 + B_{10,2} \delta \overline{TG}_2 + B_{12,2} \delta \overline{WG}_3 + B_{13,2} \delta \overline{WG}_2 \\ & + (B_{14,2} + B_{15,2}) \delta \overline{P}_2 + B_{18,2} \delta \overline{WS}_2 + B_{19,2} \delta \overline{WS}_3 \\ & = A_{11,2} \delta \overline{TM}_2 + A_{12,2} \delta \dot{X}_3 + A_{13,2} \delta \dot{X}_2 \end{aligned} \quad I.5.13$$

$$i = 3$$

$$\begin{aligned} & B_{9,3} \delta \overline{TM}_3 + B_{10,3} \delta \overline{TG}_3 + B_{12,3} \delta \overline{WG}_4 + B_{13,3} \delta \overline{WG}_3 \\ & + G_2 \delta \overline{P}_3 + B_{17,3} \cdot \left(\frac{\partial HS_4}{\partial TS_4} \right) P_4 \cdot \delta TS_4 + B_{18,3} \delta \overline{WS}_3 + B_{19,3} \delta \overline{WS}_4 \\ & = A_{11,3} \delta \overline{TM}_3 + A_{13,3} \delta \dot{X}_3 \end{aligned} \quad I.5.14$$

where

$$G_2 = [B_{14,3} + B_{15,3} + B_{17,3} \cdot (1 - W_3) \cdot \left(\frac{\partial HS_4}{\partial P_4} \right)_{TS_4}].$$

I.5.4 Equation (I.4.23) Linearization

Equation (I.4.23) is repeated as follows:

$$\begin{aligned} & WS_i \cdot HS_i - WS_{i+1} \cdot HS_{i+1} + Q_{ms} = A \cdot \frac{d}{dt} \left[\frac{1}{\bar{V}_i} \cdot \bar{U}_i \cdot \right. \\ & \left. (X_{i+1} - X_i) \right] + A \cdot \left[\frac{1}{\bar{V}_i} \cdot HS_i \cdot \dot{X}_i - \frac{1}{\bar{V}_{i+1}} \cdot HS_{i+1} \cdot \dot{X}_{i+1} \right] \\ & (i = 1, 2, 3). \end{aligned} \quad I.5.15$$

Assume the $\bar{U}_i$ and $\bar{V}_i$ are functions of $\bar{P}_i$ and $\overline{HS}_i$.

Let

$$\delta HS_i = \left(\frac{\partial HS_i}{\partial P_i} \right)_{\text{quality}} \cdot \delta P_i \quad (i = 2, 3).$$

Note: It is 0% quality at boundary 2 and 100% quality at boundary 3. The linearized form of Equation (1.4.23) is as follows:

$$\begin{aligned}
 & B_{20,i} \delta HS_i + B_{21,i} \delta WS_i + B_{22,i} \delta HS_{i+1} + B_{23,i} \delta WS_{i+1} \\
 & - B_{24,i} \delta X_{i+1} + B_{24,i} \delta X_i - B_{25,i} \delta \overline{TM}_i + B_{26,i} \delta \overline{P}_i \\
 & + B_{27,i} \delta \overline{P}_{i+1} + B_{28,i} \delta \overline{P}_{i-1} = A_{14,i} \cdot \delta HS_i + A_{16,i} \delta HS_{i+1} \\
 & + (A_{15,i} + A_{17,i}) \delta \overline{P}_i + A_{18,i} \delta \dot{X}_{i+1} + A_{19,i} \delta \dot{X}_i
 \end{aligned}$$

where

$$\begin{aligned}
 B_{20,i} &= WS_i - E_{4,i} \cdot \left(\frac{\partial \overline{TS}_i}{\partial HS_i} \right) \cdot L_i \quad i = 1 \\
 &= 0 \quad i = 2, 3
 \end{aligned}$$

$$B_{21,i} = HS_i + W_i \cdot E_{5,i}$$

$$\begin{aligned}
 B_{22,i} &= - WS_{i+1} - E_{4,i} \left(\frac{\partial \overline{TS}_i}{\partial HS_{i+1}} \right) \cdot L_i \quad i = 3 \\
 &= 0 \quad i = 1, 2
 \end{aligned}$$

$$B_{23,i} = - HS_{i+1} + (1 - W_i) \cdot E_{5,i}$$

$$B_{24,i} = - E_{4,i} \cdot (\overline{TM}_i - \overline{TS}_i)$$

$$B_{25,i} = - E_{4,i} \cdot L_i$$

$$D_i = - E_{4,i} \cdot L_i \cdot \left[\left(\frac{\partial \overline{TS}_i}{\partial P_i} \right)_{HS_i} + \left(\frac{\partial \overline{TS}_i}{\partial P_{i+1}} \right)_{HS_{i+1}} \right]$$

In $B_{26,i}$, $B_{27,i}$, and $B_{28,i}$, we let

$$\delta \overline{P}_i = \frac{1}{2} (\delta \overline{P}_{i-1} + \delta \overline{P}_i)$$

for $i = 1$

$$B_{26,1} = D_1 - \frac{1}{2} \cdot WS \cdot \left(\frac{\partial HS_2}{\partial P_2} \right)_{\text{quality}}$$

$$B_{27,1} = -\frac{1}{2} \cdot WS \cdot \left(\frac{\partial HS_2}{\partial P_2} \right)_{\text{quality}}$$

$$B_{28,1} = 0$$

$i = 1$

$$B_{26,2} = D_2 - \frac{1}{2} \cdot WS \cdot \left[\left(\frac{\partial HS_3}{\partial P_3} \right)_{\text{quality}} - \left(\frac{\partial HS_2}{\partial P_2} \right)_{\text{quality}} \right]$$

$$B_{27,2} = -\frac{1}{2} \cdot WS \cdot \left(\frac{\partial HS_3}{\partial P_3} \right)_{\text{quality}}$$

$$B_{28,2} = \frac{1}{2} \cdot WS \cdot \left(\frac{\partial HS_2}{\partial P_2} \right)_{\text{quality}}$$

$i = 3$

$$B_{26,3} = D_3 + \frac{1}{2} \cdot WS \cdot \left(\frac{\partial HS_3}{\partial P_3} \right)_{\text{quality}}$$

$$B_{27,3} = 0$$

$$B_{28,3} = \frac{1}{2} \cdot WS \cdot \left(\frac{\partial HS_3}{\partial P_3} \right)_{\text{quality}}$$

$$A_{14,i} = A \cdot L_i \cdot V_i^{-2} \cdot \left[\bar{V}_i \cdot \left(\frac{\partial \bar{U}_i}{\partial HS_i} \right)_{P_i} - \bar{U}_i \cdot \left(\frac{\partial \bar{V}_i}{\partial HS_i} \right)_{P_i} \right] \quad i = 1$$

$$= 0 \quad i = 2, 3$$

$$A_{15,i} = A \cdot L_i \cdot V_i^{-2} \cdot \left[\bar{V}_i \left(\frac{\partial \bar{U}_i}{\partial P_i} \right)_{HS_i} - \bar{U}_i \left(\frac{\partial \bar{V}_i}{\partial P_i} \right)_{HS_i} \right]$$

$$A_{17,i} = A \cdot L_i \cdot V_i^{-2} \cdot \left[\bar{V}_i \left(\frac{\partial \bar{U}_i}{\partial P_{i+1}} \right)_{HS_{i+1}} - \bar{U}_i \left(\frac{\partial \bar{V}_i}{\partial P_{i+1}} \right)_{HS_{i+1}} \right]$$

$$A_{16,i} = A \cdot L_i \cdot V_i^2 \cdot \left[V_i \cdot \left(\frac{\partial \bar{U}_i}{\partial HS_{i+1}} \right) P_{i+1} - U_i \cdot \left(\frac{\partial \bar{V}_i}{\partial HS_{i+1}} \right) P_{i+1} \right] \quad i = 3$$

$$= 0$$

$$i = 1, 2$$

$$A_{18,i} = A \left[\frac{\bar{U}_i}{V_i} - \frac{HS_{i+1}}{V_{i+1}} \right]$$

$$A_{19,i} = -A \left[\frac{\bar{U}_i}{V_i} - \frac{HS_i}{V_i} \right]$$

The linearized equations for each node are:

for $i = 1$

$$\begin{aligned} B_{20,1} \cdot \left(\frac{\partial HS_1}{\partial TS_1} \right) P_1 \cdot \delta TS_1 + B_{21,1} \delta WS_1 + B_{23,1} \delta WS_2 \\ - B_{24,1} \delta X_2 - B_{25,1} \delta \bar{TH}_1 + B_{26,1} \delta \bar{P}_1 + B_{27,1} \delta \bar{P}_2 \\ = A_{14,1} \cdot \left(\frac{\partial HS_1}{\partial TS_1} \right) P_1 \cdot \delta TS_1 + (A_{15,1} + A_{17,1}) \delta \bar{P}_1 + A_{18,1} \delta \dot{X}_2 \quad (37) \end{aligned}$$

$i = 2$

$$\begin{aligned} B_{21,2} \delta WS_2 + B_{23,2} \delta WS_3 - B_{24,2} \delta X_3 + B_{24,2} \delta X_2 \\ - B_{25,2} \delta \bar{TH}_2 + B_{26,2} \delta \bar{P}_2 + B_{27,2} \delta \bar{P}_3 + B_{28,2} \delta \bar{P}_1 \\ = (A_{15,2} + A_{17,2}) \delta \bar{P}_2 + A_{18,2} \delta \dot{X}_3 + A_{19,2} \delta \dot{X}_2 \quad I.5.16 \end{aligned}$$

$i = 3$

$$\begin{aligned} B_{21,3} \delta WS_3 + B_{22,3} \cdot \left(\frac{\partial HS_4}{\partial TS_4} \right) P_4 \delta TS_4 + B_{23,3} \delta WS_4 + B_{24,3} \delta X_3 \\ - B_{25,3} \delta \bar{TH}_3 + G_3 \delta \bar{P}_3 + B_{28,3} \delta \bar{P}_2 \\ = G_4 \delta \bar{P}_3 + A_{16,3} \cdot \left(\frac{\partial HS_4}{\partial TS_4} \right) P_4 \cdot \delta TS_4 + A_{19,3} \delta \dot{X}_3 \quad I.5.17 \end{aligned}$$

where

$$G_3 = B_{26,3} + (1 - W_3) \cdot B_{22,3} \cdot \left(\frac{\partial HS_4}{\partial P_4} \right)_{TS_4}$$

$$G_4 = A_{15,3} + A_{17,3} + (1 - W_3) \cdot A_{16,3} \cdot \left(\frac{\partial HS_4}{\partial P_4} \right)_{TS_4}.$$

The nine linearized algebraic equations are

$$\delta WG_i = \delta WC_{in} \quad (i = 1, 2, 3) \quad I.5.18$$

$$\delta \overline{TG}_i = F_i \cdot \delta TG_i + (1 - F_i) \cdot \delta TG_{i+1} \quad (i = 1, 2, 3) \quad I.5.19$$

$$\delta WS_2 = KK_2 \cdot (\delta \bar{P}_1 - \delta \bar{P}_2) \quad I.5.20$$

where

$$KK_2 = \frac{WS}{2(\bar{P}_1 - \bar{P}_2)}$$

$$\delta WS_3 = KK_3 \cdot (\delta \bar{P}_2 - \delta \bar{P}_3) \quad I.5.21$$

where

$$KK_3 = \frac{WS}{2(\bar{P}_2 - \bar{P}_3)}$$

$$\delta WS_4 = KK_4 \cdot (\delta \bar{P}_3 - \delta \bar{P}_4) \quad I.5.22$$

where

$$KK_4 = \frac{WS}{2(\bar{P}_3 - \bar{P}_4)}.$$

The linearized equations from (I.5.4) to (I.5.22) can then be arranged in a matrix form.

I.6 Results and Discussion

The linearized equations presented in the above section were used to calculate the frequency response and time response of the main steam generator (no reheater) for selected inputs. In obtaining these results, two different sets of equations were used. In the first set, the feedwater pressure, $\bar{P}_0$, (see Figure I.3) is held constant; hence the description "constant pressure model" (CPSG). In the second set, the variable WS_1 (feedwater flow) is held constant; hence, the description "constant flow model" (CFSG). The frequency response and time responses for the constant pressure and constant flow models were calculated for each input selected.

The code SFRMOD⁽⁵⁾ was used to calculate the frequency response. The input to SFRMOD is of the form:

$$A_1 \frac{d\bar{x}}{dt} + A_2 \bar{x} = \bar{f} \quad \text{I.6.1}$$

where

A_1 = matrix of constant coefficients related to differential terms

A_2 = matrix of constant coefficients related to algebraic terms

$\bar{x}$ = vectors of system variables

$\bar{f}$ = forcing vector for the selected input.

Before calculating the time response, it was necessary to transfer the system of algebraic and differential equations (I.6.1)

describing the steam generator to the following form

$$\frac{d\bar{X}_d}{dt} = A\bar{X}_d + \bar{f}_d \quad \text{I.6.2}$$

where

$\bar{X}_d$ = vector of state variables

A = matrix of constant coefficients

$\bar{f}_d$ = forcing terms for the system input.

This was performed using the computer code PUREDIF. ⁽⁵⁾ The input to PUREDIF consists of the A_1 and A_2 matrices and $\bar{f}$ vector as in Equation (I.6.1). The A_1 , A_2 , and A matrices appear in Table I.2 and Table I.3.

TABLE I.2

THE VARIABLES ASSOCIATED WITH CPSG FOR A_1 AND A_2 MATRIX

Number	Variables
1, 2, 3	$\delta\bar{P}_1, \delta\bar{P}_2, \delta\bar{P}_3$
4, 5, 6, 7	$\delta TG_1, \delta TG_2, \delta TG_3, \delta TG_4$
8, 9, 10	$\delta\bar{TM}_1, \delta\bar{TM}_2, \delta\bar{TM}_3$
11, 12	$\delta X_2, \delta X_3$
13	δMST
14, 15, 16	$\delta\bar{TG}_1, \delta\bar{TG}_2, \delta\bar{TG}_3$
17, 18, 19, 20	$\delta WS_1, \delta WS_2, \delta WS_3, \delta WS_4$

TABLE I.3
THE STATE VARIABLES ASSOCIATED WITH CPSG FOR REDUCED A MATRIX

Number		State Variables	
1, 2, 3, 4		$\delta P_1, \delta P_2, \delta P_3, \delta TG_4$	
5, 6, 7		$\delta TH_1, \delta TH_2, \delta TH_3$	
8, 9		$\delta X_2, \delta X_3$	
10		δMST	
11, 12, 13		$\delta TG_1, \delta TG_2, \delta TG_3$	

A1(I,J)			
1	1 2.61053E-01	1 11 3.38096E 01	2 2 1.01479E 00
2	11 2.28625E 01	2 12 1.47444E 01	3 3 3.13884E-01
3	12 1.05238E 01	3 13-5.05814E-01	7 7 1.00000E 00
8	8 7.41049E 03	8 11-7.91655E 03	9 9 4.71117E 03
9	11-7.91655E 03	9 12-1.09647E 04	10 10 7.51123E 03
10	12-1.09647E 04	11 1 4.14207E 02	11 11-6.98903E 02
12	2 9.45497E 02	12 11 1.57921E 04	12 12 8.68772E 03
13	3 3.67355E 02	13 12 9.52171E 03	13 13-4.25746E 02
14	11-3.11099E 02	14 14 3.67501E 01	15 11 3.11099E 02
15	12-3.11099E 02	15 15 1.97891E 01	16 12 3.11099E 02
16	16 2.59237E 01		

A2(I,J)			
1	17-1.00000E 00	1 18 1.00000E 00	2 18-1.00000E 00
2	19 1.00000E 00	3 19-1.00000E 00	3 20 1.00000E 00
4	4-7.50000E-01	4 5-2.50000E-01	4 14 1.00000E 00
5	5-6.70000E-01	5 6-3.30000E-01	5 15 1.00000E 00
6	6-3.00000E-01	6 7-7.00000E-01	6 16 1.00000E 00
7	7 2.00000E 00	8 1-8.77428E 01	8 8 3.54908E 03
8	14-9.37232E 02	8 17 8.58445E 01	8 18 2.86148E 01
9	2-9.14281E 01	9 9 2.35927E 03	9 15-5.95840E 02
9	18 4.74777E 01	9 19 2.33845E 01	10 3 1.42560E 02
10	10 3.89902E 03	10 13-1.83209E 03	10 16-9.49973E 02
10	19 3.52938E 01	10 20 8.13522E 01	11 1 1.13677E 02
11	2 2.59346E 01	11 8-2.61185E 03	11 11-1.39502E 03
11	17-4.71644E 02	11 18 7.48713E 02	12 1-2.59346E 01
12	2 3.78754E 01	12 3-2.76181E 01	12 9-1.76343E 03
12	11 1.54129E 03	12 12-1.54129E 03	12 18-8.24805E 02
12	19 1.02896E 03	13 2 2.76181E 01	13 3-1.25829E 02
13	10-2.94904E 03	13 12 1.81334E 03	13 13 2.04697E 03
13	19-1.08763E 03	13 20 1.48584E 03	14 4 5.95539E 02
14	5-5.95539E 02	14 8-9.37232E 02	14 11 1.45888E 03
14	14 9.37232E 02	15 5 5.95539E 02	15 6-5.95539E 02
15	9-5.95840E 02	15 11-1.61186E 03	15 12 1.61186E 03
15	15 5.95840E 02	16 6 5.95539E 02	16 7-5.95539E 02
16	10-9.49973E 02	16 12-1.89635E 03	16 16 9.49973E 02
17	1 6.18079E 00	17 17 1.00000E 00	18 1-7.24500E 00
18	2 7.24500E 00	18 18 1.00000E 00	19 2-7.24500E 00
19	3 1.85951E 00	19 19 1.00000E 00	20 3-1.37705E 00
20	20 1.00000E 00		

TABLE 1.3 (Continued)

THE REDUCED A1-MATRIX IS											
1	1-2.0806900E 01	1	2 1.3222581E 01	1	5 6.2250166E 00	1	8 3.3246164E 00				
2	1 6.5625935E 00	2	2-8.1229670E 00	2	3 2.4341621E 00	2	5 3.2062525E-01				
2	6 5.0771427E 00	2	8-4.2665825E 00	2	9 4.4378195E 00	3	1-2.6294289E 00				
3	2 5.6758375E 00	3	3-1.1549730E 01	3	5-3.3963531E-01	3	6 2.2627077E 00				
3	7 2.8572784E 01	3	8-2.1591740E 00	3	9-1.5588333E 01	3	10-1.9833328E 01				
4	4-2.0000000E 00	5	1-1.9712412E-01	5	2 1.4782137E-01	5	5-5.3030968E-01				
5	8-2.7431257E-02	5	11 1.2647772E-01	6	1 4.7485876E-01	6	2-2.6879144E-01				
6	3-8.7153971E-02	6	5 4.1298445E-02	6	6-1.3141069E 00	6	8 7.3300046E-01				
6	9-7.1094406E-01	6	12 1.2646991E-01	7	1 5.9283441E-01	7	2-3.4774709E-01				
7	3-8.5797608E-02	7	5 7.6574564E-02	7	6-5.1015300E-01	7	7-5.1910526E-01				
7	8 4.8681015E-01	7	9-4.4591367E-01	7	10 2.4390888E-01	7	13 1.2646112E-01				
8	1-2.3641872E-01	8	2 1.1217344E-01	8	5-4.8073113E-02	8	8-2.5674574E-02				
9	1 4.0627545E-01	9	2-2.3232561E-01	9	3-4.1429687E-02	9	5 5.2477356E-02				
9	6-3.4961307E-01	9	8 3.3361596E-01	9	9-3.0558920E-01	10	1 6.8181896E 00				
10	2-4.9869909E 00	10	3-1.6296978E 00	10	5 8.8068461E-01	10	6-5.8672705E 00				
10	7 1.7732300E 01	10	8 5.5988035E 00	10	9-1.6029999E 01	10	10-1.2308591E 01				
11	1-2.0013561E 00	11	2 9.4958210E-01	11	4 2.4830109E 01	11	5 2.5095078E 01				
11	8-3.9918015E 01	11	11-4.7107468E 01	11	12 3.2246902E 01	11	13-3.5471603E 01				
12	1 1.0103191E 01	12	2-5.4155455E 00	12	3-6.5127712E-01	12	4-1.0479436E 02				
12	5 1.5806589E 00	12	6 2.4610168E 01	12	8 8.7103363E 01	12	9-8.6259186E 01				
12	12-7.5017975E 01	12	13 1.4970625E 02	13	1-4.8762445E 00	13	2 2.7884445E 00				
13	3 4.9725193E-01	13	4 7.6581772E 01	13	5-6.2984949E-01	13	6 4.1961641E 00				
13	7 3.6651230E 01	13	8-4.0041618E 00	13	9 7.6815918E 01	13	13-1.1323302E 02				

The time response calculations were performed using the computer code MATEXP.⁽⁶⁾ The input to MATEXP consists of the A matrix and $\vec{f}_d$ vectors, as defined in Equation (I.6.2).

Results of the frequency response and time response calculation at 100 percent load point are shown in Figures I.4 to I.23 for the following inputs: helium flow, helium inlet temperature, and feedwater flow. The five runs were labeled in Table I.4

The selected outputs to be plotted for each run were: (a) main steam outlet temperature (TS4), (b) Helium outlet node average temperature (TG1B), and (c) main steam outlet average pressure at node 3 (P3B).

TABLE I.4
DESCRIPTION OF EACH ANALYSIS RUN

Run No.	Description of Perturbed Input Variable	Constant Flow Model or Constant Pressure Model
1	Helium flow rate (1 lb/sec)	CFSG
2	Helium flow rate (1 lb/sec)	CPSG
3	Helium inlet temperature (1°F)	CFSG
4	Helium inlet temperature (1°F)	CPSG
5	Feedwater flow rate (1 lb/sec)	CFSG

I.6.1 Discussion of Results for Run No. 1 (Figure I.4 to I.7).

Run No. 1 shows the CFSG model time response and frequency response for a 1 lb/sec increase in helium flow rate. Increased helium flow rate increases the heat transfer rate from the helium to the secondary coolant through the tube wall. For constant outlet steam header pressure and feedwater flow rate, the steam pressure initially rises and then approaches the original value. This is because there is no increase in feedwater flow to sustain the rise. The main steam temperature (TS4) increases significantly (1.44°F), whereas the temperature rise for the helium outlet node (average) is relatively low (0.63°F). The steady values of each output in time response after 300 sec are close to the magnitude of the frequency response at low frequency (0.004 rad/sec) just as they should be.

Fig. 1.5. Frequency Response of Main Steam Temperature to Helium Flow (CFSG).

Fig. 1.4. CFSG Time Response to a 1 lb/sec Step Increase of Helium Flow.

Fig. 1.6. Frequency Response of Helium Outlet Temperature to Helium Flow (CFSG).

Fig. 1.7. Frequency Response of Main Steam Pressure to Helium Flow (CFSG).

The response curves of the outputs are almost monotonic and settle down within 165 seconds (0.006 rad/sec in frequency domain). The time lag is primarily due to the thermal capacitance of the metal, water/steam mass and the thermal resistance from the primary to the secondary coolant.

1.6.2 Discussion of Results for Run No. 2 (Figure I.8 to I.11)

Run No. 2 shows the CPSG model time response and frequency response for a 1 lb/sec increase in helium flow rate. The system

Fig. I.8. CPSG Time Response to a 1 lb/sec Step Increase of Helium Flow.

Fig. 1.9. Frequency Response of Main Steam Temperature to Helium Flow (CPSG).

Fig. 1.10. Frequency Response of Helium Outlet Temperature to Helium Flow (CPSG).

Fig. I.11. Frequency Response of Main Steam Pressure to Helium Flow (CPSG).

responses for "P3B" and "TG1B" are similar to those of Run No. 1 (CFSG). There is large difference in the early transient of "TS4" response. For the CPSG model, the fast rise of main steam pressure (P3B) reduces the feedwater flow rate which makes the response of "TS4" faster than for the CFSG model in Run No. 1.

I.6.3 Discussion of Results for Run No. 3 (Figure I.12 to I.15)

Run No. 3 shows the CFSG model time response and frequency response for a 1°F increase in helium inlet temperature. Increased temperature of the primary helium coolant increases the rate of transfer of thermal energy from the primary to the secondary coolant. "P3B" increases initially, then returns to the original value due to the constant

Fig. I.12. CFSG Time Response to a 1°F Step Increase of Helium Inlet Temperature.

Fig. 1.13. Frequency Response of Main Temperature to Helium Inlet Temperature (CFSG).

Fig. 1.14. Frequency Response of Helium Outlet Temperature to Helium Inlet Temperature (CFSG).

Fig. I.15. Frequency Response of Main Steam Temperature to Helium Inlet Temperature.

feedwater flow. All the temperature nodes increased following the increase of helium inlet temperature.

I.6.4 Discussion of Results for Run No. 4 (Figure I.16 to I.19).

Run No. 4 shows the CPSG model time response and frequency response for a 1°F increase in helium inlet temperature. The system transient responses for this run are similar to those of Run No. 3.

ORNL DWG 78 7774

Fig. I.16. CPSG Time Response to a 1°F Step Increase of Helium Inlet Temperature.

ORNL DWG 78 7775

Fig. I.17. Frequency Response of Main Steam Temperature to Helium Inlet Temperature (CPSG).

Fig. 1.18. Frequency Response of Helium Outlet Temperature to Helium Inlet Temperature.

Fig. 1.19. Frequency Response of Main Steam Pressure to Helium Inlet Temperature (CPSG).

I.6.5 Discussion of Results for Run No. 5 (Figure I.20 to I.23).

Run No. 5 shows the CFSG time response and frequency response for a 1 lb/sec increase in feedwater flow rate. For constant helium flow rate and inlet temperature, the temperature in the secondary coolant circuit drops (-2.74°F) following the increased feedwater flow rate. The helium outlet temperature also drops (-0.68°F). For constant outlet steam header pressure, the main steam pressure increases by 0.71 psi in order to sustain the increase in the feedwater flow rate.

Fig. I.20. CFSG Time Response to a 1 lb/sec Step Increase of Feedwater Flow.

Fig. 1.21. Frequency Response of Main Steam Temperature to Feedwater Flow (CFSG).

Fig. 1.22. Frequency Response of Helium Outlet Temperature to Feedwater Flow (CFSG).

Fig. I.23. Frequency Response of Main Steam Pressure to Feedwater Flow (CFSG).

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CHAPTER II
NONLINEAR MODELING OF A TIGR STEAM GENERATOR

Ming-Huei Lee

A nonlinear model is important when large disturbances or various operation levels are involved in the simulation of a steam generator. It is expected to be more accurate than the linear model for these conditions. The nonlinearity of the system parameters and the spatial effects cause the problem to be quite complex. Moreover, since the uncertainties in the system parameters are significant, the results will have uncertainties. Therefore, in the nonlinear model, the spatial effect, the nonlinear parameters and the uncertainties in system parameters are three major areas of concern.

Practical methods for handling spatial effects require the use of lumps to represent the spatial distributions. However, as the number of lumps is increased, the computing time is also increased. It is important to know whether a few-lump model can provide the necessary accuracy. In this study, the number of lumps will be varied to find the number required to reach asymptotic performance as the number of lumps increases.

The system parameters which are involved in major nonlinear terms include the thermal conductivities, heat capacities, and viscosities of fluid (steam, water, helium) and/or tube metal, surface tension of water, suppression factor and some other factors in the heat transfer correlations. These parameters may be evaluated using available subroutine programs or by using empirical equations which are obtained by least square methods.

In studying the uncertainties in system responses due to parameter uncertainties, the heat transfer coefficients and two phase multiplier factors are most important, since their inaccuracy could be up to 20%. Other important parameters with significant uncertainties will also be studied for their sensitivities.

The dynamic equations for the transient state were derived from the fundamental differential equations. The conservation of energy, the conservation of momentum, and the conservation of mass equations were developed into a moving boundary model. The developed model was compared to different models from a literature survey so that an assessment of model assumptions and approximations could be made.

The literature was reviewed to find the data and correlations required for the steam generator model. A short summary of the information from the literature and other preparations for the model development follow:

A. Survey on heat transfer correlations and/or friction factors.

1. For single phase

- a. Seban and McLaughlin's⁽¹⁾ correlation
- b. Mori and Nakayama's⁽²⁾ correlation
- c. Pressure Drop and Heat Transfer in Coils summarized by Srinivasan et al.⁽³⁾
- d. Srinivasan's⁽⁴⁾ correlation
- e. Ito's⁽⁵⁾ correlation.

All of the above correlations are for helical coils - one of the features of the HTGR steam generator. The correction factor for helical tubes from (e) or (a) was

selected to use in this study, since the applicable range of Reynold numbers and curvature is adequate for HTGR steam generator.

2. For two phase, steam/water
 - a. Borishanskii, et al's.⁽⁶⁾ correlation (straight tube)
 - b. Owhadi, Bell, and Crain's⁽⁷⁾ correlation (coil tube)
 - c. Crain and Bell's⁽⁸⁾ correlation (coil tube)
 - d. Chen's⁽⁹⁾ correlation (straight tube)
 - e. Rohsenow's^(10,11) correlation (straight tube)
 - f. Miropolskii's⁽¹²⁾ correlation (straight tube)

(a) through (e) are for nucleate boiling and subcooled boiling. (f) is for film boiling.

Chen's correlation, Rosenhow's correlation, and Borishanskii's corrections were selected for use in the nucleate boiling and subcooled boiling region. The last two correlations were evaluated at high pressure and high temperature. The Miropolskiy's correlation was selected to use in the dryout region for high pressure and high temperature. The correction factor for helical coils is the same as for single phase, which is based on the discussion in (b) or the suggestion of Collier.⁽⁹⁾

3. For single phase, steam
 - a. Miropolskii and Shitsman⁽¹³⁾ correlation
 - b. Bishop et al's.⁽¹⁴⁾ correlation

The correction factor for coil tube is the same as for single phase water.

4. For helium side heat transfer and pressure drop through banks of helical tubes, the Gilli's⁽¹⁵⁾ correlations were selected to use. Grimison's⁽¹⁶⁾ straight tube bank data are necessary for evaluating Gilli's correlations.

Development of the two phase pressure gradient in homogeneous models by employing Colebrook⁽¹⁷⁾ friction coefficient, Ito's curve pipe correction factor, and McAdams^(9,18) two phase viscosity's definition was made.

- B. Obtaining the empirical equations for the nonlinear parameters.

The thermal conductivities of the fluid and tube metal, the viscosities of fluid, the surface tension of water, suppression factor and so on are necessary to obtain their empirical equations for practical use. Some of these equations may be found in the literature survey; some of them were obtained by using linear regression methods to fit the numerical tables.

- C. Steady State Program.

The steady state program has been developed in the multilump case so that the steady state distributions of the state variables can be obtained. This was done for three purposes: (a) To study the differences of the results by using different correlations and to compare them to the HTGR steam generator design data; (b) to evaluate the adequate average state variables for the transient study in a few lump models; (c) to provide the initial conditions to the transient case.

The steady state program contains the main program and the following subroutines:

- 1) SUBROUTINE FRCTN
- 2) SUBROUTINE TWOFRG
- 3) SUBROUTINE SUBFRG
- 4) SUBROUTINE EQCKT
- 5) SUBROUTINE TWOEK1
- 6) SUBROUTINE TWOEK2
- 7) SUBROUTINE SUBEK
- 8) SUBROUTINE VSCST
- 9) SUBROUTINE VISFG
- 10) SUBROUTINE SUBVIS.

1), 2), and 3) are used to evaluate the Colebrook friction factors by an iteration method for single phase steam, two phase steam/water and single phase water respectively. 4), 5), 6), and 7) are used to evaluate the equivalent thermal conductances, tube temperature and tube surface temperature by an iteration method for single phase steam, two phase steam/water dryout region, two phase steam/water nucleate boiling region, and the single phase water region respectively. 8), 9), and 10) are used to evaluate the dynamic viscosities of the above three regions.

The steady state program is now being tested and debugged.

In the future, the following are planned:

1. Comparison of steam generator models using different assumptions, approximations, and correlations.
2. Steam generator dynamic simulation.

3. Sensitivity to the system parameters.
4. Study of small and large disturbances.
5. Extension of the model from a few lumps to many lumps.

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CHAPTER III

GAS-COOLED STEAM GENERATOR LINEAR MODEL

David G. Penfro

This phase of the project is concerned with the development and investigation of a linear, moving-boundary model for the Fort St. Vrain steam generators. In this progress report, a brief history of the work in this area will be reviewed before describing the current project in greater detail.

General Atomic uses a linear model of the HTGR-ISS in part of the design process of the plant control system. This mathematical model is called LAP (Linear Analysis Program). It was developed by combining linear models of the reactor, primary coolant, and secondary coolant systems. Each of these models was derived from first principles using the basic conservation equations except for the steam generator. GA was unable to develop a satisfactory steam generator model by the time the rest of the system models were ready, so an empirical or transfer function model was substituted. However, a model based on the physical system still was considered to be more desirable. The development and evaluation of this model is the purpose of the current project.

General Atomic developed a lumped-parameter, three-node, moving-boundary model for the steam generator. The lumped parameter approach was chosen for its simplicity and ease of solution. The moving boundary technique attempts to account for movement of the beginning and end of the boiling zone as steam generator conditions change. This is accomplished by dividing the main steam section into three regions with moving boundaries between the economizer and evaporator zones and the evaporator

and superheater zones. This representation allows a fairly simple model that overcomes the difficulties encountered by fixed boundary models.

The project at The University of Tennessee is based on the original GA model just described. A version of LAP and the necessary data including steady state parameters were supplied by GA. To improve the physical validity of the time responses of the original GA steam generator model, S. I. Chang at UT made several basic modifications. These included the revision of equations to allow for replacement of node-boundary state variables by node-averaged state variables where applicable. A sketch of the steam generator model with variables as modified by Chang is shown in Figure III.1. As indicated by the figure, both node-boundary state

Fig. III.1. Steam Generator Nodal Structure.

node-averaged variables remain in the model equations. As suggested by GA, these are related by using a weighting factor, W_i , as follows.

$$\bar{Y}_i = Y_i W_i + Y_{i+1} (1 - W_i) \quad , \quad 0 \leq W_i \leq 1, \quad \text{III.1}$$

where Y_i and Y_{i+1} are the boundary quantities and $\bar{Y}_i$ is the node-averaged quantity. The weighting technique is very significant since it is used for temperatures on the primary side and for flow rates, pressures, and enthalpies on the secondary side. In addition, internal energies, specific volumes, and temperatures on the secondary side are calculated from the previously-averaged pressures and enthalpies. GA suggested specific values for each of the three nodal weight factors at which they arrived by physical and intuitive reasoning. These suggested weight factors influenced the calculation of the steady state parameters they supplied. Chang used these weight factors and the rest of the GA-supplied data to produce time responses that seemed reasonable and valid. However, when he varied the weight in the helium entrance node, (W_3), to investigate its effect, the time responses were similar in character to the previous ones but markedly different in magnitude. This result was a subject for some concern for at least two reasons. First, the primary side influence was not thought to be so significant. Also, GA arrived at the suggested weights fairly arbitrarily so there was little reason to believe they were the ideal ones to use.

This area of concern with weighting factors along with a basic need to verify the model modifications independently were the reasons behind this phase of the project at UT. The initial purposes were to verify the unexpected significant dependence on the primary side weighting

factors, next to investigate these effects on both the primary and secondary sides, and then to offer some explanation for the dependence or to suggest changes to improve the weighting technique. Since the beginning of this work, the investigation has broadened to include a more general sensitivity study of the model¹ and examination of model assumptions made in its development.

The first task was to verify the work done on the model at UT by S. I. Chang. An attempt was made to duplicate his development and compare the results at each stage. First, the model equations were derived from the basic conservation equations. Next, the equations were linearized and set up in the model form with specified state variables and perturbation variables. All discrepancies found were resolved by mutual agreement and the final model form was decided upon. The next task involved a review of LAP and its STMGGEN subroutine to insure that the model in its latest form was being implemented properly. After the program was corrected for necessary changes, it was revised into a simplified form with all extraneous statements in STMGGEN that remained from the GA version deleted. Finally, the input data for LAP as supplied by GA was checked for accuracy. The entire process just described that was done in preparation for the model investigation was time-consuming but necessary. Although several mistakes were located and corrected, no major errors were discovered that completely invalidated results obtained previously. The major benefit was an added degree of confidence that could be placed in the steam generator model.

In the investigation of various model assumptions, the assumption that the gas dynamics are negligible was questioned. The appropriate

modification to the model was made to include such effects but no results have been obtained at this time.

As a first step in the investigation of the dependence on the weight factors in Chang's model, some of the steam generator time response results that he had obtained were verified. Although slightly different due to the corrections made, the responses were still affected greatly by the variation of W_3 . The responses of main steam temperature (TS4) and helium outlet node temperature (TG1B) to a 1°F helium inlet step and a 1 lbm/sec helium flow step are shown in Figures III.2-5. As shown, the asymptotic values of some variations can more than double when W_3 is increased from 0.45 to 0.55. (Note: the GA value suggested for use is 0.30. Some responses for this value are shown later.)

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Fig. III.2. Helium Inlet Temperature Perturbation (1°F).

Fig. III.3. Helium Inlet Temperature Perturbation (1°F).

Fig. III.4. Helium Flow Rate Step (1 lbm/sec).

Fig. III.5. Helium Flow Rate Step (1 lbm/sec).

This behavior led to an examination of the equations and the program for an explanation of this dependence. Included in the input data, GA supplied steady-state values for the helium node boundary temperature (TG_i) but supplied no values for the helium node-averaged temperatures ($\overline{TG}_i$). These were calculated in the subroutine by Chang using whatever weight factors the user chose to input. This meant that steady state values of $\overline{TG}_i$ were being varied as the weights were varied. This change affected the heat transferred out of the primary side as well as other terms where $\overline{TG}_{iss}$ appeared in the equation coefficients. So it seemed that the variation of $\overline{TG}_{iss}$ with weight was responsible for the large dependence on weight factors.

To avoid this problem, it was necessary to calculate steady-state values of $\overline{TG}_i$ that were not weight-dependent. This was accomplished by calculating $\overline{TG}_{iss}$ with the original GA weight factors. In this way, the steady-state values of $\overline{TG}_i$ and TG_i were constant and the weights could be varied independently. Results using this approach are shown in Figures III.6-9. As shown, the time responses were no longer very dependent on weighting factors. This method seemed adequate until the question was raised that variation of weight factors while $\overline{TG}_{iss}$ and TG_{iss} were fixed violated the relation in Equation (III.1). To avoid this, it was decided to calculate $\overline{TG}_{iss}$ with the original weights as before but to recalculate TG_{iss} with each new set of weight factors input.

Fig. III.6. Helium Inlet Temperature Perturbation (1°F).

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Fig. III.7. Helium Inlet Temperature Perturbation (1°F).

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Fig. III.8. Helium Flow Rate Step (1 lbm/sec).

Fig. III.9. Helium Flow Rate Step (1 lbm/sec).

Results using this method are shown in Figures III.10-13. As shown, the responses are more weight-dependent than those of the second method (Figures III.6-9) but less weight-dependent than those of the first method (Figures III.2-5). The third method is actually more closely related to the first since weight changes lead to changes in various steady-state parameters in both schemes. As a result, the flaws inherent in the first method due to steady-state values being adjusted are present in the third as well. However, from the results, the effect of changing $T_{G_{iss}}$ is not as important as changing $\overline{T_{G_{iss}}}$.

The previous discussion of weighting schemes illustrates flaws in all three methods. However, when dealing with a linear model, several

Fig. III.10. Helium Inlet Temperature Perturbation (1°F).

Fig. III.11. Helium Inlet Temperature Perturbation (1°F).

Fig. III.12. Helium Flow Rate Step (1 lbm/sec).

Fig. III.13. Helium Flow Rate Step (1 lbm/sec).

approximations and assumptions have been made already. The question of importance is what additional approximations or assumptions are implied when weighting factors or any other parameters are adjusted. For example, the methods given above modify the model in ways ranging from the amount of heat transferred from the primary side into the tube metal (method 1) to the helium boundary outlet temperature (method 3).

The next step in the investigation will be to classify the possible weighting factor schemes along with the assumptions they imply and the results they produce. In addition, methods need to be developed for the variation of the weighting factors on the secondary side independently from the primary. Examination of the relationship between primary and secondary sides through the heat balance should produce these methods. If such parameters as pressure and enthalpy on the secondary side could possibly be weighted independently of each other, another weighting factor effect could be observed. All of these results have led to a broadening of the scope of the work in the project to a consideration of the variation of other parameters besides the weight factors. A more general sensitivity study will be performed where various parameters that are fairly arbitrary (like weight factors) or not precisely known (like heat transfer coefficients) can be examined and the accompanying assumptions and consequences outlined.

Since two of the techniques discussed for variation of the primary weight factors involved changing steady-state parameters, possibly the most accurate way to determine the effects of such variations would be to recalculate a completely new set of steady-state values for each

proposed variation. This method will be investigated to see if it is feasible to develop such a capability at UT.

In conclusion, results from the investigation of the effects of variation of weighting factors on the steam generator response have been obtained and have led to a broader sensitivity study of the model. A more general classification of the assumptions made and the results obtained when parameters are varied is desirable and will be developed. Assumptions such as the neglect of gas dynamics have been selected for further verification. These future studies should yield valuable information about significant parameters and assumptions used in this steam generator model.

CHAPTER IV NONLINEAR, NODAL SYSTEM MODEL

J. G. Thakkar

This section briefly describes work being done in formulating the nonlinear system models, for the Fort St. Vrain nuclear steam supply system. The helium circulator model has been completed and some improvements were made in the reactor, steam generator and reheater models.

IV.1 Steam Properties

ASME steam tables were added in the steam generator program using available function generating capability, in CSMP III, of the forms $y = f(x)$ and $y = f(x,z)$. This is a convenient way to calculate thermodynamic properties of both saturated and superheater steam. The results were found to be comparable with the previous model where steam properties were calculated using polynomial fits. The comparison of two cases for the change in main steam outlet temperature (for 1°F step change in Helium inlet temperature) is shown in Figure IV.1.

IV.2 Reactor

The reactor model was reformulated using the prompt jump model⁽¹⁾ i.e., setting

$$\frac{dP}{dt} = \frac{\partial \rho}{\partial P} P + \sum \lambda_i C_i = 0.$$

This gives an algebraic equation for the reactor power, which allows us to take larger time steps while integrating the heat transfer equations. For small reactivity perturbations (~5¢) results from the

Fig. IV.1. 1°F Step Change in Helium Inlet Temperature.

prompt jump model give less than 2% error compared with the reference case. For larger reactivity perturbations ($\sim 40\text{c}$) the error increases to about 20%. This is probably due to the comparatively large neutron life time ($\sim 1.85 \times 10^{-4}$ sec) in the HTGR. (The prompt jump model gives accurate results for shorter neutron life times, i.e., in fast reactors⁽¹⁾).

IV.3 Reheater

The reheater model was updated using new constants and steady state values from the General Atomic memo.⁽²⁾ This is being tested

for perturbations in primary inlet temperature, primary flow, and steam flow. The preliminary results are shown in Figures IV.2-3.

IV.4 Helium Circulator

Fort St. Vrain has four identical helium circulators, two in each loop. Each circulator unit consists of an axial flow compressor, a single stage steam-turbine main drive, and a single stage water-turbine auxillary drive. The schematic of the system is shown in Figure IV.4.

The circulator steam turbine operates on cold reheat steam from the exhaust of the high pressure turbine. The circulator turbines are in series with the main turbine. Steam enters the blower turbine at about 875 psia and 748°F and leaves it at 685 psia and 611°F. Steam from the exhaust of the circulator turbine flows to the reheater section of the steam generator. About 5% of the steam flow bypasses the circulator turbine, at full load, to provide a control margin. The bypass flow is adjusted to the desired value by maintaining a nearly constant pressure ratio across the blower turbine.

Helium from the steam generator exit enters the circulator at about 638°F and 686 psia and leaves the circulator at 647°F and 700 psia and enters the reactor inlet plenum.

The mathematical model of the circulator is based on conservation equations and performance maps for the turbine and the circulator.⁽³⁾

The circulator dynamic characteristics are evaluated from the conservation of momentum equation

$$I \frac{dw}{dt} = T_t - T_c$$

Fig. IV.2. Response of an Isolated FSV Reheater.

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REHEATER TRANSIENTS AT 100% LOAD FOR STEP CHANGES IN HP INLET TEMP., HE FLUID RATE, & STEAM FLOW RATE
STEAM OUTLET TEMPERATURE

REHEATER OUTPUT PRESENTATION FOR TEST

Fig. IV.3. Response of an Isolated FSV Reheater.

Fig. IV.4. Schematic of Blower Turbine Helium Circulator.

I = mass moment of inertia of the blower turbine rotor shaft

w = angular velocity

T_t = Torque exerted by steam

T_c = Torque exerted by helium.

T_t and T_c are calculated from energy balances, as follows

$$T_t = P_t / w$$

$$T_c = P_c / w$$

P_t = power transmitted to the turbine

P_c = power required by the compressor.

$$P_t = w_{\text{steam}} \times \Delta_{hs} \times \eta_T$$

w_{steam} = steam flow rate through the turbine

Δh_{hs} = isentropic steam enthalpy drop through the turbine

η_T = turbine efficiency

$$P_C = w_{\text{He}} * \Delta h_{\text{He}} / \eta_C$$

w_{He} = Helium flow rate through the compressor

Δh_{He} = Helium enthalpy rise through the compressor

η_C = compressor efficiency.

η_C , η_T , Δh_{hs} , and ΔP_{He} are calculated from the compressor and the turbine performance maps.

The computer program for the circulator model has been completed and is being debugged and tested for the steam flow perturbation.

Future Work

1. Model the steam turbines
2. Review the steam generator model, to explain the differences with the linear steam generator model currently used in LAP, particularly for the primary inlet temperature perturbation.
3. Improve pressure drop and heat transfer correlations in the models.
4. Couple all subsystems of Fort St. Vrain to obtain an overall model for the plant and test it for different perturbations and sensitivity to changes in different system parameters.
5. Prepare for correlation of model predictions with trip test results.

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