AN ASSUMED MODE APPROACH TO FAST REACTOR
CORE SEISMIC ANALYSIS

by

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NOMENCLATURE

The following symbols and notations are used in the text of this report.

Script

$\mathcal{L}$	Lagrangian function
$\mathcal{T}$	Kinetic energy function
$\mathcal{U}$	Total energy function
$\mathcal{V}$	Potential energy function
$\mathcal{W}$	Work function

Upper Case

A	Deformable body kinetic energy matrix
B	Interassembly fluid kinetic energy matrix
C	Deformable body potential energy matrix
E	Young's modulus
I	Moment of inertia; integral of assumed modes
L	Length of deformable body
M	Number of degrees of freedom
N	Number of deformable bodies
Q	Generalized force

Lower Case

a	Element of matrix A
b	Element of matrix B
c	Element of matrix C
d	Contact energy generalized force
q	Generalized coordinate for assumed mode

u	Assumed displacement of deformable body
w	Inertial displacement of deformable body
x	Spacial coordinate of deformable body
y	Reference frame displacement

Prefix

Δ	Change in the quantity, gap size
δ	Virtual quantity

Greek

ρ	Mass density per unit length
μ	Function used in defining assumed modes
α	Energy loss fraction in impulse

Subscript

C	Contact point
D	Deformable body
F	Interassembly fluid
j	Index over number of bodies
k	Index over number of degrees of freedom

Mathematical

$\dot{q}$	Derivative of q with respect to time, t .
u'	Derivative of u with respect to the spacial variable x
$\hat{Q}$	Impulsive form of Q
$\langle \quad \rangle^n$	n^{th} order singularity function (see page 7)

Abstract

This report describes the need for a time history approach, rather than a response spectrum approach, to the seismic analysis of fast breeder reactor core structures, and advocates the use of a Rayleigh-Ritz/Assumed Mode formalism for developing mathematical models of reactor cores. Various factors including structural nonlinearity, fluid inertia, and impact which necessitate abandonment of response spectrum methods are discussed. The use of the assumed mode formalism is described in some detail as it applies to reactor core seismic analysis. To illustrate the use of this formal approach to mathematical modeling, a sample reactor problem with increasing complexities of modeling is presented. Finally, several problem areas -- fluid inertia, fluid damping, coulomb friction, impact, and modal choice -- are discussed with emphasis on research needs for use in fast reactor seismic analysis.

I. Introduction

The seismic analysis of fast breeder reactor (FBR) cores is a subject of rapidly growing interest but very little established technology. The purpose of this report is to attempt to define the scope of this subject and to propose an analytical formalism within which the technology can develop.

In general, any engineering analysis consists of three steps. First a suitable abstract, simplified physical model is conceived which is intended to include all of the effects considered important by the analyst and to exclude all of the details which are unimportant. The second step involves construction of a mathematical model, i.e., some series of equations or inequalities based on constitutive relations, boundary conditions, symmetry conditions, etc., which describe the abstract physical model. The third step in the analysis consists of solving the mathematical model for the desired information to an acceptable degree of accuracy.

The steps are closely interrelated. The abstract physical model must be sufficiently detailed to provide the required information and accuracy in the final step while also remaining amenable to mathematical modeling. The mathematical model must reflect the physical phenomena contained in the physical model to a sufficient degree that the answers obtained in the final step have the required accuracy but it also must not be so complex as to preclude solution of the equations. Finally, the degree of accuracy in solution of the mathematical model should not go beyond the approximations used in the original abstract physical model or the mathematical model.

In this report we shall attempt to identify the physical phenomena which are likely to be important in the seismic analysis of FBR cores, and

we shall propose a formalism for generating mathematical models based on the abstract physical model. The purpose of this formalism is to provide a consistent basis for comparing the results of analyses involving different physical effects, different mathematical models of the same effects, and different solution techniques for the same mathematical model. A second purpose of the formalism is to provide for economy of effort in the development and communication of the technology. Finally, we will illustrate the use of the formalism in various approaches to FBR core seismic analysis and discuss areas where further research is needed.

II. Seismic Analysis Requirements

The seismic analysis of FBR cores differs from that of most engineering structures in that, in addition to the elastic and inertial properties of the structural members, several additional effects must be included in the analysis. FBR cores are nonlinear structures because changes in load cause changes in the contact pattern between assemblies and a resulting change in structural stiffness. This effect probably occurs quite infrequently in cores with large inter-assembly gaps such as the GCFR and the effect might be treated on an individual basis. On the other hand, in LMFBRs with small gaps at load pads, the gap patterns are likely to be changing often during a seismic event and may have to be treated on a statistical basis.

A problem related to changing structural stiffness is the effect of impacting members. Recent Japanese work indicates that it may not be possible to solve equations which model individually the many rapidly occurring impacts in even a single row model of an LMFBR. However, individual modeling of duct impacts is probably feasible for the GCFR.

The structural damping of the various fuel, control, blanket, and shielding assemblies in a FBR probably is significantly greater than in normal steel structures and the usual use of "modal damping" may not be appropriate. In addition to this structural damping, LMFBRs have considerable viscous damping due to the rapid flow of liquid metal in narrow gaps between ducts during a seismic event. To this must be added the effect of Coulomb friction on the load pads of LMFBRs. Low quantities of oxygen in the environment leads to high friction coefficients and the effect of friction is believed to be dominant in determining the static loading of LMFBR cores.¹ Friction should be anticipated as a major factor in a dynamic analysis also.

Finally, the inertial effect of the coolant between assemblies in LMFBRs is a major contribution to the core dynamics. A study of the vibration of closely spaced rods² indicates, when extrapolated to a typical LMFBR core, that this is the dominant inertial effect. This interduct fluid coupling is possibly the reason that recent Japanese work found good agreement between one-fourth scale experiments and a single degree of freedom analytical model.

The information required from a FBR core seismic analysis will depend on needs defined by designers but most commonly these include: (1) alignment of control assemblies during the seismic event; (2) peak stresses in various components; and (3) reactivity changes due to the seismic event.

If an analytical formalism is to be useful in developing the technology without being unduly restrictive, it must satisfy certain general requirements. The following requirements have been identified:

- (1) It must provide consistent mathematical models of all the pertinent physical phenomena;

- (2) It must have the ability to include both information derived from fundamental laws of nature and from empirical observation;
- (3) It must lend itself to providing mathematical models which can be solved with a reasonable expenditure of effort;
- (4) It must be capable of providing mathematical models which can be solved to within the accuracy demanded by designers; and
- (5) It should lend itself to building on previous efforts when attempting more detailed solutions.

The most common approach to mathematical modeling of structures for seismic analysis is the "lumped mass" model. This formalism is conceptually quite simple, allows a pictorial presentation of the first steps in the mathematical model, and yields a diagonal inertial matrix in the equations of motion. While the approach does introduce error due to the inconsistent treatment of the inertial and stiffness properties, the advantages outweigh the disadvantages in the seismic analysis of most structures. For the LMFB core, however, this is no longer the case. The dominant inertial property is the coolant coupling between assemblies, and lumped mass models cannot adequately handle this phenomena. Even if the inertial coupling is handled separately, the advantage of the diagonal inertia matrix is lost and the pictorial representation of the mathematical model loses much of its meaning.

A second approach is an attempt to use a general purpose finite element code for both the mathematical modeling and solution. This is not recommended for the following reasons:

- (1) Few codes capable of handling the fluid inertia, nonlinear stiffness, and impact characteristics are available;

(2) Experience with finite element codes for static core restraint analysis indicates that running times are likely to be prohibitive; and

(3) Many of the general purpose codes are proprietary which impedes the communication of the developed technology.³

III. The Assumed Mode/Rayleigh-Ritz Formulation

The assumed mode method, which is closely allied to the Rayleigh-Ritz procedure, appears to best meet the requirements outlined in Section II. In this section we will present the assumed mode formalism as it relates to fast breeder reactor core seismic analysis. It is based on a Lagrangian approach and while the notation will appear somewhat complex, the application, as illustrated by the examples in Section IV, is quite straightforward.

Let N be the number of deformable bodies, i.e., the total numbers of fuel assemblies, control assemblies, blanket assemblies, reflectors, and any other core components whose deformation might be considered important to the seismic analysis such as the support plate or the core barrel. Denote the deformations of these bodies with respect to an inertial reference frame by:

$$w_j(x_j, t) \quad 0 \leq x_j \leq L_j$$

where $j = 1, N$ and x_j is the coordinate of the j^{th} body carrying over its length L_j .

The kinetic energy of the deformable bodies, $\mathcal{T}_D$, is:

$$\mathcal{T}_D = \sum_{j=1}^N \mathcal{T}_{Dj}(w_j)$$

$$\text{where } \mathcal{T}_{Dj} = \frac{1}{2} \int_0^{L_j} \rho_j(x_j) \dot{w}_j^2(x_j, t) dx_j \quad (3.1)$$

with $\rho_j(x_j)$ the mass per unit length of the j^{th} body and a "dot" representing time differentiation.

The potential energy of the deformable bodies, $\mathcal{V}_D$, is given by:

$$\mathcal{V}_D = \sum_{j=1}^N \mathcal{V}_{D_j}(w_j)$$

where, for example, if the j^{th} body is a fuel assembly represented by a beam

$$\mathcal{V}_{D_j} = \frac{1}{2} \int_0^L EI(x_j) (w_j''(x_j, t))^2 dx_j \quad (3.2)$$

with $EI(x_j)$ representing the bending stiffness of the beam and the expression $( )''$ representing two derivatives with respect to x_j .

In addition it may be necessary to consider other energy terms. For example the effect of an incompressible fluid on a single duct due to the motion of the duct is an inertial term which has a kinetic energy:

$$\mathcal{K}_{F_j} = \frac{1}{2} \int_0^L \gamma_{jj} \rho_F \dot{w}_j^r(x_j, t)^2 dx_j \quad (3.3)$$

where ρ_F is the mass density of the fluid, $\dot{w}_j^r$ is the velocity of the duct relative to the velocity of the fluid at infinity, and γ_{jj} is a weighting factor for the geometry of the fluid flow. For a single duct in an infinite fluid, γ_{jj} is the cross sectional area of the duct. Fluid inertia will be discussed in more detail in Section V.

In cases where consideration of interassembly contact, and in particular changes in these contacts, is important, it is necessary to consider the potential energy of the assembly duct local deformation at contact. Such

an energy might have the form:

$$V_{c_{j1}} = \frac{1}{2} k_{j1} \left\langle w_j(x_j^1, t) - w_{j+1}(x_{j+1}^1, t) - \Delta_{j1} \right\rangle^2 \quad (3.4)$$

where x_j^1 is the location of the contact point 1 on duct j , k_{j1} is the contact stiffness, Δ_{j1} is a gap size, and $\langle x \rangle^n$ is a singularity function; i.e. $\langle x \rangle^n = 0$ for $x \leq 0$, $\langle x \rangle^n = x^n$ for $x > 0$.

The Lagrangian function $\mathcal{L}(\dot{w}, w, t)$ is obtained by summing all of the kinetic energy terms and subtracting from it the potential terms. The assumed mode method makes a substitution for the deformation functions $w_j(x_j, t)$; for seismic analysis we choose the transformation:

$$w_j(x_j, t) = \sum_{k=1}^M u_{jk}(x_j) q_k(t) + y(t) \quad (3.5)$$

In this expression M is chosen by the analyst and represents the number of degrees of freedom that a particular model of the system will have; a choice which has large bearing on the eventual complexity of obtaining solutions. The functions $u_{jk}(x)$, $j = 1, N$, $k = 1, \dots, M$ represent the k^{th} mode of deformation for body j . $q_k(t)$ is termed the k^{th} mode. $y(t)$ is the motion of the reference frame in which the generalized coordinates are measured and for seismic analysis will be determined by the structural response of the reactor building and support to the earthquake.

Substitution of (3.5) into the various energy expressions in the Lagrangian, and performing the spatial integrations, will leave the Lagrangian a function of the generalized coordinates, q_k , the generalized velocities, $\dot{q}_k$, and the excitation velocity $\dot{y}(t)$.

$$\mathcal{L} = \mathcal{L}(q, \dot{q}, \dot{y}(t))$$

The equations of motion are then given in the form:

$$\frac{d}{dt} \frac{\partial}{\partial \dot{q}} (\mathcal{L}) - \frac{\partial}{\partial q_i} (\mathcal{L}) = Q_i(q, \dot{q}, t) \quad i = 1, \dots, M \quad (3.6)$$

where the functions $Q_i(q, \dot{q}, t)$ are called generalized forces.

For fast reactor core analysis the generalized forces will consist of various dissipative forces such as structural damping, fluid viscous damping, and friction forces.

Generalized forces are most easily formulated in terms of the concept of virtual work. Let $f_j(w, \dot{w}, t)$ be the force per unit length acting on the j^{th} body. In general it will depend on the positions w and velocities $\dot{w}$ of the body and its neighbors. Then the virtual work

$$\delta W = \sum_{j=1}^N \int_0^{L_j} f_j(w, \dot{w}, t) \delta w_j(x_j) dx_j \quad (3.7)$$

where

δw_j is a virtual displacement of the j^{th} body.

Then

$$\delta w_j(x) = \sum_{k=1}^M U_{jk}(x_j) \delta q_k \quad (3.8)$$

where

δq_k is now a virtual displacement of the k^{th} generalized displacement,

and

$$f_j(w, \dot{w}, t) = f_j \left(\sum_{k=1}^M u_{jk} q_{k_j}, \sum_{k=1}^M u_{jk} \dot{q}_k, t \right) \quad (3.9)$$

Substituting (3.8) and (3.9) into (3.7) and performing the spatial integrations yields

$$\delta \mathcal{W} = \sum_{k=1}^M Q_k(q, \dot{q}, t) \delta q_k$$

As an example, consider the viscous drag on a fuel assembly which is proportional to the square of the velocity of the assembly relative to the reference frame.

$$f_i = \mu (\dot{w}_j - \dot{y})^2$$

$$\delta \mathcal{W}_j = \int_0^{L_j} \mu (\epsilon u_{jk}(x) \dot{q}_k)^2 u_{ji}(x) dx_j$$

and

$$Q_i = \int_0^{L_j} \mu \left(\sum_{k=1}^M (U_{jk}(x) \dot{q}_k) \right)^2 U_{ji}(x) dx_j$$

At this point it is valuable to emphasize some of the characteristics of this formulation. Consideration of additional effects of the system is obtained by adding additional terms to the kinetic energy, potential energy, or generalized forces without necessarily changing the number of generalized coordinates or the corresponding model functions. For any given model of the system, as many modes of motion can be chosen as are deemed necessary for the required information or as can be solved within the allotted expenditure of time and computational capability. For example, a solution with one or more modes might indicate that there is no need for further study- or it might indicate that a more detailed analysis is necessary.

To further illustrate the form of the equations of motion, we will consider a model with deformation energies given by (3.1), (3.2), a fluid kinetic energy given by (3.3), and duct interaction given by (3.4) with $x^1 = L_j$. Moreover we will choose all the L_j equal to L , the $\Delta_{il} = \Delta$, and the $k_{jl} = K$. We will further simplify the expressions by not explicitly showing the functional dependence of some expressions where the meaning is clear.

Substitution of 3.5 into 3.1 yields:

$$\mathcal{R}_{D_j} = \frac{1}{2} \int_0^L \rho_L \left[\sum_{k=1}^M u_{jk} \dot{q}_k + \dot{y} \right]^2 dx$$

$$\frac{\partial}{\partial \dot{q}_i} (\mathcal{R}_{D_j}) = \int_0^L \rho u_{ji} \left(\sum_{k=1}^M u_{jk} \dot{q}_k + \dot{y} \right) dx$$

$$\frac{d}{dt} \frac{\partial}{\partial \dot{q}_i} (\mathcal{R}_{D_j}) = \sum_{k=1}^M \int_0^L \rho u_{ji} u_{jk} dx \ddot{q}_k + \int_0^L \rho u_{ji} dx \ddot{y}$$

Similarly (3.1) gives:

$$\frac{\partial}{\partial q_i} (\mathcal{V}_{D_j}) = \sum_{k=1}^M \int_0^L EI u_{jk}'' u_{ji}'' dx_j q_k$$

For the fluid kinetic energy we use (3.3) with

$$\dot{\mathbf{w}}_j^r = \sum_{k=1}^M u_{jk} \dot{q}_k$$

Then

$$\frac{d}{dt} \frac{\partial}{\partial \dot{q}_i} (\mathcal{R}_{F_j}) = \sum_{k=1}^M \int_0^L \gamma \rho_f u_{jk} u_{ji} dx \dot{q}_k$$

The contact energy (3.4) gives:

$$\frac{\partial}{\partial q_i} (\mathcal{V}_{C_j}) = \sum_{k=1}^M \delta_{jk} K (u_{jk}(L) - u_{j+1,k}(L)) (u_{ji}(L) - u_{j+1,i}(L)) q_k$$

Assembling these components in equations (3.6) gives

$$A\ddot{q} + B\dot{q} + Cq + d(q) = f \ddot{y}(t) \quad (3.10)$$

Where A, B, C, D, are MxM symmetric matrices with components:

$$a_{ik} = \sum_{j=1}^N \int_0^L \rho u_{ji} u_{jk} dx$$

$$b_{ik} = \sum_{j=1}^N \int_0^L \gamma \rho_f u_{ji} u_{jk} dx$$

$$c_{ik} = \sum_{j=1}^N \int_0^L EI u''_{ji} u''_{jk} dx$$

d and f are M vectors with components:

$$d_i = \sum_{j=1}^N K \left\langle u_{jk}^{(\ell)} - u_{j+1,k}^{(\ell)} - \Delta_{j\ell} \right\rangle \left(u_{ji}^{(\ell)} - u_{j+1,i}^{(\ell)} \right) q_k$$

$$f_i = \sum_{j=1}^N \int_0^L \rho u_{ji} dx$$

and q is an M vector with components q_i .

Equation (3.10) is linear if the d vector remains linear during the motion, i.e. if no gaps open or close. In that case a modal analysis of (3.10) can be performed using available numerical procedures and the seismic response determined by response spectrum methods. This would be the normal technique for seismic analysis of structures but few FBR cores are likely to remain linear.

A second approach is termed a static seismic analysis. In this method an appropriate constant acceleration $\ddot{y}$ is chosen and equation (3.10) is

solved for an equilibrium solution, i.e., $\ddot{q} = \dot{q} = 0$. Equation (3.10) then had the form $Cq + d(q) = f\ddot{y}$ (3.11). While this solution technique involves considerably less calculational effort than a dynamic solution, it leaves serious questions as to the relation of the solution to the actual maximum displacements in a seismic event. Typically, FBR seismic analysis specifications allow static solution techniques only when it can be demonstrated that they are conservative,⁴ a difficult task for the FBR core structure.

The remaining solution technique is to determine the time history of the generalized coordinates, $q(t)$, by integration of (3.10) for a specific seismic input, $\ddot{y}(t)$, and initial conditions, usually $\ddot{q}(0) = \dot{q}(0) = 0$. Unfortunately, a single seismic input is normally not sufficient and several such calculations should be made. Moreover, the calculations will usually include large numbers of impact calculations as gaps in the core close during the seismic event. For this reason it is useful to review the formulation of the impulse-momentum equations in the assumed mode formalism. Consider an impact at some point interior to the system. The virtual impulsive work done is

$$\delta \hat{\mathcal{W}} = \hat{F} \delta w_1(x_1^*, t) - \hat{F} \delta w_2(x_2^*, t)$$

where $\hat{F}$ is the impulse, δw_1 and δw_2 are the virtual displacements of the two impacting bodies and x^* locates the point of impact on the bodies.

Then

$$\delta \hat{\mathcal{W}} = \hat{F} \sum_{k=1}^M \left[u_{1k}(x_1^*) - u_{2k}(x_2^*) \right] \delta q_k$$

and we define the generalized impulse by

$$\hat{Q}_i \equiv \hat{F} \left[u_{1i}(x_1^*) - u_{2i}(x_2^*) \right] \equiv \hat{F} U_i^* \quad (3.11)$$

where U^* is used to denote the bracketed expression. Lagrange's equations for impulses then have the form

$$\Delta \left(\frac{\partial \mathcal{T}}{\partial \dot{q}_i} \right) = \hat{Q}_i \quad (3.12)$$

where Δ denotes the change during the impulse. For the form of the equations given by (3.10), equation (3.12) becomes

$$(A + B) (\dot{q}(2) - \dot{q}(1)) = \hat{F} U^* \quad (3.13)$$

where $\dot{q}(2)$ is the generalized velocity vector after impact, $\dot{q}(1)$ is the generalized velocity vector before impact, and $\hat{Q}$ is the vector of generalized impulses.

In most impact problems, $\dot{q}(1)$ is known and $\dot{q}(2)$ and $\hat{F}$ are unknown; an auxiliary condition is needed in addition to (3.11) and (3.13). We will examine briefly some of the conditions that have been used for impact studies.

Let $R = (A + B)^{-1}$. Then the energy change of the system during the impact (relative to a fixed reference frame) can be written:

$$\Delta \mathcal{U} = \frac{1}{2} U^{*T} R U^* \hat{F}^2 + \left(U^{*T} \dot{q}(1) + U^{*T} R f \dot{y} \right) \hat{F}. \quad (3.14)$$

Note that $U^{*T} \dot{q}(1)$ is the relative velocity at the point of impact before the impulse; we will denote this expression v_1^* and consider the case $\dot{y} = 0$ first. The expression $U^{*T} R U^*$ is always positive and we will use the notation $\beta \equiv U^{*T} R U^*$.

One possible impact condition is that no energy is gained or lost during impact. Then (3.14) gives

$$\Delta \mathcal{U} = 0 \Rightarrow \hat{F} = \frac{-2v_1^*}{\beta} \quad \text{or} \quad \hat{F} = 0$$

The latter expression implies no impact.

Another condition is that some fraction of the initial energy is lost:

$$\Delta \mathcal{U} = -\alpha \mathcal{U}(t_1)$$

then (3.14) becomes a quadratic equation for $\hat{F}$.

The maximum energy that can be removed in an impact occurs when:

$$\frac{\partial}{\partial F} (\Delta \mathcal{U}) = 0 \rightarrow \hat{F} = \frac{-v_1^*}{\beta}.$$

The same conditions can also be used when $\dot{y} \neq 0$ but v_1^* must be replaced by $v_1^* + U^{*T} R \dot{y}$. The additional term accounts for the fact that the generalized coordinates are measured on a moving reference frame. Alternatively the energy can be written relative to the moving reference frame and then the $\dot{y}$ term in (3.14) does not appear. When impact occurs with the boundary of the system, the analysis follows the same form except that U^* involves only one contacting body, i.e.,

$$U_i^* = u_{1i} (x_1^*)$$

Another approach is to base the auxiliary condition on the velocity change of the point of impact. Let v_2^* denote the relative velocity after impact of the contact point. Then the condition that this velocity reverse during the impact,

$$v_2^* = -v_1^*$$

gives

$$\hat{F} = \frac{-2v_1^*}{\beta}$$

which is equivalent to no energy change. The condition $v_2^* = 0$ can be shown to be equivalent to maximum energy loss. The condition $v_2(x^*) = -\alpha v_1^*$ gives

$$\hat{F} = \frac{-(1+\alpha) v_1^*}{\beta}$$

IV. Sample Problem

To illustrate the methods discussed in Section III we have chosen a grossly simplified model of a typical demonstration size LMFBR. The reactor consists of 691 identical, uniform assemblies arranged on a triangular pitch consisting of sixteen hexagonal rows with 30 assemblies removed from the outer row, five at each corner. The assemblies are 4.5 m in length, are pinned at the bottom, and have load pads at 2.7 m from the bottom and at the top. For purposes of simplifying the analysis in this example, we shall assume that the constraint rings and support plate move together during the seismic event; i.e. a rigid core barrel attached to a rigid support plate.

The individual assemblies have the following properties:

length:	$L = 4.5 \text{ m}$
bending stiffness	$EI = 4.3 \times 10^5 \text{ N-m}^2$
ACLP height	$a = 2.7 \text{ m}$
mass density/unit length	$\rho A = 50 \text{ kg/m}$
contact stiffness at ACLP	$K_A = 2 \times 10^7 \text{ N/m}$
contact stiffness at TLP	$K_T = 2 \times 10^8 \text{ N/m}$

We will develop several mathematical models of increasing sophistication and consider various solutions to the resulting equations. It should be recognized, however, that the purpose of these examples is to illustrate modeling techniques using the assumed mode formalism; the solutions, while they may bear some qualitative resemblance to the behavior of an LMFBR, should not be construed to represent quantitative predictions for the behavior of any particular reactor.

The first model considered has a single degree of freedom. A central group of 625 assemblies is assumed to pivot in unison about their support point while 66 assemblies in the outer row are assumed to contact the inner group at the ACLP and the restraint ring at the TLP, bending as the inner

assemblies pivot, see Figure 1 . A pivot motion about the bottom can be represented by a function

$$\mu_1(x) = x/L$$

while a bending of a uniform beam has the form

$$\mu_2(x) = [b x^3 - (a^2 b + 2ab)x - L \langle x-a \rangle^3] / 2a^2 b^2$$

where $b = L - a$ and $\langle x \rangle^n$ is a singularity function. Using these functions, the displacement functions for $x \geq 0$ (3.5) may be written.

$$w_j(x,t) = u_{j1}(x) q_1(t)$$

where $u_{j1}(x) = \mu_1(x)$ for assemblies in the central group and $u_{j1}(x) \equiv \mu_2(x)$ for assemblies on the outer row. It will be necessary to evaluate the following integrals to obtain the coefficients in equation (3.10).

$$I_1 = \int_0^L \mu_1^2(x) dx = 1.5 \text{ m}$$

$$I_2 = \int_0^L \mu_2^2(x) dx = 2.331 \text{ m}$$

$$I_3 = \int_0^L (\mu_1''(x))^2 dx = 0. \quad (4.1)$$

$$I_4 = \int_0^L (\mu_2''(x))^2 dx = 0.5716 \text{ m}^{-3}$$

$$I_5 = \int_0^L \mu_1(x) dx = 2.25 \text{ m}$$

$$I_6 = \int_0^L \mu_2(x) dx = -2.906 \text{ m}$$

With this information we can calculate

$$a_{11} = \rho A (625 I_1 + 66 I_2) = 5.46 \times 10^4 \text{ kg}$$

$$b_{11} = 0$$

$$c_{11} = EI (625 I_3 + 66 I_4) = 1.62 \times 10^7 \text{ N/m}$$

$$d_1 = 0$$

$$f_1 = \rho A (625 I_5 + 66 I_6) = 6.07 \times 10^4 \text{ kg}$$

This simple model, governed by the equation

$$a_{11} \ddot{q}_1 + c_{11} \dot{q}_1 = f_1 \ddot{y} \quad (4.1)$$

is linear with a natural frequency of 2.74 Hz and a modal participation factor, f_1/a_{11} , of 1.11. A response spectrum analysis based on this linear model could be done given the response spectrum at the support plate. However, such an analysis would yield displacements of several tenths of a meter in an LMFBR for most response spectra currently being considered for operating base earthquakes, and this is inconsistent with the available gap space in the reactor.

The first modification which must be made to the model is to include "gap" elements at the TLP between the central group of assemblies and the outer row and at the ACLP between the outer row and the restraint ring. We assume that the stiffnesses of these contact elements at the ACLP are $33 K_A$ and at the TLP $33 K_T$ representing contact between 33 assemblies at each point. We shall set the available gap at 2.4 mm at the TLP and 1.2 mm at the ACLP. Now $\mu_1(L) = 1$, $\mu_2(L) = 0$, $\mu_2(a) = .6$ and the ACLP restraint ring remains fixed. With these assumptions $d_1(q)$ in equation (3.10) can be written

$$d_1(q_1) = 66 \times 10^7 \langle .6 q_1 - .0012 \rangle + 66 \times 10^7 \langle - .6 q_1 - .0012 \rangle \\ + 66 \times 10^8 \langle q_1 - .0024 \rangle + 66 \times 10^8 \langle - q_1 - .0024 \rangle$$

The new equation of motion is given by (4.1) with the above expression for $d_1(q_1)$ added to the lefthand side. It is nonlinear, more precisely piecewise linear, and a response spectrum analysis is no longer appropriate since the response is not expected to remain in the linear range of the equation.

An alternate solution technique is to consider a static solution to this equation with a constant acceleration. While correct practice allows the use of such a solution only under quite restrictive conditions, it is constructive to consider this solution technique. We will assume a static load of 1 g (9.81 m/sec^2) and let $\ddot{q}_1 = 0$. The resulting equation is solved iteratively by assuming a gap state and verifying that state in the solution.

First assume $0 < q_1 < .002\text{ m}$ so that all gaps are open. Then

$$q_1 = \frac{f_1}{c_{11}} (9.81) = 3.69 \times 10^{-2} \text{ m} > .002 \text{ m}$$

so the condition is not satisfied. Next assume $q_1 > .0024\text{ m}$ so that two of the gaps close. Then

$$q_1 = \frac{f_1 (9.81) + 66 \times 10^7 (.0012) + 66 \times 10^8 (.0024)}{c_{11} + (.6) 66 \times 10^7 + 66 \times 10^8} = .0024567 \text{ m}$$

which satisfies the assumption. With this displacement, the forces in the gap elements are

$$66 \times 10^7 (.6 \times .0024567 - .0012) = 1.81 \times 10^5 \text{ N at the ACLP}$$

and

$$66 \times 10^8 (.0024567 - .0024) = 3.74 \times 10^5 \text{ N at the TLP.}$$

These forces are interpreted as total force across the gap and individual forces on assemblies would be obtained by dividing by the number of assemblies in contact, 33 in this case.

A second approach to solving this model is to perform a time history analysis using some specified support plate acceleration. For this example

we chose an input acceleration consisting of four sine waves with frequencies of 9.55, 4.77, 2.39, and 1.19 hz, relative amplitudes of 1., .25, .23 and .38 respectively and zero phase shift at time zero. Figure 2 illustrates the form of the assumed acceleration normalized to 1 g maximum acceleration.

A FORTRAN program for integrating differential equations of the type considered in seismic analysis was written using a Newmark integration technique.⁵ Figure 3 illustrates the displacement response $q_1(t)$ for this single degree of freedom model during the first .25 sec subject to the excitation in Figure 2. Figures 4 and 5 give the corresponding forces on the gap elements.

The next approach to modeling of this reactor is to introduce more degrees of freedom. Without much additional modeling effort, a five degree of freedom model can be introduced assuming again that the center sector of 625 assemblies moves only by rotating about its base but that the outer row is divided into two sectors of 33 assemblies each which are allowed to move independently of each other and of the center sector. We assume that the modes of these outer sectors consist of a rotation about their base and a first mode bending as before.

The deformation is given by

$$w_1(x,t) = u_{11}q_1 + u_{12}q_2 + u_{13}q_3 + u_{14}q_4 + u_{15}q_5$$

$$w_2(x,t) = \sum_{j=1}^5 u_{2j} q_j$$

$$w_3(x,t) = \sum_{j=1}^5 u_{3j} q_j$$

where

$$u_{11} = \mu_1, u_{1j} = 0, j = 2, 5$$

$$u_{22} = \mu_1, u_{23} = \mu_2, u_{21} = u_{24} = u_{25} = 0$$

$$u_{34} = \mu_1, u_{35} = \mu_2, u_{31} = u_{32} = u_{33} = 0$$

In addition to the integrals given in 4.1 we will need to calculate

$$I_7 = \int \mu_1 \mu_2 = -1.48 \text{ m}$$

$$I_8 = \int \mu_1'' \mu_2'' = 0$$

With these results the mass matrix has the form:

$$M = \begin{bmatrix} 625I_1 & 0 & 0 & 0 & 0 \\ & 33I_1 & 33I_7 & 0 & 0 \\ & \text{Sym} & 33I_2 & 0 & 0 \\ & & & 33I_1 & 33I_7 \\ & & & & 33I_2 \end{bmatrix}$$

and the stiffness matrix had the form:

$$K = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 \\ & \text{Sym} & 33I_3 & 0 & 0 \\ & & & 0 & 0 \\ & & & & 33I_3 \end{bmatrix}$$

In addition to the gap elements used in the one degree of freedom model, additional gaps are used in this model between the TLP of the outer sectors and the restraint ring and between the central sector and the outer sectors at the ACLP.

The gap equations have the form:

$$\begin{aligned}
 d_1(q) = & 66 \times 10^7 \langle .6q_1 - .6q_2 - q_3 + .0005 \rangle \\
 & + 66 \times 10^7 \langle -.6q_1 + .6q_4 + q_5 + .0005 \rangle \\
 & + 66 \times 10^8 \langle q_1 - q_2 - .0024 \rangle \\
 & + 66 \times 10^8 \langle -q_1 + q_4 - .0024 \rangle
 \end{aligned}$$

with similar expressions for $d_2(q)$ through $d_5(q)$. A .5 mm interference was used at the ACLP between the central sector and the outer sector to give the reactor core some thermal bowing load.

This mathematical model of the reactor core was then integrated using the same code described above along with the same support plate motion. The results are depicted in Figures 6 - 11.

Comparison of the central sector motions of the two models, Figures 3 and 6, shows quite good agreement. Figures 7 and 8 represent the motion of the right-hand outer sector of 33 assemblies for the 5 degree of freedom model. This should be compared with Figure 3 which also represents this same motion in the one degree of freedom model. Clearly the one degree of freedom model does not adequately reflect the "rattling" of these outer assemblies.

Figure 9, representing the force between the central sector and the righthand outer sector at the TLP, compares quite favorably with Figure 4. However, there is significantly more impacting at the ACLP-restraint ring interface in the five degree of freedom model, Figure 10, than in the one degree of freedom model, Figure 5. Finally, Figure 11 represents the force between the left hand outer sector and the central sector at the ACLP. These are the largest forces given by this model.

A final observation should be made concerning the computational difficulties of the two models. Little effort was expended on optimizing the integration algorithm used to compute the time histories, but with the method used, the five degree of freedom model required 516 times as much computer time as the single degree of freedom model. This was primarily due to the necessity of reducing the integration time-step from 500 to 5 microseconds.

V. Research Requirements

There are several areas where further research is needed to provide adequate modeling capability for seismic analysis. In this section these needs are briefly reviewed and some approaches to resolving them are discussed.

A. Fluid Inertia. There has been considerable research into the effects of a fluid environment on the free and forced oscillations of simple structures in a fluid medium. It is well known, for example, that a simply supported beam oscillating in a fluid in which the boundaries of the fluid are far from the beam behaves as if the beam had attached to it an "apparent mass" proportional to the density of the fluid and the cross sectional area of the beam. When the boundaries of the fluid move close to the beam, this effect is increased; Chen has shown that for circular rods vibrating in a circular cavity filled with fluid this "added mass" increased as $r/(R-r)$ where r is the diameter of the rod and R is the diameter of the cavity.² When several circular rods are vibrating close together, there is an inertia coupling effect in which the force on one rod is affected by the acceleration of its neighbors. The result is that the system of rods has most of its natural frequencies reduced by a factor which increases rapidly as the gap between the rods decreases. This theoretical work for circular rods has also been confirmed experimentally by Moretti and Lowery.⁶

Extrapolations of these results for a gap/diameter ratio of .04 which is typical of LMFBR fuel assemblies, indicates an added mass effect that is several times as great as the mass of the fuel assemblies. Such a dominant inertia effect will have important implications concerning the seismic response of a reactor core. However, further work must be done on the validity of the results for circular rods in the hexagonal geometry and the small gap/diameter ratio environment of the reactor core.

B. Fluid Damping: A second major area of concern is the energy dissipated in the inter-assembly fluid. The mass flow of this fluid is governed by the relative motion of adjacent assemblies and thus can be associated with the generalized velocities in an assumed mode analysis. It may be anticipated that the energy dissipated in this flow is dependent on the fluid viscosity and two characteristic distances, probably the gap size between assemblies and the width of a duct flat. It is most likely possible to develop a theoretical model of the damping effect of the inter-duct fluid based on some type of flow assumptions. However, a parallel experimental effort is also needed since the amount of damping in a reactor core is likely to be critical for modes with frequencies near the natural frequencies of the reactor vessel (about 5-10 hz), and a theoretical model will depend strongly on the flow assumption which cannot be reliably predicted.

C. Friction: A second dissipation mechanism, which will govern the magnitude of seismic loads and displacements, is Coulomb friction. The lack of oxides on the assembly load pads induces a high friction coefficient (~ 0.5) which will tend to impede relative motion of assemblies and dissipate energy in the event such motion occurs. An extensive program for determining the friction characteristics of load pads in a sodium environment is being conducted; the seismic need is for methods of including this effect in the mathematical model. Such methods might include the use of an "equivalent damping coefficient" for the various modes involving sliding friction as well as the elimination of some possible modes because of friction locking.

It is also possible that the Core Restraint Test Facility at W-ARD might be used to evaluate the Coulomb-friction-induced hysteresis characteristics of a reactor core. Such information could be used as a basis for choosing coulomb damping factors in a seismic model.

D. Impact. The problem of adequately modeling impacts between adjacent assemblies and between assemblies and restraint rings in a liquid metal environment must also be addressed. The extensive computational requirements of a time history solution to nonlinear equations of even a moderate number of degrees of freedom demand a simple model of impact. Two approaches to the problem present themselves. The first involves identifying the time of impact and determining the initial conditions for the post-impact time history from impulse-momentum equations as discussed in Section III. An evaluation of what additional condition is appropriate to impose on the impact needs to be made. The problem of impacting beam-like elements has been addressed by several investigators, and, based on this earlier work, it is likely that the appropriate condition can be developed.

A second approach is to introduce a stiff spring, as in Section IV, or a spring and dashpot at the point of assumed contact and simply continue the time history solution past this nonlinear gap closure. This approach has conceptual simplicities which will manifest themselves in simpler coding of a solution algorithm, but also causes computational problems related to the small time step needed to follow the rapidly changing forces and velocities. A program should be conducted to evaluate the relative merits of these two approaches to impact modeling.

E. Assumed Modes. Finally, the results of any assumed mode analysis depend to some extent on the choice of the assumed modes and an effort is

needed to develop guidelines for choosing these modes in reactor seismic analysis. For example, friction locking may constrain the system and eliminate modes that would otherwise be important. The inertial coupling of the interassembly coolant may so lower the natural frequencies of modes with a great deal of relative assembly motion as to make them unimportant. Various symmetries in the core will reduce the number of modes required and others may have small modal participation factors which reduce their importance. Several of these ideas are apparent in the choice of modes for the very simple model considered in Section IV but it is important to verify these concepts with more detailed models including all of the appropriate physical phenomena.

VI. Conclusions

While large portions of this report are expository, the implications of this work can be summarized in three simple statements:

- A. Linear response spectrum analysis techniques are not appropriate for the seismic analysis of fast breeder reactor cores which use the current design concepts.
- B. The Rayleigh-Ritz/Assumed Mode formalism is capable of modeling the various physical phenomena necessary to understand the seismic response of fast reactor cores and presents good prospects of describing this response in a mathematical model of sufficient simplicity to allow numerical solutions.
- C. An adequate understanding of the seismic behavior of fast reactor core structures requires further research in the areas of fluid inertia effects, fluid damping effects, coulomb friction effects, impact modeling techniques, and the art of choosing modal functions.

Figure 1. Diagram of LMFBR Core Model

Figure 2. Core Support Plate Time History

Figure 3. One Degree of Freedom Displacement Response

Figure 4. One Degree of Freedom Central Sector/Righthand Sector TLP Force

Figure 5. One Degree of Freedom Righthand Sector/Restraint Ring ACLP Force

Figure 6. Five Degree of Freedom First Mode Response

Figure 7. Five Degree of Freedom Second Mode Response

Figure 8. Five Degree of Freedom Third Mode Response

Figure 9. Five Degree of Freedom Central Sector/Righthand Sector TLP Force

Figure 10. Five Degree of Freedom Righthand Sector/Restraint Ring ACLP Force

Figure 11. Five Degree of Freedom Central Sector/Lefthand Sector ACLP Force

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