MASTER

THE EFFECTIVE COUPLING IMPEDANCE
AND THE "MICROWAVE" INSTABILITY

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Introduction

Recent observations on the growth of beam density fluctuations^{1,2} have demonstrated the existence of longitudinal instabilities of individual bunches at very high frequencies. It was suggested^{1,2} that a quantitative description could be obtained by treating the different sectors as if each was a distinct coasting beam and applying the usual coasting beam theory,³⁻⁵ while substituting local values of current and energy spread for the separate segments. This ad hoc procedure can be justified if the underlying source of the bunched induced instability was a combination of low Q, high frequency resonators.⁶ The underlying agreement is that these conditions imply such a fast wakefield that communication between bunch sectors is weakened, thereby allowing them to behave independently of each other. The fast growth rate implied by the high frequency unstable oscillations allows the instability to evolve to significant proportions within a small part of a synchrotron cycle. From this it can be inferred that a single bunch could develop an unstable condition in spite of the damping character for unstable waves propagating along the top of a bunch.⁶

1. D. Boussard, CERN Report Lab II/RF/Int. 75-2 (1975).
2. S. Hansen and A. Hofmann, CERN ISR Performance Report, ISR-TH/RF-AH/SH, 25.6.1975 and A. Hofmann, ISR-TH/AH/amb, 10.12.1975 (private communication).
3. V.K. Neil and A.M. Sessler, Rev. Sci. Instrum. 32, 256 (1961).
4. A.G. Ruggiero and V.C. Vaccaro, CERN Report, ISR-TH/68-33 (1968).
5. E. Keil and W. Schnell, CERN Report, ISR/TH/RF/69-48 (1969).
6. H.G. Hereward, Proc. 1975 ISABELLE Summer Study, Brookhaven, BNL 20550, p. 555.

The dependence of the threshold impedance and growth rate on the local characteristics of a bunch was qualitatively confirmed by numerical simulation⁷ of a particle bunch interacting with high frequency resonators of low quality factor (i.e. low Q). However, when the quality factor of the resonant objects was changed, while keeping the frequency of the resonators fixed, a very strong impact on the results was observed, thus indicating that the simple picture of local bunch segments behaving independently is incomplete.

Here we present the main results of a theory⁸ for the fast bunch blow-up. The predictions of this theory are consistent with both computational and experimental observations.

The Effective Coupling Impedance

The basic postulate for the fast instability of a single bunch is that a single azimuthal beam mode, say n_0 , can excite fields at all modes, n . The limit of this coupling is determined by the bunch spectrum, i.e. the bunch length. The coupling strength between a field mode n to a beam mode n_0 is assumed to be given by the usual coupling impedance of mode n . Writing

$$\ell = n - n_0,$$

and denoting the coupling impedance of mode n by Z_ℓ , it can be shown that the effective coupling impedance for Gaussian bunches is given by

$$Z_{\text{eff}} = \sum_{\ell=-\infty}^{\infty} Z_\ell e^{-\ell^2 \theta_{\text{rms}}^2}, \quad (1)$$

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7. E. Messerschmid and M. Month, Nucl. Instrum. Methods 136, 1 (1976).
 8. The original paper is to be submitted to Nucl. Instrum. Methods.

where the exponential term represents the "weighting" of the impedances Z_ℓ due to the bunch spectrum. The bunch length is represented in units of the azimuthal angle θ_{rms} .

In a rough way the Gaussian distribution function can be approximated by a double step of the same area and unit height, resulting in a cut-off mode number

$$\ell_c = \frac{1}{2B} = \frac{\sqrt{\pi}}{2} \frac{1}{\theta_{rms}} \quad ; \quad (2)$$

B is the bunching factor defined by the ratio of bunch length to bunch separation. Thus we approximate Z_{eff} by

$$Z_{eff} = \sum_{\ell=-\ell_c}^{\ell_c} Z_\ell \quad . \quad (3)$$

In the resonance case, the effective impedance is purely resistive, since the reactive component averages to zero. Thus, writing Z_R as the peak resistive impedance (i.e. at resonance), we have for Z_ℓ

$$Z_\ell = Z_R \frac{1}{1 + [(2\ell/n_o)Q]^2} \quad , \quad (4)$$

where Q is the cavity quality factor. In this case Z_{eff} is given by

$$Z_{eff} = Z_R \left\{ 1 + 2 a^2 \sum_{\ell=1}^{\ell_c} \frac{1}{a^2 + \ell^2} \right\} \quad , \quad (5)$$

where $a = n_o/2Q$ is the damping time constant. In the limit of high Q the effective impedance becomes Z_R , and in the limit of low Q it becomes $Z_R/3$ for small bunching factors B .

In terms of the effective coupling impedance, as discussed above the threshold criterion is found to be

$$\left| \frac{Z_{eff}}{n_o} \right| \leq F \frac{E}{e} \frac{|\eta|}{I_o} \left(\frac{\Delta E}{E} \right)_h^2 \quad (6)$$

where

E is the particle energy

η is the energy slipping factor equal to $\gamma_t^{-2} - \gamma^{-2}$, where γ is the energy in proton mass units, and γ_t corresponds to the transition energy,

I_0 is the dc current,
 $(\Delta E/E)_h$ is the relative full energy spread at half maximum
 in the center of the bunch, and
 F is a form factor, depending on the nature of the energy distribution function.

The growth rate may be obtained from

$$\alpha = \frac{f_0 e I_0}{(\Delta E)_h} Z_{th} \left[1 + \left(\frac{Z_{eff}}{Z_{th}} - 1 \right)^2 \right]^{\frac{1}{2}} \sin \varphi \quad (7)$$

where

$$\varphi = \frac{1}{2} \arctan \left(\frac{Z_{eff}}{Z_{th}} - 1 \right)$$

f_0 is the revolution frequency of a central particle and Z_{th}/n_0 is given by the rhs of Eq. (6).

Numerical Example

The real part of the coupling impedance, corresponding to Z_R as introduced in Eq. (4), for a cylindrical cavity resonating at the TM_{010} mode, is found to be⁹

$$Z_R = Q G(x)$$

where $G(x)$ for relativistic particles only depends on the number x equal to the length to diameter ratio of the cavity:

$$G(x) = 128 \frac{\sin^2(1.202x)}{x}$$

If we conjecture that the coupling impedance is dominated at high frequencies by resonators, it is not improbable that the sum of many such "low" Q elements can be simulated by a single strong "very low" Q resonator. Then the total coupling impedance is the sum of the single, say m , impedances. For $x = 1$ we find

$$Z_R = 111.4 \text{ m}\Omega \quad (9)$$

9. E. Messerschmid, BNL Report, ISA 75-14 (1975); and E. Keil and B. Zotter, CERN Report, ISR-TH/70-30 and 70-33 (1970).

With Eq. (9), the effective impedance for the Gaussian bunch spectrum becomes

$$\frac{Z_{eff}}{n_0 m} = 55.7 \times \frac{1}{a} \left[1 + \sum_{l=1}^{\infty} \frac{e^{-\frac{1}{2}(lB)^2}}{1 + (l/a)^2} \right] .$$

This impedance is plotted in Fig. 1 (full lines) and compared, as function of $1/a$, with the impedance for the case of a single rectangular bunch spectrum (dashed lines).

$$\frac{Z_{eff}}{n_0 m} = 55.7 \times \frac{1}{a} \left[1 + 2 \sum_{l=1}^{\infty} \frac{1}{1 + (l/a)^2} \right] . \quad (10)$$

It is obvious that the latter becomes a valid approximation for $l_c = 1/2B > 1$. The interesting feature of this impedance is that for large l_c it does not change very much over a wide range of values of the parameter of $1/a$. For large values of $1/a$ it becomes independent of the bunching factor and for decreasing values of $1/a$ it becomes a linear function of $1/a$. A comparison of the effective impedance in the limit of small $1/a$ with the impedance for the case of an "unbunched" beam (that is $l_c = 0$) yields the bunching factor, in other words

$$\frac{Z_{eff}(l_c = 0)}{Z_{eff}(l_c = \frac{1}{2}B > 1)} = B$$

as is evident from Fig. 2.

For the numerical example used in references 7 and 8, where an ISR bunch was simulated ($l_c = 3$), Fig. 2 shows the effective impedance derived from the numerical simulation and compared with the effective impedance as obtained from Eq. (10). We have also included the results for a very short bunch ($l_c = 27$), and for a coasting beam ($l_c = 0$) having the same rms energy spread as the bunched beam. The corresponding growth rates of the simulated bunched beam instabilities are plotted in Fig. 3 as a function of the inverse damping time constant $1/a$ and compared with Eq. (7).

Discussion of the Results and Some Conclusions

We have presented here the results of a theory for the fast longitudinal blow-up of individual particle bunches. The dispersion

relation turned out to have an exceedingly simple form. It has a form like the coasting beam dispersion relation except with the usual coupling impedance for a particular frequency replaced by a quantity which we have termed the effective coupling impedance and which we have denoted by Z_{eff} .

There are in particular two properties of the bunch-environment interaction system which have allowed us to arrive at such a simple picture. First, the rapid evolution of the unstable oscillation modes means that phase motion can be neglected. That is, synchrotron motion is negligible and only direct energy loss by the bunch to the cavity plays a significant role in the dynamics. Second, we recognized that for this type of high frequency interaction, a given beam mode may couple to many field oscillation modes and that the range of such coupling was determined by the bunched character of the interacting beam. Thus we arrived at the concept of an effective coupling impedance, essentially a sum of the usual impedance function over the bunch frequency spectrum. The novel nature of this Z_{eff} is that, to completely define it, we must include not only the properties of the resonators but also the characteristics of the bunched beam.

The theory was tested using a computer simulation model. A comparison of the theory with the simulation experiments has yielded remarkably good agreement. There are slight discrepancies in the range of intermediate values of the damping time constant. For large bunching factors, it is clear that the dominant inaccuracy is due to the small number of lines in the beam spectrum (low l_c). This is confirmed by the observation that the agreement of the results is better for small bunching factors.

The general solutions of the dispersion relation for instability thresholds and growth rates appear consistent with experimental observations of the fast instability of proton bunches. Since electrons and protons are equivalent on the time scale for the fast instability, it might be of interest to test our theory against the bunch lengthening observations made in electron storage rings.

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Figure Captions

- Fig. 1. Effective coupling impedance of a single resonator, as function of the damping time constant $1/a$, for various values of l_c .
- Fig. 2. Effective impedances, obtained from theory and numerical simulation, as a function of $1/a$.
- Fig. 3. Growth rates, obtained from theory and numerical simulation, as a function of $1/a$. The parameters underlying the simulations were: $\gamma = 27.7$, $\eta = 0.011$, $f_0 = 318$ kHz, peak rf voltages 8.27 kV ($l_c = 3$), 410 kV ($l_c = 27$) and 0 (for coasting beam), rf frequency $= 30 \times f_0 = \text{constant}$, $m = 70$, $n_0 = 6300$, $I_0 = 0.18$ mA per bunch, elliptic distribution function of $(\Delta E/E)_h = 2.75 \times 10^{-4}$.