27
5-9-77
25 cp
w NTIS

MASTER

Internal Disruptions in Tokamaks

Bruce V. Waddell
Gary L. Jahns
James D. Callen
H. Richard Hicks

OAK RIDGE NATIONAL LABORATORY

OPERATED BY UNION CARBIDE CORPORATION FOR THE ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

DISTRIBUTION OF THIS REPORT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161
Price: Printed Copy \$4.00; Microfiche \$3.00

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the Energy Research and Development Administration/United States Nuclear Regulatory Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Contract No. W-7405-eng-26

Fusion Energy Division

INTERNAL DISRUPTIONS IN TOKAMAKS

Bruce V. Waddell
Gary L. Jahns
James D. Callen
H. Richard Hicks

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Date Published - May 1977

Prepared by the
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37830
operated by
UNION CARBIDE CORPORATION
for the
ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

9b

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

ABSTRACT

We hypothesize that sawtooth oscillations are the result of a cyclic process in which the plasma core is resistively heated until the safety factor drops below unity, causing the $m = 1$ tearing mode to become unstable, to grow with an accelerating growth rate, and ultimately to flatten the electron temperature and safety factor profiles. A model based on this hypothesis compares favorably with experimental data from ORMAK in explaining 1) the rate at which a sawtooth rises, 2) the radial dependence of the precursor and main sawtooth oscillation amplitudes, 3) the accelerating growth of the $m = 1$ precursor oscillations, and 4) the repetition time of the sawteeth.

Since their discovery in 1974, "sawtooth oscillations," which are sometimes referred to as "minor" or "internal disruptions," have remained one of the intriguing mysteries of tokamak discharges. Von Goeler et al.¹ observed that as a function of time, the soft x-ray emission from the plasma core exhibited a sawtooth structure (see Fig. 1). In the ORMAK device, the sawteeth are characterized by a period or repetition time of approximately 1 msec and a disruption time of about 50-100 μ sec. The growth rate of the $m = 1$ oscillations preceding the disruptions is $1-5 \text{ msec}^{-1}$; here m denotes the poloidal mode number. The importance of sawtooth oscillations regarding overall plasma confinement has not presently been clarified. We present a series of arguments and comparisons with experiment which indicate that a reasonably consistent model of the sawtooth oscillations can be developed from the concept that the slow rise of a given sawtooth is due to ohmic heating while the disruptive part is due to the nonlinear growth of the $m = 1$ tearing mode, modified by diamagnetic drifts.

An explanation of the disruptive part of the sawtooth has provided the motivation for several theoretical efforts. Von Goeler et al. observed that the growth rate of the $m = 1$ ideal MHD internal kink mode is approximately seven times too large to explain the observed growth rate of the $m = 1$ oscillations.¹ Furthermore, Rosenbluth showed that in the low β limit, the ideal internal kink instability should saturate at an amplitude much lower than the amplitude inferred from experiment.² Later, however, Kadomtsev argued heuristically that resistivity should prevent the saturation of the internal kink at low amplitudes and thus allow the magnetic field to evolve in such a way that the current density, safety factor, and

ORNL-DWG 76-17992

Fig. 1. Sketches of a sawtooth oscillation and the corresponding time evolution of the temperature and safety factor profiles.

temperature profiles flatten inside the "singular" or "mode rational" surface where the safety factor equals unity.³ Recently, it was shown numerically by four independently developed codes that Kadomtsev's model is essentially correct.⁴⁻⁷ Specifically, the results presented in Ref. 6 indicate that even in the absence of the pressure gradient term responsible for the internal kink mode, the $m = 1$ tearing mode can, in its nonlinear phase, flatten the current density so that the safety factor q becomes approximately equal to unity inside the original $q = 1$ singular surface. Furthermore, this process occurs on a time scale consistent with the time for the disruptive part of the sawtooth.

We use the results of Refs. 4-7 to construct a model for sawteeth which is consistent in many respects with experimental observations. The essential elements of the model are described in Fig. 1, where a schematic diagram of a sawtooth x-ray signal as a function of time is presented. The time evolution of the $m = 0$ components of the temperature and safety factor profiles is also sketched at selected times corresponding to the time t_1 through t_5 designated in the x-ray signal schematic. We assume that at time t_1 when a given sawtooth begins to rise, the temperature and safety factor profiles inside a radius r_0 are nearly flat; here r_0 is some radius greater than the radius r_s of the singular surface. Then during the next few hundred microseconds, the heat between r_s and r_0 begins to diffuse away during the time interval t_1 - t_2 , producing the heat pulse analyzed in Ref. 8. Simultaneously, the plasma inside r_s begins to reheat resistively as indicated at time t_2 . As the resistive heating continues during the time interval t_2 - t_4 , the increase in the temperature produces a decrease in the resistivity, which leads to an increase in the current and a

corresponding reduction in the safety factor q to a value less than unity at the origin. Specifically, the poloidal magnetic field evolves through a magnetic diffusion equation, which has a time scale longer than the heating time scale. (The deformation of the q profiles in Fig. 1 is exaggerated for clarity of presentation.) We define $\Delta q = 1 - q(0)$, where $q(0)$ is the value of the safety factor at the plasma center. As Δq increases, the shear at the singular surface increases, and hence the tearing mode growth rate increases until the magnetic island produced by the tearing mode again flattens the temperature, current density, and safety factor profiles during the time interval t_4 - t_5 . Then the process is repeated.

The assumption that the temperature profile flattens during the time interval t_4 - t_5 is based on our recent results using the code briefly described in Ref. 6 but with the time evolution of the electron temperature included. Numerical results for $q(0)$ initially 0.9 show that the temperature evolves as shown in Fig. 2, where the normalized temperature is plotted as a function of the radius (normalized to the wall radius) along a line through the x-point and the center of the $m = 1$ magnetic island. The time is normalized to the skin time. The numerical results also indicate that $q(0)$ can perhaps increase to a value slightly greater than one during the disruption, but this fact has not been included in the following analysis.

In order to verify the preceding model by comparison with experiment, we need to explain the following observations:

- 1) the temperature at which sawteeth occur during a given discharge,
- 2) the rate at which a sawtooth initially rises,

Fig. 2. Numerical code results giving the evolution of the temperature profile as a function of time during the disruption.

- 3) the radial dependence of both the amplitude of the $m = 1$ oscillations and the amplitude of the sawteeth,
- 4) the frequency and growth rate of the $m = 1$ oscillations preceding the disruptions and the rate at which the growth rate accelerates,
- 5) the repetition time or period of the sawteeth.

The temperature of the plasma core at which sawteeth occur during a given discharge is consistent with the requirement that $q(0)$ must be less than one for the $m = 1$ tearing mode to be unstable. Because

$$B_{\theta} = \mu_0 J_z r/2$$

and

$$\eta J_z = E_z = \frac{V}{2\pi R}$$

near the plasma center, requiring that $q(0) = 1$ gives

$$1 = \frac{r}{R} \frac{B_z}{B_{\theta}} = \frac{2B_z}{\mu_0 R J_z} = \frac{4\pi B_z}{\mu_0 V} \eta. \quad (1)$$

Here, B_{θ} is the poloidal magnetic field, μ_0 is the vacuum magnetic permeability, J_z is the toroidal current density, r is the radial coordinate, η is the resistivity, E_z is the toroidal electric field, V is the voltage at the limiter, R is the major radius, and B_z is the toroidal magnetic field. In relating the voltage at the limiter to the toroidal current density at the plasma center, we have assumed that the toroidal electric field is constant across the plasma. Consequently, if we employ the usual expression for the Spitzer resistivity, Eq. (1) gives an expression for the minimum temperature of the plasma core at which sawteeth occur. In Fig. 3, the temperature predicted by Eq. (1) is compared with the measured temperature

ORNL-DWG 76-17993

Fig. 3. Comparison of the theoretical and observed electron temperature at which sawteeth begin to occur for various discharges.

at which sawteeth occur for various discharges.⁹ The agreement is quite good in view of the difficulty in estimating the effective ion charge near the plasma center.

An explanation of the rate at which a sawtooth rises can be obtained through an analysis of the one-dimensional electron heat balance equation

$$\frac{3}{2} \frac{\partial}{\partial t} n T_e = \frac{1}{r} \frac{\partial}{\partial r} r \chi \frac{\partial}{\partial r} T_e + \eta J_z^2 - Q_{ei} + Q_{fe} - Q_R \equiv Q. \quad (2)$$

Here, n is the particle density, T_e is the electron temperature, χ is the electron thermal conductivity perpendicular to the magnetic field, ηJ_z^2 is the ohmic heating rate, Q_{ei} is the electron-ion energy transfer rate, Q_{fe} is the fast ion-electron heating rate due to injection, and Q_R is the impurity radiation loss rate. Near the plasma center, where the temperature is flat, the transport term should be negligible. Then since the central density $n(0)$ is approximately constant,¹⁰ the rate at which the electron temperature changes at the plasma center is given by

$$\left. \frac{\partial T_e}{\partial t} \right|_{r=0} = \frac{2}{3} \frac{1}{n(0)} \left(\eta J_z^2 - Q_{ei} + Q_{fe} - Q_R \right)_{r=0}. \quad (3)$$

The quantities ηJ_z^2 and Q_{ei} can be easily evaluated in terms of experimental observables. For example,

$$\eta J_z^2 = E_z J_z = \frac{V}{2\pi R_0} J_z;$$

J_z is determined by invoking $q(0) = 1$. In addition, the time rate of change of the experimental x-ray intensity can be converted to the time rate of change of electron temperature.¹¹ The quantity $Q_R|_{r=0}$ is neglected because the concentration of impurities at the plasma core is difficult to ascertain, although often it appears to be small. Consequently, in Fig. 4, the experimental $\left. \partial T_e / \partial t \right|_{r=0}$ is plotted as a function of the right side of Eq. (3),

ORNL-DWG 76-17994

Fig. 4. Comparison of the observed heating rate in the plasma core with the theoretical prediction given by Eq. (3).

neglecting Q_R . For the discharges with no injection heating, Q_{ei} is included. On the other hand, for the typically higher density injection heated shots, where the electron and ion temperatures are about equal, Q_{ei} is small and approximately cancels Q_{fe} so that only nJ_z^2 is plotted. Considering the approximations involved, the agreement between experiment and theory illustrated in Fig. 4 is reasonable.

With respect to the radial dependence of the amplitudes of the oscillations, the experimental results are compared in Fig. 5 with our latest numerical results. The amplitude of the $m = 1$ temperature oscillations as observed in ORMAK¹¹ is plotted in Fig. 5(a) as a function of chord radius. The singular surface is located at about 8 cm. The observed amplitude T_e of the sawtooth oscillations is plotted as a function of chord radius in Fig. 5b. In Fig. 5c, the numerically obtained harmonics of the temperature perturbation for small island widths are plotted as a function of the radial coordinate normalized to the radius r_w of the wall. The singular surface is located at $r/r_w = 0.2$. We observe that the shape of the amplitude of the $m = 1$ fundamental agrees reasonably well with the observed amplitude. In Fig. 5d, the harmonics of the temperature perturbation are plotted at a time when the temperature and current have become flat. The shape of the dominant $m = 0$ component of the temperature also agrees well with the shape of $\hat{T}_e$, including the phase reversal at the $q = 1$ singular surface. However, if diamagnetic effects were included in the numerical algorithm, the structure of the temperature perturbation near the singular surface might change somewhat. Nevertheless, both Fig. 2 and Fig. 5d explicitly indicate the probable origin of the heat pulse analyzed in Ref. 8.

Fig. 5. Comparison of the experimentally observed radial dependence of the $m = 1$ oscillation amplitude and the sawtooth amplitude with the numerical results. (a) Experimental $m = 1$ oscillation amplitude estimated from the x-ray amplitude at a time immediately preceding the internal disruption. (b) Experimental sawtooth temperature amplitude just after disruption. (c) Fourier harmonics of the temperature perturbation vs the radial coordinate for small island widths (numerical results). (d) Fourier harmonics of the temperature perturbation vs the radial coordinate at a time when the temperature has become flat (numerical results).

In order to analyze the growth rate of the $m = 1$ oscillations preceding the disruption, we must evaluate the linear growth rate of the tearing mode including diamagnetic effects, which are not incorporated in the present version of the code. The following expressions for the growth rates were derived employing the low β , large aspect ratio, cylindrical geometry approximations described in Refs. 4, 6, 12, and 13. These approximations should be valid within the plasma core for ORMAK parameters. Furthermore, the electron-ion collision frequency is much greater than the growth rates of interest, so that a fluid description is reliable. In the absence of diamagnetic drifts and the pressure gradient term responsible for the internal kink mode, the tearing mode growth rate is given by

$$\gamma_T = \alpha^{2/3} / (S^{1/3} \tau_{Hp}) \quad (4)$$

where

$$\alpha \equiv r_w \left. \frac{dq}{dr} \right|_{r_s},$$

$$S \equiv \tau_R / \tau_{Hp},$$

r is the radial coordinate, r_w is the radius of the wall, r_s is the radius of the singular surface, $\tau_R (\equiv r_w^2 / \bar{\eta})$ is the skin time, $\bar{\eta}$ is the magnitude of the resistivity, $\tau_{Hp} [\equiv m \rho^{1/2} / (k B_z)]$ is the poloidal MHD time, ρ is the mass density, k is the toroidal wave number, and B_z is the magnetic field in the toroidal direction. Also, the electron and ion diamagnetic drift frequencies are defined by

$$\omega_{*i} = \frac{1}{e B_z} \frac{m}{n r_s} \left. \frac{dP_{oi}}{dr} \right|_{r_s}$$

and

$$\omega_{*e} = \frac{1}{eB_z} \frac{m}{r_s} \left(\frac{1}{n} \frac{dP_{oe}}{dr} + 0.71 \frac{dT_e}{dr} \right) r_s,$$

where e is the magnitude of the electron charge, P_{oi} is the total unperturbed ion pressure, P_{oe} is the unperturbed electron pressure, n is the particle density, and T_{oe} is the unperturbed electron temperature. The growth rate γ of the tearing mode including diamagnetic drifts^{12,14,15} is given by

$$\gamma = G \gamma_T, \quad (5)$$

where

$$\begin{aligned} G &= Z_1 + Z_2, \\ Z_1 &= (\bar{Z} + 1/2)^{1/3}, \\ Z_2 &= -(\bar{Z} - 1/2)^{1/3}, \\ \bar{Z} &= (\omega_*/\gamma_T)^6, \end{aligned}$$

and

$$\omega_{*i} = \omega_{*e} \equiv \omega_*.$$

The electron and ion diamagnetic drift frequencies are taken to be equal for simplicity. In the limit $\gamma < \omega_{*i}/2, \omega_{*e}/2$,

$$\gamma = \frac{\gamma_T^3}{\omega_{*i}\omega_{*e}}. \quad (6)$$

The frequency corresponding to the preceding growth rates is simply the Doppler shift due to the bulk toroidal rotation of the plasma. Equations (5) and (6) were obtained in the limit $|\gamma_k \tau_{Hp}/\epsilon| < 1$ (see Ref. 12), where γ_k is the growth rate of the ideal internal kink in the absence of diamagnetic effects and ϵ is the tearing layer width. Our estimates indicate that this is the appropriate limit for ORMAK parameters.

From the preceding expressions for the growth rate, we see that it is necessary to compute the shear parameter α , which in turn requires the

calculation of the poloidal magnetic field B_θ . As implied by our model, B_θ and α are in general functions of time. We assume that the $m = 0$ components of B_θ and the temperature T_e evolve independently of the tearing mode activity over a sufficient length of time so that they can be described by a one-dimensional equation in the radial coordinate. Combining Faraday's law and Ohm's law, we obtain the equation for B_θ

$$\frac{\partial B_\theta}{\partial t} = \frac{\partial}{\partial r} \eta J_z, \quad (7)$$

where η is the resistivity and J_z is the toroidal current density. We linearize all quantities about the profiles pictured in Fig. 1 at time t_2 and restrict our attention to the region enclosed by the singular surface r_s . We denote the unperturbed quantities at time t_1 by "o" subscripts and the perturbed quantities by tildes. Then selecting the dominant contributions due to the perturbed resistivity and noting that $\tilde{\eta}^o = (\partial \eta_o / \partial T_{eo}) \tilde{T}_e^o$ and that the unperturbed electric field $\eta_o J_{zo}$ is approximately constant, we obtain

$$\frac{\partial \tilde{B}_\theta^o}{\partial t} = \frac{\partial}{\partial r} \tilde{\eta}^o J_{zo} = -\frac{3}{2} \eta_o J_{zo} \left(\frac{1}{T_{eo}} \frac{\partial \tilde{T}_e^o}{\partial r} - \frac{1}{T_{eo}} \frac{\partial T_{eo}}{\partial r} \frac{\tilde{T}_e^o}{T_{eo}} \right). \quad (8)$$

Here, the "o" superscripts indicate that only the $m = 0$ components of the perturbed quantities are being treated. Thus, in order to calculate $\tilde{B}_\theta^o$, the perturbation in the electron temperature must be calculated. We also observe that because Eq. (8) is a magnetic diffusion equation, $\tilde{B}_\theta^o / B_{\theta o} \sim (t/\tau_R)(r_w/r_s)^2 (\tilde{T}_e^o / T_{eo})$; i.e., a significant temperature perturbation does not produce a large change in B_θ and hence q . This means that the growth rate is smaller than one might expect although, as we shall show, frequently it is rapidly growing in time. Because

$$q(r,t) \sim q_0(r) (1 - \tilde{B}_\theta^0/B_{\theta 0})$$

and because $\partial \tilde{B}_\theta^0 / \partial t$ must vanish at the $q = 1$ singular surface, we can immediately obtain an equation for α , namely

$$\frac{\partial \alpha}{\partial t} = \frac{3}{2} r_w \frac{q_0}{B_{\theta 0}} \frac{\eta_0 J_{z0}}{T_{co}(r_s)} \left(\frac{\partial^2 \tilde{T}_e^0}{\partial r^2} \right)_{r_s} \quad (9)$$

For the case of interest, Eq. (2) cannot be solved for $\tilde{T}_e^0(r,t)$ in closed form. In addition, we wish to focus our attention on correlation of the growth rate with experiment rather than on the evolution of the temperature perturbation. Consequently, using experimental observations, we construct a model for $\tilde{T}_e^0(r,t)$. As previously argued, the data indicate that the temperature at the plasma center increases linearly with time. Furthermore, as illustrated by discharge 13477 in Ref. 8, the radius r_s of the $q = 1$ singular surface does not appear to change significantly with time, and the $m = 0$ component of the temperature perturbation vanishes at this surface. Therefore, Eq. (8) implies that $\left. \frac{\partial \tilde{T}_e^0}{\partial r} \right|_{r_s}$ vanishes. Thus, as a model for $\tilde{T}_e^0$, we write

$$\tilde{T}_e^0 = \tilde{T}_e^0(o,t) \left(1 + ar^2 + br^3 \right),$$

where

$$\tilde{T}_e^0(o,t) = T_{eo} t / \tau_h$$

and the heating time τ_h is given by

$$\tau_h = (3/2 n T_{eo}) / Q.$$

Then, requiring that $\tilde{T}_e^0(r_s,t) = \frac{\partial \tilde{T}_e^0(r_s,t)}{\partial r} = 0$ gives

$$\gamma_e^0(r,t) = \gamma_e^0(o,t) \left[1 - 3(r/r_s)^2 + 2(r/r_s)^3 \right].$$

Now, using Eq. (9), we immediately obtain

$$\alpha = \alpha_o + \bar{\alpha} t^2,$$

where α_o is the initial or unperturbed value of α and

$$\bar{\alpha} = 9 \frac{q_o}{B_{\theta o}} \frac{r_w}{r_s^2} \frac{\eta_o J_{zo}}{\tau_h}.$$

As we shall later show, the growth rate of the $m = 1$ oscillations is too small to be explained by γ_T , the tearing mode growth rate in the absence of diamagnetic drifts. Thus, using Eq. (6), which includes diamagnetic effects and implies that $\gamma < \gamma_T$ since $\gamma_T < \omega_{*i}, \omega_{*e}$, we see that

$$\gamma = \frac{(\bar{\alpha}_o + \bar{\alpha} t^2)^2}{\omega_{*i} \omega_{*e} \tau_{Hp}^2 \tau_R}.$$

The total electron temperature T_e and the corresponding x-ray intensity I can be written as

$$T_e = T_{eo} + \tilde{T}_e^0 + \tilde{T}_e^1$$

and

$$I = I_o + \tilde{I}^0 + \tilde{I}^1,$$

where the "o" subscript indicates the unperturbed value, the tilde indicates the perturbation, and the "o" and "1" superscripts respectively indicate the $m = 0$ and $m = 1$ components of the perturbation. In general, for a given chord

$$I = F(T_e),$$

where F is a complicated function, and thus

$$\gamma^1 = \gamma_e^1 \left. \frac{\partial F}{\partial T_e} \right|_{T_e = T_{eo}} + \gamma_e^0 .$$

Consequently, denoting the initial value by the subscript i,

$$\frac{\gamma^1}{\gamma_i} \approx \frac{\gamma_e^1}{\gamma_{ei}} ,$$

since the change in γ_e^0 is small compared to T_{eo} . Therefore,

$$\ln \tilde{A}/\tilde{A}_i = \int_0^t \gamma dt' = \frac{\alpha_o^2 t + 2\alpha_o \bar{\alpha} t^3/3 + \bar{\alpha}^2 t^5/5}{\omega_{*i} \omega_{*e} \tau_{Hp}^2 \tau_R} ,$$

where $\tilde{A}$ is the magnitude of the $m = 1$ oscillation in the x-ray intensity at time t and $\tilde{A}_i$ is the initial intensity at $t = 0$. The terms responsible for driving the internal kink mode could also possibly contribute to the growth rate in a way such that α_o in Eq. (2) would effectively increase. However, within the accuracy of our measurements, it is unnecessary to include this effect.

Then, if α_o dominates,

$$\ln \tilde{A}/\tilde{A}_i = \gamma_o t ,$$

where

$$\gamma_o = \frac{\alpha_o^2}{\omega_{*i} \omega_{*e} \tau_{Hp}^2 \tau_R} .$$

For $\gamma_T \gtrsim \omega_{*i}/2 \sim \omega_{*e}/2$, Eq. (5) should be used to evaluate γ_o . On the other hand, if the $\bar{\alpha}$ term is dominant, then

$$\ln \tilde{A}/\tilde{A}_i = Ct^5 ,$$

where

$$C = \frac{\bar{\alpha}^2}{5 f \tau_{Hp}^2 \tau_R \bar{\omega}_*^2},$$

$$f = 1.71 T_i / T_e,$$

and

$$\bar{\omega}_* = \frac{m}{e B_z r_s} \left. \frac{dT_{eo}}{dr} \right|_{r_s}.$$

Assuming that the safety factor q increases linearly from the magnetic axis to the singular surface, we estimate that

$$\alpha_o \sim r_w \left[\frac{1 - q(o, t_o)}{r_s} \right] = \frac{9}{2} \frac{\eta_o J_{zo}}{B_{\theta o}(r_s)} \frac{r_w}{r_s^2} \frac{t_o^2}{\tau_h},$$

where t_o is the repetition time. The quantity $B_{\theta o}(r_s)$ can be evaluated by observing that J_{zo} is essentially constant inside r_s so that $B_{\theta o}(r_s) = \mu_o J_{zo}/2$. In order to obtain $|\bar{\omega}_*|$, we estimate that $|\partial T_{eo}/\partial r|$ is given by $\hat{T}_e/r_s$, where the sawtooth amplitude $\hat{T}_e$ is given by $\hat{T}_e = \tilde{T}_e^o(o, t_o) = T_{eo} t_o / \tau_h$. The heating time τ_h is taken directly from experiment.

In Fig. 6a, $\tilde{A}$ for different chords is plotted on a semilogarithmic grid as a function of t for ORMAK discharge number 11141; in Fig. 6b, $\tilde{A}$ for different chords is plotted as a function of t^5 for ORMAK discharge number 13477. We conclude that for discharge 11141, α_o is dominant, while for discharge 13477, $\bar{\alpha}$ is dominant. In Fig. 7, $\tilde{A}$ for the chord passing nearest to the singular surface is plotted as a function of both t and t^5 . The time is normalized to the maximum value of t for which $\tilde{A}$ was measured, and $\tilde{A}$ has been scaled to facilitate the presentation. We observe that some

Fig. 6. (a) The amplitude of the $m = 1$ oscillations for various chords as a function of t for Shot 11141. (b) The amplitude of the $m = 1$ oscillations for various chords as a function of t^5 for Shot 13477.

Fig. 7. Normalized amplitude of the $m = 1$ oscillations (for various discharges) for the chord nearest the singular surface vs t/t_M and $(t/t_M)^5$, where t_M is the largest time for which the amplitude was measured.

discharges exhibit clearly a t or t^5 dependence while others show what can be interpreted as a transition from t to t^5 behavior.

In Table 1, the theoretical and experimental values of γ_0 and C are compared for various discharges. We conclude that when $\ln \tilde{A}$ is clearly proportional to t , the theoretical γ_0 agrees well with experiment, while for the cases where $\ln \tilde{A}$ is clearly proportional to t^5 , C agrees well with experiment. Furthermore, the growth rate γ_T for the tearing mode in the absence of diamagnetic drifts is consistently too large to explain the data, which is to be expected since the estimated diamagnetic frequency exceeds the experimental growth rate. We also note that for all the discharges which are heated by neutral beam injection, $\ln \tilde{A}$ is essentially proportional to t^5 and, as indicated in Table 1, the growth of the $m = 1$ oscillations can be described as "fast." This correlation is consistent with the experimental observation that neutral beam injection often flattens the temperature and density profiles somewhat and thus perhaps decreases the shear parameter α . On the other hand, for discharges which are only heated resistively, $\ln \tilde{A}$ is sometimes proportional to t and the growth of $m = 1$ oscillations can be described as "slow." Often, however, resistively heated shots exhibit some t^5 dependence for large t and the growth of the $m = 1$ oscillations is characterized as "moderate." It should be emphasized that the designation fast, moderate, or slow refers to the rate at which the $m = 1$ growth rate changes.

Thus, it appears that the growth of the $m = 1$ oscillations can be explained by elementary tearing mode theory. Unfortunately, however, the magnitude of the frequency cannot be predicted precisely. As argued in Ref. 12, a reasonable hypothesis is that the frequency is simply the

Table 1. Comparison of theoretical and experimental values of c and γ

Discharge	C_{theory} (msec) ⁻⁵	C_{exp} (msec) ⁻⁵	γ_o theory (msec) ⁻¹	γ_o^{exp} (msec) ⁻¹	Description of growth of $m = 1$ oscillations	Basic time dependence of $\ln A/A_i$	γ_T (msec) ⁻¹	Injection
13477	0.09	0.06-0.12	1.4	3-6	fast	t^5	11	Yes
10950 Pulse 5	23	7-25	6	3-12	slow	t (some t^5)	22	No
10950 Pulse 6	23	6-7	6	1	moderate	combi- nation	22	No
13474	0.11	0.14	2	0.6	fast	t^5	9	Yes
11411	15	2-3	9	5	moderate	t	26	No
11388	15	9-11	6	4	moderate	combi- nation	18	No

Doppler shift due to the bulk rotation of the plasma, which is not described by the equations in Ref. 12 and about which little is known, although there is evidence that the plasma does rotate toroidally.¹⁶

This situation, however, prompts us to consider another limit of the dispersion relation as discussed in Ref. 12. In the limit where $\gamma_k < 0$ and $|\gamma_k \tau_{Hp}/\epsilon| > 1$, the growth rate γ of the instability is given by

$$\gamma = 2^{1/3} \frac{\gamma_T^3}{(\gamma_k^4 \omega_{*i} \omega_{*e})^{1/3}} \quad (10)$$

and the frequency is ω_{*e} (without a Doppler shift). Although the frequency is qualitatively correct, we conclude that the limit does not apply to ORMAK for several reasons. First, although both the sign and the magnitude of γ_k in toroidal geometry are in dispute,^{17,18} it appears that even if $\gamma_k < 0$ for ORMAK, $|\gamma_k \tau_{Hp}/\epsilon| < 1$. Second, because it is shown in Refs. 17 and 18 that γ_k is proportional to α^{-1} , the growth rate as given by Eq. (10) is proportional to $\alpha^{10/3}$, which implies a possible $t^{23/3}$ dependence for $\ln \tilde{A}/\tilde{A}_1$. There is no evidence of such a rapid time dependence in ORMAK. It should be pointed out, however, that a more accurate determination of γ_k in toroidal geometry might show that γ_k is proportional to $(\alpha + \alpha_c)^{-1}$, where α_c is constant. If α_c were large enough, this would imply a t^5 time dependence for $\ln \tilde{A}/\tilde{A}_1$ and, consequently, one would have difficulty determining the appropriate limit of the dispersion relation for ORMAK.

An additional important caveat is that one can also argue that coupling between the $m = 1$ and $m = 2$ tearing type modes might also result in an accelerating growth rate. This point is presently under investigation.

Finally, we return to Eq. (6) for our expression of the growth rate and use it to calculate the repetition time of the sawteeth. An inspection of a large number of discharges indicates that for a clear majority of the cases, the growth of the $m = 1$ oscillations can be described as fast or moderate, so that we expect $\ln A$ to be proportional to t^5 at least for large t . Consequently, in order to determine an expression for the repetition time t_o , we consider the limit in which $\bar{\alpha}$ dominates α_o . Then denoting the initial (final) island width by $W_i (W_f)$, we have

$$\ln W_f/W_i = \frac{1}{2} \int_0^{t_o} \gamma(t) dt = \frac{1}{2} C t_o^5.$$

In the preceding equation, t_o is determined by observing that it is the time at which W becomes approximately equal to r_s . Therefore,

$$t_o = D\bar{t}, \quad (11)$$

where

$$D = \left(\frac{1}{19} \ln W_f/W_i \right)^{1/5}$$

and

$$\bar{t} = \left(\frac{r_s}{r_w} \right)^{6/5} \left(\frac{T_i}{T_e} \right)^{1/5} \tau_{Hp}^{2/5} \tau_h^{2/5} \omega_*^{2/5} \tau_R^{3/5}.$$

Since the quantity D is very weakly dependent on the radius of the singular surface, we take it to be constant. In Fig. 8, the experimental repetition time is plotted as a function of $\bar{t}$. The experimental slope D is 0.73 (msec)^{-1} . If we take W_f to be between r_s and $2r_s$, or about 8 cm, and W_i to be between one tenth of the tearing layer width (0.01 cm) and the tearing layer width (0.1 cm), then the theoretical slope is between 0.75 (msec)^{-1}

Fig. 8. Experimental sawtooth repetition time compared with the theoretical prediction given by Eq. (11).

and 0.81 (msec)^{-1} . The agreement with experiment is surprisingly good, considering the assumption that $\bar{\alpha}$ dominates α_0 for all shots. Since t_0 is used to estimate ω_* , Eq. (11) is an implicit formula for t_0 . If we solve explicitly for t_0 , we obtain

$$t_0 = d(\bar{n} \bar{B}_z \bar{T}_i \bar{T}_e \bar{r}_s^2 \bar{R}^2 / \bar{V}^3)^{1/3} \text{ msec},$$

where

$$d = 13(\ln W_f/W_i),$$

$\bar{n}$ is the density in units of 10^{13} cm^{-3} at the $q = 1$ singular surface, $\bar{B}_z$ is the magnetic field in tesla, $\bar{T}_i$ and $\bar{T}_e$ are the electron and ion temperatures in kiloelectron volts at the singular surface, $\bar{r}_s$ is the radius of the singular surface in meters, $\bar{R}$ is the major radius in meters, and $\bar{V}$ is the voltage in volts. The theoretical value of d is about 20-25; the experimental value is 19.

The preceding arguments and comparisons with experiment support the concept that sawtooth oscillations are the result of a cyclic process in which the plasma core is resistively heated until the safety factor drops below unity, causing the tearing mode to become unstable, to grow with an accelerating growth rate, and ultimately to flatten the electron temperature and safety factor profiles. The requirement that the safety factor near the plasma center must be less than unity in order for the tearing mode to become unstable is consistent with the electron temperature of the plasma core at which sawteeth occur. Although the frequency cannot be predicted precisely, the agreement between the theoretical and experimental growth of the $m = 1$ oscillations is reasonable, considering the accuracy to which the relevant quantities can be calculated. The rapid rate at which the growth rate often increases can be explained by the rapid rate of

change of the shear at the singular surface. Furthermore, the radial dependence of the $m = 1$ oscillations and the sawtooth oscillations agrees well with the numerical results. Finally, the theoretical prediction for the repetition time of the sawteeth agrees very well with the experimental observations.

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

ACKNOWLEDGMENTS

The development of this model was stimulated in part by a discussion between one of the authors (J. D. Callen) and Yu. N. Dnestrovskii during a U.S.-Soviet exchange visit. The inclusion of the time evolution of the electron temperature in the code described in Ref. 6 was carried out primarily by D. A. Monticello and R. B. White at the Princeton Plasma Physics Laboratory. The authors thank M. Murakami and J. Harris for providing Fig. 3. They also express their appreciation to M. N. Rosenbluth, J. L. Dunlap, and G. Bateman for several useful discussions and for constructive criticism of the original manuscript.

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

REFERENCES

1. S. von Goeler, W. Stodiek, and N. Sauthoff, Phys. Rev. Lett. 33, 1201 (1974).
2. M. N. Rosenbluth, R. Y. Dagazian, and P. H. Rutherford, Phys. Fluids 16, 1894 (1973).
3. B. B. Kadomtsev, Fizika Plazmy 1, 710 (1975).
4. A. F. Danilov, Yu. N. Dnestrovskii, D. P. Kostomarov, and A. M. Popov, Fizika Plazmy 2, 167 (1976).
5. A. Sykes and J. A. Wesson, Phys. Rev. Lett. 37, 140 (1976).
6. B. V. Waddell, M. N. Rosenbluth, D. A. Monticello, and R. B. White, Nucl. Fusion 16, 528 (1976).
7. Masafumi Azumi, Japan Atomic Energy Research Institute, Ibaraki, Japan (1976), private communication.
8. J. D. Callen and G. L. Jahns, "Experimental Measurement of Electron Heat Diffusivity in a Tokamak," ORNL/TM-5481, Oak Ridge National Laboratory, Oak Ridge, Tennessee (June, 1976); also published in Phys. Rev. Lett. 38, 491 (1977).
9. Thermonuclear Division Annual Progress Report for Period Ending December 31, 1975, ORNL-5154, Oak Ridge National Laboratory, Oak Ridge, Tennessee (June, 1976).
10. TFR Team, Paper A8 in Proc. Sixth International Conference on Plasma Physics and Controlled Nuclear Fusion Research (1976).
11. J. L. Dunlap, G. L. Jahns, and J. H. Harris, Bull. Am. Phys. Soc. II 20, 1255 (1975); J. L. Dunlap et al., to be published.

12. G. Laval, B. V. Waddell, and M. N. Rosenbluth, paper presented at the 1976 Annual Meeting on the Theoretical Aspects of Controlled Nuclear Fusion, Madison, Wisconsin, April 7-9, 1976; to be published elsewhere.
13. M. N. Rosenbluth, D. A. Monticello, H. Strauss, and R. B. White, Phys. Fluids 19, 1987 (1976); also Proc. Fifth International Conference on Controlled Fusion, Vol. 1, p. 595 (1974).
14. M. N. Bussac, D. Edery, R. Pellat, and J. L. Soule, Paper B3 in Proc. Sixth International Conference on Plasma Physics and Controlled Nuclear Fusion Research (1976).
15. Bruno Coppi, Massachusetts Institute of Technology (1976), private communication.
16. D. J. Sigmar, J. F. Clarke, R. V. Neidigh, and K. L. Vander Sluis, Phys. Rev. Lett. 33, 1376 (1974).
17. M. N. Bussac, R. Pellat, D. Edery, and J. L. Soule, Phys. Rev. Lett. 35, 1638 (1975).
18. Young-ping Pao, Phys. Fluids 19, 1796 (1976).

INTERNAL DISTRIBUTION

ORNL/TM-5840

1. J. D. Callen
2. J. F. Clarke
3. R. A. Dory
4. G. G. Kelley
5. O. B. Morgan
6. M. W. Rosenthal
- 7-31. B. V. Waddell
- 32-34. Laboratory Records Department
35. Laboratory Records, ORNL - RC
36. Y-12 Document Reference Section
- 37-38. Central Research Library
39. Fusion Energy Division Library
40. Fusion Energy Division Reports Office
41. ORNL Patent Office

EXTERNAL DISTRIBUTION

42. Plasma Physics Library, Plasma Physics Laboratory, Princeton Univ., Forrestal Campus, P.O. Box 451, Princeton, NJ 08540
43. Controlled Thermonuclear Research Library, Lawrence Livermore Laboratory, P.O. Box 808, Livermore, CA 94550
44. Q Division Library, c/o F. L. Ribe, Los Alamos Scientific Laboratory, P.O. Box 1663, Los Alamos, NM 97544
45. Controlled Thermonuclear Research Library, c/o Weston M. Stacey, Jr., Argonne National Laboratory, 9700 S. Cass Ave., Argonne, IL 60439
46. CTR Computer Center, c/o Dr. John Killeen, Lawrence Livermore Laboratory, P.O. Box 808, Livermore, CA 94550
47. Librarian, Culham Laboratory, U.K. Atomic Energy Authority, Abingdon, Oxon, OX14 3DB, United Kingdom
48. Ruth Lengye, Bibliothek, Max-Planck Institut für Plasmaphysik, 8046 Garching bei München, Federal Republic of Germany
49. Library, Centre de Recherches en Physique des Plasmas, 21 Avenue des Bains, 1007, Lausanne, Switzerland
50. A. M. Dupas, Documentation S.I.G.N., Département de la Physique du Plasma et de la Fusion Contrôlée, Association EURATOM-CEA sur la Fusion, Centre d'Etudes Nucléaires, BP 85 Centre Du TRI 38041 Grenoble Cedex (France)
51. Bibliothèque, Service du Confinement des Plasmas, C.E.A., B.P. No. 6, 92, Fontenay-aux-Roses (Seine) France
52. Library, International Centre for Theoretical Physics, Trieste, Italy
53. Library, Laboratorio Gas Ionizzati, Frascati, Italy
54. V. E. Ivanov, Physical-Technical Institute of the Ukrainian Academy of Sciences, Sukhumi, U.S.S.R.
55. L. M. Kovrizhnikh, Lebedev Institute of Physics, Academy of Sciences of the U.S.S.R., Leninsky Prospect 53, Moscow, U.S.S.R.
56. L. M. Kovrizhnikh, Lebedev Institute of Physics, Academy of Sciences of the U.S.S.R., Leninsky Prospect 53, Moscow, U.S.S.R.
57. Library, Inst. for Plasma Physics, Nagoya Univ., Nagoya, Japan 464

58. Library, FOM-Institut voor Plasma-Fysica, Rijnhuizen, Jutphaas, Netherlands
59. Plasma Physics Group, Department of Engineering Physics, Australian National University, P.O. Box 4, Canberra A.C.T. 2600, Australia
60. Thermonuclear Library, Japan Atomic Energy Research Institute, Tokai, Naka, Ibaraki, Japan
61. Dr. D. G. McAlees, Exxon Nuclear Co., Inc., Research & Technology Laser Enrichment Department, 2955 George Washington Way, Richland, WA 99352
62. CTR Reading Room, c/o Prof. D. W. Kerst, Dept. of Physics, Sterling Hall, Univ. of Wisconsin, Madison, WI 53706
63. CTR Reading Room, c/o Prof. I. B. Bernstein, Yale Univ., 200 Mason Laboratory, Dept. of Engineering & Applied Science, New Haven CT 06510
64. Center for Plasma Physics and Thermonuclear Research, c/o D. W. Ross, Physics Dept., Univ. of Texas, Austin, TX 78712
65. CTR Reading Room, c/o Prof. B. D. Fried, Physics Dept., Univ. of California, Los Angeles, CA 90024
66. CTR Reading Room, c/o Prof. David C. Montgomery, Physics & Astronomy Dept., Univ. of Iowa, Iowa City, IA 52240
67. Magneto-Fluid-Dynamics Library, c/o Dr. Harold Grad, Courant Inst. of Math. Sci., New York Univ., 251 Mercer St., New York, NY 10012
68. CTR Reading Room, c/o Prof. Allan N. Kaufman, Physics Dept., Univ. of California, Berkeley, CA 94720
69. Dr. David A. Dingee, Fusion Programs, Battelle-Northwest, Battelle Boulevard, Richland, WA 99352
70. CTR Reading Room, c/o Prof. C. S. Liu, Dept. of Physics and Astronomy, Univ. of Maryland, College Park, MD 20742
71. CTR Reading Room, c/o Prof. T. Kammash, 103 Research Admin. Bldg., N. Campus, Univ. of Michigan, Ann Arbor, MI 48105
72. CTR Reading Room, c/o Dr. Ravi N. Sudan, Phillips Hall, Cornell Univ., Ithaca, NY 14850
73. Prof. Marshall N. Rosenbluth, Institute for Advanced Study, Princeton, NJ 08540
74. CTR Reading Room, c/o Prof. R. Gross, Plasma Research Lab., Columbia Univ., New York, NY 10027
75. CTR Reading Room, c/o Prof. Roy Gould, California Inst. of Tech., M.S. 116-81, Pasadena, CA 91125
76. Dr. Nicholas A. Krall, Science Applications, Inc., P.O. Box 2354, 1200 Prospect St., La Jolla, CA 92037
77. CTR Reading Room, c/o Dr. Jay P. Boris, Plasma Physics, Naval Research Laboratory, Washington, DC 20390
78. Professor A. Simon, Dept. of Mechanical & Aerospace Sciences, University of Rochester, Rochester, NY 14627
79. CTR Library, c/o Dr. Alan F. Haught, United Technologies Research Labs, East Hartford, CT 06108
80. Dr. H. K. Forsen, Exxon Nuclear Co., Inc. 777-106th Avenue, NE, C-000777, Bellevue, WA 98009
81. Dr. George Vahala, Physics Dept., College of William & Mary, Williamsburg, VA 23185
82. Dr. Robert E. Price, Division of Magnetic Fusion Energy, G-234, Energy Research and Development Administration, Washington, D.C. 20545

83. Dr. R. C. Davidson, Division of Magnetic Fusion Energy, G-234, Energy Research and Development Administration, Washington, D.C. 20545
84. Dr. Oscar P. Manley, Division of Magnetic Fusion Energy, G-234, Energy Research and Development Administration, Washington, D.C. 20545
85. Mr. E. E. Kintner, Division of Magnetic Fusion Energy, G-234, Energy Research and Development Administration, Washington, D.C. 20545
86. Dr. L. D. Pearlstein, L-388, Lawrence Livermore Laboratory, P.O. Box 808, Livermore, CA 94550
87. Dr. J. P. Friedberg, Los Alamos Scientific Laboratory, Los Alamos, NM 87544
88. Dr. David J. Rose, Dept. of Nuclear Engineering, MIT, Cambridge, MA 02139
89. Dr. Gareth E. Guest, General Atomic Co., P.O. Box 81608, San Diego, CA 92138
90. Dr. Claude Mercier, Service du Theorie des Plasmas, Centre d'Etudes Nucléaires, Fontenay-aux-Roses (Seine) France
91. Dr. J. B. Taylor, Culham Laboratory, UKAEA, Abingdon, Oxon, OX14 3DB, United Kingdom
92. Dr. D. Pfirsch, Institute for Plasma Physics, 8046 Garching bei München, Federal Republic of Germany
93. Dr. V. D. Shafranov, I. V. Kurchatov Inst. of Atomic Energy, 46 Ulitsa Kurchatova, P.O. Box 3402, Moscow, U.S.S.R.
94. Dr. A. Rogister, Institute for Plasma Physics, KFA, Postfach 1913, D-5170, Jülich 1, Federal Republic of Germany
95. Dr. J. G. Cordey, Culham Laboratory, UKAEA, Abingdon, Oxon, OX14 3DB, United Kingdom
96. Dr. David Baldwin, L-388, Lawrence Livermore Lab., P.O. Box 808, Livermore, CA 94550
97. CTR Reading Room, c/o Prof. Bruno Coppi, Dept. of Physics, MIT, Cambridge, MA 02138
98. Dr. Harold P. Furth, Princeton Plasma Physics Lab., Princeton Univ., P.O. Box 451, Princeton, NJ 08540
99. Dr. Paul H. Rutherford, Princeton Plasma Physics Lab., Princeton Univ., P.O. Box 451, Princeton, NJ 08540
100. Dr. Herbert H. Woodson, Dept. of Electrical Engineering, Univ. of Texas, Austin, TX 78712
101. Research & Technical Support Div., Oak Ridge Operations, Energy Research and Development Administration, P.O. Box E, Oak Ridge, TN 37830
- 102-128. Technical Information Center, Energy Research and Development Administration, P. O. Box 62, Oak Ridge, TN 37830