APPLICATION OF UNCERTAINTY ANALYSIS OF IGNITION PERFORMANCE TO THE ENGINEERING TEST REACTOR

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ABSTRACT

The design of future Engineering Test Reactor (ETR) to demonstrate ignition is complicated by the uncertainties in the projected database for ignited plasmas. Application of uncertainty analysis to ETR design utilizing a figure-of-merit defined as the probability of ignition[1] is presented. Performance evaluation from the uncertainty analysis in density-temperature space can locate an optimum operating window for ignition.

I. INTRODUCTION

A primary goal for the future Engineering Test Reactor (ETR) is to demonstrate ignition of the plasma. Due to the uncertainties in the empirical scaling laws from experimental data and in the projected database for reactor-grade plasmas, an accurate assessment of the requirements of a reactor design satisfying this goal is very difficult. The uncertainty spreads in the design input data result in uncertainty distributions of the plasma performance figures-of-merit with mean values, standard deviations, and skewness depending on their functional dependence on the input data and their distributions.

We have developed systematic methodologies to evaluate the uncertainty in plasma ignition performance by both a Monte Carlo scheme and an analytic model. The concept of "probability of ignition" as a figure-of-merit for ignition studies had been introduced in previous work by the authors[1]. In this paper, we focus on the application of the uncertainty analysis to reactor ignition performance studies and possible implications for the design of near-term ETR tokamaks. The concept of uncertainty analysis and the associated methodologies developed should have applicability to near-term and future tokamak design studies.

II. FIGURE-OF-MERIT

In this section, we examine a figure-of-merit appropriate for studying the plasma performance in ignition operation. This figure-of-merit is then employed in our uncertainty analyses of plasma performance in Sections IV and V.

The relative likelihood of ignition in a plasma is conventionally measured by the parameter "ignition margin" defined by

$$M_I = \frac{P_\alpha}{\frac{W_{th}}{\tau_E} + P_{rad}} \qquad (1)$$

where P_α = alpha heating power
W_{th} = plasma thermal stored energy
τ_E = energy confinement time
P_{rad} = radiation power loss.

The plasma is in thermal equilibrium with an ignition margin of unity, when the alpha heating power balances the total plasma power loss. Usually, an ignition margin greater than unity is arbitrarily assigned in reactor design to attempt to compensate for unspecified uncertainties involved in the ignition process.

We have proposed a figure-of-merit called probability of ignition[1] to characterize the ignition performance of a tokamak design. The uncertainty analysis generates a spread of distribution of the ignition margin due to the uncertainty in the input data. The probability of ignition is defined as the fraction of outcome having an ignition margin greater than unity.

$$P(I) = \text{Probability } (M_I > 1.0) \qquad (2)$$

This figure-of-merit is a statistical result that incorporates the database uncertainties into the power balance calculations. It provides a better analytic measurement for evaluating ignition performance and the level of confidence that ignition can be achieved.

III. MODELING

A brief description of the uncertainty analysis modeling similar to Ref. 1 is outlined in this section.

A. Evaluation of Database

The major difficulty in the analysis of plasma performance is the assessment of an accurate database, especially in the reactor relevant density and temperature regimes where no experimental verification is available. There are basically two types of uncertainty involved in evaluating plasma performance, namely, the interpretation of the physical laws governing the system and the identification of the plasma conditions.

The evaluation of the uncertainty spread of the input variables as represented by their standard deviations are based on published results[2-7] or simply estimated consistent with general experimental observations. These uncertainty spreads are not conclusive and are certainly subject to future modifications. Table 1 shows a list of uncertainty variables and our estimation of their normalized standard deviations.

B. Monte Carlo Analysis

The Monte Carlo model consists of several modules: (1) the formulation of the input uncertainty variable distributions into mathematical representation in terms of the cumulative probability, (2) the random samplings of the values of the input variables weighted with their distribution probability, and (3) the tokamak power balance calculations. The random samplings and power balance calculations are repeated for sufficient iterations for a specified statistical fluctuation tolerance in the result.

Table 1. A List of Input Uncertainty Variables and Their Normalized Standard Deviations

uncertainty variable	$\langle x_i \rangle$	$\sigma_i/\langle x_i \rangle$
density profile exponent α_n [a]	0.5	0.68
temperature profile exponent α_T [a]	1.0	0.50
elongation κ	2.3	0.13
triangularity δ	0.6	0.33
fusion reactivity $\langle\sigma v\rangle$	$\langle\sigma v\rangle$ formulation	0.04
fraction fast alpha retained f_α	0.99	0.10
thermal alpha fraction C_α	0.055	0.68
effective charge Z_{eff}	1.5	0.68
H-mode enhancement[b] H_{fac}	2.0	0.34
energy confinement time τ_E	τ_E formulation[c]	0.30
Troyon coefficient[d] g	0.03	0.33
Greenwald coefficient[e] ν_G	0.75	0.40
bremsstrahlung radiation P_b	P_b formulation	0.05
synchrotron radiation P_s	P_s formulation	0.10
wall reflectivity[f] R_w	0.9	0.20

[a] profiles of the form $x = x_0 \left(1 - \left(\frac{r}{a}\right)^2\right)^\alpha$

[b] where $\tau_E = H_{fac} \times \tau_{E_{L\text{-mode}}}$

[c] Kaye-Goldston + neo-Alcator scaling

[d] where $\beta = g\ I/aB$

[e] where density limit is $n_{e,max} = \nu_G\ I/\pi a^2$

[f] for reflection of synchrotron radiation

The Monte Carlo method has the capability of modeling any form of non-symmetrical distributions of the input uncertainty variables in the random sampling. It also takes into account the correlation between the functional dependence of the plasma performance figures-of-merit on the uncertainty input variables. As a result, it generates results accurately within the statistical fluctuation. Hence, the Monte Carlo model can be used to benchmark any approximate models for the uncertainty analysis.

C. Sensitivity Analysis

A simple analytic model derived from sensitivity analysis[9] is also developed to provide a methodology for extensive usage of the uncertainty analysis in parametric system studies, which is not feasible with the Monte Carlo method.

The plasma performance figure-of-merit is treated as a function U of several random variables, that is $U = U(x_1, x_2, \ldots x_n)$. We can obtain the approximate mean and standard deviation of U as[9,10]:

$$\langle U\rangle = U(\langle x_1\rangle, \langle x_2\rangle, \ldots \langle x_r\rangle) + \frac{1}{2}\sum_{i=1}^{n}\left(\frac{\partial^2 U}{\partial x_i^2}\right)\sigma_i^2 \qquad (3)$$

$$\sigma_U^2 = \sum_{i=1}^{n}\left(\frac{\partial U}{\partial x_i}\right)^2 \sigma_i^2 + \sum_{i\neq j}\left(\frac{\partial^2 U}{\partial x_i \partial x_j}\right)^2 \sigma_i^2\,\sigma_j^2 \qquad (4)$$

where $\langle U\rangle$ and σ_U denote the mean and standard deviation of a random distribution U, respectively. The first terms of Eqs. (2) and (3) are the linear approximation while the second terms are the second-order non-linear correction.

In this simpler method, the resulting distribution of the utility function will be distributed in Gaussian form. As a result, we are able to construct a Gaussian distribution based on the mean and standard deviation calculated from Eqs. (3) and (4) to approximate the actual utility function distribution.

We expect this analytic model to work best for symmetrical input and output distributions. The accuracy of the model may decrease in handling extremely non-symmetrical uncertainty input distributions and/or the resulting highly non-symmetrical utility function distribution. The validity of this sensitivity modeling is discussed in the following section.

IV. COMPARISON OF MONTE CARLO AND SENSITIVITY MODELS

The power balance and ignition margin calculations for our studies are computed with a zero-dimensional profile-averaged tokamak power balance code MUMAK.[11] A 4 m major-radius ETR tokamak design with major parameters listed in Table 2 are used for the calculations.

Table 2. Parameter Set Used for the 4 m tokamak ETR design

major radius R_o	4.04 m
minor radius a	1.41 m
elongation κ	2.3
triangularity δ	0.6
plasma current I_p	16.4 MA
magnetic field B_o	4.98 T
electron temperature $\langle T_e\rangle$	8.9 keV
electron density $\langle n_e\rangle$	1.45×10^{20} m^{-3}
ignition margin M_I	1.30[a]

[a]Under Kaye-Goldston + neo-Alcator scaling

A Monte Carlo analysis is performed with 2000 runs of independently randomly sampled uncertainty variables according to their respective distributions. We normalize the ignition margin histogram to a probability distribution such that the area under it is unity. A sensitivity analysis with the same plasma parameters and uncertainty spreads is also performed to construct a Gaussian distribution of the ignition margin. The resulting ignition margin probability distribution from both the sensitivity and Monte Carlo models are compared in Fig. 1.

We see that the two models agree with each other fairly well within the statistical fluctuation of the Monte Carlo results. The mean value of the ignition margin from the composite distribution of the input variables is smaller than that calculated simply from the mean values of the uncertainty inputs. The probability of ignition based on our estimation of database for the uncertainties is 46% and 43% from the Monte Carlo and sensitivity models, respectively. Accordingly, an illusive comfortable ignition margin of ~1.3 actually turns out to have less than half the chance of achieving ignition.

The probability of ignition parameter takes into account the uncertainties in the database and presents a more analytic and relevant figure-of-merit for assessing the chance of fulfilling the goal of plasma ignition. The validity of the sensitivity model is checked by good agreement with the Monte Carlo model. Thus, we are able to apply the sensitivity model with confidence to our extensive uncertainty analysis of ignition performance in reactor design in Section 5.

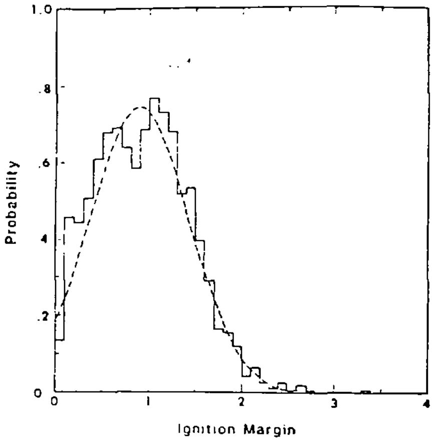

Fig. 1 Probability distribution of the ignition margin from Monte Carlo (——) and sensitivity (---) models.

V. APPLICATION IN REACTOR DESIGN IGNITION STUDIES

Here, we apply the sensitivity model to classify regions of various probability of ignition in (n,T) space and locate an optimum operating point that has the largest chance of achieving ignition.

A 5.8 meter major radius ETR tokamak design is used for our example here. The major parameters for the design are shown in Table 3. The same mean values and uncertainties database of the input uncertainty parameters of Table 2 are employed with the exception of changing f_a to 0.95 and Z_{eff} to 1.78. We also remove the density limit in the burn phase where the radiation power << total plasma heating power and the density limit is probably not applicable.

Figure 2 shows the ignition margin in (n,T) space for the machine using the mean values of the uncertainty variables. The beta limit curve is also shown for reference. The ignition margin has a maximum value of over 1.2 at about $n_e > 1 \times 10^{20}$ m^{-3} and T ~ 11-15 keV. A feasible ignition window exists below the beta limit in this range of density and temperature. This would be the desirable design window if there are no uncertainties in the input variables.

Table 3. Parameter Set Used for the 5.8 m tokamak ETR design

major radius R_0	5.80 m
minor radius a	1.96 m
elongation κ	2.2
triangularity δ	0.6
plasma current I_p	20.0 MA
magnetic field B_0	5.13 T
electron temperature $\langle T_e \rangle$	10.0 keV
electron density $\langle n_e \rangle$	1.10×10^{20} m^{-3}
ignition margin M_I	1.176[a]

[a]Under Goldston + neo-Alcator scaling

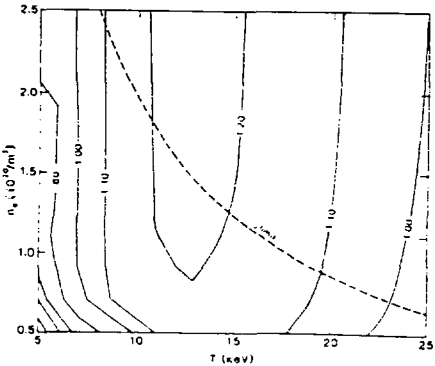

Fig. 2 Contour plot of the ignition margin in density (n_e) and temperature ($T=T_e=T_i$) space. The mean beta limit curve is also shown.

Figure 3 shows the probability of ignition in the same (n,T) space. The calculations there assume that the beta limit is a "soft limit" such that no major disruption will occur and the density and temperature are reduced to maximum allowable levels if the beta limit is exceeded. The probability of ignition tends to be larger at region of higher ignition margin. The region of highest probability of ignition of about 50% is partially coincided with the maximum region of the ignition margin below the beta limit. An ignition margin of 1.2 only has a 50% chance of achieving ignition for the set of uncertainties database used.

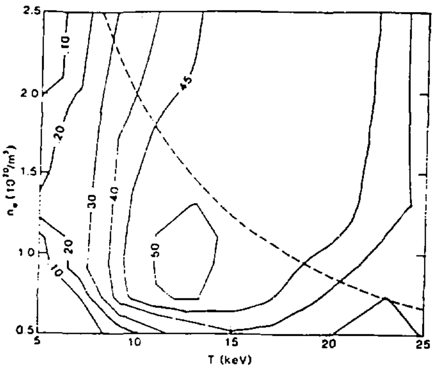

Fig. 3 The probability of ignition for a "soft beta limit".

In Fig. 4, we examine the probability of ignition by treating the beta limit as a "hard limit". In this case, the plasma will be disrupted, and hence no ignition if the beta limit is exceeded. As a result, the probability of ignition P(I) is modified to:

$$P(I) = P'(I) \times \text{Prob (below beta limit)} \quad (5)$$

where P'(I) is the probability of ignition calculations excluding the uncertainty in beta limit. A distinct maximum probability of ignition of about 0.45 is observed around $n_e = 0.7 - 1.0 \times 10^{20}\ m^{-3}$ and T - 10 - 13 keV, far away from the mean beta limit. The location of the mean beta limit and its uncertainty spread have a strong influence in the probability of ignition values. The design region for best chance of ignition does not coincide with the maximum ignition margin. The probability of ignition decreases rapidly for regions above the mean beta limit. Below the mean beta limit, the probability of ignition in the "hard limit" assumption is slightly less than that of the "soft limit" except in regimes far away from the beta limit.

Lastly, we check the variation of the probability of ignition with a reduction of the uncertainties attributed to a better understanding of the database. The calculations of the hard beta limit case are repeated by reducing the standard deviations of all the uncertainty input variables by 50%. The resulting probability of ignition is shown in Fig. 5. The qualitative behavior of the contours of the probability of ignition remains about the same. The maximum value occurs relatively closer to the beta limit. The maximum probability of ignition increases roughly by a factor of 1.7 to a fairly desirable value of 75%. This shows the need to have a better understanding of the database in order to obtain a higher probability of ignition.

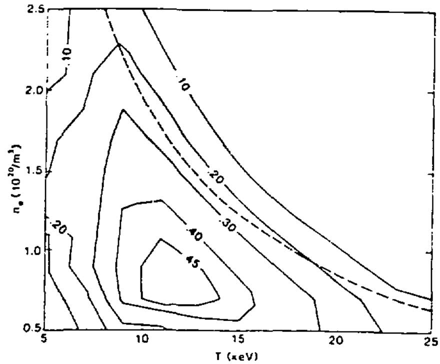

Fig. 4. The probability of ignition for a "hard beta limit".

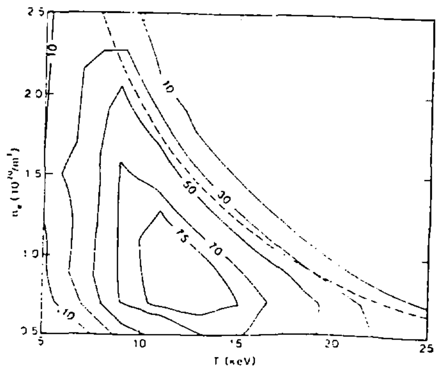

Fig. 5 The probability of ignition for a "hard beta limit" with the uncertainty standard deviations reduced by 50%.

VI. CONCLUSION

A Monte Carlo model and an approximate sensitivity model are applied to include the uncertainties in the design parameters in order to compute the probability of ignition. The probability of ignition is shown to be a better working parameter than the ignition margin for evaluating the confidence in achieving ignition. The validity of the sensitivity model allows extensive parametric system studies with the probability of ignition. Contours of probability of ignition in (n,T) space can then be located. The optimum operating point in (n,T) space for achieving ignition can then be determined. Our sample study results indicate a significant risk of not achieving ignition for conventionally applied ignition margin of 1.0 - 1.5. The calculations of the probability of ignition and the location of an optimum ignition design point can be updated, and possibly with improved higher probability of ignition, from a better assessment of physics database.

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