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Ronald H. Cohen

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TANDEM MIRROR RATE CODE AND CYCLIC
PURGING OF ALPHAS IN TANDEM REACTORS*

Ronald H. Cohen

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ABSTRACT

A set of coupled rate equations for densities and energies in a tandem mirror machine have been incorporated into a fast-running code. The code is suitable for parameter-searching and studying time-dependent processes. The code has been used to study buildup of thermalized alphas in a tandem mirror reactor, and cyclic schemes for limiting the alpha population. The principal findings are: Q_{av} is drastically reduced as alphas build up from a steady-state in which alphas were artificially eliminated; running in a pulsed mode to clean out alphas improves the time-averaged Q significantly, but not enough; elimination of 80% of the alphas by nonadiabatic loss and running in pulsed mode allows a reasonable time-averaged Q .

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This report is intended as both a status report on the tandem mirror rate code and a summary of recent results obtained with the code on alpha-particle studies.

I. INTRODUCTION

In the tandem mirror concept^{1,2}, small, hot, dense end cells provide electrostatic plugging for less hot, less dense ions in a large central cell. Electrons are electrostatically confined in the overall three-cell system.

Pastukhov³ approximately solved the Fokker-Planck equation to obtain analytic energy and density endloss expressions for electrons in a conventional mirror machine. The procedure can be applied with minor modifications to any species confined by an electrostatic potential several times its mean energy. Hence, in the tandem mirror, Pastukhov-like expressions can be written for the electrons and the central-cell ions. A complication in the case of the electrons is that, while Pastukhov assumed a square-well magnetic field, the field seen by the electrons in the tandem mirror can at best be described as three tandem square wells with two different depths.

Logan⁴ has given a steady-state model for tandem mirror machines based on the Pastukhov expressions. The plug ions are not describable by a Pastukhov expression; for the

Tandem Mirror Experiment, Logan assumes that the plug density loss rate is governed by the requirement of marginal stability of the Drift-Cyclotron Loss Cone (DCLC) mode, while the plug energy is assumed to be fixed at the mean injection energy. For tandem mirror reactors, the plug density loss rate and mean energy are given by empirical formulas derived from some Fokker-Planck runs. The non-square well nature of the electron problem is essentially ignored, but the resulting error in the computed electron confining potential is expected to be small.

Cohen et al⁵ have extended Pastukhov's calculation to apply to electrostatically confined ions as well as electrons, to plasmas with an arbitrary number of multiply charged ion species, and to non-square well fields (bounce-averaged Fokker-Planck equation). A particular result of Reference 5 is that, for a plasma with a singly charged, single ion species, the loss rate for ions in a given potential is related to the loss rate for electrons in the same magnitude (but opposite sign) potential by a factor which, for large mirror ratio, is almost, but not exactly, $1/2$. (Logan⁴ took the factor to be $1/2$.)

The code described in this report is based upon the rate equations in Reference 5. It differs from Logan's model in several ways. Most significantly, it is a time-dependent, rather than steady-state, model. Also, it uses a more refined (and hopefully more accurate) set of

equations for the electrostatically confined species, and can specifically handle multiple ions species (e.g. deuterium, tritium and alphas) in the center cell. Finally, the plug model differs in several respects from Logan's; in particular, the plug energy is described by a rate equation rather than an algebraic expression. These differences will be spelled out in the next section.

II. THE RATE CODE

The rate equations used in the code are given in Appendix A. Their content is summarized below.

A. Center Cell

The endloss rates for electrostatically confined ion species in the center cell are the generalized Pastukhov expressions given in Reference 5. To these, we add sources, energy exchange with other species, charge exchange with the sources, and loss of particles (and their mean energy) which burn to form fusion products. Sources can have multiple energy components. Energy exchange between electrostatically confined components is assumed to be given by the usual expression for Maxwellian species.

When a species is injected at an energy larger than the confining potential energy (such as is the case for alphas produced by D-T fusion), the injected particles drag down in

energy; a fraction are dragged to below the potential height and become trapped and thermalized. While they slow down, the particles transfer energy to the various species in the central cell. In the code, these effects are presently included by specifying the fraction which survive drag and the energy transfer fractions as user-supplied constants. Any source (not just a fusion product) can be injected above the potential barrier, provided that a suitable set of these constants is supplied.

Presently, the only fusion reaction considered is D-T. The D-T cross section is computed by interpolation from tabulated values for Maxwellian species. Other fusion reactions could be added, provided that tables of cross sections and reasonable sets of constants for drag survival and energy transfer fractions could be given.

B. Electrons

The electron losses are treated by using the "double square well" bounce-averaged Pastukhov-like expression given in Appendix A of Reference 5. To the endloss expressions we add a source (with strength given by the sum of the ion sources times their charge number, and a specified energy) and energy exchange between species. Supplementary electron heating (e.g. by r.f. or electron beams) is not included at present but could easily be added.

C. Plug Ions

There is presently a choice of two models to describe loss of plug ions: a collisional model and a DCLC model. The collisional model is based on the notion that neutral-beam injected, high energy particles are dragged by electrons down to an energy where ion-ion collisions become comparable with electron drag, and then quickly lost due to pitch angle scattering. The model is then that ions are lost at energy

$$E_{\text{out}} = \eta T_e^{3/2} / E_p^{1/2} \quad , \quad (1)$$

where T_e is the electron temperature and E_p is the mean plug energy, and that the particle lifetime is the time to be electron dragged from the injection energy to E_{out} . The constant η is determined by numerically matching to a sample steady state Fokker-Planck code run. The predictive value of Eq. (1) is that, once η is determined, the model gives reasonable agreement with Fokker-Planck results over a range of plug parameters and for time-dependent runs, as well. We take $\eta = 48$ (all energies in keV) in all reactor runs described in this report.) For comparison, we note that Logan's collisional model for the plugs, based on empirical fits to Fokker-Planck runs with a limited range of parameters, is

$$(n\tau)_p \approx 10^{10} E_p^{3/2} \quad ,$$

$$E_p = 3.9 (T_e E_{inj})^{1/2},$$

where E_{inj} is the injection energy (in keV).

The DCLC model, valid for TMX or other experiments where the plugs have 2XIIB-like parameters (plasma radius of order 2 to 3 gyroradii), is based on an estimate of the flux necessary to stabilize a mirror deuterium plasma with that radial scale length and density n ,

$$J_{stab} = 2.3 \times 10^7 \lambda \frac{T_e^{3/2} n_p}{E_p L_p}. \quad (2)$$

The parameter λ is taken to be 1.3 as in 2XIIB. In the code's DCLC model, if the sum of the central cell ion losses and plug losses (the latter computed as in the collisional model described in the preceding paragraph) are insufficient to supply the stabilizing flux given by Eq. (2), the plug loss rate is increased to make up the difference (a mock-up of unstable operation). Whether or not the plug rate is increased, we assume that DCLC acts to diffuse a particle to the edge of the ambipolar hole before collisions could pitch-angle scatter the particle into the loss cone at energy E_{out} ; hence, in the energy rate equation, the energy removed by an escaping particle is assumed to be the energy at the ambipolar hole edge, $q\phi/(R-1)$, where q is charge, ϕ is the ambipolar potential, and R is the plug mirror ratio. This model is used in TMX-type runs only, not in reactor runs.

D. Potentials

The ion and electron confining potentials are obtained from charge neutrality conditions. The ion potential (potential from center to middle of solenoid) is derived from the condition that the electron densities in the plugs and central cell be in the same ratio as the total ion charge densities. The electron potential (potential from center to end of system) is determined from the condition that the net endloss of charge be zero.

E. Comparison With Fokker-Planck Runs

Here, we review the comparison of the rate code and Fokker-Planck results presented at the Sherwood Theory Meeting. The input parameters for these runs are as follows: Plug length, 1 m; central cell length, 10 m; central cell volume, 1 m³; mirror ratio from center to plugs, 8.78; mirror ratio from plug center to throat, 3; plug current, 29.18 amps; central cell current, 32 amps; ion species deuterium only in both plug and center; injection energy 2.4 meV for plug ions, all others 1 keV; no charge exchange. Table 1 gives a comparison of the rate equation results with those obtained from 2-D and 1-D Fokker-Planck runs (the 1-D code solves the radial Fokker-Planck equation assuming lowest normal mode for the angular variation). The value $\eta = 48$ of the constant in Eq. (1) was chosen to make the plug densities given by the rate

equations and 2-D Fokker-Planck agree; the remaining parameters also agree satisfactorily. Comparisons for time-dependent runs are shown in Fig. 1. The steady state of table 1 is obtained; then the plug source is halved and the subsequent evolution plotted. The transient behavior and new steady-state values are all in reasonable agreement; it is particularly noteworthy that the rate equations work at least as well as the 1-D Fokker-Planck in predicting transient response. (It was noted in Reference 5 that the rate equations do better than 1-D Fokker-Planck for a single cell with specified confining potential; we now see that the rate equations perform as well as 1-D Fokker-Planck for the whole system, even given the oversimplicity of the analytic plug model.)

Fig. 2 illustrates the sensitivity of the system of rate equations to the choice of plug model. Fig. 2 was produced by assuming that the plug lifetime is a specified multiple of the electron drag time, with the specified multiple chosen to make table 1 come out the same. We see that it is easy to make bad plug models; the one normally used in the code and described in Section II C is apparently a good one.

F. Comparison With Quasilinear Code

Berk and Stewart (unpublished) have combined Pastukhov-like equations for the center cell ions and the electrons with their quasilinear code for the plug ions. We compare those results with those from the rate code with the DCLC model described in Section II C, in Table 2. Two cases are presented, one where the endloss from the center cell plus the (relatively small) collisional loss from the plugs is enough to just stabilize the plugs, and another where the center cell endloss is reduced by a factor of 5 so that the plugs have to supply much of their own stabilizing flux. The good agreement confirms the validity of the DCLC model Eq. (2).

III. ALPHA PARTICLE STUDIES

Due to their larger charge, alpha particles are much better confined by the electrostatic potential in the central cell of a tandem mirror reactor than are deuterium or tritium at the same energy. Consequently, in reactor scenarios with an appreciable fraction of burn, those fusion alphas which survive against mirror loss long enough to be dragged to an energy below the confining potential energy ($2e\phi_1$) thermalize, and accumulate to appreciable density level. This contaminates the D-T fuel, by increasing the plasma pressure for given D and T densities and by

increasing the scattering rates for D and T.

A. Options

Some options one might consider for dealing with this problem are: (1) live with it; (2) lower the burn fraction enough to reduce the steady-state alpha fraction to an acceptable level; (3) invoke a mechanism that dumps alpha particles before they become electrostatically confined; (4) run the reactor in a pulsed mode to periodically clean out the alphas; (5) arrange the radial density profile so that alphas preferentially diffuse outward.

Figure 3 demonstrates the futility of option 1. In Fig. 3, a steady state in which the alpha population has been set to zero (but the alpha energy retained for purposes of calculating Q) is obtained; beginning at $t = 0$, alphas are allowed to accumulate. Q in the final steady state is nearly an order of magnitude below the initial value. (In this report, Q is defined as the ratio of fusion power, 17.6 meV per fusion alpha, to total beam input power; Q_{av} is the ratio of the time integrals of these quantities from the beginning of the run to the present.)

Option (2) is not promising, as lowering the burn fraction itself lowers Q . Option (5) is being pursued by G. Logan, and appears to be quite promising. A means of achieving option (3) is to make the field gradients steep enough that nonadiabatic scattering causes loss of an

appreciable fraction of alphas. This mechanism acts only on high energy particles; hence 3.5 meV alphas can be scattered without affecting deuterium or tritium. (However, one loses the heating value of the lost alphas.) Nonadiabatic scattering of alphas in a solenoid with mirrors at the ends has been studied by R. Cohen (unpublished); it appears that it would be easy to design tandem mirror magnets so as to lose 80% of the alphas, but hard to lose much more. Studies with the rate code, discussed below, indicate that an 80% loss of alphas does not allow satisfactory Q in steady-state operation, but appreciable Q_{av} values can be obtained in pulsed operation. In the remainder of this report, we consider option (4), pulsed purging of alphas, both singly and in conjunction with nonadiabaticity. This is a natural problem to attack with the time-dependent code.

B. Pulsed Purging

Figure 4 shows a buildup starting from small initial densities, with alphas included from the beginning. It may be noted that both Q and Q_{av} have maxima. Shutting off the reactor by turning off all sources when the Q curve in Fig. 4 has its maximum, one obtains the reactor cycle shown in Fig. 5. Q_{av} is slightly greater (about 2.6) than the maximum in Fig. 4. This is because fusion reactions continue for a short while after the sources are turned off. This Q value is appreciably higher than the steady-state

value of 1.2, but not as large as one would like in a reactor.

While, in the presence of a confining potential, alphas are better confined than deuterium or tritium, in the absence of a potential, the reverse is true (assuming mirror confinement and only collisional loss), since the collision rate is proportional to $z^2/A^{1/2}$. One might suppose that the Q_{av} value cited in the preceding paragraph could be improved upon by a cyclic scheme in which the potential were briefly lowered to allow selective leakage of alphas. Unfortunately, in between the normal operating regime where the potential is large compared to the thermalized alpha and D, T temperatures and the regime of negligible potential lies a regime where $e\phi_i \approx T_\alpha \approx T_D$. In this regime, D and T confinement times are only marginally enhanced over collision times, but alpha confinement is appreciably better. Hence it is essential that the potential be switched between the large and small regimes in a time short compared to the central cell ion collision time. It is for this reason that simply turning off the plug source to lower the plug density and thereby drop the potential won't do. As shown in Fig. 6, most of the central cell D and T is lost while the potential is dropping.

Several schemes for rapidly dropping the plug potential have been suggested: (1) Drop the outer mirror, allowing loss of plug density in a transit time; (2) Puff a large

amount of cold deuterium into the central cell, raising the central density and therefore lowering the potential; (3) Rapidly cool the electrons; the potential goes down since $\phi_i \propto T_e$ and since lowering T_e spoils plug confinement; (4) Allow r.f. turbulence to spoil the plugs. Sample cycles are shown in Figs. 7-10. Dropping a mirror is simulated in the code by enhanced loss and $E_{out} = E_i$. r.f. turbulence is simulated by enhanced loss and $E_{out} = \text{energy at ambipolar hole}$. Cooling electrons is simulated by injecting electrons with negative temperature. These schemes still suffer from the fact that the potential is not raised sufficiently rapidly after the purge. While the potential is being raised, either the densities or the temperatures, or both, drop to values small compared to those before the potential was dropped. Consequently, the plug source must do an appreciable amount of work, essentially the same as in a buildup from scratch, to restore the fusion power to a high level. Hence the attainable Q_{av} is not much different than the one resulting from turning off the machine completely (Fig. 5). It may be noted that the cycles considered all become periodic and independent of initial conditions after sufficient repetition.

C. Nonadiabaticity Plus Pulsed Purging

In the runs plotted in Figs. 3-10, it was assumed that 30% of the fusion alphas are lost due to collisions while slowing down, and the remaining 70% become electrostatically

trapped. These values are obtained from Fokker-Planck runs (Rensink, unpublished). Fig. 11 shows a run in which only 14% are trapped, simulating the effect of an 80% loss of alphas due to nonadiabaticity. Also, in this run, only 20% of the alphas are included in calculating energy transfer to other species, since the nonadiabatically lost alphas take almost all of their energy with them. In steady-state, the density is about the same as for adiabatic alphas (the higher DT product in the source strength and the larger ϕ/T compensating for the lower trapping fraction), and Q has an unimpressive value of around 2. However, the peak Q is about 4.3. Hence pulsed operation in conjunction with nonadiabaticity appears to be a reasonable route.

IV. SUMMARY

A rate code for tandem mirror systems, based on Pastukhov-like equations for endloss of central cell ions and electrons and a simple collisional model for plug ions, has been developed and agrees satisfactorily with Fokker-Planck runs. A plug model including the Drift-Cyclotron Loss-Cone mode for 2XIIB-like plug parameters is also available; it agrees well with quasilinear code calculations.

The code is suitable for studies of startup, impurity buildup, and other time-dependent problems. Since it runs fast (about three seconds for a minute of reactor time, including plot generation; TMX runs at about real time), it is suitable for parameter searching.

The code has been used to study alpha particle buildup and pulsed mode schemes for limiting alpha buildup. For a sample set of reactor parameters for which Q in the absence of any alpha buildup would be 7, we find the steady-state Q in the presence of alphas to be 1.2. In a buildup from scratch, Q reaches a peak value of 4.5, while Q_{av} , defined from the time integrals of fusion power and source power, peaks at 2.5. Several pulsed schemes for purging alphas were considered; none achieved Q_{av} much higher than this value. No scheme based solely on dropping the ion confining potential could do much better, unless a means could be found for rapidly raising, as well as lowering, the potential. Pulsed operation combined with nonadiabatic loss of alphas resulted in a run with Q_{av} about 4.3.

The code is easy to run and easy to modify. Suggestions for additional problems are welcome.

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APPENDIX A

RATE EQUATIONS

I. Plug Ions:

$$\frac{dN_p}{dt} = -\frac{N_p}{\tau_p} + \sum_s S_s \quad (A1)$$

$$\frac{dE_p}{dt} = -\frac{1}{\tau_p} (E_{out} - E_p) - \frac{E_p - \frac{3}{2} T_e}{\tau_{dr}} + \sum_s \left(1 + \frac{\sigma_x}{\sigma_i}\right) (E_{inj} - E_p) \frac{S_s}{N_p} \quad (A2)$$

Where the sum s runs over all plug sources, and where

N_p = total number of plug ions

τ_{dr} = electron drag time

$$\tau_p = \tau_{dr} \ln \frac{E_{inj} - \frac{3}{2} T_e}{E_{out} - \frac{3}{2} T_e}$$

$E_{out} = \eta T_e^{3/2} / E_p^{1/2}$; η = const determined by matching to a steady-state Fokker-Planck run

σ_x / σ_i = ratio charge exchange/ionization cross sections

S_s = plug source (volume-integrated) ionization current

Except: E_{out} , τ_p modified as described in text for DCLC options

II. Central Cell Ions

$$\begin{aligned} \frac{dN_j}{dt} = & -\frac{N_j}{\tau_{el}(j, \phi_i, Z/2, R)} + \sum_s S_s \left\{ f_s \left[1 + \left(\frac{\sigma_x}{\sigma_i} \right)_s \right] - \left(\frac{\sigma_x}{\sigma_i} \right)_s \right\} \\ & - S_\alpha \delta_{jD,T} \end{aligned} \quad (A3)$$

$$\begin{aligned} \frac{dT_j}{dt} = & -\frac{2}{3} \frac{z_j e \phi_i}{I \tau_{el}(j, \phi_i, Z/2, R)} + \sum_k \frac{T_k - T_j}{\tau(k, j)} \\ & - \frac{S_{\alpha} \delta_{jD, T}}{N_j} (T_{\text{reac}} - T_j) + T_{hj} \\ & + \sum_s \frac{S_s}{N_j} \left\{ -T_j f_s \left[1 + \left(\frac{\sigma_x}{\sigma_i} \right)_s \right] + f_s T_{\text{trs}} + \left(\frac{\sigma_x}{\sigma_i} \right)_s T_{\text{injs}} \right\} \quad (A4) \end{aligned}$$

where the sum k runs over all species in the solenoid, the sum s runs over all sources for species J , and where

$$Z = \frac{\sum_k \lambda_{jk} n_k z_k^2}{\sum_k \lambda_{jk} n_k z_k^2 (m_j/m_k)}$$

$n_k = N_k / \text{solenoid volume}$

$R = \text{mirror ratio from center of solenoid to center of plug;}$

$\delta_{jD, T} = 1 \text{ if } j = D \text{ or } T; 0 \text{ otherwise.}$

The endloss lifetime is given by

$$\tau_{el}(j, \phi, \zeta, \rho) = \tau_j \frac{\sqrt{\pi}}{4} \frac{\ln(4\zeta\rho + 2)}{\zeta} \frac{z_j e \phi}{T_j} \frac{2\zeta\rho + 1}{2\zeta\rho} \exp\left(\frac{z_j e \phi}{T_j}\right) \frac{1}{I(T_j/z_j e \phi)} \quad (A5)$$

with

$$\tau_j = \frac{m_j^{1/2} T_j^{3/2}}{\sqrt{2} \pi e^4 z_j^2 \sum_k \lambda_{jk} n_k z_k^2}, \quad \lambda_{jk} = \text{coulomb log}$$

$$I(x) = 1 + \frac{(\pi x)^{1/2}}{2} \exp\left(\frac{1}{x}\right) \left[1 - \text{erf}(x^{-1/2}) \right] = 1 + \frac{x}{2} - \frac{x^2}{4} + \dots \quad (A6)$$

$$\tau(k,j) = \frac{3}{8\sqrt{2}\pi} \frac{m_k}{m_j^{1/2}} \frac{T_j^{3/2}}{\lambda_{jk} n_k z_j^2 z_k^2 e^4} \left(1 + \frac{T_k}{T_j} \frac{m_j}{m_k} \right)^{3/2} \quad (A7)$$

$$T_{\text{reac}} \equiv \frac{2}{3} \times \text{mean energy of D or T lost in fusion reactions.}$$

$$T_{\text{injs}} = \frac{2}{3} \times \text{injection energy}$$

$$T_{\text{trs}} = \min \left(\frac{2}{3} z_s e \phi_i, T_{\text{injs}} \right).$$

For sources injected above the potential barrier (including alphas), i.e.
for $T_{\text{injs}} > T_{\text{trs}}$, ("hot" sources)

f_s = fraction which survive to become electrostatically trapped

T_{hj} = rate of energy transfer to species j from hot sources:

$$T_{\text{hj}} \equiv \sum_s \frac{S_s}{N_j} \kappa_{sj} (T_{\text{injs}} - T_{\text{trs}})$$

where the sum runs over all hot sources.

For sources injected below potential barrier, $f_{\text{tr}} \equiv 1$, and note that $T_{\text{sj}} = 0$.

At present

$$\kappa_{sj} = \kappa_{si} N_j z_j^2 / \sum_k (N_k z_k^2)$$

for ions, and κ_{sj} , κ_{se} , f_{trs} , $(\sigma_x/\sigma)_s$ are specified as input constants.

III. Alphas

Same as above, except $S_\alpha = \langle \sigma v \rangle n_D n_T V_C$ where V_C is central cell volume, n_D , n_T are densities, $n_D = N_D/V_C$, and $\langle \sigma v \rangle =$ D-T cross-section averaged over Maxwellians.

IV. Electrons

Electron endloss is described by bounce-averaged expressions as described in Appendix B of Ref. 2. We depart slightly from Ref. 2 by defining the bounce-average quantities relative to the plugs rather than the solenoid and hence, in Eqs. (B5) and (B6) of Ref. 5, take 1 and 2 to be the plugs and central cell, respectively. Hence we have

$$\begin{aligned}
 \frac{dN_e}{dt} &= - \frac{N_{ep}}{\langle \tau \rangle} + S_e \\
 \frac{d}{dt} \left(T_e + \frac{2}{3} \langle \phi \rangle \right) &= - \frac{2}{3} \frac{N_{ep}}{N_e} \frac{1}{\langle \tau \rangle} \left[\frac{\phi_e + \phi_i}{I \left(\frac{T_e}{\phi_i + \phi_e} \right)} - \langle \phi \rangle \right] \\
 &\quad - \sum_i \frac{T_e - T_i}{\tau_{(e,i)}} \frac{N_i}{N_e} \\
 &\quad + \frac{S_e (T_{inj} - T_e - \frac{2}{3} \langle \phi \rangle) + \frac{2}{3} S_{ec} \phi_i}{N_e} \\
 &\quad - \frac{J_{str}}{N_e} \left[T_e + n (\phi_e + \phi_i) \right] + E_{he}
 \end{aligned} \tag{A8}$$

where the sum on i runs over all ion species in the solenoid and plugs,
 $N_{ep} = N_p Z_p$ is the number of plug electrons, S_{ec} is the rate of injection
of electrons in the solenoid ($= 1/e \times$ sum of solenoid ion injection
currents), $T_i = 2 E_i/3$ for plug species, $\tau(e,i)$ is given by Eq. (A7),
 $\langle \tau \rangle$ is given by Eq. (A5), i.e. $\langle \tau \rangle = \tau_{el}(e, \phi_i + \phi_e, \langle Z \rangle, R_p)$, with
 $\tau_e = m_e^2 v_e^3 / (4\pi e^4 n_{ep} \lambda_{ee} \langle \epsilon \rangle)$; R_p is the mirror ratio from the center
of the plug to the throat, T_{inje} is the electron injection temperature,
 J_{str} is the externally supplied stream, which cools the electrons
 $T_e + \eta(\phi_e + \phi_i)$ per injected stream particle, with η a specified constant,
 E_{he} is the rate of energy transfer to electrons from hot (injected above
potential barrier) species,

$$\langle s \rangle = 1 + \frac{L_c}{2L_p} \frac{\lambda_{eec}}{\lambda_{eep}} \frac{n_c}{n_p} \frac{\phi_i + \phi_e}{\phi_e}$$

$$\langle \phi \rangle = \begin{cases} \frac{\phi_i}{1 + \frac{N_{ep}}{N_{ec}}} & \phi_e > 0 \\ 0 & \phi_e \leq 0 \end{cases}$$

and

$$\langle Z \rangle = \frac{\zeta_p + \frac{N_{ec}}{N_{ep}} \frac{\phi_i + \phi_e}{\phi_e} \frac{\lambda_{eec}}{\lambda_{eep}}}{1 + \frac{n_c}{n_{ep}} \frac{\phi_i + \phi_e}{\phi_e} \frac{L_c}{2L_p} \frac{\lambda_{eec}}{\lambda_{eep}}} \quad (A9)$$

with

$$\zeta_c = \frac{1}{2} \left[1 + \frac{1}{n_{ec}} \sum_{\substack{\text{solenoid} \\ \text{ions}}} n_j z_j^2 \frac{\lambda_{ej}}{\lambda_{eec}} \right]$$

and similarly for ζ_p . In evaluating Eq. (A9), we take $n_{ec}/n_{ep} = \exp(-\phi_i/T_e)$ $[1 - \exp(-\phi_e/T_e)]$ in accordance with Ref. 2, in order to ensure a suitable limit as $\phi_e \rightarrow 0$ (required for "empty solenoid" problems). In the code, J_{str} may either be set at a specified level, or adjusted to the minimum value required for DCLC stability up to a maximum specified level.

V. Empty Solenoid Limit

In order to handle problems where an empty solenoid and non-empty plugs are expected, the bounce-averaged electron expressions should pass smoothly to the expressions for an isolated mirror as solenoid density is dropped. Strictly speaking, the bounce-average theory of Ref. 6 does not contain this limit, since, when there are just a few electrons in the plug, they must be part of the nonmaxwellian tail of the Pastukhov distribution function. In the code, we take $\phi_e < 0$ as the criterion for an empty solenoid and ignore the solenoid contributions to $\langle Z \rangle$, $\langle \epsilon \rangle$, and $\langle \phi \rangle$. The resulting energy and number endloss expressions are nearly continuous at $\phi_e = 0$, having a discontinuity of order $\exp(-\phi_i/T_e)$, which is typically very small when $\phi_e = 0$.

VI. Potentials

$$\phi_i = T_e \ln \frac{n_p z_p}{\sum_j n_j z_j}$$

$$\phi_e \text{ from: } \sum_j z_j \frac{N_j}{\tau_{cj}} + z_p \frac{N_p}{\tau_p} = \frac{N_{ep}}{\langle \tau \rangle}$$

where $\sum_j$ runs over central cell ion species.

VII. Low ϕ_i/T

An option available in the code and used in all runs where $\phi_i/T < 1$ is expected is to replace the Pastukhov expression for I , Eq. (A6), by the following (so that τ_{el} remains finite as $\phi_i/T \rightarrow 0$):

$$I(x) = \frac{1 + x/2}{1 + x^2/4} \quad (A10)$$

For electrons, we simply set an upper limit to I_e of $2(\phi_i + \phi_e)/T_e$.

TABLE I

Comparison of Steady-State Results

	1-D	2-D	Rate Eqs.
n_p	$7.69 \times 10^{14} \text{ cm}^{-3}$	8.17×10^{14}	8.17×10^{14}
$\langle E_p \rangle$	1795 keV	1864	1558
n_c	$1.91 \times 10^{14} \text{ cm}^{-3}$	1.90×10^{14}	1.78×10^{14}
T_c	39.9 keV	37.0	36.2
T_e	84.3 keV	90.2	85.0
ϕ_i	117 keV	131	130
ϕ_e	491 keV	518	476

TABLE 2

Comparison of Rate Code using Eq. 2 with Quasilinear Code.

$$L_p = 0.4 \text{ m}, \quad L_c = 5.5 \text{ m}, \quad \pi r_c^2 = .302 \text{ m}^2,$$

$$R = 20, \quad R_T = 40, \quad S_p = 8 \text{ amps/plug}, \quad \frac{\sigma_x}{\sigma_i} = 2, \text{ plug; } 0, \text{ center}$$

	$S_c = 144 \text{ amps}^*$		$S_c = 28.8 \text{ amps}^{**}$	
	Quasilinear	Rate Eq.	Quasilinear	Rate Eq.
n_p	5.34×10^{13}	5.42×10^{13}	1.95×10^{13}	2.17×10^{13}
n_c	1.37×10^{13}	1.30×10^{13}	5.52×10^{12}	5.36×10^{12}
E_p	28.1 keV	27.3 keV	33.5 keV	35.7 keV
T_c	107 ev	94 ev	89.5 ev	86.2 ev
T_e	248 ev	231 ev	215 ev	218 ev
ϕ_i	335 ev	331 ev	271 ev	304 ev
ϕ_e	1.48 keV	1.38 keV	1.24 keV	1.23 keV

* Marginally stable without extra plug loss

** Much extra plug loss required

Fig. 1. Comparison of rate code with Fokker-Planck codes. Steady-state of Table 1 is obtained; at $t=0$, plug source is halved.

Fig. 2. Rate code, old plug model. Same conditions as Fig. 1.

Fig. 3. Degradation of Q with alpha buildup, starting from steady-state in which

alphas were removed. Parameters: $L_p = 1$ m, $L_c = 150$ m, $B_p/B_c = 8.8$,

$B_{\text{throat}}/B_c = 24$; plug source current, central cell D and T source

currents all 40 amps; $(W_{\text{inj}})_p = 2400$ keV; all other species injected

at 1 keV.

Fig. 4. Buildup from scratch with alphas. Parameters for Figs. 4-10 same as for Fig. 3, except $(W_{inj})_p = 1800$ keV.

Fig. 5. Buildup and turn off all sources at maximum Q_{av} .

→ Plug source
off

Fig. 6. Turn off plug source at $t = 40$ sec.

Drop R, plug R restored, Source on
 source off source off

Fig. 7. Cycle of dropping mirror ratio.

Fig. 8. Cycle of puffing in cold D^+

Fig. 9. Cycle of cooled electrons. Electrons "injected" at $\sim 10^5$ keV, cooling phase; $\sim 2 \times 10^4$ keV, sustaining phase; 1 keV, normal phase.

Fig. 10. Cycle of "r.f." plug dumping.

Fig. 11. Buildup with 80% of alphas removed at high energy.

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University of California | Livermore, California | 94550