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SOME EFFECTS OF MAGNETIC WALLS ON THE TRANSVERSE MOTION
OF AN INTENSE BEAM OF CHARGED PARTICLES*

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I. INTRODUCTION

Following a suggestion of L. J. Laslett,¹ we examine the focusing properties of a magnetic medium surrounding a beam of relativistic particles. The magnitude of the magnetic forces generated by the wall is proportional to the beam current. For a current of the order of 10 kA, significant magnetic field gradients are obtained.

In particular we treat a pipe of circular cross section and inner radius b . The wall material has sufficiently high magnetic permeability that the tangential component of the magnetic field $\vec{B}$ may be taken as zero on the wall surface ($r = b$). The wall is interrupted with slots either above and below the beam, or on either side. We will show that the magnetic field generated by this structure has desirable focal properties for incoherent motion of particles within a beam centered on the axis of the pipe. However, the presence of the structure leads to unstable coherent transverse motion of the beam centroid.

With the slots cut above and below the beam the quadrupole component of the magnetic field generated by the wall results in focusing in the vertical (y) direction and defocusing in the horizontal (x) direction. The opposite is true for slots located on either side of the beam. Thus by alternating

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structures with slots above and below and structures with slots on either side, we can construct a passive system of alternating gradient elements. For incoherent motion, the magnitudes of the focusing and defocusing gradients are equal, and such a series of elements can lead to stable motion. But for coherent motion, the magnitude of the effective defocusing gradient is three times that of the effective focusing gradient, and such a series of elements leads to unstable motion.

In Sections II and III we perform a magneto-static calculation to determine the magnetic forces acting on the particles. Considering these forces alone, equations for both coherent and incoherent motion are derived in Section IV. In Section V we consider the electric forces arising from the beam's charge, and the equations of motion are modified to include these forces.

The results are insensitive to the radial distribution of charge and current within the beam. For convenience we take this distribution to be uniform for $r < a$ and zero for $r > a$.

II. MAGNETIC FIELD FOR CENTERED BEAM

We first obtain an expression for the magnetic field inside the pipe when the beam is centered on the pipe axis. We identify this magnetic field with a superscript o . A subscript s is used to identify the field from the beam in the absence of walls, and a subscript w is used to identify the field generated by the walls. The total field is the sum of these two and carries no subscript.

We employ cylindrical coordinates and c. g. s. electromagnetic units. The slots are of width $2b\delta$ and are centered at $\theta = \pi/2$ and $\theta = 3\pi/2$. In order to satisfy the condition

$$\int \vec{B} \cdot d\vec{T} = 4\pi I, \quad (1)$$

in which I is the total beam current, we must postulate B_θ in the gaps at $r = b$. Elsewhere on the surface $r = b$ we have $B_\theta = 0$. Thus we have

$$B_\theta^0(r = b, \theta) = \begin{cases} \pi I/b\delta, & \text{in the slots,} \\ 0, & \text{elsewhere.} \end{cases} \quad (2)$$

We expand this function in a Fourier series, obtaining

$$B_\theta^0(b, \theta) = \frac{2I}{b} \left\{ 1 + 2 \sum_{n=1}^{\infty} (-1)^n \frac{\sin 2n\delta}{2n\delta} \cos 2n\theta \right\}. \quad (3)$$

This expression gives the total azimuthal component of the magnetic field at $r = b$. The contribution from the self-field of the beam is $2I/b$. The contribution from the magnetic wall is therefore given by

$$B_{w\theta}^0(b, \theta) = \frac{4I}{b} \sum_{n=1}^{\infty} (-1)^n \frac{\sin 2n\delta}{2n\delta} \cos 2n\theta. \quad (4)$$

Inside the pipe the vector potential $A_w^0(r, \theta)$ has only an axial (z) component of the form

$$A_{wz}^0(r, \theta) = \sum_{n=1}^{\infty} A_n r^{2n} \cos 2n\theta. \quad (5)$$

We obtain $B_{w\theta}^0(r, \theta)$ from Eq. (5), evaluate it at $r = b$, and equate the resulting expression to that given by Eq. (4). This procedure yields expressions for A_n in Eq. (5). With these expressions inserted in Eq. (5), we obtain

$$A_{wz}^0(r, \theta) = -2I \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\frac{r}{b}\right)^{2n} \frac{\sin 2n\delta}{2n\delta} \cos 2n\theta. \quad (6)$$

From the $n = 1$ term in this expression we obtain the quadrupole magnetic field generated by the wall, namely

$$B_{\text{quad}}^0 = -4I \frac{\sin 2\delta}{2\delta} \frac{r}{b^2} (\sin 2\theta \hat{r} + \cos 2\theta \hat{\theta}). \quad (7)$$

In rectangular coordinates $x = r \cos \theta$, $y = r \sin \theta$, we have

$$B_{x\text{quad}}^0 = -4Iy/b^2, \quad (8a)$$

$$B_{y\text{quad}}^0 = -4Ix/b^2, \quad (8b)$$

in which we have used the approximation $\sin 2\delta = 2\delta$.

Note that, as stated above, the field components given by Eqs. (8) provide a focusing force in the y direction (plane containing the slots) and a defocusing force in the x direction. The current in these equations is in abamps (1 abamp = 10 A). For a current of 10^4 A, $b = 7$ cm, we obtain a gradient $\partial B_y/\partial x = \partial B_x/\partial y$ of 80 gauss/cm.

The $n = 2$ term in the summation in Eq. (6) represents the octopole component of B^0 . This component provides restoring (focusing) forces in the y direction proportional to y^2 , and in the x direction proportional to x^2 .

III. MAGNETIC FIELD FOR A DISPLACED BEAM

If the axis of the beam is displaced from the axis of the pipe by a distance ξ in the y direction, additional field components are generated both by the beam and by the wall. The source of these additional field components may be taken as a distribution of surface current located at $r = a$ of the form

$$K(\theta) = \frac{I\xi}{\pi a} \sin \theta, \quad (9)$$

The surface current in Eq. (9) represents the difference between the current distribution of the displaced beam and that of the centered beam. The field arising from this surface current is identified with a superscript 1. Subscripts have the same meaning as in the previous section.

The field arising from the surface current in the absence of any walls is given by

$$\bar{B}_s^1(r, \theta) = \frac{2I\xi}{a^2} \begin{cases} (\cos \theta \hat{r} - \sin \theta \hat{\theta}) \equiv \hat{x}, & r < a, \\ (a/r)^2 (\cos \theta \hat{r} + \sin \theta \hat{\theta}), & r > a. \end{cases} \quad (10)$$

In this calculation the field generated by the wall may be found by setting the total $B_\theta^1 = 0$ everywhere on the surface $r = b$. The presence of the slots is unimportant since there is no condition equivalent to Eq. (1) to be satisfied. Proceeding as above for the centered beam, we find the magnetic field $\bar{B}_w^1$ to be given by

$$\begin{aligned} \bar{B}_w^1(r, \theta) &= \frac{2I\xi}{b^2} (\cos \theta \hat{r} - \sin \theta \hat{\theta}), \\ &= \frac{2I\xi}{b^2} \hat{x}. \end{aligned} \quad (11)$$

We further note that for a beam displaced a distance η in the x direction, we obtain expressions for $\bar{B}_s^1$ and $\bar{B}_w^1$ by a rotation of coordinates. We have

$$\bar{B}_s^1 = -\frac{2I\eta}{a^2} \hat{y}, \quad r < a, \quad (12)$$

$$\bar{B}_w^1 = -\frac{2I\eta}{b^2} \hat{y}. \quad (13)$$

IV. EQUATIONS OF MOTION WITH MAGNETIC FORCES

To obtain the equations of motion for a particle in the displaced beam we take its position $\bar{R}(t)$ to be

$$\bar{R}(t) = \bar{r}_0(t) + \xi(t) \hat{y} + \eta(t) \hat{x}, \quad (14)$$

in which $\bar{r}_0$ is the position of the particle relative to the beam axis. For motion in the y direction we have

$$\frac{d^2 y_0}{dt^2} + \frac{d^2 \xi}{dt^2} = \frac{e\beta}{\gamma m} \left[B_x^0(y_0) + \xi \left. \frac{\partial B_x^0}{\partial y} \right|_{y_0} + B_x^1 \right], \quad (15)$$

in which β is the particles' speed (in the z direction) in units of the speed of light, e is the particles' charge, $\gamma^2 = (1 - \beta^2)^{-1}$, and m is the particles' rest mass. Recall that $B_x^0 = B_{xs}^0 + B_{xw}^0$ and $B_x^1 = B_{xs}^1 + B_{xw}^1$. Inside the beam ($r < a$) we have $B_{xs}^0 = -2Iy/a^2$. In this linear calculation we retain only the quadrupole component of B_{xw}^0 given by Eq. (8a). Equations (10) and (11) give respectively B_{xs}^1 and B_{xw}^1 . When these equations are substituted into Eq. (15) we obtain

$$\frac{d^2 y_0}{dt^2} = - \frac{2eI\beta y_0}{\gamma m} \left[\frac{1}{a^2} + \frac{2}{b^2} \right], \quad (16a)$$

$$\frac{d^2 \xi}{dt^2} = - \frac{2eI\beta}{\gamma m b^2} \xi. \quad (16b)$$

For motion in the plane containing the slots, the effective focusing gradient from $\bar{B}_w$ is twice as large for incoherent motion as for coherent motion.

For motion in the x direction we have

$$\frac{d^2 x_0}{dt^2} + \frac{d^2 \eta}{dt^2} = - \frac{e\beta}{\gamma m} \left[B_y^0(x_0) + \eta \left. \frac{\partial B_y^0}{\partial x} \right|_{x_0} + B_y^1 \right]. \quad (17)$$

Inside the beam $B_{ys}^0 = 2Ix/a^2$, B_{yw}^0 is given by Eq. (8b), and B_{ys}^1 and B_{yw}^1 are given respectively by Eqs. (12) and (13). When these expressions are substituted into Eq. (17) we obtain

$$\frac{d^2 x_0}{dt^2} = - \frac{2eI\beta x_0}{\gamma m} \left[\frac{1}{a^2} - \frac{2}{b^2} \right] , \quad (18a)$$

$$\frac{d^2 \eta}{dt^2} = \frac{6eI\beta}{\gamma m b^2} \eta . \quad (18b)$$

From Eqs. (16a) and (18a) describing incoherent motion we see that the defocusing gradient from $\bar{B}_w$ in the x direction has the same magnitude as the focusing gradient in the y direction. But from Eqs. (18a) and (18b) describing coherent motion, we see that the effective defocusing gradient from $\bar{B}_w$ in the x direction is a factor of three greater than the effective focusing gradient in the y direction.

V. ELECTRIC FORCES

For completeness we now consider the forces on the beam that arise from the beam's electric charge. When the beam is on axis the electric field inside the beam is $\bar{E}^0 = 2\lambda r/a^2$, where λ is the charge per unit length. In these units $\lambda = I/\beta$. Thus we have

$$E_x^0 = 2Ix/a^2\beta , \quad (19a)$$

$$E_y^0 = 2Iy/a^2\beta . \quad (19b)$$

We retain the superscript convention employed previously. If the surrounding medium is azimuthally symmetric, there is no additional electric field from the wall when the beam is on axis.

For a beam displaced a distance ξ in the y direction, we must consider the electric field arising from a surface charge distribution $\sigma(\theta)$ at $r = a$.

Analogous to Eq. (9) we have

$$\sigma(\theta) = \frac{I\xi}{\pi\beta a^2} \sin \theta . \quad (20)$$

In order to obtain the electric field arising from this surface charge distribution we must specify the electrical properties of the surrounding wall. We consider that the magnetic material fills a annular region $b < r < d$. Furthermore the magnetic material has a dielectric constant ϵ . For this configuration the electric field $\vec{E}^1$ inside the beam is given by Eq. (34) of ref. 2. The field is in the negative y direction and uniform in the region $r < a$. In the present notation we have

$$E_y^1 = - \frac{2I\xi}{\beta a^2} \left[1 - \left(\frac{a}{d}\right)^2 H \right], \quad (21)$$

in which the quantity H is given by

$$H = 1 + \frac{(\epsilon - 1)(d^4 - b^4)}{b^2[d^2(\epsilon + 1) + b^2(\epsilon - 1)]}. \quad (22)$$

We note the following limiting values of H : $H = 1$ if $\epsilon = 1$, $H \rightarrow d^2/b^2$ as $\epsilon \rightarrow \infty$ and as $d \rightarrow \infty$ we have

$$\lim_{d \rightarrow \infty} H/d^2 = (\epsilon - 1)/b^2(\epsilon + 1). \quad (23)$$

For a beam displaced a distance η in the x direction we obtain the electric field E_x^1 inside the beam by a rotation of coordinates. We have

$$E_x^1 = - \frac{2I\eta}{\beta a^2} \left[1 - \left(\frac{a}{d}\right)^2 H \right]. \quad (24)$$

The magnetic force considered in Eqs. (15) and (17) must be augmented by electric forces F_y and F_x given by

$$F_y = e \left[E_y^0(y_0) + \xi \left. \frac{\partial E_y^0}{\partial y} \right|_{y_0} + E_y^1 \right], \quad (25)$$

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2. R. J. Briggs, and V. K. Neil, Plasma Physics (Journal of Nuclear Energy Part C) 8, 255, (1966).

$$F_x = e \left[E_x^0(x_0) + \eta \frac{\partial E_x^0}{\partial x} \bigg|_{x_0} + E_x^1 \right]. \quad (26)$$

With these electric forces included, Eqs. (16a), (16b), (18a) and (18b) become

$$\frac{d^2 y_0}{dt^2} = \frac{2eIy_0}{\gamma m a^2} \left[\frac{1}{\gamma^2 \beta} - 2\beta \left(\frac{a}{b} \right)^2 \right], \quad (27a)$$

$$\frac{d^2 \xi}{dt^2} = \frac{2eI\xi}{\gamma m b^2} \left[\frac{H}{\beta} \left(\frac{b}{d} \right)^2 - \beta \right], \quad (27b)$$

$$\frac{d^2 x_0}{dt^2} = \frac{2eIx_0}{\gamma m a^2} \left[\frac{1}{\gamma^2 \beta} + 2\beta \left(\frac{a}{b} \right)^2 \right], \quad (28a)$$

$$\frac{d^2 \eta}{dt^2} = \frac{2eI\eta}{\gamma m b^2} \left[\frac{H}{\beta} \left(\frac{b}{d} \right)^2 + 3\beta \right]. \quad (28b)$$

Including the electric forces in the equations of motion does not significantly alter the conclusions of the previous section. The incoherent motion can still be made stable provided $(\gamma a/b)^2 \gg 1$, while the coherent motion remains unstable.

We conclude that the series of structures studied here is not satisfactory for providing focusing of an intense beam of particles. However, the characteristics of the transverse forces suggest a method of constructing a passive, current-dependent, deflector for an intense beam. This concept will be investigated in a future work.

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