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ERDA SUMMER STUDY OF HEAVY IONS FOR INERTIAL FUSION

**Claremont Hotel, Oakland/Berkeley, California
JULY 19-30, 1976**

**FINAL REPORT
December 1976**

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Editors:
Roger O. Bangerter
William B. Herrmannsfeldt
David L. Judd
Lloyd Smith

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Roger C. Bangerter (LLL)

David L. Hall (LBL)

to chairman of the Study

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I. DIGEST

The use of intense short bursts of high-energy heavy ions to ignite thermonuclear macro-fusions of deuterium-tritium pellet targets for the generation of commercial electric power has several attractive features. The pulse rates, reliability, efficiencies, and operating lifetimes of modern high-energy accelerators are appropriate, and stored beam energies in the relevant range [$1-10$ megajoule (10^6 joule)] have been attained. The short, well-defined ranges of heavy ions ($\sim 100\%$) at particle energies up to ~ 100 MeV are well suited to target requirements. Recent studies at the Argonne and Brookhaven National Laboratories have indicated that the high peak beam power requirement ($\sim 10^{12}$ watts) might be met. At a February 1976 meeting of accelerator and inertial-fusion experts at ERDA, it was agreed that a summer study should be held to explore this possibility further.

The study, sponsored by ERDA and conducted by the Lawrence Berkeley and Lawrence Livermore Laboratories, was held at the Claremont Hotel, Oakland/Berkeley, July 12-30, 1976. Over 50 specialists in high-energy accelerators, target and reactor design, atomic, molecular and plasma physics were assembled to consider the feasibility of commercial power production rather than break-even experiments and to identify areas requiring research and development.

CONCLUSIONS

The central result was an affirmation that ion-beam fusion power merits serious attention. The accelerator experts found no fatal flaws in the systems they studied. Many of the requirements are within the present state of the art, and others may be reached by extrapolations of known techniques with good likelihood of success. Target experts developed pellet requirements in which they have high confidence, and also less demanding targets that may be acceptable. Reactor designers began to consider a wide range of concepts required for the use of heavy ions. Considerable enthusiasm was generated among the participants.

However, it is clear that present information is inadequate to establish technical feasibility of ion-beam fusion power with reasonable confidence, or to select the optimum type of accelerator system. A concerted effort on target, reactor, and accelerator designs, a number of physics experiments, and some hardware development and testing are needed.

Because specific system designs have not yet been developed, capital and operating cost estimates for a complete power plant, based largely on general experience, are very rough. The sample accelerator systems considered could cost a few hundred million dollars for an electric plant output in the 1000 megawatt range. Substantial experience exists in accurately estimating accelerator costs after a system has become well defined.

AREAS REQUIRING RESEARCH AND DEVELOPMENT

1. *Targets.* Target requirements considered were: 1-10 MJ beam energy; 100-600 TW (10^{12} watt) peak beam power; $2-10 \times 10^{-6}$ sec pulse length; 20-50 MJ/g energy deposition; 1-10 mm target spot size; and 1-10 pulse/sec. More design work is needed to ease the most demanding requirements on accelerators and to exploit their special advantages. The possi-

bility, unique to high-efficiency accelerators as igniters, of using lower gain pellets that generate tritium directly by using the D-D reaction is an example. This would avoid the need for a tritium breeding blanket in the reactor. Construction of prototype targets and conceptual design of a target factory are also needed.

2. *Accelerator.* A nominal beam vessel radius was considered. A range of design concepts developed for laser fusion must be extended. Provision of pressure and temperature within given ranges around the target at ignition may be required, depending on the outcome of detailed studies of ion-beam neutralization and stability near the target.

3. *Target assembly and transport, alignment, and development.* Construction, and testing at both conventional and high-current pulsed-power sources of heavy ions in low charge states, and of structures for the very low-velocity accelerating stage, are needed to determine accelerator parameters and to identify an optimum system. Most of this work can be based on extrapolations of known systems.

4. *Accelerator design.* Cross sections for charge-to-charge state in ion-ion collisions are poor. Both theoretical, computational, and experimental work is urgently needed. Estimates indicate that this process, which limits beam lifetime, may rule out use of a synchrotron, which accelerates more slowly than a linac. Other cross sections are also needed and should be measured or calculated.

5. *Accelerator systems.* It appears possible to provide the high beam power, corresponding to currents of several thousand amperes, by two distinct approaches. One, allowing many variations, uses a conventional 50-100 MeV ion source feeding a radio-frequency linac. Acceleration to full energy is achieved in the linac or a synchrotron. Current multiplication is obtained with an accumulator ring, longitudinal bunch compression, and/or a multiplicity of beams on the target. The other approach provides a single-pass system from injector to target, using an induction linac fed by one or more high-current sources. Beam compression and pulse-shaping result from controlled ramping of accelerating voltage pulse shapes.

Only a few selected examples from this wide spectrum could be examined in detail during the study. Ongoing intensive work on many options not studied, and on some factors not yet considered, is essential to define an optimum system needed for realistic cost estimates. Development, construction, and testing of hardware and experiments on existing accelerator systems may be needed.

6. *Beam transport, focusing, and neutralization.* Superconducting-magnet transport and focusing systems will probably be required and will need study and tests. The advantages and disadvantages of beam neutralization have not yet been explored. A crucial problem, central to all ion-beam igniter systems, is the study of beam propagation and avoidance of possible instabilities, within the reactor vessel near the target. Intensive theoretical, computational, and perhaps experimental work, with the help of plasma physics experts, is urgently needed.

II. Introduction

Fusion scientists have recognized for several years that intense ion beams might be used to initiate inertially confined thermonuclear micro-explosions. However, serious consideration of ion initiated fusion was delayed by the lack of a well defined beam of accelerating very intense ion beams. During the past few years Alfred W. Maschke of the Brookhaven National Laboratory and Ronald L. Martin and Richard C. Arnold of the Argonne National Laboratory considered the application of high energy accelerator technology to this task. They discussed their ideas with inertial fusion and accelerator experts and became convinced that this technology is capable of producing ion beams of the appropriate particle energy, atomic number, power, and total energy to ignite fusion micro-explosions.

If accelerators can indeed produce such beams, there are several reasons for becoming enthusiastic about the possibility of a marriage of inertial fusion and accelerator technologies. The demonstrated repetition rate capabilities, efficiencies, reliability, and operating lifetimes of large present day accelerators make them appropriate for inclusion in working power plants. Accelerators are the products of a highly developed mature technology. Although they have been custom designed in the past, and have not been reduced in unit price to mass production, even the largest have cost less than a large modern power plant.

The favorable response to several presentations and discussions of these concepts in 1975 led to a desire to present them to ERDA officials. A meeting of accelerator designers and inertial fusion experts was held at ERDA headquarters in Germantown, Maryland, on February 15, 1976. This meeting was organized jointly by Glenn W. Kuswa of the Division of Laser Fusion and William B. Hermannsfeldt of the Division of Physical Research. In addition to ERDA staff members, representatives from Argonne, Berkeley, Brookhaven, Fermi, Livermore, Los Alamos, and Sandia Laboratories participated. It was concluded at this meeting that ion-beam fusion merited serious consideration and that ERDA would sponsor a summer study.

At the request of ERDA, the Lawrence Berkeley and Lawrence Livermore Laboratories jointly undertook to organize and conduct the study. Roger O. Bangerter (LBL) and David L. Hald (LBL) were designated as summer study coordinators. Detailed planning began in March with the help of an organizing committee which included

Glenn W. Kuswa, ERDA
 Ronald L. Martin, ANL
 Alfred W. Maschke, BNL
 John B. Jackolls, LBL
 Floyd Smith, LBL

The committee was assisted by William B. Hermannsfeldt (ERDA). The organizing committee established

the following objectives for the summer study:

- Assess the feasibility of energy production with ion-beam fusion.
- Identify areas needing research and development.
- Prepare a report on these matters.

In order to accomplish these objectives the committee selected a body of scientists representing many disciplines. Experts in conventional accelerator technology, collective acceleration, pulsed-power technology, fusion target design, reactor design, plasma physics, atomic and molecular physics, and commercial power production were invited to attend the study. A list of participants will be found at the end of this report (Section III).

The committee gave much thought to the identification, in advance, of technical problem areas to provide a framework within which the participants could focus their efforts, and to the designation of leaders in each area. In this way the study's structure evolved into its working group's three subject areas, and those who agreed to serve as leaders, are listed here.

Working Group Leaders

1. Target and Reactor Design
Robert L. Heath, LLNL
2. Ion Sources
Glenn W. Kuswa, ERDA
3. Low Velocity Acceleration
William B. Hermannsfeldt, ERDA
4. Atomic and Molecular Physics
Thomas J. Donnelly, LLNL
5. Accelerator Parameters
John B. Jackolls, LBL
6. Beam Manipulations
David L. Hald, LBL
7. Induction Linac
Thomas J. Donnelly, LLNL
8. Final Focusing and Transmission to Target
William B. Hermannsfeldt, ERDA
9. Systems and Cost Studies
Thomas J. Donnelly, LLNL
10. Alternatives
Thomas J. Donnelly, LLNL

A number of the participants contributed to the work of more than one group.

Section III of this report contains a brief overview of the technical problems considered by these groups. The main results of the study are described in the 16 detailed technical summaries of their work (Section IV). Each of these has been prepared under the supervision of its working group leader. Section V, on conclusions of the study and areas requiring research and development, is a verbatim copy of the short summary report submitted

to NASA by the working group letter on August 18, 1979.

In addition to this material, a considerable body of more detailed information will be found in the several groups of Appendices (Section VI), which are keyed to the corresponding Technical Summaries. The numbering scheme for these is apparent from examination of the table of contents.

III. Overview

This section provides a brief overview of the technical problems addressed during the summer study, and is a guide to the division of effort among the working groups. For this reason it is organized in section with titles descriptive of these groups, which follow the introductory paragraphs below.

For several years the requirements for fusion target ignition by laser and electron beams have been studied by target design specialists. The short and well-defined ranges of very heavy ions (mass number > 100) at energies up to 100 GeV are particularly advantageous. Based on previous experience, estimates of energy density, total energy, particle range, pulse length, and pulse-shape requirements for ion beams have recently been made by target experts. The primary task addressed during the study was to consider applications and extensions of existing accelerator technology needed to meet the requirements of a system suitable for commercial power production.

A high energy ion igniter system requires a source of the particles to be accelerated, and a means of raising their energy, by one or more stages, to a level appropriate for injection into the main accelerator. After this acceleration the ion beam must be transported to the reactor vessel and focused onto a small target spot at its center. A wide variety of techniques for performing each of these functions has been developed over nearly half a century. Taken together, they constitute the existing state of the accelerator art. In the course of these developments, which for the most part have followed lines dictated by the goals of basic physics research, insights have been gained regarding many possible extensions for which motivations were lacking in the past. In describing the application of these insights, based on broad knowledge and experience, to the problem of producing the beams required for heavy-ion fusion, it is useful to indicate which aspects of the functions mentioned above are outside presently known capabilities.

1. Target and Beamer Design

An important activity of the working group on target and reactor design during the study was to interact with accelerator physicists. Together they expressed target requirements, as indicated above, in forms suitable for the parametrization of accelerator systems, and worked to extend these so as to vary the balance of relative confidence between accelerator and target areas. Target experts also considered some new possibilities, specific to high-efficiency ion-beam igniters, which emerged during the study. In particular, the use of lower-gain targets, which directly generate more tritium by the D-D reaction than they consume, was considered. In addition to continuing examination of various reactor design strategies, reactor specialists took cognizance of the concern developed by accelerator and plasma physicists during the

study regarding the degree of vacuum, and the character of background gas and plasma in the reactor vessel, at pulse-arrival time. Consideration of systems providing lower and higher pressures was initiated, and the properties of several first-wall materials were examined.

2. Ion Sources

Practical experience with intense sources of heavy ions in low charge states is limited; aside from Calutron sources developed during World War II for electromagnetic separation of isotopes, most heavy-ion source development has been for accelerator systems in which a relatively high charge state for the main acceleration is required, resulting in low currents and negligible space-charge forces. However, enough general knowledge has been gained to indicate the lines of attack in source development, and to allow estimates of attainable source brightness, emittance, and total current, for extensions of conventional heavy-ion sources. The working group on ion sources provided such information, and concluded that the development of appropriate sources should be possible. In addition, a similar exploration of high-current pulsed-power sources of heavy ions useful for injection into an induction linac was made and outlines of the needed development program were indicated.

3. Pre-Accelerating Acceleration

The pre-accelerating stages of present machines still use certain low-velocity acceleration systems developed decades ago. However, the requirement of high mass (for most favorable energy deposition) and low charge state (to minimize space-charge repulsion) leads to ion velocities during the pre-acceleration stage that are below the range covered by present accelerating structures and focusing systems. New development is needed; its direction is here again indicated by past experience, and its probable success can be estimated with some confidence. It appears that a succession of stages of known types of structures can be designed, with the help of existing computer codes, to meet the requirements. A first approximation to such a design was developed during the study; factors affecting the choices of structures were discussed, and the character of needed future work was indicated.

4. Atomic and Molecular Physics

The majority of existing ion accelerators employ bare nuclei of hydrogen isotopes, for which only a single ionic charge state is possible. Linear accelerators using heavy ions expose small currents to background gas for short times, but for some circular accelerators beam loss by electron exchange with the background gas can be an important limitation. In particular, with intense heavy-ion beams in a specific low-charge state, it is necessary to avoid appreciable loss of particles not

only from interactions with the gas but also by a mechanism new to accelerator design; even in a perfect vacuum two beam ions may exchange an electron, resulting in the loss of both. The former processes set an upper limit on the product of background pressure and exposure time, while the latter (in a given system) fixes an upper limit on the acceleration time and beam density. The cross sections for this latter mechanism are not well known, but estimates indicate that the conventional synchrotron acceleration or storage times of as long as a second may be ruled out. The cross sections for gas interactions of heavy ions in low charge states are also poorly known. During the study the members of this group collected existing information, made some additional calculations and estimates, and considered the theoretical, computational, and experimental activities needed to obtain adequate information about these important processes.

5. Accelerator Parameters

The total beam energy per target pulse lies within reach of present capabilities in the sense that large systems exist (CERN, FNAL) which contain circulating proton beams of comparable total energy. However, a new requirement is that this energy must reach the target in a pulse lasting of order 10 nsec, more than two orders of magnitude shorter than the one-turn extraction time from such a ring. This requirement for a peak beam current in the several thousand ampere range lies well outside existing experience. Current multiplication can be obtained by several methods: (1) increase of velocity during acceleration; (2) multiturn injection (stacking) in a circular device (synchrotron or accumulator ring); (3) longitudinal compression of a beam bunch in an accelerator or drift line; and (4) physical separation of a long beam into a multiplicity of short ones, all brought on target at the same time. The manner in which these are combined to attain the total needed multiplication of current can serve as a means of categorizing or parametrizing a variety of possible combinations of components. These components constitute the accelerating and beam-pulse-forming systems considered before and during the study, as well as others yet to be invented. Alternatively, the systems may be divided into three groups utilizing an rf linac, synchrotron, or induction linac as the main component. The working group on accelerator parameters, considering primarily the first two of these, developed the framework just described and considered fundamental limitations associated with the invariant property of beam brightness (phase-space density), and with space-charge effects, which limit the capabilities of all systems. During the study this group was also able to present a few illustrative examples, but time was inadequate to allow detailed studies. Although some of the components and combinations are based on familiar concepts and design principles, certain features of each system require at least additional study and in some cases exploration of new techniques or new hardware.

6. Beam Manipulations

To make these systems work, a variety of beam manipulations must be performed. One must extract

particle beams from one component; transport them to another, focus and inject them there, reduce bunch lengths, perhaps chop and divide them, and possibly change their molecular state and charge-to-mass ratio. Extraction, multiturn injection, change of rf "bucket size" during acceleration, transport, and focusing are all familiar operations within present technology, but special demands are placed on them by the present system requirements. Limitations on performance of these beam manipulations are not well known. During the study members of this working group considered some of these limitations, particularly those associated with space-charge forces, tune shift in synchrotrons, lifetime against charge-state change, and phase-space requirements. They applied these considerations to the description of a few sample systems, but time was not adequate to explore the full range of variations.

7. The Induction Linac

An ion-beam igniter system utilizing an induction linac has certain properties that make it distinctly different, and in some respects potentially more appealing than the others. It offers the unique possibility of a single-pass system from source to target with modest vacuum requirements, good electrical efficiency, and freedom from the complexities of multiturn injection, stacking, and extraction from a circular device at high energy. The general design principles and engineering technology of induction linacs are well established, although in the past they have been applied only to acceleration of electrons. Here again the present requirements go beyond those previously encountered, particularly with regard to pulse length and pulse shaping. Because the advantage of the induction linac is its ability to accelerate short, very high current pulses, the development of a heavy-ion version of a known type of pulsed-power ion source, or a method for accumulating many pulses from sources of more conventional type, is required. The working group studied this topic, considering all aspects of the system from source to final pulse shaping in a preliminary survey, and presented an illustrative set of design parameters. Many problems remain to be considered in detail; a future program is outlined. No fundamental difficulties were encountered, and the induction linac appears at present to be an attractive alternative deserving further study in parallel with work on other options. In addition, the application of induction-linac technology to the compression and shaping of pulses and to the control of space-charge repulsion during transport from accelerator to reactor may be very important for use with any type of main accelerator.

8. Final Focusing and Transmission to Target

The focusing of a beam of high-energy particles onto a very small spot at a considerable distance beyond the last physical focusing element is a familiar operation at high-energy accelerator laboratories. However, present beams are not usually so intense as to be influenced by space charge, and in addition an excellent high-vacuum environment is usually attainable. For the present application new problems arise; the lens system must not only be free of excessive aberrations in the presence of

significant space-charge forces, but also its structure must not be directly exposed to explosions inside the reactor. Furthermore, the pressure, temperature, and composition of background gas and plasma inside the reactor vessel are not well known and will vary significantly among different reactor designs. Material from preceding explosions that cannot be pumped out in time may interact with the converging beam pulse to produce partial charge and current neutralization varying in space and time. Study of the possibility that these interactions could generate harmful beam instabilities, or alternatively that neutralization might avoid some instabilities, constituted a major activity of this working group. Methods of selecting parameters so as to suppress a specific mode of instability are generally known in plasma physics. Under some combinations of beam and reactor conditions, certain instabilities can be expected to develop. Continuing effort is needed to establish consistent beam and reactor parameters to assure that the beam can be focused on the target spot.

9. *Systems and Cost Studies*

It became apparent during the study that the working group assembled to estimate capital investment and power cost for an ultimate large-scale electric generating plant based on ion-beam fusion would not be able to do so with meaningful accuracy. Too many options for the accelerator system remain open, and none of them well defined in sufficient detail to provide more than guesses regarding their capital and operating costs. However, previous work on reactor-system design for laser ignition provided a base from which this group could conduct

comparative studies of such components. An interesting result was the conclusion that if an accelerator system with high efficiency can be designed, the use of lower-gain targets which directly generate more tritium than they consume might become feasible. This would avoid the need for a tritium-breeding blanket, greatly reducing the required tritium inventory and perhaps also the cost and complexity of the reactor.

10. *Alternatives*

Members of this working group were experts on collective acceleration, plasma physics, pulsed-power technology, and related matters. They undertook to survey studies and developmental work aimed at achieving inertial fusion by collectively accelerated ions and by methods other than high-energy heavy-ion bombardment. Ongoing programs based on use of lasers and electron beams were not included. The motivation for including a group on these activities in the study, and a summary of their conclusions in this report, is the possibility that, if successful, one of these alternatives might be cheaper than a large accelerator. Their summary shows that although some such devices may be useful for very low-repetition-rate experiments in smaller reactor vessels, the lack of high-repetition-rate capability makes them less promising for full-scale power-producing systems at present. Programs to explore the feasibility of developing this capability are suggested. A wide variety of alternative devices and systems, ranging from well-developed existing hardware to speculative concepts without experimental foundations, are briefly described.

IV. Technical Summaries

1. TARGET AND REACTOR DESIGN

John H. Nuckolls

In order to generate useful fusion energy with heavy-ion accelerators, three major developments must occur in the target/reactor areas.

- An efficient fusion microexplosion must be generated.
- A target factory must be constructed that will manufacture cheap targets.
- A reactor first wall must be constructed that will withstand the neutron, x-ray, and plasma effects.

These objectives appear to be feasible if practical heavy-ion accelerators can be constructed that have the appropriate power, energy, voltage, pulse shaping, and focusing properties.

TARGETS

None of the ion-beam-imploded target designs that have been published^{1,2} or reported^{3,4} are adequate for electrical-energy production (see Appendix 1-1). Either the gain is too small or the required power is too high. In addition, there are serious problems involving fluid instabilities driven by irradiation asymmetries. Consequently, the success of practical heavy-ion fusion depends not only on the success of advances in accelerators, but also on the success of advances in targets. Recently, the Livermore group has designed a number of targets having energy gains in excess of 100. Such targets appear suitable for commercial power production when imploded by appropriate heavy-ion accelerators.

After considering the various proposals for target designs and estimating the potential growth in performance reasonably to be expected in the next few years, the target panel agreed on the following target input (accelerator) requirements for commercially useful power plants.

The gains greater than 100 implied by Case 1 are achieved despite the long deposition length of the ion beam relative to that for laser light because the massive absorber acts as a hydrodynamic tamper which is much less lossy than the ablation to vacuum in a laser implosion. This compensates for the extra energy required to heat the absorber. In both cases, high gains are achieved via use of low-density, lightweight pushers driven by shaped pulses.

In Case 2 our confidence is significantly reduced because the shells are made thinner in order to reduce the required power and energy. Consequently, there is a greater risk of failure due to fluid instabilities.

Cases 3 and 4 exploit the potentially very high efficiency of certain types of accelerators. In Case 3, the fuel is mostly deuterium. Some T and He³ seeding is used, but the amounts of these materials that are consumed are generated in the burning deuterium. Consequently, the necessity of breeding tritium in a lithium blanket is eliminated. Note that because of the high efficiency requirement this very important option is unique to heavy-ion accelerator fusion. In Case 4, the DF fuel is a non-cryogenic gas. This provides an option that may be exercised if it should turn out that fabrication of cryogenic targets is too expensive.

The number of beams should be at least two and may be 100 or more. The focused beam radius should not be less than 1 mm. In general for a given range, the beam radius will be determined by the specific energy requirement in Table 1-1.

The accelerator should be capable of focusing to the 1 mm (or larger) beam spot from the wall of the reactor chamber, which is 10-15 m from the target for a single chamber, and 5-7.5 m for eight chambers. The total area of the beam and vacuum ports should not exceed 10% of the area of the reactor wall.

The required pulse shape is approximately as shown in Fig. 1-2, where E is the total energy, P is the peak power, and Δt is the characteristic

Fig. 1-1. Range vs energy for various ions incident on gold. (XBL 7611-9911)

Table 1-1. Target Input (Accelerator) Requirements^a

Case No.	E (MJ)	Δt (ns)	P (TW)	ϵ (J/g)	γ (%)	Rep rate (s^{-1})	Voltage (GeV)	Special features	Confidence level
1	10	10	600	3×10^7	10	?	100(0)	nominal	high
2	1	6	100	2×10^7	10	20	40(0)	advanced	moderate
3	10	10	600	3×10^7	50	20	100(0)	D burner	moderate
4	10	10	600	3×10^7	50	10	100(0)	non-cryo-genic	moderate

^aE is the total beam energy, Δt is the pulse length at peak power, P is peak power, ϵ is energy/mass deposited in the target, and γ is accelerator efficiency.

^bInstead of uranium, lower Z charged particles may be used provided the voltage is reduced to give the same range (see Fig. 1-1). For energies greater than 100 GeV, target preheat may be a serious problem.⁵

time in Table 1-1. This pulse shape may be approximated by a series of shorter Gaussian or square pulses of varying height. Typically ≥ 5 shorter pulses are required for an adequate approximation.

Fig. 1-2. (XBL 7612-11072)

Further work is needed in the target area to evaluate the advanced designs. Improvements here could have a great impact on heavy-ion accelerator design. There is some uncertainty concerning the range of the heavy ions (see Appendix 1-2 and Refs. 6 and 7). However, this uncertainty is believed to be relatively small and requires experiments in hot plasmas and with high-intensity beams. Further theoretical study is merited.

It has been suggested that target performance might be degraded by preheat resulting from nuclear

fragmentation of the ion beam.⁸ This does not appear to be a problem.⁵

Target Fabrication

The principal problem in the target-fabrication area concerns the allowable target cost. The cryogenic nature of the pellet tends to increase this cost. We require microexplosions in the 0.1 to 1 ton yield range so that the tolerable target fabrication costs are 5 to 50 cents per target (assuming the electricity sells for 20 mills/kWh and that 2 mills/kWh may be allocated to target fabrication). We have also included a non-cryogenic target option to cover the possibility that the cryogenic-related problems are much more difficult than anticipated.

In order to make more reliable estimates of target costs, prototypes of the targets should be constructed, and the target factory design should be roughed out. The details of the various target designs are contained in Ref. 5.

REACTOR

A practical fusion reactor depends on solution of at least two difficult problems: neutron damage of the first wall, and thermal ablation of the

Table 1-11. Approaches to Reactor Problems

First wall option	Residual pressure at 300°K (Torr)	Coolant	Neutron load (MW/m ²)	Tritium breeder blanket	Confidence
Wetted (Li)	> 1	Li	~1	Li	moderate
Gas buffer	10 - 100	Li	~1	Li	moderate
Li "waterfall"	1 - 100	Li	< 0.1	Li	high
Dry (Pyro C)	10 ⁻³	Li	~1	Li	moderate
Magnetic/C	10 ⁻³	Li	~1	Li	moderate
Dry (Pyro C)	10 ⁻³	He gas	~0.1	none (in target)	moderate
Magnetic/C	10 ⁻³	He gas	~0.1	none (in target)	moderate

inner surface of this wall. A number of approaches to these problems have been proposed.

In all approaches except one, the neutron damage has been minimized by making the reaction chamber relatively large - about 15 m radius for a 1.0 Mc chamber - so that the neutron loading is about 1 MKW/m² with D-T burning targets. A wall lifetime of a few years has been estimated under these conditions. However, the lifetime has not been measured experimentally and there are a number of important uncertainties (see Appendix 1-3). The neutron damage rate is reduced more than an order of magnitude so that the wall lifetime should equal that of the reactor in the lithium "waterfall" scheme where the wall is protected by a thick lithium layer. The wall is also shielded from the x-ray pulse. If the falling lithium is given a small inward velocity component, the entire pellet blast may be absorbed without transmitting a strong shock to the wall. The reactor diameter may then be reduced several fold, and the lithium mass and tritium inventory correspondingly reduced. The lithium "waterfall" approach is viable only if the ion beam can propagate and focus through the associated lithium vapor (pressure > 1 Torr). This is believed to be feasible.⁹

In the wetted-wall design, the ablation problem is solved via a layer of Li liquid which is continually renewed by flow through a porous stainless-steel wall. In the gas-buffer designs, ablation is reduced by having a sufficiently high residual gas mass so that the wall is not heated by extremely hot plasma (see Appendix 1-4).

The dry-wall and magnetic schemes are suitable for very low pressure applications (in case the transport focusing requirements include such low pressures). If a magnetic field is not used to shield the wall from hot plasma, then highly conducting pyrolytic graphite may be needed to avoid too rapid ablation rates.

In the D-T burning approaches, the tritium consumed in the target burning is regenerated by neutron reactions in the lithium blanket. This blanket, which is about 1 m thick, absorbs the high neutron fluxes generated by the fusion microexplosions.

Most of the fusion energy is carried by the 14-MeV neutrons and deposited in this blanket. The resulting heat is converted to electricity via conventional thermal electrical systems. In the deuterium-burning options the lithium breeder blanket is unnecessary since the small amount of tritium seeded into the pellet is regenerated by the burning deuterium. However, a neutron absorbing blanket is still required to shield the external environment. In this case, most of the fusion energy is contained in the plasma. Electricity may be generated by MHD systems, by compressing a magnetic field at present, or by more conventional means.

Only the lithium "waterfall" reactor option can be characterized as having a high confidence level. The other options involve important uncertainties, which are optimistic that a practical reactor is feasible. In the near future, the calculations and design analyses should be refined, and when a microexplosion source becomes available, a test facility should be constructed.

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Appendices 1-1, 1-2, 1-3, and 1-4 are pertinent to this Technical Summary.

An adequate source of low-charge-state heavy ions is an essential basic component of the high-intensity accelerator required for the inertial fusion application envisioned here. The minimum specifications for this ion source depend upon the specific details of the accelerator design adopted; for the purposes of discussion the requirements for an rf linac system will be taken as ~ 100 nA of singly charged ions with a mass > 100 (amu) with a normalized emittance

$$\left[\epsilon_n = \frac{\text{Area} \left(\frac{1}{2} r_0^2 \right)}{\pi} \right] \geq 0.02 \text{ cm-mrad.}$$

For an induction linac, a pulsed source is required of at least 20 A with a charge state $q \sim 1$ to 3 and a mass $M > 100$ and only modest emittance requirements. No suitable ion sources generally fulfilling these requirements are now available. However, it was the consensus of the members of the ion source committee that it appears feasible to develop ion sources (or at least selected ions) meeting either of these stated minimum performance figures. To the extent that the performance of the ion source can be made to exceed the minimum requirements, accelerator design is made easier and operation more flexible.

A number of conference proceedings^{1,2} and general reviews of ion sources^{3,4} have recently been published for background detail and these generally include brief discussions of low-current, high-charge-state, heavy-ion sources. Hence the object here is primarily to note needed definitions and briefly discuss those constraints that are particularly relevant to the intense low-charge-state heavy-ion sources required here. The constraints of classical ion sources will be covered first and the case of pulsed plasma sources (e.g., from an ion diode) will be included as a separate subtopic. Additional details in either case are given in Appendices 2-1, 2-2, and 2-3.

CONSTRAINTS FOR CLASSICAL ION SOURCES

The functions of a classical ion source are most conveniently discussed in terms of:

- 1) Plasma production
- 2) Extraction
- 3) Beam transport into a suitable accelerator.

1) Production of a Plasma

Production of a plasma of a suitable density and species purity for the available ion flux to match the space-charge-limited extraction current appears to be the subject of most uncertainty and the one most in need of development for a low-charge-state heavy-ion source. The work needed appears to be primarily practical rather than fundamental and in a few selected cases, does not appear particularly difficult; e.g., in the case of a gaseous material such as Xe allowing use of almost any of the standard ion-source arc configurations, or the case of a low-work-function material such as Cs allowing contact ionization. However, for most

other materials (e.g., CaF_2 , BaF_2) or selected plasma ion species (e.g., H^+ , He^+) it is quite difficult to obtain high vapor pressure materials such as Hg, to quite difficult for low vapor pressure materials such as Au, Ba , or Cs , or of all-ion ion species from a heated filamentary cathode is also a possibility at low current density. In addition the requirement for use of a single ion species for timed acceleration tends to preclude use of an arc discharge, since ion species populations are typically apportioned over several adjacent charge states according to a Saha-type equation.⁵ Similarly, use of material in the form of a molecular compound or use of molecular ions generally would be expected to lead to a variety of ion states, which again reduces the useful fraction of the total ion output. The case of a surface contact ionized material such as Cs is unique in this regard, with nearly 100% of the ions generally available in the $q=1$ state. The special requirements for use of the $q=1$ ions are covered in Appendix 2-1.

2) Extraction of Ions

The most fundamental restriction for any classical ion source follows from the Child-Langmuir space-charge limited (SCL) current density:

$$\sum_{\text{all ions}} n_i M_i q_i^2 \leq \frac{4}{9} \frac{V_0^{3/2}}{V_0} = n_{\text{SCL}} \text{ cm}^{-2} \quad (1)$$

where M_i is the ion mass (amu), q_i is the charge state (integer), V_0 is the total applied extraction voltage in kV, and $\frac{4}{9}$ is the effective extraction gap in cm. Experimental tests of the effective extraction gap show $d \sim d_0 + r$ for an inter-electrode gap d_0 and a circular aperture of radius r . Proton sources have been optimized to approximately 500 nA/cm² implying the possibility (for similar arc parameters) of attaining up to $i \sim 500/\sqrt{M/q}$ mA/cm² for heavy ions. However, practical considerations involving the voltage required and the associated gap and aperture scaling tend to generally reduce this, unless one goes to a short-gap multi-aperture system. In particular, the total current for a single aperture of radius r for a single ion species follows from Eq. (1) as:

$$I = \pi r^2 J_{\text{SCL}} = \frac{(1.77) \sqrt{V_0}^{3/2}}{\sqrt{M/q} (d_0/r + 1)} \text{ mA} \quad (2)$$

with the limitation of the extraction voltage $V_0 \leq V$ (breakdown) $\sim 60/\sqrt{d_0}$ kV.⁶ If one adopts the common good optics requirement that $d_0 \geq r$ and requires large currents from a single aperture, one finds that voltages above or near 100 kV are required, which in turn require large spacing and result in much lower than the maximum current density. In this case use of a multi-aperture system would appear advantageous, allowing higher current densities, with perhaps similar or only slightly higher normalized emittances as discussed below.

3) Beam Transport

Successful beam transport into an accelerator depends largely upon minimizing the normalized emittance of the heavy-ion beam from the ion source

and achieving nearly complete space charge neutralization if transported any appreciable distance. The formal definition of the normalized emittance used here is

$$\epsilon_n = \frac{\text{Area}(\pi r_{\perp}^2)}{q} \text{ in cm-rad} \quad (3)$$

The magnitude of ϵ_n thus depends linearly on beam size, and on the total ion transverse energy. For similar source sizes ϵ_n is often used as a figure of merit for comparison of source performance, since the ion transverse energy is the sum of the initial plasma ion temperature, any uncorrelated radial drift components, extraction optics aberrations, and nonlinear space charge divergence. A good working value of normalized emittance for a single-aperture source is ~ 0.02 cm-rad, while multi-aperture systems tend more typically to have $\epsilon_n \sim 0.1$ cm-rad, depending fairly sensitively on the transparency of the extraction grid system. It is expected that optimized heavy-ion sources will have relatively lower transverse energies than proton sources and thus normalized emittances at least as low as for proton sources and possibly much lower for sources using surface contact ionization.

The space-charge forces of the beam represent a problem in any transport more than a few beam diameters. Solutions to this problem include use of background gas ionization at perhaps $\sim 5 \times 10^{-5}$ Torr or use of extensive magnetic focusing to cancel the divergent tendency from space charge. Electrons from a plasma source can be used instead of a background gas. This problem does not appear fundamental and appears best treated on a case-by-case basis because of widely different possible background gas ionization properties and a strong dependence on ionization with beam velocity for the heavy ions and extraction voltage considered.

In conclusion, development of a 100 mA, $\epsilon_n \sim 0.02$ cm-rad heavy-ion source with $q=1$ appears a distinct challenge, but appears feasible at least for gases with higher-vapor-pressure materials. Use of a single-aperture system would be desirable but it may be necessary to go to a multi-aperture system to work in the most favorable range of extraction voltages. As a specific example, a 100-mA source of Cs⁺ could be met by a 4.5-cm-diam, 1.25-cm-gap, single-aperture source operating at 90 kV, or by a 2-cm (total) diam array of seven 0.5-cm-diam apertures with a 0.25-cm gap operating at 25 kV.

*Plasma Ion Source for 20 A Output
(with $q=1$)*

The first candidate for this system would be to simply scale up a classical multi-aperture heavy ion source (or set of source modules) to achieve the necessary area for the needed total current. The comments given in the previous section above are then generally applicable. For the case of a 20 A beam, an expected $j_{\text{scat}} \sim 30 \text{ mA/cm}^2$, with only $\sim 50\%$ in the desired charge state, and a transparency of 67%, would require a total area of $\sim 2000 \text{ cm}^2$. This sounds large, but the 50 A multi-aperture sources of the type used for neutral D beam production⁹ have an area of 245 cm² per module

and up to 12 modules have been used at LBL (2940 cm²) on a single experiment. One module was tested with Xe, achieving¹⁰ a total output of 9 A (electrical current) likely with $q \sim 2$ to 3, for $j \sim 37 \text{ mA/cm}^2$ (electrical). Thus the major problems with this approach are engineering to a larger scale and beam "collection and transport" for injection into a suitable linear induction accelerator. The effective source emittance from these sources should be ~ 0.5 cm-rad despite their large size. However, the caveat should be added that normally the LBL sources operate without prompt charge exchange to a neutral beam, so a serious space charge problem would be present unless arrangements can be made to achieve prompt and nearly complete space charge neutralization.

The second candidate for this requirement would be to utilize the plasma output from some non-classical source such as an ion diode. Relatively high heavy-ion current densities of order 100 A/cm² appear available from these at several hundred keV, based upon scaling from work with H⁺ or Al³⁺. However, pulse times tend to be short ($\leq 50 \text{ nsec}$) because the diodes are shorted out by plasma closure and breakdown; though magnetically insulated units may be capable of the desired duration. An additional positive factor for this approach is the possibility of directly providing a 4-5 MeV beam needed for linear induction accelerator operation. The problems with this approach arise because of likely poor emittance values (perhaps $\epsilon_n \sim 10$ cm-rad) and from the dense plasma output available. Removal of the space charge neutralizing electrons from the beam is required at or before the accelerator entrance aperture. The topic of ion diodes is discussed in much more detail in Appendix 2-2.

AREAS OF RECOMMENDED RESEARCH AND DEVELOPMENT

As a result of the study, a number of areas for needed research and development have been identified. These areas are subdivided into work for classical ion sources and work for pulsed diodes in the following list:

Classical Ion Sources

1) Paper studies and atomic cross-section measurements should be pursued as rapidly as possible to establish criteria limits for heavy-ion-species selection. The purpose being to establish immediately the principal ion species of interest and from these select the easiest elements for use, rather than unnecessarily working on some materials with very low vapor pressure that might present more difficulty.

2) In parallel with the above, work should be started on selected "best" candidates from an ion source point of view. This would certainly include a Cs⁺ contact ionization source; in this case one would expect to be able to build a very low-emittance high-current-density $q=1$ source. Work should also include tests with materials such as Ne, Xe, and perhaps Hg in the existing ion source types (e.g., duoplasmatrons or other reflex arc variations). The goal here being early experience with relatively high current heavy-ion beams and an early check on scaling.

3) Backup studies should also be started on general plasma sources for heavy ions (particularly masses > 200 amu), with work to include species control for high fraction utilization.

4) Work should be included on molecular ion candidates such as $(\text{H}_2)^+$ if dissociative beam trapping appears necessary to ease Liouville theorem limitations on ion accumulation.

5) Heavy-ion-beam space-charge neutralization studies should be started to assure satisfactory beam transport to a suitable accelerator entrance structure. This study should also assess likely losses, including the fraction of the beam outside of the useful emittance limits, the fraction in unwanted charge states, and transport losses to the accelerator.

Pulsed Ion Sources

1) The plasma output from pulsed-ion diodes should be carefully evaluated for possible use with induction linacs. This should include measurements of desired ion species fraction, energy spread, and emittance. Beam transport and a satisfactory means of electron removal at the accelerator structure entrance should be important aspects of this research and development.

2) The large-area LBL-type pulsed-ion source should also be evaluated in more extensive tests to assess the species purity, available useful current density, and emittance for representative gases such as Xe.

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Appendices 2-1, 2-2, and 2-3 are pertinent to this Technical Summary.

3. LOW VELOCITY ACCELERATION

Donald E. Young

Target requirements dictate a high-intensity current of heavy ions with small transverse emittance from the ion source. This beam must be transported and accelerated to the linear accelerator. Then the momentum of the ions has been increased to a value that can be accommodated in the more usual β .

$$\beta = v/c = \frac{\text{velocity of ion}}{\text{velocity of light}}$$

and

$$\lambda = c/f \quad \text{frequency}$$

structure of the standing-wave Alvarez linac, the accelerator design becomes conventional and the particular problems associated with the low-beta section of the accelerator can be considered solved.

The main problem in the low-beta section of the accelerator is the need to provide strong focusing forces to preserve the beam emittance against the defocusing space-charge force. A secondary problem results from the need for capturing a large fraction of the dc beam into the rf accelerating sections following the dc accelerating column after the ion source.

The focusing forces necessary to preserve the transverse beam quality are usually provided by magnetic quadrupole elements. A design based on the use of magnetic quadrupoles will be presented. The design is complicated by the difficulty of incorporating the focusing elements into the rf accelerating structure in this region where the velocity of the heavy ions is small.

Computational problems exist for calculating the performance of various combinations of quadrupole focusing and accelerating structures.¹ The six-dimensional phase space acceptance and the expected growth due to space-charge forces are of interest. Such a program is in use at IBL and has been used for design studies of the low-beta section of the SuperHILAC.

The problems inherent in the low-beta section are the same problems that are encountered in accelerating low-charge states of heavy ions in the heavy-ion accelerators such as the SuperHILAC (LBL) and the UNILAC (GSI, Darmstadt, Germany). The performance of the equipment in use, or proposed for upgrading the performance of these accelerators, serves as a guide to the state-of-the-art. Although it appears possible to design this section of the accelerator using conventional equipment, clearly the improvement in the performance that might be possible with further development can well justify the effort.

THE PROPOSED SYSTEM

A block diagram of the low-beta accelerating section of the accelerator is shown in Fig. 3-1.

Fig. 3-1. Proposed low-beta accelerating section. (ABL 7612-11074)

Following extraction of the ions from the ion source, the beam is accelerated in a dc high-voltage accelerating column. It is accepted practice for these large beam currents to use high-gradient accelerating columns and to avoid the use of matching lenses between the source and column by using Pierce geometry² from the source to the end of the accelerating column. This type of column is in use on most of the injection systems for operating high-energy proton synchrotrons. A dc accelerating voltage of 750 kV has been chosen, since this is common practice. Suitable equipment and columns exist at this voltage. This choice also allows several sources and columns to be installed to provide redundancy for necessary source repairs. Normal air insulation allows repairs to be carried out expeditiously.

The ion beam from the dc accelerating column is focused with magnetic quadrupole lines in a simple transport system, which includes an rf buncher. The buncher allows more particles to be trapped in the accelerating phase-space volume in the accelerating system following the buncher. This is done by making use of the transformation of velocity modulation into particle-density modulation along a drift space. A simple buncher can increase the capture of beam into the rf accelerator by a factor of two or more. At the low particle velocities considered here, the buncher gap space must be short and the construction modified from the usual design. However, the use of a tunable stub to excite the gap in the beam line probably would lead to a reasonable design.

The choice of an rf accelerating structure in this low-beta region is dominated by the requirement of providing magnetic quadrupole elements of sufficient strength to give adequate acceptance into the system. Additional space can be provided for the focusing system by choosing a lower frequency in the accelerating structure, but this results in a very large structure and an inefficiency in using the rf in later higher-frequency sections. Compromises are possible and ultimately some optimization is desirable. In this study, the following structures were considered.

The Alvarez Structure

The higher beta sections of the linac will use an Alvarez structure with quadrupoles inserted in the drift tubes. The largest portion (length and cost) of the linac will consist of this structure and an operating frequency will be chosen to minimize the cost of the system. In the low-beta section a subharmonic of this frequency should be used. Frequencies below 50 MHz result in a structure that is considered too large. Reducing the size by capacitive loading with larger drift tubes

would lead to a less efficient structure and still result in inadequate space for quadrupoles. If the distance from gap to gap is made some multiple of 8λ , say 28λ , the required quadrupole gradient becomes feasible, but the phase-space acceptance area is reduced. This structure was ruled out in the low-beta section of the accelerator.

The AFP Structure

In an Alvarez accelerator, longitudinal stability is assured by accelerating on a negative phase angle; this results in transverse defocusing, which is overcome by external focusing devices such as quadrupoles. If the sign of accelerating phase is suitably alternated, then focusing and defocusing will exist both transversely and longitudinally, providing a net focusing force in all three planes. The principle of an alternating phase-focusing accelerator is not new, and recent work at LASL⁴ has revived interest in this method.

Initial work by Swenson⁴ was with a strictly periodic gap-phase-relationship. Computer studies of particle dynamics have shown poor acceptance for the present application.

The Spiral Structure

The spiral structure is a $8\lambda/2$ -mode acceleration device studied recently by groups at Cal-Tech⁵ and LASL.^{6,7} Quadrupoles are located not in the drift tubes but between cavities that contain one to three drift tubes.

Several linacs were modeled using spiral resonators in several configurations. All of these linacs required a large number of spirals, and the transverse acceptance was poor. Furthermore, the quadrupole length between the cavities was rather long because of the low velocity of the injected ions, and the length of the accelerator tended to be dominated by the quadrupoles.

It was felt that the spiral structure was not a good choice for the low-beta section.

The Wideröe Structure

The Wideröe is a structure that in recent designs includes focusing quadrupoles in the drift tubes. Versions of Wideröes are currently in operation at Manchester, at Orsay and now at the IMLAC in Darmstadt. The IMLAC design solves the difficult problem of quadrupole alignment by placing the quadrupoles only in the even-numbered drift tubes mounted directly from the tank wall.

In the Wideröe structure for very-low-velocity particles, the spacing between the drift-tube gaps, $8\lambda/2$, becomes so small as to make quadrupoles unrealistically thin. However, the longitudinal phase advance from gap-to-gap can be any odd multiple of π , and a $\pi - 3\pi$ sequence of alternating gap-to-gap lengths produces a satisfactory solution to the drift-tube length problem. In a $\pi - 3\pi$ accelerator, where the quadrupole length is 8λ long, the required gradient is less than 10 kG/cm for an operating frequency below 30 MHz.

Table 3-1. Design parameters for the rf accelerating structures in the low-beta section (for $258 U^{+1}$).

Parameter	Structure type				
	Split ring	Split ring	Wideröe	Wideröe	Alvarez
Periodicity	$38\lambda/2$	$8\lambda/2$	$58\lambda/2, 8\lambda/2$	$8\lambda/2, 8\lambda/2$	8λ
Frequency (MHz)	25	25	25	25	100
β_{in}	0.0026	0.007	0.01	0.02	0.05
β_{out}	0.007	0.01	0.02	0.05	0.154
Energy (MeV)	0.75-5.4	5.4-11.1	11.1-44.5	44.5-280	280-2000
Quad. length (cm)	15	21	15-30	30 -	--
Quad. gradient (kG/cm)	10	7	10- 5	5 -	--
Aperture radius (cm)	1.5	1.5	1.5	1.5	--
Cells	39 (3 gaps/cell)	19 (3 gaps/cell)	179	--	--
No. of quads	39	19	80	--	--
Total length (m)	12	5.5	31	--	--

A combination of π and $n = 3\pi$ structures appeared to be a good choice leading into the Alvarez section of the linac.

4.2. The Split-Ring Structure

Recently work has been done at Cal-Tech⁸ and ANL in the development of a structure, called the split-ring, which is suitable for heavy-ion acceleration in the low-beta region. Although it was intended to use this structure for the development of a superconducting accelerator, it can be used at normal temperatures as well. The cavities, which are heavily loaded to reduce the size at low frequencies, consist of two drift tubes and three accelerating gaps per cavity. This allows quadrupole elements to be placed between the cavities.

The split-ring structure is proposed as a possible solution to the problem of accelerating the ions to a velocity where they can be accepted into the more efficient Wideröe structure. However, this structure has not been used in this application and will require further development and design. Although this is considered a good choice at this time, there are other possibilities that could be investigated.

A partial list of design parameters is shown in Table 3-1 for singly charged U ions. It is intended that the table illustrate the feasibility only of designing a system of low-beta structures that meets the requirements of capturing, transporting, and accelerating the heavy ions from the ion source to the conventional linac system.

PERFORMANCE

The calculated performance parameters are given in Table 3-11.

Table 3-11. Performance parameters

258 U^{+1}	Ion source	Low-beta section	2-GeV linac
Intensity out (mA)	100	50	50
Normalized emittance—in (cm-mrad)	—	≤ 0.04	≈ 0.1
Normalized emittance—out (cm-mrad)	0.02	0.1	0.1
$\Delta p/p$			$\approx 1.3 \times 10^{-4}$
Δz (cm)			≈ 0.7

It is not unreasonable to expect that the linac will provide an ion current of 50 mA from an ion-source current of 100 mA. The major loss of current will result from the fraction of the ion-source current that is the desired species and the fraction of the dc beam that can be captured in the rf accelerating system.

A factor of 5 growth in the four-dimensional

transverse emittance is assumed from the ion source to the output of the linac. This is less growth than usually occurs in injectors for high-energy synchrotrons. This growth occurs in the dc high-gradient accelerating column and in the low-beta section of the linear accelerator. Special efforts should be taken to reduce aberrations and to preserve the beam quality. Estimates have been made of the seriousness of space-charge forces in the low-beta section of the accelerator using Ohnuma's formula.¹⁰ At a current of 20 to 100 mA a 1-cm aperture radius might be saturated. Further consideration should be given to this problem for a particular design. In particular, no thought was given to the transport system following the Cockcroft-Walton column.

The momentum spread and bunch length from the linear accelerator are important performance parameters. According to the usual damping formulas,

$$\frac{\Delta p}{p} = \pm \frac{2 \cdot 10^{-4}}{(T_{\text{GeV}})^{5/8}}$$

$$\Delta z = \pm 0.63 (T_{\text{GeV}})^{1/8} \text{ cm}$$

The momentum spread is so small that the longitudinal self-force can increase it substantially as the bunches travel from the linac to the next stage. Ways of minimizing this effect require further study.

TECHNICAL PROBLEMS

1. Ion sources must be developed to achieve higher intensities of the desired heavy-ion species with acceptable emittance, i.e., to maximize ion-source brightness. The extraction of the beam from the ion source and the acceleration in a high-gradient accelerating column requires matching of the source to the column. This equipment needs to be improved beyond the present technology so as to preserve the beam quality and improve the performance of the accelerator.

2. The low-beta section of the accelerator requires a buncher to maximize the capture of the dc ion beam into the rf linear accelerator. The requirements in this application are unique and a buncher must be designed and built for this application.

3. The split-ring rf structure has been proposed for the first rf accelerating stage. This approach needs to be examined more closely and compared with other choices. Development work should proceed on the split-ring structure or alternatives for the particular accelerator design chosen.

4. It would be desirable to develop quadrupoles with stronger gradients suitable for the limited space available in the low-beta section of the accelerator. This would allow greater acceptance into the low-beta section and a preservation of the beam quality from the dc accelerating column.

The present feasibility study indicates that it is possible to design a low-beta section of the

accelerator that will accelerate the heavy-ion beam from the ion source to the more conventional Alvarez linac accelerator. Parameters have been given that will allow further design studies to be made for the higher-energy sections of the accelerator. Problem areas have been listed so that work can proceed to revise and improve the design.

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There are no Appendices pertinent to this Technical Summary.

4. ATOMIC AND MOLECULAR PHYSICS

4.1. INTRODUCTION

We have identified two major and several minor categories of atomic and molecular data relevant to the design of heavy-ion accelerators for inertial fusion. The major ones are

- A. The cross section for the charge transfer between ions, e.g.,

and

- B. The cross section for the ionization of the projectile by background gases, e.g.,

The minor ones are

- C. The cross section for the ionization of background gases by the projectile, e.g.,

- D. Molecular binding energies needed to prevent premature dissociation of molecular ions during acceleration; and

- E. The cross section for the photodissociation of molecular ions.

Categories A, B, and C apply to both atomic and molecular projectiles, and D and E apply to molecular projectiles only.

The current status of our knowledge on these data is summarized below. Technical details upon which our conclusions are based are presented in Appendices 4-1 and 4-2.

ION-ION CHARGE-TRANSFER PROCESS (CATEGORY A)

The relative velocity between the ions due to their thermal motion is expected to be of the order of 5×10^7 cm/sec ($\sim \frac{1}{2}$ keV/nucleon or roughly 110 keV for a U^+ ion). At this velocity, charge transfer of the type (1) will be a dominant process (as compared to ionization, $U^+ + U^+ \rightarrow U^{++} + U^+ + e^-$) and its cross section will limit the lifetime of the ion beam in a storage ring as well as in an accelerator.

No theoretical or experimental data are available on the charge transfer between ions. As explained in Appendix 4-1, from some scanty data on the reverse process ($A^0 + A^{++} \rightarrow A^+ + A^+$), we conclude that the ion-ion charge-transfer cross section for heavy ions at relative kinetic energies of 50-100 keV would be of the order of geometric cross sections, i.e., $5 \cdot 10^{-16}$ cm².

Although this cross section plays a crucial

role in the design of storage rings and accelerators, it is one of the least known cross sections. We conclude, therefore, that the experimental and theoretical study of ion-ion charge transfer process should be given the highest priority.

Since the valence-electron configuration is expected to play an important role in the charge-exchange process (see Appendix 4-1), it would be worthwhile to study various options (1^+ through U^+) so that we can identify ions with small charge-transfer cross sections.

IONIZATION OF THE PROJECTILE BY BACKGROUND GASES (CATEGORY B)

To assess the vacuum requirement for accelerators and storage rings, we must know the charge-changing cross section of the ion, particularly at high velocities ($\beta = v/c > 0.1$). At such high velocities, ionization by background gas molecules is the dominant process. Fortunately, this is the case since the range of β values for which this is valid. Based on recent theoretical work by Gillespie (see Appendix 4-2), we can estimate the upper limit of such ionization cross sections, and the results at $\beta^2 = 0.4$ are presented in Table 4-1.

Table 4-1. Atomic and molecular cross sections relevant to the design of heavy-ion accelerators and storage rings.

Process	$\beta = v/c$	Cross section (cm ²)	Uncertainty
Charge transfer			
$1^+ + 1^+ \rightarrow 1^{++} + 1^0$	0.001	5×10^{-16}	Large
$U^+ + U^+ \rightarrow U^{++} + U^0$	0.001	1×10^{-15}	Large
Ionization of ion			
1^+ by H_2	0.66	2×10^{-18}	Upper limit
by N_2	0.66	2×10^{-17}	Upper limit
by p	0.66	5×10^{-18}	Reliable ^a
U^+ by H_2	0.66	3×10^{-18}	Upper limit
by N_2	0.66	3×10^{-17}	Upper limit
Ionization of background gases			
H_2 by 1^+	0.66	3×10^{-17}	Upper limit
by U^+	0.66	7×10^{-17}	Upper limit
by p	0.66	3×10^{-19}	Reliable ^b
N_2 by 1^+	0.66	1×10^{-16}	Upper limit
by U^+	0.66	3×10^{-16}	Upper limit
by p	0.66	1.5×10^{-18}	Reliable ^b

^aExtrapolated from the experimental data on $\bar{A}e$ by F. F. Rieke and W. Prepejchal, Phys. Rev. A **6**, 1507 (1972).

^bBased on the experimental data by Rieke and Prepejchal, cited above.

To assure the reliability of the Born calculations, it would be desirable to verify theoretical predictions by experiment. Although no heavy-ion accelerator suitable for our purpose exists now, the theory can be checked under the experimental conditions attainable at present (e.g., lower β , lighter ions). Appendix 4-3 lists the lowest charge states and the highest energy currently available for various ions at the Berkeley SuperHILAC/Bevalac. A reliable assessment of the vacuum requirement would lead to a more definite estimate on the cost of the accelerator and storage ring.

We conclude, for Category B, that a theoretical study is adequate for our needs. To estimate the Born cross sections, atomic form factors and incoherent scattering functions for the ions and gas molecules are needed. Such data are not readily available in the literature now, but can be generated with a minor effort. A carefully selected class of experiments at available facilities can be carried out in parallel with the theoretical study.

IONIZATION OF BACKGROUND GASES BY PROJECTILE (CATEGORY C)

Since the projectile carries a large number of electrons, the cross sections for the ionization of N_2 , O_2 , H_2 , etc. by the ion are rather large, even at $\beta^2 = 0.4$. It is necessary, therefore, to investigate the consequences of the ionization of background gases (e.g., beam instability caused by secondary electrons or vacuum breakdown). The cross section can be estimated by the same theoretical method as in Category B. Estimated cross sections are listed in Table 4-1, and the details are given in Appendix 4-2. Note that U^* is ~ 200 times as effective as a proton in ionizing nitrogen gas.

Again, the study of this problem is best handled by theoretical calculations. These calculations are similar to those discussed in the last section, and hence, the experimental verification of theoretical predictions for ionization of the projectile would also indirectly verify the validity of the cross sections discussed in this section.

DATA ON MOLECULAR IONS (CATEGORIES D AND E)

If it is concluded that a molecular dissociation scheme (such as the HEARTHTIRE concept proposed by Argonne) offers a substantial advantage over atomic ions in increasing the ion density in a

storage ring, then molecular data of the types D and E are needed.

A theoretical method to estimate the binding energy of molecular ions necessary for survival through the electromagnetic fields of an accelerator has been worked out on H_2^+ .¹ The same theory can easily be adapted for the Hr^+ ion, for example, to conclude that about 1 eV molecular binding energy would be sufficient to survive a magnetic field of 20 kG (equivalent to 4×10^6 V/cm electric field at $\beta = 0.66$).

The cross sections for the photodissociation of molecules are of the order of 10^{-17} cm². More precise values can be determined experimentally for those specific molecular ions that are found to satisfy the binding-energy criterion of Category D.

SUMMARY AND SUGGESTIONS

The most urgently needed cross section (to estimate the attenuation of an ion beam in a storage ring) is that for the ion-ion charge-exchange process. The estimated cross section for heavy ions is $5 \cdot 10^{-16}$ cm², but the uncertainty is large. We suggest that both theoretical and experimental studies be undertaken immediately. To assess the vacuum requirement in an accelerator or storage ring, the ionization cross section for the collision of the ion with the background gas must be known. The cross sections at $\beta^2 = 0.4$ obtained from the Born approximation are listed in Table 4-1. A better understanding of the vacuum requirement is mandatory for an accurate cost estimate for an accelerator and storage ring. Therefore, it is desirable to initiate a theoretical study of the ion-molecule ionization cross sections. To establish the reliability of this theoretical work, it is necessary to have a parallel experimental study with a selected class of presently available lighter ions at lower β .

The data on molecular ions should be studied as the needs are further identified.

Reference

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Appendices 4-1, 4-2, and 4-3 are pertinent to this Technical Summary.

5. ACCELERATOR PARAMETERS

Lee C. Teng

TARGETING REQUIREMENTS

The principal target- and reactor-vessel parameters that determine the required accelerator performance are:

1. The pulse shape of the beam power delivered to the target. Specifically, the most demanding are the peak power level and its time duration. About 60% of the beam energy is delivered at peak power. The other 40% is delivered prior to peak power according to some prescribed pulse shape extending over about 4 times the peak-power duration. For this study, we will concentrate only on the peak-power delivery.

2. The specific energy deposition in the target. This is the energy deposited per gram of target material, and determines the maximum beam-spot size and the range of beam particles in the target material. In addition, physical design of the target also imposes limits on the allowable beam-spot radius.

3. The radius of the reactor vessel and the maximum radius of the beam entry ports on the vessel wall. These determine the aperture of the final beam-focusing elements and, together with the spot size on target, the maximum allowable emittances and momentum spread of the beam.

For purposes of the study, we have chosen to consider two cases; one which we consider to have a high confidence of success, and the other which puts less demand on the accelerator but can probably be made to work. The parameters for these two cases are:

	High confidence	Minimal
Peak power (P)	600 TW	100 TW
Duration of peak power (τ)	10 nsec	6 nsec
Total beam energy ($W = P\tau/0.6$)	10 MJ	1 MJ
Specific energy deposition	30 MJ/g	20 MJ/g

The geometrical parameters of the target and the reactor vessel were taken to be:

Number of beams (n)	2 - 100
Beam-spot radius (r)	1 - 20 mm
Radius of reactor vessel (R)	10 m
Radius of beam port (R)	30 cm

ION TYPE, ENERGY, AND BEAM EMITTANCES

The targeting requirements, plus a range-energy relation, permit the construction of tables giving the beam requirements as functions of ion type and kinetic energy, T . We use a range-energy relation given for particles¹ in gold (R_{Au}) which, it should be noted, is questionable by a factor of 1.5 - 2 for

normal matter and even more so for matter in a hot ionized state.

For all three spatial degrees of freedom we use a unified invariant emittance defined by

$$\frac{\epsilon}{\pi} = q \frac{p}{mc},$$

where q is any one of the three Cartesian coordinates and p is the corresponding conjugate momentum variable. For the two transverse degrees of freedom this agrees with the usual normalized emittance; for the longitudinal degree of freedom this is the usual emittance expressed in mc units and hence has the same dimension of length as the transverse emittances.

The maximum allowed transverse emittance, ϵ_t , of the beam is given by the relation

$$\frac{\epsilon_t}{\pi} = \beta \gamma \frac{rR}{\lambda}.$$

The increase, Δr , of the radius of the beam spot on target due to chromatic aberration is given approximately by

$$\frac{\Delta r}{R} \sim \frac{\Delta p}{p}.$$

In order that $\Delta r < r$, we must have $\Delta p/p < r/R$. This gives the maximum allowed longitudinal emittance, ϵ_z , as

$$\frac{\epsilon_z}{\pi} = \frac{\gamma \beta c}{2} \left(\beta \gamma \frac{\Delta p}{p} \right) < \beta \gamma \frac{\gamma \beta c}{2} \frac{r}{R}.$$

At the very small beam spot on target, the space-charge forces, if unneutralized may be sizable. The ratio k of the unneutralized space-charge term to the emittance term in the equation giving the variation of the beam envelope at the target is an indication of the relative importance of space-charge forces in achieving the desired spot size. This is given by

$$k = \frac{2}{\beta \gamma} \frac{e^2}{mc^2} \frac{I}{ec} \frac{r^2}{(\epsilon_t/\tau)^2},$$

where I is the beam current.

In Tables 5-1 to 5-111, these parameters are given for singly charged Ca^{40} , I^{127} , and U^{238} ions, assuming two beams. For higher charge states we have $I \propto e$ and $k \propto e^2$. Inspection of the tables shows immediately the reason for the interest in heavy ions and low-charge states. For light ions or high-charge states one is forced to high beam currents, leading to severe space-charge problems in accumulating and transporting the beams. On the other hand, the difference between U and I ions at comparable kinetic energies is not very great, so that any ion with atomic number higher than 100 is a potential candidate for use.

In addition to being in a low-charge state, if the ions are to be accelerated in a resonant-type accelerator, they must remain in the same charge state throughout acceleration. Both intra-beam and beam-residual gas interactions tend to alter the charge state of the ions. The lifetime

Table 5-I. Targeting and beam requirements for two beams ($n=2$) of Ca_{40}^{+} ions.

T	I	R_{Au}	r	ϵ_t/π	ϵ_g/π	k
(GeV)	(kA)	(g/cm ²)	(mm)	(cm·mrad)	(cm·mrad)	
High Confidence Case P=500 TW, $\tau=10$ nsec, E=30 MJ/g, W=10 MJ						
2	150	0.45	3.4	3.41	178	7.35
4	75	1.4	1.9	2.77	197	1.25
6	50	2.8	1.4	2.43	205	0.44
8	37.5	4.5	1.1	2.24	211	0.21
10	30.0	6.9	0.88	2.04	209	0.11
Minimal Case P=100 TW, $\tau=6$ nsec, E=20 MJ/g, W=1 MJ						
2	25	0.45	1.33	1.32	41.4	1.22
4	12.5	1.4	0.75	1.07	45.8	0.21
6	8.33	2.8	0.53	0.94	47.5	0.073
8	6.25	4.5	0.42	0.87	49.0	0.034
10	5.00	6.9	0.34	0.79	48.5	0.019

Table 5-II. Targeting and beam requirements for two beams ($n=2$) of Fe_{56}^{+} ions.

T	I	R_{Au}	r	ϵ_t/π	ϵ_g/π	k
(GeV)	(kA)	(g/cm ²)	(mm)	(cm·mrad)	(cm·mrad)	
High-Confidence Case P=600 TW, $\tau=10$ nsec, E=30 MJ/g, W=10 MJ						
6	50	0.25	4.6	4.44	226	0.84
10	30	0.48	3.3	4.17	268	0.23
20	15	1.2	2.1	3.80	327	0.038
40	7.5	3.5	1.23	3.27	361	0.0060
60	5.0	6.5	0.90	3.04	378	0.0020
80	3.75	10.5	0.71	2.85	381	0.0009
Minimal Case P=100 TW, $\tau=6$ nsec, E=20 MJ/g, W=1 MJ						
6	8.33	0.25	1.78	1.72	52.5	0.140
10	5.00	0.48	1.29	1.61	62.2	0.038
20	2.50	1.2	0.81	1.47	76.0	0.0064
40	1.25	3.5	0.48	1.27	83.8	0.0010
60	0.833	6.5	0.35	1.18	87.9	0.0003

Table 5-III. Targeting and beam requirements for two beams ($n=2$) of U_{238}^{+1}

T	I	R_{Au}	r	ϵ_t/π	ϵ_l/π	k
(GeV)	(kA)	(g/cm ²)	(mm)	(cm·mrad)	(cm·mrad)	
High-Confidence Case $P=600$ TW, $\tau=10$ nsec, $E=30$ MJ/g, $W=10$ MJ						
20	15	0.37	3.8	4.91	325	0.055
40	7.5	0.86	2.5	4.65	411	0.0092
60	5.0	1.5	1.88	4.40	451	0.0031
80	3.75	2.1	1.59	4.38	494	0.0014
100	3.00	2.9	1.35	4.25	511	0.0008
200	1.50	7.5	0.84	4.06	575	0.0001
Minimal Case $P=100$ TW, $\tau=6$ nsec, $E=20$ MJ/g, $W=1$ MJ						
10	5	0.17	2.16	1.96	56.8	0.054
20	2.5	0.37	1.47	1.90	75.5	0.0092
40	1.25	0.86	0.96	1.80	95.4	0.0015
60	0.833	1.5	0.73	1.71	104.8	0.0005
80	0.625	2.1	0.62	1.70	114.7	0.0002
100	0.500	2.9	0.52	1.64	118.8	0.0001

of a specific charge state is a crucial question that should be studied as soon as possible.

During the final transport, especially after entering the reactor vessel, the beam may be traveling in a poor vacuum and extremely "dirty" environment. The ions could then be fully stripped and the beam neutralized. The behavior of a fully stripped and neutralized beam is another crucial question that should be investigated immediately and thoroughly.

For this study we shall assume that the charge state of the ions remains unchanged and the beam remains unneutralized all the way to the target.

POTENTIALS OF ACCELERATOR SYSTEMS

Since the goal of this short study is to assess the feasibility of using heavy ions for imploding pellets, we have concentrated for the most part on U_{238}^{+1} ions, and on the minimal case (Table 5-III), disregarding questions of costs and efficiencies. The reason for proceeding in this way is that an unfavorable conclusion would seem to deny any possibility of using heavy ions, whereas a clearcut favorable conclusion would leave open the possibility of choosing all the parameters to optimize cost and efficiency, and of increasing the beam intensity to approach the high-confidence case with its greater likelihood of success at the target end.

Although many accelerator configurations have been suggested, we find only two distinct approaches. One is the single-pass, high source-current approach exemplified by the induction linac and the more speculative collective-effect accelerators. For

this approach no supplementary current amplification is necessary. The other approach involves relatively conventional sources, rf linacs, and ring accelerators, and very much lower initial beam current. For this approach, a subsequent current amplification by a large factor is needed to obtain the required beam current on target. We concentrated largely on this latter, more conventional approach, leaving the induction linac to a group of experts in that well-developed but relatively specialized technology. It is of course quite possible that the former approach may well be demonstrated to be more economical, and more efficient and reliable.

Performance of Conventional Sources and rf Linacs

Studies made by the Low-Beta Acceleration Group gave the following performance parameters as achievable with high confidence for such a system.

Ion type	U_{238}^{+1}
Beam current	0.05 A
Beam bunch frequency	25 MHz
Number of ion/bunch	1.25×10^{10}
Emittances of each beam bunch	
Transverse ϵ_t/π	0.1 cm·mrad
Longitudinal ϵ_l/π	0.012 cm·mrad
6-D density $\equiv \frac{\text{No./bunch}}{\left(\frac{\epsilon_t}{\pi}\right)^2 \frac{\epsilon_l}{\pi}}$	1.04×10^{14} (cm·mrad) ⁻³

Targeting Requirement

We take as our exemplary case that given in the box in Table 5-III — namely the minimal targeting requirement with two 40 GeV U^{92} beams. The maximum allowable emittances of each beam are

Transverse ϵ_t/π	1.80 cm-mrad
Longitudinal ϵ_L/π	95.4 cm-mrad

With the 6-D density supplied by the linac, if the total available 6-D volume is filled we can get for each beam

$$1.04 \times 10^{14} \times (1.80)^2 \times 95.4 = 3.2 \times 10^{16} \text{ ions.}$$

This is more than 700 times the required 0.45×10^{13} ions. In practice, it is of course difficult if not impossible to fill the entire available 6-D phase volume. But the factor 700 gives one confidence that even with rather inefficient filling the necessary number of ions can be achieved.

CURRENT AMPLIFICATION AND CLASSIFICATION OF ACCELERATOR SYSTEMS

For the exemplary case a current of 1250 A is required for each one of the two beams. To obtain this beam current the 0.05 A beam from the linac must be amplified by a factor of 25 000. There are four ways to amplify current.

Velocity Increase due to Acceleration

If the bunch spacing is held fixed during acceleration as in a synchrotron the average beam current will increase linearly with velocity (or β). It is conceivable that the bunch spacing can even be reduced as the beam is accelerated in a linac. The beam current will, then, increase faster than the velocity. In some sense, this mechanism is operative in an induction linac. Adhering to only conventional accelerators, this implies that to the extent permitted by the space-charge limit and the repetition rate, it is advantageous to transfer the beam from the linac into a synchrotron at the lowest possible energy and do the remaining acceleration in the synchrotron. This is also more economical, since the capital cost/GeV of a linac is considerably higher than that of a synchrotron.

We shall denote this current amplification factor due to acceleration by A . This amplification is limited by the space-charge detuning in the synchrotron at injection. The tune shift, $\Delta\nu$, is given by

$$\Delta\nu = \frac{e^2}{mc^2} \frac{1}{(\beta\gamma)^2} \frac{I}{ec} \frac{L/2\pi}{\epsilon_t/7\pi}$$

where L is the circumference of the synchrotron and I is the peak current in the bunches. To avoid losing beam on major resonances, $\Delta\nu$ must be smaller than 0.25. An amplification factor of 10 in average current is probably close to maximum, since the rf accelerating system must be frequency modulated.

Stacking in Transverse Phase-Space

This is commonly known as multi-turn injection.

With a 4-D beam, transverse emittance of 0.1 cm-mrad and an available 4-D transverse phase volume of 1.80 cm-mrad, one can, in principle, stack a maximum of

$$\left(\frac{1.80}{0.1} \right)^2 = 324$$

turns. To stack the beam to this extent in both transverse planes by the conventional orbit manipulation scheme is difficult; values of $\beta \sim 10^{-3}$ in either plane represent standard practices. Charge-exchange or molecular dissociation injection schemes can, in principle, give full stacking in both planes, but depends on finding appropriate ion and molecule types, and a high efficiency exchange mechanism. The scheme proposed for Hearshfire is

$$\Delta H^+ = I^+ \cdot H^+$$

induced by Xe flashlamps. In any case, we shall denote the current gain factor obtained from transverse stacking by S .

Bunching in Longitudinal Phase-Space

Two schemes have been employed. In the first scheme, the beam is bunched in one single step using a buncher. The most efficient buncher should supply a linear voltage ramp. To produce a large bunching factor for a 40-GeV U^{92} beam the buncher voltage requirement is rather severe. In the second scheme, the beam is bunched adiabatically by an rf system that can operate over a wide range of frequency to pass from lower to higher harmonic numbers in binary steps. With proper adiabatic manipulations of the rf voltage when passing from one harmonic number to the next, the beam is bunched progressively by the shorter buckets at higher harmonic numbers. For this scheme the technical goal demand on the rf system is not as severe, but the time required for bunching is rather long.

We denote the bunching factor so obtained by B . A value for B of the order of 10 is common; schemes for achieving greater values have not yet been tested.

Multiple Beam Beams

With K bunch beams each with current I/K hitting the target on the same spot, the effect is clearly identical to that of one beam with current I .

The four parameters A , S , B , K , together with the initial beam current, I_0 , from the linac serve to classify the various accelerator systems considered. The final current, I_f of each beam is

$$I_f = A \cdot S \cdot B \cdot K \cdot I_0$$

and the total power on target is

$$P = n \cdot I_f \cdot L$$

where n is the number of beams. If the beam-on-target duration is τ (considering only the peak-power duration) the total physical length of the beams before the B -fold bunching is $(\beta c) \cdot B \cdot K \cdot n$. Hence, the circumference of the accelerating/stacking/bunching ring must be larger than this value. We shall see later that with reasonable magnetic

guide-field strength, the ring circumference generally turns out to be much larger. This suggests that one may, by increasing the number of branches per beam, approach the high-confidence targeting requirement; or if the minimal requirement is indeed adequate, the use of one single accelerator system as the igniter for several reactors at the same time becomes possible.

Table 3-B gives three examples of the above method of classification: in each case rather large values of S , B , or K are involved. When classified in this manner the trade-offs between these parameters are immediately clear. For a given maximum attainable I_0 it is best to employ all four multiplicative factors A , S , B , and K to the same degree of exploitation.

ONE EXAMPLE

For the exemplary case considered above we need a total current amplification of $A \cdot S \cdot B \cdot K = 25,000$. A more-or-less equal-exploitation could be

$$\begin{aligned} A &= 4 \\ S &= 30 \quad (6(\text{horizontal}) \cdot 5(\text{vertical})) \\ B &= 52 \\ K &= 4 \end{aligned}$$

To see that this set of parameters is indeed internally consistent we have:

Injection Energy, Circumference, and Space-Charge Limit of Synchrotron

A B -ratio (A) of 4 puts the injection energy (output energy of linac) at 1 GeV. The circumference of a 40-GeV U₃₀ synchrotron with an average peak guide-field of, say, 12 kG is 2440 m. Stacking 30 turns ($S = 30$) in the transverse phase space one gets a beam current at injection of 1.5 A. The space-charge time shift at injection corresponding to a transverse emittance of $\epsilon_q/\pi = 1.8$ cm-mrad is

$\Delta t = 0.24$, within the allowable limit. Since the maximum transverse stacking allowed is 324 turns, to stack 30 turns should present no special difficulty.

Bunching and Number of Branch Beams

The remaining current amplification factor $25,000/4 \cdot 30 = 208$ is to be shared between the bunching factor, B , and the number of branches, K , making up one beam. A value of $B = 52$ is credible regarding the technological aspects, and is certainly far below the theoretical maximum-allowable value of $95.4/0.012 \approx 8000$. One could reduce B further by increasing K (say, $B = 26$, $K = 8$), and may indeed have to do so if the technological requirements for obtaining $B = 52$ turn out to be overly severe, but a large number of branch beams is costly, and difficult and less reliable to operate.

Length of Beam

The total length of beam before bunching is $(\text{arc}) \cdot B \cdot K \cdot n = 39''$ m, which is only $\sim 1/6$ of the circumference of the synchrotron. Thus, we can feed six reactors with this single accelerator system or, alternatively, readjust the parameters to approach the high-confidence targeting requirement.

To summarize, in this accelerator system one starts with a 1-GeV, 50-mA U₃₀ linac injecting 30 turns into a synchrotron to give a current of 1.5 A. Accelerated to 40 GeV in the synchrotron the beam current will be 6 A. Sections of the beam each 50 m long are longitudinally bunched to 0.90π (6 nsec) to give a current of 312 A and are then extracted from the synchrotron. Four sections of the beam are transported and focused to hit the target simultaneously on the same spot to give an equivalent beam current of 1250 A.

Reference

1. See Fig. 1 of Technical Summary 1, J. Mackolls, Target and Reactor Design (this report).

There are no Appendices pertinent to this Technical Summary.

Table 3-B. Parametric classification scheme for accelerator systems applied to three examples.

Example	A	S	B	K	I_0 (mA)	I (A)
Hearthfire ²	8	4000	3.75	100	0.5	6000
Maschke Expt. I	5	10	5000	1	20	5000
Maschke Expt. II ²	1	10	5000	1	167	5000

²Parameters are taken from R. L. Martin and R. C. Arnold, "Heavy Ion Accelerators and Storage Rings for Pellet Fusion Reactors," RLM/RCA-1 (February 9, 1976). In this example $S = 250$ (injection into synchrotron) $\times 17.5$ (injection into storage ring), and

$$B = \frac{5 \text{ usec}/100}{8 \text{ nsec}} = 3.75$$

³Since only ranges of parameters were given by A. W. Maschke (distributed at the ERDA Summer Study of Heavy Ions for Inertial Fusion - unpublished report), the values shown represent one particular set chosen within the specified ranges.

6. BEAM MANIPULATIONS

Mark J. Barton

We concerned ourselves with examining fundamental accelerator problems that might have a bearing on the feasibility of developing heavy-ion beams with parameters suitable for the target requirements. The three fundamental questions are:

- Are space-charge limits reasonable?
- Are there lifetime limiting processes to be concerned with?
- Do phase space requirements on target fit with the available brightness from rf linacs and can schemes be devised which match the source to the target?

The space-charge limit in circular rings is usually stated in terms of the incoherent frequency shift of betatron oscillation frequencies and is given by the formula:

$$\Delta\nu = \frac{1}{I_2} \cdot \frac{R}{n} \cdot \frac{1}{\beta^2 \gamma^2} \quad (1)$$

where I is the beam current, I_2 is the Alfvén current (mc^2/e) for the particle, R is the major radius of the accelerator, n the normalized emittance, and β the usual relativistic factor. The $\Delta\nu$ is usually limited to $1/4$ for long-term stability of the beam. Because the I_2 is so large for heavy ions in the low-charge states, it is relatively easy to store large currents and stored energy in circular rings. The $\Delta\nu = 1/4$ limit might be drastically violated for a few turns during a rapid bunching of the beam. A consensus of opinion was that since the frequency is only shifted for the incoherent motion, closed orbit errors that appear as coherent motion at resonances would not be a problem and a $\Delta\nu$ of several integers could be tolerated.

Some parameterizations of space-charge limits on injection energy and stored energy for an intermediate synchrotron are available.¹

Space-charge problems can also occur in the beam transport between the accelerator and the target vessel. If this transport system is made up of quadrupole lenses, the power that can be transmitted through the channel is given by a formula²

$$P = 1.6 \cdot 10^{12} \left(\frac{A}{\pi} \right)^{4/3} \epsilon^{2/3} (\beta\gamma)^{7/3} (\gamma - 1) B^{2/3} \quad (2)$$

where B is the quadrupole pole tip field in teslas and ϵ the emittance in meter-radians. For low-charge-state heavy ions, formidable amounts of power can be transmitted. For example ($B = 1$ tesla):

$$U^{3+}, 100 \text{ GeV}, \epsilon = 4 \text{ cm-mrad}, P = 1400 \text{ TW}$$

$$U^{1+}, 40 \text{ GeV}, \epsilon = 3 \text{ cm-mrad}, P = 140 \text{ TW}$$

but there seems to be a problem for high-charge states; e.g.

$$U^{55}, 40 \text{ GeV}, \epsilon = 1 \text{ cm-mrad}, P = 0.6 \text{ TW}$$

A solution to this problem might be to use solenoids. Now in a solenoid the phase space to match a small-beam emittance would appear as the small circle in Fig. 6-1, while for a space charge solution, the emittance must be matched to the shaded ellipse. If that matched solution were used, then n

Fig. 6-1. (XBL 7612-11095)

the absence of space charge or with nonlinear space charge, or because of momentum dispersion, etc., the entire large circle would be filled. Thus, the emittance would be blown up by a large factor. The details of how bad this effect might be is a numerical question that cannot be resolved in a short time. A brute-force solution would be to make the focusing strong enough that there is an acceptably small ratio between the size of the two circles, which corresponds to what was done in the quadrupole case in formula (2) of Ref. 2. This analysis is included in the appendix by Courant.³ It appears from his results that if, indeed, this brute-force solution is necessary, then solenoids are probably not viable for transport.

The most serious limitation on lifetime of beams appears to be collisions of the type

$$U^{1+} + U^{1+} \rightarrow U^{2+} + U^{0+}$$

which might have cross sections as large as 10^{-15} cm^2 . A rough estimate of the lifetime is given by:

$$\tau = 4 \left(\frac{Ze}{I} \right) \frac{B^{3/2} \epsilon^{1/2}}{\sigma \gamma} \quad (3)$$

where B is the local wavelength of the focusing system. In an example case:

$$I = 8 \text{ A}$$

$$B = 15 \text{ m}$$

$$\sigma = 10^{-15}$$

$$\epsilon = 3 \text{ cm-mrad}$$

$$40 \text{ GeV}, U^{1+}$$

$$\tau \approx 125 \text{ msec}$$

This lifetime is about a factor of 100 shorter than that due to ionization loss in a gas of 10^{-11} Torr . Such a short lifetime suggests short storage times and that the current should be kept low as long as possible. If long life at high currents is required, the situation can be improved by using weak focusing, but such a solution is expensive.

The question of phase-space matching was examined in conjunction with the group studying parameters. This work was primarily an exercise

in checking feasibility for an example target. The target chosen was the so-called "minimal" target for which we require:

- $U^{\dagger}$, 40 GeV
- 6 nsec pulse length
- 2 beams
- 1200 A/beam
- $\Delta p/p = \pm 0.003$ momentum spread
- $\epsilon = 3$ cm $\cdot$ mmrad, or
- $\epsilon_{\text{normalized}} = 1.8$ cm $\cdot$ mmrad.

These parameters were derived by the parameters group using information supplied by the target group. A circular accumulator ring or accelerator would be required to supply these requirements from the linac. The linac group provides a beam of

- $U^{\dagger}$, 40 GeV
- $I = 50$ mA
- $\epsilon_n = 0.1$ cm $\cdot$ mmrad
- $\Delta p/p = 2 \times 10^{-5}$

The phase-space density of this beam must be greater than that required on target. Thus,

$$\frac{0.05}{2 \times 10^{-4} (0.1)^2} > \frac{1200}{3 \times 10^{-3} (1.8)^2} \left(\frac{\beta_i}{\beta_f} \right)^{3/4}$$

This indicates that the phase-space matching is possible and one is simply left to use imagination in devising rings to do the job. This comparison is for a 40 GeV linac and stacking ring but is valid also for a lower-energy linac and synchrotron. For example, an accumulator ring could stack the beam for 18×18 turns building up the current from 50 mA to about 16 A and the emittances go from 0.1 cm $\cdot$ mmrad to 1.8 cm $\cdot$ mmrad. A modest bunching of about 75 brings the beam up to the required 1200 A. Interestingly enough, such a ring would have to be nearly an order of magnitude larger than that required to accumulate enough charge. This size is because the magnetic field must be practically low. Thus the ring is capable: (at the expense of a longer injection pulse) of storing nearly an order of magnitude more charge and stored energy than needed. This is a margin available to cover up losses or dilution that might occur. Thus one could attach a very high degree of confidence in this approach. Another concept using two accumulator rings that shows a specific method of doing stacking was presented by the author.⁴

A synchrotron could be interposed between the linac and the accumulator or, if the final bunching can be done in the synchrotron, the linac-synchrotron combination could do the whole job. At least three synchrotrons were considered during the study. In one described by Teng,⁵ a 2 GeV linac would be used to inject into a synchrotron. Only 40 turns need to be stacked. When accelerated to 40 GeV and bunched by a factor of 150, 12 beams would be available of which only two are needed. Here again, we see that the ring is big enough to hold more beam than required. Yanglai Cho examined a linac-synchrotron-accumulator system. In this

system, a 1-GeV linac injects into a 10-Hz rapid-cycling synchrotron of 720 m radius. Only about two turns are injected. The beam in this synchrotron is accelerated to 40 GeV and then put into a smaller high field, accumulator ring. This ring would have a 105 m radius and use 60 kG average field. Seven turns per accelerator pulse and three pulses would be stacked. The beam would be bunched by a factor of 27 and then quickly compressed by about 8. Twenty-four beams would be extracted and targeted. Another synchrotron was examined in detail by T. K. Khoe. He uses a linac of 1 GeV and uses it to inject into a synchrotron. Parameters of the synchrotron were developed and are given in Table 6-1.

Table 6-1. Synchrotron Properties

Maximum energy	40 GeV
Repetition rate	10 Hz
Normalized beam emittance	1.88 cm $\cdot$ mmrad
Maximum field	11 kG
Number of particles	1.9×10^{14}
Magnet radius	420 m
Machine radius	720 m
Required maximum energy gain per turn	50 MeV
Rotation frequency	8.9 - 35.4 kHz
Vacuum	$P \leq 10^{-11}$ mm Hg

Note: At the end of the acceleration, the beam is debunched and rebunched with harmonic number $h=20$. The length of each bunch is 150 nsec, determined by the space-charge limit at 40 GeV. The 20 bunches are extracted simultaneously and each compressed to 6 nsec. The bunching voltage is approximately 1 GV.

Of course, the Hearthfire project, which was not explicitly examined as part of the study, forms an example of a synchrotron used as an accelerator. It has the feature of a very large number (100) of output beams. With the type of linac performance projected at the summer study, only about 24 turns of beam would need to be stacked to achieve the desired performance. The proponents of that concept have pointed out, however, that even if linac performances are short of projected values the desired beam brightness can be achieved using an irreversible process such as molecular dissociation or charge exchange. The group participants concurred in this view but are encouraged by the linac experts who suggest current densities that may eliminate the need for such processes.

All of these schemes using synchrotrons are attractive, at least for scientific breakeven type experiments, because they might be cheaper in initial cost. They are understood and will probably work as projected. A major difficulty is the short lifetime due to ion-ion collisions and a better understanding of this process would be necessary

before investing much money in this approach. A major disadvantage of this approach would be the large power required to operate the accelerator. A rapid-cycling synchrotron would probably be the least efficient of all the accelerator choices and for a real power plant might push the target gains or sizes into an undesirable region.

In summary, the conventional-accelerator technology required to match an rf linac of the suggested performance to a target with the specified requirements seems reasonable and can be approached with a variety of viable techniques with a high degree of confidence.

References

1. D. Berley and G. J. Danby, Appendix 6-3 of this report.
2. A. W. Maschke (distributed at the ERDA Summer Study of Heavy Ions for Inertial Fusion - unpublished report).
3. E. D. Courant, Appendix 6-2 of this report.
4. M. A. Barten, Appendix 6-1 of this report.
5. L. C. Teng, Technical Summary 5 of this report.

Appendices 6-1, 6-2, 6-3, 6-4, and 6-5 are pertinent to this Technical Summary.

7. INDUCTION LINAC

Dennis Keefe

TECHNOLOGY

The Induction Linac utilizes a sequence of singly pulsed accelerating cavities arranged in a straight line. Each cavity is loaded with ferromagnetic material (thin-laminated iron or ferrite) driven by a modulator (pulse-forming network and spark-gap/thyratron switch) and acts as a transformer in which the particle beam plays the role of the secondary winding. The phasing of the pulsed cavities is adjusted to accelerate a particle of particular q/m in correspondence with the mean accelerating field E (MV/m). Thus, only a portion of the structure is energized at one time, and this region propagates down the accelerator in synchronism with the particle bunch.

The technology is relatively simple but not widely known, simply because only four machines have been built in this country [Astron I (2 MeV), Astron II (5 MeV), ERA (4 MeV), and NBS (0.25 MeV)]. The ERA induction linac is shown in Fig. 7-1.

Fig. 7-1. Photograph of the ERA 4-MeV induction linac. (CBB 716-2082)

In addition, two other electron machines are in the planning stages at LLL, while conceptual designs for induction linacs for very special purposes have appeared in the literature.^{1,2} Basically the technology seems best matched to applications where a very high current (100 to 10 000 A) is required for a short time (10 to 2000 nsec) with rather precise voltage control. In addition, the ability to run at repetition rates up to 100 Hz or so seems to come naturally for very little extra expenditure on additional charging supplies. If the beam current can be matched to the cavity voltage and impedance, the pulsed-energy conversion efficiencies can approach 50% in the absence of eddy-current and hysteretic losses. Achievement of high impedances for accelerating low-current beams is an outcome of the NBS work on parallel driven cores (see Table 7-1).

Table 7-1. Induction linac parameters achieved (or designed).

Machine	I (A)	τ (nsec)	Rep. rate (Hz)	V/gap (kV)	E (MV/m)
Astron II	1000	300	30	10	
ERA Inj.	1-5000	50	1(10)	250	0.25
NBS	1000	1700	1(10)	250	
ERA (des)	--	10	200	350	5.5
Excav (des)	5000	1000	360	80	0.11

INDUCTION LINAC AS AN IGNITER

The short-pulse, high-current features make the induction linac worthy of consideration as an igniter for D-T pellets and a preliminary set of workable self-consistent parameters were presented.³ That example was explored, however, in the context of a scientific break-even experiment and has the features shown in Table 7-11.

Table 7-11. Parameters of example of Ref. 3.

Ion	$U \times 10$
Q =	0.2 MJ
T_f =	25 GeV
$\bar{E}$ =	3.0 MV/m
L =	800 m
I_j =	84 A
I_f =	8400 A
τ_i =	1000 nsec
τ_f =	10 nsec
V_{col} =	2.5 GV
P_f =	20 TW

The work of the ERDA Summer Study Group has been directed, on the other hand, to parameters appropriate to a power plant of ≥ 1 GW. Because the number of parameters that can be varied is rather large, and some of the correlations somewhat subtle, the focus of attention was on just two examples - which in hindsight are almost certainly not optimum. Table 7-111 shows the cases studied, the range of variation of parameters that still needs exploration, and an indication of how far the examples lie above the case of minimal performance. A realistic average voltage gradient of 2.5 MV/m was assumed. Note that the repetition rate up to 10 Hz certainly, and perhaps 100 Hz, probably can be considered as a free parameter, and this point will not be addressed further.

Comparison of Tables 7-II and 7-III will reveal that the increased requirements of energy (E) and power (P) by a factor of 15 to 20 over the Ref. 3 example have forced one in the direction of lower charge state and hence a longer accelerating column.

Table 7-III. Cases addressed in summer study.

Specific cases	Range to explore	Factor above level of minimum interest
Q = 3 MJ	1 - 10	3
P _{tgt} = 450 TW	100 - 600	4.5
T = 25 GeV	10 - 100	
A = 238	100 - 250	
q = 5(1)	1 - 5	
N(U) = 7.5 × 10 ¹⁴		
τ _f = " nsec		
ε = 6π 8γ cm·mrad		
V _{col} = 8.3(25) GV		
L _{col} = 3.3(10) km		

In addition, it will emerge that the increased power will impose rather special requirements on the beam handling in the front 1 to 2% of the accelerator column that were not in the earlier example. The remaining 98 to 99% of the accelerating column, however, is a good match to the characteristic features of pulse length and current level inherent in the induction linac technology.

LIMITING FEATURES AND CHOICE OF PARAMETERS

A convenient approach to setting parameters is to consider the end point of the accelerator when acceleration is essentially complete and before the beam is subdivided and the segments finally compressed in time to 7 nsec for delivery to the target. The most strongly limiting effect is the beam-power transmission limit discussed by Maschke⁴ and later by Courant.⁵ Using Courant's Eq. (1),

$$P = 2.6 \times 10^{15} B^{11/5} Y^{5/3} \frac{(B_0^{2/3})}{q^{4/3}} \text{ Watts (1)}$$

for a normalized emittance ε of 6π cm·mrad. For a practical pole-tip field strength of B₀ = 1 T, this yields maximum transmitted power and current for charge states 3 and 1 as:

$$P_3 = 35 \text{ TW} \rightarrow I_3 = 4.2 \text{ kA}, \tau_3 = 85 \text{ nsec}$$

$$P_1 = 150 \text{ TW} \rightarrow I_1 = 6.0 \text{ kA}, \tau_1 = 20 \text{ nsec}$$

at the 25-GeV end point. (How these powers can be transformed up to the goal of 450 TW on target will be discussed later). These currents are within the

comfortable range for induction acceleration and as one proceeds backward up the accelerator, it is desirable to maintain the current as high as possible. However,

$$P_1(B) = V_1(I) \cdot I_1(B) = 1/2 \frac{M_0 c^2}{q} I_1(B) \quad (2)$$

Combining this with Eq. (1) it is clear that one can remain everywhere within the power limit if I₁(B) is made to vary as B^{3/5} (viz., as B^{3/6} or B^{5/6}, where S is the approximate distance along the accelerator measured from the front end of the column). This would require a pulse-length variation of

$$\tau = B^{-5/5} \quad \text{or} \quad S^{-5/6} \quad (3)$$

How close this can be approached with reasonable ramping voltages remains to be studied in detail. Since Eq. (3) impresses only a lower limiting condition, larger values of τ are permitted if desired. In particular, in the case of q=1 it is undesirable to compress all the way to τ₁ = 20 nsec, but instead to maintain a constant value after τ = 50 to 70 nsec has been reached in order to avoid the steep gradients needed to counterbalance space-charge effects in the bunch.

For a major fraction of the machine (> 95%) both the currents and pulse lengths are in ranges that are well adapted to the induction linac technology, and it is possible therefore to achieve rather good efficiency in converting electrical energy into beam energy. There is an engineering trade-off to be examined between the capital cost and the efficiency desired (see Appendix 7-1).

From Fig. 7-2 it can be seen that the Maschke power-transmission-limit formula imposes a requirement that the pulse length be rather long and the current quite low (tens of amperes) in the front few percent of the machine (referred to for convenience as the Transition section). Thus, apart from the main induction linac, there are three sections of the system for special consideration:

- Ion Source
- Front few percent of accelerator - Transition section
- Transmission and bunching between the end of the accelerator and the target.

Ion Source

For the ion source there are several choices: magnetically insulated diode (see Fig. 7-3), reflex diode, or stacked conventional sources. The high-voltage diodes have the outstanding advantages of creating the ions at high energy and of having very large current density. They have not yet, however, been studied enough to be sure that they can provide adequate brightness. Since the currents needed are in the range of a few tens of amperes (compared with kilocumperes available), there is considerable hope of achieving this goal by operating with larger spacing and with careful control of the anode plasma conditions. At this time, still early in the development history of these diodes, there seem to be some reasons for antici-

Fig. 7-2. Plot of beam parameters vs percentage accelerator length for U^{+1} and U^{+3} beams, as limited by quadrupole-focusing capability with 1 T on the pole tips (from Maschke-Courant formula). (XBL 7612-11086)

Fig. 7-3. Sketch of a magnetically insulated diode, courtesy of Shearer. (XBL 769-3933)

peating that the magnetically insulated diode is to be preferred. The geometry allows electrons to drift (by $\mathbf{E} \times \mathbf{B}$) indefinitely around without having a preferred location for accumulation of charge, thus giving hope of operation on the microsecond, and longer, time scale. Also, the magnetic field is transverse to the direction of motion of the ions and so leads to a deflection in one plane which can be corrected for exactly. In the reflex diode there is some concern about instabilities of the virtual cathode that could lead to energy modulation of the ions; this situation could be ameliorated by using a second image cathode pierced to allow ion extraction. A more fundamental objection is that the magnetic field in which the ions are born is parallel to the direction of

motion of the ions, which leads to an undesirable coupling of the two transverse degrees of freedom, and hence a dilution of useful emittance in the later transport system of the accelerator. This dilution seems not to be serious at the 1 kg level but could become so at the 10 kg for more level required.

Stacked conventional sources can give adequate brightness if operated in the range of 30 to 100 nsec. The size required is rather large but not unduly so, approximately 15 to 30 cm radius. There is a considerable body of experience in operation and performance to be drawn upon, and long-pulse operation is assured. A slight disadvantage is the relatively low voltage (about 100 kV) which will cause the front-end, low energy transport and acceleration system to become longer.

Induction Linac

Reference to Fig. 7-2 shows that for U^{+3} the pulse length needed in the first few percent of the accelerator exceeds the 1 nsec or so that seems most suitable for induction linac cavities. Possible methods of handling this problem include using a conventional rf linac to inject about 10 bunches into an accumulator ring at a few hundred MeV and accelerating thereafter in the induction linac (see Appendix 7-2).

Alternatively, five or so beams of lesser current could be extracted from a magnetically insulated diode or parallel conventional sources, separately accelerated to a few hundred MeV, and reassembled via magnetic septa into the required high-current beam. Appeal to various neutralization schemes would, of course, provide other solutions; these are, as yet, unexplored. For pulse lengths, up to 20 nsec, are certainly feasible—at the expense of decreased accelerating field—and may well be a viable method of proceeding. At the present level of investigation, however, it seems prudent to suggest that other methods should be considered. It should also be pointed out that the Maschke formula is based on conditions that are not valid at low energies. In other words, a realistic transport and accelerating system still needs to be studied and defined in detail.

Bunching

Pulse shaping can be rather easily accomplished in the induction linac (c.f. FRA injector experience where a ramped cavity could be turned on to produce a programmed ramp in the energy of the beam with time). For instance, towards the end of the accelerator it is envisioned that addition of an $\mathbf{E}$ term (or higher-order, odd derivative) to the $\mathbf{B}$ ramp will be required to contain the space-charge forces at the beam ends, and so have a moderately uniform transverse space-charge force throughout most of the length of the bunch (Fig. 7-4). This need not be accomplished by detailed shaping of pulses in every cavity but simply by adding occasional short-pulse ferrite kickers.

Splitting of the beam into 2 or 4 segments, say, for separate transport to the target can be accomplished in a number of ways. Fast kickers provide one approach; alternatively, modest differ-

ential acceleration of sequential portions of the beam can be used to create a momentum spread later to be transformed into spatial spread by passage through a dc bending magnet.

Fig. 7-4. Accelerating and bunching voltage profiles. (XBL 769-3932)

In the final time compression and transport of individual beams to the target a design pole-tip field target less than 1 T is assumed. To bunch to 7 nsec and transmit 450 TW on target thus requires:

$q = 1$	2 beams	$B_0 = 1.85 \text{ T}$
$q = 3$	4 beams	$B_0 = 5.2 \text{ T}$

These "pole-tip" fields are within the state of the art for superconducting magnets.

For the final stages of time compression it is essential to use ferrite because of its fast time response. Values of E in excess of 10^6 V/meter , sec are easily obtainable. A factor of two is available by employing bi-polar pulses.

Finally, the ability to tailor pulse shapes, within reason, on the nanosecond time scale permitted by ferrite allows one to consider ramping the beam power and approach to some degree the most desirable profile for optimum pellet efficiency.

CONCLUSION

At this time the induction linac seems to be a worthwhile candidate for an efficient pellet igniter (see Appendix 7-1). A comprehensive conceptual design is needed in order to properly evaluate its promise. The more urgent tasks include

Scientific

- Lay out transport system; examine emittance dilution.
- Develop realistic pulse-compression scenario (in accelerating and bunching sections).
- Prepare reference front-end transition section.
- Examine beam instabilities and their control.
- Measure high-voltage diode brightness.
- Explore designs for $q > 3$, etc.

Engineering

- Ferromagnetic alternatives when scale is large.
- How to fit together cavities and transport magnets.
- Switches, modulation, component reliability.
- Other urgent tasks.

References

1. Ehlens, Hartwig, Keefe, and Salsig, IEEE Trans. Nucl. Sci. NS-20, 1010 (1973).
2. Avery and Keefe, "An Electron Accelerator for Tunneling Through Hard Rock," LBL-4251, October 1975.
3. D. Keefe, "Linear Induction Accelerator to Produce Intense Short Pulses of Very High Ions," LBL publication BEV-3201, April 1976.
4. A. W. Maschke (distributed at the ERDA Summer Study of Heavy Ions for Inertial Fusion - unpublished report).
5. E. D. Courant, Appendix 6-2 of this report.
6. J. Leiss, Appendix 7-1 of this report.
7. I. Godlove, Appendix 7-2 of this report.

Appendices 7-1, 7-2, and 7-3 are pertinent to this Technical Summary.

8. FINAL FOCUSING AND TRANSMISSION TO TARGET

William B. Herrmannsfeldt

From the point at which the final time compression and/or stripping of the ion beam occurs, until the beam has been transported to the target pellet, extremely high currents need to be considered along with properties of the medium through which the beam is to be transported. In order that the several treatments that follow can be compared, standard parameters have been used: e.g., pellet target radius of a few millimeters, i.e., $0.1 < r_t < 1.0$ cm; reactor chamber radius 10 m; radius of the beam at the chamber wall, $r_0 \sim 10$ cm. The target chamber is assumed to have experienced an "event" 1 sec earlier resulting in gas ionized at a temperature of 1 eV and a pressure of $\sim 10^{-3}$ Torr.

The following paragraphs summarize the discussions* to be found in several appendices representing the contributions from the individual members of the study. Although these appendices generally treat the same or related sets of topics, "Propagation and Neutralization of the Beam in the Chamber," it seems both necessary and useful to include all of them even at the risk of some redundancy. The variety of treatments generally finding the same or similar conclusions give some confidence that these extrapolations from experience are valid. Adequate verification and reconciliation of these approaches requires significant time and effort and is the major subject of the research and development needed for the final transport and focusing systems.

The typical case to be considered consists of a beam of U^+ ions at 70-100 GeV with $\sim 3 \times 10^{14}$ ions in a pulse lasting 10 nsec. Assuming $\beta = v/c \sim 0.5$, the pulse is about 150 cm long.

For sufficiently low background gas pressures, certainly below 10^{-5} Torr and perhaps below 10^{-3} Torr, plasma effects on the ion beam are unimportant. For such cases, we will first consider the vacuum case. We will later consider the plasma cases, but recognize at the outset that vacuum conditions adequate to make the vacuum case treatment valid may be unattainable.

VACUUM CASE

Electrostatic Self Repulsion

We observe that its own electrostatic repulsion will not keep the beam from converging. Consider the radial motion of a particle on the edge of a zero-emittance beam. The transverse energy of such a particle is

$$E = \frac{1}{2} M v_1^2 - 2ZeQ \log r = \text{const}$$

where Q = charge per unit length = Nze .
 $N \sim 2 \times 10^{12}/\text{cm}$. The minimum radius reached by the beam is given by r_m , where:

$$\frac{1}{2} M v_1^2 - 2ZeQ \log r_0 = -2ZeQ \log r_m,$$

$$\log(r_0/r_m) = \frac{\frac{1}{2} M v_1^2}{2ZeQ^2} = \frac{\frac{1}{2} M v_{1\parallel}^2}{2Nz^2 e^2} \frac{V_1}{V_{1\parallel}}$$

$V_1/V_{1\parallel}$ gives the ratio of the initial aperture radius to the chamber radius;

$$\frac{a}{R} = \frac{V_1}{V_{1\parallel}} = f^{-1},$$

thus:

$$f^2 \log(r_0/r_m) = \frac{\frac{1}{2} M v_{1\parallel}^2}{2Nz^2 e^2} = \frac{mc^2}{4Ne^2} \frac{M}{z^2 m} \frac{V_{1\parallel}^2}{c^2},$$

and since

$$\frac{Ne^2}{mc^2} = 1.2 \times 10^{12} \times 2.8 \times 10^{-13} \approx 0.35$$

$$\frac{M}{m} = 240 \times 1860 \approx 4.4 \times 10^5$$

$$f^2 \log(r_0/r_m) = \frac{4.4 \times 10^5}{4 \times 0.35} \times 0.5^2 = 8 \times 10^4.$$

Assuming a 10-cm-radius beam at the chamber wall and a 10-m-radius chamber, $f=100$ and

$$\log(r_0/r_m) \leq 8.$$

For a spot at the target $0.1 < r_m < 1.0$ cm, $100 > r_0/r_m > 10$, and $4.6 > \log(r_0/r_m) > 2.3$, so that focusing to a spot of 0.1 to 1.0 cm from 10 m away is comfortable. For much higher charge-to-mass ratios, neutralization must be applied. This analysis has ignored the possibility of multiple beams and also neglects the self-magnetic field of the ion beam that cancels the space-charge repulsive forces by $(1-\beta^2)$, i.e., a 25% reduction at $v/c = 0.5$.

The final focusing system is expected to consist of quadrupole magnets (either in a doublet or triplet array) located just behind the protective wall of the reactor chamber. The last quadrupole element has the largest bore, and by being nearest to the chamber, has the most severe environment. For multiply charged beams of modest energy (e.g., 10^2 at 40 GeV) it is possible to use a conventional dc iron laminated magnet.

However, for most of the cases discussed here, it is expected that the required field strengths are sufficiently high that a very strong pulsed quadrupole is necessary. The strengths available from superconducting quadrupoles are attractive but the operating conditions are quite severe for superconductivity. Some examples of final quadrupole configurations are presented in Appendix 8-7. For example, a doublet consisting of elements 3 m long, closely spaced, would need a pole-tip field of about 5 T to focus a 100 GeV U^+ beam to a point

*The author wishes to thank K. R. Symon for his help in summarizing this topic.

in the center of a 10-m-radius chamber.

Electrostatic Target Repulsion

The total charge in the beam is $\sim 10^{14} \times 5 \times 10^{-10} = 5 \times 10^4$ esu and the potential produced on a target assembly of radius 1 cm $\approx 1.5 \times 10^6$ V. If the radius is 1 mm the potential is 1.5×10^5 V, still much less than the 70 GeV in the beam. Fully ionized beam particles would, however, be stopped, although we have already shown that fully ionized beams need neutralization. Obviously, this neutralization would also neutralize the target before such potentials could build up.

Stability

Beam stability in the vacuum condition is discussed in Appendix 8-5. The most dangerous instability appears to be an electrostatic wobble mode, for which the growth rate $\Gamma \approx k\omega_B$; here $k = 2\pi/\lambda$, r = beam radius, and ω_B = beam plasma frequency,

$$\omega_B^2 = 4\pi n \frac{e^2}{M} = 85\sqrt{n} \text{ sec}^{-1}.$$

If $n = 10^{14}/\text{cm}^3$, $\omega_B = 8.5 \times 10^6$, and the maximum growth ($kr \sim 1$) as the particle moves along the beam is

$$\begin{aligned} \exp \int 8.5 \times 10^6 \sqrt{n/n_f} dt &\sim \exp 8.5 \times 10^8 \int_{r_0}^{r_f} \frac{r_f}{r} \frac{dr}{r} \\ &\approx \exp 8.5 \times 10^8 \frac{L}{V} \frac{r_f}{r_0} \log(r_0/r_f) \\ &\approx \exp 8.5 \times 10^8 \times \frac{1000}{1.5 \times 10^{10}} \times \frac{0.1}{10} \times 4.6 \\ &\approx \exp 2.6 \approx 13.5 \end{aligned}$$

Hence this electrostatic mode forms no barrier to the transport of singly charged ions. Since ω_B scales as $[Z(E_f/E)^{1/2}/M]^{1/2}$, multiply charged ions or ions of smaller mass may suffer a current limitation from this mode.

In conclusion, it appears that beams of singly charged U at energies of 70 GeV, and currents of ~ 5000 A should be able to hit the target if the background gas is neutral and at densities $< 10^{14}/\text{cm}^3$. If Z is increased, and M decreased, by factors of ~ 5 , the wobble instability may limit the beam current.

ENVIRONMENT IN THE TARGET CHAMBER

In Appendix 8-1 calculations are presented, predicting that the temperature of the plasma remaining in the chamber after the previous explosion will decay to about 1 eV in about 1 sec. The density will depend upon design choices in the vacuum system, but pressures from 0.001 Torr to 0.1 Torr seem reasonable. Lower pressures could perhaps be achieved if necessary, but the pumping system would be expensive. Higher pressures could be provided if desired, but might cause significant stresses in the reaction chamber. It could be

somewhat expensive (~ 10 MK, time averaged) to heat the background plasma significantly, although it would not be difficult technically. The beam itself may cause heating of the plasma in the beam path.

To get an idea of gas-pressure ranges that are relevant, assume a beam of 5000 A, U^{+1} , 1 mm² cross section near the pellet, and a fully ionized background gas, $Z_0 = 1$. Then for $p > 10^{-2}$ Torr,[†] the background electron density n_0 will exceed the beam electron density $92 n_b$ near the pellet. Near the wall, the beam density is 10^4 times smaller and the corresponding pressures are $p > 10^{-6}$ Torr. At pressures below these values, plasma effects are unimportant; there are not enough electrons to neutralize the beam. Another interesting pressure threshold occurs where the mass density of the gas becomes comparable with the mass density in the beam. For 5000 A of U on H, this occurs near the pellet at $p \sim 1$ Torr, and near the wall at $p \sim 10^{-4}$ Torr. At such pressures, the plasma can interact dynamically with the beam. Finally, we note that at pressures above 10^{-3} Torr, the gas can strip electrons from U^{+1} on its way to the pellet, and above 0.1 Torr the U will be fully stripped when it arrives at the pellet. These examples are summarized in Table 8-1.

Table 8-1. Threshold pressures (5000 A, U^{+1} , into hydrogen).

	Near wall	Near pellet
Beam size	100 cm ²	0.01 cm ²
Gas electron density = beam electron density	10^{-2} Torr	10^{-7} Torr
Gas mass density = beam mass density	10^{-4} Torr	1 Torr
Beam charge-state changed	10^{-3} Torr	10^{-1} Torr
Beam fully stripped	10^{-1} Torr	10^{-1} Torr
Coulomb scattering > 1 mm	10 Torr	--

TIME SCALES

In Table 8-II, we give formulas and example values for some time scales and distance scales of interest. Subscript "g" refers to the gas, "b" to the beam, and "e" to the electrons. The density n is in particles cm^{-3} , Z is charge number, $\beta = v/c$, T is temperature (eV), r_b is the beam radius (cm).

The ionization rate in the example shown is such that the beam will ionize the background gas

[†]Gas densities are given as reference pressures, i.e., pressures at $T = 300^\circ\text{K}$ which give that density. 1 Torr = $3 \times 10^{16} \text{ cm}^{-3}$.

Table 8-11. Time and distance scales.

Ionization rate:

$$1.3 \times 10^{-19} \frac{n_b^2 g_{b2}^2}{g_b^2} \quad \text{ion pairs per cm per beam ion}$$

$$\sim 1 \text{ cm}^{-1} \text{ for } \bar{z}_b = 1, \bar{z}_b = 1 \text{ at } 10^{-1} \text{ Torr}$$

Heating time:

$$1.7 \times 10^{-12} \frac{\bar{r}_b^2}{n_b \bar{z}_b} \quad (\text{sec})$$

$$\sim 10^{-6} \text{ sec for } T_e = 260 \text{ eV } (\bar{r}_{th} = 0.75),$$

$$\bar{z}_b = 92, \quad n_b = 10^{18} \text{ cm}^{-3}$$

$$\sim 2 \cdot 10^{-9} \text{ sec for } T_e = 500 \text{ eV } (\bar{r}_{th} = 0.11),$$

$$\bar{z}_b = 92, \quad n_b = 10^{18} \text{ cm}^{-3}$$

Plasma frequency:

$$9 \times 10^7 n_e^{1/2} \text{ Hz} \sim 5 \times 10^8 \text{ Hz}$$

if $p > 10^{-5} \text{ Torr}$

Resistive diffusion time:

$$\sim \pi \frac{\bar{r}_b^2}{c^2} \sim 6 \times 10^{-6} T_e^{3/2} \frac{\bar{r}_b^2}{\bar{z}_b} \quad (\text{sec}) \sim 10^{-6} \text{ sec}$$

for $\bar{r}_b = 0.1 \text{ cm}, \quad T_e = 1 \text{ eV}, \quad \bar{z}_b = 1$

Corona expansion:

$$10^7 \text{ cm/sec} \sim 0.5 \text{ cm in } 50 \cdot 10^{-8} \text{ sec}$$

sufficiently to provide neutralizing electrons even if the gas is initially neutral. For much smaller values of $n_b g_{b2}^2 / g_b^2$, that would not be the case.

The time for the beam to heat the plasma electrons by Coulomb collisions to $\sim V_e^2/c^2 = \bar{r}_{th}^2$ is shown for two values of T_e of interest. For $T_e = 260 \text{ eV}$, $\bar{r}_{th} = 0.75 = \bar{r}_b$ for U at 100 GeV ; the beam pulse does not last long enough to heat the electrons to this temperature. However, the electrons nearest the target ($n_b = 10^{18} \text{ cm}^{-3}$) can be heated to 500 eV ($\bar{r}_{th} = 0.1$) in less than the pulse duration if the U is fully stripped. The electrons will apparently be heated well above 1 eV over a substantial part of the beam path in the chamber even if the U ions are not stripped.

Since the corona about the pellet expands less than 0.5 cm during the pulse, the corona will have no effect on the propagation of the beam to the target pellet.

NEUTRALIZATION

The plasma frequency (Table 8-11) gives the time scale for electrostatic (charge) neutralization of the beam if the plasma pressure is high enough (first line, Table 8-1). This time scale is much shorter than any other time of interest ($< 10^{-6} \text{ sec}$) for any likely pressure, so the charge density in the beam will be zero for plasma pressures exceeding those in the first line, Table 8-1.

Current neutralization is a trickier problem. In Appendix 8-5 it is shown that inductive effects result in an electron current that initially cancels the beam current if the electron density is high enough. This current then dies away on the resistive diffusion time scale as discussed in Appendices 8-1, 8-4, 8-5, and 8-6. We see from Table 8-11 that this time is roughly the pulse time near the pellet for $T_e \sim 1 \text{ eV}$, in a hydrogen plasma. For $\bar{z}_b = 1$, the time is shorter. Under these conditions, the beam current will be unneutralized, or only partially neutralized right up to the pellet. However, calculations in Appendix 8-1 suggest that beam heating of electrons near the pellet raises T_e to values higher than 1 eV so that current neutralization will be greater than predicted by the formulas in Table 8-11. The formula in Table 8-11 is derived assuming negligible conductivity outside the beam. Since this is probably incorrect, the actual time may exceed that given by the formula by perhaps a factor of 10^2 . In that case, the net current will always be zero if the plasma density is high enough (first line, Table 8-1). However, a calculation in Appendix 8-5 suggests that the neutralizing current may flow in a channel larger than the beam cross section, which may be important for stability calculations. Frictional (Coulomb) drag by the beam can result in a final equilibrium electron current after resistive diffusion given in Appendix 8-2 as:

$$I_e = - \frac{\pi \bar{r}_{th}^2}{c^2 \bar{z}_b} I_b$$

For $\bar{z}_b = 1$, $\bar{z}_b = 92$, $\bar{r}_{th} = 0.1$, this gives $I_e = -0.1 I_b$, and the current stays neutralized forever. The heating time to reach $\bar{r}_{th} = 0.2$ is about a beam pulse length. For $\bar{r}_{th} = \bar{r}_b$, we get $I_e = -0.1 I_b$, and we have a net back current $-0.1 I_b$. The beam would then be highly unstable in the decaying self-magnetic field. The time to reach these temperatures is much too long. However, even for $\bar{r}_{th}$ slightly larger than 0.2 , instabilities can be expected after the resistive diffusion. Fortunately, at such electron temperatures, the resistive diffusion time becomes longer and this particular problem does not appear to arise.

STABILITY

Two possible instabilities have been analyzed in Appendices 8-1, 8-3, and 8-5. When the beam is not current neutralized, kink instabilities may arise that can cause the beam to miss the target if they grow to large enough amplitude. An analysis in Appendix 8-1 gives for the number of e-foldings of the fastest growing mode

$$n \sim 10^{-3} \pi^2 n_b \left(\frac{\Delta R}{r_b} \right),$$

where R is the distance over which the mode can grow, perhaps the last 10 cm if the beam current is not neutralized near the pellet. If $n > 10$, as it may be for a fully stripped beam, it seems likely that initial perturbations may grow to unacceptable amplitudes. The above formula is derived neglecting the conductivity outside the beam. That assumption is probably incorrect in this case. If the plasma outside the beam is conducting, and if its density is greater than given in line 2, Table 8-1, the beam has to drag the plasma with it as it kinks, and the growth rate is reduced, perhaps by a factor of 10 as shown in Appendix 8-3. The kink instability is then less serious, although still of some concern. As noted previously, short-wavelength electrostatic wingle modes may grow even near the walls (see Appendix 8-5), thus providing a pre-excited kink amplitude of appropriate wavelength, which is then amplified by a further factor n near the pellet.

The Weibel instability can also occur, as described in Appendix 8-5, if the beam is not current neutralized. This results in breaking the beam cross section into current filaments. If the transverse temperature is not zero (it appears to be about 100 eV for typical parameters), a beam with transverse velocity passes across randomly spaced transverse-magnetic fields as it crosses the current filaments. The resulting transverse diffusive scattering can cause the particle to miss the target. Preliminary estimates suggest the effect may be serious if the Weibel instability is present. However, since a transverse temperature also tends to stabilize the instability, further study is required.

Other instabilities, for example the two-stream instability, may be important, but have not yet been studied in this application.

Appendices 8-1, 8-2, 8-3, 8-4, 8-5, 8-6, and 8-7 are pertinent to this Technical Summary.

9. SYSTEMS AND COST STUDIES

Glenn W. Rumba

INTRODUCTION

During the formative stages of a new program there is some reluctance to emphasize cost studies because of the uncertainties. We intend that the cost data and conceptual designs be taken with the realization that important gaps in the data base have forced us to make rather brave extrapolations. We have assumed that a power plant based on heavy-ion fusion will employ an ion-acceleration system capable of supplying 10 MJ of ions to a fusion target at 1 to 10 pulses/sec, and the energy gain of the pellet is ≥ 100 . We have also assumed that tritium will be a primary pellet fuel requiring breeding of T_2 in a Li blanket. If pellet designs using predominantly deuterium become possible after further experimental and theoretical work, the design of an economically competitive and environmentally satisfying reactor might be greatly facilitated.

As a consequence of previous studies of reactors for the inertial-confinement fusion program, the following suggestions have been considered.

- Netted Wall - interior cavity surface protected by a thin film of liquid metal that is replenished between microexplosions by radial inflow through porous cavity wall from blanket region.
- Sacrificial Wall - cavity wall includes liner of carbon or other refractory material. Time interval of energy deposition and energy deposition per unit area controlled by target design and cavity size, respectively, so that ablation of cavity wall is limited to acceptable values for reasonable cavity lifetimes.
- Gas Buffer - cavity includes low-pressure gas that absorbs most of the x-ray and target debris energy and transmits it relatively slowly to the cavity wall.
- Magnetic Protection - beam-focusing magnets and cavity wall protected from ionized target debris by solenoid concentric with ion-beam path.
- Use a very large chamber, perhaps an underground cavity, to increase dispersion in arrival time and lower wall loading to a tolerable level. The thermal inertia of such a system would be quite large.
- A system based upon exhausting material from a large mass around the pellet could be considered. The reactor-vessel walls could be maintained at a high temperature to avoid deposition of the working fluid.
- Various hybrid schemes could be envisioned in which neutrons are used to produce fissile fuel from natural uranium or thorium. The

major power production would utilize this fuel in off-site plants. This option may offer significant advantages, especially for near-term applications because an overall lower accelerator efficiency or lower pellet gain could be economically tolerated. In addition, if a plant failure should occur, there would be no immediate effect on generating capacity.

Several of the above designs have been examined in sufficient detail to make some cost estimates, and are treated in several sections below. It should be emphasized that a reasonable approach to the program would call for building some small test systems, say 1 m in diameter, and operating them with low-yield pellets to model the wall loading expected in larger systems. Materials data from studies being funded from the Magnetic Fusion Energy Program in ERDA will be utilized as applicable.

ECONOMIC CONSIDERATIONS

Crude estimates (consistent with available cost information) were made of power-production costs based on generalized concepts considered by the study group. Items included in the economic analyses are:

- Capital amortization (direct and indirect)
- Fusion-target costs
- Operation and maintenance costs
- Reactor-component replacement costs
- Accelerator-component replacement costs

Items included in direct capital costs are:

- Accelerator system (ion source, accelerator, storage ring and/or accumulator, beam transport system, and focusing)
- Reactor cavities and primary coolant loop
- Electric generators
- Miscellaneous (land, structures and site facilities, land and site preparation, miscellaneous plant equipment, and lithium-tritium processing equipment)
- Target fabrication facility

Indirect capital costs include:

- Interest during construction
- Construction facilities and equipment
- Engineering and management

Fusion-target costs are estimated to contribute 3 mill/KWh to power-production costs for each one dollar of target cost for a one ton (high-explosive equivalent) target yield.

Operation and maintenance costs are estimated to be ~ 2 mill/KWh of power-production costs.

Reactor-component replacement costs will depend on the rate of damage (primarily embrittlement) to reactor components due to 14-MeV neutron irradiation. It was assumed that reactor-blanket components can

withstand 4 MW year/m^2 of 14-MeV neutron irradiation before requiring replacement, resulting in a contribution to power-production costs of $\sim 2.6 \text{ mill/KWh}$.

Reliable estimates of accelerator-component replacement costs are not available. A contribution of 0.4 mill/KWh to power-production costs was assumed for this item.

Estimates of direct capital costs are given in Table 9-1. Assuming 7.5% interest rate and 10-year construction period, total indirect capital costs are estimated to be 56% of direct capital costs.

Table 9-1. Estimated direct costs for 1000 Mw/ electric power plant

	Cost (millions of \$)
Land and site preparation	20
Structures and facilities	130
Miscellaneous plant equipment	25
Reactor chamber and primary coolant loop	100
Li ⁺ gun and tritium processing equipment	15
Target factory	10
Electric generator	100

Accelerator to produce $\sim 10^7$ Joules of ions²

Indirect costs will approximately double the above costs for a 9-year construction interval.

²Meaningful estimates not available; extrapolations from past experience indicate possible costs from a few hundred million to several hundred million dollars.

COST DATA FOR SELECTED APPROACHES

The costs data assembled in Table 9-1 were obtained by examining previous laser-fusion-system studies and costs of light-water reactors.

The true costs could vary a large factor from the given values. Since the requirements for a heavy ion fusion accelerators are very different from previously constructed machines, meaningful accelerator cost estimates cannot be made at present. Extrapolations from past accelerator experience indicate that the types of systems considered during the study might cost from a few hundred million dollars to several hundred million dollars, and perhaps up to the order of a billion dollars for those capable of supporting several separate reactors. The factor that could greatly change the costs would be the introduction of primarily deuterium burning pellets. In this case the cost of pellet manufacture, tritium recovery, and lithium breeding blankets would be greatly reduced. However, since pellets using primarily D₂ will not be as high in gain as D₂ pellets, the accelerators will have to be more efficient. An alternative, which is not listed in Table 2-1, but is mentioned elsewhere in this report, is collective ion acceleration. If this technique can be made to work it could be useful for experiments with pellets, but the fact that the collective process is only 5-10% efficient at converting electron energy to ion energy reduces the attractiveness for reactor applications. For instance, supplying power to an ion burst by collective acceleration could increase the costs associated with the pulser by a factor of ~ 10 in comparison with an "ion diode." It is presupposed that ion diodes can be developed that suppress most energy input into the electrons.

DETAILS OF SELECTED CONCEPTS

Two concepts were given some detailed thought as part of the summer study. These are described in Appendices 1-3 and 1-4. Time did not permit a detailed costing estimate to be made for either concept. Appendix 1-3 examines the possibility of using a number of materials with different thermal conductivities and melting points to absorb energy from the fusion pellet. Appendix 1-4 is a preliminary examination of the use of lithium vapor in the reactor chamber to moderate the flow of energy to the reactor wall. If the reaction chamber is sufficiently large, possibilities exist for making a chamber withstand the required number of shots. This study is by no means exhaustive, and the other reactor-chamber concepts should be closely examined. Before a sound design for a large system could be formulated, small test systems will be required.

There are no Appendices directly associated with this Technical Summary.

1. ALTERNATIVES

Target and Ion

The task includes efforts to achieve inertial fusion by collective means and by means other than high-energy heavy ion bombardment. Ongoing efforts including lasers and electron beams are not included.

The objective is to produce 100 TW and a few MJ of particles with a short range in the target; for example, 0.1 cm in Au, which is the range of 12 MeV B, 10 MeV protons, or 1 MeV electrons. It is also necessary that the particle beam be focused on a target of 1×10^{-4} m at a distance of 1 cm. Up to 100 separate beams are acceptable.

Conventional accelerators represent a developed mature technology by means of which these objectives can probably be achieved with some modest-risk extrapolation of existing systems. However, the cost of such machines is substantial - of the order of 10^5 to meet the above requirements.

The purpose of considering alternatives is that they may be cheaper. We shall be concerned with two phases:

1. Pellet physics experiments to establish the principles of pellet implosions. These experiments do not require a rapid-pulse-repetition capability. In addition, the 10-cm focusing distance can be relaxed because protection of the front of the accelerator is not essential.
2. Accelerator development where pulse repetition rate capability and the 10-cm focusing distance are essential.

For the first phase there exists a pulse power technology that can meet the objectives. For example, the Aurora machine, which has been in operation at least 5 years, produces 10 TW and about 2 MJ. It has only been operated as an electron machine; however, smaller machines have produced ion beams during the past 2 years, so that there is enough information to make a modest-risk extrapolation to a break-even experiment on inertial fusion.¹ This involves the equivalent of about 10 Aurora machines, each with a reduced pulse length and energy storage by about a factor of 10 to give a net energy of about 2 MJ with a power of 100 TW. The beams would be focused to deliver the energy to a 1.5-mm target. The cost of the system has been estimated to be about 30 MS.² At Sandia Laboratories PROTO II is scheduled to be completed by Fall, 1976. It is a 160 KJ, 10 TW machine at 1.5 MV. IBFA is a 40 TW, 200 KJ machine at 1.5 MV that has been approved at a cost of 5 MS and is in the final design stage.

In order to meet the focusing requirements of the reactor phase, it will probably be necessary to produce smaller currents of heavy ions. Several collective accelerator schemes to meet this requirement are described. For example, the ionization front accelerator could produce 12 GeV U^{90+} ions with an emittance of $\epsilon/n = 12$ cm-mrad and thus be focused on a 2.5-mm target at a distance of 10 m.

Each module of the system is a 10 MV, 300 KJ, 50-nsec machine similar to one module of the Aurora machine, and would deliver about 10 KJ of heavy ions. The cost of the full modular system would be about 100 MS. It should be emphasized that this and other collective accelerator schemes are based on physical principles not yet established in laboratory experiments.

The major deficiency of pulse-power systems for the second phase is the lack of a pulse-repetition technology. Indeed this is the major reason for the present study of conventional accelerators where this capability is well established. The pulse-power machines were mainly developed for simulation requirements where 5-10 pulses/day are sufficient. The development of pulse-repetition rates of 1/sec appears to be feasible except for some serious material problems in the diode. The alternatives group does not have the requisite expertise to estimate the costs of development and production of a pulse-power machine with a repetition rate of about 1/sec. A suitable panel should be convened for this purpose to make estimates and comparisons with conventional accelerator technology.

The problems of current and charge neutralization associated with beam propagation in the reactor chamber are critical to the entire concept. Although they have been studied by the pulse-power community for many years, studies with appropriate beam parameters have not been done. The necessary capability with pulsed beams presently exists and this group concludes that such studies should be supported with a high priority.

In this report further alternatives are considered that have not previously received the support and study devoted to electron and ion beams. They include alternative collective accelerators: microparticle acceleration, plasma focus, HIF, spherical space-charge accelerator, etc. This group believes that support at a serious level should be provided for new ideas and that this support should be insulated from the demands of large-scale efforts. This is essential to guarantee that physical principles will continue to develop and so that alternatives will be available if some of the large-scale efforts are unsuccessful.

ION DIODES

For the first phase of the problem there exists a mature pulse-power technology that was originally developed for electron beams. In the last few years pulse-power machines have been applied to accelerate ions in a diode. There are three forms of ion diodes:

- Magnetically insulated ion diodes¹
- Reflex triodes or double diodes³
- Pinched-electron-beam diodes.⁴

For protons, currents of 200 KA have been achieved at about 1 MV for 50 nsec in the pinched-electron-beam diode at the Naval Research Laboratory. Current densities of 1 KA/cm^2 have been observed. For magnetically insulated diodes the half-angle

divergence of the ion beam at the cathode has been determined¹ to be about 3° . An upper bound to the emittance can be estimated as $\epsilon/\pi \sim 150$. Break-even experiments with protons have been proposed based on existing pulse-power technology.¹ They involve 10 MA of protons at 10 MV and a total proton energy of 2.5 MJ. The angular divergence based on present data is estimated to be 0.5° . Thus by geometric focusing the beam could be delivered to a 1.5-mm target at a distance of about 24 cm from the cathode.

It is not possible to produce high-energy heavy ions (e.g. 12 GeV U^{90+}) with ion diodes. This would require a 200 MV diode. The maximum operational diode voltage is about 10 MV. Most of the ion-diode work to date has been done with protons. Heavier ion sources could be developed, but the existing technology cannot produce 10-100 GeV ions. The second phase of the problem involves a repetition-rate capability and a focusing requirement (1-10 mm target at a distance of 10 m). The solution of the latter problem by going to small currents of very high-energy ions is attractive for conventional accelerators, but is not possible with a modest extrapolation of ion-diode technology. However, there are some possibilities for increasing the brightness of beams from ion diodes to meet the focusing requirement. For example, if the beam contains some high-Z ions, radiation cooling of intense space-charge-neutralized ion beams, which has been studied theoretically,⁵ appears to be an attractive possibility.

Ion-diode technology provides a present capability for studying beam propagation and neutralization in gas or plasma. This is a central problem for the target area in any application of ions to inertial fusion and should be investigated.

For the first phase of the problem of inertial fusion, all of the requirements can be met with pulse-power technology and ion diodes with modest extrapolations. Based on the cost of the Aurora machine, a 100 TW machine that produces 2 MJ of 10-MeV protons could be built for about 30 MS. For the second phase, research needs to be done on methods of increasing beam brightness and a repetition-rate capability needs to be developed.

COLLECTIVE ACCELERATORS

There are several collective accelerator schemes that might be used with pulse-power machines to produce small currents of high-energy heavy ions to meet the focusing requirement of the second phase.

Ionization Front Accelerator

When an intense electron beam is launched from a diode into a neutral gas, ions are accelerated to an energy that is greater than the electron energy.⁶ This has been explained in terms of the net space charge at the front of the beam where ionization is incomplete.⁷ A potential well for ions is produced that accelerates. Electric fields of 1 MV/cm have been observed, but only over distances of about 5 cm. Insufficient control over particle ionization is available by control of gas density. An experiment is in progress at Sandia Laboratories where

the gas density is too low for significant particle ionization and ionization is controlled by photons and a complex of light pipes of appropriate lengths.⁸ An accelerator based on this principle has been studied for a heavy-ion fusion reactor.⁹ It assumes only an existing pulse-power technology. The objective is to accelerate U^{90+} to 12 GeV in an ionization front accelerator. The system involves 100 modules, each producing electron beams of 10 MV, 500 kA for 50 nsec. Each beam of 1 cm diam involves 150 kJ of electrons that would accelerate 5×10^{12} ions to 12 GeV so that the total ion energy per beam would be 10 kJ. The emittance can be estimated from the oscillation energy of ions in the potential well to be $\epsilon/\pi \sim 12$ cm-mrad. This would permit focusing on a 5-mm target at a distance of 10 m. Since each module is similar to a module of the Aurora machine, the cost of the system can be estimated to be about 100 MS. Key problems for this alternative are

- Adequate control of the ionization front for which an experiment is in progress.
- Ion-source development
- Recombination and charge exchange - considerably less serious than for conventional accelerators because acceleration time is only 50 nsec.
- Repetition-rate capability.

Negative-Energy Wave Accelerators

Several proposals have been advanced that involve a single high-amplitude wave of variable phase velocity to accelerate ions. The accelerator is similar to a conventional linear accelerator except that the wave is not produced externally, but is a mode of the unneutralized electron beam that grows when energy is taken from it (negative-energy wave) so that the energy source to accelerate ions is the electron beam. The variable phase velocity can be obtained by using a Doppler-shifted cyclotron wave and varying the static magnetic field¹⁰ or by using a beam mode and varying the cross section of the drift tube.¹¹ At present there is no experimental basis for negative-energy wave accelerators, but there are active programs at Austin Research Associates and Cornell University. Some preliminary estimates for the Auto-Resonant-Accelerator¹⁰ indicate that ions such as U^{90+} could be accelerated from 200 MeV to 30 GeV. The magnetic field would have to change from 100 kG to 5 kG over a distance of about 25 m. An electron beam of 10 MeV, 100 kA, and 400-nsec pulse length would be required. Since the electron beam would follow the magnetic-field lines, it must expand as it propagates with a corresponding expansion in the ion bunch so that it is more difficult to keep the emittance low than in an ionization front accelerator. The injector to produce 200 MeV U^{90+} ions could presumably be a 4 MV ion diode.

Vacuum Diodes

The highest-energy ions and the largest number of ions per pulse by collective acceleration have been obtained with an insulated-anode high-vacuum diode.¹² 130 MeV fluorine ions (7 MeV/nucleon) have been observed in pulses of 10^{11} particles.

Although this is probably the least understood of collective accelerator experiments, the results are sufficiently impressive to warrant further study of these diodes as injectors.

MICROPARTICLES

Acceleration of small particles¹⁵ to speeds of $10^7 - 10^8$ cm/sec has been investigated since the beginning of the fusion program. We shall not consider acceleration by ablation, which is of course fundamental to the target problem in all schemes for inertial fusion. We consider the possibility of accelerating microparticles so that they play the same role as heavy ions.

At TRK¹⁴ there is a microparticle accelerator that has been in operation for the past 16 years. Particles of radius about 0.1μ are accelerated through a potential of 3 MV. They reach a velocity of $v \sim 3 \cdot 10^8$ cm/sec. Assuming a density of 1 g/cm^3 the particle energy is 2×10^{-7} J or 1630 GeV. The energy/ion is 3 keV. From this we can infer that Z = number of electron charges/total number of particles = 10^{12} , and $q = 5 \times 10^3$ electron charges. The electric field at the surface of the particle is $E = q/a^2 = 7.5 \cdot 10^8$ V/cm and the potential of the particle is $\phi = q/a = 7.5$ kV. The small spherical particle is optimum for electrostatic acceleration. Other shapes such as a disk that has a similar surface-to-volume ratio encounter stress magnification at the edge and would require a very high potential to charge them.

If sufficient control could be exercised over the Z and mass of such dust particles a large spherical accelerator and implosion chamber could easily be designed. For example, in a 10-m radius sphere $4 \cdot 10^{12}$ particles of 0.1μ radius and 10μ spacing could be accelerated through 10 MV to produce 10 MJ of particle energy. Techniques for producing small numbers of charged microparticles have been developed at TRK. Research on the production and control of large numbers of charged microparticles¹⁵ would be necessary to develop this scheme for inertial fusion.

HIPAC (HEAVY ION PLASMA ACCELERATOR)

This device was originally proposed at AVCO, 16 the theory was developed and small-scale experiments were carried out. It consists of a torus with a rising toroidal magnetic field that sweeps electrons inward. A hot-filament thermionic cathode served as the electron source. A toroidal electron plasma of density 10^9 cm^{-3} and total charge about $10 \mu\text{C}$ was established and confined for about 5 msec .¹⁷ A radial potential well of a few MV was created. Calculations indicated that a radial potential well of 1 GeV should be possible. The original objective was to strip and accelerate heavy ions in the radial direction. This configuration is not very suitable for imploding a spherical target.

Recent experiments at Maxwell Laboratories¹⁸ have shown that almost all of the confined electrons are trapped and do not go around the torus. A toroidal electric field of 0.5 V/cm produced a negligible toroidal current although about $100 \mu\text{C}$ of charge was confined. It is thus possible to conceive of an ion accelerator where electrons are

magnetically contained and their space charge contains and focuses ions that can be accelerated around the torus as in a storage ring. A toroidal electric field can be applied of suitable magnitude so that it accelerates ions around the torus, but not the electrons, provided that $u_e > u_i/2$ where u_e is the electron magnetic moment, u_i is the ion magnetic moment, and Z is the ion charge. If the space-charge electric field is $E = 10^7 \text{ V/cm}$, ions of $Z = 60$ would be contained for energies up to $2\pi ER/2 \sim 100 \text{ GeV}$ for a torus major radius of $R = 3.3 \text{ m}$. For a 1-cm minor radius of the electron beam, the maximum emittance of the ion beam would be $\epsilon_i \sim 10 \text{ cm}^2\text{-mrad}$. This type of collective accelerator leads to a very small storage ring for 100 GeV ions. Of course a considerable amount of research would be necessary to develop this application and there are no active programs at this time.

SPHERICAL SPACE-CHARGE ACCELERATOR

A parabolic potential well is produced in spherical geometry by injecting electron beams. This arrangement was originally proposed and studied as an electrostatic containment device.¹⁹ More recently it has been studied as an ion-implosion scheme.²⁰ In a parabolic well, ions initially at rest at any radius will be accelerated and all arrive at the center of the sphere at the same time. Focusing in time and space is determined by the initial temperature and angular momentum of the ions. If τ_e is the electron-beam pulse time and Δt is the spread in time over which ions arrive at the center, the power amplification of the electron beam source is $\tau_e/\Delta t$. For an initial ion temperature $T_i \sim 0.026 \text{ eV}$, calculations show a power-amplification factor $\tau_e/\Delta t \sim 10^4$.

PLASMA GUNS

There has been some recent progress in plasma guns.²¹ From a small 20 kJ bank up to 100 keV D^+ has been observed at equivalent currents of 10 kA. Damage patterns and calorimetric data at 2-3 m from the gun show focal spots of a few cm^2 and several kJ of energy deposited. The scaling up of plasma guns merits consideration for this problem, particularly because the Linus technology²² of megagauss magnetic fields has been developing rapidly in the last few years.

PLASMA FOCUS

In recent years a low-density mode of operation has been studied that involves very large currents of high-energy²³ electrons and ions (energies like 100 times the bank voltage). At the LeWille laboratory in France 4×10^{16} ions have been observed with energies of 0.5 - 1.5 MeV. They were concentrated in a cone with a half angle of about 14° . This amounts to 5 kJ of ions from a 400 kJ plasma focus, which is comparable with the best results to date with ion diodes at the Naval Research Laboratory. Although the plasma focus is one of the most surprising developments in plasma physics to date and has been studied for many years, the understanding of the physics is still marginal. In the study of inertial fusion the high-energy density may yield many surprises and the plasma focus still provides the simplest available technology for research on the physics of high density.

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There are no Appendices pertinent to this Technical Summary.

V. Conclusions and Needed Research and Development

This section consists of a verbatim copy of the short summary report submitted by the organizing committee to ERDA on August 18, 1976.

ERDA SUMMER STUDY OF HEAVY IONS FOR INERTIAL FUSION

July 19-30, 1976

CONCLUSIONS OF THE STUDY AND AREAS REQUIRING RESEARCH AND DEVELOPMENT

August 18, 1976

Introduction

This brief report of the conclusions of the study, and of areas requiring research and development, has been prepared by the study's organizing committee and is submitted with their concurrence. A complete report of the study is expected to be ready for issue in September. It will include detailed technical summaries of results obtained by each of the ten working groups at the study, and a substantial volume of supplementary information in a series of appendices.

Summary

The summer study was held to assess the advisability of launching a program directed toward the use of high-energy heavy-ion beams as the ignition mechanism for inertial fusion. This possibility was opened by conceptual design studies totalling less than five man-years initiated by a very small number of high energy accelerator physicists, together with target design studies by inertial fusion scientists. The study considered parameters appropriate to a workable power plant, rather than a break-even experiment.

The majority of the participants are expert in current accelerator technology, but had not worked on this application before coming to the study. They drew on their knowledge of this well-established technology, and their experience in extending it, to evaluate the likelihood of achieving the target requirements, which are themselves still under development. No fatal flaws were discovered in the schemes which were studied; many of the general requirements are within the present state of the art. The central result, based on two weeks of effort by some fifty specialists, was an affirmation that this approach merits serious attention. In particular, a broader range of accelerator systems than thus far considered deserves investigation.

It is clear that the information available now is inadequate to arrive at firm conclusions. A concerted effort on target, reactor, and accelerator designs, a number of physics experiments and calculations, and some hardware development are needed. Some of the problem areas were identified before and during the study and are summarized below.

Considerable enthusiasm has been generated among the participants, and it is a conclusion of the study that a prompt expansion of effort along these lines is warranted. It is also desirable to establish a working group to meet for a few days of concentrated effort at frequent intervals, rotating locations among the interested Laboratories. This group could help to coordinate the program, exchange detailed information, identify needed changes of emphasis, and provide continuing information to the supporting ERDA Divisions.

Target and Reactor Requirements

The target and reactor physicists supplied the accelerator experts with ranges of requirements for commercial power plants. These ranges were 1 to 10 MJ of input energy at peak power levels of 100 to 600 TW, with 2 to 10 nanosecond pulse lengths at peak power. The focal spot radius and particle range are related by an energy deposition requirement of 20 to 30 MJ/gm; however, radii less than 1 mm are not useful. The most promising targets require pulse shaping. The confidence level of the target designers is moderate at the 1-MJ, 100-TW level and high for 10 MJ, 600 TW. A nominal 10-m reactor vessel radius and repetition rates $\geq 1/\text{sec}$ were considered.

Classes of Accelerators

The requirement least familiar from past accelerator experience is that of supplying such high peak beam power, corresponding to peak current in the several thousand ampere range. Two distinct approaches to meeting this requirement were discussed during the study. One of these, allowing many variations, consists of utilizing conventional ion sources (50-100 mA) in pulses of $\sim 10^{-6}$ sec feeding an r.f. linac. Acceleration to full energy is achieved in the linac or with a synchrotron. Current multiplication is obtained by accumulation of many turns in a ring device (storage ring or fast accumulator), longitudinal compression, and/or a multiplicity of beams on target. The second approach provides a single-pass system from injector to target, using a linear induction accelerator fed by one or more high-current pulsed-power ion sources. Bunch compression and beam-pulse shaping is accomplished by controlled ramping of the accelerating pulse shape and by supplementary manipulation at high energy.

Areas Requiring Research and Development

1. Targets and Reactor Systems

Additional work on target design is needed. Progress in this area will have an impact on accelerator design, cost, and feasibility. Heavy-ion systems have some particularly advantageous properties which allow unexplored approaches to target design. For example, a high-efficiency accelerator might be used to drive a deuterium-burning target, eliminating the need for breeding tritium in a lithium blanket. Prototypes of targets should be fabricated to identify critical problems, and conceptual design of a target factory should be studied.

As a result of the study, a wider range of reactor designs needs to be examined. For example, systems providing relatively high vacuum around the target at ignition, as well as those with pressure a fraction of an atmosphere, may need consideration, depending on the outcome of detailed studies of ion beam focusing, neutralization, and stability near the target.

2. Ion Sources and Low-velocity Acceleration

Intense sources of heavy ions (calutrons) have been used in the past; however, the emerging beam quality is not well known, but is suspected to be poorer than desired. A development program is therefore desirable. Extrapolations from known performance of conventional (duoplasmatron) types gives encouragement that current and brightness (50-100 mA, ~ 0.02 cm-mrad) needed for injection into low-velocity r.f. accelerator structures can be obtained. The use of less familiar molecular ions might prove to be advantageous; development of sources for these, and experiments on their photodissociation, would be required. The high-current pulsed-power sources now being developed for lighter ions deliver currents required by linear induction accelerators, but measurements of their beam quality are required. Also, development of heavy-ion versions, together with appropriate pre-accelerating structures, are needed.

Because of the low ratio of charge to mass, and hence low ion velocity, present high-voltage column and low-velocity r.f. linac techniques require some extrapolations. Many of these are subject to

calculation using existing knowledge, which is adequate to provide some assurance that the requirements can be met. Construction of the first stages of a system along lines proposed during the study is desirable for tests of performance and reliability.

3. Atomic, Molecular, and Nuclear Physics

Available estimates of cross-sections for change of charge state in ion-ion collisions, for various ions and energies up to 150 keV, have significant uncertainty. These cross-section values are of great importance to accelerator design studies; the estimates indicate that the upper limit on the time for acceleration and manipulation may be so short as to rule out the slower options (~ 1 sec) but to allow the faster ones (~ 1 ms). Theoretical, computational, and experimental work that, taken together, may improve our confidence in these estimates is urgently needed.

Experiments to measure a variety of cross-sections for interactions of beam particles with the background gas are needed to provide vacuum requirements and to estimate wall bombardment effects.

The range-energy relation for very heavy ions in solids at normal temperatures is well enough known for target design estimates. Corrections for ionized target materials are needed for detailed design work. Nuclear fragmentation processes can be studied with existing or proposed heavy ion beams and may be of interest for certain target configurations.

4. Accelerator Systems

During the study an approach was made to parametrizing a wide variety of possible systems with respect to the means of achieving the very large current multiplication required. Many, but not all, of the various fundamental and practical constraints on performance were considered in a brief examination of a few selected cases. The results of this work were encouraging, but its incompleteness strongly emphasizes the need for an ongoing intensive study of the many options not examined and of factors not considered in the few cases which were studied.

Accelerator systems based on an r.f. linear accelerator and a circular accumulator provided the most promising case examined in detail during the study. Such systems are well understood, utilize demonstrated technology, accumulate beam rapidly, and are efficient at currents of 50 to 100 mA. Although their repetition rate capability would allow them to serve as ignition sources for fusion reactor plants, it is recognized that they are more expensive than systems employing circular accelerators. These systems also show promise, although they require a longer time for acceleration. Therefore their feasibility is contingent on uncertainties in ion-ion cross sections for change of charge state. Components for beam accumulation and manipulation in both types of systems must be better defined as a basis for more detailed examination of feasibility, design features, costs, and efficiency.

The linear induction accelerator offers the unique possibility of a high-current single-pass system from source to target with good electrical efficiency and modest vacuum requirements. Its principles are well established; several have been built and operated to accelerate electrons, but their application to ion acceleration has not been demonstrated. Further intensive design study is necessary to permit evaluation of feasibility, costs, and efficiency.

Longitudinal compression of beam bunches by r.f. occurs naturally, or can be induced, to a limited extent in existing accelerators, and is well understood. Much greater compression factors may be desirable in r.f. components of igniter systems. Practical limitations are not well known; the possibility of performing relevant experiments on a proton accelerator should be explored.

5. Final Transport and Focusing

Quadrupole beam transport systems can handle the needed total beam power for heavy ions in low charge states. Consideration of economy and the need for high field strengths may indicate that these magnets should be superconducting. High power beams of fully stripped ions may pose a serious problem in quadrupole transport systems, even if a multiplicity of beams is used, unless neutralization is employed. The use of solenoid magnets for such beams should be studied. The advantages and disadvantages associated with beam neutralization by naturally occurring processes or by electron injection were not resolved during the study; further examination is indicated.

Focusing of beams on target spots of the required size by quadrupole lens systems located outside the reactor's inner wall seems feasible. The behavior of un-neutralized beams between reactor wall and target is predictable in sufficiently high vacuum. However, the presence of background gas, and of residual plasma from the preceding shot, will tend to produce a degree of charge and current neutralization. Although some work during the study indicated that beam instabilities near the target were not likely to be a serious problem, further work with the help of plasma physicists is required.

6. Alternatives

Included among the study's participants were several workers familiar with, or engaged in, studies and developmental work aimed at achieving inertial fusion by means other than lasers or heavy ion beams produced by proven types of accelerators. The potentials of these programs are speculative in varying degrees and some are without any experimental foundation at present. No attempt was made to reach conclusions about them during the study.

7. Costs of Power-producing Systems

Specific accelerator system and target-reactor system designs were not developed during the study. Therefore capital and operating cost estimates of an ultimate power generating installation are very rough at present and are based largely on general experience. These indicate in a qualitative way that the accelerator systems considered could cost a few hundred million dollars for an electric power plant in the thousand megawatt range. Economic analyses of complete systems should be undertaken as more definite designs are developed in future studies.

The accelerator systems will be largely based on existing technology in which substantial experience exists in accurately estimating costs, reliability, and useful life after a system has become well defined.

Organizing Committee:

Glenn W. Kuswa, ERDA
 Ronald L. Martin, ANL
 Alfred W. Maschke, BNL
 John H. Nuckolls, LLL
 Lloyd Smith, LBL

Assisted by:

William B. Herrmannsfeldt, ERDA
 and SLAC

Co-chairmen:

* (Note by Editors):

A fuller statement on this point will be found in the Overview, and an extended discussion in the Technical Summary, under the headings "Final Focusing and Transmission to Target."

Roger O. Bangertel
 Roger O. Bangertel, LLL

David L. Judd
 David L. Judd, LBL

VI. Appendices

APPENDIX 1-1 ION-BEAM TARGET DESIGN

Milton J. Clauser

Ion-beam targets consisting of a high Z shell containing DT gas^{1,2} appear to be useful in demonstrating feasibility of ion-beam fusion. They would also be useful in reactors with very efficient accelerators (~50%) since the gain appears to be about 20-30. To date, the study has concentrated on finding targets with low-power ignition thresholds, rather than targets with high gains. Consequently, higher gains may well be possible with these targets. No particular pulse shape is required for these targets; the calculations assumed a constant power during the implosion. The optimum ion energy for these targets is around 10 MeV for protons, 8 GeV for iodine, and 20 GeV for uranium, all of which have a range in gold of ~0.4 g/cm². Table A1-1.1 shows several typical results. All of these calculations assume that the energy-deposition profile has a Bragg peak at the endpoint of the range. However, additional calculations for the first target in the table show that the beam power required for ignition increases only 15-20% when the Bragg peak is omitted (i.e., when a flat profile was used).

Table A1-1.1.

Diameter (mm)	Gold thickness (mm)	DT mass (μ g)	Proton energy (MeV)	Beam power (TW)	Yield (MJ)	Gain (~)
1. 2.44	0.22	100	10	120	15	15
2. 2.44	0.22	200	10	250	35	25
3. 1.44	0.22	50	10	160	6	14
4. 6.38	2.69	50	50	800	6	2.3

As the target size is increased the beam power required to obtain a given gain appears to remain fairly constant. Larger gains appear to be obtainable with larger targets, but at the expense of higher beam powers. Beams of higher voltage (particle energy) can be used (see last line of Table A1-1.1), however they result in lower gain. More material is added to the outside of the target, which has the main effect of reducing the particle energies (to 10 MeV for protons, for example).

Initial studies of instabilities in these targets³ have shown that when a deposition profile

with a Bragg peak is used, Rayleigh-Taylor instabilities may limit the performance that can be achieved with these targets. When the beam is defocused somewhat, the Bragg peak is reduced, thereby reducing the pressure and density gradients that produce the instability. A related problem, which will also require further study, is the effect of asymmetries produced by multiple-beam irradiation of the targets.

On the positive side, there are a number of techniques, such as pulse shaping, which have yet to be exploited, and which may increase the gain of these targets.

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APPENDIX 1-2 ION-STOPPING POWER OF IONIZED MEDIA

David Mosher

Here, it is desired to evaluate the modifications to the usual stopping-power formula for energetic ions due to free electrons in the stopping medium. For both bound and free electrons, one may write in cgs units

$$\frac{dE}{dx} = - \frac{4\pi z^2 e^4}{m_0 v^2} \left\{ N_b \ln A_b + N_f \ln A_f \left[\operatorname{erf}(y) - \frac{2}{\pi^{1/2}} y e^{-y^2} \right] \right\}, \quad (1)$$

where:

$$E = \text{ion energy} = 1/2 M_i v^2,$$

- m_0 = electron rest mass,
 z = charge state of test ion with atomic number Z_A ,
 N_b = density of bound electrons in medium,
 N_f = density of free electrons,
 $y = [m_0 E / N_f \theta]^{1/2}$,
 $\theta = kT_e$ of the free-plasma electrons.

The quantity in square brackets in the above equation becomes important, i.e., appreciably different from unity, when the electron thermal velocity becomes comparable to V . For E measured in MeV and θ in keV

$$y^2 = \frac{0.545}{A} \frac{E_m}{\theta_k}$$

where A is the test ion atomic number. Thus, the thermal correction is significant for protons of a few MeV and uranium ions of about 0.5 GeV. The quantity in square brackets can be accurately approximated by

$$\left[\right] = \left(1 + \frac{4}{3} y^2 \right)^{-1}$$

For the heavy ions of interest here, the thermal correction can be safely ignored except during the last few percent of the range, so that the behavior of

$$\frac{dE}{dx} = - \frac{4\pi z^2 e^2}{m_0 V^2} N_e \left\{ \left(1 - \frac{z}{Z_A} \right) \ln A_b + \frac{z}{Z_A} \ln A_f \right\} \quad (2)$$

will be investigated. Here Z is the ionization level of the background plasma of atomic number Z_A , and N_e is the total electron density.

The Coulomb-logarithm for bound electrons may be written

$$A_b = \frac{2m_0 V^2}{(I)}$$

where (I) is the average ionization potential over bound states. For neutral atoms, the empirical expression

$$(I)_0 = 9Z_A \left(1 + \frac{1.8}{Z_A^{2/3}} \right) \text{ eV}$$

may be employed. As the plasma becomes ionized, (I) must increase above this value since the low-energy bound states are missing. In the limit of a single K-shell electron remaining bound, (I) would have to be its ionization potential. A reasonable variation of (I) with ionization level might be

$$(I) = (I)_0 \quad \text{when} \quad (I)_0 > 13.7 \text{ eV}$$

$$(I) = 13.7 \text{ eV} \quad \text{when} \quad (I)_0 < 13.7 \text{ eV}$$

Free electrons can accept energy from the test ion for impact parameters as large as a Debye shielding length so that

$$A_f = \frac{2m_0 V^2}{I_f}$$

where

$$I_f \approx \frac{ze^2}{\lambda_D}$$

The combined logarithm term in Eq. (2) may now be written

$$\ln A = \ln A_s + \ln \left(\frac{(1)}{(I)_0} \right) + \frac{z}{Z_A} \ln \left(\frac{(1)}{I_f} \right) \quad (3)$$

Here $\ln A_s$ is the usual expression associated with deposition in cold, neutral media and is in the range of -9 for the ion beams of interest here. The second term in Eq. (3) is the correction due to changes in the mean ionization level (I) as loosely bound electrons are stripped. The final term is the contribution due to collisions with free electrons. The correction terms $= \ln A - \ln A_s$ do not depend on the beam parameters and may be evaluated once an ionization model for the medium is chosen.

When the beam traverses low-density gases that might be present in the reaction chamber, beam transit times are much less than the time required to achieve corona-model ionization equilibrium. When interacting with high-density pellet material, Saha-ionization equilibrium is appropriate. In either case, choosing an ionization level consistent with stripping to binding energies comparable to the electron temperature is reasonable. Figure A1-2-1

Fig. A1-2-1. (XBL 7612-11090)

illustrates the variation in the correction to stopping power as a function of electron temperature with this assumption for various electron densities and for a low- and high-atomic number medium.

The correction in low-atomic-number medium is primarily due to free electrons, while that for high- Z materials (because of the predominance of bound electrons even at elevated temperatures) is

mostly associated with changes in $\langle I \rangle$. The $Z_A = 6$ results shown are therefore sensitive to the ionization model employed in the below-100 eV range. The $Z_A = 81$ results are sensitive to assumptions made about the variation of $\langle I \rangle$ with Z in the above-100 eV regime.

In any case, factors-of-two or so increases in ion-stopping power are predicted when the beam traverses low-density media heated to above-100 eV electron temperatures.

APPENDIX 1-3 ION-BEAM REACTOR FIRST-WALL DESIGN

Clark Hovington

A study of a rudimentary first-wall design for a heavy-ion beam initiated, inertially confined fusion reactor with a fusion yield of 4×10^9 J per microexplosion with a prf of 1 sec^{-1} is presented. Several materials are investigated, including stainless steel, niobium, lithium, and isotropic and anisotropic graphite. If a chamber pressure of less than 0.1 Torr is required for beam penetration, the best first-wall material is pyrolytic graphite from a thermodynamic basis, and isotropic graphite from an economic basis. For higher allowable chamber pressures, a lithium wetted wall is acceptable.

FUSION REACTOR PERFORMANCE CHARACTERISTICS

To investigate possible reactor designs we assumed a fusion yield of 4×10^9 J per microexplosion, with a prf of 1 sec^{-1} . We also assumed that 32% of the yield is in x-ray and debris energy, divided between 2% in x-ray energy and 30% in debris energy.

THERMODYNAMIC ANALYSIS

A thermodynamic analysis was performed on various first-wall materials as a function of first-wall radius for the microexplosion. This analysis assumed that all the x-ray and debris energy was deposited at the surface of a semi-infinite solid. The initial temperature of the solid was assumed to be 800°K, and the thermal properties of the solid were assumed to be invariant with temperature. During the times when no energy flux was acting on the surface, the surface was assumed to be adiabatic. Finally, the solid was assumed to undergo no phase change, and the surface position did not move even when the surface temperature exceeded the melting point or the boiling point of the material.

The materials considered for the first wall included lithium (wet wall), niobium, stainless

steel, and two types of graphite. The first type of graphite was assumed isotropic, while the second type was assumed to be highly anisotropic, with the direction of maximum thermal conductivity in the reactor's radial direction.

The peak temperature at the surface of the first wall as a function of the reactor radius for the various materials is shown in Fig. A1-3-1.

Fig. A1-3-1. Peak liner surface temperature as a function of reactor radius with a 4000 MJ microexplosion. (NBL 7612-11078)

The stainless steel first wall will always have a peak temperature above its melting point, and the lithium will always have a peak surface temperature above its boiling point at 760 Torr. Niobium does not exceed its melting point for reactor radii greater than 16 m. The isotropic graphite does not exceed its sublimation point at 760 Torr for reactor radii greater than 13 m. The pyrolytic graphite can operate with a first-wall radius as small as 10 m.

Another consideration in the first-wall design is the mass of first-wall material lost during a shot cycle. This mass transfer will, in part, determine the vacuum pumping requirements for the reactor system. This mass-transfer analysis assumed only surface vaporization due to the difference in the vapor pressure of the first-wall material and cavity pressure as a function of the surface and cavity gas temperatures and time. The initial cavity partial pressure of the first-wall material is assumed to be the vapor pressure of the material

at 800°K. When the partial pressure in the chamber due to the vaporization of the first-wall material is the same as the vapor pressure, mass transfer from the wall is assumed to cease. Mass transfer from the cavity to the first wall during the portion of the shot cycle in which the surface temperature is decreasing was not considered.

The mass lost from the surface is shown in Fig. A1-3-2. Using as a criterion a maximum mass loss from the first-wall surface of 1 kg/shot, a lithium or stainless steel first wall will not be allowed, while the niobium and isotropic graphite first-wall radii must be greater than 16 m. Note that the pyrolytic graphite first-wall radius can be as small as 10 m.

Fig. A1-3-2. First-wall mass loss per 4000 MJ microexplosion as a function of reactor radius for several candidate first-wall materials. (XBL 7612-11079)

Another consideration for the first-wall materials is the thickness of material lost per shot. This loss, assuming no condensation of the gas in the cavity back onto the surface, will determine the lifetime of the first wall (neglecting fatigue, neutron damage, and hydrogen isotope chemical-reaction considerations). The design criterion of maximum allowable thickness loss of 10^{-3} μm per shot corresponds to about 1 cm of material per year.

The thickness loss of first-wall material per shot is shown in Fig. A1-3-3. Using the 10^{-3}

$\mu\text{m}/\text{shot}$ criterion, a niobium or isotropic graphite first-wall radius must be greater than 16 m, while the pyrolytic graphite first-wall radius must be greater than 11 m. Note that stainless steel is unacceptable for this reactor, while a lithium first wall at 20 m requires the replacement of a layer 0.2 mm thick per shot.

Fig. A1-3-3. First-wall recession per 4000 MJ microexplosion as a function of reactor radius for several candidate first-wall materials. (XBL 7612-11080)

RESULTS

Using the somewhat arbitrary criteria that the mass loss must be less than 1 kg/shot and the surface recession must be less than 10^{-3} $\mu\text{m}/\text{shot}$, the minimum radius for each material is given below.

Material	Radius, m
Niobium	16
Stainless steel	-
Lithium	-
Graphite	16
Pyrolytic graphite	11

^aDoes not meet criteria

If the cavity pressure is limited by ion-beam injection considerations to a maximum of the order

at 10⁻⁶ Torr prior to each microexplosion, then lithium is not a viable candidate for a first-wall material. If the maximum cavity pressure is not a limitation on ion-beam injection, then the lithium wet wall is a viable alternative to the dry wall since the recirculation of 70 kg of lithium between shots is not anticipated to be troublesome. Since the maximum allowable chamber pressure between shots is currently an open question, the use of lithium as a first-wall material will not be discussed further in this Appendix.

DESIGN OPTIONS

For a given blanket design (including structure) several first-wall designs are apparent. Basically, they consist of using a liner which is mounted independently of the blanket so that the liner can be replaced without removing all the blanket. There is one geometry that will allow easy removal of the liner without removing much of the blanket. This is a hollow cylindrical blanket with covers on the top and bottom. Thus, to change all the liners only the top cover of the blanket need be removed. Since the impulse load from the explosions can be handled for millions of shots (without consideration of neutron damage) by a 2-3 cm thick cylindrical shell of either niobium or stainless steel at 800°K, the following designs are possible:

- (1) A stainless steel or niobium liner 3-4 cm thick.
- (2) A niobium inner liner 2 cm thick in a 2-3 cm thick stainless steel shell.
- (3) A graphite inner liner 2 cm thick in a 2-3 cm thick stainless steel shell.

The outer surfaces of these liners can be cooled by a gas or a liquid metal. Note that the inner liner is not a load-carrying member. It is used to reduce the vacuum pumping loads and increase lifetime. It also decreases the cost of the liner by using a cheaper material (stainless steel) as a structural material to support the more expensive material (niobium), or to support a weak (in tension) material (graphite).

FIRST-WALL COSTS

The fabricated costs of the first-wall materials are given below.

Stainless steel ¹	\$ 30/kg
Niobium ²	\$120/kg
Graphite (isotropic) ³	\$ 22/kg
Pyrolytic graphite ⁴	
High grade (hot pressed and annealed at 3000°K)	\$12,000/kg
Low grade	\$ 3,000/kg
Pyrolytic graphite ⁵	
High grade	\$ 1,200/kg
Low grade	\$ 306/kg

The volume of the first wall is based on a "square" cylinder with top and bottom covers. Thus,

$$V = 6\pi R^2 t$$

where R is the radius of the first wall and t is the thickness of the vessel. The radius, thickness, and mass for several first-wall material combinations are given in Table A1-3-I.

Table A1-3-I. First-wall radius, thickness, and mass for several first-wall material combinations.

Material combination	Material	Thickness (cm)	Radius (m)	Mass (kg)
Stainless	stainless	4	NA	NA
Niobium	niobium	4	16	1.64×10^6
Niobium/Stainless	niobium	2	16	0.82×10^6
	stainless	3		1.13×10^6
Graphite/Stainless	graphite	2	16	0.16×10^6
	stainless	3		1.13×10^6
Pyrolytic graphite/Stainless	pyrolytic graphite	2	16	0.08×10^6
	stainless	3		0.53×10^6

The cost of the first-wall liner for various material combinations is given in Table A1-3-II below, using data from Table A1-3-I. The cost of the first-wall liner per unit of electrical power for various material combinations is also given in Table A1-3-III below. Note that this cost assumes a liner lifetime of 4000 MW_e-years (1500 MW_e-years).

Table A1-3-II. Cost of first-wall liner.

Material combination	Cost	
	M\$	Mill/kWh _e
Stainless	NA	NA
Niobium	197	17
Niobium/Stainless	132	11
Graphite/Stainless	37.4	3.2
Pyrolytic graphite/Stainless ^a (high grade)	106	9.1
Pyrolytic graphite/Stainless ^a (low grade)	38.6	3.3

^aCurrent pyrolytic graphite costs reduced by an order of magnitude.

The isotropic graphite over stainless steel combination is the cheapest first-wall liner. This is somewhat misleading as the blanket cost for this combination is 2.1 times that for the pyrolytic graphite over stainless steel liner combination.

because of the difference in the reactor radii. Also, the costs of the reactor housing and other portions of the reactor system are proportional to a function of the reactor radius. Thus it is not clear that the isotropic graphite over stainless steel combination liner results in a cheaper reactor system than the pyrolytic graphite over stainless steel combination liner.

In terms of cost per kWh, the isotropic graphite combination liner is cheaper than other material combination liners. Figure A1-3-4 shows the cost

Fig. A1-3-4. Cost per kWh of a pyrolytic graphite over stainless steel liner combination as a function of pyrolytic graphite cost for a 4000 MJ yield, 1 s^{-1} microexplosion in 11 m radius reactor. (XBL 7612-11076)

per kWh for a pyrolytic graphite stainless steel combination as a function of the cost of pyrolytic graphite. Note that for the same or less cost per kWh as an isotropic graphite stainless steel combination liner, pyrolytic graphite can cost no more than 280 \$/kg, or 13 times that of isotropic graphite. The cost per kWh of the blanket is independent of the blanket radius since the lifetime of the blanket is proportional to the inverse of the blanket radius squared, while the cost of the blanket is proportional to the blanket radius squared. The cost per kWh of the reactor housing should depend only on the reactor radius. Thus, in terms of cost per kWh for the total reactor system, the pyrolytic graphite maximum allowable cost may be more than 13 times that of isotropic graphite.

GRAPHITE ISSUES FOR INERTIALLY CONFINED FUSION REACTORS

Olander has used pyrolytic graphite targets in his molecular beam research.^{6,7} He has found that the pyrolytic graphite remains highly anisotropic even at surface temperatures as high as 2700°K (which conforms to the performance of a 10 m radius reactor) even when bombarded with oxygen molecules and hydrogen atoms. Olander has found that hydrogen attacks pyrolytic graphite at high temperatures to

form acetylene (C_2H_2). Assuming that all the hydrogen isotope debris from the microexplosion forms C_2H_2 , the surface recession rate for a 10-m-radius graphite liner is about 10^{-5} m per shot, or a loss of 0.3 mm per year. Thus, from a first-wall lifetime consideration, the hydrogen isotope attack of graphite is not important.

Rovner³ has estimated that the lifetime of pyrolytic graphite with a neutron flux of 1.2 MW/m^2 should be about 1 year. This is only a guess on Rovner's part, and should be confirmed using a pulsed 14 MeV neutron source with the same thermal cycle and flux as expected in an inertially confined fusion reactor.

Rovner and Hopkins⁸ discuss other aspects of graphite in fusion reactors. Varnado and Carlson⁹ discuss the possibility of a graphite first wall in an electron-beam-induced fusion reactor, while Hovingh¹⁰ discusses the possibility of a graphite first wall in a laser-fusion hybrid reactor. Varnado, Carlson, and Hovingh feel that graphite is a viable candidate as a first-wall material for inertially confined fusion reactors.

LEAD FORMATION ON FIRST WALL

Assuming that pellets will use lead shells instead of the gold used in Clauser's calculations¹¹ (because lead is cheaper than gold), the lead-pellet debris will deposit on or in the first-wall material. The lead may be vaporized from the first wall and pumped into the vacuum system. The amount of lead that will be vaporized from the first wall, assuming all the lead is located on the liner surface, is a minimum for the pyrolytic graphite over stainless steel liner combination because this liner operates with the lowest peak temperature. For the liner combinations and design radii of interest, more than 100 kg of lead can be vaporized each shot. This is an overestimate of the amount of lead vaporized, as the calculations do not consider the amount of lead vapor condensed on the first-wall surface while the wall is cooling down after the energy has been deposited.

The range of lead ions in graphite as calculated by Haggmark of Sandia-Livermore is shown in Fig. A1-3-5. The evaporation rate of the lead on the graphite surface may well be diffusion limited, so the diffusion coefficient of lead in graphite should be estimated, and then measured.

If more lead is deposited per shot than re-vaporized, the lead will reduce the range of the debris and x-rays into the first wall with the result that the thermodynamic model given above is more realistic. The disadvantage of the lead is that it is a poorer thermal conductor than pyrolytic graphite, niobium, or graphite. Thus, the surface temperature of the lead will be higher than estimated above, with the result that more lead will be vaporized from the surface. Also the lead will reach an equilibrium thickness since the melting point of the lead is less than the wall temperature before the shot (600°K vs 800°K). This equilibrium thickness will be dependent on the surface tension and flow characteristics of the lead on the first-wall material.

Fig. A1-3-5. Range as a function of energy of lead ions in graphite. (XBL 7612-11062)

CONCLUSIONS

A reactor vessel for a 4×10^9 J ion-beam-initiated, inertially confined fusion reactor can be designed for reasonable lifetimes. The best first-wall material from a performance consideration is pyrolytic graphite. It will need to be developed with high thermal conductivity in the radial direction (as opposed to nosecones for re-entry vehicles where the thermal conductivity is low in the radial direction and high in the axial direction). Using pyrolytic graphite as a first-wall liner, an 11-m radius chamber appears to be possible with a liner lifetime of a year. From economic considerations an isotropic graphite over stainless steel liner is the desirable first-wall material combination, provided that pyrolytic graphite cannot be fabricated for less than \$280/kg. An isotropic graphite liner of 16 m radius will have a lifetime of a year.

Maniscalco has identified some of the factors, which should be investigated, that affect the feasibility of the reference reactor concept.¹² In addition to those listed by Maniscalco, some work should be performed on how the cost and performance characteristics of pyrolytic graphite depend on quality and quantity of production.

For the low pre-shot cavity pressure case, isotropic graphite is the desirable and feasible liner material at least until proven otherwise. For the high pre-shot cavity pressure case (> 1 Torr), the wet-wall concept may be a viable competitor to an isotropic graphite liner, depending on the relative costs.

This Appendix does not cover the complete analysis of the first-wall liner. Additional analysis needs to be performed on the first wall. Most of the other problems such as blanket design, pellet fabrication and injection systems, vacuum system, tritium recovery system, etc., will need

to be studied. Hopefully this Appendix will be a seed from which reactor designs for ion-beam fusion systems can grow.

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APPENDIX 1.4 GAS-FILLED REACTOR

Robert J. Burke

High-energy heavy ions can penetrate a substantial thickness of gas without being scattered away from a 1-mm-diam target. This tolerance for gas may be employed to reduce the vacuum-pumping requirement in the reaction chamber and to alleviate the problems of removing the energy carried by the charged-particle component of the fusion reactions. Sputtering by energetic charged particles and very

high wall-heating rates by x-ray or fast-plasma impact would be avoided. Charged-particle energy would be absorbed in the atmosphere and then transmitted to the wall by radiation. Tolerance for gas avoids the pumping problem associated with ablation of a wetted wall.

The main questions about the viability of a substantial gas density in the reaction chamber concern the possibility of collective effects that can deviate or diffuse the beam. It is also necessary to maintain a large pressure differential between the chamber and the accelerator system, but this may be accomplished by using a gas that has low vapor pressure at room temperature or differential pumping techniques. The charge state of the high-energy ions would be quickly changed in the gas. This poses no difficulty for fully stripped ions, but ions that are accelerated and transported in a lightly stripped state require a good vacuum. Transport of lightly stripped ions and a gas-filled chamber can be made compatible by stripping the ions at the end of the transport lines. Transport at a high charge state would be restricted to a short distance (where charge neutralization is expected) and the final focusing magnets would work on highly charged ions.

Lithium has several good properties for the chamber gas. It is a relatively weak scatterer,

condenses at about 200°C, has a vapor pressure of 1 Torr at 750°C, and has high opacity in the temperature range that is involved in eventual transmission of heat to the wall. In addition, its low ionization potential would be valuable if it is desired to apply direct conversion techniques to a low-temperature plasma heated by the charged particle energy.

The question of the duration of the wall heating period is complicated by the necessity for opacity data in the 1-eV range. Analytic estimates based on an opacity maximum of $10^5 \text{ cm}^2/\text{g}$, which occur in this range, indicate that the wall heating period could be hundreds of milliseconds. Numerical calculations at LLL do not disagree with this conclusion, but further work is required. For instance, one calculation indicated that less than 1% of a $2 \times 10^9 \text{ J}$ energy input (10^{10} J total for d-t) was transmitted to a 10-m wall through lithium at a density of $10^{18}/\text{cm}^3$ after 50 msec. However, for the same energy input and gas density but with a 5-m radius chamber, almost all the energy was transmitted to the wall in less than 1 msec. This indicates high sensitivity of radiation transport to the temperature (since the smaller chamber had about an order of magnitude less gas to absorb the energy) and a need for further study to obtain reliable numbers.

APPENDIX 2-1

1. CLASSICAL ION SOURCES

John Faso

SOURCE REQUIREMENTS

Once I_f , the final (uncompressed) current required to ignite the pellet is specified, the total output of the ion source, I_0 , is given by

$$I_0 = \frac{I_f}{\Gamma \eta} \quad (1)$$

where Γ is the fraction of the total current I_0 that has the correct mass and charge, and η is the fraction of I_0 that survives acceleration and transport to the pellet. To obtain a reasonably large value of Γ , it will be necessary to use a single isotope of whichever element is chosen as the feed material if unwanted isotopes constitute a significant fraction of the whole. For a plasma source of ions that utilizes a molecular vapor for the production of predominantly singly charged atomic ions (e.g., UCl_4 for U^+), Γ will be about 0.5 due to the production of extraneous ion species. (Values of Γ ranging from 0.45 to 0.59 can be found in the literature^{1,2} for U^+ ion sources using UCl_4 as the feed material.) For plasma sources using atomic vapors or gases that yield large numbers of multiply charged ions, little or no work seems to have been done to maximize the Γ yield; we shall assume that $\Gamma \approx 0.5$ for these sources also. For a surface-contact ionization source of the porous-emitter type, $\Gamma \approx 1.0$.

Most of the attenuation of the beam during acceleration and transport occurs in the linac, which transmits about 60% of a bunched beam. We may assume that additional attenuation elsewhere reduces the value of η to ~ 0.5 . Thus the assumed ion source current requirements are

$$I_0 \geq 4I_f \quad (\text{plasma source of ions}) \quad (2)$$

$$I_0 \geq 2I_f \quad (\text{porous-emitter surface-ionization source}). \quad (3)$$

The attainable current density from heavy-ion plasma sources may be estimated by noting (as John Osher, LLNL, has done) that 500 mA/cm^2 is readily attainable with hydrogen plasma sources. The current density may thus be expected to scale as

$$j_0 \sim 500 (q/A)^{1/2} \text{ mA}/\text{cm}^2. \quad (4)$$

Thus, for $q=1$ and $A=200$, $j_0 \sim 35 \text{ mA}/\text{cm}^2$. That this estimate is conservative is shown by the fact that an analyzed 1 A Te^+ beam has been obtained from a plasma-surface ion emission area of 29 cm^2 in an Oak Ridge grid-type source.³ The total beam current (Te^+ plus unspecified impurities from a plasma fed by TeCl_4) is greater than 1 A; hence the current density at the plasma ion-emission surface is $j_s > 34 \text{ mA}/\text{cm}^2$.

For a cesium porous-emitter, surface-ionization source, Kuskevics and Thompson⁴ have obtained up to 9 mA of 10 keV Cs^+ ions at a stated current density

of 80 mA/cm^2 . A more typical value of current density (one at which the source operates stably) is 35 mA/cm^2 .

For plasma sources, the maximum single-aperture beam current that can be obtained from a given extraction geometry can be estimated by using the empirical formula which Kelley⁵ writes in the form

$$I = \frac{5.44 \times 10^{-8} \pi^{3/2}}{A^{1/2} (z+a)^2/a} A \quad (5)$$

for a singly charged ion. For the present discussion it is more convenient to rewrite (5) in the form

$$I = \frac{1.71 \times 10^{-4} a^{1/2} V^{3/2}}{A^{1/2} (1+z/a)^2} \text{ mA}, \quad (6)$$

where V is the extraction voltage, a is the initial beam radius, and z is the length of the extraction gap. According to Kelley, this formula is usually accurate to better than 5%. The ratio z/a is usually restricted to values ≥ 1 and frequently to values ≥ 2 to limit aberrations and the diverging effect of the aperture ($1/2$ angle divergence = $a/4z$).

Once a value of the ratio z/a has been chosen and the required value of I_0 has been determined by putting the appropriate numbers in Eq. (1), V_0 is given by Eq. (6). The value of a is then chosen to give a reasonable current density, which may be defined as the value given by the expression (4). z is then calculated from the chosen ratio z/a . If the voltage V_0 can be held across the gap z , conceptual design of the source is complete, provided the emittance of the ion beam does not exceed the acceptance of the accelerators that follow. If V_0 exceeds the breakdown voltage, z can be increased and r may be increased proportionately to keep z/a constant.

In the absence of aberrations, the two-dimensional momentum-normalized emittance of the beam⁶ at the exit of the source is given by

$$\epsilon_n = \beta_0 a_e r_0' \text{ cm-mrad} \quad (7)$$

where a_e is the beam radius at the exit, r_0' is the value of dr/dz at $r=0$ and $\beta_0 a_e r_0'$ is the two-dimensional phase space area, which is bounded by an ellipse. The beam area $r=a_e$ at the exit of the source can be less than, equal to, or greater than the radius $r=a$ at the ion emission surface. We shall assume $a_e = 0.9a$.

For the preliminary reference design, the value $\epsilon_n \leq 0.10$ has been chosen for the exit of the low-beta linac. When emittance dilution is taken into account, the emittance at the source must be at least a factor of five better, i.e., $\epsilon_n \leq 0.02 \text{ cm-mrad}$ at the source exit.

Some preliminary calculations of single-aperture ion-source design characteristics are shown in Table A2-1-1-1. For the atomic ions, the average voltage gradients, V_0/z , are rather high, of the order of 100 kV/cm.

Table A2-1-i-1. Design characteristics of single-aperture ion sources.²

Ion	Γ	n	z/a	I_f (mA)	I_o (mA)	V_o (kV)	j_o (mA/cm ²)	a (cm)	ϵ_z (cm·mrad)	r_o' (mrad)	$V_o/z^{1/2}$ (kV cm ^{-1/2})	α (mrad)	B_4 (mA cm ⁻² mrad ⁻²)
¹³² Xe ⁺	0.5	0.5	2	50	200	245	43.5	1.21	0.02	8.30	157	112.5	1.0×10 ⁵
¹³³ Cs ⁺	1.0	0.5	2	50	100	154	43.4	0.86	0.02	14.8	117	112.5	1.0×10 ⁵
²⁰⁵ Tl ⁺	0.5	0.5	2	50	200	283	34.9	1.35	0.02	8.63	172	112.5	1.0×10 ⁵
²³⁸ U ⁺	0.5	0.5	2	50	200	298	32.4	1.40	0.02	8.74	178	112.5	1.0×10 ⁵
¹²⁸ (HI) ⁺	0.5	0.5	2	8.0	32	71.3	7.1	1.20	0.02	17.0	46	112.5	1.6×10 ⁴
¹²⁸ (HI) ⁺	0.5	0.5	4	8.0	32	141	7.1	1.20	0.02	12.1	91	62.5	1.6×10 ⁴
¹²⁸ (HI) ⁺	0.5	0.5	7	8.0	32	240	7.1	1.20	0.02	8.8	170	35.7	1.6×10 ⁴

I_f = final (uncompressed) current required to ignite the pellet

I_o = I_f/Γ = total ion current out of ion source

Γ = fraction of I_o having correct charge and mass

n = fraction of ΓI_o which survives acceleration and transport to pellet

j_o = $500 A^{-1/2} (80 A^{-1/2} \text{ for } HI^+)$

a = initial beam radius = $(I_o/\pi j_o)^{1/2}$

z = extraction gap length

a_e = 0.9 a = beam radius at source exit

ϵ_z = $8\gamma a_e r_o'$

r_o' = dr/dz at $r=0 = \epsilon_z (8\gamma a_e)^{-1}$

α = $a_e/4z$ = initial beam divergence due to aperture effect

B_4 = $2I_o/(\pi \epsilon_z)^2$

The literature offers seemingly contradictory evidence on whether the extraction voltages V_o listed in Table A2-1-i-1 for the atomic ions exceed the breakdown voltage V_B . Cranberg⁷ has shown that for breakdown in vacuum V_B obeys a law of the form $V_B = kd^{1/2}$, where d is the gap length. Coupland et al.⁸ have found that $V_B = 60 d^{1/2}$ kV for their single-aperture, three-electrode helium ion source. Huguenin et al.⁹ have found that, by using titanium alloys for the fabrication of electrodes, an average gradient greater than 100 kV/cm can be held at 600 kV (at pressures of 10^{-6} and 10^{-4} Torr) in the CERN Linac preinjector. Figure 16 of reference 9 gives data on voltages and gaps of some preinjectors, from which it may be inferred that for high-gradient preaccelerators for proton synchrotrons $V_B > 150 d^{1/2}$ kV. For the ANL preinjector¹⁰ (which is, in fact, a single-gap 750 keV ion source with a gap length of 17.8 cm) $V_B > 178 d^{1/2}$ kV. A comment by O. B. Morgan, of ORNL, during a discussion of a paper by Wroe at the 1974 Linear Accelerator Conference,¹¹ indicates that a 300-keV dc hydrogen-ion beam has been extracted from a dioplasmatron with a 1-cm extraction gap; this implies $V_B > 300 d^{1/2}$ kV for this Oak Ridge source.

CHOICE OF ATOMIC ION

If contamination of accelerators and transport

lines by cesium can be tolerated, ¹³³Cs⁺ produced by diffusion of cesium vapor through a porous emitter and ionization on the surface of the emitter is the ideal choice, since only ions of the correct charge and mass are obtained in any significant amount ($\Gamma \approx 1$). If cesium contamination cannot be tolerated, ¹³²Xe⁺ would be the next best choice because xenon is an inert gas easily introduced into a source plasma. Since multiply charged ions will also be produced, Γ cannot be as high as for cesium. In the absence of experimental efforts to maximize the +1 charge state, it is necessary to defer a final judgment on the suitability of Xe⁺ but, because xenon is an inert gas, it is clearly preferable to any other ion for which Γ is comparable.

As with cesium, contamination of accelerators and transport lines would be a problem with Hg⁺, but mercury shares with xenon the advantage of being easy to introduce into a source plasma in isotopically pure elemental form. As a result of the uranium isotope separation effort at Oak Ridge during the 1940s and 1950s, a great deal is known about producing large currents of ²³⁸U⁺ from UCl₄ and other compounds. The available information on isotope separators shows that the use of uranium, thallium, or other ions obtained from halides would pose a number of serious problems because of the

reactions of the halides and their elemental constituents with ion source and accelerator components. Large ion currents can be obtained as or more easily with cesium, xenon, and mercury than with uranium or thallium, and all of the former would pose fewer problems than the latter.

CHOICE OF SOURCE TYPE

For Cs^+ , a porous-emitter surface-ionization source is the obvious choice since, unlike a plasma source, it produces only low-temperature, singly charged ions of cesium from a solid emitter of fixed shape and position.

For ions such as Xe^+ , Hg^+ , U^+ , etc., which cannot be produced by surface ionization, any plasma source with the ability to produce a quiescent plasma of adequate density and uniformity over an emission surface several centimeters in diameter will do, provided that the discharge and extraction electrodes are not rapidly destroyed by physical and/or chemical sputtering or altered in a detrimental way by the buildup of deposits which, e.g., clog apertures or (if they are insulating deposits) disturb the plasma or the ion beam by distorting electric fields as they become charged by ions or electrons.

MOLECULAR IONS FOR DISSOCIATION INJECTION INTO A STORAGE RING

The discussion above was concerned with singly charged atomic ions. If a molecular ion such as HI^+ is used for dissociation injection into a storage ring, as in the Heartfire concept, the current requirement at the source will be reduced to ~ 32 mA for a design value of $I_f = 8$ mA. This reduction, by a factor of 6.25 from the 200 mA required for the beams from plasma sources of atomic ions, has also been applied to the current density to give the value of 7.1 mA/cm^2 shown in Table A2-1-i-1. It may be necessary to reduce the current density still further to minimize dissociation of HI .

Wilson¹² has obtained a $75 \mu\text{A}$, 120 keV HI^+ beam with an initial diameter of 0.4 cm (0.6 mA/cm^2). The beam current was limited by the 300 kV accelerator, which was operated at reduced voltage to get a better separation of I^+ and HI^+ with a 60 cm $\text{E} \times \text{B}$ velocity separator (Wien filter). The beam was obtained with a low-current $\text{E} \times \text{B}$ source (axial magnetic field, radial electric field) with extraction along the magnetic field.¹³ The arc voltage was 100 V , the arc current $\sim 100 \text{ mA}$, and the source magnetic field was $\sim 200 \text{ G}$. The beam composition was as follows:

H^+	6%
H_2^+	6%
H_3^+	0.05%
I^+	35%
I_2^+	1.7%
I_3^+	0.05%

I_2^+	0.2%
HI^+	50%
HI_2^+	0.08%

Preliminary calculations of single-aperture HI^+ ion-source design characteristics are shown in Table A2-1-i-1 for three values of the ratio z/a . Because the current density is lower for HI^+ , z/a can be larger than is practical for the atomic ion cases shown in Table A2-1-i-1.

For equal beam intensities, physical and chemical sputtering and the buildup of detrimental deposits should not be as severe for HI and for ions derived from HI as for metal halides such as UCl_4 and the ions derived from them. Since the current requirement is lower for HI^+ than for atomic ions such as U^+ derived from metal halides, the problems of sputtering and material buildup will be proportionately less severe.

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3. J. Luce, "Grid Ion Sources," Oak Ridge National Laboratory Electromagnetic Research Division Conference Notes (6) Y-B20-57, p.11 (January 13, 1950).
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11. O. B. Morgan, a comment during discussion of a paper by H. Wroe, Minutes of the 1964 Conf. on Proton Linear Accelerators held at Midwestern Universities Research Association, p.484.
12. R. G. Wilson, Hughes Research Laboratories, Malibu, California (private communication), July 1976.
13. R. G. Wilson, Ion Mass Spectra (Wiley-Interscience, New York, 1974), pp.1-8.

APPENDIX 2-1

ii. PENNING ION SOURCES

David Clark

Penning sources used by heavy-ion accelerators today were developed from the Calutron sources first developed during World War II for the uranium separation process at Oak Ridge. Present sources were summarized by Bennett¹ and by Winter and Wolf.² They are considerably smaller in size than the Calutron, with slit sizes of $1 \times 10 \text{ mm}^2$ to $3 \times 20 \text{ mm}^2$. The output currents are 5–50 mA total current, compared to 100–1000 mA, for Calutrons. The emphasis in these accelerator sources is in the production of high-charge states such as Au^{9+} and U^{11+} at the Berkeley SuperHILAC³ and the GSI Unilac at Darmstadt.⁴ The high-charge state reduces the accelerator size for a given energy. As a result the arc voltages are high: 500 V–2 kV and the plasma is noisy. The magnetic fields used are 3–17 kG. Penning sources have also been developed for cyclotrons, such as the heavy-ion machines at Oak Ridge,⁵ Berkeley,⁶ Dubna and Kurchatov,⁷ and elsewhere.¹ The emittances of some of these sources have been summarized.⁸ As shown in Table A2-1-ii-1 (from Ref. 8), the emittance areas are about the same as that of a Calutron: $\epsilon_{\eta} = 2\gamma A/\pi = 0.1 \text{ cm}^2\text{-mrad}$. However, as shown, the output current of a Calutron is 30–100 times greater for one charge state. This is a result of the much larger Calutron slit area, and its lower arc power which concentrates most of the current in one charge state: $Q=1$. So for the application of U^{1+} beams for inertial fusion, the Calutron source appears to be the best choice among Penning sources.

References

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7. B. N. Maslov, IEEE Trans. Nucl. Sci. **NS-73**, No. 2, p.1035 (1976).
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APPENDIX 2.1

iii. DUOPLASMATRON AND REFLEX-ARC SOURCES

Emery Zajec

The ion-source requirements for the initial fusion study indicate a beam of 50 to 100 particle mA, of mass in the range of 200, a charge of 1 or 2 with a normalized emittance of less than $0.025 \text{ cm}^2\text{-mrad}$.

Some data on duoplasmatron operation with Xe and Ar is shown in Table A2-1-iii-1. The output is listed as total charged beam collected into a biased Faraday cup. The estimated xenon singly charged beam is 9 mA. Also listed is a reflex-arc type of source giving 100 mA of separated H^+ with a normal-

ized emittance of $0.02 \text{ cm}^2\text{-mrad}$, or for a multiple-aperture version, 700 mA total beam at a normalized emittance of $0.2 \text{ cm}^2\text{-mrad}$.

Table A2-1-iii-1

		Low	1 (mA)	keV	ϵ_{η} ($\text{cm}^2\text{-mrad}$)	Plasma cup area
1.	Duoplasmatron	Xe	17	35	0.045	0.13 cm^2
	Duoplasmatron	Ar	25	35	0.08	0.13 cm^2
2.	Reflex arc	H^+	100	25	0.02	
	Reflex arc	H^+	700	20	0.2	

The task of producing beams of mass 200 and currents of 100 mA from a source seems to be within reach. A perhaps more difficult problem to be faced is the one of transporting and accelerating this very low β beam without incurring large space-charge losses.

APPENDIX 2.1

iv. BERKELEY XENON ION SOURCE TEST

Richard M. Mobley

A Berkeley 40 kV–65 A D_2 source was run with Xe as the source gas on July 24, 1976. A schematic of the source is shown in Fig. A2-1-iv-1. The short run at 30 kV extraction potential across a 50 mm gap yielded 9 A of charge current. Full smooth pulses of 10 msec were obtained, indicating that a matched plasma meniscus condition held at the extraction boundary.

Figure A2-1-iv-1. Extended-area ion source. Problem areas: (1) Want an ion heavier than Xe. (2) Xe^{+3} may be > 50% at extractor. Downstream collisions will neutralize. (3) Downstream accelerator has to contend with a "compensated stream." (XBL 7612-1100")

Table A2-1-ii-1. Comparison of present Penning source emittance measurements with some previous work.⁸

Lab	Ion	Acceleration voltage kV	Beam current		Energy per nucleon keV/A	β 10^{-3}	Anode slit dimensions mm $\times$ mm	Emittance		Normalized emittance ^a		Percentage of beam %
			Peak current μ A	Duty factor				$n \cdot 10^{-6}$ Axial	m·rad Radial	$n \cdot 10^{-6}$ Axial	m·rad Radial	
88-Inch Cyclotron (LBL) (B)	Ar ^{4+ + 8+}	10	200 $\pm$ 7.5	1	1.00 $\pm$ 2.00	1.46 $\pm$ 2.06	$\left\{ \begin{array}{l} 13 \times 1 \\ 0.7 \text{ mm} \\ \text{inside the} \\ \text{arc bore} \end{array} \right.$	74-117	50-78	0.14-0.22	0.084-0.15	> 90
(C)	Ar ⁴⁺ Ar ⁵⁺ Ar ⁶⁺	10 10 10	1100 400 200	1 1 1	1.00 1.25 1.50	1.46 1.63 1.79		~ 135	180-270	0.20-0.24	0.26 - 0.49	> 90
(P)	Ar ⁵⁺	10	240	0.33	1.25	1.63	Same as (C)					
Alice (5) (IPN Orsay)	Kr ⁸⁺	135	205	0.29	12.86	5.23	20 $\times$ 1	89		0.47		100
	Kr ⁸⁺	135	275	0.29	12.86	5.23	20 $\times$ 1	127	233	0.67	1.16	100 (axial) 80 (radial)
	Kr ⁸⁺	135	343	0.29	12.85	5.23	20 $\times$ 1	185		0.97		100
Super (6) HILAC (7) (LBL)	Ne ³⁺	750	300	0.15	113.	15.5	19 $\times$ 1.3		25.8		0.40	90
	Ar ³⁺	14.3	300	0.31	1.07	1.51	4.1 $\times$ 1.3	200	310	0.302	0.47	90
Grenoble Cyclotron (8)	Ne ⁴⁺	10	510	0.37	2.86	2.47	5 $\times$ 1		454		1.19	
	Ne ⁵⁺	10	46	0.37	3.57	2.76	5 $\times$ 1		134		0.37	
	Ne ⁴⁺	10	380	0.37	2.00	2.06	5 $\times$ 1		668		1.38	
	Ne ⁵⁺	10	54	0.37	2.50	2.31	5 $\times$ 1		127		0.29	

*Normalized emittance = $\beta \times$ measured emittance.

By comparison to D_2 results, using the scaling law

$$\frac{I_2}{I_1} \propto \sqrt{\frac{M_1}{M_2} \frac{Z_2}{Z_1}},$$

we deduce the average charge state of Xe to be 2.8. We expect to obtain 15 A at 40 kV by $V^{3/2}$ scaling.

There are 105 beamlets of $0.2 \times 7 \text{ cm}^2$, and the emittance in the "poor" plane should be less than that for D_2 :

Charge current 105 mA/beamlet (Xe^{+1} , Xe^{+2} , Xe^{+3})

Particle current 60 mA/beamlet, 40 mA/cm².

$$\epsilon_n < \frac{5}{\pi} \times 0.001 \text{ cm} \cdot \text{mrad}$$

Geometrical focusing is used in both planes to focus a neutralized beam to $\sim 4 \times 10^{-2} \text{ cm}^2$ (all beamlets) at 3 m. With Xe, we expect a high degree of charge-exchange neutralization ($> 90\%$), the rule of thumb being that $< 10 \text{ keV/nucleon}$ yields a "slow" atom for electron exchange collision with the back-ground gas.

For application to heavy-ion accelerators, the neutral Xe atoms would have to be re-ionized and quickly accelerated to $\sim 1 \text{ MeV}$ (Xe flash lamps?). The Berkeley CTR group is scheduled to have a 125 kV source of reduced area operating in August 1976. Consideration of this type of source for heavy-ion acceleration is continuing.

APPENDIX 2-2 PULSED HIGH-VOLTAGE ION DIODES

James W. Shearer

Pulsed diodes are driven by such large voltages that electrons are always field-emitted from the cathode. In the simple space-charge-limited diode, these electrons carry the bulk of the current. The ion current never exceeds $\sim 2\%$ of the total current, as shown by Langmuir's bipolar space charge solution of Poisson's equation. In the ion diodes this efficiency is enhanced by suppressing the electron current. The three ways of doing this are called the magnetically insulated diode, the reflex diode, and the pinched diode.

In the magnetically insulated diode (Figs. A2-2-1, A2-2-2, and Table A2-2-1*) the electrons

*The figures and tabulations in Figs. A2-2-1 through A2-2-6, and Tables A2-2-1 through A2-2-5 were excerpted from C. Olson's presentation to the Summer Study of Heavy Ions for Inertial Fusion, July 19-30, 1976, LBL.

experience an $\mathbf{E} \times \mathbf{B}$ drift from the combined effects of the diode voltage plus an externally applied transverse magnetic field $\mathbf{B}$. The heavy ions, however, are accelerated across the diode because their Larmor radius is much larger than the anode-cathode gap. Thus, the electron current is inhibited, and the ions carry up to 100% of the current.

Fig. A2-2-1. Ion diodes - Magnetic insulation. (1) Large B_0 inhibits electron flow ($\mathbf{E} \times \mathbf{B}$ drifts result); ions relatively unaffected. (2) Cylindrical geometry prevents charge (e) buildup. (3) Anode must be ion source. (4) Cathode must be transparent to ions. (XBL 7612-11068)

Proposed by:

1. Winterberg, *Physics of High Energy Density* (Academic Press, New York, 1971), p.370.
2. Rostoker, *IEEE Trans. Nucl. Sci.* NS-19, No. 2, 301 (1971).
3. Sudan and Lovelace, *Phys. Rev. Lett.* 31, (1174) (1973).

Fig. A2-2-2. Ion diodes - Magnetic insulation theories. B_c critical field defined (electrons just shut off) for B just $> B_c$, n_e is enhanced + enhances J_i . (XBL 7612-11071)

Critical field (B_c) calculations:

1. Ron, Mondelli, and Rostoker, *IEEE Plasma Sci.* PS-1, 85 (1973).
2. Lovelace and Ott, *Phys. Fluids* 17, 1263 (1974).

Ion flow calculations:

3. Sudan and Lovelace, *Phys. Rev. Lett.* 31, 1174 (1973).
 $B \gg B_c$, $J_i \approx J_i^{CL}$
4. Antonsen and Ott, *Phys. Fluids* 19, 52 (1976).
 $B \gg B_c$, $x^* \rightarrow L$, $J_i \gg J_i^{CL}$ (actually diverges)
5. Bergeron, *Appl. Phys. Lett.* 28, 306 (1976).
1-D cold-fluid model, $J_i/J_i^{CL} \sim 3-6$ (finite)

Table A2-2-1. Ion diodes - Magnetic insulation; experiments. Demonstration of magnetic cutoff of diode;^{a,c} demonstration that B stops A and K plasma motion.^{b,c}

Ion	ϵ_i (keV)	I_i (kA)	J_i (A/cm ²)	τ (nsec)	η	I_i/I_i^{CL}	B_z (kG)	Ref.
p	200	4	50	50	high	2	0-11	^d
p	--	5	70	--	--	--	--	^e
	150-300	--	40	100's	--	--	0-15	^f
p	150-200	5	70	50	70%	2.3	8	^g

^aGolden, Orzechowski, and Bekefi, J. Appl. Phys. 45, 3211 (1974).

^bOrzechowski and Bekefi, Phys. Fluids 19, 43 (1976).

^cBekefi, Orzechowski, and Bergeron, Proc. Int. Conf. E-Beams, 1, 303 (1976).

^dDreike, Eichenberger, Humphries, and Sudan, J. Appl. Phys. 47, 85 (1976).

^eWiley, Humphries, and Sudan, Bull. Am. Phys. Soc. 21, 691 (1976).

^fLuckhardt, Renk, and Fleischmann, Bull. Am. Phys. Soc. 21, 691 (1976).

^gHumphries, Sudan, and Wiley (to be published).

In the reflex diode (Figs. A2-2-3, A2-2-4, and Table A2-2-11) the electrons oscillate between two cathodes, passing through the thin anode with each pass. This reflex motion increases the ratio of electron space charge to electron current; therefore the space-charge limitation suppresses the magnitude of the electron current. The ion current passes out of the diode in the first pass; its magnitude is not suppressed by space charge. Efficiencies up to 50% are possible with this device. Details of contemporary research are shown in Figs. A2-2-3, A2-2-4, and Table A2-2-11.

Fig. A2-2-3. Ion diodes - Reflex (similar to PIG discharge with axial extraction). (1) Electrons reflex (net I_e reduced by $\sim N^{-1}$, N = number of reflections). (2) B_z prevents electron loss at edges. (3) Anode must be highly transparent and ion source

screen/varnish
nylon filaments
foils: Mylar, aluminum

(4) Cathode must be transparent to ions.
mesh
plasma
virtual cathode

Proposed by:

1. Humphries, Lee, and Sudan, Appl. Phys. Lett. 25, 20 (1974).

Fig. A2-2-4. Ion diodes - Reflex theories. As electrons slow down during reflexing, space charge builds up near anode + enhances J_i . (XEL 7612-11063)
Predict divergent I_i (resonance at $N = N_c$):

1. Humphries, Proc. II Symp. Ion Sources, LBL, VII-I-1 (1974).
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4. Creedon, Smith, and Prono, Phys. Rev. Lett. 35, 91 (1975).
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6. DiCapua, Huff, and Creedon, Proc. Int. Top. Conf. E-Beams, 1, 555 (1975).

Predict finite I_i :

7. Golden, Kapetanokos, Lee, and Goldstein, Proc. Int. Conf. E-Beams, 1, 635 (1975).
8. Miller, Mendel, Swain, and Goldstein, Proc. Int. Conf. E-Beams, 1, 619 (1975).

Table A2-2-III. Ion diode - Reflex experiments.

Ion	ϵ_i	I_i	J_i (A/cm ²)	τ (nsec)	n	I_i/I_i^{BP}	B_m (KG)	Ref.
p	100 keV	500 A	12	50	-	-	-	?
p	130 keV	6 kA	20	50	42	-	1.2	?
Ar ⁺⁺	1.8 MeV	5 kA	30	130	20	-	1.5	?
p	400 keV	1 kA	-	50	-	-	0.25	?
D	400 keV	1.5 kA	16	50	-	-	-	?
p	400 keV	3 kA	60	-	38%	-	4-20	?
p	400 keV	4.5 kA	85	50	-	5-10	6	?
p	500 keV	3 kA	100	60	-	-	-	?
D	1 MeV	200 kA	1k	40	-	59	2-8	?
D	50 keV	36 kA	3k	-	~ 5%	-	16	?
p	300 keV	100 kA	2-3k	100	~ 1%	~ 8	10	?

^aHumphries, Lee, and Sudan, Appl. Phys. Lett. 25, 20 (1974).^bHumphries, Lee, and Sudan, J. Appl. Phys. 46, 18⁷ (1975).^cHumphries, Sudan, and Condit, Appl. Phys. Lett. 26, 66⁷ (1975).^dKapetanokos, Golden, and Young, Nucl. Fusion (to be published).^eGolden and Kapetanokos, Appl. Phys. Lett. 28, 3 (1976).^fBlack, Golden, and Kapetanokos, Bull. Am. Phys. Soc. 20, 1581 (1975).^gGolden, Kapetanokos, Lee, and Goldstein, Proc. Int. Top. Conf.

E-Beams, 1, 635 (1975).

^hMiller, Mendel, Swain, and Goldstein, Proc. Int. Top. Conf. E-Beams, 1, 619 (1975).ⁱKapetanokos, ERDA Meeting on Ion Beams, May 1976.^jProno, Shearer, and Briggs, Proc. Int. Top. Conf. E-Beams, 1, 575 (1975).^kProno, Shearer, and Briggs, Phys. Rev. Lett. 32, 21 (1976).

In the pinched diode (Fig. A2-2-5, Fig. 1, Tables A2-2-III and A2-2-IV) the electrons experience

Table A2-2-III. Ion diodes - pinched IREB simulations.

ϵ_i	I_i	I_e	Diode	Ref.
500 keV	300 kA	285 kA	solid K, R = 19 cm, d = 0.8 cm	?
1.5 MeV	225 kA	340 kA	solid K, R = 19 cm, d = 2 cm	?
10 MeV	530 kA	700 kA	spherical, R = 25 cm	?
20 MeV	9.4 MA	5.2 MA	spherical, R = 80 cm	?

¹Poukey, J. Vac. Sci. Tech. 12, 1214 (1975).²Poukey, Freeman, Clauser, and Yonas, Phys. Rev. Lett. 35, 1086 (1975).³Poukey, Proc. Int. Top. Conf. E-Beams, 1, 247 (1975).

Fig. A2-2-5. Ion diodes - Pinched IREB. (1) Electrons drift to axis to make pinch. (2) Net $I_e < I_i^{BP}$

$$\frac{I_i}{I_e} \gg \left(\frac{I_i}{I_e}\right)_{\text{bipolar}}$$

(3) Anode must be ion source

IREB heated

preformed plasma (laser/anode)

(XBL 7612-11008)

Table A2-14. Ion diodes: pinched (PRC) experiments.

Ion	V_1	I_1 (kA)	J_1 (kA/cm ²)	I_2 (kA)	η	Ref.
D	800 keV	6				
D	1.7 MeV	60			15%	
p	100 keV	5	2			
p	600-700 keV	50		200	<20%	
p	1 MeV	2-3	2	400	<50%	

Blaugrund, Cooperstein, Boller, and Goldstein, Bull. Am. Phys. Soc. 20, 1252 (1975).

Miller, Mendel, Skain, and Goldstein, Proc. Int. Top. Conf. 1, 649 (1975).

Miller (private communication).

Cooperstein, Stephanakis, Boller, Lee, and Goldstein, Hiff Conf. Plasma Sci., Austin, Texas, May 21-26, 1976.

and $\mathbf{B}$ drift towards the central axis, where in this case the magnetic field $\mathbf{B}$ is the self-field of the electron ion current itself. This "pinch" effect increases the path length of each electron, which corresponds to an increase of the ratio of electron space charge to electron current, just as in the reflex diode. The ion current is not suppressed because its Larmor radius is larger than the gap distance, just as in the magnetic diode.

1. *Self-Field Pinch: Estimating the ratio*

$$\frac{V_0}{V_1} = \frac{n_1}{n_0} = \frac{I_1}{I_0} = \frac{(eN/V_1)^2}{(eN/V_0)^2}$$

The choice of an appropriate ion diode for the pulsed ion current injector for the heavy-ion induction linac was influenced by the following considerations.

- *Scalability.* Very little experimental evidence is available, but from what is known about the pinched diode, the predicted angular emission pattern is an awkward match to the accelerator requirement.
- *Plasma.* The formation of a heavy-ion plasma is necessary to provide a space-charge limited source of ions. This requires good vacuum techniques to eliminate surface contamination by lighter elements. The anode geometry of such a source would be difficult to fit into the reflex diode because there the anode is in the center of the reflexing electrons. The magnetically insulated diode might be used to suppress proton currents by raising the magnetic field.
- *Neutralization Problem.* The ion beams from the reflex and pinched diodes are emitted as neutralized plasmas ("plasma wind"). However, the transverse magnetic field of the magnet-

ically insulated diode prevents this electron emission. Unneutralized beams are required in the accelerator for obvious reasons.

- *Current Density.* The required A/cm^2 for the accelerator source is less by an order of magnitude or more than the capabilities of the present ion diodes. This leaves a margin for future development of good emittance sources with adequate pulse-repetition rates.
- *Voltage.* Present diode experiments are in the range 0.1-1.8 MV; injector requirements lead to the desirability of higher voltage. 5-10 MV is an existing capability in some electron diodes; extension to ion diodes appears to be straightforward.
- *Pulse Length.* Extension from current practice (~ 0.1 μ sec) to the 2 μ sec requirement depends on inhibiting plasma extension across the gap. The transverse magnetic field of the magnetically insulated diode should be effective in lengthening this pulse.

Thus, it was concluded that the magnetically insulated diode was the most suitable candidate for the induction linac. Figure A2-15 and Table A2-15 give a comparison of the measured proton

Fig. A2-15. Pulsed diode source option.

(XBL 76-3653)

Table A2-15

	Measured	Extrapolated	Required
Diode I (A)	5000	700	21
Diode V (MV)	0.2	5.0	10
Pulse τ (nsec)	0.05		2.0
Field x gap Bd (kG/cm)	>2	>20	>20
Current density (A/cm ²)	7	10	1
Area	70	70	21
Other diodes			Reflex, pinched

(H^*) experiments with two sets of U^{*3} parameters. The extrapolated U^{*3} is scaled from the H^* experiments according to the space charge law:

$$J \sim \left(\frac{q}{M}\right)^{1/2} \left(\frac{V^{3/2}}{d^2}\right)$$

where the transverse magnetic field is held constant. Comparison of these parameters with the required current for injection shows that there is considerable margin available for future development, as mentioned above.

The reflex diode is not as attractive as the magnetically insulated diode due to the problem of generating the heavy-ion plasma in the restricted space of the thin anode. The pinched diode is not as attractive due to its emittance properties, and the difficulty of control of a self-pinched device. However, both of these alternative devices are conceivable, and cannot be ruled out completely.

Needed research and development in this area may be summarized as follows:

- *Existence*. Measurements are not now available for $\Delta p/p$ or for ϵ . Such data are sorely needed to establish injection parameters between source and accelerator.
- *Time Duration*. 2 μ sec pulse capability needs development; should be possible especially for larger gaps and lower current densities.
- *Species Selection* at anode needs development, especially for multiply charged ions.
- *Electron Removal* from beam requires some care, especially at high current densities.
- *Voltage Regulation* needed for 5 MV power supply to maintain low $\Delta p/p$.

APPENDIX 2.3

1. SOURCE OPTIONS FOR THE INDUCTION LINAC

Richard M. Mobley

The induction linac approach calls for quite different ion-source characteristics than accelerator schemes starting with a conventional linac. Following Keefe's comments we have roughly:

Desired Source Characteristics

Charge state	$q = 3$
Current	$20 \text{ A} + 100\text{'s A}$

Duration	1 μ sec + several μ sec
T_0	5 MV
$\Delta p/p$	~ 0.1
ϵ_n	3 cm-mrad (A/n Py)
Aperture	$10^2 - 10^3 \text{ cm}^2$
Repetition rate	10 Hz
Species	A ~ 200

Such ion sources do not exist. Possible directions we have discussed include pulsed diodes (reflex triodes, magnetically insulated diodes), extended-area arc sources, laser-induced arcs, laser plumes, and cluster sources.

PULSED DIODES

Sudan (Cornell) has measured half angles on a single beamlet of 1.27-cm diam from a reflex triode, with the following parameters:

Al^{++}	3.6 MeV	20 A/cm ²
$\pm 5^\circ$ fwhm	150 nsec	

Shearer's summary on pulsed diodes is given in Appendix 2-2.

EXTENDED-AREA ARC SOURCES

The possibility of large input aperture for the induction linac, or several front ends in parallel, suggested large-area multiple-aperture ion sources of the type developed for tokamak heating. For example, a Berkeley CTR source with 100 slots in a $7 \times 35 \text{ cm}^2$ area was run with Xe, producing the following preliminary results:

Area of one beamlet	$7 \times 0.2 \text{ cm}^2$
Particle A (Xe^{+1} , Xe^{+2} , + ?)	60 mA/beamlet
ϵ_n	$3/n \times 0.001 \text{ cm-mrad/beamlet}$

Total current 6 A over 250 cm. Other details are given in Appendix 2-1-iv.

LASER-INDUCED ARCS

Hirshfield (Yale) reported on laser-induced arcs on metal cathodes including tungsten (Gatlinburg Proceedings, 1975). The arc has properties of a vacuum spark, using a 2 J laser and a 50 V, 1 kA arc. Microwave attenuation indicates a density of $> 10^{11}$ ions for $> 1 \text{ msec}$. Charge states are probably low. He plans to extract a neutralized stream from a 10-cm diam area. This approach has several advantages - high rep rate, nondestructive arc, long enough pulse length. This option would be similar to extended-area arc sources, with the possibility of higher current, higher voltage extraction for a short pulse.

LASER SOURCES

References here are to work by Culham and Lomell (Gatlinburg Proceedings 1971). High-power laser beams applied to heavy metals produce the highest charge states in the narrowest angles of

the laser plume. The Lemeil group (Tonon et al.) report charge states of $^{14-16}$ with Fe. The problem with this work is that it is directed to low-current heavy-ion acceleration. In that case, they use high laser power on a small spot, generating high electron temperatures, high ion charge states, and collective effects. Lincoln (NASA, Ames) has studied laser plumes from insulators (graphite, Al_2O_3). With $\sim 150 J/cm^2$ on a 0.1-cm spot these insulators vaporize. Careful time-of-flight spectroscopy work has led to gas dynamic modeling of the vapor plume. Positive ions in singly charged states are apparently due to laser-neutral ionization in the expanding cloud via electric field acceleration of residual electrons. Power densities were $\sim 600 kW/cm^2$, pulse lengths $\sim 400 \mu sec$. In this regime, neutral vapor is the main component. The proposed model indicates that after adiabatic free-jet expansion, a condition of uniform velocity over the full angular range is obtained. In other words, the emittance could be excellent.

CLUSTER SOURCES

Obert of Karlsruhe, on a visit to Berkeley, described their cluster source development which has been going on for 20 years. In their scheme, liquified clusters of $\sim 10^3$ atoms are given a single charge. They run 100 mA of H_2 with 1 MW acceleration for 10 particle A of H_2 . This is a 100 kW dc tokamak heating scheme. The low e/m of the cluster is very promising for induction linac front ends. Angular spread from free expansion nozzles, after using a skimmer and before 100-Atom 10-V ionization is $\sim 2^\circ$. At 1 MW, a Xe cluster with one charge has a velocity of ~ 10 cm/sec. Variation in cluster size may not be too bad for the $\Delta p/p = \pm 0.1$ range quoted by the induction linac group. The Karlsruhe group previously concentrated on rare gases, and Obert considers that Cs or Hg would work; what is needed is free expansion to a supercooled condition. A Fontenay group is also active in this field.

References

1. D. Keefe, Technical Summary 7 of this report.
2. K. Lincoln, Int. J. Mass Spectrom. Ion Phys. **16**, 191 (1975).

APPENDIX 2-5

ii. REFERENCE SEARCH ON LASER SOURCES

K. Prelac

Reference: H. F. A. Harrison, Conference on Heavy Ion Sources, USAEC, Washington, March 1970, Wash-1159, p.162.

Summary: Report on work done by N. J. Peacock and R. S. Pease [J. Phys. D **2**, 1705 (1969)].
Laser: 500 MW, 10 J (20 nsec), on Fe.
Results: Estimated plasma density (at the end of the pulse 2) of $2 \cdot 10^{20} cm^{-3}$, equivalent electron temperature of 100-120 eV. Number of ions emitted per pulse about 10^{15} (all species).
Observed spectra from Fe^{14+} and Fe^{15+} the most probable states being $7+$ or $8+$.

Reference: G. F. Tonon, Proceedings International Conference Multiply Charged Heavy-Ion Sources,

Gatlinburg, October 1971, IEEE Trans Nucl. Sci. NS-19, No. 2, p.172.

Summary: A review article.

Theory: Expressions for the plasma temperature T_e and mass flow from the surface average ion energy, ion flux (ppp), ion flux in charge state Z, extracted current (Child's law).
Conclusions: A Fe^{16+} ion source with 10^{12} ions per pulse seems possible at present (1972). A U^{30+} ion source with 10^9 ions per pulse can be envisioned in the not too distant future.
Laser flux: $10^{12} - 10^{13} W/cm^2$ (Nd).
Results of this source: Fe target, $3 \cdot 10^{11} W/cm^2$, $I_0 = 50$ mA, $\Delta t = 10$ nsec.
Be target: only information is the emittance (not norm.) of $20 cm^2$ mrad.

Reference: N. G. Basov, et al., JETP Lett. **5**, No. 6, 141 (1967); **6**, 291 (1967).

Summary: Identification of lines Ca^{13+} and Ca^{14+} , Al^{11+} ; no other information.
Laser: Nd-glass, 10 J, 15 nsec.

Reference: C. Faure, et al., Phys. Lett. **34A**, No. 6, 313 (1971).

Summary: Source of multiply charged Al and Fe ions.

Laser: Nd-glass, 3 J, 100 nsec (30 nsec), flux density $2 \cdot 10^{11} W/cm^2$ (spot $5 \cdot 10^{-5} cm^2$)

Results: $Al^{1+} \rightarrow 10^{14} cm^{-2}$, $Al^{11+} \rightarrow 10^{12} cm^{-2}$, Al^{16+} detected.

Conclusions: It is possible to envision the production of ions up to U^{60+} with flux densities of $10^{14} W/cm^2$.

Comment: Due to the very small spot size the total number of ions is very small.

LASER-INITIATED VACUUM ARC

Reference: J. L. Hirshfield, Proceedings International Conference on Multiply Charged Heavy-Ion Sources, Gatlinburg, October 1975, p.1006.

Summary: Laser-initiated vacuum arc using C, Cu, and W electrodes (cathode). Discharge current pulses up to 10 kA, 1 msec were employed to yield fully ionized source plasmas of liters volume, with densities $10^{13} cm^{-3}$.

Laser: CO₂, 2 J, 100 nsec, 1 pps
Arc: 100 V, $10^4 A$

Results: Measurements of target loss: $\sim 100 \mu g/asec$.

Conclusion: By using an appropriate extraction scheme it could be possible to produce ion streams in kA range.

Comments on published results on laser sources:

Relatively low yields of multiply charged ions have been achieved so far, especially in very high states where often only spectral lines have been observed. Extrapolation to e.g. 10^{14} ions of U^{60+} with an acceptable emittance, is highly speculative (according to G. F. Tonon, about 10^{-6} of the uranium ions should be in $45+$ state).

Vacuum arcs, laser-initiated, offer a better solution because of large-volume, high-density plasmas, but no data were found on ion currents and charge-state distributions.

APPENDIX 4-1 ESTIMATE OF THE CROSS SECTION FOR THE CHARGE TRANSFER BETWEEN IONS

Yong-Ki Kim

There has been no theoretical or experimental study (as of July 1976) of the charge transfer between ions

though there are some data¹⁻³ on the reverse process

Of these, only the data on He and Ar ions^{1,2} cover a wide range of ion velocities relevant to our interest. The data on He and Ar, however, exhibit opposite trends.^{1,2} The He cross section¹ shows a distinct maximum, whereas the Ar data² are almost flat in the projectile energies from 10 to 40 keV.

Process (1) is endothermic. The energy needed, ΔE , is the difference between the second and the first ionization potentials (IP) of the neutral atom. Both ΔE and the Coulomb repulsion between the ions are insignificant fractions of the relative kinetic energy of the ions ($\sim 1/2$ keV/nucleon). This ion energy range is one of the most difficult cases for which to guess the cross sections.

As outlined below, a few guesses I have made all lead to the same order of magnitude ($5 \cdot 10^{-16}$ cm²) for the charge-transfer cross section.

Geometric Cross Section

The geometric cross sections of ions can be obtained easily from the values of $\langle r^2 \rangle$ for the valence orbitals. The values of $\langle r^2 \rangle$ for singly charged ions are not very different from those of the neutral atoms unless the principal quantum number of the valence orbital changes by stripping the outermost electron (e.g., Cs⁺). The following data (Table A4-1.1) are based on a relativistic calculation.⁴

Table A4-1.1

Ion	$\pi \langle r^2 \rangle (10^{-16} \text{ cm}^2)$
I ⁺	6.4
Cs ⁺	4.5
Au ⁺	2.8
U ⁺	18.8

Principle of Detailed Balancing

Process (1) is likely to produce many excited neutrals. Hence, a rigorous application of the principle of detailed balancing is difficult. If we assume, however, that all products are in their ground states, and further take (from Ref. 2) the

cross section for Process (1') for Ar to be $8 \cdot 10^{-16}$ cm² (at $v_{ion} \sim 3 \cdot 10^7$ cm/sec), then we get

$$\begin{aligned} \sigma(1) \times (\text{multiplicity of Ar}^+)^2 &= \sigma(1') \\ &\times (\text{multiplicity of Ar}^{++}) \times (\text{multiplicity of Ar}) \end{aligned}$$

Therefore

$$\begin{aligned} \sigma(1) &= 8 \cdot 10^{-16} \text{ cm}^2 \times (3 \times 3) \times \frac{1}{(2 \times 3)^2} \\ &= 2 \cdot 10^{-16} \text{ cm}^2 \end{aligned}$$

This would be a lower limit for Process (1) because of the possibility of producing excited Ar. The geometric cross section of Ar is $2.9 \cdot 10^{-16}$ cm², and the actual cross section for the charge transfer between Ar⁺ ions is more likely to be of the order of the geometric cross section.

Potential-Curve Crossing

For the charge transfer between slowly moving ions, the Landau-Zener method based on the potential-curve crossing is useful.⁵ According to Ref. 6, the maximum cross section for the charge transfer can be estimated by

$$\sigma_{\max} = 0.452 \pi R_X^2$$

where R_X is the distance between the ions when the two potential curves for the diatomic systems $A^{++} + A$ and $A^+ + A^+$ cross each other.

For our purpose, we can assume that the interaction potentials are:

between A^{++} and $A \rightarrow$ flat, and

between A^+ and $A^+ \rightarrow$ Coulomb repulsion.

Then, the two potential curves will cross (see Fig. A4-1.1) at

$$R_X(R) = \frac{14.39}{\Delta E} \text{ (eV)}$$

Fig. A4-1.1. Schematic potential curves for the charge transfer between ions. The effect of the polarization at large R is ignored. $V_1 = e^2/R$. $V_2 = \text{const}$. ΔE as $R \rightarrow \infty$ = second IP - first IP. The crossing point is defined by $\Delta E = e^2/R_X$. (XBL 7612-11064)

The ionization potentials are known⁷ well enough to calculate ΔE and hence $\sigma_{\max}$:

Table A4-1-11

Ion	ΔE (eV)	R_X (Å)	$\sigma_{\max}$ (10^{-16} cm ²)
I^+	8.68	1.66	4
Cs^+	21.2	0.679	0.7
Au^+	11.3	1.27	2
U^+	6	2.4	8

The cross section for Cs^+ may be an underestimate because a more rigorous treatment should account for the large dipole polarizability of the neutral Cs , leading to a larger R_X and hence to a larger $\sigma_{\max}$.

Thus, the cross section for Process (1) is likely to be of the order of the geometric cross section, but there is a possibility that closed-shell ions such as Cs^+ and Au^+ may have smaller cross sections.

References

1. On He^{++} see, for example, H. S. W. Massey and H. B. Gilbody, *Electronic and Ionic Impact Phenomena* (Oxford University Press, London, 1974), Vol. IV, 2nd Ed., pp. 2848-2850.
2. On Ar^{+4} , see H. Klinger, A. Müller, and E. Salzborn, *J. Phys.* B **8**, 230 (1975). See Ref. 3 also.
3. On rare gases, see J. B. Hasted, A. R. Lee, and M. Hossain, *Proceedings of the Third International Conference on the Physics of Electronic and Atomic Collisions*, Ed. M. R. C. McDowell (North-Holland, Amsterdam, 1964), p. 802. The velocity range covered by this paper is too low for our purpose.
4. J. P. Desclaux, *Atom. Data Nucl. Data Tables* **12**, 311 (1973).
5. See, for example, L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon Press, London, 1965), 2nd Ed., pp. 553-554.
6. See, for example, R. E. Olson, F. T. Smith, and E. Bauer, *Appl. Opt.* **10**, 1848 (1971).
7. For $Z \leq 89$, see C. E. Moore, *Ionization Potentials and Ionization Limits Derived from the Analyses of Optical Spectra*, NSRDS-NBS 34 (U.S. Government Printing Office, 1970). The ionization potential of U is 6.19 eV according to R. W. Solarz et al. (Lawrence Livermore Laboratory), *J. Opt. Soc. Am.* (to be published). Accurate value of the ionization potential for U^+ is not known, but my estimate based on a relativistic calculation is 12.2 ± 0.5 eV.

APPENDIX 4-2

CLOSURE ESTIMATE OF THE EXCITATION AND IONIZATION CROSS SECTIONS IN HIGH-ENERGY COLLISIONS FOR I^+ AND U^+ IONS INCIDENT ON HYDROGEN, NITROGEN, AND OXYGEN

George H. Gillespie

INTRODUCTION

Among the limiting factors in the use of storage rings, for the accumulation of heavy-ion currents for applications to inertial confinement fusion, are the collisions of beam particles with ambient air atoms and molecules. Two types of collisions are of particular concern. Collisions in which the charge of a beam particle is changed result in particles being lost from the beam. At high energies, electron pick-up cross sections become negligibly small so that all charge-changing processes of significance are those for which electrons are stripped off. For example, in the *Hearthfire*¹ concept for ion-beam fusion, the electron loss cross sections for I^+ ions are needed to help establish the vacuum requirements for the heavy-ion storage ring. Collisions in which the ambient air becomes ionized are also important in establishing vacuum requirements since they affect breakdown and pressure bump phenomena.²

In this note, an upper bound for these cross sections is estimated utilizing an extension of the Bethe theory^{3,4} for total inelastic scattering by structureless charged particles. The extension of the theory, which considers the structure of the incident ion explicitly, is briefly outlined in the next Section on the theory of the cross sections.⁵ Expressions are derived there for bounds of both types of cross sections for ion collisions on an atomic or molecular target. The following section then applies the results to obtain upper bounds for these cross sections in the specific case of I^+ and U^+ collisions on hydrogen, nitrogen, and oxygen. Some concluding remarks are presented in the last section.

THEORY OF THE CROSS SECTIONS⁵

Basic Formulas and Theoretical Overview

The differential scattering cross section in Born approximation for a collision involving a momentum transfer of magnitude K between two atomic systems, initially in their respective ground states, but in final states labelled by n and m , may be written as

$$d\sigma_{nm} = 8\pi a_0^2 \frac{q^2}{\beta^2} \left| F_n^{(1)}(K) \right|^2 \left| F_m^{(2)}(K) \right|^2 \frac{d(a_0 K)}{(a_0 K)^5}, \quad (1)$$

when exchange effects are neglected. The incident particle's velocity is contained in the parameter $\beta = v/c$; a_0 and α are the Bohr radius and fine-

structure constant, respectively. The functions $F_n^{(j)}(K)$ are the atomic form factors for the incident ($j=1$) and target ($j=2$) particles. For the j th atom or ion with nuclear charge $Z_e^{(j)}$ and a total of $Z_e^{(j)}$ electrons, the form factor is given by

$$F_n^{(j)}(K) = \left\langle n \left| \left[Z_N^{(j)} - \sum_{\ell=1}^{Z_e^{(j)}} \exp(i\mathbf{K} \cdot \mathbf{r}_{\ell}^{(j)}) \right] \right| 0 \right\rangle_j, \quad (2)$$

where $\mathbf{r}_{\ell}^{(j)}$ is the coordinate for the ℓ th electron in the j th system. The nuclear-charge term in (2) has been included here so that the elastic form factor ($n=0$) at $K=0$ is the total charge of the atomic system, i.e.,

$$F_0^{(j)}(0) = Z_N^{(j)} - Z_e^{(j)}. \quad (3)$$

(When the nuclear-charge term is omitted for the $n=0$ form factor, it is usually referred to as the atomic scattering factor or atomic form factor.) The inelastic atomic form factor is related to the generalized oscillator strength, $f_n^{(j)}(K)$, by

$$f_n^{(j)}(K) = E_n^{(j)} \left| \frac{F_n^{(j)}(K)}{a_0 K} \right|^2, \quad (4)$$

where $E_n^{(j)}$ is the excitation energy in rydbergs of the state n . The ordinary oscillator strength, $f_n^{(j)}$, is defined by the limit of equation (4) as K goes to zero.

The total cross section for the excitation of the incident particle to the state n , and of the target particle to the state m , is given by an integral over the momentum transfer,

$$\sigma_{nm} = 8\pi a_0^2 \frac{\alpha^2}{\beta^2} \int_{K_{\min}}^{K_{\max}} |F_n^{(1)}(K)|^2 |F_m^{(2)}(K)|^2 \frac{d(a_0 K)}{(a_0 K)^3}. \quad (5)$$

The integration limits are determined by the kinematic constraints of the collision. The momentum transfer is

$$(a_0 K)^2 = \frac{\alpha^2 M^2 \beta^2}{2} \left\{ 1 - \frac{(E_n^{(1)} + E_m^{(2)})}{M\beta^2} - \left[1 - \frac{(2E_n^{(1)} + E_m^{(2)})}{M\beta^2} \right]^{1/2} \cos\theta \right\}, \quad (6)$$

where M is the reduced mass (in rydbergs) and θ is the scattering angle. The minimum and maximum values of K appearing in (5) correspond to scattering angles of 0 and π , respectively.

Excitation and Ionization of the Incident Ion

The first cross sections of interest are those for which the incident +ion is stripped of one or more of its electrons. An upper bound on this cross section may be obtained by summing over all final

states of the +ion that are not the ground state. This sum will include a number of final states for which the +ion is excited, but still a bound state of charge +1, as well as all states in which one or more electrons are removed. This cross section will be referred to as the total excitation and ionization cross section of the ion, $\sigma(\text{ion})_{\text{ex}+\text{ion}}$. It is given by

$$\sigma(\text{ion})_{\text{ex}+\text{ion}} = 8\pi a_0^2 \frac{\alpha^2}{\beta^2} \sum_n \sum_{n \neq 0} \int_{K_{\min}}^{K_{\max}} |F_n^{(1)}(K)|^2 |F_m^{(2)}(K)|^2 \frac{d(a_0 K)}{(a_0 K)^3}. \quad (7)$$

Writing the integrals appearing in Eq. (7) as the sum of three terms,

$$I_{nm} = \int_0^{\infty} |F_n^{(1)}(K)|^2 |F_m^{(2)}(K)|^2 \frac{d(a_0 K)}{(a_0 K)^3}, \quad (8)$$

$$J_{nm}(\beta^2) = \int_0^{a_0 K_{\min}} |F_n^{(1)}(K)|^2 |F_m^{(2)}(K)|^2 \frac{d(a_0 K)}{(a_0 K)^3}, \quad (9)$$

$$K_{nm}(\beta^2) = \int_{a_0 K_{\max}}^{\infty} |F_n^{(1)}(K)|^2 |F_m^{(2)}(K)|^2 \frac{d(a_0 K)}{(a_0 K)^3}, \quad (10)$$

then the total excitation and ionization cross section for the ion becomes

$$\sigma(\text{ion})_{\text{ex}+\text{ion}} = 8\pi a_0^2 \frac{\alpha^2}{\beta^2} \sum_n \sum_{n \neq 0} [I_{nm} - J_{nm}(\beta^2) - K_{nm}(\beta^2)]. \quad (11)$$

The integrals defined by Eqs. (8) through (10), are all well behaved at both small and large K^2 , provided that $n \neq 0$; i.e., the final state of the ion is not the ground state. The integral I_{nm} is a constant independent of the incident velocity of the ion and provides the leading contributions to the cross section at high energies. The integrals $J_{nm}(\beta^2)$ and $K_{nm}(\beta^2)$ are velocity dependent. Their contributions will reduce the cross section at lower energies from their asymptotic values. Because of the approximate treatment of the leading term, they will not be addressed here.

Since the integration limits for the evaluation of I_{nm} are independent of the states n and m , the orders of integration and the summation over final states appearing in (11) may be interchanged. The completeness of the energy eigenstate basis may then be used to reduce the matrix-element sum to a matrix element involving the ground state only. The result is

$$\sum_n \sum_{n \neq 0} I_{nm} = \sum_{m \neq 0} \sum_{n \neq 0} I_{nm} + \sum_{n \neq 0} I_{n0}, \quad (12)$$

where

$$\sum_{m \neq 0} \sum_{n \neq 0} I_{nm} = \int_0^{\infty} \left[Z_e^{(1)} S_{inc}^{(1)}(K) \right] \left[Z_e^{(2)} S_{inc}^{(2)}(K) \right] \frac{d(a_0 K)}{(a_0 K)^3}, \quad (13)$$

and

$$\sum_{n \neq 0} I_{n0} = \int_0^{\infty} \left[Z_e^{(1)} S_{inc}^{(1)}(K) \right] \left| F_0^{(2)}(K) \right|^2 \frac{d(a_0 K)}{(a_0 K)^3}. \quad (14)$$

$S_{inc}^{(1)}(K)$ is the incoherent scattering function for the ion, $S_{inc}^{(2)}(K)$ and $F_0^{(2)}(K)$ are the incoherent scattering function and elastic form factor, respectively, of the target atom.

Excitation and Ionization of the Target Atom

The second type of cross sections of interest are those for which the ambient air atoms (or molecules) are ionized, irrespective of what the incident ion does. Again an upper bound on this cross section may be obtained by summing over all final states of the target atom that are not the ground state. As before this sum will include a number of final states of the target atom that are not ionized states, as well as all ionized states. This cross section will be referred to as the total excitation and ionization cross section of the target atom (or molecule), $\sigma(\text{atom})_{\text{ex+ion}}$. It is given by

$$\sigma(\text{atom})_{\text{ex+ion}} = 8\pi a_0^2 \frac{\alpha^2}{\beta^2} \sum_{m \neq 0} \sum_n \left[I_{nm} - J_{nm}(\beta^2) \cdot K_{nm}(\beta^2) \right]. \quad (15)$$

The leading-order contributions for this cross section come from the I_{nm} terms when $n \neq 0$, which are given by (13), and from the combination $J_{0m} - J_{0m}(\beta^2)$, for $n = 0$. (In all cases the terms arising from $K_{nm}(\beta^2)$ contribute to the next order and will not be considered further.) $I_{0m} - J_{0m}(\beta^2)$ is defined by

$$\left[I_{0m} - J_{0m}(\beta^2) \right] = \int_{a_0 K_{\min}}^{\infty} \left| F_0^{(1)}(K) \right|^2 \left| F_m^{(2)}(K) \right|^2 \frac{d(a_0 K)}{(a_0 K)^3}. \quad (16)$$

For optically allowed transitions, the lower limit in Eq. (16) may be evaluated by examining the small K^2 behavior of the form factors, as given by (3) and (4). Expanding the lower integration, $a_0 K_{\min}$, then yields the leading corrections to the cross section. Summing over all excited final states of the target atom yields (for singly charged ions)

$$\sum_{m \neq 0} \left[I_{0m} - J_{0m}(\beta^2) \right] \approx M_{\text{tot}}^{(2)^2} \epsilon_n \left[2 \frac{\beta}{\alpha} \right] - L^{(2)}(-1) + 1/2(I_1 - I_2), \quad (17)$$

where

$$I_1 = 2 \int_{\lambda}^{\infty} \left| F_0^{(1)}(K) \right|^2 Z_e^{(2)} S_{inc}^{(2)}(K) \frac{d(a_0 K)}{(a_0 K)^3}, \quad (18)$$

$$I_2 = 2 \int_0^{\lambda} \left[M_{\text{tot}}^{(2)^2} (a_0 K)^2 - \left| F_0^{(1)}(K) \right|^2 Z_e^{(2)} S_{inc}^{(2)}(K) \right] \frac{d(a_0 K)}{(a_0 K)^3} - 2M_{\text{tot}}^{(2)^2} \epsilon_n(\lambda) \quad (19)$$

and $L^{(2)}(-1)$ is defined by

$$L^{(2)}(-1) = \sum_{n \neq 0} \left(E_n^{(2)} \right)^{-1} \epsilon_n \left(E_n^{(2)} \right) f_n^{(2)}. \quad (20)$$

The integrand in I_2 is well behaved as $K \rightarrow 0$ since $Z_e^{(2)} S_{inc}^{(2)}(K)$ approaches $M_{\text{tot}}^{(2)^2} (a_0 K)^2$ for small K . The combination $I_1 - I_2$ should be independent of λ , for λ of the order of unity. For simplicity, λ will be taken to be 1. The parameters $L^{(2)}(-1)$, I_1 and I_2 , occur as the combination.

$$\epsilon_n c_{\text{tot}}^{(2)} = - \left[\frac{2L^{(2)}(-1) - (I_1 - I_2)}{M_{\text{tot}}^{(2)^2}} \right]. \quad (21)$$

Combining this result with Eq. (13) yields for the resulting cross section

$$\sigma(\text{atom})_{\text{ex+ion}} = 4\pi a_0^2 \frac{\alpha^2}{\beta^2} \left[M_{\text{tot}}^{(2)^2} \epsilon_n \left(4c_{\text{tot}}^{(2)} \frac{\beta^2}{\alpha^2} \right) + 2 \sum_{n \neq 0} \sum_{m \neq 0} I_{nm} \right]. \quad (22)$$

ESTIMATE OF THE CROSS SECTIONS

I^+ and U^+ on Hydrogen

The incoherent scattering function and elastic form factor for atomic hydrogen may be evaluated analytically,

$$Z_e^{(2)} S_{inc}^{(2)}(K) = \left\{ 1 - \frac{1}{\left[1 + \left(\frac{a_0 K}{2} \right)^2 \right]^4} \right\}, \quad (23)$$

and

$$\left| F_0(K) \right|^2 = \left\{ 1 - \frac{1}{\left[1 + \left(\frac{a_0 K}{2} \right)^2 \right]^2} \right\}^2. \quad (24)$$

There are no calculations of these two functions for either I^+ or U^+ ions. However, reasonable approximations may be obtained by using the incoherent-scattering functions and atomic-scattering factors of atoms with the same number of electrons, i.e. Te and Pa,⁷ together with the ion nuclear charge in (2). A summary of these form factors is reproduced with permission of the authors as Tables A4-2-III to A4-2-VI. Tables A4-2-III and A4-2-IV are incoherent-scattering functions for free atoms (Z_0 Sinc). The relationship between $\sin \theta/\lambda$ and $a_0 K$ is given simply by

$$a_0 K = 6.6496 \cdot \sin \theta/\lambda$$

Table A4-2-V gives atomic scattering factors for He, C, N and O, normal gas constituents. Scattering factors for other elements can be obtained from Table A4-2-VI, which lists coefficients for an empirical analytic function given as follows:

$$f(a_0 K) = \sum_{i=1}^4 A(i) \exp(-B(i) \cdot (a_0 K)^2) + C$$

(The last two columns in Table A4-2-VI list the maximum and minimum percentage errors of this analytic fit.)

The form factors $|F_0(K)|^2$ are then obtained by

$$|F_0(K)|^2 = |Z_n - f(K)|^2$$

which must be used for elements other than hydrogen (next section). Note that limiting values of these form factors are given as follows:

$$Z_e \text{ Sinc}(K) \xrightarrow{a_0 K \rightarrow \infty} Z_e$$

$$|F_0(K)|^2 \xrightarrow{a_0 K \rightarrow \infty} Z_n^2$$

Using these tables, the integrals required for Eqs. (13), (14), (18), and (19) can be evaluated numerically. In addition, values of $L^{(2)}(-1) = -0.073$, and $M_{\text{tot}}^{(2)} = 1$, have been evaluated for atomic hydrogen.

The results for the required integrals and $\ln c_{\text{tot}}^{(2)}$ are given in Table A4-2-I. ($L^{(2)}(-1)$ gives a negligible contribution). The cross sections for collisions on H_2 are then estimated to be (simply multiplying the results for atomic H by 2):

$$\sigma(\text{ion})_{\text{ex+ion}} \approx \begin{cases} \frac{9.7 \times 10^{-19}}{\beta^2} \text{ cm}^2, & I^+ \text{ on } H_2, \\ \frac{1.3 \times 10^{-18}}{\beta^2} \text{ cm}^2, & U^+ \text{ on } H_2; \end{cases} \quad (25)$$

$$\sigma(\text{atom})_{\text{ex+ion}} \approx \begin{cases} \frac{3.7 \times 10^{-17} (\ln \beta^2 + 3.50)}{\beta^2} \text{ cm}^2, & H_2 \text{ on } I^+, \\ \frac{3.7 \times 10^{-17} (\ln \beta^2 + 6.70)}{\beta^2} \text{ cm}^2, & H_2 \text{ on } U^+. \end{cases} \quad (26)$$

Table A4-2-I. Parameters for I^+ and U^+ cross sections in high-energy collisions with H_2 .

Ion	$\sum_{m \neq 0} \sum_{n \neq 0} I_{nm}$	$\sum_{n \neq 0} I_{n0}$	I_1	I_2	$\ln c_{\text{tot}}^{(2)}$
I^+	11	1.9	260	-37	300
U^+	13	4.9	550	-82	630

I^+ and U^+ on Nitrogen and Oxygen

For the case of atomic nitrogen and oxygen, the elastic form factors and incoherent-scattering functions may be obtained from tabulated values⁸ and the integrals again evaluated numerically.

$M_{\text{tot}}^{(2)}$ may also be extracted from these tables since for small momentum transfer,

$$Z_e^{(2)} S_{\text{inc}}^{(2)}(K) = M_{\text{tot}}^{(2)} (a_0 K)^2, \quad a_0 K \ll 1. \quad (27)$$

The determination of $L^{(2)}(-1)$ is considerably more difficult. However, it may be estimated from experimental data on electron-atom collisions. The cross sections for those collisions are precisely the same form as Eq. (22), only $\sum_{n \neq 0} \sum_{m \neq 0} I_{nm} \neq 0$ since a free electron has no structure (and hence no excited states).⁸ Using the experimental data for N_2 and O_2 targets, together with numerical integrations for I_1 and I_2 appropriate for electrons ($F_0^{(1)}(K) \equiv 1$), the values of $L^{(2)}(-1)$ appropriate for nitrogen and oxygen are in the range of 5 to 15. Compared to the theoretical uncertainties in the other terms in the cross sections, these give negligible contributions. The results are summarized in Table A4-2-II. The cross sections for N_2 and O_2 targets are then (again simply multiplying the atomic results by 2):

$$\sigma(\text{ion})_{\text{ex+ion}} = \begin{cases} \frac{9.3 \times 10^{-18}}{\beta^2} \text{ cm}^2, & I^+ \text{ on } N_2, \\ \frac{1.1 \times 10^{-17}}{\beta^2} \text{ cm}^2, & I^+ \text{ on } O_2, \\ \frac{1.4 \times 10^{-17}}{\beta^2} \text{ cm}^2, & U^+ \text{ on } N_2, \\ \frac{1.7 \times 10^{-17}}{\beta^2} \text{ cm}^2, & U^+ \text{ on } O_2; \end{cases} \quad (28)$$

Table A4-2-IV. Incoherent scattering factors for aspherical free atoms. From D. T. Cromer, LASL Report LA-4079 (1969), p.6.

SIN(THETA)/																
LAMBDA	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
0.005	0.005	0.004	0.003	0.002	0.010	0.009	0.009	0.007	0.007	0.017	0.016	0.015	0.012	0.012	0.04	0.01
0.010	0.019	0.015	0.011	0.010	0.039	0.035	0.029	0.026	0.027	0.067	0.063	0.059	0.050	0.047	0.04	0.04
0.015	0.038	0.033	0.021	0.242	0.332	0.782	0.688	0.617	1.321	1.263	1.206	1.050	1.019	0.98	0.98	0.98
0.100	1.270	1.221	0.977	0.686	2.284	2.291	2.205	2.118	2.005	7.118	6.902	6.701	6.482	6.273	6.07	5.87
0.150	2.310	2.159	1.885	1.741	3.324	3.587	3.880	3.645	4.492	4.5	3.450	4.005	4.369	4.369	4.32	4.28
0.200	2.751	2.925	2.789	2.491	4.047	4.520	4.960	5.074	5.801	5.860	5.958	5.781	5.764	5.72	5.68	5.62
0.250	3.026	3.628	3.628	3.457	5.230	5.808	6.785	7.182	7.189	8.312	8.715	8.332	8.606	8.606	8.56	8.52
0.300	3.503	4.400	4.257	3.582	6.451	7.293	8.420	8.818	9.818	10.218	10.818	10.818	10.818	10.818	10.78	10.74
0.350	3.730	4.488	4.828	3.639	7.523	7.937	9.860	9.939	11.961	11.901	12.196	12.687	12.687	12.78	12.78	12.78
0.400	3.946	4.690	4.175	6.832	8.459	8.867	9.829	10.382	10.684	13.140	13.140	13.140	13.140	13.140	13.140	13.140
0.450	4.165	4.876	4.611	7.151	9.225	9.667	10.622	11.158	13.335	14.093	14.574	15.716	16.050	16.31	16.31	16.31
0.500	4.320	5.000	4.800	7.352	9.523	9.953	10.922	11.458	13.632	14.393	14.874	16.016	16.350	16.61	16.61	16.61
0.550	4.468	5.207	4.755	7.552	10.294	10.404	11.652	12.499	14.885	15.509	16.111	17.377	18.022	18.68	18.68	18.68
1.000	4.438	5.747	6.901	7.703	10.652	11.286	12.72	13.050	15.400	16.095	16.721	18.488	19.039	19.58	19.58	19.58
1.500	4.586	5.780	7.482	8.288	11.592	12.280	13.990	14.750	17.430	18.134	19.232	21.097	21.777	22.44	22.44	22.44
2.000	4.910	5.930	7.957	8.922	12.937	14.411	15.487	16.718	19.585	20.379	22.204	23.642	24.21	24.81	24.81	24.81
3.000	4.997	5.992	7.957	8.922	12.937	14.411	15.487	16.718	19.585	20.379	22.204	23.642	24.21	24.81	24.81	24.81
4.000	5.000	5.999	7.957	8.922	12.879	13.632	15.716	16.648	20.328	21.399	22.152	24.807	25.799	26.71	26.71	26.71
5.000	5.000	6.000	7.958	8.995	12.957	13.937	15.880	16.843	20.641	21.580	22.515	25.210	26.238	27.18	27.18	27.18
6.000	5.000	6.000	8.000	9.000	12.957	13.995	15.880	16.843	20.641	21.935	22.919	25.856	26.830	27.80	27.80	27.80
SIN(THETA)/																
LAMBDA	GA	GB	GC	GD	GE	GF	GG	GH	GI	GJ	GK	GL	GM	GN	GO	GP
0.005	0.011	0.011	0.010	0.009	0.022	0.021	0.017	0.015	0.014	0.015	0.015	0.014	0.013	0.013	0.01	0.01
0.010	0.044	0.042	0.038	0.036	0.087	0.083	0.068	0.059	0.056	0.062	0.060	0.056	0.054	0.051	0.11	0.11
0.015	0.059	0.044	0.079	0.086	1.694	1.654	1.395	1.256	1.217	1.334	1.323	1.261	1.230	1.230	0.21	0.21
0.100	2.761	2.639	2.789	2.771	3.999	4.064	3.672	3.559	3.499	3.852	3.917	3.307	3.903	3.903	5.87	5.87
0.150	4.483	4.486	4.794	4.851	5.876	6.014	5.735	5.783	5.793	6.207	6.416	6.610	6.610	6.610	7.06	7.06
0.200	5.919	5.926	6.459	6.459	7.719	7.719	7.182	7.182	7.182	7.597	7.597	7.000	7.000	7.000	7.87	7.87
0.250	8.599	8.912	9.401	9.940	11.100	11.236	11.213	11.213	11.531	11.591	12.008	12.415	13.171	13.564	14.81	14.81
0.300	11.082	11.338	12.033	12.440	13.899	14.176	14.317	14.782	14.883	15.443	15.444	15.746	16.466	16.876	18.00	18.00
0.350	13.290	13.536	14.168	14.562	16.279	16.658	16.949	17.685	17.858	18.489	18.760	19.499	19.777	21.17	21.17	21.17
0.400	15.212	15.468	16.099	16.493	18.409	18.888	19.179	19.815	19.988	20.619	20.890	21.520	21.798	23.19	23.19	23.19
0.450	16.967	17.215	17.835	18.185	19.911	20.373	20.484	22.123	22.557	23.779	24.059	24.685	24.960	26.50	26.50	26.50
0.500	18.465	18.741	19.391	19.747	21.474	21.916	22.095	22.389	23.855	24.318	25.006	26.252	26.927	27.269	28.75	28.75
0.550	19.734	20.076	20.778	21.149	22.820	23.234	23.787	25.312	25.819	27.491	28.113	28.612	29.298	30.85	30.85	30.85
1.000	20.951	21.224	22.005	22.389	24.110	24.546	25.251	26.621	27.148	29.203	29.725	30.226	30.912	32.47	32.47	32.47
1.500	22.346	22.629	23.410	23.804	25.529	26.000	26.685	28.150	28.676	30.730	31.252	31.753	32.024	33.68	33.68	33.68
2.000	24.800	25.108	26.504	26.910	31.870	32.522	33.167	35.081	35.715	38.255	38.695	40.181	40.827	43.44	43.44	43.44
3.000	28.536	29.377	31.034	31.850	35.043	35.825	36.601	38.691	39.643	42.605	43.338	44.798	45.526	48.68	48.68	48.68
4.000	32.436	30.340	32.137	33.020	36.957	37.426	38.291	40.491	41.693	45.003	45.817	47.438	48.233	51.60	51.60	51.60
5.000	39.973	37.873	39.670	40.553	42.969	43.438	44.303	46.503	47.705	50.915	51.729	53.350	54.145	57.51	57.51	57.51
6.000	30.705	31.648	33.589	34.547	38.306	39.318	40.270	43.121	44.070	47.663	48.610	50.702	51.647	55.41	55.41	55.41
SIN(THETA)/																
LAMBDA	CE	PN	ND	PM	OS	OG	OT	OB	OV	OE	ER	TM	LU	HF	T	Q
0.005	0.029	0.029	0.029	0.028	0.028	0.026	0.026	0.026	0.025	0.025	0.025	0.024	0.024	0.023	0.00	0.00
0.010	0.116	0.116	0.114	0.112	0.110	0.104	0.102	0.103	0.101	0.101	0.100	0.099	0.099	0.098	0.00	0.00
0.015	0.216	0.216	0.214	0.212	0.210	0.204	0.202	0.203	0.201	0.201	0.200	0.199	0.199	0.198	0.00	0.00
0.100	5.025	4.876	4.849	4.811	4.788	4.831	4.802	4.854	4.621	4.588	4.555	4.634	4.679	4.679	4.67	4.67
0.150	7.634	7.489	7.463	7.432	7.395	7.420	7.387	7.425	7.204	7.166	7.122	7.176	7.245	7.245	7.24	7.24
0.200	10.190	10.051	10.036	10.010	9.984	10.007	9.979	9.867	9.830	9.792	9.751	9.762	9.815	9.815	9.81	9.81
0.250	12.640	12.501	12.486	12.460	12.434	12.457	12.429	12.317	12.280	12.242	12.204	12.215	12.268	12.268	12.26	12.26
0.300	14.713	14.554	14.503	14.461	14.452	14.469	14.441	14.279	14.240	14.187	14.189	14.009	14.019	14.037	14.01	14.01
0.350	21.377	21.644	21.759	21.856	21.910	22.269	22.317	22.282	23.310	23.287	22.412	22.440	22.850	23.010	23.01	23.01
0.400	24.334	24.331	24.493	24.436	24.374	25.155	25.315	25.284	25.283	25.471	25.532	25.838	26.110	26.263	26.26	26.26
0.450	26.750	26.818	27.012	27.017	27.325	27.768	27.948	27.914	28.141	28.167	28.378	28.439	28.644	28.696	28.69	28.69
0.500	29.672	29.723	29.959	29.959	30.459	30.944	31.168	31.168	31.678	30.930	30.954	33.332	33.624	33.624	33.62	33.62
0.550	31.143	31.311	31.556	31.776	31.973	32.643	32.703	32.645	33.340	33.216	33.369	33.778	34.070	34.361	34.36	34.36
1.000	33.083	33.304	33.581	33.832	34.062	34.583	34.842	35.021	35.290	35.440	35.620	36.059	36.350	36.641	36.64	36.64
1.500	40.001	40.515	41.033	41.523	42.199	42.886	43.335	43.713	44.101	44.563	44.808	45.471	45.817	46.101	46.10	46.10
2.000	46.750	47.179	47.608	48.037	48.466	49.143	49.481	49.859	50.247	50.635	50.923	51.686	52.031	52.315	52.31	52.31
3.000	49.175	49.921	50.640	51.328	52.136	53.000	53.345	53.673	55.803	56.568	57.250	58.672	59.375	60.01	60.01	60.01
4.000	52.197	52.996	53.788	54.579	55.369	56.942	57.731	58.526	59.314	60.101	60.887	62.451	63.227	64.01	64.01	64.01
5.000	54.083	54.934	55.778	56.621	57.461	59.130	59.994	60.863	61.640	62.400	63.288	64.636	65.755	66.53	66.53	66.53
6.000	56.361	57.303	58.243	59.181	60.119	61.990	62.924	63.858	64.789	65.719	66.647	68.497	69.419	70.34	70.34	70.34
SIN(THETA)/																
LAMBDA	B	CS	IN	PI	TS	FB	PD	AT	AC	TH	PA	U	NP	PU	U	Q
0.005	0.021	0.020	0.019	0.016	0.018	0.017	0.017	0.035	0.034	0.034	0.033	0.033	0.032	0.030	0.00	0.00
0.010	0.083	0.078	0.076	0.074	0.073	0.068	0.068	0.066	0.066	0.064	0.063	0.062	0.060	0.059	0.00	0.00
0.015	0.141	0.141	0.142	0.139	0.1573	1.571	1.522	1.497	2.548	2.540	2.485	2.487	2.438	2.424	0.24	0.24
0.100	4.449	4.590	4.590	4.423	4.575	4.658	4.682	4.702	5.968	6.039	5.873	5.851	5.846	5.774	0.24	0.24
0.150	7.238	7.244	7.258	6.917	7.460	7.694	7.945	8.076	9.092	9.289	9.434	9.136	9.143	9.088	0.24	0.24
0.200	9.772	9.772	9.786	9.429	9.970	10.204	10.455	10.706	11.722	11.919	11.722	11.618	11.618	11.5		

Table A4-2-V. Hartree-Fock scattering factors. From D. T. Cromer and J. B. Mann, LASL Report LA-3816 (1968), pp. 5,7.

[illegible][illegible][illegible][illegible]

Table A4-2'V (continued)

N CONFIGURATION		ZP 1, 25 2, 15 2, ,									
SIN(THETA)/LAMBDA		+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0	7.000	6.991	6.983	6.918	6.865	6.774	6.682	6.574	6.453	6.322	
0.1	6.150	6.021	5.876	5.715	5.552	5.386	5.219	5.052	4.887	4.724	
0.2	4.543	4.407	4.254	4.106	3.963	3.825	3.693	3.566	3.444	3.328	
0.3	3.218	3.113	3.014	2.919	2.830	2.746	2.666	2.591	2.520	2.454	
0.4	2.391	2.333	2.277	2.225	2.177	2.131	2.088	2.047	2.010	1.974	
0.5	1.941	1.904	1.880	1.852	1.826	1.801	1.776	1.756	1.735	1.715	
0.6	1.548	1.529	1.514	1.494	1.476	1.460	1.441	1.428	1.415	1.402	
0.7	1.450	1.438	1.427	1.416	1.405	1.394	1.384	1.374	1.364	1.354	
0.8	1.444	1.435	1.425	1.416	1.407	1.397	1.386	1.377	1.370	1.361	
0.9	1.352	1.343	1.335	1.326	1.317	1.308	1.299	1.290	1.282	1.273	
1.0	1.274	1.265	1.256	1.246	1.237	1.228	1.219	1.210	1.202	1.194	
1.1	1.176	1.167	1.159	1.150	1.141	1.132	1.124	1.115	1.106	1.097	
1.2	1.089	1.080	1.071	1.063	1.054	1.046	1.037	1.029	1.020	1.012	
1.3	1.003	0.995	0.986	0.978	0.970	0.961	0.953	0.945	0.937	0.928	
1.4	0.920	0.912	0.904	0.896	0.888	0.880	0.873	0.865	0.857	0.849	
1.5	0.842										

N CONFIGURATION		ZP 1, 25 2, 15 2, ,									
SIN(THETA)/LAMBDA		+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0	8.000	7.992	7.986	7.956	7.909	7.797	7.711	7.612	7.500	7.377	
0.1	7.244	7.173	7.092	6.976	6.855	6.732	6.606	6.478	6.348	6.216	
0.2	5.620	5.452	5.285	5.122	4.961	4.805	4.652	4.503	4.359	4.220	
0.3	4.085	3.906	3.831	3.711	3.596	3.485	3.380	3.279	3.183	3.091	
0.4	3.003	2.920	2.841	2.765	2.695	2.626	2.562	2.500	2.442	2.387	
0.5	2.335	2.259	2.189	2.123	2.061	2.003	1.948	1.896	1.846	1.797	
0.6	1.944	1.915	1.888	1.862	1.838	1.814	1.792	1.771	1.751	1.731	
0.7	1.713	1.696	1.679	1.663	1.648	1.633	1.619	1.605	1.592	1.579	
0.8	1.507	1.555	1.544	1.533	1.522	1.511	1.501	1.491	1.481	1.472	
0.9	1.462	1.453	1.444	1.435	1.426	1.416	1.409	1.401	1.392	1.384	
1.0	1.376	1.368	1.360	1.352	1.344	1.336	1.328	1.320	1.312	1.305	
1.1	1.297	1.289	1.281	1.274	1.266	1.258	1.251	1.243	1.235	1.228	
1.2	1.220	1.212	1.205	1.197	1.190	1.182	1.174	1.167	1.159	1.152	
1.3	1.144	1.137	1.129	1.122	1.115	1.107	1.099	1.092	1.084	1.077	
1.4	1.069	1.062	1.054	1.047	1.040	1.032	1.025	1.017	1.010	1.003	
1.5	0.996										

Table A4-2'VI. Coefficients and maximum errors for analytic approximation of Hartree-Fock scattering factors. From D. T. Cromer and J. B. Mann, LASL Report LA-3816 (1968), pp. 66-69.

Li	1	0.99274	4.33779	0.87402	1.24006	0.84240	98.7088	0.23101	212.008	0.05986	0.402	-1.185
Li+1	0.08475	0.00436	0.86773	1.53730	0.80588	4.28524	0.17420	9.8143	-5.93560	0.085	-0.136	
Be	2	2.22744	0.04965	1.55249	42.9165	1.40060	1.66379	0.82900	100.361	-1.76339	0.355	-0.180
Be+2	5.60034	-0.01336	1.19705	0.39000	1.03057	0.97441	0.20153	4.90662	-0.11950	0.113	-0.025	
B	3	2.03374	73.0888	1.41491	0.97864	1.11659	59.8995	0.73273	0.08538	-0.30409	0.082	-0.19
C	4	1.92019	12.7180	1.87812	28.6406	1.57415	0.89645	0.37108	0.50337	0.26337	0.027	-0.026
N	5	12.7713	0.02064	3.28546	10.7018	1.76483	30.7773	0.54709	1.48044	-1.13926	0.465	-0.20
O	6	2.95648	13.8964	2.45240	5.91765	1.90510	0.34557	0.78135	34.7811	0.20413	0.025	-0.066
O -1	3	3.22563	18.4991	3.01717	6.65480	1.42573	0.50589	0.90255	61.1889	0.62362	0.101	-0.192
F	7	1.30393	11.2651	1.01753	4.66504	1.35754	0.33760	0.83645	27.9898	0.88398	0.031	-0.062
F -1	3	3.63220	5.27756	3.51057	1.67353	1.26064	0.44225	0.94071	47.3437	0.65340	0.102	-0.208
NE	3	3.71272	3.91091	3.52631	6.93126	1.19237	0.40463	0.83080	23.9566	0.73728	0.031	-0.062
Na	8	5.26400	4.02579	2.17549	10.4796	1.36690	0.64222	1.08599	133.617	1.09912	0.119	-0.255
Na+1	3	3.99477	3.11047	3.73745	7.14318	1.13877	0.40692	0.65118	15.7319	0.84267	0.010	-0.020
Mg	9	4.59229	4.41162	2.68206	1.36549	1.72235	0.34685	0.73025	32.5281	1.26883	0.281	-0.663
Mg+2	4	4.30491	2.55961	3.14719	5.60660	1.12859	0.41574	0.49034	11.4840	0.92893	0.003	-0.006
AL	4	4.35047	3.88665	2.82451	1.20535	2.27309	4.26051	1.15511	107.170	1.28499	0.113	-0.265
Al+3	4	4.17448	1.93818	3.30760	4.14553	1.20296	0.22875	0.52814	4.28524	0.70879	0.004	-0.002
SI	5	5.79411	2.71034	3.22300	3.61775	2.42795	0.86937	1.32149	85.3401	1.23139	0.020	-0.020
Si+4	4	4.43918	1.64167	3.20345	3.43757	1.19453	0.21490	0.41563	6.5365	0.74630	0.002	-0.001
P	6	4.92073	1.37778	4.14396	2.70198	2.01697	0.21316	1.53660	67.1086	0.37870	0.112	-0.056
S	7	7.18742	1.43280	4.08671	0.02865	5.15858	22.1101	1.64403	55.4651	-3.87732	0.092	-0.164
CL	8	8.93977	-0.00053	7.53181	1.11119	6.07100	18.3846	1.87128	55.3686	-0.31340	0.131	-0.261
Cl -1	18	18.0862	0.01297	7.42021	1.12976	6.46337	19.3079	2.43918	59.0633	-16.4565	0.109	-0.231
AR	16	16.0752	-0.01456	8.32256	0.83310	4.91326	14.9177	2.18515	37.2256	-16.2972	0.121	-0.267
K	8	8.11756	12.6864	7.48062	0.76409	1.07795	211.222	0.97218	37.2727	1.35009	0.416	-0.194
K +1	7	7.70659	0.59947	7.37245	11.3765	5.67225	-0.08359	1.90688	26.7668	-6.65819	0.091	-0.201
CA	8	8.60272	10.2636	7.50769	0.62794	1.75117	149.301	0.96216	60.2274	1.17430	0.404	-0.176
Ca+2	13	13.2063	0.96466	11.0586	-0.38204	7.73221	9.42976	1.72057	20.3361	-15.7176	0.058	-0.125
SC	9	9.06482	8.77431	7.55526	0.73406	2.05017	123.880	1.28745	36.8990	1.03349	0.262	-0.115
SC+3	13	13.4008	0.29854	8.02730	7.96290	1.65943	-0.28604	1.77934	16.6662	-6.66662	0.034	-0.072
TI	9	9.54969	7.60579	7.60067	4.05899	2.11223	109.099	1.75438	27.5715	0.91762	0.137	-0.063
TI+4	17	17.7344	0.20601	8.73814	7.04716	5.25691	0.15762	1.92134	15.9768	-14.4654	0.011	-0.013
Ti+6	19	19.5114	0.78055	8.23473	6.67101	4.01361	-0.23653	1.52080	12.9464	-13.2803	0.018	-0.017
V	10	10.0661	6.77211	6.16420	3.40322	2.23551	98.5954	2.21730	22.5720	0.84574	0.048	-0.044
V +2	9	9.34513	6.49985	7.68333	0.39491	2.94531	15.9868	0.26998	41.0832	0.75143	0.096	-0.042
V +3	5	4.93161	6.39535	7.74190	0.38134	2.15343	15.1908	0.01686	63.9690	0.65657	0.106	-0.046
V +5	16	16.8887	0.67900	8.14208	5.40135	2.03011	0.37278	-0.57580	0.94044	1.17430	0.002	-0.007
CR	10	10.7557	6.01555	7.51402	0.37426	3.50115	19.0654	1.54902	47.4599	0.95226	0.055	-0.050

Table A4-2-VI (continued)

LM2	4.56436	4.56437	4.56438	4.56439	4.56440	4.56441	4.56442	4.56443	4.56444	4.56445	4.56446	4.56447	4.56448	4.56449	4.56450	4.56451	4.56452	4.56453	4.56454	4.56455	4.56456	4.56457	4.56458	4.56459	4.56460	4.56461	4.56462	4.56463	4.56464	4.56465	4.56466	4.56467	4.56468	4.56469	4.56470	4.56471	4.56472	4.56473	4.56474	4.56475	4.56476	4.56477	4.56478	4.56479	4.56480	4.56481	4.56482	4.56483	4.56484	4.56485	4.56486	4.56487	4.56488	4.56489	4.56490	4.56491	4.56492	4.56493	4.56494	4.56495	4.56496	4.56497	4.56498	4.56499	4.56500	4.56501	4.56502	4.56503	4.56504	4.56505	4.56506	4.56507	4.56508	4.56509	4.56510	4.56511	4.56512	4.56513	4.56514	4.56515	4.56516	4.56517	4.56518	4.56519	4.56520	4.56521	4.56522	4.56523	4.56524	4.56525	4.56526	4.56527	4.56528	4.56529	4.56530	4.56531	4.56532	4.56533	4.56534	4.56535	4.56536	4.56537	4.56538	4.56539	4.56540	4.56541	4.56542	4.56543	4.56544	4.56545	4.56546	4.56547	4.56548	4.56549	4.56550	4.56551	4.56552	4.56553	4.56554	4.56555	4.56556	4.56557	4.56558	4.56559	4.56560	4.56561	4.56562	4.56563	4.56564	4.56565	4.56566	4.56567	4.56568	4.56569	4.56570	4.56571	4.56572	4.56573	4.56574	4.56575	4.56576	4.56577	4.56578	4.56579	4.56580	4.56581	4.56582	4.56583	4.56584	4.56585	4.56586	4.56587	4.56588	4.56589	4.56590	4.56591	4.56592	4.56593	4.56594	4.56595	4.56596	4.56597	4.56598	4.56599	4.56600	4.56601	4.56602	4.56603	4.56604	4.56605	4.56606	4.56607	4.56608	4.56609	4.56610	4.56611	4.56612	4.56613	4.56614	4.56615	4.56616	4.56617	4.56618	4.56619	4.56620	4.56621	4.56622	4.56623	4.56624	4.56625	4.56626	4.56627	4.56628	4.56629	4.56630	4.56631	4.56632	4.56633	4.56634	4.56635	4.56636	4.56637	4.56638	4.56639	4.56640	4.56641	4.56642	4.56643	4.56644	4.56645	4.56646	4.56647	4.56648	4.56649	4.56650	4.56651	4.56652	4.56653	4.56654	4.56655	4.56656	4.56657	4.56658	4.56659	4.56660	4.56661	4.56662	4.56663	4.56664	4.56665	4.56666	4.56667	4.56668	4.56669	4.56670	4.56671	4.56672	4.56673	4.56674	4.56675	4.56676	4.56677	4.56678	4.56679	4.56680	4.56681	4.56682	4.56683	4.56684	4.56685	4.56686	4.56687	4.56688	4.56689	4.56690	4.56691	4.56692	4.56693	4.56694	4.56695	4.56696	4.56697	4.56698	4.56699	4.56700	4.56701	4.56702	4.56703	4.56704	4.56705	4.56706	4.56707	4.56708	4.56709	4.56710	4.56711	4.56712	4.56713	4.56714	4.56715	4.56716	4.56717	4.56718	4.56719	4.56720	4.56721	4.56722	4.56723	4.56724	4.56725	4.56726	4.56727	4.56728	4.56729	4.56730	4.56731	4.56732	4.56733	4.56734	4.56735	4.56736	4.56737	4.56738	4.56739	4.56740	4.56741	4.56742	4.56743	4.56744	4.56745	4.56746	4.56747	4.56748	4.56749	4.56750	4.56751	4.56752	4.56753	4.56754	4.56755	4.56756	4.56757	4.56758	4.56759	4.56760	4.56761	4.56762	4.56763	4.56764	4.56765	4.56766	4.56767	4.56768	4.56769	4.56770	4.56771	4.56772	4.56773	4.56774	4.56775	4.56776	4.56777	4.56778	4.56779	4.56780	4.56781	4.56782	4.56783	4.56784	4.56785	4.56786	4.56787	4.56788	4.56789	4.56790	4.56791	4.56792	4.56793	4.56794	4.56795	4.56796	4.56797	4.56798	4.56799	4.56800	4.56801	4.56802	4.56803	4.56804	4.56805	4.56806	4.56807	4.56808	4.56809	4.56810	4.56811	4.56812	4.56813	4.56814	4.56815	4.56816	4.56817	4.56818	4.56819	4.56820	4.56821	4.56822	4.56823	4.56824	4.56825	4.56826	4.56827	4.56828	4.56829	4.56830	4.56831	4.56832	4.56833	4.56834	4.56835	4.56836	4.56837	4.56838	4.56839	4.56840	4.56841	4.56842	4.56843	4.56844	4.56845	4.56846	4.56847	4.56848	4.56849	4.56850	4.56851	4.56852	4.56853	4.56854	4.56855	4.56856	4.56857	4.56858	4.56859	4.56860	4.56861	4.56862	4.56863	4.56864	4.56865	4.56866	4.56867	4.56868	4.56869	4.56870	4.56871	4.56872	4.56873	4.56874	4.56875	4.56876	4.56877	4.56878	4.56879	4.56880	4.56881	4.56882	4.56883	4.56884	4.56885	4.56886	4.56887	4.56888	4.56889	4.56890	4.56891	4.56892	4.56893	4.56894	4.56895	4.56896	4.56897	4.56898	4.56899	4.56900	4.56901	4.56902	4.56903	4.56904	4.56905	4.56906	4.56907	4.56908	4.56909	4.56910	4.56911	4.56912	4.56913	4.56914	4.56915	4.56916	4.56917	4.56918	4.56919	4.56920	4.56921	4.56922	4.56923	4.56924	4.56925	4.56926	4.56927	4.56928	4.56929	4.56930	4.56931	4.56932	4.56933	4.56934	4.56935	4.56936	4.56937	4.56938	4.56939	4.56940	4.56941	4.56942	4.56943	4.56944	4.56945	4.56946	4.56947	4.56948	4.56949	4.56950	4.56951	4.56952	4.56953	4.56954	4.56955	4.56956	4.56957	4.56958	4.56959	4.56960	4.56961	4.56962	4.56963	4.56964	4.56965	4.56966	4.56967	4.56968	4.56969	4.56970	4.56971	4.56972	4.56973	4.56974	4.56975	4.56976	4.56977	4.56978	4.56979	4.56980	4.56981	4.56982	4.56983	4.56984	4.56985	4.56986	4.56987	4.56988	4.56989	4.56990	4.56991	4.56992	4.56993	4.56994	4.56995	4.56996	4.56997	4.56998	4.56999	4.57000	4.57001	4.57002	4.57003	4.57004	4.57005	4.57006	4.57007	4.57008	4.57009	4.57010	4.57011	4.57012	4.57013	4.57014	4.57015	4.57016	4.57017	4.57018	4.57019	4.57020	4.57021	4.57022	4.57023	4.57024	4.57025	4.57026	4.57027	4.57028	4.57029	4.57030	4.57031	4.57032	4.57033	4.57034	4.57035	4.57036	4.57037	4.57038	4.57039	4.57040	4.57041	4.57042	4.57043	4.57044	4.57045	4.57046	4.57047	4.57048	4.57049	4.57050	4.57051	4.57052	4.57053	4.57054	4.57055	4.57056	4.57057	4.57058	4.57059	4.57060	4.57061	4.57062	4.57063	4.57064	4.57065	4.57066	4.57067	4.57068	4.57069	4.57070	4.57071	4.57072	4.57073	4.57074	4.57075	4.57076	4.57077	4.57078	4.57079	4.57080	4.57081	4.57082	4.57083	4.57084	4.57085	4.57086	4.57087	4.57088	4.57089	4.57090	4.57091	4.57092	4.57093	4.57094	4.57095	4.57096	4.57097	4.57098	4.57099	4.57100	4.57101	4.57102	4.57103	4.57104	4.57105	4.57106	4.57107	4.57108	4.57109	4.57110	4.57111	4.57112	4.57113	4.57114	4.57115	4.57116	4.57117	4.57118	4.57119	4.57120	4.57121	4.57122	4.57123	4.57124	4.57125	4.57126	4.57127	4.57128	4.57129	4.57130	4.57131	4.57132	4.57133	4.57134	4.57135	4.57136	4.57137	4.57138	4.57139	4.57140	4.57141	4.57142	4.57143	4.57144	4.57145	4.57146	4.57147	4.57148	4.57149	4.57150	4.57151	4.57152	4.57153	4.57154	4.57155	4.57156	4.57157	4.57158	4.57159	4.57160	4.57161	4.57162	4.57163	4.57164	4.57165	4.57166	4.57167	4.57168	4.57169	4.57170	4.57171	4.57172	4.57173	4.57174	4.57175	4.57176	4.57177	4.57178	4.57179	4.57180	4.57181	4.57182	4.57183	4.57184	4.57185	4.57186	4.57187	4.57188	4.57189	4.57190	4.57191	4.57192	4.57193	4.57194	4.57195	4.57196	4.57197	4.57198	4.57199	4.57200	4.57201	4.57202	4.57203	4.57204	4.57205	4.57206	4.57207	4.57208	4.57209	4.57210	4.57211	4.57212	4.57213	4.57214	4.57215	4.57216	4.57217	4.57218	4.57219	4.57220	4.57221	4.57222	4.57223	4.57224	4.57225	4.57226	4.57227	4.57228	4.57229	4.57230	4.57231	4.57232	4.57233	4.57234	4.57235	4.57236	4.57237	4.57238	4.57239	4.57240	4.57241	4.57242	4.57243	4.57244	4.57245	4.57246	4.57247	4.57248	4.57249	4.57250	4.57251	4.57252	4.57253	4.57254	4.57255	4.57256	4.57257	4.57258	4.57259	4.57260	4.57261	4.57262	4.57263	4.57264	4.57265	4.57266	4.57267	4.57268	4.57269	4.57270	4.57271	4.57272	4.57273	4.57274	4.57275	4.57276	4.57277	4.57278	4.57279	4.57280	4.57281	4.57282	4.57283	4.57284	4.57285	4.57286	4.57287	4.57288	4.57289	4.57290	4.57291	4.57292	4.57293	4.57294	4.57295	4.57296	4.57297	4.57298	4.57299	4.57300	4.57301	4.57302	4.57303	4.57304	4.57305	4.57306	4.57307	4.57308	4.57309	4.57310	4.57311	4.57312	4.57313	4.57314	4.57315	4.57316	4.57317	4.57318	4.57319	4.57320	4.57321	4.57322	4.57323	4.57324	4.57325	4.57326	4.57327	4.57328	4.57329	4.57330	4.57331	4.57332	4.57333	4.57334	4.57335	4.57336	4.57337	4.57338	4.57339	4.57340	4.57341	4.57342	4.57343	4.57344	4.57345	4.57346	4.57347	4.57348	4.57349	4.57350	4.57351	4.57352	4.57353	4.57354	4.57355	4.57356	4.57357	4.57358	4.57359	4.57360	4.57361	4.57362	4.57363	4.57364	4.57365	4.57366	4.57367	4.57368	4.57369	4.57370	4.57371	4.57372	4.57373	4.57374	4.57375	4.57376	4.57377	4.57378	4.57379	4.57380	4.57381	4.57382	4.57383	4.57384	4.57385	4.57386	4.57387	4.57388	4.57389	4.57390	4.57391	4.57392	4.57393	4.57394	4.57395	4.57396	4.57397	4.57398	4.57399	4.57400	4.57401	4.57402	4.57403	4.57404	4.57405	4.57406	4.57407	4.57408	4.57409	4.57410	4.57411	4.57412	4.57413	4.57414	4.57415	4.57416	4.57417	4.57418	4.57419	4.57420	4.57421	4.57422	4.57423	4.57424	4.57425	4.57426	4.57427	4.57428	4.57429	4.57430	4.57431	4.57432	4.57433	4.57434	4.57435	4.57436	4.57437	4.57438	4.57439	4.57440	4.57441	4.57442	4.57443	4.57444	4.57445	4.57446	4.57447	4.57448	4.57449	4.57450	4.57451	4.57452	4.57453	4.57454	4.57455	4.57456	4.57457	4.57458	4.57459	4.57460	4.57461	4.57462	4.57463	4.57464	4.57465	4.57466
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Table A4-2.VI (continued)

St.N	M11	P111	A111	K11	L11	M1	P11	F11	F111	E111
GO+5	24.0290	0.0921	24.1387	2.1946	11.0511	11.7141	3.5624	21.0929	-8.7815	0.007 -0.005
TD	24.3738	2.6667	13.8664	0.4717	13.2510	14.0078	1.7844	17.4646	10.7426	0.007 -0.116
TB+3	26.7821	2.1333	25.9663	0.1057	10.4724	11.0974	3.0817	20.7042	-8.5830	0.002 -0.005
UD	24.6193	2.2200	16.2735	0.5856	13.5857	15.8667	2.7016	19.3871	11.0272	0.019 -0.027
UD	24.6193	2.2200	16.2735	0.5856	13.5857	15.8667	2.7016	19.3871	11.0272	0.019 -0.027
HD	24.3162	2.6729	14.9012	0.6152	13.3587	15.0041	2.0369	16.9871	11.8816	0.003 -0.005
HQ+3	27.1956	2.0734	19.9550	0.1475	11.0101	10.1165	6.3325	17.2589	0.7049	0.003 -0.005
EC	23.8201	2.4419	15.4796	0.6865	13.4938	13.1932	2.6800	13.6782	17.2027	0.063 0.135
EK+3	24.6193	1.9799	16.3416	0.1782	11.1113	9.7687	6.4184	18.7296	1.49325	0.003 -0.006
FD	24.3386	2.6729	15.4796	0.6865	13.4938	13.1932	2.6800	13.6782	17.2027	0.063 0.135
FD	24.3386	2.6729	15.4796	0.6865	13.4938	13.1932	2.6800	13.6782	17.2027	0.063 0.135
YD	22.3028	2.4419	16.7202	0.4088	15.5720	17.5890	6.6851	19.1781	12.9185	0.064 -0.181
YH+2	24.1313	1.9979	13.5857	0.3235	11.4357	3.4971	2.6955	20.3507	3.1782	0.001 -0.005
YH+3	29.4761	1.9879	14.3575	0.2793	11.3667	9.7998	6.5481	17.8206	2.7960	0.006 -0.008
YU	21.1366	0.4886	10.1760	2.6860	11.0537	12.2746	2.2190	10.1760	13.3489	0.067 -0.148
YU	21.1366	0.4886	10.1760	2.6860	11.0537	12.2746	2.2190	10.1760	13.3489	0.067 -0.148
IF	24.4725	0.9740	17.2295	2.4903	12.0959	12.2887	3.5546	15.4381	13.7049	0.074 -0.161
IA	24.1757	1.0439	14.4288	3.2074	12.4612	12.5054	3.7418	6.0183	13.9874	0.010 -0.178
K	31.0945	0.1085	12.5273	12.3531	12.7319	14.3298	3.7913	79.7647	14.1862	0.001 -0.191
KL	33.2961	1.0915	12.3497	1.1259	11.7894	14.1675	3.7913	79.7647	14.1862	0.001 -0.191
KL	33.2961	1.0915	12.3497	1.1259	11.7894	14.1675	3.7913	79.7647	14.1862	0.001 -0.191
IA	35.0454	1.0692	11.9980	5.4363	11.2501	14.9983	3.3612	74.7918	14.4444	0.074 -0.197
PT	36.8102	1.0462	13.0747	6.0730	11.3323	15.7018	2.3121	73.8375	14.6526	0.072 -0.178
AO	37.3027	1.00810	14.3904	6.5250	10.3625	16.5100	2.01229	73.9917	14.3992	0.059 -0.156
HU	37.8186	0.9655	17.0353	6.6756	6.5112	16.8636	2.6360	76.7228	14.2911	0.044 -0.131
HL	37.8186	0.9655	17.0353	6.6756	6.5112	16.8636	2.6360	76.7228	14.2911	0.044 -0.131
BI	37.7383	0.8755	2.3394	6.5894	5.1727	21.2637	3.71094	78.0094	14.0203	0.023 -0.063
BL	37.7143	0.8322	22.4562	6.27051	4.86549	24.4693	4.1816	72.1558	13.8301	0.013 -0.037
PO	37.6297	0.7860	23.1323	5.8664	5.59203	27.8878	4.06218	68.1617	13.5991	0.008 -0.012
AT	37.4971	0.74012	23.5635	5.42694	7.19593	29.8350	3.49526	66.1617	13.3181	0.011 -0.016
KV	37.3936	0.6858	23.9383	5.9860	6.0039	27.2222	3.49526	66.1617	13.3181	0.011 -0.016
KN	37.1902	0.6301	24.1301	4.1305	11.5026	29.2597	1.47890	77.6565	12.9938	0.031 -0.056
KA	36.9820	0.6396	24.2695	4.17857	11.8719	24.3782	2.72428	200.024	12.1642	0.053 -0.024
AC	36.8705	0.6450	24.7131	3.88776	12.3889	23.1506	3.26501	161.726	11.7494	0.050 -0.023
TH	36.7754	0.62510	25.2506	3.61658	13.0681	22.3410	3.63791	139.166	11.2497	0.046 -0.024
NP	37.3758	0.62510	25.2506	3.61658	13.0681	22.3410	3.63791	139.166	11.2497	0.046 -0.024
PA	37.2808	0.62510	25.2506	3.61658	13.0681	22.3410	3.63791	139.166	11.2497	0.046 -0.024
PU	37.3968	0.6876	26.0671	3.52325	14.8366	18.7619	3.31586	142.798	11.3632	0.027 -0.026
AM	37.6407	0.7976	26.5603	3.57718	15.4492	17.9814	2.79814	165.232	11.5358	0.022 -0.022
PI	37.6909	0.66617	27.1436	3.52195	15.7842	17.3069	2.79814	165.232	11.5358	0.023 -0.029
CF	37.5543	0.4932	27.6657	3.38133	15.8568	16.6688	3.37558	133.547	11.8531	0.031 -0.037
CM	37.5543	0.4932	27.6657	3.38133	15.8568	16.6688	3.37558	133.547	11.8531	0.031 -0.037
CF	37.6111	0.3255	29.2645	3.39285	15.4056	15.6791	2.78616	133.766	11.8531	0.027 -0.051
ES	37.4779	0.2353	30.0445	3.35234	16.5881	15.2381	2.77596	151.474	12.0698	0.029 -0.056
FD	37.3380	0.1562	30.8936	3.31193	16.6818	14.8362	2.76929	149.344	12.2983	0.031 -0.061
MD	37.1301	0.40883	31.7721	3.27142	16.7622	14.6655	2.76232	147.353	12.5741	0.033 -0.065
MD	37.1301	0.40883	31.7721	3.27142	16.7622	14.6655	2.76232	147.353	12.5741	0.033 -0.065
IA	36.8313	0.0105	31.9799	3.13608	16.4649	13.7255	3.31406	139.477	12.3313	0.037 -0.071

APPENDIX 4-5 HEAVY-ION ENERGY AND CHARGE-STATE AVAILABILITY AT THE LBL SUPERHILAC/BEVALAC

Gene R. Altmann

Some representative ion-beam energies and charge states capable of being produced at the LBL heavy-ion accelerator facilities are summarized in Table A4-5-1. To help understand this table, a schematic layout of the Bevalac is shown in Fig. A4-5-1. The SuperHILAC has two main accelerating

Fig. A4-5-1 SuperHILAC/Bevalac layout.
(NBL 7612-11075)

tanks, labeled the pre-stripper and post-stripper. The pre-stripper is designed so the maximum rf gradient will accelerate particles with e/m as low as 0.046. Thus to do experiments at 1.2 MeV per nucleon, the lowest charge-state attainable is governed by this e/m value. These charge states are listed in the "1.2 MeV/amu" column of Table A4-5-1. For energies between 1.2 MeV/amu and 5.5 MeV/amu, the lowest e/m that the post-stripper rf gradient can accelerate is 0.167, yielding the charge states listed in the next column of Table A4-5-1. For injection into the Bevatron, the strength of the bending magnets in the transfer line sets the lower limit on charge states, e/m greater than 0.29 at an energy of 5 MeV/amu. This is the lowest practical injection energy into the Bevatron.

The current values given in parentheses in Table A4-5-1 represent recent running experience for the ions and charge states listed. Currents are in electrical microamperes. The charge states normally accelerated are higher than those listed in the table, since the machine operates more efficiently and reliably at the lower power levels. The design gradients, though, allow for acceleration of the lowest charge states listed.

In the "source" column it should be noted that present capabilities exist for accelerating any gaseous and most metallic elements. Of the ions listed, Cs and I beams would require some developmental efforts. Note too that the temperatures in the PIG source used preclude the acceleration of any molecular species.

The information in Table A4-5-1 is summarized in Fig. A4-5-2, which shows solid lines of slopes 0.046, 0.167, and 0.29 for the limiting e/m values, and equilibrium charge distributions for foil and gas stripping at appropriate energies. These curves might prove useful for estimating the efficiency of production at various ion species of interest.

Fig. A4-5-2. (NBL 7612-11089)

Table A4.3-1. Lowest charge state for energies (MeV/amu) of various ions.

Ion	Source	E (MeV/amu)		
		1.2 MeV/amu (SuperHILAC pre-stripper)	1.2 < E < 8.5 (SuperHILAC post-stripper)	8.5 < E < 1 GeV/amu (Bevalac)
^{12}C	available	+1	+2	+4
^{16}O	available	^{18}O - (+1 (180 μA of +5))	+5 (35 μA of +7)	+5 (fully stripped)
^{20}Ne	available	+1 (500 μA of +3)	+4 (170 μA of +7)	+6 (fully stripped)
^{27}Al	available	+2	+5	+8
^{40}Ar	available	+2 (250 μA of +3)	+7 (175 μA of +13)	+12 (50 μA of +17)
^{40}Ca	available	^{48}Ca - (+2 (15 μA of +5))	+7 (7 μA of +14)	+12 ---
^{56}Fe	available	+5	+10	+17
^{74}Ge	available ^a	+4	+13	+22
^{84}Kr	available	+4 (50 μA of +6)	+14 (15 μA of +23)	+25 ---
^{107}Ag	available ^a	+5	+18	+32
^{127}I	development	+6	+22	+37
^{129}Xe	available	^{136}Xe - (+6 (11 μA of +8))	+22 (2 μA of +29)	+38 ---
^{133}Cs	development	+7	+23	+39
^{197}Au	available ^a	+9	+33	+57
^{209}Bi	available ^a	+10	+35	+61
^{238}U	available ^a	+11	+40	+70

^a Ion sources have produced adequate beams of these ions on test stands, but actual acceleration in the SuperHILAC has not occurred as of this date (July 1976).

APPENDIX 6-1 A LINAC-ACCUMULATOR RING SCHEME

Mark Q. Barton

A set of accelerator parameters required to illuminate a minimal target has been suggested. These are:

Power	100 TW
Ion energy	40 GeV
Ion type	U^{1+}
Pulse length	6 nsec
Emittance	$\epsilon/\pi = 3 \text{ cm}\cdot\text{mrad}$
Momentum spread	$\Delta p/p = \pm 0.003$
Number of beams	2

The scheme described below satisfies all of these requirements starting with a linac whose properties are:

$$\begin{aligned} E &= 40 \text{ GeV}, U^{1+} \\ \Delta p/p &= \pm 2 \times 10^{-5} \\ \epsilon_n/\pi &= 0.1 \text{ cm}\cdot\text{mrad} \\ I &= 50 \text{ mA} \end{aligned}$$

We note first the the six-dimensional phase-space density of this source exceeds that required by the target. To see that, we note that the current in each beam on target is 1200 A. Then for an initial beam to have a higher density than a final beam, we must have

$$\left(\frac{\Delta p}{p}\right)_{\text{init}} < \left(\frac{\Delta p}{p}\right)_{\text{final}} \left(\frac{\epsilon_n \text{ final}}{\epsilon_n \text{ init}}\right)^2 \frac{I_{\text{init}}}{I_{\text{final}}} \left(\frac{\beta_{\text{final}}}{\beta_{\text{init}}}\right) \left(\frac{p_{\text{final}}}{p_{\text{init}}}\right)$$

The normalized emittance ϵ_n/π is 1.8 cm·mrad for the final beam. Then..

$$\left(\frac{\Delta p}{p}\right)_{\text{init}} < 3 \times 10^{-5} \left(\frac{1.8}{0.1}\right)^2 \left(\frac{0.050}{1200}\right) \left(\frac{1}{1}\right) \left(\frac{1}{1}\right) = 5 \times 10^{-5}$$

which is probably easily satisfied by a linac even prior to debunching.

The straightforward method of matching this linac to the target requirement is to stack in a ring in transverse phase space for a total of $(1.8/0.1)^2$ or 324 turns. The resulting beam has a current of 32×0.050 or 16.2 A. A compression by a factor of 74 in pulse length will bring the current to the required 1200 A, at the same time increasing the momentum spread but not exceeding the target requirement.

An example that shows how the beam might be stacked is as follows. The linac beam is stacked in horizontal phase space in a ring of radius 404 m for 18 turns. This ring would require an average field of slightly more than 1 T. The details of the focusing, etc., are not important for the argument. The beam properties are

$$I = 18 \times 0.05 = 0.9 \text{ A}$$

$$\epsilon_{nh}/\pi = 1.8 \text{ cm}\cdot\text{mrad}$$

$$\epsilon_{nv}/\pi = 0.1 \text{ cm}\cdot\text{mrad}$$

$$\Delta p/p = 2 \times 10^{-5}$$

$$\begin{aligned} \Delta v_{\text{space charge}} &= \frac{1}{\gamma} \cdot \frac{2R}{\sqrt{\epsilon_{nv}} (\sqrt{\epsilon_{nv}} + \sqrt{\epsilon_{nh}})} \\ &\cdot \frac{1}{E^2 \gamma^2} = 0.06 \end{aligned}$$

$I_p = 7.4 \times 10^9 \text{ A}$, and $\beta_\gamma = 0.624$. This beam could be bunched by a factor of two on harmonic 36, then extracted with a kicker magnet with a waveform shown in Fig. A6-1-1, in which the

Fig. A6-1-1. (XBL 7612-11073)

pulse length must be just longer than the 220 nsec pulse length and the spacing is adjusted to match the spacing between 17 bunches in the ring. The ejected bunches are then stacked in vertical phase space in a ring of 381 m radius which is 17/18 of the radius of the first ring. Because of the numerical relations between the rings, the 36 bunches combine into two bunches in which

$$\epsilon_{nh}/\pi = \epsilon_{nv}/\pi = 1.8 \text{ cm}\cdot\text{mrad}$$

$$I = 32.4 \text{ A}$$

$$\Delta v_{\text{space charge}} \approx 0.25$$

$$\Delta p/p \sim 4 \times 10^{-5}$$

A modest bunch compression of 37 results in two beams of 1200 A with all of the other requirements.

APPENDIX 6-2 POWER TRANSPORT IN QUADRUPOLE OR SOLENOID- FOCUSING SYSTEMS

E. D. Courant

Maschke¹ gives a formula for the power that can be transported in a quadrupole focusing channel by a beam with space charge. It is

$$P = 1.67 \times 10^{15} \left(\frac{A}{Z}\right)^{4/3} \epsilon^{2/3} (\beta_\gamma)^{7/3} (\gamma - 1) R^{2/3} \quad (1)$$

where

P = power (W)

A, Z = mass number, charge state

$\pi \epsilon$ = emittance of beam (m·rad)

B = pole tip field of quadrupoles (T).

This formula is based on the assumption that the beam fills the aperture of a quadrupole focusing channel, and that the space-charge defocusing forces are about half the average focusing forces.

Let us rederive this formula, and produce a similar expression for solenoid focusing.

QUADRUPOLES

Let a be the semi-aperture, and also the beam radius. Then:

$$a = \sqrt{\epsilon f_{\max}} \quad (2)$$

where $f_{\max}$ is the Courant-Snyder function. Its value is determined by quadrupole gradient a : space charge, and is, in the absence of space charge,

$$f_{\max} = f_1 \sqrt{\frac{(B_0/a) \cdot a}{B_0}} \quad (3)$$

where $f_1 \approx 0$, and the field gradient is B_0/a . If space charge introduces an effective magnetic field gradient, $f_{\max}$ is increased.

Let space-charge gradient be f_2 times the quadrupole gradient. We estimate that for $f_2 \approx 1/10$ we get $f_1 \approx 10$ (exact calculations can be done).

The effective space-charge gradient is

$$G_1 = f_2 \frac{B_0}{a} = \frac{I \bar{z}_0}{2\pi a^2 B^2 \gamma^2 c} \quad (4)$$

where I is the current, and $\bar{z}_0$ is the impedance of free space. Combining Eqs. (2) and (3) we find

$$a = f_1^{2/3} \epsilon^{2/3} \left(\frac{B_0}{B_0} \right)^{1/3} \quad (5)$$

and, from (4)

$$I = \frac{2\pi a c (\delta\gamma)^2 B_0 f_2}{\bar{z}_0} \quad (6)$$

The magnetic rigidity is, for ions with Z and A ,

$$B\rho = (1/c)(A/Z)(M_p c^2/e) \cdot \delta\gamma \quad (7)$$

so that

$$I = \frac{2\pi f_1^{2/3} \epsilon^{2/3}}{\bar{z}_0} \left(\frac{M_p c^2}{e} \right) \left(\frac{A}{Z} \right)^{1/3} \epsilon^{2/3} (\delta\gamma)^{7/3} B_0^{2/3} \quad (8)$$

The power transported is

$$P = (\gamma - 1) \frac{A}{Z} \left(\frac{M_p c^2}{e} \right) I \quad (9)$$

leading to formula (1) with the numerical coefficient

$$\frac{2\pi f_1^{2/3} \epsilon^{2/3}}{\bar{z}_0} \left(\frac{M_p c^2}{e} \right)^{4/3} = 6.85 \times 10^{11} f_1^{2/3} \epsilon^{2/3} \quad (10)$$

so that Maschke's coefficient seems safe.

SOLENOIDS

In solenoids, Eq. (3) is replaced by

$$f_{\max} = \frac{2(B_0)}{B_s} \quad (11)$$

A space-charge gradient G_1 changes this to

$$f_{\max} = \frac{2(B_0)}{1 + 4G_1 \frac{B_0}{B_s}} \quad (12)$$

If we permit $f_{\max}$ to double we have

$$G_1 = \frac{5}{16} \frac{B_s^2}{B_0} \quad (13)$$

and

$$f_{\max} = \frac{4(B_0)}{B_s} \quad (14)$$

Using (4) and (13) we find

$$I = \frac{3\pi}{8\bar{z}_0} a^2 (\delta\gamma)^2 c \frac{B_s^2}{B_0} \quad (15)$$

and with a^2 from (2) and (14)

$$a^2 = \frac{4\epsilon(B_0)}{B_s} \quad (16)$$

$$I = 3/2 \frac{\pi c}{\bar{z}_0} \epsilon (\delta\gamma)^2 B_s \quad (17)$$

The ratio for current-carrying capacity of a solenoid system to a quadrupole system becomes

$$\frac{I_{\text{sol}}}{I_{\text{quad}}} = \frac{3}{4 f_1^{2/3} f_2} c^{1/3} \epsilon^{1/3} \left(\frac{c}{\bar{z}_0} \right)^{1/3} \left(\frac{e}{M_p c^2} \right)^{1/3} (\delta\gamma)^{-1/3} B_s B_0^{-2/3} \quad (18)$$

Numerically, with $r_1^{2/3} f_2 = 1/4$, $Z/A = 1/238$, $B_y = 1$, $B_z = 6$ T, $B_0 = 1$ T, $\epsilon = 6 \times 10^{-5}$, this ratio is 0.075. The solenoid system with 6 T can carry 0.075 times as much current as the quadrupole system.

Reference

1. A. W. Maschke (distributed at the ERDA Summer Study of Heavy Ions for Inertial Fusion, unpublished report).

APPENDIX 6-3 SPACE-CHARGE LIMITATIONS OF HEAVY-ION ACCELERATION IN A SYNCHROTRON

David Berley and Gordon J. Danks

The cost of a heavy-ion fusion pellet igniter is strongly affected by the space-charge forces that tend to defocus the ion beam. In this report we consider the limitations on the design of an accelerator system imposed by space-charge forces. The ion beam is first accelerated in a linear accelerator and then injected into a synchrotron, which takes the beam to full energy.

The parameters of the accelerators are the linac energy (equal to the synchrotron injection energy), the maximum magnetic rigidity to be contained in the synchrotron, and the aperture of the synchrotron. These, together with the beam emittance at the target, define the maximum energy that can be delivered to the pellet in one cycle of the synchrotron.

We have also investigated whether conventional ion sources are sufficiently bright to deliver the requisite power confined within the specified target area. The linac current, transverse emittance, and longitudinal phase space defined by Young¹ are adequate to design the rest of the system to meet the target requirement, at least in a specific case. Since this subject has been touched upon in another report by Teng,² we do not include our work in this appendix.

MAXIMUM DELIVERABLE ENERGY

The maximum energy that can be delivered to the target per synchrotron cycle is:

$$W \leq \sqrt{2} \pi \frac{\Delta v}{c^2} \frac{R_f}{R_p^{5/2}} \frac{1}{r_p} \frac{a R_q}{f} \left(\frac{Z}{A} \right)^{3/2} \quad (1)$$

$$\left(\frac{e}{c} \text{HR} \right)_{\text{tgt}}^3 \sqrt{e v_{\text{inj}}} \left[\frac{\sqrt{2} \gamma_{\text{inj}} \sqrt{1 + \gamma_{\text{inj}}}}{1 + \gamma_{\text{tgt}}} \right]$$

where

Δv = maximum synchrotron tune shift;
assumed value = 0.25

B_f = synchrotron bunching factor;
assumed value = 0.5

M_p = proton mass

r_p = classical proton radius

a = pellet radius; assumed value = 0.1 cm

R_q = beam radius at final focusing lens
focal plane; assumed value = 30 cm

f = focal length of final lens;
assumed value = 10 m

z = charge state of ion

A = atomic number of ion

γ_{inj} = Lorentz velocity factor at injection

γ_{tgt} = Lorentz velocity factor at target

[Editor's note: The authors use the term magnetic rigidity for the quantity $(e/c \text{ HR})_{\text{tgt}}$. HR refers to the usual magnetic rigidity defined by $p = ze/c \text{ HR}$. They also define V_{inj} as an injector voltage where V_{inj} is the kinetic energy of the injector.]

In the region of interest, the velocity factors (i.e. uranium ions with less than 100 GeV kinetic energy) are:

$$0.84 < \left[\frac{\sqrt{2} \gamma_{\text{inj}} \sqrt{1 + \gamma_{\text{inj}}}}{1 + \gamma_{\text{tgt}}} \right] < 1.28$$

and are approximated by 1 for the purpose of this discussion. Thus,

$$W \leq 0.680 \frac{a R_q}{f} \left(\frac{Z}{A} \right)^{3/2} \left(\frac{e}{c} \text{HR} \right)_{\text{tgt}}^3 \sqrt{e v_{\text{inj}}} M_J$$

The magnetic rigidity $(e/c \text{ HR})$ and $(e v_{\text{inj}})$ are in units of GeV/c and GeV. Units of length are cm.

It is assumed that the charge state accelerated in the linac and in the synchrotron are the same.

TRANSVERSE SYNCHROTRON APERTURE

The radial beam size in a synchrotron is

$$x = \sqrt{\beta \epsilon}$$

where β is the betatron function (assumed ≤ 25 m), and ϵ is the beam emittance.

The emittance is largest at injection. The emittance at injection is related to the emittance at the target by:

$$\epsilon_{inj} = \epsilon_{tgt} \frac{p_{tgt}}{p_{inj}},$$

where

$$\epsilon_{tgt} = \frac{aR_q}{f}$$

p_{tgt} and p_{inj} are momenta at target and injection. Therefore,

$$x \leq 2.7 \sqrt{\frac{p_{tgt}}{p_{inj}}} \geq 2.7 \left(\frac{T_{tgt}}{T_{inj}} \right)^{1/2}$$

where T_{tgt} and T_{inj} are the kinetic energy of the beam at injection and at the target.

The radial beam size for several values of p_{tgt}/p_{inj} is shown in Table A6-5-1.

Table A6-5-1.

$\frac{p_{inj}}{p_{tgt}}$	$\frac{T_{inj}}{T_{tgt}}$ (approx.)	Maximum beam radius (cm)
1	1	2.7
1/2	1/4	5.8
1/4	1/16	5.48

In order that the synchrotron magnet aperture does not become unreasonably large, we restrict

$$T_{tgt} < 16 T_{inj}$$

KINEMATIC CONSTRAINTS

The injection energy is always less than or equal to the target energy:

$$T_{tgt} = \alpha T_{inj},$$

where $1 \leq \alpha$ from aperture considerations (see previous section). Momentum-energy relations imply

$$\epsilon V_{inj} = \frac{1}{\alpha} \frac{1}{2M_p} \frac{z}{A} \left(\frac{e}{c} HR \right)_{tgt}^2 \left[\frac{2}{1 + \gamma_{tgt}} \right].$$

In the region of interest

$$0.85 < \left[\frac{2}{1 + \gamma_{tgt}} \right] < 1$$

and for the purpose of this discussion will be taken to equal 1.

Inverting ϵV_{inj} in Eq. (1),

$$W \leq 0.4967 \left(\frac{z}{A} \right)^2 \frac{aR_q}{f} \frac{1}{\alpha} \left(\frac{e}{c} HR \right)_{tgt}^4$$

$$\left[\frac{2\gamma_{inj} \sqrt{1 + \gamma_{inj}}}{1 + \gamma_{tgt}} \right]^{1/2}$$

DEPENDENCE ON ATOMIC NUMBER AND CHARGE STATE

For the same beam energy V_{inj} is independent of z and A and

$$\left(\frac{e}{c} HR \right)_{tgt} \propto \sqrt{\frac{A}{z}}$$

RANGE CONSIDERATIONS

The ion range in a dense material, like gold, should be less than 2 g/cm². This places an upper limit on the kinetic energy of the ion.

$$T_{iodine} \leq 50 \text{ GeV}$$

$$T_{uranium} \leq 100 \text{ GeV}$$

CONCLUSIONS AND SUMMARY

1. The energy that can be delivered to the target is illustrated in Figs. A6-5-1, A6-5-2, and A6-5-3 (all factors have been included).

Fig. A6-5-1. Maximum synchrotron magnetic rigidity vs injector voltage for uranium $Z = +1$. (—) Space-charge limit of energy deliverable to target. (---) Kinematic relation between magnetic rigidity and injector voltage. $T_{tgt} = \alpha T_{inj}$. (NBL 7612-11083)

Fig. A6-3-2. Maximum synchrotron magnetic rigidity vs injector voltage for uranium $Z = +10$. (—) Space-charge limit of energy deliverable to target. (----) Kinematic relation between magnetic rigidity and injector voltage. $T_{tgt} = \alpha T_{inj}$. (XBL 7612-11085)

Fig. A6-3-3. Maximum synchrotron magnetic rigidity vs injector voltage for uranium $Z = +100$ or iodine $Z = +53$ (approximate to within relativistic factors). (—) Space charge limit of energy deliverable to target. (----) Kinematic relation between magnetic rigidity and injector voltage. (XBL 7612-11081)

Table A6-3-11.

	$^{238}\text{U}^{+1}$		$^{238}\text{U}^{+10}$		$^{126}\text{I}^{+53}$ or $^{238}\text{U}^{+100}$	
	$eV_{inj} = eV_{tgt}$		$eV_{inj} = eV_{tgt}$		$eV_{inj} = eV_{tgt}$	
W	eV_{tgt}	$\left(\frac{e}{c}HR\right)_{tgt}$	eV_{tgt}	$\left(\frac{e}{c}HR\right)_{tgt}$	eV_{tgt}	$\left(\frac{e}{c}HR\right)_{tgt}$
MJ	GeV	GeV/c	GeV	GeV/c	--	--
0.001	0.44	13.96	0.44	4.4	0.44	1.46
0.01	1.4	24.86	1.4	3.0	1.4	2.86
0.1	4.3	42	4.3	14.5	Beyond useful range in material	
1	14	74	14	28.6		
10	40	140				
100	100	233				

Fig. A6-3-1, U^{+1} A = 238, Z = +1

Fig. A6-3-2, U^{+10} A = 238, Z = +10

Fig. A6-3-3, U^{+100} A = 238, Z = 100

or (in nonrelativistic approximation)

$$I^{53} \quad A = 126, \quad Z = 53$$

The solid curves are plots of the maximum magnetic rigidity vs injector voltage for constant maximum power deliverable. The dashed curves are plots of the kinematic constraints for three values of

$$\alpha = \frac{T_{tgt}}{T_{inj}} = 1, 4, 16$$

These curves are useful for cost optimization. For a given final energy the lowest linac energy is desirable, but not so low that the cost of magnet aperture dominates. A guess is that $\alpha \sim 16$ would be a practical working number.

2. Some of the information of Figs. A6-3-1, A6-3-2, and A6-3-3 is summarized in Table A6-3-11.

When using Table A-5-11, it should be noted that the injection energy is equal to the energy at target

$$T_{inj} = T_{tgt} \quad \alpha = 1$$

To obtain the energy deliverable at lower injection energy, scale the deliverable energy downward by

$$k(\alpha) = \sqrt{\frac{1}{\alpha}} W(1)$$

$$T_{inj} = \frac{1}{\alpha} T_{tgt}$$

5. Large energy can be delivered only by accelerating a low-charge state. Acceleration in a high charge state does not yield useful beam energy to the power plant. The limiting parameter becomes the ion range in material. An energy of 10 MJ per synchrotron pulse can be achieved with uranium in a charge state ≤ 3 and lower charge state for lower atomic mass number.

4. A scaled-down accelerator system capable of delivering energy in the 0.1 to 1 MJ range might consist of a 1 GeV linac and a 20 GeV/c synchrotron. Such an accelerating system, used with U^{+10} ions would be in size like the LAMPF linac injecting into an AOS synchrotron.

References

1. D. A. Young, Technical Summary 3 of this report.
2. L. C. Teng, Technical Summary 5 of this report.

APPENDIX 6-4 ESTIMATE OF LONGITUDINAL SPACE-CHARGE EFFECTS IN rf AND DRIFT BUNCHING

Lloyd Smith

PROCEDURE

The distance z from the center of the bunch and $A \equiv \Delta p/p$ are used as the dependent variables and path length s as the independent variable. The bunch is assumed to travel through a tube whose transverse dimensions are small compared to the bunch length, so that the longitudinal space-charge force can be written as:

$$F_z = -\frac{q}{\gamma^2} q^2 c^2 \frac{d\lambda}{dz} \quad (1)$$

where q is the charge state of the ions, λ is the

number of ions per unit length, and g is a geometrical factor of order unity. λ is taken to be parabolic in z :

$$\lambda = \frac{3}{4} \frac{N}{z_m} \left(1 - \frac{z^2}{z_m^2} \right) \quad (2)$$

where N is the total number of ions in the bunch and $2z_m$ is the length of the bunch. This distribution leads to a force linear in z , thus permitting the use of linear equations and elliptical shapes in phase space. Unfortunately, a uniformly populated ellipse in phase space leads to $\lambda \propto (1 - z^2/z_m^2)^{1/2}$, so that the analysis is not self-consistent; on the other hand, the form in Eq. (2) is more realistic and has a finite derivative at $z = z_m$.

The equations of motion can be written as

$$\frac{dz}{ds} = \left(\frac{1}{\gamma^2} - \frac{1}{\gamma_T^2} \right) \Delta = \frac{1}{\gamma^2} \Delta, \quad \left(\frac{\gamma^2}{\gamma_T^2} \ll 1 \right) \quad (3)$$

$$\frac{d\Delta}{ds} = -\frac{q\epsilon V h}{2\pi mc^2 \gamma^2 R^2} z + \frac{3}{2} \frac{q^2}{A} \frac{g}{\epsilon^2 \gamma^3} \frac{N r_p}{z_m^3} z, \quad \left(\sin \frac{h\pi}{R} z - \frac{h\pi}{R} z \right) \quad (4)$$

where $\Delta = \Delta p/p$ and the rest of the notation is conventional.

rf BUNCHING IN A RING

For the particle of largest amplitude, $z = z_m \sin \Omega s$ and $\Delta = \gamma^2 \Omega z_m \cos \Omega s$ where

$$\Omega^2 = \frac{1}{\beta^2 \gamma^3} \left[-\frac{q\epsilon V h}{2\pi mc^2 R^2} - \frac{3}{2} \frac{q^2}{A} \frac{g}{\epsilon^2 \gamma^3} \frac{N r_p}{z_m^3} \right] \quad (5)$$

as found by combining Eqs. (3) and (4). Then

$$\epsilon = \frac{1}{\pi} \times \text{emittance in } \left(z, \frac{\Delta p}{p} \right) = \gamma^2 \Omega z_m^2 \quad (6)$$

by squaring both sides of Eq. (6), one finds that the voltage required to hold a bunch of given ϵ , z_m and N is

$$eV = \frac{2\pi mc^2 R^2}{qh} \left[\frac{\beta^2 \epsilon^2}{\gamma_m^4} + \frac{3}{2} \frac{q^2}{A} \frac{g}{\epsilon^2 \gamma^3} \frac{N r_p}{z_m^3} \right] \quad (7)$$

Note that $V = h^{-1}$; the least voltage is required for the highest possible frequency consistent with $h z_m/R \leq 1$.

For a numerical example, consider the last stage of Barton's two-ring betatron stacking scheme.¹ In that case, $mc^2 = 223$ GeV, $\beta = 0.624$, $q = 1$, $R = 581$ m, $2z_m = 37$ m, $\epsilon = 37/2 \times 4 \times 10^{-5}$ m, $A = 258$, $N = 0.45 \times 10^{10}$, $h = 36$, and we take $g = 2$. Then

$$eV = 12[1 + 47] = 580 \text{ keV/turn at } 2.4 \text{ MHz,}$$

where the second number in the brackets is the ratio of the space-charge term to the emittance term. When the space-charge term is so dominant, the same answer can be obtained by setting the right-handed side of Eq. (4) equal to zero.

DRIFT BUNCHING

Many of the schemes considered involve a final linear voltage ramp to tilt the ellipse in phase space; the ellipse then shears as the beam travels around a ring and/or along a linear drift space and the bunch shortens to the desired final length and momentum spread. The space-charge force resists the shortening of the bunch, reducing the momentum spread and slowing the rate of collapse. It is the purpose of this section to estimate the overvoltage needed to attain a given final momentum spread and the distance needed to bunch. It is convenient to use an envelope equation similar to the more familiar one for transverse motion.

The equation of the ellipse in phase space can be written in the form:

$$\epsilon^2 = \Delta_m^2 z^2 + z_m^2 \Delta^2 - 2z\Delta\sqrt{z_m^2 \Delta_m^2 - \epsilon^2}, \quad (8)$$

where Δ_m and z_m are the maximum values on the bunching ellipse (not the axes) and ϵ is the (constant) are of the ellipse divided by π , as in the previous section. Equations for the rate of change of Δ_m and z_m can be obtained by differentiating both sides of Eq. (8), inserting Eqs. (3) and (4) for dz/ds and $d\Delta/ds$ (with $V=0$) in the result and finally equating the coefficients of z^2 , Δ^2 , $z\Delta$ to zero. One finds:

$$\frac{dz_m}{ds} = \gamma^2 z_m \sqrt{\Delta_m^2 - \epsilon^2} \quad (9)$$

$$\frac{d\Delta_m}{ds} = \frac{3}{2} \frac{q^2}{A} \frac{g}{\beta^2 \gamma^3} \frac{N r_p}{z_m^3 \Delta_m} \sqrt{z_m^2 \Delta_m^2 - \epsilon^2} \quad (10)$$

whence:

$$\Delta_m^2 = \left(\frac{\Delta p}{p}\right)_m^2 = \frac{\epsilon^2}{z_m^2} + \gamma^4 \left(\frac{dz_m}{ds}\right)^2 \quad (11)$$

$$\frac{d^2 z_m}{ds^2} = \frac{\epsilon^2}{\gamma^4 z_m^3} + \frac{3}{2} \frac{q^2}{A} \frac{g}{\beta^2 \gamma^5} \frac{N r_p}{z_m^3} \quad (12)$$

Since the behavior of the envelope is symmetric before and after the waist, we can integrate from the waist ($dz_m/ds = 0$) onward to obtain the distance and initial momentum spread as final conditions on the solution of Eq. (12). Equation (12) has a first integral for $dz_m/ds = 0$ at $s=0$, $z_m = z_T$:

$$\left(\frac{dz_m}{ds}\right)^2 = \frac{\epsilon^2}{\gamma^4} \left(\frac{1}{z_T^2} - \frac{1}{z_m^2}\right) + 3 \frac{q^2}{A} \frac{g}{\beta^2 \gamma^5} N r_p \left(\frac{1}{z_T} - \frac{1}{z_m}\right) \quad (13)$$

where $2z_T$ is the length of the bunch at the target. Equation (11) then gives the momentum spread directly as a function of bunch length:

$$\Delta_m^2 = \Delta_T^2 + 3 \frac{q^2}{A} \frac{g}{\beta^2 \gamma^5} N r_p \left(\frac{1}{z_T} - \frac{1}{z_m}\right), \quad (14)$$

where $\Delta_T = \epsilon/z_T$ = half-momentum spread at the target. If $z_{\text{initial}} \rightarrow z_T$ (large compression ratio), then

$$\left(\frac{\Delta p}{p}\right)_{\text{initial}}^2 = \left(\frac{\Delta p}{p}\right)_{\text{target}}^2 + 3 \frac{q^2}{A} \frac{g}{\beta^2 \gamma^5} \frac{N r_p}{z_T^2} \quad (15)$$

Using Barton's case again as an example, $(\Delta p/p)_{\text{target}} = 37/4 \times 10^{-5} = 1.5 \times 10^{-4}$, $z_T = 50$ cm, the required initial tilt is

$$\begin{aligned} \Delta_i &= \left(\frac{\Delta p}{p}\right)_{\text{initial}} = 1.5 \times 10^{-4} \sqrt{1 + 4.7} \\ &= 3.6 \times 10^{-4} \quad (\Delta E_i = \pm 265 \text{ MeV}) \end{aligned}$$

The drift length L can be obtained by integrating Eq. (15). The result is:

$$L = \frac{z_i^2 z_T}{\Delta_i} \left\{ \left[1 - \frac{R(1-p^2)+p^2}{R^2} \right]^{1/2} + \frac{1-p^2}{2R} \ln \left[\frac{p^2 + 2R - 1 + 2R \left[1 - \frac{R(1-p^2)+p^2}{R^2} \right]^{1/2}}{1+p^2} \right] \right\} \quad (16)$$

where Δ_i and z_i are the initial momentum spread and bunch length, R is the compression ratio z_i/z_T , and p is the ratio of momentum spreads Δ_T/Δ_i , as given by Eq. (15). For large R ,

$$L \sim \frac{\gamma^2 z_i}{\Delta_i} \left\{ 1 + \frac{1-p^2}{2R} \left[\ln \frac{2R}{1+p^2} - 1 \right] \right\} \quad (17)$$

Using the same case again as an example, $z_i = 37/2$ m, $\Delta_i = 3.6 \times 10^{-5}$, $p = 1.5/3.6$, $R = 37$,

$$L = 7.1[1 + 0.04] \text{ km}$$

which is equivalent to about three turns in the ring. Note that the braking effect does not extend the distance appreciably (0.04/1) and that the required distance is Δ_T/Δ_i times shorter than if the space-charge force were negligible, but at the expense of requiring a voltage ramp Δ_i/Δ_T times larger.

LINEAR INDUCTION ACCELERATOR

In this device, it is possible to bunch during acceleration by ramping the voltage in the accelerating units as a function of time or, in our variable, as a function of distance z from the center of the bunch. The bunching action can be described by the formalism of the previous section by using the variables $\{z, \Delta(\delta y)\}$, in which space the emit-

tance is independent of energy, and taking account of the dependence of energy on path length. Equations (3) and (4) become:

$$\frac{dz}{ds} = \frac{\Delta(\gamma)}{\epsilon\gamma^3} \quad (3')$$

$$\frac{d}{ds} \Delta(\gamma) = - \left[\frac{q\epsilon E_0 \alpha}{mc^2 \epsilon} + \frac{3}{2} \frac{gq'}{\Lambda} \frac{Nr_p}{\beta\gamma^2 z_m^2} \right] z \quad (4')$$

where the accelerating field is $\epsilon E_0(1 - \alpha z)$.

Equations (11) and (12) become:

$$\dot{z}_m^2 = [\Delta(\gamma)]_m^2 = \frac{\epsilon^2}{z_m^2} + \beta^2 \gamma^6 \left(\frac{dz_m}{ds} \right)^2 \quad (11')$$

$$\frac{d}{ds} \beta\gamma^3 \frac{dz_m}{ds} = \frac{\epsilon^2}{\beta\gamma^2 z_m^2} + \frac{q\epsilon E_0 \alpha}{mc^2 \beta} z_m + \frac{3}{2} \frac{gq'}{\Lambda} \frac{Nr_p}{\beta\gamma^2 z_m^2}, \quad (12')$$

where ϵ is the emittance in $[z, \Delta(\gamma)]$ space.

These equations cannot be solved analytically, but could be done easily by computer, given the dependence of E_p and α on path length s . It should be remembered that the initial conditions are different from the constant-energy situation; by the time the bunch is entirely in the accelerator, the leading ions have a higher energy and the trailing ions a lower energy than the central reference particle, so that the ellipse is initially tilted upward. In the absence of space charge and with $\alpha = 0$, the bunch would lengthen during acceleration in proportion to the velocity.

POST SCRIPT

The author was reminded by A. Faltens that the geometrical factor g can be modified by changing the wall structure. An effective inductance of $0.10 \text{ g}/(\beta\gamma)^2 \text{ nH/m}$ would reduce the space charge force to zero; a higher inductance would produce self-bunching. For $g \sim 2$ and $\beta\gamma < 1$, disk loading of the beam pipe looks impractical, but lumped ferrite structures could be considered.

As for self-bunching, it is easy to show from Eqs. (13) and (14) that if g (now negative) is chosen so that

$$-\frac{3q^2}{\Lambda} g \frac{Nr_p}{\Lambda^2 z_T \beta^2 \gamma} = 1 + \frac{1}{R}, \quad (18)$$

then an upright ellipse of dimensions (Δ_1, z_1) will bunch to an upright ellipse of dimensions (Δ_T, z_T) in a distance,

$$L = \frac{\pi}{2} \frac{\gamma^2 z_1}{\Delta_T} \sqrt{R},$$

where R is again the compression ratio. However, negative g creates the possibility of the bunch breaking up into smaller clumps during compression; this phenomenon should be investigated before considering self-bunching seriously.

Reference

J. M. Q. Barton, Appendix 6-1 of this report.

APPENDIX 6-5

INTRA-BEAM CHARGE EXCHANGE

Frederick E. Mills

Consider two ions of charges ze and mass M moving with momenta $p_1 = p + \delta p$, and $p_2 = p + \epsilon p$, with δ and ϵ defined appropriately. The speed of each particle in the center of mass is given by

$$\beta_{c.m.}^2 = \frac{\beta^2}{4} \left(\frac{\delta p_1}{p} + \frac{\delta p_2}{p} \right) + (\beta\gamma)^2 \sin^2 \frac{\theta}{2},$$

where θ is the angle between $\vec{p}_1$ and $\vec{p}_2$ in the laboratory. For cases of interest here $\delta p/p \ll \sin \theta/2$, so we can take $\beta_{c.m.} \approx \sin \theta/2$. If we assume that emittances ϵ_x and ϵ_y are equal, then typical center-of-mass velocities are obtained by setting $\theta = x_{max}'$ or y_{max}' . Then

$$\theta = \sqrt{\frac{\epsilon}{\pi \beta_x}},$$

and

$$\beta_{c.m.} = \frac{\beta\gamma}{2} \sqrt{\frac{\epsilon}{\pi \beta_x}}.$$

where β_x is the CSL parameter. The loss rate is

$$\alpha = \frac{dN}{Ndt} = n_{Lab} V_{c.m.} \sigma_{c.m.}$$

If there are N particles in an accelerator of radius R ,

$$n_{Lab} = \frac{N}{2\pi R \beta_x \epsilon},$$

so that

$$\alpha = \frac{N\beta\gamma c}{4\pi^2 R} \sqrt{\frac{1}{\pi \beta_x^2}} \sigma_{c.m.}$$

Consider a case for which

$$e_x = 30\pi \times 10^{-6} \text{ m}$$

$$R = 640 \text{ m}$$

$$B_x = 20 \text{ m}$$

$$B_y = 0.89 \quad \text{and} \quad N = 1.6 \times 10^{15} (10 \text{ MJ}),$$

then $\alpha = dN/Ndt = 3.4/\text{sec}$, or the mean lifetime is about 0.3 sec. This would seem to pose a severe limitation on accumulation rates.

ROUGH ESTIMATE OF CROSS SECTION

The normal ionization process by Coulomb momentum transfer can be estimated by calculating the maximum impact parameter for momentum transfer from an ion moving with $B_{c.m.}$ to an electron moving at $-B_{c.m.}$. This limit occurs when the Fourier spectrum of the Coulomb field at the electron contains no frequencies greater than the atomic frequency of the electron, usually estimated at $13.52 \text{ eV}/\hbar$. Then, $b_{\max} \sim (V_{c.m.}\hbar)/(13.52) \sim 10^{-11} \text{ cm}$ here. For larger impact parameters the electrons' motion in the atoms is adiabatic and momentum is transferred, but the electron wavefunctions are slowly distorted during the collision. The cross section for Coulomb ionization is then $\sim 10^{-22} \text{ cm}^2$ and is negligible compared to the atomic cross section.

The collision time is of the order of $r_{\text{atom}}^2/\lambda_{c.m.} \sim 4 \times 10^{-15} \text{ sec}$, which is long compared with electron frequencies.

What presumably happens is the formation of a "compound molecule" (in analogy with compound nuclei), which results in a statistical distribution of the available final states in the two objects. Since the energy available is of the order of 1 keV and the energy difference of levels and ionized states is of the order of electron volts, many states are available with similar probabilities. A large fraction of these correspond to changing the charge state of the ions. Accordingly, the cross section for charge changing must be near geometrical, or in any case nearly the cross section for formation of the compound molecule. Thus $\sigma \sim 10^{-15}$ seems reasonable.

Since $b_{\max}$ does not depend on the charge of the ion, but only on its velocity, we do not expect much improvement by using higher charge states, except by obtaining smaller "size" ions with closed-shell configurations in which the electron wavefunctions are more tightly localized.

More detailed estimates of such cross sections are discussed in this report (Y.-K. Kim, Technical Summary 4 of this report).

APPENDIX 7-1 INDUCTION LINAC ACCELERATOR STRUCTURES FOR HEAVY-ION BEAM IGNITERS

James F. Deane

INTRODUCTION

It has been proposed by Keefe¹ that induction linac accelerators may be appropriate for large-current heavy-ion beams for inertial-confinement pellet ignition. Currents of 5000 A or more, pulse lengths of 10 nsec or less, and energies of about 25 GeV or more are of interest for uranium ions. Space-charge forces for such high currents in short pulses are extremely large throughout the accelerator, and become prohibitive at low energies. To circumvent these difficulties Keefe has proposed to use much longer pulses at reduced beam currents in the lower-energy portions of the accelerator, and to compress gradually the beam-pulse length, with resultant higher instantaneous current, in the high-energy end of the accelerator.

Detailed design of such an accelerator system will be dominated by space-charge considerations, and appears to be outside of the scope of what can be accomplished during this workshop. Both transverse and longitudinal space-charge effects are important and must be considered. Longitudinal space-charge forces serve to accelerate the head of the beam and to decelerate the tail of the beam. If uncorrected, these forces would cause the pulse to spread out in time into a longer, lower current pulse.

Compensation for these longitudinal forces appears possible by use of ramped accelerating pulses. As much as 300 keV/m voltage ramp over the length of the pulse may be required. Additional voltage ramp is required to provide the pulse-length compression as the energy is increased. These compensating requirements place a lower limit on the magnitude of accelerating gradient for the acceleration. To allow both for acceleration and for space-charge compensation and bunching, this lower limit appears to be about 1 MeV/m, with a higher gradient being greatly desired.

Transverse-focusing requirements to overcome space-charge forces are also severe. If provided by solenoid, magnetic fields as high as 6 T are required even for beam radii of 30 cm or more. Quadrupole-focusing systems appear to be feasible and with achievable magnetic fields at pole tips would result in smaller beam size than solenoids.

Detailed consideration of these space-charge forces will greatly influence the accelerator design. However, the general nature of the problem and requirements for accelerator modules is clear. The purpose of this note is to discuss the design of modules for such an accelerator.

INDUCTION LINAC PRINCIPLES

The basic induction-linac concept is illustrated in Fig. A7-1-1. A cylindrical core of magnetic material is excited by a primary winding from a modulator. Because of magnetic flux change in the

Fig. A7-1-1. Induction linac module. (XBL 7012-1107)

magnetic material, an induced electric field is generated across the accelerating gap. For a constant input voltage from the modulator, this same voltage will appear across the accelerating gap as long as the magnetic material remains unsaturated. It is possible to drive several magnetic cores in parallel from a common modulator, with a common secondary loop around the cores. The cores can be stacked either longitudinally along the beam line, or radially. Longitudinal stacking is economical in use of magnetic material but results in low accelerating gradient along the accelerator. Radial stacking preserves high accelerating gradient but requires more magnetic material.

Both radial stacking and longitudinal stacking of cores have been demonstrated in existing induction linacs. These stacking arrangements allow convenient transformation of accelerator gap voltage and impedance to match those of the modulator. The relationship is the same as for transformers. If N cores are stacked, the accelerator gap voltage is N times the modulator voltage. The impedance at the gap transforms like N^2 .

By proper packaging of conductors around the magnetic cores, stray inductance can be kept very small, essentially negligible in practice. Resistive losses, however, can be very large. These resistive losses result from eddy currents set up in the magnetic material. For cores wrapped from thin metal foil and for square input pulses, the current flowing into these eddy-current losses increases linearly with time throughout the pulse. An additional loss-current "step" is also observed in most metal-foil cores. This initial current step is not well understood, and in most practical situations it is not a dominant effect. For cores made of ferrite material, resistive losses are not large and for most applications can be ignored in approximate estimates of accelerator performance. On the other hand, ferrite materials have low flux swing and are relatively expensive.

The relationships governing the minimum amount of magnetic material required and the eddy-current resistive losses of magnetic cores are given below for magnetic cores driven by square-voltage pulses. For the heavy-ion induction accelerators considered here, ramped voltage pulses are required. For this case it is a good approximation to replace Vt by $\int V(t)dt$ and Vt by $\int V(t)dt$. Figure A7-1-2 illustrates the core geometry.

Referring to Fig. A7-1-2, the equations govern-

Fig. A7-1.2. Core geometry and a square-wave modulator pulse. (XBL 7612-11069)

ing the required magnetic material and eddy-current loads are given by Eqs. (1) to (4).

$$VT = \Delta B \cdot A \cdot S \frac{2}{1 + \sqrt{a/b}} \quad (1)$$

$$\frac{i_T}{V} = \frac{\pi}{2} \frac{b}{AS} \frac{d^2}{\rho} \frac{1 + \sqrt{a/b}}{2} \quad (2)$$

$$i_T = \frac{\pi}{2} \frac{b \Delta B}{T} \frac{d^2}{\rho} \quad (3)$$

$$i(t) = \frac{\pi}{2} \left(\frac{V}{AS} \right)^2 \frac{b d^2 t}{\Delta B \rho} \left(\frac{1 + \sqrt{a/b}}{2} \right)^2 \quad (4)$$

where

V = voltage applied to core

T = length of pulse (sec)

ΔB = flux swing in magnetic material (T)

A = cross-sectional area of core (m^2)

a, b = core outer and inner radii (m)

L = core length (m)

S = magnetic material packing factor

d = metal-foil thickness (m)

ρ = foil resistivity (Ωm)

t = time (sec)

Equation (1) defines the minimum cross-sectional area A of magnetic material for a given maximum flux swing ΔB . More magnetic material can be used, resulting in reduced eddy-current loading. If the actual flux swings and cross-sectional areas are used, Eqs. (2) to (4) are valid even if more magnetic material is used than the minimum required.

In practice a magnetic material packing factor $s = 0.8$ is achieved in realistic cores. By proper core design and subdivision the factor $2/(1 + \sqrt{a/b}) \approx 0.9$ can be achieved, where a and b now refer to dimensions of the individual core sub-elements. Foil thickness of 0.5–2 mils (1.27×10^{-5} to 5×10^{-5} m) are typically used. Available flux

swings ΔB depend upon the magnetic material used. For oriented-silicon (3.25%) steel or for low-carbon steel, $\Delta B = 3$ T can be realized. For Permalloy (50% Ni, 50% Fe), $\Delta B = 1.5$ T can be realized. For demonstrated ferrite materials, $\Delta B = 0.55$ T is realized and $\Delta B = 1$ may be possible. The resistivity of these materials varies greatly. Costs of foil materials are dominated largely by metallurgical properties that determine the difficulty of rolling thin foils. Hysteresis losses, which determine the energy required to reset the cores, also vary greatly among the different materials.

INDUCTION ACCELERATORS FOR HEAVY-ION BEAMS

Design of induction linacs for ion-pellet fusion is dominated by space-charge considerations. For a given ion energy and charge state, the maximum current that can be handled for focusing systems of a given peak field is limited and is less at lower energies. Thus if the total charge required in the pulse is fixed, the pulse length must be made longer at the lower energies. This is an essential point of the induction linac approach proposed by Keefe. For the case of a fixed accelerating gradient accelerator, Fig. A7-1.3 illustrates

Fig. A7-1.3. Variation of pulse length (T) along the accelerator. (XBL 7612-11077)

trates how pulse length and current vary along the accelerator. For this example, a total of 5 MJ energy per pulse at 24 GeV is assumed with 1-T maximum pole-tip field on the quadrupole focusing system. The pulse lengths and currents are limited as described above. These considerations are discussed in other reports and will not be expanded upon here.

In general it is clear from Fig. A7-1.3 that the most difficult area for the induction accelerator is the low-energy region (the first 5 to 10% of the machine). Here pulse lengths are long, resulting in need for considerable magnetic material for a given energy gain. Additionally, beam currents are low in this region, making achievement of good accelerator efficiency difficult. However, since this difficult region is a small fraction of the total accelerator, the influence on the total accelerator performance is not severe. For the very-low-energy region (the first 1 or 2% of the accelerator) the required pulse length increases very rapidly. At some point these pulse lengths may be

so large as to become impractical. Where this point occurs is not clear. Two-microsecond pulse-length cores with 0.5 MeV/m gradient have been successfully operated. Four-microsecond cores at this gradient are certainly possible. However, for longer pulse lengths, practical accelerator cores would require reduced gradient. In this low-energy region several shorter pulse-length accelerators in parallel feeding into the single accelerator at an appropriate energy, or some other alternate injection scheme, should be considered.

For most of the accelerator where the pulse length is less than 1 μ sec, accelerator design is dominated by cost trade-off and efficiency considerations. Many costs such as focusing systems, vacuum, shielding, pulsed power systems, etc., depend linearly upon accelerator length or number of accelerator modules. The outer dimensions of magnetic cores for a given pulse length depend linearly upon accelerating gradient, and core weight almost quadratically. Eddy-current loading and magnetic-material cost are rapid functions of foil thickness and material type. Any actual accelerator design will require a cost-optimization study to determine the most economical design. Such cost-trade-off studies have not been made at this point; however, it is reasonable to project average accelerator gradients of about 2.5 MeV/m over most of the accelerator, with about 1.5 MeV/m in the first 10 or so of the accelerator. A model accelerator core for 1- μ sec pulse length, average gradient of about 1.5 MeV/m, and 0.6-m-diam aperture for focusing was examined. The outer diameter of this core would be about 2.2 m. If made with 0.5 mil, 3.25% silicon-iron, the eddy current load seen at the accelerator gap would be about 500 A. If the outer radius of this core design were changed according to Eq. (1) for shorter pulse lengths, the core eddy current load at the gap would change according to Eq. (3) and would be about 50 A for 1- μ sec pulses. For this particular scenario an overall accelerator efficiency of about 14% could be achieved for charge state 3, and about 5% for charge state 4. For this particular example the magnetic material is efficiently used in the sense that the flux swing is about the maximum available. Greater accelerator efficiency can be achieved with use of excess magnetic material. For example, Eq. (2) shows that the eddy current impedance is independent of pulse length if the core geometry is held constant. For short pulses, where the magnetic material required per core is not great, this use of core iron than the minimum necessary allows improved accelerator efficiency without too large an increase in capital cost. Overall accelerator efficiency as large as 50% might be possible.

DISCUSSION

These investigations of induction linacs for ion-beam-pellet fusion indicate that accelerators of this type can certainly be built that satisfy the requirements imposed by the reactor designers. It appears also that accelerators of this type will have attractive features. The design is relatively straightforward, and the amount of beam manipulation is minimal. Good accelerator efficiency is projected for realistic magnetic materials. The accelerator involved, however, is very large, vastly larger than any induction linac built to date.

The largest induction linac in the U.S. has energy of about 6 MeV. In the projections considered here, an energy gain per accelerator module of 0.5 MeV is projected. This is twice as large as previously demonstrated. The accelerating gradient projected is 1.5 to 2.5 MeV/m, also larger than previously demonstrated, but consistent with projections of induction linacs for other applications. The accelerator efficiency that can be achieved is directly dependent on the eddy-current losses of the magnetic materials. Since 10,000 to 20,000 tons of magnetic material would be required for the accelerator, research, development or prove-out and to reduce the cost of these materials would have large payoff. About 20,000 modules would be required for the accelerator. Development of these modules with the required variable-impedance-pulse-line characteristics are necessary. Modular reliability will be the primary factor determining reliability of the accelerator.

RESEARCH AND DEVELOPMENT

A relatively obvious second program is called for. This program should include more detailed systems studies to understand the economic and technical trade-offs in accelerator design and indicate where improved technology would pay off. Development and construction of test accelerator modules suitable for different sections of the accelerator should be initiated and should be used to test and prove out different magnetic materials, pulse lines, and switches. Development of suitable ion-beam sources and careful beam-dynamics design are essential.

A second R&D phase should include development and construction of a first portion 50 to 100 m of the low-energy end of an ion-beam accelerator. Successful development and operation of such an accelerator would remove most of the technical uncertainty regarding the performance of a large accelerator designed for pellet ignition.

Reference

1. D. Keefe, Technical Summary 7 of this report.

APPENDIX 7-2 AN INDUCTION LINAC SYSTEM WITH A LOW-ENERGY ACCUMULATOR INJECTOR

Temp. 10/10/68

A possible scenario for using the induction linac: A serious problem is the space charge limit at low energy. This leads to examination of a long pulse, low-current injector for the induction linac, which is by nature a short-pulse accelerator. This arrangement is illustrated in Fig. A7-2-1.

Fig. A7-1-1. An injection system for an induction linac. (XBL 612-11096)

A plausible set of numbers that seems to strain the present understanding by only a modest amount at each stage is as follows: We adopt a 1 msec pulse at the input to the induction linac. This leads to a peak current of 100 A to get $6 \cdot 10^{14}$ particles. This peak current, 100 A, should be just within the space-charge limit of a storage ring at an energy of about 1 GeV. (The Maschke formula gives 330 A for $A = 200$, $E = 6$ cm/mrad, and $S = 1$ T.)

The next problem is to match the accumulator to an rf linac. We adopt 100 mA for the beam current in the rf linac. This requires a 1 msec pulse. To get 100 A for 1 msec out of the accumulator we adopt a 100-turn injection into the accumulator followed by rf bunching by a factor of 10. Injection will presumably be done using 10 turns into each plane. The circulation time at $E = 0.1$ is thus 10 μsec, which gives an orbit diameter of 100 m. All of these numbers appear to be within the state of the art; or, at most, a mild stretch. The emittance after accumulation would appear to be in the range 1 to 10 cm/mrad consistent with the final requirement.

The induction linac provides both acceleration and final-pulse compression. At a nominal gradient of 3 MV/m the linac length would be of the order of 10 km. Final beam current for a 10 nsec pulse: 10 kA. An obvious advantage of this scheme is the short overall cycle time, namely, 1.1 msec, which minimizes the losses.

Variation of parameters for the "minimal" case [1 MJ on target, 20 MJ/g density of deposited energy; $\Delta t = 6$ nsec (40% prepulse), and 100 Tb; $1.5 \cdot 10^{14}$ particles split into 2 beams at the end, with 1.2 kA per beam]. For these parameters the peak current out of the accumulator is reduced to 25 A after bunching and 2.5 A before bunching. The requirements on the rf linac are reduced to 50 mA peak for 500 μsec and the accumulator stores 50 turns. These parameters look straightforward compared to present technology.

In all of the above description a charge state of 1 is assumed. For charge-state 1, the peak current at the entrance of the induction linac is in the range of 25 to 100 A, where the linac modules are very inefficient. This leads one naturally to consideration of higher charge states. A charge of +3, for example, would give reasonable overall efficiency. The peak current at final energy and

pulse duration would be sufficiently intense that the induction linac should be designed for an output pulse length of about 100 nsec. Final bunching would then be accomplished by separating the linac beam into 3 or 4 beams for final transport.

APPENDIX 7-5

EFFECTS OF LONGITUDINAL ELECTRIC SELF-FIELD IN AN INDUCTION LINAC

A. Introduction

The longitudinal component of the electric self-field of the particles in an induction linac places certain requirements on the magnitude and time profile of the accelerating electric field. We estimate this requirement by first assuming that the accelerating electric field E_a at a distance s down the machine is of the form

$$E_a = E_0 \exp(-t/t_0) \quad (1)$$

in which t_0 is the time the center of the bunch reaches the point s . The electric self-field E_s is taken to be

$$E_s = -\frac{q}{4\pi\epsilon_0} \frac{\partial}{\partial s} \quad (2)$$

in which q is the charge/unit length in the beam. The quantity g is given by

$$g = \frac{(1 + 2 \ln(b/a))}{4\pi\epsilon_0} \quad (3)$$

with a the beam radius and b the radius of the conducting wall surrounding the beam. Gaussian units are employed.

In terms of the time $t = t - t_0$ and the total time of passage of the bunch L , we take $E_s(t)$ to be given by

$$E_s(t) = \frac{Q}{2\epsilon_0 L} \left[1 - \left(\frac{t}{L} \right)^2 \right] \quad (4)$$

in which Q is the total charge in the bunch and v_0 is the velocity of the particle that reaches the position s at the time $t = t_0$.

This approximate form is valid if the spread in velocities within the bunch is small. In Eq. (4) both v_0 and L may be functions of s . The time profile of the bunch is chosen for convenience.

Replacing the spatial derivative $\partial/\partial s$ by

we have

$$E_{\text{acc}} = \frac{1.6 \times 10^{-19}}{\sqrt{1 - \beta^2}} \quad (5)$$

comparing with Eq. (4) we see that, in order to compress the field E_{acc} we must have

$$E_{\text{acc}} > \frac{1.6 \times 10^{-19}}{\sqrt{1 - \beta^2}} \quad (6)$$

or, in other words, in order to compress the bunch longitudinal dimensions at times t the machine's E_{acc} must be correspondingly greater.

As a numerical example, we choose the total number of particles to be 1.5×10^{10} , and let $v_0 = 0.99c$ for 10^{-10} at $t = 0$. (The condition (6) is most stringent at full energy after the bunch has been compressed.)

We consider first a charge state of one and $t = 10^{-10}$ sec, and obtain

$$E_{\text{acc}} > \frac{1.6 \times 10^{-19}}{10^{-10}} = 1.6 \times 10^9 \text{ V/m} \quad (7)$$

The accelerating electric field must be rising at the rate of 50 kV/m/nsec in order to overcome the electric self-field of the bunch.

As a somewhat more practical example we consider a charge state of 3 and $t = 90^{-10}$ sec, and obtain

$$E_{\text{acc}} > \frac{1.6 \times 10^{-19}}{90^{-10}} = 1.8 \times 10^9 \text{ V/m} \quad (8)$$

It must be emphasized that this calculation is useful only in estimating the effect of the longitudinal electric self-field. A self-consistent calculation is required to determine the accelerating field E_{acc} required to accomplish the desired compression.

APPENDIX 5.1
PLASM TRANSPORT IN THE TARGET CHAMBER

Dr. H. J. Taylor, 201 West 4th St., St. Louis

One of the most crucial problems in ion beam inertial fusion is the stable transport and focusing of high-powered ion beams in the target chamber. There exists the danger that the propagation of extremely well focused beams in the residual plasma of the target chamber is subject to a "kink" instability, which causes the beam trajectory to shake and thereby miss the target.

WEIGHT FLASH

After a microexplosion we expect the target chamber to be filled with plasma of electron density n_e (cm^{-3}) and temperature T_0 (eV). If R is the radius of the chamber in cm, κ is the heat conductivity, then the cooling time t_{cool} of this plasma is approximately

$$= 3.2 \times 10^{-11} \text{ g cm}^{-2} \text{ s}^{-1} = 3.2 \times 10^{-9} \text{ kg m}^{-2} \text{ s}^{-1}$$

If we let $\beta = 10^7$ cm, in $t = 10$, $z = 1$ for the
substituted gas in the chamber, that

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Then the plasma will cool rapidly until it reaches $T_e = 1$ eV for a background density of 10^{21} cm⁻³. For the most part we shall assume these parameters prior to the injection of a new batch of ions.

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During a typical beam pulse of 10 nsec the background plasma ions move a distance

$$T_{\text{eff}} = 10^{-1} \left(\frac{f_{\text{eff}}}{f_{\text{ref}}} \right)^{1/2}$$

where r_0 is the beam radius, $T_0 = T_p$ is the ion temperature, and k is the atomic weight. Thus the plasma ions are practically immobile and we neglect their dynamics. Let T or electrons be described by a simple Ohm's law.

1. 2. 3. (3)

where j is the electron current, σ the plasma conductivity, and E the electric field. From Langmuir's and Faraday's laws and (1) we obtain

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where j_0 is the injected ion beam current density. From Eq. (4) it is clear that for $t \gg \tau_0 = \pi^2 \epsilon_0^2 / 4j_0^2$, the magnetic diffusion time,

$$j = j_0 = 0 \quad (15)$$

i.e., there is no net current and therefore no magnetic field at infinity. Above detailed analysis would indicate the following additional result(s):

$$f_{\alpha} = \frac{1}{2} \left(\frac{1}{\alpha} + \frac{1}{1-\alpha} \right) \quad \text{for } 0 < \alpha < 1$$

1. *Phragmites* (common)

where n_0 is the beam density and ω_{pe} is the electron plasma frequency. These conditions are quite likely to be satisfied at the injection point, as the beam propagates and focuses, it diffuses out into the surrounding medium on the time scale of τ_d , giving rise to B_0 , as shown in Fig. 3A-1-1. This diffusion time for the return current is

Fig. 14-14. Schematic of ion beam propagation through anode plane. (a) $V_0 = 10$ kV.

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2000年12月20日，在“2000年中国城市竞争力”会议上，中国城市竞争力研究会会长、清华大学教授李俊成指出，中国城市竞争力研究会成立三年来，一直致力于中国城市竞争力的研究，并出版了《中国城市竞争力报告》。该报告指出，中国城市竞争力在2000年有所提高，主要表现在以下几个方面：一是城市经济竞争力增强，二是城市社会竞争力提高，三是城市环境竞争力改善。

$$x_{12} = \left(\frac{1}{2} + \frac{(22)}{2} \right)^{\frac{1}{2}} = \left(\frac{22}{2} \right)^{\frac{1}{2}}$$

2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023 2024 2025 2026 2027 2028 2029 2030 2031 2032 2033 2034 2035 2036 2037 2038 2039 2040 2041 2042 2043 2044 2045 2046 2047 2048 2049 2050 2051 2052 2053 2054 2055 2056 2057 2058 2059 2060 2061 2062 2063 2064 2065 2066 2067 2068 2069 2070 2071 2072 2073 2074 2075 2076 2077 2078 2079 2080 2081 2082 2083 2084 2085 2086 2087 2088 2089 2090 2091 2092 2093 2094 2095 2096 2097 2098 2099 2100 2101 2102 2103 2104 2105 2106 2107 2108 2109 2110 2111 2112 2113 2114 2115 2116 2117 2118 2119 2120 2121 2122 2123 2124 2125 2126 2127 2128 2129 2130 2131 2132 2133 2134 2135 2136 2137 2138 2139 2140 2141 2142 2143 2144 2145 2146 2147 2148 2149 2150 2151 2152 2153 2154 2155 2156 2157 2158 2159 2160 2161 2162 2163 2164 2165 2166 2167 2168 2169 2170 2171 2172 2173 2174 2175 2176 2177 2178 2179 2180 2181 2182 2183 2184 2185 2186 2187 2188 2189 2190 2191 2192 2193 2194 2195 2196 2197 2198 2199 2200 2201 2202 2203 2204 2205 2206 2207 2208 2209 2210 2211 2212 2213 2214 2215 2216 2217 2218 2219 2220 2221 2222 2223 2224 2225 2226 2227 2228 2229 2230 2231 2232 2233 2234 2235 2236 2237 2238 2239 2240 2241 2242 2243 2244 2245 2246 2247 2248 2249 2250 2251 2252 2253 2254 2255 2256 2257 2258 2259 2260 2261 2262 2263 2264 2265 2266 2267 2268 2269 2270 2271 2272 2273 2274 2275 2276 2277 2278 2279 2280 2281 2282 2283 2284 2285 2286 2287 2288 2289 2290 2291 2292 2293 2294 2295 2296 2297 2298 2299 2300 2301 2302 2303 2304 2305 2306 2307 2308 2309 2310 2311 2312 2313 2314 2315 2316 2317 2318 2319 2320 2321 2322 2323 2324 2325 2326 2327 2328 2329 2330 2331 2332 2333 2334 2335 2336 2337 2338 2339 2340 2341 2342 2343 2344 2345 2346 2347 2348 2349 2350 2351 2352 2353 2354 2355 2356 2357 2358 2359 2360 2361 2362 2363 2364 2365 2366 2367 2368 2369 2370 2371 2372 2373 2374 2375 2376 2377 2378 2379 2380 2381 2382 2383 2384 2385 2386 2387 2388 2389 2390 2391 2392 2393 2394 2395 2396 2397 2398 2399 2400 2401 2402 2403 2404 2405 2406 2407 2408 2409 2410 2411 2412 2413 2414 2415 2416 2417 2418 2419 2420 2421 2422 2423 2424 2425 2426 2427 2428 2429 2430 2431 2432 2433 2434 2435 2436 2437 2438 2439 2440 2441 2442 2443 2444 2445 2446 2447 2448 2449 2450 2451 2452 2453 2454 2455 2456 2457 2458 2459 2460 2461 2462 2463 2464 2465 2466 2467 2468 2469 2470 2471 2472 2473 2474 2475 2476 2477 2478 2479 2480 2481 2482 2483 2484 2485 2486 2487 2488 2489 2490 2491 2492 2493 2494 2495 2496 2497 2498 2499 2500 2501 2502 2503 2504 2505 2506 2507 2508 2509 2510 2511 2512 2513 2514 2515 2516 2517 2518 2519 2520 2521 2522 2523 2524 2525 2526 2527 2528 2529 2530 2531 2532 2533 2534 2535 2536 2537 2538 2539 2540 2541 2542 2543 2544 2545 2546 2547 2548 2549 2550 2551 2552 2553 2554 2555 2556 2557 2558 2559 2560 2561 2562 2563 2564 2565 2566 2567 2568 2569 2570 2571 2572 2573 2574 2575 2576 2577 2578 2579 2580 2581 2582 2583 2584 2585 2586 2587 2588 2589 2590 2591 2592 2593 2594 2595 2596 2597 2598 2599 2600 2601 2602 2603 2604 2605 2606 2607 2608 2609 2610 2611 2612 2613 2614 2615 2616 2617 2618 2619 2620 2621 2622 2623 2624 2625 2626 2627 2628 2629 2630 2631 2632 2633 2634 2635 2636 2637 2638 2639 2640 2641 2642 2643 2644 2645 2646 2647 2648 2649 2650 2651 2652 2653 2654 2655 2656 2657 2658 2659 2660 2661 2662 2663 2664 2665 2666 2667 2668 2669 2670 2671 2672 2673 2674 2675 2676 2677 2678 2679 2680 2681 2682 2683 2684 2685 2686 2687 2688 2689 2690 2691 2692 2693 2694 2695 2696 2697 2698 2699 2700 2701 2702 2703 2704 2705 2706 2707 2708 2709 2710 2711 2712 2713 2714 2715 2716 2717 2718 2719 2720 2721 2722 2723 2724 2725 2726 2727 2728 2729 2730 2731 2732 2733 2734 2735 2736 2737 2738 2739 2740 2741 2742 2743 2744 2745 2746 2747 2748 2749 2750 2751 2752 2753 2754 2755 2756 2757 2758 2759 2760 2761 2762 2763 2764 2765 2766 2767 2768 2769 2770 2771 2772 2773 2774 2775 2776 2777 2778 2779 2780 2781 2782 2783 2784 2785 2786 2787 2788 2789 2790 2791 2792 2793 2794 2795 2796 2797 2798 2799 2800 2801 2802 2803 2804 2805 2806 2807 2808 2809 2810 2811 2812 2813 2814 2815 2816 2817 2818

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

where $\beta_0 = \beta_1 = \beta_2 = 0$.

THE INTRODUCTION

After the return current has diffused out and B_z has been established, the beam satisfies the conditions for the classic kink instability whose e-folding growth rate γ is approximately

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where $k = 2\pi/\lambda$, λ is the wavelength of the perturbation and

$$\frac{3}{4} \pi r^2 \cdot \frac{1}{2} \pi r^2$$

where γ is the relativistic factor, B_1 is measured on the beam surface.

2. 1. 1950

The consequences of such fields for beam propagation are difficult to assess. A simple calculation shows that a 100 GeV fully stripped U ion would be deflected about 0.1 cm by a 1 G field over a path length of 10^3 cm. However, the effect of electrons inside the beam deriving from the ambient plasma is that this E-field is probably shielded out from the beam interior so that the beam-deflection effect should not be as large as this. This requires further study.

APPENDIX 8.2 NEUTRALIZATION IN THE TARGET CHAMBER

$$kT \ll \frac{1}{2} m_e v_b^2$$

The plasma frequency is $\omega_p = 9 \times 10^3 n_e^{1/2}$, where n_e is the electron density (cm^{-3}). Hence at any electron density above $5 \times 10^{11} \text{ cm}^{-3}$ (corresponding to 10^{-3} Torr) the time scale for electrostatic neutralization is short compared with pulse times. The electron density required to neutralize 5000 A of singly charged U in a 100 cm² entering cross section is $2 \times 10^{12} \text{ cm}^{-3}$. At the pellet, the density is 10^3 times higher, so we need a gas pressure of order at least 10^{-3} Torr to provide enough electrons unless they come from far away, which they may well do. The conclusion is that with gas in the chamber above 10^{-3} to 10^{-1} Torr, and perhaps even lower, the beam will be electrostatically neutral if the gas is ionized.

The ionization rate per beam particle is approximately

$$\frac{1}{T_{\text{pair}}} \sim \frac{dI}{dx} = 1.3 \times 10^{-13} \frac{n_g Z^2}{Z_b} \text{ ion pairs/cm,}$$

where n_g and Z are density (cm^{-3}) and atomic number of the gas, and Z_b and A_b are charge and v/c of the beam particle. For $Z_g = 1$, $n_g = 3 \times 10^{11}$ (10^{-3} Torr), $Z_b = 1/2$, $Z_b = 1$, we get 1.5×10^{-13} ion pairs cm^{-1} , and the beam will not ionize the gas. For $Z_b = 92$, or for any other combination of $n_g Z_g^2$ that increases the ionization rate by 10^3 , the beam will produce enough ions to neutralize itself. Of course, the gas may be ionized already before the beam particles arrive.

At a cross section of 10^{-12} cm^2 , the collision rate for an incident beam particle in a gas of density $3 \times 10^{11} \text{ cm}^{-3}$ is 0.003 cm^{-1} , or about 3 collisions in 10 m. Hence at pressures below about 10^{-3} Torr the incident beam probably retains its charge state, while at pressures above about 10^{-3} Torr the beam may be fully ionized before it reaches the target. This calculation clearly needs doing with the correct cross sections.

When the beam enters the target chamber, the inductive and space charge fields will result in nearly complete current neutralization in a time of the order of a plasma period. As pointed out in Appendix 8.1, the neutralizing electron current will decay on a resistive time scale. The final state, however, will be determined by balancing the drag forces of the beam and of the gas ions.

The drag force due to Coulomb collisions on the electrons due to gas ions is

$$\vec{F}_K = - \frac{4\pi}{3} e^2 \sum_K n_K \langle v_K^2 \rangle \frac{\vec{v}_e}{v_K},$$

where n_K is the gas atom density, and

$$\langle v_K^2 \rangle = \frac{1}{2} \text{Debye}^{-2} \frac{\vec{v}_e \cdot \vec{v}_K}{v_K^2},$$

Angle brackets denote averages over the velocity distribution. Taking $v_K \ll v_e$ we get, approximately

$$\vec{F}_K = - \frac{4\pi}{3} e^2 \sum_K n_K \langle v_K^2 \rangle \frac{\vec{v}_e}{v_K},$$

where v_{th} is the electron thermal velocity. This time the beam drags the electrons with a force

$$\vec{F}_b = \frac{4\pi}{3} e^2 n_b n_e \langle v_b^2 \rangle \frac{\vec{v}_b}{v_b} = \frac{4\pi}{3} e^2 n_b n_e \frac{\vec{v}_b}{v_b},$$

where

$$n_b = \frac{1}{2} \text{Debye}^{-2} \frac{\vec{v}_b \cdot \vec{v}_b}{v_b^2},$$

we assume $v_e \gg v_b$ and set $\vec{F}_b + \vec{F}_K = 0$ to obtain

$$\vec{v}_e = \frac{\vec{v}_b}{\sum_K \frac{n_K}{n_e} \frac{m_b v_b}{m_e v_K}} \frac{v_{th}}{v_b},$$

The electron current is

$$\vec{J}_e = -en_e \vec{v}_e = -\sum_K n_K n_b \frac{\vec{v}_b}{v_K},$$

we assume $n_b n_b = \frac{1}{2} n_K n_b$ and obtain

$$\vec{J}_e = -\sum_K \frac{n_K}{n_e} \frac{m_b v_b}{m_e v_K} \frac{\vec{v}_b}{v_b} = -\sum_K \frac{n_K}{n_e} \frac{\vec{v}_b}{v_K},$$

where

$$\vec{J}_b = \sum_K n_K v_b \frac{\vec{v}_b}{v_b}$$

is the beam current. The effect of drag forces is therefore to take the electron current $\vec{J}_e$, which is initially $-\vec{J}_b$, approach the value given above on the resistive time scale. The ratio

only $\sim 10^{-1}$ of order unity. If $v_b \sim c/2$, and the electron temperature is 1 eV , we have

$$\frac{v_{th}}{v_b} \approx 10^{-2}$$

and β_e becomes negligible. The beam current is then unneutralized, and may even be increased by a factor Z_b , the beam atomic number, if an initially singly ionized beam becomes fully ionized on the background gas. On the other hand, if the electron temperature is 10^3 keV , $v_{th} \sim v_b$, and $\beta_e \approx (Z_b/T_e T_b)^{1/2}$, the beam is then over neutralized by a factor of 91 for a U beam in an H or D plasma.

The electron gain energy from the beam at a rate determined approximately by

$$\frac{dv_{th}}{dt} = \frac{8\pi e^2 n_b n_b^0}{n_b^0} \ln^2 b$$

The time τ required to reach $v_{th} = v_{th}^0$ is then

$$\tau = \frac{v_{th}^0}{8\pi e^2 n_b n_b^0 \ln^2 b} = \frac{v_{th}^0}{n_b^0} \approx \frac{1}{n_b^0}$$

Putting $n_b = 1.8 \times 10^{17} \text{ cm}^{-3}$, $\ln^2 b = 10$, we get

$$\tau = 1.7 \times 10^{-15} \frac{v_{th}^0}{v_b} \text{ sec}$$

For $Z_b = 1$, $v_{th}^0 = 1/2$, $n_b^0 = 10^{17} \text{ cm}^{-3}$ (beam density at entrance) we get $\tau = 2 \times 10^{-15} \text{ sec}$. Even at the pellet, $n_b = 10^{17} \text{ cm}^{-3}$, $\tau = 2 \times 10^{-15} \text{ sec}$ is still much longer than the pulse time, so the electrons will not heat to $v_{th} = v_b$. However, if $Z_b/T_e = 92$, and $v_{th}^0/v_b = 1/92$, we have $v_{th}^0 = 0.1$, and near the pellet we get $\tau = 2 \times 10^{-14} \text{ sec}$, so the electrons may be hot enough near the pellet to maintain neutralization of the beam.

We conclude that the net beam current for U^{92} into deuterium gas may be between .92 and .92 times the entering current, but probably between .92 (unneutralized, fully stripped to over-neutralized by a small factor times the entering current).

We note finally that the pellet-corona expansion velocity is about 10^3 cm/sec ,¹ so even in $30 \times 10^{-15} \text{ sec}$, the beam expands only 0.3 cm. The pellet corona therefore will not significantly affect the beam focusing.

Reference

1. H. Nuckolls, Technical Summary 1, this report.

APPENDIX 8-3 KINK MODE IN A RESISTIVE MEDIUM

Edward Lee

An important concern is the stability of the ion beam with respect to the kink (or hose) mode during passage from accelerator to pellet. This is especially so when the density of the intervening plasma is sufficient to strip the ions to a high charge, thereby producing a short scale length for mode growth. Here I treat the case of a medium described only by scalar conductivity, which is usually a valid approximation if the electron-ion collision frequency is greater than the electron cyclotron frequency in the beam-induced field.

To model the kink mode, let

$Y(z,t)$ = transverse displacement of the beam from the intended axis of propagation

$D(z,t)$ = transverse displacement of the center of the pinch field from the axis of propagation.

Then the equation of motion for $Y(z,t)$

$$\frac{d^2 Y}{dt^2} = -\frac{1}{\tau^2} (D - Y)$$

where the betatron frequency is

$$\frac{1}{\tau^2} \approx \frac{2Z_b e^2 I}{m c^2 r_b^2} \quad (2)$$

Z_b is the charge state of the beam, r_b is beam radius, and I is net current.

The current satisfies the diffusion-type equation

$$\tau \frac{\partial I}{\partial t} + I = I_b \quad (3)$$

where I_b is beam current and

$$\tau = \frac{4\pi a^2}{c^2} \quad (4)$$

is the resistive diffusion time.

The field displacement D satisfies a diffusion equation similar to Eq. (3):

$$\tau \frac{\partial D}{\partial t} + (D - I) = I_b \quad (5)$$

To solve Eqs. (1) to (5), it is convenient to use the variable

$$x = \beta c t + z \quad (6)$$

instead of t . Then we have the transformations

$$\frac{d^2}{dt^2} = (\beta c)^2 \left(\frac{\partial^2}{\partial z^2} \right) \quad (7)$$

$$\left(\frac{\partial}{\partial t}\right)_z = \lambda \left(\frac{\partial}{\partial x}\right)_z, \quad (8)$$

Assuming a single Fourier mode in z :

$$Y_1(z) = e^{ik_z z}, \quad (9)$$

Eq. (1) gives

$$0 = \frac{k_z^2 - \omega^2}{k_B^2} Y, \quad (10)$$

where $k_B^2 = \frac{1}{2}(\partial/\partial z)^2$. Substituting this relation in Eq. (5) yields

$$\lambda \frac{\partial}{\partial x} \left(\frac{k_z^2 - \omega^2}{k_B^2} Y \right) + \frac{k_z^2 - \omega^2}{k_B^2} Y = I_b Y, \quad (11)$$

with $\lambda = \partial/\partial t$. Noting that $k_z^2/\omega^2 = \text{const}$, Eq. (11) is used to simplify Eq. (11) to get

$$\lambda \frac{\partial Y}{\partial x} + \lambda \frac{1}{k_B^2} \frac{\partial Y}{\partial x} - \lambda \frac{1}{k_B^2} Y = 0, \quad (12)$$

or

$$\frac{1}{Y} \frac{\partial Y}{\partial x} = \frac{1}{\lambda} \frac{\partial}{\partial x} \left(\frac{\omega^2 - k_z^2}{k_B^2} \right), \quad (13)$$

The essential scale lengths for unstable growth are revealed in Eq. (13). For real ω , the displacement Y increases exponentially with growth length ~ 1 backward in the beam frame. Thus we want a pulse to be not much longer than 1 . In general there is growth simultaneously in x and z . It is important to note that the resonant denominator in Eq. (12) must actually be spread by anharmonic effects. That is, the current profile produces particle orbits with a broad spread of k_z ; this yields damping of the mode via phase mixing. We find

$$\text{Re} \left\langle \frac{\omega^2 - k_z^2}{k_B^2 - \omega^2} \right\rangle = 1, \quad (14)$$

$$\text{Im} \approx \frac{1}{2} k_B^2$$

The scaling of growth in z with k_z is especially important because it shows that current neutralization overcomes the adverse effect of stripping, at least in principle. At the pulse head there is no sideways force on the particles because $I = 0$. If λ is large enough, I remains low throughout the pulse and there is no problem with beam displacement. Heating of the plasma by the beam should help in this.

APPENDIX 8.4 CURRENT NEUTRALIZATION IN PLASMA TRAVELING GAS BACKGROUND

(continued)

It is desired to estimate the degree to which a high energy ion beam remains current neutralized as it passes through, heats, and ionizes a gas background. Consider a fixed volume of gas through which the beam passes. Assume a step function beam front followed by a constant beam current. The temperature increase of the gas, or equivalently, the temperature variation from the front to back of the beam is calculated. An integration of temperature during the pulse length then yields an estimate of the degree to which the magnetic field diffuses into the newly created plasma region.

For an ion beam in charge state q_b traversing a gas of electron density N_g with velocity v_b , the stopping power of the medium is

$$\frac{dE}{dx} = \frac{4\pi e^2}{m_e v_b^2} N_g \ln \left(\frac{v_b}{v_e} \right) = \frac{4\pi}{N_g} N_g \ln \left(\frac{v_b}{v_e} \right)$$

in erg/cm or

$$\frac{dE}{dx} = 458 \frac{N_g}{v_b} \ln \left(\frac{v_b}{v_e} \right) \text{ eV/cm} \quad (1)$$

The power volume available for heating the gas is then

$$P = I \frac{dE}{dx}$$

where I is the beam particle flux equal to

$$I_b = \frac{I_b}{v_b} = \frac{I_b}{v_b} \frac{1}{v_b} = \frac{I_b}{v_b^2}$$

I_b being the beam current and r_b its radius; therefore

$$P = 9.1 \times 10^{12} \frac{I_b}{v_b^2} N_g \ln \left(\frac{v_b}{v_e} \right) \frac{I_b}{r_b^2} \text{ eV/sec/cm}^2 \quad (2)$$

where I_b is measured in amperes. Since this power increases the gas internal energy, $dE/dt = P$, where

$$E_g = \frac{3}{2} (1 + \frac{1}{Z_g}) N_g T_g + N_g x_g \text{ eV/cm}^3 = C_g N_g T_g$$

where $N_g = \frac{3}{2} N_b$, T_g is the gas temperature in eV and Z_g is its ionization level. x_g is the energy required to strip an ion to level Z_g , and

$$C_g = \frac{3}{2} (1 + \frac{1}{Z_g}) + \frac{x_g}{T_g}$$

Then

$$\frac{dT_g}{dt} = 9.1 \times 10^{12} \frac{I_b}{v_b^2} \ln \left(\frac{v_b}{v_e} \right) \frac{I_b}{r_b^2} \text{ eV/sec,} \quad (3)$$

where it has been assumed that $d/dt \approx v_p$ (1.0 cm/ns). Alternatively, for fixed time, the variation of i_b along the beam can be written

$$\frac{di_b}{dx} = - \frac{di_b}{dt} \approx -e n_p v_p$$

For certain cases, ionization potentials for the first few levels can be approximated by $e_p \approx 2\phi$ where ϕ is the binding energy associated with the $1 \rightarrow 1 + \frac{1}{2}$ transition and e_p varies with the case. In the $1 \rightarrow 2$ case

$$e_p \approx \frac{13.6 \text{ eV}}{2} \approx 6.8 \text{ eV}$$

for the low background densities under consideration, there is insufficient time to ionize to equilibrium n_p . It is assumed that straggling up to a level where $e_p \approx i_b$ is achieved so that

$$i_b \approx \frac{1}{2} n_p v_p$$

With the above approximations, $e_p \approx 1$ and

$$\frac{di_b}{dx} = - \frac{1}{2} n_p v_p \approx - \frac{i_b}{x} \quad \text{and} \quad \frac{1}{i_b} \frac{di_b}{dx} \approx - \frac{1}{x} \quad \text{eV} \quad \text{sec}^{-1} \quad (4)$$

Since the temperature variation is known, the variation of net current throughout the beam pulse can be determined from

$$i_b \frac{dT_b}{dt} = \frac{1}{2} n_p v_p T_b \quad (5)$$

where

$$T_b = \frac{1}{2} \left(1 + \frac{3}{2} \frac{i_b^2}{n_p^2 v_p^2} \right)$$

for plasma densities and temperatures of interest. The solution of i_b in (5) with $1 \rightarrow 2$ at $t = 0$ is

$$\frac{1}{i_b} = 1 + \exp \left(\int_0^t \frac{dT_b}{T_b} \right)$$

with $i_{b0} = 1 + 1 + \frac{3}{2} \frac{1}{n_p^2 v_p^2}$ for t_b and i_{b0} determined by Eq. (4). Performing the indicated integration over time

$$\frac{1}{i_b} = 1 + \exp \left[\frac{1.36 \times 10^{-5} V_b^{1/2}}{e_p n_p v_p (\text{and } i_b)^{1/2}} \right] \quad (6)$$

As an example, choose $V = 1.5 \times 10^{11}$ cm/sec, $\phi_p = 90$, $e_p = 16$ eV (appropriate for argon), $\ln 5 \approx 1.6$. Then

$$\frac{1}{i_b} = 1 + \exp \left[\left(0.55 \times 10^{-5} \frac{t}{i_b^{1/2} V_b^{1/2}} \right)^{1/2} \right] \quad (7)$$

with a temperature variation

$$T_b = 840 \frac{i_b}{V_b} t \quad \text{eV} \quad (8)$$

These variations with time refer to a particular fixed volume through which the beam passes, i_b and T_b can be considered as well to be functions of x .

The distance from the beam front) and the following geometry with the help of Fig. AS-1-1.

Fig. AS-1-1. OBI 7612-11-9-6

From Fig. AS-1-1,

$$T_b(x) = \frac{840}{V_b} i_b(x) x$$

with the identification, $x = 3t$ in Eqs. (7) and (8), for $x = 10$ cm and $t = 10^{-8}$ s,

$$\frac{1}{i_b(x)} = 1 + \exp \left[\frac{35 \times 10^{-5}}{i_b(x)^{1/2} V_b^{1/2}} \right] \quad (9)$$

$$T_b(x) = 5.6 \times 10^{-5} \frac{i_b(x)}{V_b} x \quad \text{eV} \quad (10)$$

with $x = 0.100$ cm for a typical beam.

Consider now two numerical examples. The first example corresponds to that of 1-of-100 five kA beams converging on the target. The second is the total energy incident in the form of one 500 kA beam.

From the equations, the largest net current and plasma temperature occur at $x = 0$, i.e., a distance within the beam equal to the separation of pellet from beam front. At this location

$$\frac{1}{i_b^{\max}} = 1 + \exp \left(\frac{18}{V_b^{1/2} i_b^{\max 1/2}} \right)$$

and

$$T_{b\max} = 1.4 \times 10^{-5} \frac{i_b}{V_b} \quad \text{eV}$$

Figure AS-1-2 shows the variation of T_b and i_b with x for small values of i_b and $V_b = 5$ kA. Very close to the pellet where i_b is small, i_b is insufficient to heat the plasma to temperatures required to efficiently exclude the magnetic field. Further back in the beam, the temperature is low but the beam radius is sufficiently large to main-

Fig. AB-4-2. (XBL 7612-11082)

tain good neutralization. At distances greater than about 10 cm, values of I_b and I are most probably determined by energy remaining from a previous pulse rather than the weak beam heating. The degree of current neutralization, taking the ambient background conditions into account, will be higher than determined here for regions more than a few cm from the pellet.

In summary, a 5 kA beam will probably be well neutralized except within a few cm of the pellet, provided the beam enters an approximately 1- μ or better plasma. Figure AB-4-2 shows I/I_b and T_b for

Fig. AB-4-3. (XBL 7612-11084)

a single 500 kA beam and two small values of t . Again, only within 1 cm of the pellet is non-neutralization of current appreciable. However, even there, the high beam current maintains neutralization to better than 90%. Neutralization is better than 99% at distances greater than 10 cm. Limiting values for T_b and I/I_b can be obtained from the above equations for other values of beam current.

Non-noble and low-atomic-number gases will have T_b and I/I_b values that (because of different ionization characteristics and specific heats) might vary on either side of the calculated values by factors of two. Beams of low-atomic-number ions

will experience similar neutralization characteristics and slightly lower plasma temperatures.

Ambient conditions in the chamber are related with a previous event, and I_b and I are sufficient to maintain good neutralization far from the pellet. Within about 1 cm from the pellet, good neutralization results from beam heating, except within 1 cm of the pellet when low current beams are employed.

SUMMARY

Ambient conditions are sufficient to ensure good charge and current neutralization at beam front to pellet distances in excess of a few centimeters. Provided the ambient state is characterized by $T_b \approx 10^4$ eV, close to the pellet, beam heating of the plasma is usually sufficient to maintain good current neutralization except for a low current beam in the very late vicinity of the pellet.

If $t > 0.1$ cm, the distance of the beam front from the pellet, the maximum plasma temperature and net current occur a distance x within the beam. These values are given approximately by

$$x_{max} = 1.1 \times 10^{-3} \left(\frac{I_b}{I} \right)^{1/2} \quad (1)$$

$$\frac{I_{max}}{I_b} = \frac{1}{\sqrt{t}}$$

with x in cm and I_b in amperes.

APPENDIX A-3 PROPAGATION OF ENERGETIC ION BEAMS THROUGH A VACUUM CHAMBER

$$v = 0.6 \text{ cm} \cdot \text{pellet}^{-1} \cdot \text{beam}^{-1/2}$$

INTRODUCTION

In order to realize pellet fusion using heavy ion beams as an igniter, it is necessary to propagate a beam of heavy, slightly ionized energetic ions through a chamber of ~ 10 m radius and strike a target of $r \sim 0.1$ cm. A typical case is as follows: Beams of singly charged H^+ at 70 kV with currents of ~ 3000 A, initial radius of ~ 10 cm, and a pulse length of $t \sim 10$ nsec. The length of the beam is then $\sim vt \sim 0.65 \times 10^{-4} \times 200$ cm.

These notes are primarily concerned with the stability of such beams, but a few notes on equilibrium must first be made.

Note that the beam velocity v/c follows from

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$$r = r_0 + 1.1 \cdot 10^{-4} \cdot z,$$

$$N(r) = \left[1 - \left(\frac{r-r_0}{1.1 \cdot 10^{-4}} \right)^2 \right]^{1/2} \cdot N_0,$$

and for the case $r = 10$ cm, $r_0 = 0.1$ cm, $N_0 = 10^{17}$ cm⁻³, and $z = 1.1 \cdot 10^{-4}$ cm. The corresponding value of $N(r) = 1.1 \cdot 10^{17}$ is the number of particles per unit length of the beam.

$$1.1 \cdot 10^{17} \cdot \frac{1}{1.1 \cdot 10^{-4}} = 10^{21} \text{ cm}^{-2}, \quad N_0 = 1.1 \cdot 10^{17} \cdot \frac{1}{1.1 \cdot 10^{-4}} = 10^{21} \text{ cm}^{-2}$$

for similar values of r_0 at $z = 0$.

$$N_0 = 1.1 \cdot 10^{21} \text{ cm}^{-2},$$

or for $1.5 \cdot 10^{-4}$ cm

$$N_0 = 1.1 \cdot 10^{21} \cdot 1.5 \cdot 10^{-4} = 1.65 \cdot 10^{21} \text{ cm}^{-2}.$$

A useful dimensionless quantity frequently used in the following is $N_0 e^2 / M_0^2 = 1.5 \cdot 10^{-12} \cdot Z^2$ for these beam parameters. Bulker's parameter for our parameters is $N_0 e^2 / M_0^2 = 0.1 \cdot 10^{-12}$. The total beam energy available for heating the target is $11.4 \cdot 10^{-12} \cdot 10^{21} = 1.14$ eV.

Since the beam radius decreases from $r = r_0$ to $r = r_f$ as the beam moves from $z = 0$ to $z = z_f$

$$r = r_f + (r_0 - r_f) \frac{z}{z_f},$$

$$r = r_f + r_0 \frac{z}{z_f},$$

and the beam density

$$n = \frac{N_0}{\pi r^2} = \frac{N_0}{\pi [r_f + r_0 (z/z_f)]^2}.$$

For $r_0 = 10$ cm and $r_f = 0.1$ cm this becomes

$$n = 3 \cdot 10^{21} \cdot (1 + 0.1z)^{-2} \text{ cm}^{-3}.$$

If the chamber contains a neutral gas, we note that the cross section for ionization of the beam is $\sim 10^{-17}$ cm² and the mean free path for beam ionization

$$\lambda_i = \frac{1}{n_0 \sigma_i} = \frac{10^{17}}{n_0} \text{ cm},$$

If this is $> 10^7$ cm, then the beam can proceed without secondary ionization. This requires a background density $n_0 < 10^{10}$. The cross section for ionization of the background gas by the beam is $\sim 10^{-19}$ cm², hence the degree of ionization induced in the gas by the beam, from

$$\frac{dn_0}{dt} = -n_0 [n_B \sigma_B]$$

$$z (\log n_0) = -n_B \sigma_B z = -n_B \sigma_B z_B$$

$$z = -0.6 \left(1 + \frac{r_0}{r_f} \frac{z}{L} \right)^{-2}$$

and at $z = 0$ ($r = r_0$)

$$\frac{n_1}{n_0} = 0.55$$

and the gas at r_0 is about 60% ionized. At $z = 10$ cm, $\log n_0 = -0.25$ and the gas is $\sim 15\%$ ionized. Thus only in the last 10 cm of its track does the beam ionize the background gas.

We may then consider two different propagation problems: first, the vacuum case, in which the beam travels through a neutral gas with no electrical effects; second, the plasma case, in which the chamber is filled with a background plasma. In the latter case the filling density exceeds $n_0 = 10^{10}$, and in the latter case we will consider the plasma as produced by a thermonuclear burn that occurred 1 sec before. The plasma temperature is then ~ 1 eV, and the gas is effectively fully, but only singly, ionized.

THE VACUUM CASE

Electrostatic Self-Repulsion

We observe that its own electrostatic repulsion will not keep the beam from converging. Consider the radial motion of a particle on the beam edge. Energy conservation yields

$$e = \frac{1}{2} v_{th}^2 - \left(\frac{Ze^Q}{\ell} \right) \ln r = \text{const.},$$

and the minimum radius r_m reached by the beam follows from

$$\frac{1}{2} v_{th}^2 - \left(\frac{Ze^Q}{\ell} \right) \log r_0 = - \left(\frac{Ze^Q}{\ell} \right) \log r_m$$

$$\log \left(\frac{r_0}{r_m} \right) = \frac{\frac{1}{2} v_{th}^2}{\left(\frac{Ze^Q}{\ell} \right)} = \frac{\frac{1}{2} v_{th}^2}{\frac{Ze^2}{N_0^2 \ell^2} \frac{v_{th}^2}{v_{th}^2}} = \frac{v_{th}^2}{N_0^2 \ell^2}.$$

Expressed in terms of $f = v/v_{th} = L/r_0$ this becomes

$$f^2 \log \left(\frac{r_0}{r_m} \right) = \frac{\frac{1}{2} v_{th}^2}{N_0^2 \ell^2} = \frac{g^2}{4\pi} \approx 1.7 \cdot 10^6.$$

This then scales as

$$\frac{M}{Z} \left(\frac{T}{E_0} \right)^{3/2} L^{-1/2}.$$

For the beam parameters under discussion $f = 100$, so that the focusing requirement becomes

$$\log \left(\frac{r_0}{r_m} \right) \leq 17,$$

which is satisfied in the present case for which

$r_0/r_m = 100$ or $\log(r_0/r_m) \approx 4.6$. Because of the N/Z dependence, however, a beam of doubly charged iodine ions of the same speed and current would have a focusing requirement of $\log(r_0/r_m) \approx 4.5$, which is hardly satisfied for this value of r_0/r_m .

Electrostatic Stretching

The particles in the beam experience an electrostatic repulsion which leads to an expansion of the beam segment. The effect of this electrostatic stretching may be calculated from the force experienced by particles at the ends of the beam. For a cylindrical segment of length L the potential at a point z along the axis is given by

$$\phi = \int \frac{\rho dv}{R} = 2\pi\rho \iint \frac{r'dr'dz'}{[(z-z')^2 + r'^2]^{\frac{3}{2}}}$$

$$= 2\pi\rho \int_{-L/2}^{L/2} \left\{ [(z'-z)^2 + r^2]^{\frac{1}{2}} - (z'-z) \right\} dz'$$

and the field for $z > 0$ ($z_{\pm} = z \pm \frac{1}{2}L$)

$$E_{z>0} = -\frac{\partial\phi}{\partial z} = 2\pi\rho \left\{ [z_{-}^2 + r^2]^{\frac{1}{2}} - [z_{+}^2 + r^2]^{\frac{1}{2}} + L \right\}$$

or at one end ($z = \frac{1}{2}L$)

$$E = \left(\frac{2Q}{rL} \right) \left(1 - \frac{r}{2L} \right),$$

for $r \ll L$ and where Q is the total charge in the beam.

An estimate of the change in length of the beam as it converges may be obtained from the equation of motion. Under the impulse approximation with L_0 the beam length at $r=r_0$ the rate of change in the beam length is

$$\frac{dL}{dt} \approx 2 \left(\frac{Ze}{M} \right) \frac{2Q}{L_0} \int_0^L \frac{dt}{r}$$

$$\approx 2 \left(\frac{Ze}{M} \right) \left(\frac{2Q}{L_0} \right) \frac{1}{r} \ln \frac{r}{r_0}$$

$$\approx -4 \left(\frac{NZ^2e^2}{Mv^2} \right) \frac{L}{r_0} v \ln \frac{r}{r_0}$$

The change δL in beam length as it propagates a distance L with a change in radius from r_0 to r_f is given by

$$\delta L \approx -4 \left(\frac{NZ^2e^2}{Mv^2} \right) \frac{L}{r_0} v \int_{r_0}^{r_f} \frac{dr}{r}$$

$$\approx -2 \left(\frac{NZ^2e^2}{Mv^2} \right) \frac{L^2}{r_0} \left(\frac{r}{r_0} \ln \frac{r}{r_0} - 1 \right) \frac{r}{r_0}$$

$$\approx 4 \left(\frac{NZ^2e^2}{Mv^2} \right) \frac{L^2}{r_0}$$

For the parameters $N = 9.6 \times 10^{11}$, $M = 238 m_p$, $Z = 1$, $v = 0.65 c$, $L = 10$ m, and $r_0 = 10$ cm, this yields

$$\frac{\delta L}{L} \approx 3 \times 10^{-3}$$

so that the fractional change $\delta L/L$ in the beam length is about 0.3%.

Electrostatic Target Repulsion

The total charge in the beam is $(200) \times (9.6 \times 10^{11}) (4.8 \times 10^{-16}) \approx 0.2 \times 10^9$ esu and the potential produced on a target assembly of radius 1 cm $\approx 2.8 \times 10^7$ eV, and $\approx 2.8 \times 10^8$ eV if the radius is 1 mm, which is still much less than the 70 GeV in the beam. Fully ionized beam particles would, however, be stopped. The beam can thus propagate without much difficulty to the target. The ratio of electrical to mechanical energy varies as $Ne^2Z^2/T \sim ZI/B^2M$ so that as the specific charge is increased and the kinetic energy decreased, the current allowed before these effects become a problem decreases as B^2M/Z .

The remaining question is then that of beam destabilization by collective effects.

Stability

The most dangerous instability appears to be wiggle mode, which in the beam frame is electrostatic and has a growth rate $\Gamma \approx kr \omega_B$, where $k = 2\pi/\lambda$, r is the beam radius, and ω_B is the beam plasma frequency

$$\omega_B = \left(4\pi n^2 \frac{c^2}{M} n \right)^{\frac{1}{2}} \approx 85\sqrt{n} \approx 4.7 \times 10^8 \left(1 + \frac{r_0^2}{r_f^2} \right)^{\frac{1}{2}}$$

The maximum number of e-foldings as the particle moves along the beam is approximately $(kr \sim 1)$

$$\int_0^{t_f} \omega_B dt \approx \left(\frac{4Z^2e^2}{M} \right)^{\frac{1}{2}} \int_0^{t_f} n^{\frac{1}{2}} dt$$

$$\approx 2 \left(\frac{Z^2e^2}{M} \right)^{\frac{1}{2}} \frac{1}{v} \int_{r_0}^{r_f} \frac{dr}{r}$$

$$\approx 2 \left(\frac{Z^2e^2N}{Mv^2} \right)^{\frac{1}{2}} \frac{L}{r_0} \log \frac{r_f}{r_0} \approx 1$$

These calculations have been carried out in a frame moving with the beam. When laboratory densities and times are used the result must be multiplied by a factor $\gamma^{3/2} \approx 1.5$, yielding a total growth factor of about four. Hence this electrostatic mode forms no barrier to the transport of singly charged ions. Since ω_B scales as $[ZI(E_0/T)^{1/2}/M]^{\frac{1}{2}}$, multiply charged ions or ions of smaller mass may suffer a current limitation from this mode. In conclusion, it appears that beams of singly charged U at energies of 70 GeV, and currents of ~ 3000 A are able to hit the target if the background gas is neutral and at densities $< 10^{14}$. If Z is increased, and M decreased, by factors of ~ 5 , the wiggle instability may limit the beam current. We have also seen that electrostatic stretching of the beam is small, amounting to about 0.3% of the initial length.

THE PLASMA CASE

Equilibrium

We now consider the equilibrium and stability of a beam passing through a plasma of $Z=1$, $n=10^{14}$ cm^{-3} , $T \approx 1$ eV. Except near the end of the beam, i.e., near the target chamber, the beam density is small compared to the plasma density, and the plasma response can be characterized by the linear dielectric constant

$$\epsilon = 1 - \frac{\omega_p^2}{\omega} (\omega - i\nu)$$

Here the plasma frequency

$$\omega_p = 5.6 \times 10^4 \sqrt{n} = 5.6 \times 10^{11} \text{ sec}^{-1},$$

and the collision frequency

$$\nu = n\nu_e = n\pi \left(\frac{Ze^2}{mv^2} \right)^2 \log \Lambda \cdot v$$

$$\approx 3.8 \times 10^8 \log \Lambda \approx 2.6 \times 10^8$$

with $\log \Lambda \approx 7$.

The response of a plasma to an external source such as the beam current j_B is given by Maxwell's equation

$$\nabla \times \vec{B} = \frac{1}{c} \frac{\partial}{\partial t} (\epsilon \vec{E}) + \frac{4\pi}{c} \vec{j}_B$$

from which follow equations for the development of the current and of the charge. The charge density satisfies

$$0 = \frac{1}{c} \frac{\partial}{\partial t} \epsilon \nabla \cdot \vec{E} + \frac{4\pi}{c} \nabla \cdot \vec{j}_B$$

$$= \frac{4\pi}{c} \frac{\partial}{\partial t} [\epsilon q_T - q_B] = 0$$

with $\epsilon = 1 - [\omega_p^2/(\omega(\omega - i\nu))]$ and where $4\pi q_T = \nabla \cdot \vec{E}$ represents the total effective charge density. From this we deduce a time-dependent differential equation for the charge density

$$\frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} + \nu \right) (q_T - q_B) + \omega_p^2 q_T = 0$$

Laplace transformation of both sides yields

$$(q_p = q_T - q_B)$$

$$(p^2 + \nu p + \omega_p^2) \tilde{q}_p = -\frac{1}{p} \omega_p^2 \left(e^{-pt_1} - e^{-pt_2} \right) q$$

for a beam with sharp edges extending from t_1 to t_2 , i.e.,

$$q_B = q[\theta(t - t_1) - \theta(t - t_2)]$$

where θ is the Heaviside step-function. Inversion yields

$$q_p = -q_B[\theta(t - t_1) - \theta(t - t_2)] + q_B[f(t_1) - f(t_2)]$$

with

$$f(t) = e^{-\frac{1}{2}\nu t} \left(\cos \omega_p t + \frac{\nu}{2\omega_p} \sin \omega_p t \right) \theta(t)$$

and the charge is neutralized effectively in one plasma period but the neutralizing charge oscillates for a collision period and all but the leading 10 cm of the beams are neutral. The conclusions are modified slightly if the finite temperature is taken into account.

For the current the situation is not quite so clear. The magnetic field satisfies

$$\left[\frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} + \nu \right) \right] \left[\nabla^2 \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} \right] - \frac{\omega_p^2}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$= -\frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} + \nu \right) \frac{4\pi}{c} \nabla \times \vec{j}_B$$

For short times, this has a solution which screens the current j_B in a distance $\sim c/\omega_p$, the collisionless screening length ≈ 0.05 cm. For times long compared to the collision period, however, the plasma current diffuses out of the current channel and forms a parabolic cone

$$r^2 = \frac{c^2}{\omega_p^2} \frac{z}{v} \approx 4 \times 10^{-4} z$$

This cone sets in after about 10 cm of the beam has passed so that its maximum separation is ~ 0.5 cm and only for the last 2 m of its flight does the beam current appear. Thus, the beam current is effectively screened by the plasma, until the beam has shrunk to ~ 0.5 cm, i.e., in the last 20 cm of its path.

Moreover, this calculation has underestimated the effective shielding, because as the beam passes through the plasma, the ohmic heating induced by the plasma current will raise the temperature. As it travels, the beam deposits energy at a rate

$$\frac{d\epsilon_j}{dt} = \frac{\eta_j^2 A}{\text{unit length}}$$

$$\approx 4\pi \frac{\nu c^2}{\omega_p^2} \frac{10^{-7}}{10^2} j^2 A = \frac{8.4 \times 10^5}{\pi R^2 T^{3/2}} \text{ W/cm},$$

T being the plasma temperature in electron volts. Much of this energy is transported out of the beam by electrons at a rate

$$-\frac{d\epsilon}{dt} = n\nu_e 2\pi R \approx nT^{3/2} 5.93 \times 10^7 \times 1.6 \times 10^{-19}$$

$$\times 2\pi R \text{ W/cm} \approx 1.9 \times 10^3 \pi R T^{3/2} \text{ W/cm}$$

and the rate of increase of temperature of the plasma in the beam, allowing for both electron and ion heating, is

$$\frac{d\epsilon}{dt} = 3n \frac{dT}{dt} \Lambda \approx 3 \times 1.6 \times 10^{-19} n \pi R^2 \frac{dT}{dt}$$

$$= 4.6 \times 10^{-5} \pi R^2 \frac{dT}{dt}$$

Therefore,

$$\frac{dT}{dt} \approx 1.8 \times 10^9 \frac{T^{-3/2}}{R^4} [1 - 0.002 T^3 R^3] .$$

If $R < 8$ cm, the temperature begins to rise. If the heat-loss term is negligible, then the increase in temperature along the beam is

$$\frac{dT}{dz} = \frac{1.8 \times 10^9}{1.95 \times 10^{10}} \frac{T^{-3/2}}{R^4(z)} = 0.092 \frac{T^{-3/2}}{R^4(z)}$$

$$\frac{2}{5} \frac{d}{dz} T^{5/2} = \frac{0.092}{r^4(z) [1 + 0.2z]^4}$$

$$T = \left\{ T_0^{5/2} + \frac{5}{2} \frac{0.3}{r^4(z)} \left[1 - \frac{1}{(1 + 0.1z)^3} \right] \right\}^{2/5}$$

$$T = \left\{ T_0^{5/2} + \frac{0.75}{r^4(z)} \left[1 - \left(\frac{r(z)}{r_L} \right)^3 \right] \right\}^{2/5}$$

where $r(z)$ is the beam radius at the head of the beam and r_L the radius a distance L from the head. For $L > 10$ cm and $T_0 = 1$ eV

$$T \approx \left[1 + \frac{0.75}{r(z)} \right]^{2/5}$$

When the head of the beam has a radius of 0.5 cm, i.e., as the rear of the beam (of length 2 m) enters the chamber

$$T \approx [1 + 12]^{2/5} \approx 2.8 \text{ eV} .$$

At the pellet the beam radius is ~ 0.1 cm and the temperature ~ 36 eV. At this temperature, however, the conduction term becomes important when $0.126 RT \approx 1$. $R \approx 2.8$ cm, i.e., at $L = 56$ cm; hence for 3/4 of its length the temperature is determined by conduction losses

$$T(R) \approx \frac{7.9}{R} .$$

When the beam strikes the target, the radius of the rear of the beam is 4 cm, and $T \approx 2$ eV, and the size of the current shielding region $r_0 \approx T^{-3/4} \sim 0.17$ cm, so that even at this point the current is effectively shielded. This simple model has neglected the effect of electron collisions and the temperature profile. The effects of these processes remain to be studied.

Note, finally, that the analysis we have presented here has considered only the linear response of the plasma; during the last 30 cm of its flight, the beam density exceeds 10% that of the plasma, and at the end of the path, the beam density is equal to the plasma density. Thus in this small region, the linear treatment of the plasma may be invalid, and beam charge as well as beam current may appear.

Beam Stability. The beam may be unstable in ways that depend upon its finite size and shape, or

in ways that depend only on the velocity distribution. We consider the latter case first.

Micro-Instabilities. Two-Stream Model.

Since the beam passes at high velocity through a plasma, we expect the two-stream instability to appear. The collisionless dispersion relation can be written

$$\epsilon_L = 1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_B^2}{(\omega + k \cdot v)^2} = 0 ,$$

where

$$\omega_B^2 = 4\pi n_B \frac{e^2}{m} \approx 1.5 \times 10^6 \frac{n_B}{n_p} \omega_p^2 .$$

The usual two-stream analysis¹ yields a maximum growth rate

$$\gamma \approx 0.7 \omega_p \left(\frac{\omega_B}{\omega_p} \right)^{1/3} \approx 4.5 \times 10^9 \left(\frac{n_B}{n_p} \right)^{1/3}$$

with a frequency $\omega \approx -k \cdot v \approx \omega_p$, and the mode is convective. This mode will grow only if $\gamma > \nu \approx 2.6 \times 10^9$, at the head of the beam. As we have seen, the temperature near the rear of the beam is increased by ohmic dissipation, and has about doubled by the time the beam reaches the target and $\nu \sim T^{-3/2}$ decreases to $9 \times 10^8 \text{ sec}^{-1}$. Where the temperature is determined by heat losses, $T \sim 7.9/R$ and $\nu \sim 2.6 \times 10^9 (R/7.9)^{3/2}$. At the same time $\gamma \approx 2.13 \times 10^9 R^{-2/3}$. Thus growth will occur if

$$2.13 \times 10^9 R^{-2/3} > 1.1 \times 10^9 R^{3/2}$$

or

$$R < \left(\frac{2.13}{1.1} \right)^{6/13} \approx 1.36 \text{ cm} \approx r_c ,$$

hence when the head of the beam has reached the target. In the remaining 20 nsec, however, the instability grows

$$\begin{aligned} \int_{r_c}^{r_f} 2.13 \times 10^9 \frac{1}{R^{2/3}} dt &= \int 2.13 \times 10^9 \frac{dr}{r} R^{-2/3} \\ &= 3 \left(2.13 \times 10^9 \frac{L}{v r_0} \right)^{1/3} \approx 5.6 \end{aligned}$$

e-foldings for a gain of ~ 37 and any noise in the beam is seriously increased. If, however, the instability grows from thermal noise on the beam, the final fluctuation amplitude remains small, $\sim 10^{-3}$. Since $\omega_B^{1/3} \sim [21/\text{SyM}]^{1/6}$, and $\nu \sim T_p^{3/2}$, an increase in the plasma temperature, or in the scaling ratio for ω_B , can easily lead to a rapidly growing instability.

Electromagnetic Modes. In addition to the electrostatic modes considered above, the beam can support electromagnetic oscillations. For these the dispersion relation can be written

$$k^2 - \frac{\omega_p^2}{c^2} \epsilon_L = \frac{-\omega_B^2}{(\omega + k \cdot v)^2} - \frac{k^2 v^2}{c^2} \frac{\epsilon_T}{\epsilon_L} ,$$

where v_z is the component of beam velocity k , and the three dielectric functions are

$$\epsilon_p = 1 - \frac{\omega_p^2}{(\omega - i\nu)}$$

$$\epsilon_L = 1 - \frac{\omega_p^2}{(\omega - i\nu)} - \frac{\omega_B^2}{(\omega + k \cdot v)^2}$$

$$\epsilon_T = - \frac{\omega_p^2}{\omega(\omega - i\nu)} - \frac{\omega_B^2}{\omega^2}$$

Transverse Beam Instabilities. The dispersion relation for transverse modes obtained by setting $k \cdot v = 0$, becomes in the absence of damping

$$k^2 - \frac{\omega^2}{c^2} \left[1 - \frac{\omega_T^2}{\omega^2} \right] = - \frac{\omega_B^2}{\omega^2} \frac{k^2 v^2}{c^2} \left(\frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_T^2} \right)$$

with $\omega_T^2 = \omega_p^2 + \omega_B^2 + \omega_0^2$. This dispersion relation shows the possible excitation of a purely growing mode. For small ω we obtain

$$\omega^2 = - \frac{\omega_B^2 k^2 v^2 \omega_p^2}{c^2 \omega_T^2 \left(k^2 + \frac{\omega_T^2}{c^2} \right)}$$

or the growth rate

$$\omega = -i\Gamma \omega_B \left(1 + \frac{k_0^2}{k^2} \right)^{-1/2} < -iB\omega_B$$

with $k_0 \approx \omega_0/c$ the reciprocal of the collisionless screening length. As the beam propagates, this ranges from $3 \times 10^6 \text{ sec}^{-1}$ to $3 \times 10^8 \text{ sec}^{-1}$. The total growth for this nonconvecting mode is $\sim \exp[(\Gamma \omega_B L/v)] \sim 50$. To take into account collisional damping, we note that since $\omega \gg \nu$ the dispersion relation becomes approximately

$$\omega^2 + i \frac{\omega^3}{\nu} \frac{k_0^2}{k^2} + \omega_0^2 = 0$$

when $\omega_0 = \omega_B B$.

Writing $\omega = -i\omega_0 + x$, we find $x \approx i \omega_0^2 k_0^2 / \nu k^2$. If $(k_0^2/k^2)(\omega_0/\nu) \ll 1$, the approximations are valid and the wave is weakly damped: ω_0/ν varies from $10^{-3} \sim 0.1$ so that waves for which $k > 0.3 k_0$ can grow. This corresponds to transverse wavelengths less than 0.1 cm, and since the beam is less than 1 cm in radius in the regions of significant growth, damping is unimportant.

The growth rate scales as $[ZIB/M\gamma]^2$, hence an increase in Z/M or I could lead to serious filamentation.

If $k \cdot v \neq 0$, it is readily shown that the transverse instability will grow only if the system is also unstable to electrostatic modes and that the unstable motion is convected with the beam.

The dispersion relation becomes

$$k^2 - \frac{\omega^2}{c^2} + \frac{\omega_p^2}{c^2} \frac{\omega_B^2}{\omega^2} = \frac{\omega_B^2}{(\omega + k \cdot v)^2} \frac{k^2 v^2}{c^2} \left(\frac{1 - \left(\frac{\omega_T^2}{\omega^2} \right)}{1 - \left(\frac{\omega_p^2}{\omega^2} \right) + \left[\frac{\omega_B^2}{(\omega + k \cdot v)^2} \right]} \right)$$

If $\omega = -k \cdot v + x$ and $x \ll k \cdot v$, this expression reduces to

$$k^2 \left(1 - \frac{v_z^2}{c^2} \right) + k_0^2 = \frac{\omega_B^2 (k v_z)^2}{c^2} \left\{ 1 - \left[\frac{\omega_p^2}{(k \cdot v)^2} \right] \right\} / c^2 = x^2 \left\{ 1 - \left[\frac{\omega_p^2}{(k \cdot v)^2} \right] \right\} - \omega_B^2$$

or

$$x^2 = -\omega_B^2 \left\{ \frac{\frac{k^2 v_z^2}{c^2}}{k^2 \left[1 - \frac{v_z^2}{c^2} \right] + k_0^2} - \frac{1}{1 - \left(\frac{\omega_p}{k \cdot v} \right)^2} \right\}.$$

If $(k \cdot v) < \omega_p$, then x is purely imaginary. The largest growth rates occur as $\omega_p^2/(k \cdot v)^2 \rightarrow 1$, which is exactly the electrostatic dispersion relation.

If Z/M and I are increased or, in the case of the two-stream instability, if the collision frequency is decreased, these microinstabilities can grow. Of course, if the beam entered the system with appreciable density fluctuations, these are amplified by the unstable modes. We will first consider the effects of the electrostatic two-stream modes. If these are saturated, then the potential $\sim n_B Z e \lambda_{D1} \approx 6 \times 10^4 \text{ V}$. This is not enough to have much effect on the beam particles, but can seriously affect the plasma.

To get at this effect, we note that the hydrodynamic equations for the plasma electrons may be written

$$\frac{\partial v}{\partial t} + v \cdot \nabla v + \nabla v = - \frac{e}{m} \nabla \phi$$

to lowest order, neglecting collisions,

$$v^{(1)} = - \frac{e}{m} \frac{k}{\omega} \phi \approx \frac{e}{m} \frac{\phi}{v_{\parallel}}$$

and to second order

$$\frac{\partial v^{(2)}}{\partial t} + v v^{(2)} = - \frac{\partial}{\partial x} \left(\frac{e}{m} \frac{\phi}{v} \right)^2 = \frac{\gamma}{v} \left(\frac{e}{m} \frac{\phi}{v_{\parallel}} \right)^2,$$

or

$$\frac{v_{\perp}}{v_{\parallel}} \sim \frac{\gamma}{v} \left(\frac{e v_{\parallel}}{m \omega_{\parallel}} \right)^2 \leq 0.1,$$

and the electron drag significantly reduces the current. Since γ does not attain its maximum value until the end of the flight, only at the target is it neutralized. Near the target, in the last 100 cm of flight

$$\begin{aligned} \frac{j_{\text{drag}}}{j_B} &= \frac{n}{n_B} \left(\frac{e}{m \omega_{\parallel}} \right)^2 e^{-\gamma t/v} \sim 0.1 \frac{n}{n_B} e^{-\gamma t/v} \\ &\sim \frac{0.1}{[1 + (r_0/r_P)^2 (i/R)]^2} e^{-\gamma t/v} \leq 0.1. \end{aligned}$$

The electromagnetic instabilities induce a deflection in the beam ions. If they grow from a random fluctuation in beam current of average magnitude

$$\delta j \sim \frac{n_B e \frac{v_{\parallel}}{c}}{\sqrt{n_B \lambda_{\parallel}^2}} \sim 3 \cdot 10^{-6} j_B.$$

The associated vector potential is

$$A_z \sim \frac{v_{\parallel}^2}{c^2} \delta j \frac{1}{k_{\parallel}},$$

and the transverse acceleration

$$\frac{d}{dt} v_{\perp} \approx k_{\perp} \frac{v_{\parallel}^2}{\omega_p} \frac{2e}{M} \delta j \frac{v_{\parallel}}{c}.$$

$$\Delta v_{\perp} \approx \frac{k_{\perp}}{\gamma} \frac{v_{\parallel}^2}{\omega_p^2} \frac{2e}{M} \delta j_0 \frac{v_{\parallel}}{c} e^{\int \gamma dt}$$

and

$$\frac{v_{\perp}}{v_{\parallel}} \leq \frac{k_{\perp}}{k_{\parallel}} \left(\frac{\omega_B}{\omega_p} \right)^{2/3} \left(\frac{v_{\parallel}}{c} \right)^2 \frac{e \int \gamma dt}{\sqrt{n_B \lambda_{\parallel}^2}}$$

$$\leq \frac{k_{\perp}}{k_{\parallel}} \times 10^{-5}.$$

If $k_{\perp}$ is large, compared to $k_{\parallel}$, then the factor $v_{\parallel}^2/\omega_p^2$ must be replaced by $1/k_{\perp}^2$ and we find that $v_{\perp}/v_{\parallel} < 10^{-5}$. This straggling occurs only in the last meter of the path and hence can have negligible effect in the targeting.

Geometric Modes

We consider now those modes that depend on the finite extent of the beam. If the plasma did not

affect the external fields, the wobble and ripple modes discussed for the case of the vacuum beam would occur, but since the beam is quasi-neutral, and the current is also neutralized until the end of the path, these modes appear only for the last meter of the path, where they grow at a rate $\gamma_B \sim (n_B/n_P)^{1/2} 8 \cdot 10^3 \text{ sec}^{-1}$.

Then they amplify by

$$\int \gamma dt \sim 8 \cdot 10^3 \int_{r_0/r_P}^{r_0/r_P} \frac{r}{r} dr \sim 1.7$$

e -foldings and hence such instabilities have little effect.

To get some insight into the effect of the plasma, consider the ripple mode. We can then repeat the analysis exactly as in the vacuum case, but introducing the dielectric

$$\epsilon_P = 1 - \frac{\omega_p^2}{\omega^2}.$$

We then deduce for the growth rate

$$\gamma \sim \frac{\gamma_B}{1 + \left[\frac{\omega_p^2}{(k v)^2} \right]^{1/2} \left\{ 1 + 2 \left[\frac{\omega_p^2}{(k v)^2} \right]^{1/2} \right\}}.$$

If $k v \sim \omega_p$, this reduces to

$$\gamma \sim \frac{\gamma_B}{2 \left[1 + \left(\frac{\omega_p}{k v} \right)^2 \right]}.$$

For $\omega_p \sim \omega/2$,

$$\ln F \sim \frac{1}{2} \left(\frac{\omega_B}{\omega_p} \right)^{1/3} \gamma.$$

where γ is the two-stream growth rate. This yields $\ln F \sim 0.05 \gamma$ for our parameters, so that if γ is large this instability may matter. However, for our standard case, γ seldom exceeds ω_p , and $\ln F \ll 1$; hence the mode will not grow.

CONCLUSIONS

We have shown that a beam of $\sim 3000 \text{ A}$ can be propagated through a plasma of density $\sim 10^{17}$ and that the beam remains both electrically and magnetically neutral until very close to the target. The beam is subject to microinstabilities, which grow with $\gamma \sim \omega_B^{2/3} \omega_p^{1/3}$ and scale as

$$\gamma \sim \left(\frac{2 n_P}{M} \right)^{1/3}.$$

These growth rates may be large enough to produce substantial fluctuating electric and magnetic fields in the plasma. These fields do not affect the

retion of the beam particles but can entrain plasma electrons, thus further reducing the current. The most dangerous instability appears to be a high-frequency ripple of the beam surface, which resonates with the plasma, and may lead to a spreading of the beam to 0.4 cm in the last 10 cm before the target, but not for the standard case of ^{238}U uranium ions and a beam current of 3 kA.

The instabilities we have discussed grow only in the last 100 cm of the beam path. In this region the beam current density becomes very high and the plasma can no longer respond linearly. In fact, the plasma current density is limited to about 1 kA/cm². When the beam current exceeds this, current neutralization will probably produce instabilities in the plasma, growing at least as rapidly as an $\omega_{pe}/v_p \approx 10^{11}$ sec⁻¹.

APPENDIX 1: THE STABILITY OF A BEAM IN VACUUM

We consider two modes in a rough approximation.

$\omega < \omega_{pe}$, $\omega > \omega_{pe}$

Suppose we consider the beam as wiggling on a scale large compared with its cross section. Then the electrostatic repulsion between two elements has the form

$$\frac{1}{r_{12}} = q^2 \frac{\hat{r}_1 - \hat{r}_2}{r_{12}^3} \cdot \frac{\hat{r}_1 - \hat{r}_2}{r_{12}} \cdot d\mathbf{r}_1 \cdot d\mathbf{r}_2$$

and the total force on the element at $d\mathbf{r}_1$

$$\hat{r}_1 d\mathbf{r}_1 = -q^2 \left(\int \frac{\hat{r}_1 - \hat{r}_2}{r_{12}^3} \cdot \frac{\hat{r}_1 - \hat{r}_2}{r_{12}} \cdot d\mathbf{r}_2 \right) \cdot d\mathbf{r}_1$$

The acceleration experienced by the element $d\mathbf{r}_1$, normal to its direction is

$$a = - \frac{d^2 y}{dt^2} = - \frac{q^2}{M} \left(\int \frac{y_1 - y_2}{r_{12}^3} \cdot \frac{y_1 - y_2}{r_{12}} \cdot d\mathbf{r}_2 \right)$$

but

$$q^2 = N e^2 / v^2 \quad \text{and} \quad M_T = NM$$

the factor

$$\frac{N e^2 / v^2}{M_T} = \frac{N e^2 / v^2}{M} = \frac{2R}{4\pi} \lambda^{-2}$$

λ being the cross section. To evaluate the geometrical factor $1/\lambda$, we consider a sinusoidal perturbation about equilibrium $y = a \cos kx$ and write

$$1 = \int_{-a}^a \frac{1}{\sqrt{a^2 - y^2}} \cdot \frac{1}{\sqrt{a^2 - y^2}} \cdot dy$$

$$= \int_{-a}^a \frac{a[\cos ky_1 - \cos ky_2]}{[a^2 - y_1^2]^{1/2} [a^2 - y_2^2]^{1/2} a^2 [\cos ky_1 - \cos ky_2]^{1/2}} \cdot dy_1 \cdot dy_2$$

$$1/\lambda = 1/a \cdot \pi$$

$$1 = \int_{-a}^a \frac{[a \cos ky_1 - \cos ky_2] \cdot a \sin ky_1 \sin ky_2}{[a^2 - y_1^2]^{1/2} [a^2 - y_2^2]^{1/2} a^2 [\cos ky_1 - \cos ky_2]^{1/2}} dy_1 \cdot dy_2$$

We split the integral into two parts $y_1 < a$ and $y_1 > a$, which overlap if $ky_1 > \pi$, and note that the second term vanishes from symmetry.

$$1 = 2a \cos ky_1 \left[\int_0^1 \frac{1 - \cos ky}{y^{1/2}} dy \right.$$

$$\left. + \frac{1}{2} \int_0^1 \frac{1 - \cos ky}{y^{1/2} (1 + a^2 y^2 \sin^2 ky)^{1/2}} dy \right]$$

The cut-off at a gives a rough representation of the effect of finite cross section.

To lowest order in a the second integral is $\frac{1}{2} \log(1/a)$, while the first

$$= \frac{1}{2} \frac{1}{1^2} \int_0^1 \frac{\cos ky}{y^{1/2}} dy$$

$$= \frac{1}{2} \frac{1}{1^2} \left[-\frac{1}{2} \frac{\cos ky}{1^2} + \frac{1}{2} \frac{\sin ky}{1} \right]$$

$$= \frac{1}{2} \frac{1}{k^2} \int_0^1 \frac{\cos ky}{y^{3/2}} dy$$

The last integral is the cosine integral, $\text{ci}(k\lambda)$, which for small $k\lambda$ goes to $-\ln(k\lambda)$, where $\lambda = 1/k\lambda$. Thus

$$1 = k^2 a \cos ky_1 \left[\log \frac{1}{a} + \log \frac{1}{k\lambda} \right]$$

$$= k^2 a \cos ky_1 \log \left(\frac{1}{k\lambda a} \right)$$

$$\frac{d^2 y}{dt^2} = \frac{N e^2 / v^2}{M} k^2 \log \left(\frac{1}{k\lambda a} \right) y$$

$$\omega^2 = \frac{N e^2 / v^2}{M} - v_0^2 k^2 \log \left(\frac{1}{k\lambda a} \right) = \frac{1}{4} \frac{2R}{\pi} [k^2 v_0^2] \log \left(\frac{1}{k\lambda a} \right)$$

Now consider small-scale ripples on the surface of the charge distribution, so that the surface is modified from $x(z) = 0$ to $x = \tilde{x}(y, z)$. This will induce an effective charge density on the surface of magnitude

$$\tilde{\rho} = q^2 \cdot \tilde{n}$$

where q is the charge density, and $\tilde{n}$ a perturbation in the electric field for which

$$\nabla \cdot \tilde{n} = -\Delta \tilde{x}.$$

The equation of motion is

$$\frac{d^2 \tilde{x}}{dt^2} = -\frac{Ze}{M} \tilde{n}.$$

and at the surface

$$\tilde{n} + \frac{d^2 \tilde{x}}{dt^2} = -\frac{Ze}{M} \tilde{n} + \tilde{n} + \frac{Ze}{M} q^2 \cdot \tilde{n} = \left(1 - \frac{Ze}{M} \right) \cdot \tilde{n},$$

and

$$\tilde{n} = -\frac{\nabla^2 \tilde{x}}{\Delta}.$$

Clearly a more complete and detailed analysis of this problem is required.

APPENDIX 2: PROPERTIES OF THE ELECTROMAGNETIC DIFFUSION EQUATION

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + \nu \right) \left[\nabla^2 \tilde{B} + \frac{1}{c^2} \frac{\partial^2 \tilde{B}}{\partial t^2} \right] = -\frac{\nabla^2}{c^2} \frac{\partial \tilde{B}}{\partial t} \\ & = -\frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} + \nu \right) 4\pi \nabla \cdot \tilde{j}_B. \end{aligned}$$

Note that for rates of change of $\tilde{B}$ and $\tilde{j}$ fast on the ν scale, $\partial/\partial t \gg \nu$, this becomes

$$\nabla^2 \tilde{B} + \frac{1}{c^2} \frac{\partial^2 \tilde{B}}{\partial t^2} = -\frac{\nabla^2}{c^2} \tilde{B} \sim -4\pi \nabla \cdot \tilde{j}_B.$$

If we consider a half space with a surface current $\tilde{j}_B(t)$ flowing on $z = 0$, we gain some insight into this problem. Then

$$\tilde{B} \sim e^{-kz}, \quad \tilde{B}(0, t) = \tilde{j}_B(t)$$

if

$$\nu \ll \frac{\partial}{\partial t} \ll \omega_p, \quad k^2 \sim \frac{\omega_p^2}{c^2}$$

and the plasma shields in the collisionless screening length. For large spatial gradients, on the other hand,

$$\left(\frac{\partial}{\partial t} + \nu \right) \nabla^2 \tilde{B} - \frac{\omega_p^2}{c^2} \frac{\partial \tilde{B}}{\partial t} \sim -\left(\frac{\partial}{\partial t} + \nu \right) 4\pi \nabla \cdot \tilde{j}_B$$

and in the general case, $\partial/\partial t \sim \omega_p$, this is the usual reactive diffusion equation.

The one dimensional Green's function for a source at $(0, 0)$ is obtained by substitution of the temporal Laplace transform

$$g(x, s) = \int_0^\infty e^{-st} g(x, t) dt$$

The resulting differential equation

$$\frac{d^2 g}{dx^2} = -\frac{\nabla^2}{(1 + \nu/s)} g$$

has the solution

$$g(x, s) = g(0, s) \exp \left[-\left(\frac{\nabla^2}{1 + \nu/s} \right)^{1/2} x \right]$$

or by the inverse Laplace transform

$$g(x, t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g(0, s) \exp \left[-st - \left(\frac{\nabla^2}{1 + \nu/s} \right)^{1/2} x \right] ds$$

which can also be written as

$$g(x, t) = \frac{1}{2} \int_0^t g(0, t-u) \nabla^2 \sin \left[\left(\frac{\nabla^2}{1 + \nu} \right)^{1/2} x \right] du,$$

but which we have been able to evaluate only in the limits already discussed.

APPENDIX 3: THE ELECTROMAGNETIC DISPERSION RELATION

Consider the response of the beam to a fluctuating electromagnetic field of frequency ω and wavenumber k . We assume that the plasma is a polarizable medium of dielectric $\epsilon_0 = 1 - (\omega_p^2/\omega^2)$. Linearization of the equations of momentum conservation (M is the mass of the beam particles) yields

$$\frac{d\tilde{v}}{dt} = -\frac{e}{M} \left(\tilde{E} + \frac{\tilde{v}}{c} \times \tilde{B} \right) \quad (1)$$

and continuity

$$\frac{dn}{dt} = -n \nabla \cdot \tilde{v} \quad (2)$$

yields to first order ($\tilde{v}_\perp = \tilde{v} + \tilde{k} \times \tilde{v}$)

$$\tilde{v}_\perp = -\frac{e}{M \omega_*} \left(\tilde{E} + \frac{\tilde{v}}{c} \times \tilde{B} \right) \quad (3)$$

and

$$n_1 = -\frac{n \tilde{k} \cdot \tilde{v}}{\omega_*} = -\frac{en}{M \omega_*^2} \tilde{k} \cdot \left(\tilde{E} + \frac{\tilde{v}}{c} \times \tilde{B} \right). \quad (4)$$

From Poisson's equation

$$\frac{1}{\epsilon} = \frac{1}{\epsilon_0} + \frac{1}{\epsilon_1} \quad (1)$$

and $\epsilon_1 = k^2 \omega^2 / (\omega_p^2 - \omega^2)$ from (3)

$$\vec{E} \cdot \left(\frac{1}{\epsilon} + \frac{\vec{V} \cdot \vec{V}}{c^2} \right) = -\frac{1}{\epsilon_0} \epsilon_1 \vec{E} \cdot \vec{E} \quad (2)$$

and also

$$\left(\epsilon_1 \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial t^2} \right) \vec{E} \cdot \vec{E} = \epsilon_1 \vec{E} \cdot \vec{E} = \frac{\partial^2}{\partial t^2} \left(\vec{E} \cdot \frac{\vec{V} \cdot \vec{V}}{c^2} \right) \quad (3)$$

which by Faraday's law

$$\frac{\partial^2}{\partial t^2} \vec{E} = -\frac{1}{c} \frac{\partial^2 \vec{B}}{\partial t^2} \quad (4)$$

reduces to

$$\epsilon_1 \vec{E} \cdot \vec{E} = \frac{\partial^2}{\partial t^2} \left(\vec{E} \cdot \frac{\vec{V} \cdot \vec{V}}{c^2} \right) \cdot \vec{E} \quad (5)$$

$$= \frac{\partial^2}{\partial t^2} \left(\frac{\vec{E} \cdot \vec{V}}{c^2} \right) \cdot \vec{E} \quad (6)$$

where $V_\perp$ is the component of $\vec{V}$ perpendicular to $\vec{k}$.

Finally, to obtain the dispersion relation, we require Ampere's law with the plasma contribution to the current density given by the displacement current:

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J}_B + \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \quad (7)$$

where the beam current is given by

$$\vec{J}_B = -en\vec{V}_\perp + eV_\parallel \hat{k} \quad (8)$$

or with Eqs. (3), (4), and (6)

$$\nabla \times \vec{B} = \frac{\partial^2}{\partial t^2} \left(\vec{E} \cdot \frac{\vec{V}}{c^2} + \vec{B} \right) = \epsilon_1 \vec{V} \cdot (\vec{k} \cdot \vec{E}) \quad (9)$$

Substitution into Eq. (7) then yields for the field $\vec{E}$

$$\left(\vec{k} \cdot \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial t^2} \right) \vec{E} = \frac{\partial^2}{\partial t^2} \left(\frac{\vec{V} \cdot \vec{E}}{c^2} + \frac{\vec{V} \cdot \vec{E}}{c^2} \right) \quad (10)$$

where we have used Eq. (8) and the vector identity $\vec{k} \cdot (\vec{B} \cdot \vec{E}) = (\vec{k} \cdot \vec{E}) \vec{B} \cdot (\vec{k} \cdot \vec{E})$. If we now combine the scalar product of Eq. (10) with $\vec{k}$ with Eq. (9), we obtain the dispersion relation

$$(\vec{k} \cdot \vec{k} + \frac{\partial^2}{\partial t^2} - \epsilon_1 \frac{\partial^2}{\partial t^2}) = -\frac{\omega_p^2 k^2 V_\perp^2}{\omega^2 c^2} \frac{\epsilon_p}{\epsilon_L} \quad (11)$$

to analyze the dispersion relation we write $x = \omega/\omega_p$

$$\frac{1}{\epsilon} = \frac{1}{\epsilon_0} + \frac{1}{\epsilon_1} \quad (12)$$

$$\left(\frac{\partial^2}{\partial t^2} \right) = -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \quad (13)$$

then

$$x^4 + 1 + \frac{\epsilon_p}{\epsilon_L} x^2 = \frac{\omega_p^2}{\omega^2} \frac{(x^2 - 1)}{(x^2 + 1)^2} \quad (14)$$

Now this is equivalent to a sixth-order polynomial, and we first show that it has no complex roots for those values of $k\omega$ for which the plasma is electrostatically stable. If the plasma is stable, then ϵ_p/ϵ_L has four distinct real roots of which two are close to ± 1 and while the others are near ± 1 but lie outside the interval $[-1, 1]$. We note that $k\omega/\omega_p$ real roots require $x^2 > 1$ while $k\omega/\omega_p$ is small. The rhs may then be plotted in a solid line (Fig. 1) as $x^2 > 1$.

Fig. 1: 18-5-1, (NBI 7612-11065)

The singularities represent the zeros of ϵ_L with the zero at ± 1 from the numerator. Over this we put the parabola representing the lhs (the dotted line), whose position relative to the rhs is determined by the magnitude of ϵ_p/ϵ_L . These curves clearly cross six times. Next we write

$$\frac{\omega_p^2}{\omega^2} \frac{(x^2 - 1)}{(x^2 + 1)^2} = -\frac{\epsilon_p}{\epsilon_L} \frac{(x^2 - 1)}{(x^2 + 1)^2} \quad (15)$$

and since the rhs is small of order ϵ_p/ϵ_L , the complex roots lie near zeros of lhs, since none of these lie near the large roots $x^2 = k^2 + 1 + 7$. Hence the instability growth rates lie near these for the electrostatic modes.

Reference

1. A. Bersin, in *Advances in Plasma Physics*, A. Simon and W. B. Thompson, Eds., (Wiley-Interscience, N.Y., 1969) Vol. 4.

APPENDIX 8-6 PLASMA TIME SCALES

1. Penetration

A key quantity in the maintenance of beam neutralization is the magnetic diffusion time τ_d . If τ_d is short, the neutralizing electrons will diffuse out. According to Appendix 8-1,

$$\tau_d = 0.40 \cdot \frac{r_b^2}{c}$$

where r_b is the beam radius and c is the residual charge. If $c = 5$ and $r_b = 1$ mm (a worst case) we find $\tau_d = 10$ nsec, so unless the beam has a long neck, neutralization will be maintained. It was suggested in Appendix 8-3 that this time may be actually 10 times longer if the large inductance due to the magnetic energy outside the beam is included.

Another significant time scale is obtained from the self-pinch of a charge-neutralized but current unneutralized beam. The betatron period is

$$\tau_\beta = 2.4 \cdot 10^{-10} \cdot \frac{r_b}{c} \text{ sec.}$$

We find

r_b	1 mm	1 mm	1 cm	1 cm
c	1	5	1	5
τ_β (nsec)	24.0	5.0	240.0	50.0

The effect is significant near the pellet even for the weakest case ($c = 1$).

Whether residual (or injected) gas pressures of 0.01 to 1.0 Torr can be tolerated remains an open question. A computer particle simulation may be necessary, since the usual envelope equation used in accelerator theory may not be valid.

APPENDIX 8-7 FINAL FOCUSING OF THE ION BEAMS OF A PELLET FUSION REACTOR BY QUADRUPOLE DOUBLET

A. A. Garman

INTRODUCTION

The final focusing of each ion beam in a pellet fusion reactor will be done by a set of quadrupoles located just beyond the reaction chamber wall. These must focus a wide beam, parallel or slightly diverging, coming from the preceding part of the

transport line, to a very small area on the pellet. This Appendix treats the case in which the lens system consists of two unseparated quadrupoles with equal gradients. It is assumed that the beam is not neutralized by gas or plasma in the reactor chamber, that it has equal emittances in both transverse directions, that its envelope has zero slope at the entrance of the doublet, and that its envelope is identical in both transverse directions in the reactor chamber.

These simplifications, together with approximations to be introduced later, allow insight into one lens system that may provide a basis for more exact calculations on this and on other configurations.

The main results of the investigation are these. If the quadrupole lengths are expressed in units of the drift length between the target and the doublet, and the beam widths by this length times the angular beam width at the target, then these lengths and widths are functions of a single parameter. This parameter depends on the pole tip field, on geometrical properties of the doublet, and on beam quantities. By examining the dependences on this parameter, it is possible to see how the total volume and stored energy of the doublets vary with distance to the target, magnetic field, charge state, and number of beams. This will lead to a proposed optimum choice for the charge and beam number. This choice corresponds to a particular value of the parameter mentioned above, and with it a unique specification of the quadrupole dimensions. The optimum charge state depends only on beam properties and target size; it corresponds to a doubling of the beam cross-sectional area, due to the space-charge force near the target. The optimum number of beams is chosen to give the special parameter its optimum value when the magnetic field is held within specified limits.

It may, of course, not be practical for the accelerator to provide beams that optimize the final focusing; and consideration of interactions between the beam and the gas or plasma in the reactor chamber may lead to somewhat different conclusions.

DESCRIPTION OF THE SYSTEM

The beam reaches the edge of the quadrupole doublet of Fig. A8-7-1 from the right such that the horizontal and vertical beam envelopes have zero slope, and displacements X, Y in the two transverse directions. The doublet ends at a specified distance L from the target, and is to be designed (along with X and Y) so that the beam envelope at the target is a waist with specified equal transverse displacements r_0 in each transverse direction. The beam is defined by the following parameters:

A = atomic number

q = charge state

T = kinetic energy (GeV)

$\beta\gamma$ = momentum/ m_0c

X, Y = envelope displacements at doublet entrance (m)

R = displacement at doublet exit (m)

- r_0 = displacement at target (m)
 ϵ = transverse emittance/° (m-rad) (same in x and y directions)
 ϵ_N = normalized emittance
 I = current (amperes), $I_0 = 1\mu$
 ψ = envelope angle at doublet exit (radians)
 ϵ_0 = ϵ value for zero current.

The quadrupoles are assumed to have equal bore radii, sufficient to contain the largest beam displacement (X), and equal pole-tip magnetic fields, corresponding to a field B_0 at radius X ; thus the gradients are $\pm B_0/X$ in Q_1 , Q_2 respectively, the quadrupole lengths are l_F , l_D (see Fig. A8-7-1).

Fig. A8-7-1. Schematic diagram of an ion beam focused onto a pellet by a quadrupole doublet. (A8-7-11-9-11)

WAIST-TO-WAIST CONDITIONS

The problem is to bring a beam with zero-slope envelope at location 3 to the same condition at the target, with specified displacement r_0 . That is, the beam envelope has a waist in each transverse direction at locations 0 and 3.

Consider the time-reversed situation in which the beam progresses from the target (now a source) to the right end of the doublet. For the moment the effects of space charge are ignored.

At the target the beam phase ellipses are the same in both planes, defined by parameters (beta-tron functions)

$$\begin{aligned}
 \beta_0 &= r_0^2/\epsilon \\
 \alpha_0 &= 0 \\
 \gamma_0 &= \epsilon/r_0^2
 \end{aligned} \quad (1)$$

the ellipse is upright, with x , x' intercepts (see Fig. A8-7-1)

$$x_0 = \sqrt{\epsilon/\gamma_0} = r_0, \quad \theta_0 = \sqrt{\epsilon/\gamma_0} = \epsilon/r_0 \quad (2)$$

Let M be the transfer matrix from position 0 to position 3 in either the x or y direction. Then the ellipse parameters at 3 in that direction are given by

$$\epsilon_3 = M_{11}^2 \epsilon_0 - 2M_{11}M_{12}r_0 + M_{12}^2 r_0^2 \quad (3a)$$

$$\alpha_3 = -M_{11}M_{11}' \epsilon_0 + (M_{11}'M_{22} + M_{21}M_{12}')r_0 - M_{12}'M_{22}' r_0^2 \quad (3b)$$

$$\gamma_3 = M_{21}^2 r_0^2 - 2M_{21}M_{22}r_0 + M_{22}^2 r_0^2 \quad (3c)$$

The terms in r_0 vanish. For cases of interest the ϵ_0 term in Eq. (3b) is less than 1% of the γ_0 term, so we may ignore it. Hence to make $\alpha_3 \approx 0$ it is sufficient to require that the 22 elements of the horizontal and vertical transfer matrices be zero:

$$(M_H)_{22} = (M_V)_{22} = 0 \quad (4)$$

Matrices for hard-edged quadrupoles, with equal and opposite gradients, and for the drift space of length L are multiplied to obtain M_H and M_V . Then, from Eq. (4), the following relations follow:

$$\tan^2 \theta_F = \frac{-\tanh^2 \frac{K}{2} + 1}{\tanh^2 \frac{K}{2} + 1} \quad (5)$$

$$\tan^2 \theta_D = \frac{\tanh^2 \frac{K}{2} + 1}{\tanh^2 \frac{K}{2} - 1}$$

where

$$\begin{aligned}
 K &= \sqrt{K} L, & \theta_D &= \sqrt{K} l_D, \\
 l_F &= \sqrt{K} l_F, & K &= \frac{B_0^2}{B_0^2}
 \end{aligned} \quad (6)$$

The solution of Eq. (5) gives θ_F , θ_D as functions of K . It is convenient to scale the lengths by l_0 :

$$l_D = l_D/l_0, \quad l_F = l_F/l_0 \quad (7)$$

so that

$$\theta_D = \theta_D, \quad \theta_F = \theta_F \quad (8)$$

Thus we have obtained, as functions of K , the scaled quadrupole lengths l_F and l_D and the doublet length

$$L = l_F + l_D \quad (9)$$

The quadrupole radius will be determined by the largest transverse beam displacement, X , the horizontal displacement at location 3. We obtain X from Eq. (3a), using $X = \sqrt{\epsilon/\gamma_3}$. Again we ignore the ϵ_0 compared to the γ_0 term in Eq. (3a), use Eqs. (1) and (2) and obtain

$$X = (M_H)_{12} r_0 \quad (10)$$

Explicit evaluation gives X in terms of the scaled variable x :

$$x = \frac{\lambda}{L_0} = \frac{1}{\cosh^2 \frac{b}{L_0}} \left(\cosh^2 \frac{b}{L_0} + \frac{\sinh^2 \frac{b}{L_0}}{\lambda} \right) \quad (11a)$$

which also is a function of λ . The corresponding displacement in the other plane is given by

$$y = \frac{\lambda}{L_0} = \frac{1}{\cosh^2 \frac{b}{L_0}} \left(\cosh^2 \frac{b}{L_0} + \frac{\sinh^2 \frac{b}{L_0}}{\lambda} \right) \quad (11b)$$

The quadrupole field at the maximum beam radius is $B = B'x$. If we use eq. (6), together with

$$k = \frac{C_1 A b y}{q}, \quad L_1 = \frac{m c}{e} = 3.13 \cdot 10^{-6} \text{ m}, \quad (12)$$

and define another function of b , $b = \lambda x$, we find

$$b(\lambda) \equiv \lambda^2 x = \frac{C_2 L_0 b}{A b y q}, \quad (13)$$

$$C_2 = C_1^2 = 0.32 \text{ (T} \cdot \text{m)}^{-1}.$$

Thus the quadrupole lengths, radius, and the parameter b have been expressed as functions of the gradient variable λ . It proves expedient to use b as the characteristic independent parameter of the problem.

The dependence on b of the scaled doublet length z , the beam widths x and y , and the scaled gradient

$$k = \lambda^2 = \frac{L_0^2 dB/dr}{B_0} \quad (14a)$$

are plotted in Fig. A8-7-2 and tabulated in Table A8-7-1, for b values between 1 and 100. We shall see that practical values lie in the range from 7 to about 25.

OPTIMIZATION

Distance from Target to Doublet

For some parameter choices it may be best to place the doublet beyond the wall of the reactor, i.e., to choose a value of L larger than its practical lower limit. If all parameters in b except L are fixed, then $b \sim L$. The field volume of the doublet is $\pi x^2 L_0 = \pi L_0^2 x^2 = b^2 x^2 z$. In Fig. A8-7-3 we have graphed

$$v = 10^{-6} b^2 x^2 z \quad (14)$$

against b . It has a minimum near $b = 7$. If the original choice of L gives $b < 7$, the volume of the doublet can be reduced by increasing L until $b = 7$.

Magnetic Field Strength

Suppose all parameters in b except B are

Fig. A8-7-2. Quadrupole doublet parameters as functions of $b = 0.32 L_0 b / (A b y q)$: $k = L_0^2 dB/dr / B_0$, $L_0 = L_0 / L$, $z = (L_0 + L_1) / L$, $x = X/L$, $y = Y/L$. (XRL 612-11088)

Fig. A8-7-3. Quadrupole doublet parameters used for optimization as functions of b : $u = z^2$, $e = 0.001 b^2 u$, $v = 0.001 b^2 u$, $a = X/(L_0 + L)$. (XRL 612-11087)

Table 7-11. Quadrupole doublet parameters as functions of b : $u = 2x^2$, $v = 0.001$ b'u, $a = X/(1+u)$, $e = 1 - dR/dx / B_0$, $c = 1/2(1 + e^2)$, $z = X/4$, $\alpha = X/4$, $\beta = 1/2$.

b	u	e	c	z	α	β
4	0.927	1.144	16.872	56.536	0.147	
4.5	0.963	1.117	16.795	55.818	0.149	
5	0.112	1.096	17.750	17.442	0.188	
5.5	0.185	1.174	5.739	15.541	0.155	
6	0.273	1.265	1.729	16.003	0.146	
6.5	0.380	1.370	3.858	16.200	0.172	
7	0.506	1.493	5.274	7.063	0.122	
7.5	0.650	1.839	2.811	0.925	0.165	
8	0.819	1.578	2.454	6.174	0.163	
9	1.173	1.224	1.936	5.116	0.164	
10	1.583	0.992	1.609	4.122	0.169	
12	2.031	0.832	1.372	3.958	0.144	
14	2.514	0.716	1.197	3.286	0.177	
16	3.144	0.628	1.063	3.318	0.191	
18	3.859	0.560	0.958	3.198	0.188	
19	4.081	0.566	0.873	2.946	0.822	
20	5.212	0.425	0.744	2.686	0.844	
22	6.307	0.307	0.631	2.503	0.864	
24	7.414	0.224	0.581	2.364	0.873	
26	8.868	0.191	0.525	2.256	0.883	
28	10.417	0.161	0.481	2.168	0.891	
30	11.456	0.143	0.444	2.095	0.898	
32	12.785	0.125	0.413	2.054	0.904	
34	14.134	0.109	0.386	1.981	0.909	
36	15.416	0.106	0.364	1.936	0.914	
38	16.777	0.107	0.318	1.844	0.923	
40	22.535	0.131	0.281	1.775	0.939	
45	26.158	0.156	0.157	1.729	0.935	
50	29.858	0.124	0.236	1.676	0.940	
55	33.363	0.113	0.218	1.639	0.943	
60	37.371	0.106	0.194	1.607	0.947	
70	44.070	0.094	0.180	1.557	0.952	
80	52.731	0.081	0.162	1.517	0.956	
90	60.503	0.076	0.148	1.485	0.959	
100	68.556	0.070	0.136	1.459	0.962	

For $b > 10$, the values of u , e , c , z , α , and β are given in Table 7-12.

fixed. Then $b \propto R$. The stored energy is $\propto X^2 R^2 \propto b^2 X^2$. The quantity

$$u = 10^{-3} b^2 X^2 \quad (15)$$

is also graphed in Fig. 7-5. It shows that the stored energy has a minimum at about $b=16$. It may be that the corresponding field is impractically large; but if not, it may be a good choice. The cost of the doublet also depends on its volume, which is proportional to

$$u \propto X^2 z \quad (16)$$

so that a more refined method might be to minimize some function of e and u .

Assessment

The radius to length ratio of the quadrupoles should not exceed some maximum value (about $1/5$)¹

in order that their effects be substantially similar to those of the assumed hard-edged variety. Therefore Fig. 7-5 shows

$$a = \frac{X}{(1 + \frac{1}{2}e^2)} \quad (17)$$

which gives the aspect ratio of the shortest quadrupole. Fortunately a is fairly flat up to $b=16$, the value for minimum stored energy. So long as one keeps $b < 16$, the aspect-ratio limitation leads directly to an upper bound on β_0 .

The quantities v , c , u , and a are also tabulated in Table 7-11.

Summary

A case where k should be greater than a

Table 7-12. Quadrupole doublet parameters used for optimization as functions of b : $u = 2x^2$, $e = 0.001$ b'u, $v = 0.001$ b'u, $a = X/(1+u)$.

b	u	e	v	a
4	0.927	1.144	2.252	7.568
4.5	0.963	1.117	2.267	7.485
5	0.112	1.096	1.877	7.342
5.5	0.185	1.174	1.700	7.107
6	0.273	1.265	1.543	7.214
6.5	0.380	1.370	1.413	7.149
7	0.506	1.493	1.309	7.088
7.5	0.650	1.839	1.229	7.057
8	0.819	1.578	1.169	7.047
9	1.173	1.224	1.100	7.050
10	1.583	0.992	1.079	7.170
12	2.031	0.832	1.089	7.290
14	2.514	0.716	1.122	7.455
16	3.144	0.628	1.171	7.626
18	3.859	0.560	1.229	7.815
19	4.081	0.566	1.252	8.008
20	5.212	0.425	1.423	8.448
22	6.307	0.307	1.670	8.815
24	7.414	0.224	1.894	9.222
26	8.868	0.191	2.138	9.626
28	10.417	0.161	2.405	10.02
30	11.456	0.143	2.695	10.42
32	12.785	0.125	3.002	10.80
34	14.134	0.109	3.331	11.18
36	15.416	0.106	3.680	11.56
38	16.777	0.107	4.041	12.47
40	22.535	0.131	5.729	15.35
45	26.158	0.156	6.943	14.19
50	29.858	0.124	8.284	15.01
55	33.363	0.113	9.754	15.81
60	37.371	0.106	11.35	16.58
70	44.070	0.094	14.94	18.07
80	52.731	0.081	19.07	20.86
90	60.503	0.076	23.74	20.86
100	68.556	0.070	28.96	22.17

reactor radius of 10 m is that of U^{238} at 100 GeV, with $\epsilon_0 = 4.25$ cm-rad, $r_0 = 1.35$ mm, $\theta_0 = 30$ mrad.

qB	L	b	V (volume)
5	10	2.13	0.28
5	33		0.11
10	10	4.26	0.15
10	16.4		0.11

A case where qB should be reduced below 20 T is shown by U^{238} at 20 GeV, with $\epsilon_0 = 1.48$ cm-rad, $r_0 = 0.81$ mm, $\theta_0 = 30$ mrad:

qB	L	b	c	a
20	10	27.5	0.13	11
30	10	41.3	0.15	15
11.6	10	16	0.105	9

EFFECT OF SPACE-CHARGE

Consider the envelope in vacuum for a beam with uniform density and equal emittances and radii in both transverse directions. The envelope radius r obeys the Kapchinskij-Vladimirskij equation:

$$\frac{d^2 r}{ds^2} - \frac{g}{2r} - \frac{\epsilon^2}{r^3} = 0, \quad (18)$$

$$g = 1.27 \cdot 10^{-10} \frac{qI}{A(8\gamma)^3}, \quad (19)$$

where I is the current in kA. Equation (18) can be integrated once to give

$$\theta^2 = \frac{dr}{ds} = g \ln \frac{r}{r_0} + \epsilon^2 \left(\frac{1}{r_0^2} - \frac{1}{r^2} \right). \quad (20)$$

At the chamber wall, $r \gg r_0$. Since $\theta_0 = \epsilon/r_0$, we have

$$\theta^2(L) \approx \theta_0^2 + g \ln(L\theta_0/r_0). \quad (21)$$

A second integration of Eq. (20) gives

$$s = \int_{r_0}^r \left[g \ln \frac{r}{r_0} + \epsilon^2 \left(\frac{1}{r_0^2} - \frac{1}{r^2} \right) \right]^{-1/2} dr \quad (22)$$

from which we obtain $s(r)$, and hence $r(s)$. Suppose we evaluate r, θ from Eqs. (20) and (22) at the chamber wall, $s=L$. The beam will look as if it comes from a source a distance $\bar{L}$ from the wall, where $\bar{L} = r(L)/\theta(L)$. It is found numerically that $\bar{L} \approx L$ even in cases where space charge has produced a large change in θ . For example, if $g = 2.65 \times 10^{-4}$, $\epsilon = 4.42 \times 10^{-5}$ m-rad, $r_0 = 1.47$ mm, $\theta_0 = \epsilon/r_0 = 0.03$. Then at $L = 10$ m, $r = 0.464$ m, $\theta = 0.049$, $\bar{L} = 9.42$ m, and θ evaluated from the approximate Eq. (21) gives $\theta_{\text{approx}} = 0.048$.

Therefore a good approximation for the effect

of space charge consists of replacing r_0 by r_s calculated from Eqs. (19) and (21). Thus we now take

$$b = 0.52 \frac{1B}{\sqrt{1+\gamma}} \left(\frac{q}{\gamma} \right) \quad (23)$$

$$\lambda = L + x(b)$$

$$Y = L + y(b) \quad (24)$$

$$L_Q = L + z(b)$$

We ignore space charge in the doublet — for the fact that $L \approx \bar{L}$ shows that the effect of space charge is substantially limited to the neighborhood of the target.

Optimal Charge State

We ask for the value of q that minimizes the field volume of the doublet, $V = V_Q$, for fixed B . Equation (21) is rewritten

$$\theta^2 = \theta_0^2 [1 + \epsilon^2 q^2] \quad (25)$$

$$\epsilon^2 = \frac{1.278 \cdot 10^{-10} J}{A^2 (1+\gamma)^3} \approx 0.1 \frac{L}{r_0} \quad (26)$$

where $J = 1/q$. Let

$$\alpha = \epsilon/\theta_0, \quad \beta = 1 + \alpha^2, \quad \mu = \epsilon/q.$$

$$\nu = \alpha/\mu = \sqrt{1 - \alpha^2}, \quad \gamma = (1 - \alpha^2)^{-1/2}.$$

Then $b = b_0 \nu$, where

$$b_0 = 894 \text{ LB} \sqrt{\frac{\epsilon \gamma}{AJ \ln(L\theta_0/r_0)}} \quad (27)$$

The volume becomes

$$V = \frac{\pi L^3 \theta_0^2 b_0^2 x^2 z}{b_0^2 - b^2} \quad (28)$$

The quantity $u = x^2 z$ is graphed in Fig. A8-5. It may be fitted approximately by

$$u = x^2 z \approx 700 b_0^{-1.86} \quad (29)$$

We substitute Eq. (29) in Eq. (28) and find that V is minimum when

$$\nu = b/b_0 = 0.694, \quad \alpha = 1.39, \quad \mu = 0.963 \quad (30)$$

Then using (24) we get the optimum charge and field

$$\bar{q} = 2.64 \cdot 10^{-10} \sqrt{\frac{N(\bar{\gamma})^3}{\beta \cdot n (1.4 L_0^2 / r_0)}} \quad (31)$$

$$b = 0.20 L_0 \sqrt{\frac{\beta \gamma}{\Delta l \cdot 2n (1.4 L_0^2 / r_0)}} \quad (32)$$

Furthermore, total ϵ is chosen to make $b=16$, in which case

$$\begin{aligned} b &= 16 & \epsilon &= 6.40 \\ \bar{q} &= 0.367 & i_F &= 0.284 \\ z &= 0.651 & a &= 8.82 \\ x &= 2.505 & y &= 0.861 \end{aligned} \quad (33)$$

To make $\bar{q}$ integral we must depart from q to the nearest integral value, q . We then obtain b and

$$\epsilon = \epsilon(q) = \frac{16}{q}, \quad z = \sqrt{1 + \epsilon^2} \quad (34)$$

$$b(q) = \frac{16}{q} \cdot \frac{z}{\beta \gamma} \cdot b, \quad a = \frac{8.82}{\beta \gamma} \quad (35)$$

We can still choose B to make $b=16$, so the values of the scaled parameters in Eqs. (33) apply. These are used with $\bar{\gamma}$ from Eq. (35), and Eq. (24) to obtain the doublet dimensions.

Design of b and ϵ

It is required to deliver a particular total energy W to the target, at a particular specific energy $\bar{C}$. If R is the range in the target material, in g/cm², r_0 the radius of the impact area for each of n beams, then $W = n r_0 R \bar{C}$, whence $r_0 \propto n^{-1/2}$. Obviously the current $I \propto n^{-1}$.

The emittance will also scale as $n^{-1/2}$ if we assume beam stacking in both transverse planes. Hence

$$\epsilon_0 = \epsilon / r_0 = \text{constant}.$$

Suppose B is fixed and for each value of n we determine q from Eq. (31). Then from Eq. (32) we see that $b \propto 1/1/2 \propto n^{1/2}$, or $n \propto b^2$. The field volume of all the quadrupoles is

$$V = n \pi L_0^2 = n L^2 (1.4 \epsilon_0)^2 \pi z^2 = b^2 x^2 z^2 \pi \epsilon.$$

This is the same function that gives the stored energy, for a fixed number of beams when B varies, and is minimum at $b=16$. Thus we have the following prescription:

Choose the charge state q to be the integer closest to $\bar{q}$, Eq. (31) and the field B so that b , Eqs. (32), (34), (35) have value $b=16$. Choose the number of beams n to give B a reasonable value, using the scaling $\epsilon_0 = \text{const}$, $I \propto n^{-1}$, $r_0 \propto n^{1/2}$, $\epsilon \propto n^{-1/2}$.

We have arrived at an optimum doublet design that scales only with ϵ_0 and L , apart from small variations due to the quantization of charge.

EXAMPLES OF OPTIMIZED DESIGNS

The optimization discussed in the previous section is illustrated with eight examples, based on Tables II and III of Technical Summary by Teng.² They are chosen to show changes in total energy, atomic weight, and kinetic energy. The beam emittances are chosen to give a beam radius at the wall (R_W) of about 20 cm. Our procedure leads to the selection of a charge state such that space charge increases the beam radius by about 40%. Thus we want $R_W = 1.5 \epsilon_0 \approx 0.2$ m or $\epsilon_0 = 0.2/10 \times 1.4 = 0.014$, which we round off to 15 mrad. In each case we adjust parameters to make $b=16$. With a chamber radius $L=10$ m, the lengths are

$$\begin{aligned} L &= 10 \text{ m}, & L_D &= 3.6 \text{ m}, \\ L_F &= 2.84 \text{ m}, & L_Q &= 0.51 \text{ m}. \end{aligned}$$

Table A8-7-111. Optimum beam parameters with target radii scaled from 2-beam values of Teng's Tables. Emittances correspond to $\epsilon_0 = 15$ mrad.

	W (MJ)	A	T (GeV)	I/q (kA)	n	q	ϵ_N (cm-mrad)	r_0 (mm)
1	10	238	100	5	2	5	2.12	1.55
2	10	238	40	1.88	8	3	1.17	1.25
3	10	127	60	5	2	3	1.52	0.90
4	10	127	20	2.5	12	2	0.78	0.86
5	1	238	40	1.25	2	4	0.90	0.96
6	1	238	20	1.25	4	2	0.68	1.04
7	1	127	40	1.25	2	4	0.64	0.48
8	1	127	20	2.5	2	2	0.73	0.81

Table A8-7-IV. Doublet parameters for beams of Table A8-7-III. Chamber radius $L = 10$ m, quadrupole lengths $L_Q = 3.67$ m, $L_P = 2.84$ m. Beam angle width at target, $\theta_0 = 15$ mrad; at wall, $\theta_W = \theta_0 \theta_0$. V is the field volume, $E_S = B^2 V$ ($\propto$ stored energy).

	B (T)	$\theta = \theta_W / \theta_0$	R_W (m)	X (m)	Y (m)	V (m ³)	E_S (T ² m ³)
1	5.00	1.36	0.204	0.511	0.176	10.7	267
2	5.06	1.38	0.208	0.520	0.179	44.2	1130
3	4.75	1.36	0.203	0.509	0.175	10.6	240
4	4.20	1.49	0.223	0.559	0.192	77.6	1351
5	4.01	1.46	0.220	0.549	0.189	12.3	198
6	5.16	1.36	0.204	0.510	0.175	21.3	567
7	2.82	1.36	0.204	0.510	0.175	10.6	84
8	4.21	1.49	0.224	0.560	0.193	12.8	228

The beam radii depend on θ , which is close to 1.39, but varies somewhat due to the integral charge condition. For that value we have

$$R_W = L \theta_0 \theta = 0.15 \theta = 0.208 \text{ m},$$

$$X = R_W \cdot x = 0.375 \theta = 0.521 \text{ m},$$

$$Y = R_W \cdot y = 0.129 \theta = 0.179 \text{ m}.$$

In each case we have increased the number of beams, n (starting at 2), until the field was 5 T or less.

The beam parameters are given in Table A8-7-III, and the doublet parameters in Table A8-7-IV. The results strongly suggest that high energy is desirable, with corresponding high charge state and small number of beams.

Table A8-7-V. Dependence of doublet volume and stored energy on charge state for case 1 of Tables A8-7-III and IV. Optimum charge state and field: $\bar{q} = 5.42$, $B(\bar{q}) = 5.0$ T. Beam number $n = 2$.

q	L (m)	B (T)	θ	b	V (m ³)	$E_S = B^2 V$ (T ² m ³)
1	10	5	1.02	4.28	254	6340
1	16.4	5	1.02	7	203	5072
2	10	5	1.07	8.17	32.6	814
3	10	5	1.14	11.4	15.7	393
4	10	5	1.24	14.0	11.7	293
5	10	5	1.36	16.0	10.68	267
6	10	5	1.49	17.5	10.85	271
6	10	4.57	1.49	16.0	12.83	268
7	10	5	1.63	18.6	11.6	289
7	10	4.08	1.63	16.0	15.4	283

In Table A8-7-V the effect of departures from optimum values is illustrated for case 1 of Tables A8-7-III and IV. For this case, Eq. (31) gives an optimum charge state $\bar{q} = 5.42$. Table A8-7-V shows the doublet volume V and stored energy E_S for q values between 1 and 7, when the magnetic field $B \leq 5$ T and the target-doublet separation $L \geq 10$ m. Table A8-7-V shows

- Volume and stored energy are minimized at $q = 5$.
- When $q = 1$, both V and E_S are decreased by increasing L until $b = 7$.
- For $q = 6, 7$, stored energy is decreased by reducing B until $b = 16$.

In Tables A8-7-VI and VII the combined effect of beam number n and charge state q on the total doublet volume V is shown for cases 2 and 6 of Table A8-7-III. They show a pronounced minimum at the optimum q values $\bar{q}$. However, the dependence on beam number is rather flat, showing that the volume of each doublet scales roughly as $1/n$. Since the optimum doublets shown in Table A8-7-IV are quite large, it is probably best not to reduce the beam number very much below the optimum values.

Table A8-7-VI. Doublet volume vs beam number n and charge q for case 2 of Table A8-7-III, when $B = 5$ T.

n \ q	Volume				$\bar{q}$	$\bar{B}$
	1	2	3	4		
2	83	47	58	81	1.68	9.20
4	121	47	46	68	2.30	6.76
6	161	51	44	50	2.75	5.63
8	200	57	45	48	3.14	4.95
10	240	63	47	48	3.47	4.47

Table A8-7-VII. Dependence of doublet volume, $V = n\pi L q^2$, on number of beams n and charge state q for case 6 of Table A8-7-III, $B = 5$ T. $\bar{q}(n)$ is the optimum charge state, $\bar{B}(q)$ is corresponding optimum field.

n	q →	Volume (m ³)			$\bar{q}(n)$	$\bar{B}(q)$
		1	2	3		
2		31.3	23.1	31.3	1.59	6.76
3		38.3	22.3	27.1	1.94	5.63
4		45.5	22.9	25.4	2.17	4.94

CONCLUSIONS

Provided that the beams are largely unaffected by gas or plasma in the reactor, and that the accelerator can deliver beams with a suitably chosen charge state, it seems possible to achieve the required final focusing with reasonably sized quadrupole doublets with pole-tip fields in the neighborhood of 5 T. It also seems likely that optimization of the rest of the transport line will lead to similar optimum values.

References

1. L. J. Laslett (private communication).
2. L. C. Teng, Technical Summary 5 of this report.

VII. Participants

PARTICIPANTS (EXCEPT FROM LBL AND LLL)

* Adney, Kenneth J.	(ERDA)
* Barton, Mark Q.	(BNL)
* Berley, David	(NSF)
* Bock, Rudolf M.	(GST, Darmstadt)
* Brown, Karl L.	(SLAC)
* Burke, Robert J.	(ANL)
* Cho, Yanglai	(ANL)
* Clauser, Milton J.	(Sandia)
* Cooper, Martin J.	(ERDA)
* Courant, Ernest D.	(BNL)
* Cutler, Roger T.	(ANL)
* Danby, Gordon T.	(BNL)
* Dove, William F.	(ERDA)
* Fasolo, John A.	(ANL)
* Fenster, Stanley	(ANL)
* Foss, Martyn H.	(ANL)
* Frank, Thurman G.	(LASL)
* Furth, Harold P.	(Princeton U., NJ)
* Gillespie, George H.	(Physical Dynamics, Inc., CA)
* Godlove, Terry F.	(NRL)
* Gula, William P.	(LASL)
* Hermannsfeldt, William B.	(SLAC)
* Kamnash, Terry	(U. Michigan, MI)
* Kelmley, George G.	(ORNL)
* Kerns, Quentin A.	(FNAL)
* Khoe, Tat K.	(ANL)
* Kim, Yong-Ki	(ANL)
* Kuswa, Glenn W.	(ERDA)
* Langsdorf, Jr., Alexander S.	(ANL)
* Lawrence, George P.	(LASL)
* Leiss, James F.	(NBS)
* Marshall, Lauriston C.	(So. Illinois U., IL)
* Martin, Ronald L.	(ANL)
* Maschke, Alfred W.	(BNL)
* Metz, William D.	(Science Magazine, DC)
* Mills, Frederick E.	(ANL)
* Morton, Phil L.	(SLAC)
* Mosher, David	(NRL)
* Olson, Craig L.	(Sandia)
* Pearlman, Jay S.	(Sandia)
* Prelec, Krsto	(BNL)
* Putnam, Sidney D.	(Physics International Co., CA)
* Rostoker, Norman	(U. Calif. Irvine, CA)
* Ruthe, E. W.	(Wayne State U., MI)
* Scott, Franklin R.	(EPRI)
* Sudan, R. N.	(Cornell U., NY)

*Regular participants; no asterisk indicates shorter visit.

* Sutter, David F.	(ERDA)
* Sweeney, Mary Ann	(Sandia)
* Symon, Keith R.	(U. Wisconsin, WI)
* Teng, Lee C.	(FNL)
* Thompson, William B.	(U. Calif. San Diego, CA)
* Tidman, Derek A.	(U. Maryland, MD)
* White, Milton G.	(Princeton U., NJ)
* Winterberg, F.	(U. Nevada, NV)
* Young, Donald E.	(FNL)

LBL PARTICIPANTS

Alonso, Jose R.
Clark, David J.
* Garren, Alper A.
Gilbert, William S.
* Grunder, Hermann A.
Gutbrod, Hans H.
Heckman, Harry H.
* Judd, David L.
* Keefe, Denis
Kunkel, Wulf B.
* Lambertson, Glen R.
* Lofgren, Edward J.
* Mobley, Richard M.
* Peterson, Jack M.
* Pyle, Robert V.
* Smith, Lloyd
Zajec, Emery

LLL PARTICIPANTS

Alonso, Carol T.
* Bangert, Roger O.
Chapline, George F.
* Freytag, E. Karl
Hausmann, A. Carl
* Hovingh, Jack
Lee, Edward P.
Lindl, John D.
Longerbeam, Gordon T.
* Maniscalco, James A.
Max, Claire E.
Meeker, Donald J.
* Neil, V. Kelvin
* Nuckolls, John H.
* Osher, John E.
Post, Richard F.
* Sahlin, Harry L.
Shearer, James W.

ABBREVIATIONS

ANL	Argonne National Laboratory (IL)
BNL	Brookhaven National Laboratory (NY)
EPRI	Electric Power Research Institute (CA)
ERDA	U.S. Energy Research and Development Administration (DC)
FNAL	Fermi National Accelerator Laboratory (IL)
LASL	Los Alamos Scientific Laboratory (NM)
LBL	Lawrence Berkeley Laboratory (CA)

LLL	Lawrence Livermore Laboratory (CA)
NBS	National Bureau of Standards (DC)
NRL	Naval Research Laboratory (DC)
NSF	National Science Foundation (DC)
ORNL	Oak Ridge National Laboratory (TN)
Sandia	Sandia Laboratories (NM)
SLAC	Stanford Linear Accelerator Center (CA)