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$\pi_{\mu 2}$: An Absolute Pion Beam Flux Monitor

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ABSTRACT

($\pi \rightarrow \mu \nu$)

A two-arm $\pi_{\mu 2}$ /beam monitor telescope provides one possible means of monitoring pion ($P_{\pi} < 400$ MeV/c) beam fluxes to better than 1%.

I. INTRODUCTION

This report presents the results of a feasibility study to determine the accuracy to which one can measure the relative and absolute pion beam flux using a two-arm $\pi_{\mu 2}$ ($\pi \rightarrow \mu \nu$) decay monitor telescope. Coulson *et al.*¹ have used the $\pi_{\mu 2}$ decay process as a beam flux monitor. They observed both the incident pion and the decay muon with multiwire proportional counters (MWPC's) placed directly in a low-intensity pion beam. For higher beam intensities they suggested using a two-arm telescope similar to Fig. 1 consisting of MWPC's set to look at the $\pi_{\mu 2}$ Jacobian cusp region. The purpose of this report is to determine to what accuracy the relative and absolute pion flux can be monitored using two simple scintillation counter telescopes set to look at the $\pi_{\mu 2}$ Jacobian plateau region. (For a detailed discussion of the $\pi \rightarrow \mu \nu$ decay kinematics see Coulson *et al.*¹ or Hagedorn²). Figure 1 shows the geometrical arrangement. Table I presents the results of calculations to determine fractional changes of the rate in the $\pi_{\mu 2}$ monitor as a function of systematic changes in the beam geometry. These changes will limit the accuracy to which one can determine the relative flux.

Effects of beam systematics are minimized in two ways with this geometry. First, the angle is chosen so that the telescopes see only the plateau region of the $\pi_{\mu 2}$ phase-space Jacobian (Fig. 2) thereby minimizing the effects of systematic beam phase-space fluctuations on each arm separately; and secondly, the use of a second telescope reduces the effects of beam phase-space shifts in just one telescope by about one order of magnitude. The two-telescope geometry can thus provide an accurate relative beam monitor for any situation where the number of particles passing through the telescope is a slowly varying function of angle [e.g., a scattering monitor, or $K_{\mu 2}$ ($K \rightarrow \mu \nu$) monitor for Kaon beams].

Fig. 1.

Schematic of the $\pi_{\mu 2}$ telescope geometry. $\mathbf{X}$ and β (not shown) are respectively the displacement and rotation of the pion beam out of the plane of the telescopes.

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TABLE I

Fractional rate changes in the $\pi_{\mu 2}$ two-arm telescope, at several incident pion momenta and detector angles (θ), for variations in beam phase space and telescope opening angle. The following values were assumed in the calculation (Fig. 1): $L_0 = 200$ cm, $X = Y = 0.5$ cm, $\alpha = \beta = 5 \times 10^{-3}$ rad, $\delta = 2.5 \times 10^{-4}$ rad, momentum error = 0.01 times nominal momentum. Single-arm rate changes are listed where appropriate.

PION MOMENTUM MeV/c	MOMENTUM ERROR $\times 10^{-2}$	IN THE TELESCOPE PLANE						OUT OF THE TELESCOPE PLANE		OPENING ANGLE ERROR (δ) $\times 10^{-3}$
		Y DISPLACEMENT ARM1 + ARM2 = SUM $\times 10^{-2}$ $\times 10^{-2}$ $\times 10^{-3}$			= ROTATION ARM1 + ARM2 = SUM $\times 10^{-2}$ $\times 10^{-2}$ $\times 10^{-2}$			X DISPLACEMENT $\times 10^{-3}$	ROTATION $\times 10^{-3}$	
NOMINAL DETECTOR ANGLE (θ) = 4.0 DEG										
50.	0.47	3.72	-3.46	= 1.29	8.02	-6.91	= 0.56	-0.74	-2.59	1.86
100.	0.87	3.72	-3.46	= 1.29	7.95	-6.84	= 0.56	-0.14	-2.56	1.84
150.	0.99	3.72	-3.46	= 1.29	7.77	-6.67	= 0.55	1.35	-2.50	1.80
200.	1.07	3.72	-3.46	= 1.29	7.47	-6.37	= 0.55	4.05	-2.40	1.72
250.	1.15	3.72	-3.46	= 1.29	6.99	-5.87	= 0.55	8.06	-2.23	1.60
300.	1.27	3.72	-3.46	= 1.29	6.21	-5.05	= 0.58	14.75	-1.95	1.41
350.	1.47	3.72	-3.46	= 1.29	4.97	-3.60	= 0.69	26.50	-1.48	1.08
400.	1.82	3.72	-3.46	= 1.29	2.87	-0.81	= 1.03	47.52	-0.64	0.49
NOMINAL DETECTOR ANGLE (θ) = 5.0 DEG										
50.	0.47	2.95	-2.79	= 0.82	6.26	-5.57	= 0.35	-0.76	-1.66	1.48
100.	0.88	2.95	-2.79	= 0.82	6.17	-5.48	= 0.34	-0.07	-1.63	1.45
150.	1.02	2.95	-2.79	= 0.82	5.92	-5.24	= 0.34	1.61	-1.56	1.39
200.	1.14	2.95	-2.79	= 0.82	5.46	-4.77	= 0.34	4.83	-1.43	1.28
250.	1.30	2.95	-2.79	= 0.82	4.64	-3.90	= 0.37	10.72	-1.19	1.07
300.	1.61	2.95	-2.79	= 0.82	3.12	-2.11	= 0.50	22.22	-0.73	0.65
350.	2.29	2.95	-2.79	= 0.82	-0.06	2.33	= 1.14	47.41	0.30	-0.27
400.	4.56	2.95	-2.79	= 0.82	-8.49	22.19	= 6.85	130.27	3.63	-3.20
NOMINAL DETECTOR ANGLE (θ) = 6.0 DEG										
50.	0.48	2.45	-2.34	= 0.57	5.15	-4.68	= 0.24	-0.66	-1.15	1.23
100.	0.89	2.45	-2.34	= 0.57	5.04	-4.56	= 0.24	-0.05	-1.12	1.20
150.	1.06	2.45	-2.34	= 0.57	4.70	-4.23	= 0.23	1.86	-1.05	1.11
200.	1.25	2.45	-2.34	= 0.57	4.01	-3.51	= 0.25	5.97	-0.88	0.94
250.	1.60	2.45	-2.34	= 0.57	2.56	-1.87	= 0.35	14.93	-0.52	0.56
300.	2.52	2.45	-2.34	= 0.57	-1.01	3.01	= 1.00	38.48	0.43	-0.47
350.	7.75	2.45	-2.34	= 0.57	-15.03	51.45	= 18.21	165.00	5.51	-5.78
NOMINAL DETECTOR ANGLE (θ) = 7.0 DEG										
50.	0.48	2.09	-2.01	= 0.42	4.39	-4.05	= 0.17	-0.63	-0.84	1.05
100.	0.91	2.09	-2.01	= 0.42	4.24	-3.90	= 0.17	0.04	-0.81	1.02
150.	1.12	2.09	-2.01	= 0.42	3.79	-3.45	= 0.17	2.24	-0.72	0.90
200.	1.44	2.09	-2.01	= 0.42	2.73	-2.31	= 0.21	7.75	-0.50	0.63
250.	2.27	2.09	-2.01	= 0.42	-0.13	1.25	= 0.56	23.32	0.12	-0.16
300.	7.67	2.09	-2.01	= 0.42	-13.40	36.57	= 11.58	118.18	3.97	-4.88
NOMINAL DETECTOR ANGLE (θ) = 8.0 DEG										
50.	0.48	1.83	-1.76	= 0.32	3.84	-3.57	= 0.13	-0.73	-0.65	0.93
100.	0.93	1.83	-1.76	= 0.32	3.65	-3.39	= 0.13	0.05	-0.62	0.88
150.	1.21	1.83	-1.76	= 0.32	3.05	-2.77	= 0.14	2.79	-0.51	0.73
200.	1.78	1.83	-1.76	= 0.32	1.33	-0.85	= 0.24	10.40	-0.19	0.27
250.	4.47	1.83	-1.76	= 0.32	-5.83	10.42	= 2.29	48.11	1.35	-1.90
NOMINAL DETECTOR ANGLE (θ) = 9.0 DEG										
50.	0.48	1.62	-1.57	= 0.26	3.41	-3.20	= 0.10	-0.71	-0.51	0.83
100.	0.96	1.62	-1.57	= 0.26	3.19	-2.98	= 0.10	0.14	-0.48	0.77
150.	1.34	1.62	-1.57	= 0.26	2.36	-2.12	= 0.12	3.31	-0.35	0.56
200.	2.47	1.62	-1.57	= 0.26	-0.65	1.49	= 0.42	15.97	0.16	-0.26
NOMINAL DETECTOR ANGLE (θ) = 10.0 DEG										
50.	0.48	1.46	-1.42	= 0.21	3.08	-2.91	= 0.08	-0.69	-0.42	0.75
100.	1.00	1.46	-1.42	= 0.21	2.81	-2.64	= 0.08	0.21	-0.38	0.68
150.	1.54	1.46	-1.42	= 0.21	1.66	-1.42	= 0.12	4.12	-0.21	0.39
200.	4.38	1.46	-1.42	= 0.21	-4.65	7.38	= 1.37	28.73	0.81	-1.43

Fig. 2.
 π_{u2} phase-space Jacobian vs laboratory angle
for pion momenta between 50 and 400 MeV/c.

II. CALCULATIONS FOR BEAM PHASE SPACE SHIFTS

This section deals with the details of the calculations which were made to establish Table I. Figure 1 shows the geometry of the nominal system and labels the various parameters used in the calculation. The rate R is determined from

$$R = \int J D P d\Omega \quad (1)$$

where

J = Phase-space Jacobian for π_{u2}

D = Decay length seen by the detector

P = Pion decay probability per unit length
 $\approx (\beta\gamma\lambda_{\pi}cr)^{-1}$

$d\Omega$ = The detector solid angle at the detector origin.

The rate change calculations are as follows.

1. The beam is displaced by the distance Y in the plane of the two arms of the monitor.

The displacement has the effect of changing the solid angle of, and the decay length seen by, the detector. As long as the slope of the phase-space Jacobian vs angle is fairly constant over the angular acceptance ($\pm\gamma$), one can assume that the average value of the Jacobian does not change with the displacement Y . (This statement is valid provided that γ is made sufficiently small that the telescope does not see the Jacobian cusp region.) The nominal rate R_0 is [see Eq. (1)]

$$R_0 = J_0 D_0 P_0 d\Omega_0$$

where

$$d\Omega_0 = A / L_0^2$$

and A is some representative detector cross-sectional area $\sim (A_{S1} + A_{S2})/2$. The rate R_1 corresponding to the displacement Y is

$$R_1 = J_1 D_1 P_1 d\Omega_1$$

where

$$D_1 = D_0 [L_0 - Y / \sin(\theta)] / L_1$$

$$d\Omega_1 = A / [L_0 - Y / \sin(\theta)]^2$$

The fractional rate changes are

$$F_1 = (R_1 - R_0) / R_0$$

$$= Y / \sin(\theta) / [L_0 - Y / \sin(\theta)], \quad \text{ARM 1;}$$

$$F_2 = (R_2 - R_0) / R_0$$

$$= -Y / \sin(\theta) / [L_0 + Y / \sin(\theta)], \quad \text{ARM 2;}$$

$$F_T = (F_1 + F_2) / 2$$

$$= [Y / \sin(\theta)]^2 / \{L_0^2 - [Y / \sin(\theta)]^2\},$$

$$\text{TOTAL CHANGE}$$

$$(12)$$

Note that the F s are independent of the choice of A .

2. The beam is displaced by the distance X perpendicular to the plane of the two arms of the

monitor. The solid angle, decay length, and the Jacobian will all change by small amounts. The nominal rate is again

$$R_0 = J_0 D_0 P_0 d\Omega_0$$

where

$$d\Omega_0 = A / L_0^2,$$

The rate R_X for the X displacement is

$$R_X = J_X D_X P_0 d\Omega_X$$

where

$$D_X = D_0 \sqrt{L_0^2 + X^2} / L_0$$

$$d\Omega_X = A / (L_0^2 + X^2)$$

$$J_X = J \left(\theta' = \cos^{-1} [\cos(\theta) / \sqrt{1 + X^2 / L_0^2}] \right),$$

The fractional rate change in both telescopes is the same and therefore the total rate change is

$$\begin{aligned} F_T &= (R_X - R_0) / R_0 \\ &= J_X / (J_0 \sqrt{1 + X^2 / L_0^2}) - 1. \end{aligned} \quad (3)$$

Note that for most cases $J_X \approx J_0$.

3. The beam is rotated by α with respect to the nominal beam about an axis which is perpendicular to and at the point of intersection of the two telescope axes (the beam remains in the plane of the telescopes). The solid angle will remain constant while the decay length and the phase-space Jacobian change.

$$R_0 = J_0 D_0 P_0 d\Omega_0$$

$$R_\alpha = J_\alpha D_\alpha P_0 d\Omega_\alpha$$

where

$$\begin{aligned} D_0 &= \frac{L_0 \sin \gamma}{\sin(\pi - \theta - \gamma)} + \frac{L_\alpha \sin \gamma}{\sin(\theta - \gamma)} \\ &\approx 2 L_0 \gamma \theta / (\theta^2 - \gamma^2) \end{aligned} \quad (4)$$

$$\begin{aligned} D_\alpha &= \frac{L_0 \sin \gamma}{\sin[\pi - (\theta - \alpha) - \gamma]} + \frac{L_\alpha \sin \gamma}{\sin[(\theta - \alpha) - \gamma]} \\ &\approx 2 L_0 \gamma (\theta - \alpha) / [(\theta - \alpha)^2 - \gamma^2]. \end{aligned}$$

The fractional rate changes are

$$F_{L_0} = (D_0 J_{\alpha L_0} - D_0 J_0) / D_0 J_0, \quad \text{ARM 1;}$$

$$F_{L_\alpha} = (D_\alpha J_{\alpha L_\alpha} - D_\alpha J_\alpha) / D_\alpha J_\alpha, \quad \text{ARM 2;} \quad (5)$$

$$F_T = (F_{L_0} + F_{L_\alpha}) / 2, \quad \text{TOTAL RATE CHANGE.}$$

Note that $J_\alpha = J_0$.

4. The beam is rotated by β about an axis, perpendicular to the nominal beam but in the plane of the two telescopes, at the point of intersection of the telescope axis. The solid angle will remain constant while the decay length and phase-space Jacobian change. The nominal rate is

$$R_0 = J_0 D_0 P_0 d\Omega_0$$

whereas the rate R_β is

$$R_\beta = J_\beta D_\beta P_0 d\Omega_\beta$$

$$J_\beta = J \left(\theta' = \cos^{-1} [\cos(\theta) \cos(\beta)] \right)$$

$$D_\beta = 2 L_0 \gamma \theta' / (\theta'^2 - \gamma^2); \quad (\text{see Eq. 4})$$

Note

$$\begin{aligned} \theta' &= \cos^{-1} [\cos(\theta) \cos(\beta)] \\ &\approx \sqrt{\theta^2 + \beta^2} \end{aligned}$$

$$D_\beta = 2 L_0 \gamma \theta' / (\theta'^2 - \gamma^2).$$

The rate change is therefore

$$F_T = (J_\beta D_\beta - J_0 D_0) / J_0 D_0. \quad (6)$$

5. There is a rate error due to the uncertainty in the pion beam momentum. An increase in the pion momentum has the following two effects: a) it decreases the decay probability P and b) it increases the phase-space Jacobian J . In the momentum region considered ($400 \text{ MeV}/c > P_\pi > 50 \text{ MeV}/c$), there is no angle where the Jacobian and decay probability effects cancel. A 1% increase in the pion beam momentum corresponds roughly to a 1% increase in rate if the detectors are in the plateau region of the Jacobian (See Table I and Fig. 2.)

Table I shows the effects in the telescope due to beam phase-space fluctuations. The length of the telescopes (L_0 in Fig. 1) was assumed to be 200 cm. The angular acceptance ($\pm \gamma$) was 0.5° and the telescope angle to the beam (θ) was stepped between 4° and 10° . The following beam uncertainties were

assumed in the calculation: a) position, ± 0.5 cm, b) slope, 5×10^{-3} rad = 0.5 cm/100 cm, c) beam momentum uncertainty, 1%. One should note that the error inherent in the telescope is minimized by going to the largest angle possible while remaining within the π_{u2} Jacobian plateau region.

III. ABSOLUTE MONITORING CONSIDERATIONS

In this section, the effects which limit the telescope as an absolute monitor are considered.

1. A possible misalignment of the telescope can decrease the angle (2θ) between the two arms by the angle δ . The rate change will correspond to the effect calculated in Sec. II (3) above for F_u with an angle variation of $\alpha = \delta/2$.

$$F_T = F_u (\delta/2)$$

$$(D_{u1} J_{u1} \pm D_{u2} J_{u2} - D_{u0} J_{u0}) / D_{u0} J_{u0} \quad (7)$$

Column 11 of Table I contains the fractional rate uncertainty due to this error. The assumed tolerance for the relative alignment of the two arms was 0.05 cm/200 cm.

2. Accomplishing absolute beam monitoring to less than 1% requires a measurement of the beam phase space to this desired accuracy. This phase-space distribution must be used in a π_{u2} Monte Carlo to determine the acceptance of the detection system.

3. The area of the detectors must be measured to a precision which will not limit the accuracy of the monitor.

4. This section is concluded by considering the edge effects of the detectors (see Fig. 3). An edge effect is defined as the geometrical situation where a muon can trigger a telescope without passing completely through the counters. There is a twofold effect involved. First, there is a maximum solid angle fractional uncertainty which is

$$\left[\frac{A}{(S_1 + S_2)^2} - \frac{A}{(S_1 + S_2 + t)^2} \right] / \frac{A}{(S_1 + S_2)^2} = \frac{2t}{(S_1 + S_2)}$$

Secondly, the decay length is uncertain by the amount

$$\Delta D/D = 2t/S_2$$

These errors are minimized by minimizing the effective thickness (t) of the detectors (with scin-

Fig. 3.

Schematic of the geometry used to calculate uncertainties due to counter edge effects in the telescope. A is the cross-sectional area of the detectors.

tillators by running at much higher gain than necessary or by using thin solid-state detectors). It appears that the effect of this uncertainty on the rate can be reduced below 0.1%.

IV. OTHER CONSIDERATIONS AND CONCLUSION

This section is concerned with the monitor characteristics which will depend upon the experimental situation.

1. Effects due to beam-scattered particles: an estimate of the background in the telescope due to beam scattering can be made by looking at the time-of-flight distributions of particles triggering the muon monitor detectors. One should observe two peaks in this distribution corresponding to forward and backward center-of-mass π_{u2} decays. The fraction of events lying outside the peak regions provides the non- π_{u2} contamination estimate. The contamination effect is minimized by keeping the beam in a vacuum and limiting the beam phase space sufficiently to prevent the beam from striking upstream beam elements and the vacuum pipe.

2. The detectors must be sufficiently limited in size and removed from the beam so that pile-up difficulties are not encountered by the detectors.

3. Finally, a direct measurement of the accuracy of the monitors should be made. This may be done in a beam of known composition at a low intensity where direct counting methods are possible.

In conclusion, it appears that the $\pi_{\mu 2}$ decay monitor offers the possibility of monitoring the absolute pion flux to the level of the determination of the pion beam momentum above 0.5%. It requires one to monitor the beam phase space to an accuracy assumed in the calculation for Table I if this level of accuracy is required.

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