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DEVELOPMENT OF A LUMPED PARAMETRIC
MODEL FOR SCOPING INVESTIGATIONS OF
UNCERTAINTIES IN FAST REACTOR
PROBABILISTIC SAFETY ANALYSIS

Progress Report

by

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ABSTRACT

The objective of this research is to explore the possibility of the development of a novel reactor safety analysis methodology suitable for a parametric investigation of uncertainties in the progression of severe fast reactor accidents. The essential feature of this approach is a description of the reactor state by means of volumetric distributions. The distribution of volume of reactor materials, such as coolant, clad, and fuel, with temperature and in the case of fuel material, also with power, is termed in this report, a volumetric distribution. Stationary volumetric distributions are computed from detailed spatial temperature and power distributions of materials in the steady state reactor. Stationary volumetric distributions and other reactor physics quantities form the input for the reactor transient calculations in which the accident progression is described by the behavior of transient volumetric distributions.

This report discusses the representation of spatial temperature distributions, the theory and calculation of stationary volumetric distributions, and includes examples of single subassembly and reactor distributions. The status of reactor neutronic code development and application to this work is discussed and results are displayed.

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I. INTRODUCTION

Substantial uncertainties exist in the detailed description of fast reactor accidents with whole core involvement. The evaluation of these uncertainties may be accomplished by the use of consequence distributions which are treated by the methods of probabilistic safety analysis. Consequence distributions are obtained by calculation of the effect of variations in parameters during the progression of an accident. Explicit calculation of the accident progression is required because of the highly nonlinear effect of some parameter changes. Since thousands of accident sequence calculations may be needed for significant uncertainty investigations, the computing effort required for a single calculation must be on the order of seconds to minutes of computer time. With existing accident analysis codes, such as SAS, a typical calculation may require many minutes or hours; such codes are not suitable for the generation of consequence distributions.

The purpose of this work is to explore the possibility for developing a method capable of rapid calculation of LOF accidents. A novel conceptual approach, based on the use of volumetric distributions, is applied to describe the reactor state during the transient. Transient volumetric distributions of important reactor materials will be determined from stationary volumetric distributions and proper time dependent information, such as the reactor power trace. Reactivity feedback is calculated from the transient volumetric distributions and a proper characterization of the spatial location of the compositional perturbations.

A substantial advantage of this approach is that it does not rely on the "channel" modeling of the reactor, in which the behavior of a group of subassemblies is assumed to be coherent. The use of volumetric

distributions therefore permits the investigation of within subassembly incoherencies and accidents involving partially or totally destroyed duct structure.

This report describes the progress made during the year from October 1974 to October 1975. Work during this period fell into two general categories: first, the development or refinement of the reactor neutronics programs and data base of reactor physics quantities necessary for the description of the reactor at the onset of a transient, and second, the development and implementation of the volumetric distribution concept.

II. REACTOR NEUTRONICS CALCULATIONS

1. Introduction and Objectives

Calculation of stationary volumetric distributions involves the determination of temperature fields within the material concerned. Steady state temperature fields can be calculated from spatial power distributions, coolant flow rates, and other material properties. As mentioned earlier, one objective of this work is the explicit consideration of the effect of within subassembly incoherencies. Variations in spatial power distributions both over the reactor and within the subassembly are a primary cause of these incoherencies. These variations result from enrichment zoning, control rod perturbations, fission product buildup, the fuel management scheme adopted and other aspects of the particular reactor design.

Substantial differences exist between the initial "clean" core of the reactor and the equilibrium "dirty" core. In the initial core the fuel composition is homogeneous and distortions in the distribution result from control rod perturbations and enrichment zoning. As the reactor is operated the fissile inventory decreases in the core and the fission product poisons accumulate. Expended fuel is replaced so that a fresh subassembly may be juxtaposed with one with two thirds the allowable burnup and another with one third burnup. Finally, spectral changes result from the altered reactor compositions. The spatial power distribution of the equilibrium core may therefore be very different from the initial one. In view of these considerations a second objective of this work is a comparison of the transient behavior of the reactor in the initial and equilibrium states.

Several types of neutronics calculations are required to provide the detailed spatial power distributions necessary for the volumetric distributions and betas, Doppler constants, and other physics quantities necessary for the

kinetics calculations. In order to perform these calculations both one and two dimensional diffusion codes and several auxiliary processing codes are necessary. Much code development work had to be undertaken since the standard production codes available were not suited to the particular requirements of this investigation.

2. Reactor Power Distributions and Kinetics Parameters

In the determination of the reactor power distribution a typical calculation sequence has been followed. A basic multigroup cross section library is used in a one-dimensional diffusion code to calculate collapsed cross sections. A few group two-dimensional triangular mesh diffusion calculation is then performed using the collapsed cross sections.

Evaluation of the volumetric distributions has been based on a representation of the power variation within the subassembly by linear functions (planes). The "power planes" can be obtained from a triangular mesh neutronics calculation-which provides the average power in each of the six triangles comprising an individual subassembly. Each set of six average power-values is "fitted" to a power-plane for that subassembly. The resulting set of planes describes the spatial variation of power both across the reactor core and within individual subassemblies.

2.1 One-Dimensional Diffusion Calculations

The two dimensional calculations are performed with a small number of energy groups (6) for reasons of economy. Properly averaged cross sections are obtained by collapse within the one-dimensional calculation (with 21 groups). Kinetics parameters are also calculated with the one-dimensional code since a many group calculation is necessary to provide properly averaged effective betas, generation times, and Doppler constants. A suitable one-dimensional code has been developed by modification of the

existing QXI code. QXI was chosen because its structure was ideal for the proposed modifications and it already incorporated the capability to correctly compute dynamic reactivity.

2.2 Two-Dimensional Diffusion Calculations

The 2DB code is used for equilibrium core calculation as well as for the initial core calculation. In both calculations, the triangular mesh option is used with half reactor symmetry assumed. This permits explicit representation of control rods and provides a detailed determination of the flux and power spatial variations. In order to obtain the power distribution of the equilibrium core, a burnup calculation must be performed. Burnup calculations are often performed in R-2 geometry with only enough spatial zones to adequately represent the spatial variation of compositions with time. This method is relatively simple to implement and will provide satisfactory time dependent reactor compositions. A detailed power distribution, however, can be obtained only by determining the time dependent composition of each core subassembly individually. Approximate loading equilibrium can be achieved by performing the burnup calculation over a time interval which includes five refuelings.

3. Current Status

The power distribution for the initial critical core has been computed and used to calculate the stationary volumetric distributions.

The burnup calculation, however, cannot be performed yet since no group constant set with sufficiently accurate lumped fission products appears to be available. Mr. J. Cahalan at RAS/ANL has kindly provided this effort with a twenty-one group cross section library. Unfortunately, the lumped fission product cross sections of ENDF/B-III have zero scattering cross sections and exhibit some other, probably minor, inconsistencies.

In order to proceed with this work, an effort is under way, currently, to generate at Purdue fission product cross sections, which include scattering cross sections and which have the consistent relation between the transport and other cross sections.

III. VOLUMETRIC DISTRIBUTIONS

1. Introduction

The volumetric distributions to be developed here are the steady state or stationary volumetric distributions. Using detailed spatial temperature distributions the distributions of clad, coolant, and fuel volume with temperature are determined. These volumetric distributions together with other reactor physics quantities form the required input data for the transient calculations. The calculation of the volumetric distributions has been simplified by the use of approximate analytic models for the power distribution, and for the coolant, clad, and fuel temperature distributions. These models, since they provide the desired quantities in closed form, are much easier to use than more complex models, yet exhibit all basic dependencies.

Since the concept of volumetric distributions may be unfamiliar to those otherwise well versed in fast reactor safety methodology, basic concepts are described first to aid in understanding the results presented later.

2. Derivation of the General Expression for Material-temperature Volumetric Distributions.

The derivation of the volumetric distribution in three space dimensions is analogous to the derivation of the areal distribution in two dimensions (comp. Ref. 1), or the lineal distribution in one dimension. In each case, the variation of the appropriate geometric measure is expressed as a function of the field variable, such as temperature. The derivation of the areal distribution is given here since it is exactly analogous to the corresponding volumetric quantity but geometrically less complex.

In two dimensions the variation of the temperature within a material is denoted by $T(x,y)$. An isotherm, of a particular temperature T , through the material is given by: $T(x,y) = T$, which is the equation of a line. The areal distribution is obtained from the area within the material for which the temperature is between T and $T + dT$, which is the area between the two isotherms at T and $T + dT$. The desired area may be found by integration along the isotherm. Thus, expressing this differential area as $a(T)dT$, with $a(T)$ representing the unknown areal distribution, one obtains as definition:

$$a(T)dT = \oint_C dA ,$$

with C denoting the isotherm.

As is shown in figure 1A, the differential area element is given by $dA = dl \cdot dw$, where dl is the differential arc length, and dw is the projection of the distance between the two isotherms (called here "differential width").

Thus,
$$a(T)dT = \oint dl \cdot dw .$$

The differential width, dw , may be expressed in terms of dT and the temperature gradient by multiplying it with the derivative of $T(x,y)$ in the perpendicular direction, i.e.

$$dw \cdot \left(\frac{\partial T}{\partial n} \right) = dT .$$

Along the isotherm, the directional derivative is,

$$\begin{aligned} \frac{\partial T(x,y)}{\partial n} &= \vec{\nabla} T(x,y) \cdot \vec{n}(x,y) \\ &= \vec{\nabla} T(x,y) \cdot \frac{\vec{\nabla} T(x,y)}{|\vec{\nabla} T(x,y)|} \\ &= |\vec{\nabla} T(x,y)| , \text{ since the normal is equal to} \end{aligned}$$

the unit gradient.

Figure 1A

Figure 1B

Substituting,

$$dw \cdot |\vec{\nabla} T| = dT$$

or

$$dw = \frac{dT}{|\vec{\nabla} T|}.$$

The areal distribution is now given by,

$$a(T)dT = \oint_C d\ell \cdot \frac{dT}{|\vec{\nabla} T|}, \text{ where } |\vec{\nabla} T| \neq 0.$$

The differential arc length may be expressed in terms of x and y ,
see figure 1B,

$$d\ell^2 = dx^2 + dy^2.$$

Thus,

$$a(T) = \oint_C \frac{\sqrt{dx^2 + dy^2}}{|\vec{\nabla} T|} = \oint_C \frac{\sqrt{dx^2 + dy^2}}{\sqrt{\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2}}.$$

The line integral can be replaced by an ordinary integral by
parameterization of the isotherm. One possible parameterization is:

$$\begin{aligned} x &\rightarrow \xi \\ y &\rightarrow y(\xi), \end{aligned}$$

where $y(\xi)$ is obtained by solving the isotherm equation,

$$T(\xi, y(\xi)) = T.$$

Substituting this parametrization gives

$$a(T) = \int \frac{\sqrt{d\xi^2 + \left(\frac{dy}{d\xi}\right)^2} d\xi^2}{\sqrt{\left(\frac{\partial T}{\partial \xi}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2}}.$$

The equation of the isotherm [$T(x,y) = \text{constant}$] may be differentiated to determine $\frac{dy}{d\xi}$:

$$\frac{d}{d\xi} [T(\xi, y(\xi))] = \frac{dT}{d\xi} = 0$$

$$\frac{\partial T}{\partial \xi} + \frac{\partial T}{\partial y} \frac{dy}{d\xi} = 0$$

$$\frac{dy}{d\xi} = - \frac{\frac{\partial T}{\partial \xi}}{\frac{\partial T}{\partial y}} .$$

Using this relation in the areal distribution gives

$$a(T) = \frac{\int \sqrt{d\xi^2 + \left(\frac{\partial T / \partial \xi}{\partial T / \partial y}\right)^2 d\xi^2}}{\int \sqrt{\left(\frac{\partial T}{\partial \xi}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2}} ,$$

and finally,

$$a(T) = \int \frac{d\xi}{\left| \left(\frac{\partial T}{\partial y} \right) \right|} .$$

The choice of parameters should be made only after $T(x,y)$ is known. Some possible parameterizations may introduce singularities into the integral or cause other difficulties; then the parametrization has to be changed.

The derivation of the volumetric distribution is now given in analogy to the areal distribution. In three dimensions the variation of temperature within a material is denoted by $T(x,y,z)$. The volumetric distribution may be determined by finding the volume of material for which the temperature is between T and $T + dT$. An isotherm, of a particular temperature T , is defined by $T(x,y,z) = T$, which is the equation of a surface in two dimensions. The desired volume may be found by integrating

the volume between the two contour surfaces at T and $T + dt$:

$$V(T) dT = \oint_S (d\sigma \cdot dh) .$$

The differential surface area element, $d\sigma$, and the differential height, dh , are shown in Figure 2A.

The differential height, dh , may be expressed in terms of $T(x,y,z)$ and dT , as in the case of the areal distribution,

$$|\vec{\nabla}T| dh = dT .$$

Thus,

$$V(T) dT = \oint_S d\sigma \frac{dT}{|\vec{\nabla}T|}$$

and the volumetric distribution is defined by the integral:

$$V(T) = \oint_S \frac{d\sigma}{|\vec{\nabla}T|} .$$

The surface integral may be expressed as ordinary double integral by projecting $d\sigma$ onto the x - y plane (dA is the projected differential area):

$$|\cos \theta| d\sigma = dA ,$$

as shown in Figure 2B. The cosine is given by the dot-product between the normal $\vec{n}(x,y,z)$ on the contour-surface and the unit vector in z -direction:

$$\cos \theta = \vec{n}(x,y,z) \cdot \vec{k}$$

with

$$\vec{n}(x,y,z) = \frac{\vec{\nabla}T}{|\vec{\nabla}T|} = \frac{\frac{\partial T}{\partial x} \vec{i} + \frac{\partial T}{\partial y} \vec{j} + \frac{\partial T}{\partial z} \vec{k}}{|\vec{\nabla}T|} ;$$

then

$$\cos \theta = \frac{\frac{\partial T}{\partial z}}{|\vec{\nabla}T|} .$$

The differential surface element can now be expressed as,

Figure 2A

Figure 2B

$$d\sigma = \frac{dA}{|\cos \theta|} = dA \frac{|\vec{\nabla} T|}{\left| \frac{\partial T}{\partial z} \right|}$$

Finally,

$$V(T) = \iint_{A_I(T)} \frac{dA}{\left| \frac{\partial T}{\partial z} \right|},$$

or in cartesian coordinates,

$$V(T) = \iint_{A_I(T)} \frac{dx \, dy}{\left| \frac{\partial T}{\partial z} \right|},$$

where the area of integration $[A_I(T)]$ is defined by the projection of the isothermal surface onto the x-y plane. The term, $\left| \frac{\partial T}{\partial z} \right|$, can be calculated by solving the function $T(x,y,z)$ for z in terms of the temperature T .

3. Statement of the General Expression for Material-Power Volumetric Distributions

The variation of fuel power in three spatial dimensions is analogous to the temperature distribution and is given by $P = P(x,y,z)$. An iso-power surface is given by $P(x,y,z) = P$, which is the equation of a (two-dimensional) surface. The volumetric distribution of power can be given by using the result of the previous derivation of the material-temperature volumetric distribution and substituting the power function, P , for temperature, i.e.

$$V(P) = \oint_S \frac{d\sigma}{|\vec{\nabla} P|} = \iint_R \frac{dx \, dy}{\left| \frac{\partial P}{\partial z} \right|},$$

where S denotes the iso-power surface and R the projection of this surface onto the x-y plane.

The fact that the power distribution passes through a maximum in the axial direction presents no problem since the integration may be performed by dividing the surface S into two single valued surfaces. At the core midplane $\left| \frac{\partial p}{\partial z} \right|$ vanishes if one uses the normal cosine axial representation of reactor power. The integral will therefore be improper whenever the surface S passes through the core midplane. This aspect of the fuel-power volumetric distribution is discussed further in Section 7.

4. Approaches to the Numerical Evaluation of Volumetric Distributions

The general expression for volumetric distributions developed in the previous section shows that computation of volumetric distributions requires the previous calculation of spatial temperature (or power) fields. Once the temperature or power field is known the volumetric distribution is obtained by numerical integration. Temperature fields may be obtained directly by numerical solution of appropriate differential equations or indirectly by evaluation of analytic functions which solve the differential equations or have been fitted to numerical solutions. The coolant temperature field is computed first since it provides the necessary boundary conditions for solution of clad and fuel temperature fields.

4.1 Approaches to the Evaluation of Coolant Temperature Fields

Within an individual subassembly coolant flows in subchannels formed by spaces between fuel pins and between the outer row of fuel pins and the subassembly walls. The use of constant diameter wire wrap results in subchannels of equal size except along the subassembly periphery where the fuel pin lattice is interrupted. The axial spiraling of the wire wrap produces intersubchannel coolant mixing. Thermal transport results from temperature gradients induced by unequal subchannel cooling and power skew.

Consideration of these complexities is essential since the axial temperature rise near the subassembly wall may be only one-half of the central value. Due to insufficient knowledge of the physical processes involved and the mathematical complexity, computation of detailed three dimensional temperature fields is a formidable task. Approximate solutions are normally obtained by the introduction of simplifying assumptions. These assumptions are manifested in the characteristics of the temperature field and consequently also in the coolant temperature volumetric distributions.

4.1.1 The Detailed Multi-Flow Channel Approach

In this approach, subchannel lateral variations in temperature and other coolant properties are neglected in comparison to the much larger axial variations. Hydraulic and thermal interactions between subchannels are described by equations relating mass, momentum, and energy balances. The three-dimensional temperature field is obtained by solution of a system of coupled differential equations, one per subchannel. With over 400 subchannels involved in a detailed description of the 217 pin bundle of CRBR, solution of this system of equations requires substantial computational effort. This approach is described in more detail in the recent work² by Nijssing and Eifler in which results of HERA³ computer code are presented.

4.1.2 The "Lumped" Subchannel Approach

The "lumped" subchannel approach is a simplification of the previous approach. Temperature and other properties of the coolant are assumed to vary identically in groups of subchannels, or channels. The number of coupled equations can be substantially reduced with a concurrent reduction

in computing effort. This approach is essentially the one used in the COBRA⁴ code. The required computing time is still prohibitive for this application, however.

A further simplification in which the inter-channel coupling is neglected is the independent channel approach. The axial temperature distribution in each channel is described by a single differential equation which can be solved analytically. The three dimensional temperature field is generated by evaluating the axial temperature distribution in each channel.

4.1.3 The Analytic Function Approach

Detailed multi-flow channel calculations for assemblies with uniform and skewed power distributions have been performed using the approach described in 4.1.1. From these results² a satisfactory analytic representation of the subassembly coolant temperature distribution by a continuous function was obtained by Ott¹. This function describes the temperature rise as a product of radial and axial components which depend only on the coolant mass flow rate and the power distribution. The three dimensional temperature field for an individual subassembly can thus be easily generated by evaluating these functions. A particular advantage of this approach is that the coolant-temperature volumetric distribution can be generated by inserting the temperature function into the general expression for volumetric distributions and evaluating directly.

4.1.4 Comparison of Approaches

A comparison of the volumetric distributions generated by the independent channel approach and the analytic function approach is shown in figure 3. The coolant-temperature volumetric distribution for a single subassembly with a uniform lateral power distribution is shown

Figure 3. A comparison of results for different approximations to the coolant-temperature volumetric distribution for a single subassembly

for three cases. In case 1 the volumetric distribution is generated from a temperature distribution specified by the analytic function described in the previous section; in cases 2 and 3 the volumetric distribution is generated from a temperature distribution specified by a function solving the differential equation for an independent flow channel. In case 2 the volumetric distribution is the sum of volumetric distributions for three channels with different mass flow rates. The channels are located to describe decreased temperature rise near the subassembly periphery. Since temperature is laterally constant within a channel, the radial temperature decrease produces discontinuities (steps) in the temperature profile. In case 3 the volumetric distribution is that of a single channel representing the entire subassembly. The temperature profile is uniform and radial temperature decrease is not considered.

The large difference between the results of case 1 and case 3 is due to the neglect of the radial temperature variation. The volumetric distribution depends inversely on the temperature gradient. Neglect of the large radial component produced by the rapid temperature decrease near the subassembly wall produces a temperature gradient that is too small. A spurious increase in the reciprocal of the temperature gradient - and therefore the volumetric distribution - results. The intermediate results of case 2 are due to the first order approximation of the radial temperature variation. The steps in the volumetric distribution are caused by neglect of the radial component of the temperature gradient within each channel.

4.1.5 Conclusions

An extension of the independent channel approach with an increased number of channels will continue to demonstrate the stepped volumetric

distribution shown in case 2.

The comparison shown in figure 3 is for a uniform radial power distribution. Treatment of cases for which the power distribution is skewed requires many channels in order to minimize the step function effect. A further disadvantage is that the very important high temperature tail of the volumetric distribution is difficult to obtain with sufficient accuracy. For these reasons the independent flow channel approach has been discarded. The detailed multi-flow channel approach could be used if very accurate calculations of volumetric distributions are desired. The analytic function approach is used in this work since it provides sufficient accuracy for low computational expense.

4.2 An Approach to the Evaluation of Fuel Temperature Field

The temperature field for an individual fuel pin is determined by the power produced within the pin and the fuel surface temperature or boundary condition. Subassembly power skew and coolant temperature decrease near the subassembly periphery produce a fuel pin temperature field dependent on both angular and axial position within the pin. A detailed treatment of these complexities is difficult and beyond the scope of this work.

The problem can be simplified by neglect of the lateral variation in boundary temperature and power for individual fuel pins. The temperature rise within the pin is much greater than the variation of coolant temperature around the pin, permitting the use of a radially averaged fuel surface temperature. The use of a radially averaged value for fuel pin power is justified by the small fuel pin diameter as compared to the subassembly radius. These simplifications permit an analytic solution to the heat equation for the fuel pin. The temperature field in the subassembly is then expressed as the sum of the individual fuel pin temperature fields.

5. Representation and Numerical Calculation of the Power Distribution

Volumetric distributions are calculated from analytic functions describing the coolant and fuel temperature fields. Analytic representation of the temperature fields requires, in turn, analytic representation of the power distribution. A linear description of the lateral variation in subassembly power is adequate for the approach used to determine the temperature fields. As mentioned previously, this description is provided by subassembly "power planes." Values of the power plane coefficients are obtained from a detailed calculation of the reactor power distribution.

The triangular mesh power distribution is provided by the 2DB code using half reactor symmetry. A search was performed to obtain criticality for the initial core of CRBR. The center control rod and the twelve control rods in row six were inserted approximately 70 percent, other control rods were inactive. The power plane coefficients were computed by fitting a plane to the average power in each of the six triangles comprising an individual subassembly.

Figures 4, 5, and 6 present graphically results of the power plane calculations. Figure 4 shows average subassembly power. Figures 5 and 6 describe the spatial orientation of the subassembly power planes. In Figure 5, the arrows show the direction of greatest power decrease for each subassembly. Figure 6 shows the "tilt" of each power plane. The tilt, $\Delta\bar{\Pi}$, is given by ($\bar{\Pi} = P/\bar{P}$):

$$\Delta\bar{\Pi} = \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}} = \frac{1}{2} \frac{P_{\max} - P_{\min}}{\bar{P}} = \frac{1}{2} \frac{\Delta P_{\max}}{\bar{P}} = \frac{\Delta\bar{P}}{\bar{P}}$$

The axial variation in power is well represented by a "chopped" cosine. The three-dimensional power distribution within the subassembly

Figure 5. CRBR subassembly power plane representation. The arrows indicate the direction of steepest power decrease.

Figure 6. CRBR subassembly power plane representation. The power tilt, $\Delta\Pi$, is indicated.

$$\left(\Delta\Pi = \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}} \right)$$

can be expressed by a product of the axial and lateral variations, i.e.,

$$P(x,y,z) = (a_0 + a_x \cdot x + a_y \cdot y) \cos\left(\frac{z}{L} \pi\right)$$

where a_0 , a_x , and a_y are coefficients of the power plane given by,

$$P(x,y,0) = a_0 + a_x \cdot x + a_y \cdot y$$

In the axial variation, $\cos\left(\frac{z}{L} \pi\right)$, L is the cosine half period; L equals the active core height plus two times the extrapolation distance.

6. Numerical Evaluation of Volumetric Distributions

In this section, the relationships described in the previous two sections are presented in mathematical form along with the formulas used in the numerical computation of volumetric distribution.

All temperature and power distributions used in the calculation of volumetric distributions use the same Cartesian coordinate system, although the equivalent radial system is sometimes used for convenience. The coordinates x , y , z are relative to an origin located in the center of the subassembly at the core midplane. The hexagonal subassembly has been represented by an equivalent cylinder of the same cross sectional area in order to simplify the numerical calculations.

6.1 The Coolant Temperature Volumetric Distribution

The subassembly coolant temperature distribution is determined by the product of a radial factor, an axial factor, and the power plane. The radial factor describes the radial decrease in temperature rise due to the increased flow area along the periphery of the subassembly. The axial factor describes the increasing coolant temperature rise due to the axial variation in the heat source. The coolant temperature at any point in the subassembly can thus be given by

$$T_c(x,y,z) = C_n F_r(x,y) F_a(z) (a_0 + a_x \cdot x + a_y \cdot y) + T_{inlet}$$

where

$F_r(x,y)$ is the radial factor,

$$F_r(x,y) = 1 - \epsilon \left(\frac{\sqrt{x^2 + y^2}}{R} \right)^N,$$

with parameters ϵ , N chosen by Ott⁽¹⁾. $F_a(z)$ is the axial factor,

$$F_a(z) = \sin\left(\frac{z}{L} \pi\right) + \sin\left(\frac{H}{L} \frac{\pi}{2}\right)$$

C_n is a normalizing constant, determined by energy balance

R is radius of the cylindricized subassembly

H is the active core height

The coolant temperature-volumetric distribution, $V_c(T)$, can be obtained by substitution of the coolant temperature relation into the general expression for material-temperature volumetric distributions,

$$V_c(T) = \frac{1}{C_n} \iint \frac{dx dy}{F_r(x,y) \frac{dF_a(z)}{dz} (a_0 + a_x \cdot x + a_y \cdot y)} \cdot \frac{1}{A_I^C(T)},$$

$$V_c(T) = \frac{L}{\pi C_n} \iint \frac{dx dy}{F_r(x,y) \cos\left(\frac{z}{L} \pi\right) (a_0 + a_x \cdot x + a_y \cdot y)} \cdot \frac{1}{A_I^C(T)}.$$

The z dependence is determined by the isothermal equation,

$$T = C_n F_r(x,y) F_a(z) (a_0 + a_x \cdot x + a_y \cdot y) + T_{inlet}.$$

Rearranging and using a trigonometric identity, the isothermal equation yields:

$$\frac{T - T_{inlet}}{C_n F_r(x,y) (a_0 + a_x \cdot x + a_y \cdot y)} = \left[1 - \cos^2\left(\frac{z}{L} \pi\right) \right]^{\frac{1}{2}} + \sin\left(\frac{H}{L} \frac{\pi}{2}\right)$$

with further rearrangement the cosine term in the denominator of the integral can be isolated and the coolant-temperature volumetric distribution expressed as an explicit function of temperature:

$$V_c(T) = \frac{L}{\pi C_n} \iint_{A_I^c(T)} \frac{dx dy}{f_R(x,y) \sqrt{1 - \left(\frac{T - T_{inlet}}{C_n f_R(x,y) P(x,y,0)} - \sin\left(\frac{H}{L} \frac{\pi}{2}\right) \right)^2} P(x,y,0)}$$

The area of integration $A_I^c(T)$ is determined by the projection of the isothermal surface onto the x-y plane.

The expression for $V_c(T)$ has been evaluated for a single subassembly to demonstrate the effect of within subassembly power variation upon the coolant volumetric distribution. Figure 7 shows the effect of several degrees of power plane "tilt," $\Delta\pi$. $\Delta\pi$ is called DELTP in the following figures. The range of values chosen for $\Delta\pi$, 0.0 to 0.5, is larger than the range of values for the CRBR initial core indicated in figure 6, where the values lie between 0 and ~ 0.3 . The larger values of $\Delta\pi$ may be achieved, however, with other control rod configurations or during burnup. The slight irregularity is an artifact of the numerical integration algorithm applied. Note especially the "stretching" of the distribution as the tilt of the power plane increases. Initiation of coolant voiding during the LOF accident sequence would be sensitive to this increase in maximum coolant temperature.

6.2 The Fuel-Temperature Volumetric Distribution

The fuel temperature field in the subassembly is formed by the temperature fields of the individual pins. The subassembly volumetric distribution is thus calculated by summing the individual pin distributions. The temperature rise within the pin is determined by the product of an axial factor, a radial factor, and the power. The pin surface temperature and power are obtained

Figure 7. The coolant-temperature volumetric distribution of a single subassembly for several values of power plane tilt.

$$\left(\text{DELTP} = \Delta T = \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}} \right)$$

from the coolant temperature function and the power plane respectively at the location of each pin. The temperature field in a fuel pin is thus,

$$T_f(r,z;p) = C_p \left[1 - \left(\frac{r}{r_f} \right)^2 \right] \cos \left(\frac{z}{L} \pi \right) + T_s(z)$$

where,

p is a parameter denoting a particular pin within the subassembly.

r is the radial distance within the fuel pellet of outer radius r_f .

$T_s(z)$ is the fuel pellet surface temperature

C_p is parameter which depends on pin average power.

The fuel-temperature volumetric distribution for a single pin, $V_{f,p}(T)$ can be obtained by substitution of the fuel temperature relation into the general expression for material-temperature volumetric distributions, thus

$$V_{f,p}(T) = \int \int \frac{r dr d\theta}{A_I(T) \left| \frac{\partial T}{\partial z} \right|}$$

where p denotes an individual fuel pin. Since the temperature equation does not depend on angle, the θ integration simply produces a factor of 2π , resulting in -

$$V_{f,p}(T) = 2\pi \int \frac{r dr}{\left| \frac{\partial T}{\partial z} \right|}$$

On the isothermal surface the total differential of the temperature vanishes:

$$dT = \frac{\partial T}{\partial r} dr + \frac{\partial T}{\partial z} dz = 0$$

Thus,

$$\frac{dr}{\left(\frac{\partial T}{\partial r} \right)} = - \frac{dz}{\left(\frac{\partial T}{\partial z} \right)}$$

The expression for the volumetric distribution becomes,

$$V_{f,p}(T) = 2\pi \int \frac{-r}{\frac{\partial T}{\partial r}} dz ,$$

with the r dependence determined by the isothermal equation,

$$T = C_p \left[1 - \left(\frac{r}{r_f} \right)^2 \right] \cos\left(\frac{z}{L} \pi\right) + T_s(z) .$$

Performing the indicated differentiation and forming the ratio yields the volumetric distribution for pin "p":

$$V_{f,p}(T) = 2\pi \int \frac{r_f^2 dz}{2C_p \cos\left(\frac{z}{L} \pi\right)} ,$$

$$V_{f,p}(T) = \pi \frac{r_f^2}{C_p} \int_{z_{\min}^p(T)}^{z_{\max}^p(T)} \frac{dz}{\cos\left(\frac{z}{L} \pi\right)} ,$$

where the integration limits are determined by the axial location of T on the fuel surface and centerline. The volumetric distribution for the subassembly is formed by the sum of the volumetric distributions for individual pins, i.e.,

$$V_f(T) = \pi r_f^2 \sum_p \frac{1}{C_p} \int_{z_{\min}^p(T)}^{z_{\max}^p(T)} \frac{dz}{\cos\left(z \frac{\pi}{L}\right)}$$

The expression for $V_f(T)$ has been evaluated for a single subassembly to demonstrate the effect of the within subassembly power variation upon the fuel volumetric distribution. Figure 8 shows the effect of several degrees of power plane tilt. The sensitivity of the high temperature tail should be especially significant in determining the initiation and progression of fuel failure.

Figure 8. The fuel-temperature volumetric distribution of a single subassembly for several values of power plane tilt.

$$\left(\text{DELTP} = \Delta T = \frac{P_{\text{max}} - P_{\text{min}}}{P_{\text{max}} + P_{\text{min}}} \right)$$

6.3 The Clad-Temperature Volumetric Distribution

The clad-temperature volumetric distribution need not be separately calculated since it is directly related to the coolant distribution. Both temperature distributions are the same under the following assumptions:

(1) neglect of the temperature drop from coolant to clad, and (2) neglect of the temperature drop across the clad which is small in comparison to the variation axially and laterally within the subassembly. These assumptions have been tentatively accepted at the present time.

6.4 The Fuel Power Volumetric Distribution

The fuel-power volumetric distribution $V_f(P)$, can be obtained by substitution of the fuel power relation into the general expression for material-power volumetric distributions, thus

$$V_f(P) = 2 \iint_{A_I^f(P)} \frac{dx dy}{\left| \frac{\partial P}{\partial z} \right|},$$

where the power distribution is given by,

$$P(x, y, z) = (a_0 + a_x \cdot x + a_y \cdot y) \cos\left(\frac{z}{L} \pi\right).$$

The factor of 2 results from the axial symmetry produced by the cosine term in the power distribution. Differentiating the power distribution,

$$\frac{\partial P}{\partial z} = -\frac{\pi}{L} (a_0 + a_x \cdot x + a_y \cdot y) \sin\left(\frac{z}{L} \pi\right),$$

Using a trigonometric identity and the isopower equation,

$$P = (a_0 + a_x \cdot x + a_y \cdot y) \cos\left(\frac{z}{L} \pi\right),$$

to eliminate the z dependence, the denominator of the integrand becomes:

$$\left(\frac{\partial P}{\partial z} \right) = \frac{\pi}{L} (a_0 + a_x \cdot x + a_y \cdot y) \left[1 - \frac{P^2}{(a_0 + a_x \cdot x + a_y \cdot y)^2} \right]^{1/2}.$$

Thus the fuel-power volumetric distribution is given by:

$$v_F(P) = \frac{2L}{\pi} \int \int_{A_I^F(P)} \frac{dx dy}{[(a_o + a_x \cdot x + a_y \cdot y)^2 - P^2]^{1/2}},$$

where the area of integration $A_I^F(P)$ is determined by the projection of the isopower surface onto the $x - y$ plane.

Volumetric distributions for a single subassembly are shown in figures 9 and 10. The power plane skew, $\Delta\Pi$, is varied over two sets of values to illustrate the sensitivity of the fuel power volumetric distribution to this parameter. In each figure the distribution for a subassembly with flat power, i.e. $\Delta\Pi = 0$, is provided for comparison. The latter distribution is singular at the point of maximum power, the core midplane. As can be seen from these figures, even moderate values of $\Delta\Pi$ can substantially increase the maximum power in the fuel, which may be very significant in an accident analysis.

7. Volumetric Distributions for a Demonstration-Type Reactor

The volumetric distributions for CRBR with the initial core have been completed using 120° reactor symmetry. This is accomplished by summing the volumetric distributions for the 66 subassemblies involved. Figures 4 and 6 show that the subassembly power and power tilt vary, in general, more between core rings than between subassemblies within the same ring. Accordingly, the volumetric distributions have been calculated for individual core rings in order to permit the effect of the reactor power distribution upon the volumetric distributions to be more readily observed. The following subassembly groupings were used:

Figure 9. The fuel-power volumetric distribution for a single subassembly for several values of power plane tilt

$$\left(\text{DELTP} = \Delta \Pi = \frac{P_{\text{max}} - P_{\text{min}}}{P_{\text{max}} + P_{\text{min}}} \right)$$

Figure 10. The fuel-power volumetric distribution for a single subassembly for several values of power plane tilt.

$$\left(\text{DELTP} = \Delta \Pi = \frac{P_{\text{max}} - P_{\text{min}}}{P_{\text{max}} + P_{\text{min}}} \right)$$

<u>Subassembly Grouping Number</u>	<u>Core Rings</u>	<u>Number of Subassemblies</u>
1	1-3	10
2	4	8
3	5	10
4	6	8
5	7	14
6	8	16
total 120° core segment	1-8	66

The results are displayed by comparing the volumetric distributions (two per graph) with the composite volumetric distribution for the total reactor core. The curves have been scaled so that the total volume represented by each curve is equal to that for 10 subassemblies. Figures 11, 12, and 13 display the coolant-temperature volumetric distributions; figures 14, 15, and 16 display the fuel-temperature volumetric distributions; and figures 17, 18, and 19 display the fuel-power volumetric distributions.

These plots show that, for the core loading investigated here, the inner core rings (1-3) determine the maximum temperature (and power) of the distributions along with the shape of the high-temperature (and high power) tail. This portion of the volumetric distributions is expected to be of primary importance in its effect upon reactor transient behavior. The outer core rings 7 and 8, with 14 and 16 subassemblies respectively, contain three times the volume of rings 1 to 3 with 10 subassemblies. The outer core has highly skewed subassembly power distributions--the lowest $\Delta T = 5\%$ --producing a large "spreading" of the distributions. This heavily weighted spreading will strongly influence the first moment of the volumetric distributions. The moment, which characterizes the incoherence of the reactor, is expected to have a

Figure 11. Coolant-temperature volumetric distributions for the CRBR (BOL). Distributions for subassembly rings 1, 2, and 3 and ring 4 are compared with the total core distribution. The curves have been normalized to represent 10 subassemblies.

Figure 12. Coolant-temperature volumetric distributions for the CRBR (BOL). Distributions for subassembly ring 5 and ring 6 are compared with the total core distribution. The curves have been normalized to represent 10 subassemblies.

Figure 13. Coolant-temperature volumetric distributions for the CRBR (BOL). Distributions for subassembly ring 7 and ring 8 are compared with the total core distribution. The curves have been normalized to represent 10 subassemblies.

Figure 14. Fuel-temperature volumetric distributions for the "BBR (BOL)". Distributions for subassembly rings 1, 2, 3 and ring 4 are compared with the total core distribution. The curves have been normalized to represent 10 subassemblies.

Figure 15. Fuel-temperature volumetric distributions for the CRBR (BOL). Distributions for subassembly ring 5 and ring 6 are compared with the total core distribution. The curves have been normalized to represent 10 subassemblies.

Figure 16. Fuel-temperature volumetric distributions for the CRBR (BOL). Distributions for subassembly ring 7 and ring 8 are compared with the total core distribution. The curves have been normalized to represent 10 subassemblies.

Figure 17. Fuel-power volumetric distributions for the CRBR (BOL). Distributions for subassembly rings 1, 2, and 3 and ring 4 are compared with the total core distribution. The curves have been normalized to represent 10 subassemblies.

Figure 18. Fuel-power volumetric distributions for the CRBR (BOL). Distributions for subassembly ring 5 and ring 6 are compared with the total core distribution. The curves have been normalized to represent 10 subassemblies.

Figure 19. Fuel-power volumetric distributions for the CRBR (BOL). Distributions for subassembly ring 7 and ring 8 are compared with the total core distribution. The curves have been normalized to represent 10 subassemblies.

significant effect on reactor transient behavior.

The CRBR volumetric distributions can be used to demonstrate the effect of grouping subassemblies into coherent "channels." In the channel modeling used by safety analysis codes such as SAS the power distribution is assumed to be laterally uniform within the channel. Although the SAS code does not use the volumetric distribution concept, equivalent multi-channel volumetric distributions can be generated using the approaches described in this work.

A typical 10 channel fuel-power volumetric distribution has been calculated for the CRBR initial core and compared to the equivalent continuous distribution. Subassemblies have been grouped into channels in the following manner:

Channel Number	Subassembly Ring	Number of Subassemblies (for 120° symmetry)
1	1	2
2	2	4
3	3	4
4	4	8
5	5	10
6	6	8
7	7	8
8	7	6
9	8	10
10	8	6

For a single channel with no lateral power variation, ($a_x = a_y = 0$), the expression for the fuel-power volumetric distribution,

$$v_f(P) = \frac{2L}{\pi} \iint_{A_I^f(P)} \frac{dx dy}{[a_o^2 - P^2]^{1/2}}$$

can be integrated formally. The region of integration $A_I^f(P)$ is formed by the projection of 217 fuel pins onto the $x - y$ plane for each subassembly in the channel. Thus,

$$v_f^{\text{channel}}(P) = \frac{2L}{\pi} \frac{n_s (217 \pi r_f^2)}{\sqrt{a_o^2 - P^2}},$$

where n_s denotes the number of subassemblies in the channel.

The distribution is singular at $P(x,y,z) = a_o$, which occurs at the core midplane ($z = 0$). The multi-channel distribution is formed by summing the distributions of the individual channels. The 10-channel volumetric distribution will therefore contain 10 singularities which correspond to the maximum power in each channel. If the power has a lateral variation (i.e. if $\Delta\Pi \neq 0$), the volumetric distribution is nonsingular. The distribution is nonsingular even for very small values of $\Delta\Pi$ as is shown in figure 9.

The 10 channel and the continuous fuel-power volumetric distributions are shown in figure 20. The "average" agreement of the two curves appears to be very good, particularly on the low-power wing of the distribution. However, if fuel becomes progressively involved in a major accident, the involvement starts at the upper end of the distributions. The continuous distribution involves the fuel more gradually, whereas, the singularities in the multi-channel distribution tend to increase the fuel involvement in sudden "steps," each "step" corresponding to a singularity in figure 20.

Figure 20. A comparison of approaches to the fuel-power volumetric distribution.

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