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NEUTRAL BEAM CURRENT REQUIREMENTS FOR MX

B. W. Stallard
M. E. Rensink

January 14, 1977

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NEUTRAL BEAM CURRENT REQUIREMENTS FOR MX

Abstract

The neutral beam current required to sustain the MX plasma is calculated for the plasma modeled by (a) a flat-top radial density profile and (b) a diffuse radial profile. Estimates of the plasma length are obtained from Fokker-Planck calculations and a three-dimensional guiding-center

equilibrium code. Beam trapping efficiency is calculated from a model taking into account the spatial dimensions of the beam and plasma. The sensitivity to variations of magnetic field and plasma beta is calculated for the plasma parameters n_i , beam current, and axial scale length.

Introduction

This report was prepared as a supplement to the MX Major Project Proposal¹ in response to recommendations of the Technical Review Panel.² The work extends earlier calculations that led to the neutral beam parameters chosen for the MX reference design. It also indicates tradeoffs and options by varying parameters around this point design, always adhering to the following basic physics objectives stated in the MX proposal:

- The n_i and β in MX should be as large as possible to provide a convincing demonstration that mirror physics is well in hand.
- The physical dimensions (L/p_i and R/p_i) should be large

enough to test theoretical scaling laws for all kinds of known instabilities.

Because of the complex geometry and the large number of plasma phenomena, a precise calculation of the required neutral beam current corresponding to a given set of plasma conditions is impossible with computer codes now available. Instead, we must rely upon simplified analytical models that synthesize results from a number of different codes, each of which emphasizes some important portion of the physics.

The main inputs required by the analytical models are the plasma volume, the beam trapping efficiency, and the confinement parameter n_i .

To calculate the plasma volume, the radial and axial density profiles must be known. To determine the radial profile, a buildup code with neutral beam injection (Buildup III) was used. This code, however, does not treat the axial dimensions.

Estimates for the axial profiles were obtained from both Fokker-Planck models and a three-dimensional guiding-center equilibrium code. These codes do not include the effects of instability modes such as the drift-cyclotron loss-cone (DCLC) mode.

Beam-trapping efficiency calculations require both the radial and axial profiles of the beams and plasma. The profiles calculated from the radial buildup, Fokker-Planck, and three-dimensional equilibrium codes were used to estimate trapping efficiency.

The confinement parameter $n\tau$ was estimated by various theoretical prescriptions based on the theory of marginal stability of the DCLC mode, or the Fokker-Planck classical confinement calculations, as appropriate.

For all calculations the magnet length L_m has been kept constant at 3.4 m. Reducing L_m decreases beam access, the ion energy adiabatic limit, and the axial scale length L_m/ρ_i . By reducing L_m/ρ_i , it is more difficult to observe the effects of the convective and Alfvén ion cyclotron (AIC) modes, and to determine if

their effects are important for confinement as predicted by theory. There is little cost advantage in reducing L_m , since a 10% reduction in L_m results in a cost savings of only $\approx 4\%$.

To see this, we note that cost scaling with L_m at the design point varies approximately as L_m^2 . Items whose costs are proportional to L_m^2 include the winding materials, magnet structure and winding labor. There are also costs which scale as $L_m^{3/2}$, $L_m^{3/2}$, and L_m^3 , but the average scaling is L_m^2 . The costs associated with the magnet size are about \$12 million from a total of \$59.1 million, without contingency and escalation. Thus, a 10% variation in L_m reduces costs by $(1-(0.9)^2) \times \frac{12}{59.1} \times 100\% = 3.9\%$.

For the results presented, deuterium extraction at 80-keV energy is assumed. If an extraction energy of 120-keV deuterium is considered, the main advantage is higher $n\tau$ for plasma parameters of the point design ($\beta = 0.5$, $B = 20$ -kG and $R_p/\rho_i = 13$). The disadvantage is that L/ρ_i decreases, which makes it more difficult to observe effects of the convective and AIC modes.

For an extraction energy of 40 keV and the parameters of the point design, L/ρ_i increases, but cost increases also because more beam power is required. If magnetic field is reduced so that $\beta = 0.5$ is achieved

for no increases in cost, L/ρ_1 is about the same as for 80-keV extrac-

tion. In addition, $n\tau$ is reduced in any case.

Analytic Models for the MX Plasma

To calculate beam current requirements as plasma parameters are varied, two analytic models have been used, differing only in the assumed shape of the radial density profile.

Model 1, called the flat-top profile, is more appropriate for large plasmas. This model profile (Fig. 1(a)) arose from calculations using the radial buildup code. The code is one-dimensional, and assumes symmetry about the magnetic axis. Radial density profiles are calculated taking into account finite larmor radius effects and hot plasma losses on cold gas at the plasma surface. Calculations for plasmas of the size of MX show a plasma of roughly uniform density but bounded by a steep radial density gradient of scale length three to four ion larmor radii. The degree of flatness of density in the interior varies with the profile of the injected beams. The steep boundary gradient arises from surface erosion by cold gas, i.e., the plasma "leans" on the gas in equilibrium. The flat interior profile appears to be stable against the DCLC, while the boundary has a similar scale length to 2XIIB and, hence, significantly lower con-

finement. Thus, $n\tau$ for the interior exceeds that in the boundary, and the average $n\tau$ is weighted by the relative volumes.

A flattened profile is a likely shape for the MX plasma in the absence of anomalous radial diffusion of the plasma (greatly exceeding diffusion by coulomb collisions). The density tends to dip in the center, since the plasma is thick to the beam. In a minimum B field, inward fluting is a likely result for any hollowing of the

Fig. 1. Plasma density profile (flat-top model (a), diffuse model (b)).

profile, thus keeping the profile uniform in the interior.

For comparison with the flat-top model we consider a second plasma model with "diffuse" radial profile (Fig. 1(b)) and scale length R_p . For this scale length the warm plasma required for marginal stability is calculated from DCLC theory. This determines the electron temperature and the ion energy confinement parameters $n\tau_E$.

FLAT-TOP PROFILE

For the flat-top profile we assume different values for $n\tau$ on the boundary and in the interior. For the boundary, since the axial scale length is similar to 2XIIB plasmas, we scale $n\tau_{\text{drag}}$ from the experiment. In 2XIIB, typical values are $n\tau_{\text{drag}} \approx 7 \times 10^{10}$, $B \approx 6$ kG and $\beta \approx 0.5$.

Now, electron temperature scales according to Eq. (B3) of the MX proposal as:

$$T_e \propto (n\tau_i L_p / \Delta)^{1/3} \propto (8B^2 L_p / \Delta)^{1/3}, \quad (1)$$

where n and T_i have their usual definition, L_p is the plasma length, and L is the ratio of unconfined warm plasma to hot plasma as calculated by quasi-linear theory. The confinement parameter varies as $T_e^{3/2}$.

Scaling $n\tau$ to MX, assuming the same Δ and β as in 2XIIB:

$$\begin{aligned} n\tau_{\text{drag}}(\text{MX}) &= n\tau_{\text{drag}}(2\text{XI}) \times \frac{B(\text{MX})}{B(2\text{XI})} \\ &\times \left(\frac{L_p(\text{MX})}{L_p(2\text{XI})} \right)^{1/2} = 7 \times 10^{10} \left(\frac{20}{6} \right) \times \left(\frac{170}{75} \right)^{1/2} \\ &= 3.5 \times 10^{11} \text{ cm}^{-3} \cdot \text{s}. \quad (2) \end{aligned}$$

The confinement in the boundary $n\tau_B$ is given by adding the effects of ion-ion coulomb scattering:

$$\frac{1}{n\tau_B} = \frac{1}{n\tau_{\text{drag}}} + \frac{1}{n\tau_{ii}}$$

The scattering time for deuterium, including beta enhancement of the mirror ratio, is determined by Fokker-Planck calculations to be

$$n\tau_{ii} \approx 2 \times 10^{10} T_i^{3/2} \log \frac{1}{\sqrt{1-\beta}}.$$

An additional particle loss occurs by charge exchange on cold gas at the plasma surface. For a total neutral gas current I_G on the surface the charge exchange loss is:

$$I_{\text{cxs}} = I_G \left(\frac{c}{\sigma_T} \right)_{\text{eff}}, \quad \text{where} \quad \left(\frac{c}{\sigma_T} \right)_{\text{eff}}$$

is the ratio of charge exchange to the total cross section for cold gas. This term includes the effects of Franck-Condon production by dissociation of neutral molecules.

In the flat interior, even though the density gradient predicts stability for the DCLC mode, a radial gradient in T_e will drain some electron heat into the boundary from the

interior, thus causing some cooling of the ions. There may also be a residual turbulence level from the poor confinement in the boundary. These effects have not been calculated. However, with such effects in mind, we assume for confinement in the interior a value of $n\tau_{int} \approx 1.1 \times 10^{12} \text{ cm}^{-3} \cdot \text{s}$, rather than the somewhat higher values calculated from the Fokker-Planck codes (Appendix A).

For an injected neutral beam current I_b , and with trapping efficiency η_t , the particle balance equation is:

$$\eta_t \frac{I_b}{b} = \frac{en^2 \pi \left(R_p^2 - \frac{3}{2} \rho_i^2 \right)^2 L_2}{n\tau_{int}} + \frac{en^2 \pi R_p \left(\frac{3}{2} \rho_i \right) L_2}{n\tau_B} + I_{cxs}. \quad (3)$$

The factor one-third in the second term gives the average for n^2 in the boundary layer of thickness $3\rho_i$.

The length L_2 is the plasma length weighted by the square of plasma density along a flux tube. This length, estimated from Fokker-Planck equilibrium calculations, is described in Appendix A. Estimates for the beam-trapping efficiency (η_t) are described in Appendix B.

We calculate the steady-state gas n_0 at the plasma surface from a rate equation which describes the gas

buildup in the volume, V , exterior to the plasma. The gas species is assumed to be molecular for saturated walls. The gas has thermal velocity v_0 in equilibrium with the walls:

$$V \frac{dn_0}{dt} = I_s + \Gamma(I_{cx} + I_{cxs}) - \frac{1}{4} n_0 v_0 (A_p + A_{BH}). \quad (4)$$

In this equation, I_s is the gas from the neutral beam sources. The second term is due to wall reflux of charge exchange neutrals, I_{cx} from charge exchange of the injected beam on the plasma ions, and I_{cxs} from charge exchange of cold gas I_G on the plasma at the surface. At steady-state $\Gamma = 1/2$, i.e., one molecule returns from the wall for every two incident atoms. The third term is pumping of gas incident on the plasma of surface area A_p and into an aperture in the plasma chamber, given by its equivalent black-hole surface area A_{BH} . As defined earlier, the charge exchange at the surface is given by:

$$I_{cxs} = I_G \left(\frac{\sigma_x}{\sigma_T} \right)_{\text{eff}} = \frac{1}{4} n_0 v_0 A_p \left(\frac{\sigma_x}{\sigma_T} \right)_{\text{eff}}. \quad (5)$$

Using Eqs. (4 and 5), the steady-state solution for I_{cxs} is:

$$I_{\text{cxs}} = \left(I_s + \frac{1}{2} I_{\text{cx}} \right) \left(\frac{\sigma_x}{\sigma_{\text{T}}^{\text{eff}}} \right)$$

$$\times A_p / \left[A_p \left(1 - \frac{1}{2} \left(\frac{\sigma_x}{\sigma_{\text{T}}^{\text{eff}}} \right) + A_{\text{BH}} \right) \right] \quad (6)$$

The plasma surface area is $A_p = 2\pi R_s L_s S_A$, where S_A is the ratio, greater than unity, of the surface area of the plasma to a cylinder of the same length and radius, and R_s and L_s are, respectively, the radius and length of the plasma surface. The greater surface area is a result of the minimum-B magnetic field.

The confinement parameter averaged over the volume is:

$$\frac{1}{\langle n\tau \rangle} = \frac{V_B}{V_{\text{int}} + V_B} \frac{1}{n\tau_B} + \frac{V_{\text{int}}}{V_{\text{int}} + V_B} \frac{1}{n\tau_{\text{int}}} \quad (7)$$

where the boundary volume in terms of the peak density is $V_B = \frac{1}{3} 2\pi R_p^2 (3\rho_1)L_2$, and the interior volume is $V_{\text{int}} = \pi \left(R_p^2 - \frac{3}{2} \rho_1 \right) L_2$. The required beam current is given by:

$$I_b = \frac{e n^2 (V_B + V_{\text{int}})}{\langle n\tau \rangle} \frac{1}{\eta_t - \frac{1}{I_{\text{cxs}}}} \quad (8)$$

DIFFUSE PROFILE

The energy confinement parameter $n\tau_E$ for the diffuse profile is calculated from the electron temperature. The formula for confinement is:

$$n\tau_E \quad n\tau_{\text{drag}} = 1.4 \times 10^{12} T_e^{3/2} = 9.2 \times 10^9 (BL_p/\Delta)^{1/2} \quad (9)$$

The last relation in Eq. (9) is obtained by evaluating Eq. (B3) of the Proposal¹ for $B = 20$ kG and $n = 8$. Again, we assume $\frac{1}{n\tau} = \frac{1}{n\tau_E} + \frac{1}{n\tau_{ii}}$.

Analogous to Eq. (3) the required beam current is given by:

$$n_t I_b = \frac{e n^2 \pi R_p^2 L_2}{n\tau} + I_{\text{cxs}} \quad (10)$$

Parameter Variations

We first calculate the sustaining current for the point design using the flat-top and, for comparison, the diffuse profile models. For these calculations the assumed plasma length

and beam-trapping efficiency are, respectively, $L_2 = 70$ cm, and $\eta_t = 0.63$, as discussed in Appendices A and B.

Table 1 shows various plasma parameters for the two models. The

Table 1. Sensitivity of the plasma parameters to the assumed plasma model.

Year	1960	1961	1962	1963	1964
1960	100	100	100	100	100
1961	100	100	100	100	100
1962	100	100	100	100	100
1963	100	100	100	100	100
1964	100	100	100	100	100

trapping efficiencies and charge-exchange currents used in the diffuse-model calculation are assumed to be the same as in the flat-top model. The values are probably not greatly in error from more precise calculations. The calculated current for the diffuse-profile is about 20% less than for the flat-top because of the absence of the low-confinement boundary layer. It is also smaller than the 750-A in the proposal, mainly because of the longer plasma length (about $1/3$ the mirror-to-mirror spacing) assumed in the proposal.

We have also considered variations of a factor of 2 in both directions of the value of Δ for the diffuse-model. The theoretical curves in Fig. 2 of the MX Proposal¹ show this range of variation depending on the ambipolar hole size. The required beam current varies 100 A or more with these variations.

Using the flat-top model, beam current requirements have been calculated for variations of magnetic

field and plasma beta. We have not included variations of plasma parameters with R_p/ρ_i . For large plasmas the flat-top model seems appropriate rather than the diffuse profile. If R_p/ρ_i is reduced, current requirements are then decreased. However, it is not clear to what extent the flat region may be reduced relative to the constant-thickness, lossy boundary before turbulence in the boundary adversely affects confinement in the interior. The solution to this difficult problem is not yet available. Work on computer codes with a quasilinear treatment of the DCLC, and a radially varying profile are in progress.

For constant plasma beta ($\beta = 0.5$) and a constant ratio $R/\rho_{i \text{ vac}} = 13$, the magnetic field was varied. Several plasma parameters, normalized to the point design, are plotted in Fig. 2. The volume-averaged confinement $\langle n\tau \rangle$, beam current I_b , and axial scale length L/ρ_i decrease as the magnetic field is reduced. Although

$I_b \approx B^2$ at constant beta (Eq. 3), the beam current does not drop nearly so fast, since the net trapping $\eta_{t \text{ net}} \equiv \eta_t - I_{\text{exs}}/I_b$ also decreases from $\eta_{t \text{ net}} = 0.55$ at 20 kG to $\eta_{t \text{ net}} = 0.23$ at $B = 8$ kG. As B decreases, the plasma is less dense, resulting in less trapping, and leading to greater charge-exchange loss on cold gas from the increasing surface area ($R/\rho_i \text{ vac} = \text{const}$).

Also plotted in Fig. 2 is the variation of the ion lifetime τ_{B-C} estimated from the Baldwin-Callen theory for the convective loss-cone mode (NX Proposal¹, p. B-18). At the point design, $\tau_{B-C} \geq \tau_{\text{drag}}$. As B is

Fig. 2. Variation of plasma parameters as a function of magnetic field of constant $\beta = 0.5$. Parameters of the point design: $I_b = 760A$, $\langle n_i \rangle = 7.4 \times 10^{11} \text{ cm}^{-3}\text{-s}$, $L/P_i = 105$, $\tau_{B-C} \geq \tau_{\text{drag}}$.

Fig. 3. Variation of plasma parameters as plasma beta is varied at constant magnetic field $B = 20$ kG.

reduced the effects of this mode, if important, rapidly disappear due to the shorter scale length. Thus, reducing magnetic field to reduce current requirements makes observation of the convective mode more difficult, and decreases n_i .

For constant magnetic field Fig. 3 shows the parameter variations as β is reduced. The boundary confinement decreases approximately as $R^{1/2}$. The confinement in the interior is held constant at $10^{12} \text{ cm}^{-3} \text{-s}$ for all values of β . This is consistent with a flat interior profile stable against the DCLC. It appears that the convective mode can be turned on or off by varying β or magnetic field.

Maximum Beta at Highest Current Density Beam Focus

We have also estimated the maximum plasma beta achievable in MX if all beams are aimed centered at $x = 0$. The full beam width (1/e power) is 15 cm. Since the plasma is only a few Larmor radii in dimension as in 2XIIB, a diffuse-profile is appropriate. We assume a Gaussian-like profile with radial dimension at one-half the peak density equal to $\frac{15}{2} + r_{1 \text{ vac}} \approx 10$ cm. (This is consistent with 2XIIB measurements.)

This fits a Gaussian profile with scale length $R_p = 12$ cm. The corresponding volume for calculating loss weighted by the square of density is

$$V = \pi \frac{R_p^2}{2} L_2.$$

Using Eq. (10) with $n\tau$ scaled from 2XIIB the estimated achievable beta at 750 A injection is $\beta \sim 1.7$. The plasma parameters are shown in Table 1.

Comments and Conclusions

In this report we have presented the results of calculations that have refined earlier estimates of plasma volume (length) and beam-trapping efficiency for the MX device. As noted by the MX Technical Review Panel, precise calculations of neutral beam requirements cannot now be made, even assuming that all the important physics is known. Fully self-consistent, multidimensional computer codes incorporating all the important physics do not exist, thus our calculations are necessarily subject to uncertainty.

After examining the effects of varying mirror length, and plasma parameters beta and magnetic field, we reached the following conclusions:

- The present MX design is well suited to make a significant scale-up in mirror confinement $n\tau$ and ion energy for a high-beta plasma.
- For large plasmas in MX the density is likely to have a flat-top radial profile. A probable result is that confinement will be greatest near the plasma axis and decrease at large radii near the boundary. In the flat interior the plasma may be stable against the DCLC mode.
- The plasma dimensions L/ρ_i and R/ρ_i are large enough to test the theoretical scaling laws for the important mirror

instability modes. By varying beta and magnetic field from the point design it should be possible to turn the convective mode "on or off," according to theory, and determine if the mode is important for confinement. To do this at high β , it is important that the full design current be available.

- There is little advantage in

reducing the length of the magnet since this offers little cost savings, and also adversely affects beam access, adiabaticity, and L/ρ_1 .

- Concerning the possibility of attaining a field-reversed state, plasma beta exceeding unity may be obtained by aiming the beams on axis for highest current density.

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2. Open System Technical Review Panel, Rept. ERDA-76/140 (1976).
3. R. L. Miller, *Monte Carlo Simulation of Neutral Beam Injection into Fusion Reactors*, Nuclear Engineering Program, University of Illinois, Urbana, Illinois, Rept. C00-2218-38 (1975).

Appendix A. Fokker-Planck and Three-Dimensional Equilibria Calculations

The neutral-beam current is directly proportional to the plasma length that must be sustained. To calculate the axial profile, we use a bounce-averaged Fokker-Planck code. This code uses the long-thin equilibrium approximation which assumes that the plasma radial dimensions are small compared to the axial dimensions. This assumption is not strictly valid for MX, since the plasma is more nearly spherical. To check the results of the Fokker-Planck model, a three-dimensional guiding-center equilibrium code is used which does not depend on the long-thin approximation, and can model plasmas with the aspect ratio of MX.

Our Fokker-Planck model describes the plasma on a given flux tube, which for these calculations we assume is on the magnetic axis of the system ($r = 0$). We assume that the electrons form a neutralizing background with a Maxwellian velocity distribution characterized by a single temperature, T_e . The ion distribution function is obtained by solving a bounce-averaged kinetic equation which properly averages the collision, and source and loss terms over the orbital-bounce motion of the ions along field lines. Nonclassical loss processes (e.g., wave turbulence) are not directly included in the model, but the electron temperature may be chosen arbitrarily low to account for the presence of a stabilizing plasma stream. Ambipolar potential effects are included, with the magnitude of the potential drop from midplane to mirror throat being taken as $\phi_m = 4.5 T_e$.

For a typical problem we specify the vacuum magnetic field profile, $B_V(z)$, and the form of the source and loss terms at each point along the flux tube. For our MX studies we have used a parabolic fit for the vacuum magnetic field:

$$B_V(z) = B_V(0) \left[1 + (R_V - 1) z^2 / L_V^2 \right],$$

where $B_V(0) = 20$ kG is the central field strength, $R_V = 2$ is the vacuum mirror ratio, and $L_V = 135$ cm is the magnetic scale length along the axis of the system. A comparison of this model with the actual magnetic field is shown in Fig. A-1. We include the finite-beta modification of the magnetic field by using the long-thin equilibrium approximation to relate the vacuum magnetic field to the actual magnetic field when plasma is present:

$$\frac{B_v^2(z)}{8\pi} = \frac{B_v^2}{8\pi} + p_i(B),$$

where $p_i(B)$ is the pressure profile in terms of field strength as derived from the distribution function for the plasma. Thus, in solving the bounce-averaged Fokker-Planck equation we allow the magnetic field profile to change in accordance with changes in the plasma distribution function.

Our source and loss terms represent ionization and charge-exchange processes associated with neutral beam injection. The source terms are of the form:

$$S(v, \vartheta, z) = n_b(z) n_i(z) (\overline{\sigma_x v} + \overline{\sigma_i v}) S_b(v, \vartheta),$$

where n_b and n_i are the densities of the neutral beam and plasma, $\overline{\sigma_x v}$ and $\overline{\sigma_i v}$ are the (averaged) rate parameters for charge exchange and ionization, and $S_b(v, \vartheta)$ is a shape factor which specifies the energy E_b , and orientation ϑ_b , of the energetic neutral beam. The beam density $n_b(z)$ is assumed spatially uniform from the midplane out to some point z_b , which we specify as an input parameter. Also, we assume that the neutral beam source actually consists of two energetic components (at 80 keV and 100 keV) with about equal intensities, so that the mean energy for trapped injected ions is about 58 keV. This approximates the beam expected from the MX sources. The loss terms associated with charge exchange on the neutral beam are of the form:

$$L(v, \vartheta, z) = n_b(z) \overline{\sigma_x v} f_i(v, \vartheta, z),$$

where $f_i(v, \vartheta, z)$ is the local ion distribution function.

In a thick plasma there is a significant flux of secondary neutrals from charge exchange on the beam. These may be subsequently re-ionized before they leave the plasma volume, accounting for a significant fraction of the overall

Fig. A-1. Vacuum magnetic field on the axis. For the parabolic fit $B_v(0) = 20$ kG, $R_v \approx 2$, $L_v = 135$ cm.

source current. In our model we account for this secondary source by explicitly introducing another neutral beam source:

$$S(v, \vartheta, z) = n_{\text{bex}}(z) \frac{1}{v} f_i(v, \vartheta, z),$$

where the shape of this source in velocity space, $f_i(v, \vartheta, z)$, is now representative of the secondary charge-exchange neutrals, and the density n_{bex} of this secondary neutral beam is some multiple of the primary neutral beam density. The ratio of the densities of charge exchange and injected neutral beams is calculated self-consistently by the radial buildup code at any radial point in the plasma.

Input parameters for a "standard case" representative of MX at $r = 0$ are the following:

- Beam injection angle $\vartheta_b = 80^\circ$.
- Electron temperature $T_e = 1.33$ keV.
- Spatial extent of the beam $z_b = 40$ cm.

The beam extent z_b needs explanation. Although the injected beams are expected to have a Gaussian axial profile with scale length 34 cm, the beam intensity on axis ($r = 0$) has a greater scale length because of greater beam attenuation near the plasma center ($z = 0$). An approximate scale length for $r = 0$ is $z_b = 40$ cm.

The beam current density for the standard case is adjusted to obtain β_1 close to the value 0.5 of the point design. Values of 0.5 often cannot be reached in these model calculations because the mirror-mode limits the maximum beta when the long-thin approximation is used.

In addition to calculations for the standard case, the sensitivity of the model to the input parameters has been examined for the following variations:

- Electron temperature; $T_e = 1.33, 2.67, 4$ keV.
- Spatial extent of the beam; $z_b = 34, 40, 48$ cm.
- Plasma beta.

Results of this study are summarized in terms of the following simple plasma parameters:

- Mean ion energy, $\bar{E}_i$.
- Particle confinement parameter:

$$n\tau_p = - \frac{\frac{dz}{B} n^2(z)}{\frac{dz}{B} J_1(z)}.$$

- Midplane beta:

$$\beta_1 = \frac{8\pi p}{B_0^2} \quad \Big|_{z=0}.$$

- Maximum beta for mirror-mode stability

$$B + 4\pi \frac{dp_1}{dB} > 0 \quad \max(\beta_1).$$

- Spatial profiles of plasma density and magnetic field for defining two characteristic lengths:

$$L_1 \equiv \frac{\int \frac{n}{B} dz}{\frac{n_0}{B_0}},$$

$$L_2 \equiv \frac{\int \frac{n^2 dz}{B}}{\frac{n_0^2}{B_0}},$$

where n_0 , B_0 are the midplane values. The number of ions in a flux tube is given by $n_0 L_1 A_0$, where n_0 and A_0 , respectively, are the density and cross-sectional area of the flux tube at the midplane. Similarly, L_2 is used to calculate flux tube losses which are dependent on the square of density. The results of the Fokker-Planck runs are tabulated in Table A-1.

STANDARD CASE

The Fokker-Planck results can best be understood by considering the "standard case," and examining the effect of various parameter changes. The results of this case are as follows:

- $\bar{E}_1 = 34.6$ keV.
- $n\tau_p = 2.08 \times 10^{12} \text{ cm}^{-3}\text{-s}.$

Table A-1. Fokker-Planck calculations of plasma parameters. Variables are beam dimensions, electron temperature, and beta.

θ_b	Z_b , cm	T_e , keV	$\bar{E}_i$, keV	β	β_{max}	n_p , 10^{12}	L_1 , cm	L_2 , cm
80°	40	1.33	34.6	0.3939	0.4659	2.08	74	49
80°	48	1.33	34.3	0.4175	0.4863	2.07	74	50
80°	34	1.33	35.2	0.3746	0.4428	2.07	72	48
80°	40	2.67	53.9	0.0706	0.3817	2.88	104	77
80°	40	1.33	37.4	0.0286	0.2696	1.71	107	79
80°	40	2.67	51.5	0.2532	0.4244	3.27	90	64
80°	40	4	64.4	0.4007	0.4591	4.43	74	49

• $\beta_1 = 0.39$; $\max \beta_1 = 0.47$ for mirror mode stability.

• $L_1 = 74$ cm, $L_2 = 49$ cm.

The density and magnetic field profiles for this case are shown in Figs. A-2 and A-3. We note that the calculated ion energy is less than the 50-keV mean energy assumed in the proposal. There are several reasons why $\bar{E}_i$ may rise above the calculated value. First, the Fokker-Planck model neglects any effects of ion cyclotron turbulence which can raise the mean ion energy. Of course, if turbulence is too severe, confinement suffers. Second, the Fokker-Planck code would produce somewhat higher ion energy if parameters appropriate to radii > 0 were used. At larger radii the strength of the injected beam increases relative to the charge-exchange flux originating in other regions of the plasma. Since the injected beam is of higher energy, the energy

Fig. A-2. Fokker-Planck calculation of axial density profile for standard case. Plasma parameters are listed in the text.

Fig. A-3. Fokker-Planck calculation of the magnetic field with finite beta effects.

deposited per trapped ion will increase at larger radii and raise $\bar{E}_i$. Third, if the flat-top profile is really stable in the interior against the DCLC, T_e will increase because no stream which cools the electrons will be present. This will reduce drag losses of the ions and raise $\bar{E}_i$. We might also expect energy confinement to increase above the assumed value of $10^{12} \text{ cm}^{-3}\text{-s}$.

ELECTRON TEMPERATURE

The sensitivity of $\bar{E}_i$ to the electron temperature is summarized in Table A-2.

We see that the plasma length does not change, but the particle lifetime and mean ion energy do increase with T_e . This is expected for a plasma dominated by electron drag.

Table A-2. Variation of plasma parameters with electron temperature.

T_e , keV	$\bar{E}_i$, keV	$n\tau_p$, $\text{cm}^{-3}\text{-s}$	L_2 cm
1.33	34.6	2.08×10^{12}	49
2.67	50.3	3.48×10^{12}	49
4.00	64.4	4.43×10^{12}	49

SPATIAL BEAM WIDTH

The effect of varying the spatial extent of the beam along the magnetic axis is summarized in Table A-3 for $T_e = 1.33$ keV.

Table A-3. Variation of plasma parameters with beam length.

z_b , cm	$\bar{E}_i$, keV	$n\tau_p$, $\text{cm}^{-3}\text{-s}$	l_2 , cm	β_1
34	35.2	2.07×10^{12}	48	0.3746
40	34.6	2.07×10^{12}	49	.3939
48	34.3	2.07×10^{12}	50	0.4175

We see that the plasma length is not significantly increased by increasing the width of the beam. Plasma beta is also not significantly changed. An increase in current of 41% $\left(\frac{48-34}{34} \times 100\right)$ increases β_1 by 12%.

FINITE BETA EFFECT

The variation of plasma length with beta was the strongest effect observed in our Fokker-Planck studies of MX. Results are summarized in Table A-4.

Table A-4. Variation of plasma parameters with beta.

β_1	$\bar{E}_i$, keV	$n\tau_p$, $\text{cm}^{-3}\text{-s}$	l_2 , cm
0.03	37.4	1.71×10^{12}	79
0.17	36.5	1.90×10^{12}	68
0.39	34.6	2.08×10^{12}	49

Confinement $n\tau_p$ increases with β_1 due to the enhancement of the mirror ratio by plasma diamagnetism. In the long-thin equilibrium approximation this is given by $R = R_v/\sqrt{1 - \beta_1}$. The plasma length decreases with increasing β_1 primarily because the plasma density is a function of magnetic field strength, and the scale length for the axial variation of magnetic field decreases rapidly as one approaches the mirror-mode beta limit.

THREE-DIMENSIONAL GUIDING-CENTER EQUILIBRIUM

The three-dimensional guiding-center equilibrium code uses the actual MX vacuum magnetic field together with an analytic plasma pressure model to compute the spatial equilibrium configuration of the plasma, including all geometrical effects. With this code we can test the validity of the long-thin plasma equilibrium model. The vacuum magnetic field is obtained from the MAFCO code using the MX coil design parameters. The plasma pressure model is obtained by fitting the Fokker-Planck code distribution functions to an appropriate analytic model. We assume a pressure tensor of the form:

$$P(B, \psi) = \hat{P}(B)w(\psi) \quad , \quad \hat{P} = (p_{\parallel}, p_{\perp}),$$

where ψ labels the flux surface, and the pressure tensor components $p_{\parallel}(B)$ and $p_{\perp}(B)$ give the pressure variation with field strength along the flux surface. The function $w(\psi)$ determines the radial distribution of the plasma at the midplane. For MX we have used the radial profile:

$$w(r) = (1 - r^2/a^2)^2,$$

where $a = 30$ cm. The axial distribution of the plasma is modeled by the analytic form:

$$\hat{p}_{\perp}(B) = NB^{-K} (B_m - B)^{K+1/2} [2(K+1)B_m + B],$$

where N is a normalization constant, B_m is the magnetic field strength at the mirror, and K is a shape parameter which we choose to fit the pressure profiles obtained from the Fokker-Planck code results. For MX we have taken $K = 1.5$, as indicated by the curves shown in Fig. A-4 for $p_{\perp}(B)$. With this model the resultant magnetic field and plasma density profiles on the magnetic axis are shown in Figs. A-5 and A-6 for $\beta_{\perp} = 0.36$. The plasma lengths for this case are $L_1 = 89$ cm and $L_2 = 64$ cm. The long-thin plasma equilibrium model used in the Fokker-Planck code for this case yields the plasma lengths $L_1 = 74$ cm and $L_2 = 49$ cm, so the more realistic three-dimensional model predicts larger plasma volumes.

Fig. A-4. Plasma pressure profile used in three-dimensional, guiding-center equilibrium code.

Fig. A-5. Magnetic field calculated by three-dimensional, guiding-center equilibrium code.

Fig. A-6. Normalized axial plasma density profile calculated by three-dimensional, guiding-center equilibrium code.

EXPECTED PLASMA LENGTH IN MX

Plasma lengths calculated using the bounce-averaged Fokker-Planck model appear to be lower limits. The plasma shape in MX is more spherical than cylindrical. The Fokker-Planck calculation, using the long-thin approximation, overestimates the deepening of the magnetic well by finite beta, but underestimates the plasma length. This is borne out by a calculation at similar

beta using the three-dimensional equilibrium code for a plasma of 30-cm radius. For this calculation the length was $L_2 = 64$ cm compared to 49 cm from the Fokker-Planck.

We can also estimate lengths for MX by scaling 2XIIB measurements according to beam scale length as follows:

$$L_1(MX) \approx L_1(2X) \times \frac{L_B(MX)}{L_B(2X)} \approx 38 \times \frac{34}{12} = 108 \text{ cm},$$

$$L_2(MX) \approx L_2(2X) \times \frac{L_B(MX)}{L_B(2X)} \approx 24 \times \frac{34}{12} = 68 \text{ cm}.$$

We might expect the length in MX to be somewhat longer relative to 2X, since the injected beams have a greater angular spread and are injected at angles less than 90° . The plasma also should be more collisional.

In the proposal the plasma length was assumed to be about one-third of the mirror-to-mirror spacing (113 cm). From the Fokker-Planck results, and also realizing the limitations of the model, the plasma length for calculating sustaining current is assumed to be $L_2 \approx 70$ cm.

Appendix B. Beam Trapping Efficiency

Calculation of the trapping efficiency for the injected beams must take into account the geometry of the beams and plasma. The radial buildup code calculation of trapping efficiency takes into account the radial variation, but neglects axial variations. We use a simplified model to estimate trapping efficiency. Besides beam attenuation, charge exchange of the beam on the plasma must be included since only ionization adds particles. The trapping efficiency is given by:

$$\eta_t = 1 - \langle e^{-n\sigma_1 \ell} \rangle - I_{cx}/I_b,$$

where the brackets indicate an appropriate average of the total attenuation over the beam and plasma dimensions, and I_{cx}/I_b is the ratio of charge-exchange current to injected current which emerges from the plasma.

The charge-exchange calculation is complicated by re-ionization of charge-exchange neutrals as they exit the plasma. An exact solution must take into account multiple ionization and charge-exchange events, and the spatially varying angular distribution of the neutrals. An approximate solution is to assume that all charge-exchange neutrals are directed heading in the same direction as the beam (forward scattering). With this assumption only impact ionization need to be taken into account. The trapping efficiency is:

$$\eta_t = 1 - \langle e^{-n\sigma_1 \ell} \rangle. \quad (B-1)$$

Forward scattering of charge exchange is more probable at higher energy, since the charge-exchange cross section rapidly decreases with energy, and head-on collisions have an increasingly smaller probability of charge exchange. For MX energies (50 keV) 50% of the charge-exchange neutrals are created with velocity vectors within approximately 50° of the beam direction. In spite of this still rather large angle, the calculation of trapping efficiency, assuming forward scattering, agrees to within 5% of a Monte Carlo computer calculation³ for a pencil beam along a diameter in a plasma of the thickness of MX.

Taking into account the beam plasma geometry and assuming symmetry in the beam and plasma about the midplane, the trapping efficiency is given by

$$\eta_t = 1 - \frac{\int_0^{L_z} dz \int_0^{L_y} J_b(y, z) e^{-n(y, z) \sigma_i \ell(y, z)} dy}{\int_0^{L_z} dz \int_0^{L_y} J_b(y, z) dy} \quad (B-2)$$

The beam is assumed to have axial and transverse dimensions L_z and L_y , respectively.

The neutral beams injected into MX have a Gaussian current distribution in z with scale length 34 cm (1/e power point). We have evaluated Eq. (B-2) for $L_z = 34$ cm. Fokker-Planck calculations (Appendix A) show that beam injected beyond 34 cm is not trapped efficiently. Beam current density is assumed to be uniform in y and to fill the plasma to radius $L_y = R_p = 30$ cm. Fokker-Planck calculations predict a Gaussian-like axial density profile. To evaluate Eq. (B-12), the plasma density is assumed uniform in radius, and Gaussian in z with scale length L_{pz} . The path length ℓ is calculated for a plasma with elliptical cross section whose dimensions vary in z according to the vacuum magnetic field. Beams are injected in the 45° plane as shown in Fig. B-1.

The variation of η_t with plasma scale length is shown in Fig. B-1. Trapping efficiency decreases rapidly as expected for plasmas with scale lengths shorter than the beam. The plasma length L_2 for a Gaussian profile is $L_2 = 1.25 L_{pz}$. Thus, for the estimated length in MX ($L_2 = 70$ cm), $L_{pz} = 56$ cm and $\eta_t \approx 0.63$. These estimates are only approximate since the beam, in general, will not have a square radial profile and some portion of the beam may be injected into the boundary.

Fig. B-1. Variation of beam trapping efficiency with plasma length for: beam injection in the 45° plane (a); injected beam and cross section at several axial positions (b); plasma axial density profile of scale length L_{pz} (c).