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SPALL STUDY IN ONE DIMENSION

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SPALL STUDY IN ONE DIMENSION

ABSTRACT

A SOC one-dimensional calculation of an underground nuclear test is presented to exemplify the shock propagation and spall phenomenology commonly predicted. Then to examine the effects of spherical divergence on spall, a series of SOC calculations, at different radii of curvature, are conducted and prediction of depth and velocity for the first spall zone are compared with predictions from simple analytic theory. The excellent agreement in this comparison verifies that the SOC code accurately represents the physics of spalling. This study also indicates that the total spall depth is independent of divergence because of the compensating effect that subsequent convergence has on the reflected wave. The latter result implies that the total depth of spall calculated for each underground nuclear tests must be critically examined and evaluated. Finally, SOC calculations for nuclear detonations in tuff and granite are performed to demonstrate the significant effect that variations in material response under shock loading have on shock propagation and spall.

INTRODUCTION

The primary goal of this study is to develop and to inject some experience and understanding into the phenomena of spall associated with one-dimensional calculations. One-dimensional calculations, employing the SOC code,¹ have been extensively used by the containment group in K-Division to predict ground and surface motions^{2,3} as well as spall depths for underground nuclear tests. A recent example of such a SOC calculation will be presented to illustrate the calculational procedure and the type of predictions obtained. Although there are certain limitations and questions regarding the use of SOC calculations in representing underground nuclear events, this report will be confined to an examination of only the spall portion of such calculations.

Initially it is important to prove that the SOC code can accurately predict the physics of spalling. This step will be pursued by comparing SOC predictions of spall depth and spall velocity with corresponding values based on analytical solutions. This comparison was made assuming a linear (elastic)

P- μ and K-P relationship and tensile strength of 2.0 MPa for the medium in which the stress wave propagates. The analytical solutions used in the above comparisons will be derived from elementary spall theory.⁴ The close agreement obtained from the comparison provides the confidence needed to accept spall calculations using the SOC code in more realistic (nonideal) media.

Another area of concern is the divergence effect or depth-of-burial (DOB) of the explosive on the total depth of spall. The present criterion for successful containment of a nuclear explosion⁵ is that it be buried at a minimum depth of $122 W^{1/3}$ m, where W equals the yield of the nuclear explosive in kilotons.* Consequently, DOB's of 122 and 1220 might be expected for yields of 1 kt and 10^3 kt, respectively. These DOB's represent a large range of curvature for the explosive-produced shock wave that arrives at the surface. The effect of curvature on spall depth and spall velocity will be examined with a series of SOC runs in spherical geometry. This same series of SOC runs will be the basis for comparison with analytic values for the first spall depth and spall velocity that have been discussed above.

Finally, the spall dependence on material behavior under shock loading will be investigated. For this purpose SOC calculations for a nuclear detonation in granite and a Pahute Mesa tuff are performed. The P-V (or μ) and K-P curves for these materials were obtained from the PMUGEN code of Butkovich.⁶ It will be shown that the varying of the elastic limit for the tuff can have a significant influence on the resultant stress pulse and thus on the depth of the first spall.

BACKGROUND

This section will briefly review a SOC calculation for a recent nuclear event to illustrate the general explosive phenomenology predicted. The energy released in a nuclear explosion is sufficient to vaporize a significant amount of medium in the immediate vicinity of the working point (WP). A value of 11.72 MJ/kg (2800 cal/g) is used⁷ as the estimate of the energy necessary to vaporize totally the geological media composed of $\text{SiO}_2\text{-H}_2\text{O}$ mixtures. For such a medium with a bulk density of $\sim 2.1 \text{ g/cm}^3$, the vaporization radius will be $2.0 W^{1/3}$ m. The high-energy gas contained in this sphere of vaporized medium

* In practice, the minimum DOB for a nuclear test in the range of a few kilotons or less is 183 m (600 ft).

is the driving mechanism for expansion of the cavity and generation of the radial shock wave. The cavity expansion and resultant radial shock wave propagation in the direction of the surface are explosive phenomena that the SOC code attempts to simulate.

To illustrate the above dynamic processes, SOC calculational results⁸ will be given for a nuclear explosion in the intermediate yield range at a DOB of 1231 m below the surface of Pahute Mesa. Figure 1 presents the available geophysical log data and the eight-layer division of the geology between the WP and the surface used in the SOC calculations. A more detailed list of the physical properties and layering are shown in Table 1. The porosity and water content values in Table 1 are calculated from gravimeter density measurements, grain density measurements from core samples, and assumptions of 50% saturation above the water table and 100% saturation below. The division of the geology into the layers shown was based on strong impedance mismatches, the water table, and density changes. Values from Table 1 provide sufficient input to the PMUGEN⁶ code to generate P- μ and K-P tables for each geologic layer. All the elastic limits used in the calculation were default values generated by the PMUGEN code. The P- μ and K-P tables for each layer were then used in SOC to describe cavity growth and shock propagation.

Figure 2 shows the cavity growth and shock propagation vs time. The first arrival is the elastic precursor, which propagates with sonic velocities through the various layers. The main shock follows with a slowly decreasing velocity until it arrives at the water table. Compaction of the 17% air-filled porosity for the layer above the water table severely attenuates the

Table 1. Layering and physical properties of geology between WP and surface used to characterize Fontina Site.

Material No.	Range from WP (m)	Depth (m)	Material name	ρ_0 (Mg/m ³)	ρ_g (Mg/m ³)	Porosity ϕ	Saturation	Gas porosity ψ	Water content (wt%)	Velocity (m/s)	Poisson's ratio ν
58	1011.5-1231	0-219.5	Ash flow tuff	1.624	2.431	42	50	21	12.9	2029	0.25
57	789-1011.5	219.5-422	Ash flow tuff	2.26	2.54	13.7	50	7	3.05	3350	0.25
56	636-789	442-595	Ash flow tuff SWL	1.81	2.47	34	50	17	9.3	1860	0.25
55	4355-636	595-795.5	Bedded tuff	2.08	2.58	31.6	100	0	15.21	2900	0.3
54	400.4-435.5	795.5-830.6	Ash flow tuff	2.34	2.56	14.1	100	0	6.05	4220	0.3
53	305.6-400.4	830.6-925.4	Bedded tuff	2.00	2.57	36.3	100	0	18.15	2880	0.3
52	124.6-305.6	925.4-1106.4	Lava flow	2.25	2.53	18.3	100	0	8.13	4214.5	0.3
51	0-124.6	1106.4-1231	Zeolitized ash flow tuff	2.19	2.56	23.7	100	0	10.83	3000	0.3

Fig. 1. Geophysical log summary for emplacement hole U20f.

Fig. 2. SOC-calculated shock phenomenology for Fontina Event.

peak stress and the velocity of the main shock. When the peak stress in the main shock decays to the elastic limit of the material, it will also propagate at sonic velocities for the remaining distance to the surface. The elastic limits for geologic media at Pahute Mesa are typically in the 10 to 70 MPa range. Spherical divergence and reflections off geological layers will further attenuate the main shock front before it reaches the surface. Peak stresses of 10 to 20 MPa or less are typical for the main shock that arrives at the surface. Figure 3 illustrates the main shock profile vs radial distance at 400 ms, prior to its interaction with the surface.

To aid in understanding the spall results shown in Fig. 2, the following paragraphs give a brief description of events generating those results. A more thorough development of the spall phenomena will be given in subsequent sections. In one-dimensional calculations the incident and reflected waves are always normal to the surface. The reflected pulse must be opposite in sense to the incident pulse, since at the surface the stress normal to the surface must be zero. Thus, a compressional wave would be reflected as a tensional wave (and vice versa). In one-dimensional spherical geometry the main shock front will be reflected as a tensile wave that will converge back toward the WP. For deeply buried explosions with nuclear yields in the megaton range the main shock may be several hundred metres in length. Therefore, the reflected wave will interact with that portion behind the main shock front that is still propagating toward the surface. The first spall will occur at that location below the surface where stresses in the outgoing (compressive) wave and reflected (tensile) wave first satisfy the condition:

$$\sigma_T + \sigma_c + \sigma_{TS} + \sigma_{OB} = 0,$$

where σ_T is the tensile stress (negative) in the reflected wave, σ_c is the radial stress (positive) in the outgoing wave, σ_{TS} is the minimum dynamic tensile strength, and σ_{OB} is the overburden stress (positive). Typically dynamic tensile strengths of 0 to 1000 kPa are chosen for the geologic layers of Pahute Mesa. As material in the first spall thickness rises it leaves behind a new free surface. However, portions of the outgoing and reflected waves will be trapped in the first zone of spalled material. Consequently, the pulse length of the main shock will be shorter in proportion to the depth of spall. With each spall a new free surface is created and the above process will be repeated until stresses in the reflected wave are no longer sufficient to satisfy the condition expressed in the above equation.

Fig. 3. Radial stress vs radial distance in vertical direction at 400 ms.

To clarify further the spall depth illustrated in Fig. 2 let us examine the calculational results in more detail. A conservative value of zero was assigned the tensile strengths for the different layers in Fig. 2. An upper limit on the total spall depth is one-half the length of the main shock that arrives at the surface. The total spall depth of 231 m shown in Fig. 2 is consistent with that requirement because it is less than one-half the 615-m length for the radial stress pulse shown in Fig. 3, prior to its arrival at the surface. As shown in Fig. 3, the overburden contributes to the radial stress in the shock wave in proportion to the depth, because the calculation is for the vertical direction with gravitational forces acting. The overburden would be constant for a shock propagating in the horizontal direction from the WP. The decay of the peak radial stress vs radial distance is shown in Fig. 4. It was pointed out earlier that the large attenuation seen about 636 m corresponded to shock compaction of the 17% air-filled porosity in the layer above the water table. The rapid attenuation above 1100 m is a result of interaction with the tensile wave from the surface. Figure 5 illustrates the radial stress history at the radial distance of 1011.5 m, which corresponds to a point just above the total spall depth of 231 m. At that point the peak radial stress of 9.3 MPa arrives at 0.425 s, and by 0.55 s the tensile wave from the surface causes significant reduction in the radial stress. The tensile wave continues to increase in magnitude until by 0.645 s the material spalls. The spall is short-lived as recompaction occurs shortly thereafter. In general the form in Fig. 5 is typical of the radial stress histories occurring at other radial distances in the spall region.

Regions are shown in Fig. 2 where the tangential stress (σ_θ) exceeds or is less than the cavity pressure (P_c) as a function of time to illustrate the changing status calculated for the medium between the WP and the surface. Shock wave reflections at low impedance layers can be used to partially explain the regions below the water table where σ_θ is less than P_c or the pressure (P) equals zero, for times earlier than 0.5 s. For a more detailed description of this aspect of the calculations, the reader is referred to Ref. 9.

Fig. 4. Peak radial stress vs radial distance.

Fig. 5. Radial stress history at radial distance of 1011.5 m.

BASIC ANALYTICAL THEORY

This section will present the derivations for the depth and velocity of the first spall zone for a sawtooth and triangular stress pulse. The derivation for the sawtooth is straightforward and the simplest to visualize. It illustrates clearly the physical principles involved in spalling and relates the functional dependence of parameters of the stress pulse to the spall depth and spall velocity. However, the sawtooth wave is not a good approximation to the radial stress pulse arriving at the surface, since the pulse generally has a rise time that may be an appreciable fraction of the total pulse duration or length. The case of the triangular stress pulse will be developed to provide a closer fit to the experimental and SOC calculational stress profile arriving at the surface. The analytical relations for the depth and velocity for the first spall zone will be used for comparison in the following section to determine whether SOC calculations accurately represent the physics of spalling for the first spall zone.

Sawtooth Wave

Consider a material that is strong in compression and will fail in tension at much lower but predetermined value of stress. Now assume that a sawtooth shaped compressive pulse, whose maximum stress σ_0 is greater than the tensile strength σ_t of the material, is propagating toward the surface of the material. After reaching the surface it will begin to reflect as shown in Fig. 6. The resultant stress at the front of the reflected wave will be given by the difference of the reflected stress (σ_0) and the compressive stress (σ_1) acting at that point. At some distance from the surface the resultant stress at the front will equal the tensile strength:

$$\sigma_t = \sigma_0 - \sigma_1, \quad (1)$$

and a fracture will occur at that point. From geometry, Fig. 4, we obtain:

$$\sigma_t = \sigma_0 \frac{\lambda - ct}{\lambda}, \quad (2)$$

Fig. 6. Sawtooth wave interaction with free surface.

where λ is the pulse length, ct is the linear distance between σ_0 and σ_1 , because c is the propagation velocity and t is the time between these two values.

Further examination of Fig. 6 will indicate:

$$ct = 2\delta_1, \quad (3)$$

where δ_1 is the thickness for the first spall zone. Combining Eqs. (1), (2), and (3), and solving for δ_1 give:

$$\delta_1 = \frac{\sigma_t}{\sigma_0} \frac{\lambda}{2}. \quad (4)$$

When the material fails at a distance of δ_1 from the surface a portion of the pulse is trapped in this spall zone. Conservation of momentum dictates that the momentum of the spalled material equals the momentum of the trapped portion of the pulse:

$$\rho V_{s1} \delta_1 = \frac{\sigma_0 + \sigma_1}{2} t, \quad (5)$$

where ρ is the density and V_{s1} is the velocity of the first spall zone. Combining Eqs. (1), (3), and (5) and solving for V_{s1} give:

$$V_{s1} = \frac{2\sigma_0 - \sigma_t}{\rho c}. \quad (6)$$

A portion of the stress pulse remains in the original material and will strike the new free surface formed by the first spall. The above process will be repeated until the peak stress in the wave front is reduced to a value less than σ_t . It can be easily shown for a sawtooth wave that the thickness of each spall zone is equal to δ_1 . For each spall the amplitude of the wave front is reduced by σ_1 and, consequently, the number of spall zones is given by σ_0/σ_t . For example, if σ_0 and σ_t are 15.0 and 2.0 MPa, respectively, there would be seven spall zones in all. If the total pulse length $\lambda = 150$ cm, the thickness of each spall would be [Eq. (4)] 10 cm and the seven spall zones would result in a total spall depth of 70 cm. The remaining 10-cm portion of the stress wave would reflect off the final spall surface and propagate back into the medium as a tensile wave. The above one-dimensional plane version of the spall phenomena illustrates the basic concepts, and the following subsection

will consider another pulse type that better approximates the pulse shape of a radial shock arriving at the surface from an underground nuclear event.

Triangular Pulse

Close to an explosive source the velocity for the radial shock wave will exceed the sonic velocity for geologic materials and the rise time to the peak stress may occur on a microsecond or submicrosecond time scale.¹⁰ As the shock expands the wave slows down and the rise time from first arrival to peak stress increases.^{11,12} The rise time may increase to become a significant portion of the total pulse duration by the time the shock reaches the surface. It is for the purpose of accounting for the effect of rise time on spall depth that the triangular stress pulse is considered. The results of derivations that follow will be used later in evaluating one-dimensional SOC calculations.

Again, consider a material strong in compression in which a triangular compressive pulse with maximum stress σ_0 is propagating toward the surface, Fig. 7. As the wave reflects from the surface the increasing stress in the ramp more than cancels out the reflected tensile wave and keeps the material in a state of compression. This condition will persist until the peak stress of the wave reflects, and then material near the surface will go into tension. As this wave continues to reflect off the surface, the difference between the peak reflected stress (σ_0) and the compressive stress (σ_1) at that point increases until it reaches the tensile stress:

$$\sigma_t = \sigma_0 - \sigma_1, \quad (7)$$

at which time a fracture will occur. Examining the geometry, Fig. 7, we can surmise the relation:

$$\sigma_1 = \sigma_0 \left(\frac{\lambda - b - 2\delta_1}{\lambda - b} \right), \quad (8)$$

where λ is the total pulse length, b is the length of the ramp, and δ_1 is the thickness of the first spall zone. Combining Eqs. (7) and (8) and solving for δ_1 yield:

$$\delta_1 = \frac{\sigma_t}{\sigma_0} \frac{\lambda - b}{2}. \quad (9)$$

Fig. 7. Triangular wave interaction with a free surface.

A portion of the stress pulse will be trapped in this spall zone as it separates from the rest of the material. An interesting feature is that no part of the initial ramp or rise time of the incident pulse is contained in the first spall zone. Equating the momentum of the spalled material to the momentum of the trapped portion of the pulse gives:

$$\rho V_{s1} \delta_1 = \frac{\sigma_0 + \sigma_1}{2} \frac{2\delta_1}{c} \quad (10)$$

where ρ is the density, c is propagation velocity, and V_{s1} is velocity of the first spall zone. Inserting the value of σ_1 from Eq. (7) into Eq. (10) and dividing by $\rho \delta_1$ give for V_{s1} the relation:

$$V_{s1} = \frac{2\sigma_0 - \sigma_t}{\rho c} \quad (11)$$

Comparing the results of the sawtooth and triangular stress pulses illustrates certain similarities and differences. The relations for the first spall velocity — Eqs. (6) and (11) — are identical, although the first spall depth will be greater for the sawtooth case assuming equivalent values for σ_0 , σ_t , and λ . The most prominent difference is that for the sawtooth wave the leading edge of the pulse is trapped in the first spall whereas for the triangular case a middle segment of the pulse is trapped in the first spall zone. In the triangular case a net tension exceeding σ_t occurs adjacent to and just left of the first spall surface. This net tension will give rise to a number of spalls of infinitesimal thicknesses until the tension has been reduced to a value less than σ_t . Equations (9) and (11) for spall depth and velocity, respectively, will be employed later in evaluation of SOC calculations of the first spall zone.

SOC SPALL CALCULATIONS

Elementary spall theory from the previous section shows that spall depth and motion are influenced by pulse shape and length, peak stress, tensile strength, density, and propagation velocity. In addition, because the DOB for a contained nuclear explosive may vary from ≈ 183 m for a kiloton to ≈ 1200 m

or more for yields in the megaton range, divergence or radius of curvature of the shock wave at the surface may affect spalling. Loading and unloading P- μ behavior, K-P failure surface, and elastic limit are some other material response properties affecting the magnitude and the profile of the stress wave arriving at the surface and thus indirectly spall formation. This report identifies some basic features or trends that variation of many of the above factors have for plane and spherical SOC calculation on spall generation. The above list is by no means exhaustive because other factors (e.g., overburden, location of the water table,¹³ layering,¹⁴ etc.) do affect spall depths and velocities, but they are not included in the following study on divergence. Although much remains in understanding spall phenomenology in the geological environment, it is hoped that this report will provide an initial step toward the understanding and the interpretation of one-dimensional spall calculations.

Divergence Effects and Correlation of First Spall Depth and Velocity with Analytical Theory

In order to compare the depth and the velocity of the first spall zone with those predicted from analytical theory for the triangular wave, a pressure profile (Fig. 8) was applied to the inner boundaries for a series of 100-m thick elastic media that had different radii. The pressure profile shown in Fig. 8 is similar in shape and duration to a radial stress measurement made on a recent nuclear event.¹⁵ Radii for the inner boundaries were 100, 300, 500, 700, and 900 m. In addition to these five problems the case of a 100-m thick elastic medium in plane geometry was run. The plane geometry problem represented the limiting case for the one-dimensional spherical geometry with an infinite radius. All six problems were calculated in the horizontal mode to eliminate the effects of overburden or gravitational forces. The omission of gravitational effects was made to simplify the comparison with analytical solutions for the depth and the velocity of the first spall zone. It was believed that the first five problems covered satisfactorily the DOBs normally encountered in the testing program at NTS, neglecting the effect of overburden.

The reasons for using a pressure profile at the inner boundary were primarily two-fold. First, over the range of radii being considered we seldom obtain the ideal triangular pulse that is amenable to comparison with theory. Secondly, by holding constant the input pressure profile for all problems, the effects of divergence are more readily evident and comparisons simplified.

Fig. 8. Pressure profile input used at inner boundary.

The P- μ and K-P curves of the elastic medium are shown in Figs. 9 and 10, respectively. Because the peak input pressure is only 15.0 MPa, the assumption that the medium is linear elastic in compression is not too bad an approximation for tuff, sandstone, and rhyolite types of materials.^{16,17} A wide range of shear strengths vs pressure exists for dry rocks¹⁷ and the results depicted in Fig. 10 are characteristic for that type of medium. The assigned 2.0-MPa tensile strength shown in Fig. 10 is two to four times greater than that commonly used for media near the surface of Pahute Mesa, but was adopted to increase the thickness of the first spall. All six problems were run with 500 uniformly thick 20-cm zones. Thus 20 cm is the limit of accuracy for defining the spall depth, because the spall could occur anywhere within the 20-cm zone. The greater the spall depth the smaller the relative uncertainty that 20 cm represents. Other physical properties used to characterize the elastic medium are summarized in Table 2.

Table 2. Physical constants assumed for elastic materials used in spherical divergent studies.

Density, Mg/m ³	2.1
Water fraction, %	0
Elastic limit, MPa	150
Bulk modulus, GPa	9.0
Poisson's ratio	0.3
Sound speed, m/s	2613.15
Tensile strength, MPa	2

The application of the pressure profile (Fig. 8) to the inner boundaries of the elastic medium would be expected to generate a triangular radial stress pulse. Figure 11 shows the radial stress vs distance for all six problems at 36 ms after the pressure pulse was first applied at the inner boundary. At this time the wave has just about reached the outer boundary or free surface. The stress curve identified by S2 corresponds to the problem where the inner and outer radial boundaries of the elastic medium are initially at 100 and 200 m, respectively. Similarly, S4 would correspond to inner and outer radial boundaries initially at 300 and 400 m, respectively, and so on. Finally, the curve designated SP represents the problem run in plane geometry. Thus, the abscissa in Fig. 11 corresponds to the distance the wave has travelled from the inner boundary.

Fig. 9. Loading and unloading P - μ relationship for spherical divergent study.

Fig. 10. K-P failure curves used in spherical divergent study.

Fig. 11. Radial stress vs distance at 36 ms.

Examination of Fig. 11 illustrates the attenuating effect of divergence on the radial stress profile with distance. The radial stress in the shock wave decreases slightly faster than the ratio R_i/R_0 , where R_i and R_0 correspond to the inner and outer radii of the 100-m thick medium, respectively. Consequently the attenuation is greatest for S2 and becomes progressively smaller as the radii increase. Dispersion contributes to the lowering of the stress profile as evidenced by a peak stress of less than 15.0 MPa for the plane problem, SP. Another attenuating factor is the portion of the wave energy that generates tangential stresses that do not contribute to spalling at the surface.

This application of pressure (Fig. 8) on the inner boundary results in displacement of that boundary. As material near the inner boundary is radially displaced, there is an increase in tangential stress. When the tangential stress equals the 2.0-MPa tensile strength assigned the medium, radial cracks at or near the inner boundary form and propagate toward the surface. Crack formation occurs well behind the peak stress in the shock wave, and the crack propagates at a much slower rate than the shock in the medium. Radial crack formation and propagation are particularly pronounced for S2. These effects diminish as the inner and outer boundary radii increase and disappear altogether for the plane wave case because radial divergence is absent. This phenomenon will be discussed in greater detail later in the report.

The stress profiles, Fig. 11, provided the values for σ_0 , λ , and b used in Eqs. (9) and (11) for the first spall depth and velocity. Comparison of theory for a triangular wave and predictions from SOC calculations for first spall depth and velocity are summarized in Table 3. An examination of those

Table 3. Computational and analytical results for the divergent spall study.

Problem name	Radial rise length "b" (m)	Stress wave-length "λ" (m)	Profile peak ^a "σ _R " MPa	SOC code δ ₁ (m)	Theory δ ₁	SOC code V _{S1} (m/s)	Theory V _{S1} (m/s)	Total spall depth (m)	Spall origins
S2	6.3	56.3	7.0	6.0-6.2	7.1	2.08-2.53	2.18	25.6-25.8	(σ _R , σ _T)
S4	5.8	56.3	10.6	4.8-5.0	4.8	3.30-3.73	3.47	27.2-27.4	(σ _R)
S6	6.0	57.3	11.7	4.4-4.6	4.4	3.69-4.11	3.87	27.2-27.4	(σ _R)
S8	6.3	58.3	12.3	4.2-4.4	4.2	3.88-4.29	4.09	27.4-27.6	(σ _R)
S10	6.3	58.3	12.7	4.2-4.4	4.1	3.88-4.41	4.23	26.2-26.4	(σ _R)
SP	5.8	58.2	14.0	3.8-4.0	3.8	4.73-4.99	4.71	26.8-27.0	(σ _R)

^aPeak stress-values not from Fig. 9 but taken from SOC output when the peak stress was about 5 m from the outer boundary.

results shows close agreement between theory and SOC predictions, with the exception of the first spall depth, δ_1 , for the S2 problem. This discrepancy is attributed to comparing SOC result in spherical geometry with theory derived for plane geometry. As the inner and outer radii increase, this approximation becomes more accurate. The velocity range given for the first spall velocity corresponds to the lowest and highest velocity seen in the first spall zone. The lowest velocity generally occurred at or near the outer boundary, while the highest velocity always occurred for the spalled zone closest to the point at which material fracture or separation took place. This result is easily explained with the rubber band analogy. As a rubber band is stretched past its breaking point, those parts of the rubber band with the highest velocity are adjacent to the point of rupture.

After the first spall occurs there exists a new free surface from which a second spall may be initiated. The above process continues with each new spall trapping a portion of the wave. Eventually, either all of the wave is trapped in the spalled material or the reflected tensile wave does not have sufficient magnitude to exceed the tensile strength (2.0 MPa in the present example) of the material. The upper limit on the total spall depth is half the wavelength of the shock wave. All six SOC problems were run a sufficient length of time to determine the total spall depth in each case. The final two columns in Table 3 give the total spall depths and mechanisms, radial or tangential stress, responsible for producing subsequent spalls. With the exception of S2 case all of the spalls that occurred were induced by the radial stress. In the case of S2 only those spalls occurring in the first 8.6-m distance from the surface are induced by radial tensile stresses. The other 17.0 m included in the S2 results for the total spall depth, Table 3, are a consequence of the tangential stress exceeding the 2.0 MPa tensile strength and inducing radial fractures. The rapid increase in tangential stress close to the surface in the S2 case is the result of particle motion of the medium having a greater tangential effect because of the larger divergence. In essence radial fractures produced by tangential stress are not truly spalls because they are not parallel or concentric with the free surface.

Examination of the total spall indicates that they are relatively independent of divergence. This may seem strange since the magnitude of peak stress near the free surface for S2 is only half that for SP (Fig. 11). The answer to this paradox is presented in Appendix A, which illustrates the role that convergence in spherical geometry plays in the reflected (tensile) wave.

In essence, convergence increases the magnitude of the tensile wave resulting in a larger total spall depth than would appear warranted by the peak stress of the shock wave arriving at the surface. Although the examples in Appendix A are explicitly for a sawtooth stress pulse, the same principles and argument (albeit somewhat more complex) would apply to a triangular or other pulse shape in spherically symmetric geometry.

The results presented in Table 3 establish that SOC calculations accurately represent the first spall depth and velocity. To that extent the validity of SOC to describe the physics of spall has been demonstrated; such a demonstration was one of the primary goals of this report. A more fundamental question is how well do SOC calculations represent spall at the surface for underground nuclear events. If we neglect the effect that layering¹⁴ and the water table¹³ can have on spherical divergence and/or pulse shape, there remains the obvious fact that a shock-wave reflection off the ground surface in a subsurface nuclear explosion will not converge back toward the WP. This aspect of the SOC one-dimensional spherical calculations can produce a significant error in calculating the total spall depth. The magnitude of the error depends upon the DOB and the length of the stress pulse. At large DOB (>1000 m), typical of Pahute Mesa events, the radial stress wave near the surface is starting to look planar, and if the pulse length is relatively short all the pulse will be trapped in the spalled region before convergence becomes important. One example of this situation was Fontina³ for which SOC calculations indicated a total spall depth of only 231 m. As the pulse length increases, the effect of convergence becomes more important and much greater spall depths than expected will be calculated. Such was the case for Kasseri² where spall depth of 395 m was calculated with SOC. As the stress pulse length increases, we can definitely conclude that the error in determination of the total spall depth for underground nuclear events will increase due to the unrealistic convergence that occurs for SOC one-dimensional spherical calculations.

Shock Propagation and Spall Dependence on Material Response

This subsection will examine shock propagation and spall in a granite and a tuff medium. The SOC-calculated shock propagation and spall results from a 0.5-kt nuclear detonation at a DOB of 200 m in granite will be compared with those obtained in the previous study on divergent effects in an elastic medium. This comparison may be obscured to some extent because the calculations are performed assuming the presence of gravitational forces or overburden,

which was neglected in the previous studies. The validity of this comparison depends upon the equivalence of using a pressure input at 100 m from the WP to represent the shock propagation to the surface from a nuclear explosion. This question will be addressed by taking the radial stress profile in time at 100 m from the WP in the nuclear calculation and applying it as a pressure input to the outer 100-m thickness of the granite medium. The radial stress profile at the time the shock wave arrives at the outer boundary and resultant spall depths will be compared with results from the nuclear explosive calculations in granite to check the equivalence of the pressure input approach.

The granite was originally selected for the above spall study because of its elastic properties at low stress levels, but granite is not representative of the geology normally used at NTS for nuclear testing. To study a more typical medium the physical properties of a tuff, located near the surface of Pahute Mesa for the Muenster Event,¹⁸ was selected. Then a SOC calculation was performed for a 1-kt nuclear explosive at a DOB of 122 m. One important parameter commonly varied for tuff is the elastic limit. The profound effect that this parameter has on shock propagation will be exemplified by assigning tuff an elastic limit of zero and 40.0 MPa for two different SOC calculations. These two values fairly well represent the full range of the elastic limits for NTS tuffs near the surface.

Table 4. Physical properties of granite used in 0.5-kt spall study.

Initial density, Mg/m ³	2.60
Grain density, Mg/m ³	2.65
Water fraction, %	0.0
Porosity, %	1.9
Elastic limit, GPa	5.0
Bulk modulus, GPa	47.6
Poisson's ratio	0.3
Sound speed, m/s	5439.8
Tensile strength, MPa	2.0

0.5-kt Granite Study — Physical properties of the granite chosen in the current study are summarized in Table 4. Some of the properties listed were used as input for PMUGEN⁶ to obtain the P-V curves shown in Figs. 12, and 13, and the K-P results shown in Fig. 14. Experimental P-V values for

Fig. 12. P-V curves to 25.0 GPa used in 0.5-kt granite study of spall.

Fig. 13. P-V curve to 100 GPa used in 0.5-granite study of spall.

Fig. 14. K-P failure curves for granite used in 0.5-kt granite study of spall.

Fig. 15. Radial stress in shock front vs time at 100 m from explosive center.

Westerly granite¹⁹ are plotted in Fig. 13 to compare with the theoretical curve. The P-V and K-P curves are tabularized for use in SOC.

The extreme pressures and temperatures generated in a nuclear explosion are sufficient to vaporize the device materials and some of the surrounding medium. For silicate rocks, a value of 70 metric tons per kiloton of energy release has been calculated⁷ for the amount completely vaporized. For 0.5 kt of nuclear yield and a granite density of 2.60 Mg/m^3 , the vaporization radius is 1.476 m. A common practice for initiating SOC calculations is to distribute the nuclear explosive energy uniformly throughout the spherical volume of vaporized rock and to initiate the calculation at that point. This practice has been shown to be accurate at distances greater than a few cavity radii²⁰ and consequently was adopted for the present study. As the sphere of vaporized rock expands it produces a shock wave that radially propagates into the surrounding granite. A history of the radial stress calculated for the shock wave at 100 m from the WP is given in Figs. 15 and 16. Although the rise time compares favorably with the 0.4 ms assigned the triangular pressure input in the divergent study, the initial rate of decay behind the peak is much greater. In addition the duration of the radial stress pulse is nearly 15 ms longer for the 0.5-kt calculation than the 20.4-ms duration assigned the pressure input for the divergent series calculations. These differences will be shown to affect the radial stress incident on the surface, first spall depth and velocity, and total depth of spall.

Underlying the comparison of the present 0.5-kt and divergent series calculations is the basic question of whether a pressure input can be credibly used to represent a shock wave. To address this question the radial stress history shown in Figs. 15 and 16 was digitized and used as the pressure input at 100 m in the above problem. Figure 17 shows the radial stress profile at the time the shock arrives at the outer boundary. The solid line corresponds to the original 0.5-kt calculation, and the dashed line corresponds to the pressure input with time at 100 m. As the results show, the two curves coincide except for minor differences. Table 5 shows that, surprisingly, even such minor differences produce variations in values for spall depths and velocities. Actually the minor differences shown in Fig. 17 can partially be attributed to the failure of the digitization process to represent accurately every facet of the stress profile at 100 m in the 0.5-kt calculation. However, it is believed that the agreement is close enough to establish the equivalence of the two problems.

Fig. 16. Radial stress vs time at 100 m from explosive center.

Fig. 17. Radial stress vs distance at 35 ms.

Table 5. Spall depths and velocities of granite for 0.5-kt and pressure-time input.

Problem	Spall depth $\delta_1(\sigma_R)$ (m)	Spall velocity V_{S1} (m/s)	Spall depth $\delta_1 + \delta_2(\sigma_R)$ (m)	Spall velocity V_{S2} (m/s)	Total spall depth, δ_T (m)
0.5 kt	7.8-8.0	0.38-0.53	15.0-15.2	0.29-0.35	40.5-40.7
0.5 ktpt ^a	9.2-9.4	0.38-0.56	13.7-13.9	0.29-0.32	40.2-40.4

^aktpt: pressure-time input problems.

1.0-kt Tuff Study — A layer of Timber Mountain tuff located near the surface of Pahute Mesa at the Muenster site was selected for the following study. Physical properties of that tuff layer are listed in Table 6, and some of those properties were used to generate⁶ the P- μ results shown in Figs. 18 and 19 and the K-P results shown in Fig. 20. Comparing physical properties in Table 6 with tuffs of areas 12^{6,21,22} and 16⁶ indicates that Timber Mountain tuff has approximately 5 to 10% greater *in situ* density and 5% greater grain density. The tuffs in areas 12 and 16 are in the 95 to 100% saturation range and generally have 2500 m/s or greater sonic velocities. Although these are not minor differences there exist far greater differences in physical properties

Table 6. Physical properties of Timber Mountain tuff layer used in 1-kt SOC study.

Initial density, Mg/m ³	2.1
Grain density, Mg/m ³	2.52
Water fraction	0.045
Porosity, %	20.4
Saturation, %	46.3
Elastic limit, MPa	0.0 and 40.0
Bulk modulus, ^a GPa	1.7 and 9.01
Poisson's ratio	0.3
Sound speed, ^a m/s	1151.6 and 2632.5
Tensile strength, MPa	0.5

^aThe two values shown for the bulk modulus and sound speed correspond to the elastic limits of 0.0 Pa and 40.0 MPa, respectively.

Fig. 18. P- μ curves to 1.2 GPa for Timber Mountain tuff used in 1.0-kt study.

Fig. 19. P- μ curves to 100 GPa for Timber Mountain tuff used in 1.0-kt study at DOB = 122 m.

Fig. 20. K-P failure curves for Timber Mountain tuff used in 1.0-kt study at DOB = 122 m.

Fig. 21. Radial stress vs distance at time shock arrives at surface.

for tuffs at a given site (Table 1). While our choice of Timber Mountain tuff in this SOC study was arbitrary it is not that unrepresentative of NTS tuffs.

The effects of the elastic limit on shock propagation were examined by conducting one SOC run with the elastic limit set at zero and another set at 40.0 MPa. These two values represent the full range of the elastic limits generally assigned to NTS tuffs near the surface. Both SOC calculations were performed for a 1.0-kt nuclear explosion at a DOB of 122 m. The DOB is less than the prescribed 183 m designated for 1 kt primarily to provide a peak stress, in the shock front arriving at the surface for the case in which the elastic limit equals zero, well above the 0.5-MPa tensile strength assigned the medium. This calculation is primarily a test problem to illustrate the effect of elastic limit on wave shape, first spall depth, and first spall velocity.

The presence of an elastic limit not only has a profound effect upon the peak radial stress arriving at the surface, but the radial stress profile well behind the front. In addition, the shock wave for the case of a 40.0-MPa elastic limit arrives at the surface much sooner. These facts are illustrated in Fig. 21, which shows the radial stress profile at a time just prior to the arrival of the two shock waves at the surface. In the case of an elastic limit of zero, the radial stress at the front is only about 1.2 MPa and monotonically increases as the WP is approached at the time of 89 ms. This profile for the radial stress results in a large depth for the first spall of between 9.4 and 9.6 m and does not happen until 98 ms. Thus, there is a delay of approximately 9 ms after the shock is incident at the surface before the first spall forms.

Table 7 summarizes the first spall depth and velocity for both SOC calculations. The shape of the stress profile indicates a large amount of

Table 7. Depth and velocity for first spall in 1.0-kt tuff study.

Problem	Spall depth ^a	Spall velocity
	$\delta_1(\sigma_r)$ (m)	V_{S1} (m/s)
Elastic limit = zero	9.4-9.6	5.4-6.2
Elastic limit = 40.0 MPa	2.4-2.6	7.4-8.0

^a Failure due to radial cracks from source region extending to the surface occurs shortly after the first spall develops.

energy from the explosive still resides in the vicinity of the WP. As the elastic limit increases the shock velocity increases, and more of the energy is released earlier. This is exemplified by the results at 43 ms for an elastic limit of 40.0 MPa and with the peak radial stress in the shock front at 4.4 MPa. The peak stress in the shock front is higher than occurs at any other position in the shock wave behind the front. Consequently, much more of the explosive energy has already propagated well out of the WP region than for the case of zero elastic limit. The sharp rise and subsequent decay of the radial stress pulse in this case results in a shallow depth for the first spall of 2.4 to 2.6 m at a time of 45 ms. Consequently there is a delay of about 2 ms between the time the shock arrives at the surface and the occurrence of the first spall. The velocity of the first spall is greater for the case of an elastic limit equal to 40.0 MPa. The higher spall velocity can be attributed to the greater stress in the shock front resulting in more momentum being trapped in the spalled region. Radial cracking extending from the source region arrives at the surface within a few milliseconds after the first spall in both problems. Consequently, material failure near the surface can be attributed primarily to tangential stresses generated by the cavity growth and divergence effects rather than to the reflected radial stress component of the shock wave that induces spall of the surface.

SUMMARY AND CONCLUSIONS

The SOC one-dimensional calculation of a recent underground nuclear event is presented to illustrate the type of input used, assumptions made, and calculational results obtained following the explosion. The SOC code has been and is being extensively used for calculating ground motion, surface motion, and total spall depth in the underground test program. Probably the spall phenomenon is one of the least understood aspects of those calculations. This study is a preliminary effort to identify some of the important factors affecting spall at the surface.

Initially we would like assurance that the physics of the spall process is accurately represented by SOC calculations. Then, because the DOB may vary considerably for different nuclear tests, the radius of curvature of the shock wave at the surface will vary accordingly. Consequently, the influence of divergence on spall depth is a definite concern. Finally, variations in

material response to shock propagation will affect the shock profile and possibly the total spall depth.

The fact that the SOC code accurately represents the physics of spall was established by the excellent agreement obtained between SOC calculational values for the depth and velocity of the first spall zone and predictions from simple analytic theory of spall for a triangular stress pulse. The above comparison used a pressure input at the inner boundary of a 100-m thick elastic medium. In a series of SOC calculations the radius of the outer boundary was varied from 200 to 1000 m, which is a variation that nearly spans the DOB range used in the underground test program. This series of calculations indicated that the total depth of spall is independent of the radius of curvature, because the attenuating effect of divergence on the outgoing shock wave is compensated for by the increase in amplitude of the reflected shock wave as it converges. Obviously, the shock wave in an underground nuclear test will not converge back toward the WP after reflection at the surface. This discrepancy in the one-dimensional simulations become important as the length of the stress pulse increases. The accuracy of calculated total depth of spall must be evaluated for each nuclear test in light of this unrealistic convergence effect.

SOC calculations for nuclear detonations in tuff and granite illustrate the significant effect that variations in material response to shock loading have on shock propagation and spall. The equivalence of using a pressure-time input to represent shock propagation from a nuclear detonation is established in the granite study. SOC calculations of a nuclear detonation in tuff indicate that variation in the elastic limit profoundly influences the shape of the stress pulse arriving at the surface. Increasing the elastic limit increases the magnitude of peak stress in the shock front and decreases the time of arrival of the shock at the surface. Consequently, as the elastic limit increases the rate at which energy is propagated away from the WP region increases. The shape of the stress pulse for both the granite and tuff study does not correspond very closely to the triangular pulse shape used in the divergent study and thus is not conducive to comparison with prediction from analytic theory.

The following conclusions can be drawn about SOC calculations of spall depths associated with underground nuclear tests. The total depth of spall is dependent upon the pulse duration and profile, tensile strength assumed, and response of the material to shock propagation, but is independent of the

effect of divergence. This latter fact is attributed to the unrealistic compensating increase that convergence has on the reflected wave off the surface. Based on SOC calculations, we must conclude that the total spall depth will always be an overestimate. The amount of the overestimate increases as the depth of spall increases. Consequently, each calculation must be evaluated separately as to the extent that the total spall depth can be believed.

This report represents a preliminary step toward understanding the spall phenomenon owing to the fact that other factors (e.g., layering, water table, overburden, etc.) influencing calculations of the total spall depth have not been adequately examined in the present study.

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APPENDIX A. RESOLUTION OF THE CONVERGENCE PARADOX

To exemplify the convergent effect on the spall pattern and total depth we shall compare the propagation of a sawtooth wave in an elastic medium for three different geometries. The three cases are hypothetical SOC runs that retain the general shape of the sawtooth wave. In practice the use of an artificial viscosity term produces a finite ramp at the wavefront but for sake of simplicity the ramp will be omitted in the following discussion. The three geometries to be considered are:

(1). A one-dimensional spherical geometry consisting of divergence for the outgoing (compressive) wave and convergence of the reflected (tensile) wave. This is the normal operating mode for the SOC code used in containment studies and was employed throughout the report.

(2). The one-dimensional plane geometry consisting of pure propagation without modification of the wave shape. SOC was used in this geometry in the divergent effects study earlier.

(3). A one-dimensional spherical geometry consisting of divergence for the outgoing wave and divergence also for the reflected wave. The assumption of spherical divergence of the reflected wave departs from what is actually possible in a SOC calculation but is a closer approximation of the shock interaction with the surface for an underground event.

The three cases enumerated above will be presented in corresponding order. The reader is reminded that the following cases are hypothetical and heuristic in content.

Case 1. Spherical Geometry with Divergence and Convergence

Figure A1 will aid in the following discussion. As the sawtooth wave propagates radially outward, the peak stress in the front lies along the curve $\sigma_{PI}(r,t)$. If the medium were purely elastic, the peak tensile stress in the reflected wave would lie along the curve $\sigma_{PR}(r,t)$. However, when the tensile wave has propagated a distance, δ_1 , the first spall occurs, because the medium has been assigned a tensile strength, σ_t . The residual wave will reflect from the new spall surface, and the magnitude of the leading edge will increase along the dashed curve adjacent to $\sigma_{PR}(r,t)$. The above process is repeated until the wave is totally trapped in the spalled region or its magnitude is reduced below the spall threshold.

Fig. A1. Diagram of sawtooth wave interaction with free surface in spherical (diverging and converging) geometry for elastic medium with tensile strength of σ_t .

For simplicity and conciseness certain liberties have been taken in presenting Fig. A1 as a representation for spall in spherical geometry. To present the true picture a series of figures, similar to Fig. 6, would be necessary illustrating the interaction of the incident and reflected wave for each spall. The net result of such a series of graphic displays would be a spall depth slightly less than one-half the wavelength. Thus the spall depth shown in Fig. A1 is a factor two expanded version of the true picture. Another facet is that every point of the incident wave decays along a curve parallel to $\sigma_{PI}(r,t)$. The net result of this decay is a first spall depth slightly smaller than the δ_1 shown in Fig. A1. The quantitative result of the later effect would be slightly thinner spall thickness than shown and possibly an additional spall or two. However, the total spall depth would be roughly as shown. The above discussion has been presented to clarify the approximations incorporated in Fig. A1.

Case 2. Plane Geometry

Consider the pure propagation of a sawtooth wave in plane geometry illustrated in Fig. A2. It is assumed the wave propagates without distortion and arrives at the surface with peak stress, σ_0 , in the front. The same argument presented previously for the reflected wave applies in this case. Again, the factor two expansion applies to the total spall depth for the same reasons previously discussed. With that exception the diagram is essentially accurate in every other detail.

Comparing Figs. A1 and A2 highlights certain features. In the plane wave case each spall thickness is greater, there are few number of spalls, and the total spall depth is less. Further discussion is deferred following description of the Case 3.

Case 3. Spherical Geometry with Diverging Incident and Reflected Waves

The peak stress in the outgoing wave lies along the $\sigma_{PI}(r,t)$ curve shown in Fig. A3. In this regard the diagram is identical to Fig. A1. The rate at which the peak stress decays is extrapolated beyond the free surface. Application of this extrapolated curve to the reflected wave front describes the attenuation that takes place for the diverging tensile wave. The same arguments for the factor two on total spall depth shown and slightly smaller δ_1 given in Case 1 applies to this figure.

Fig. A2.. Diagram of sawtooth wave interaction with free surface in plane geometry for elastic medium with tensile strength of σ_t .

Fig. A3. Diagram of sawtooth interaction with free surface in spherical (diverging) geometry for elastic medium with tensile strength of σ_t .

Comparing Fig. A3 with Figs. A1 and A2 leads to the following conclusions: In going from Case 1 to Case 3 the corresponding spalls progressively increase in size, the spalls decrease in number, and a decrease in the total depth of spall occurs. The decrease in total spall depth is in agreement with conclusions drawn earlier regarding the interpretation given SOC calculation in spherical geometry. The conclusions regarding the thickness of each corresponding spall zone and the relative number of spalls for each case are based on the hypothetical sawtooth wave used. However, it is felt that using a more realistic triangular stress pulse and including the effect of overburden would not alter the basic conclusions. Attempting to incorporate those factors would have been an undue complication and definitely obscured our simple picture.

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