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PLASMA HEATING BY INDUCED
SCATTERING OF MAGNETOSONIC WAVES

BY

LIU CHEN

PLASMA PHYSICS
LABORATORY

MASTER

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

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Plasma Heating by Induced Scattering of Magnetosonic Waves

Liu Chen

Plasma Physics Laboratory, Princeton University,
Princeton, New Jersey 08540

ABSTRACT

It is shown that, in a plasma with $\beta < 1$ and $T_e/T_i \lesssim 4$, parametric excitations of magnetosonic waves through nonlinear ion Landau damping may be utilized to heat both electrons and ions. Nonlinear saturation and associated plasma heating rate of this decay process are also considered.

I. INTRODUCTION

Recently, there has been increasing interest in using magnetosonic (compressional Alfvén or fast) wave to heat plasmas in both low- β ($\beta \ll 1$) and high- β ($\beta \lesssim 1$) controlled thermonuclear fusion research devices. Here, $\beta = 8\pi n_0 (T_e + T_i) / B^2$. n_0 is the plasma density. T_e and T_i are, respectively, electron and ion temperatures, and B is the static confining magnetic field. In low- β devices such as tokamaks, due to the relatively long confinement time, heating may be achieved by linear collisionless dissipations; i.e., electron transit-time, electron Landau, and ion cyclotron dampings [1]. In high- β devices such as tormac and theta pinches, however, one relies on nonlinear mechanisms such as shock heating to obtain an enhanced (anomalous) resistivity and, therefore, a relatively fast heating time scale [2]. In this paper we show that in a plasma with $\beta \lesssim 1$ and $T_e/T_i \lesssim 4$ a finite-amplitude magnetosonic wave (or an external pump) can parametrically excite other magnetosonic waves at lower frequencies. Since for $T_e/T_i \lesssim 4$, the ion-sound wave ($\omega \approx k_{\parallel} C_s$, $C_s^2 = T_e/m_i$) is heavily damped here, the decay is made through nonlinear ion Landau damping; i.e., it is an induced scattering process. This process transfers wave energy into electrons as well as ions and, therefore, can be utilized as an efficient heating scheme by itself or as a complement to other schemes mentioned above. We note here that a similar decay process occurs for the kinetic Alfvén wave in a low- β plasma [3].

The linear theory of this parametric instability is presented in Sec. II. In Sec. III, we consider its nonlinear evolution and saturation using the weak turbulence theory. We then estimate the associated plasma heating rate. Final conclusions are in Sec. IV.

II. LINEAR THEORY

Let us first consider the linear aspect of this decay instability. The plasma is assumed to be infinite and homogeneous with isotropic velocity distributions. $\underline{B} = B \underline{e}_z$. The pump field, $\underline{E}_0(\underline{x}, t)$, is itself a self-consistent magnetosonic wave; i.e.,

$$\underline{E}_0(\underline{x}, t) = (E_{0\perp} \underline{e}_y + E_{0\parallel} \underline{e}_z) \exp[i(\underline{k}_0 \cdot \underline{x} - \omega_0 t)] + c.c. \quad (1)$$

Here, $\underline{k}_0 = k_{0\perp} \underline{e}_x + k_{0\parallel} \underline{e}_z$. $\omega_0^2 = k^2 v_A^2 (1 + \bar{\beta})$, $\bar{\beta} = \beta - \beta_e/2$. The notations here are standard. Subscript o denotes quantities associated with the pump. We also assume $m_e/m_i \ll (k_{\parallel}/k_{\perp})_0^2 \ll 1$ and $(k_{\perp} \rho_{e,i})_0^2 \ll 1$. Hence, for β not too small we have $(k_{\parallel} v_i)_0^2 \ll \omega_0^2 \ll \Omega_i^2$, $(k_{\parallel} v_e)_0^2 \ll \Omega_i$ is the ion cyclotron frequency. v_e and v_i are, respectively, the electron and ion thermal velocities. Note also $|E_{0\parallel}/E_{0\perp}| \approx |k_{\perp} k_{\parallel} C_s^2 / \Omega_i \omega|_0 \ll 1$.

To consider the parametric interactions, we assume that $\underline{E}_0$ is weak so that only interactions up to $O(|\underline{E}_0|^2)$ need to be kept. That is, we consider interactions among the pump $(\pm \omega_0, \pm \underline{k}_0)$, the stoke $(\omega_-, \underline{k}_-) = (\omega - \omega_0, \underline{k} - \underline{k}_0)$, the anti-stoke $(\omega_+, \underline{k}_+) = (\omega + \omega_0, \underline{k} + \underline{k}_0)$ and the low-frequency ion-sound wave $(\omega, \underline{k})$. Here, $|\omega| \ll |\omega_0|$ and $(\omega_{\pm}, \underline{k}_{\pm})$ are magnetosonic waves.

The wave equations of the two sidebands $(\omega_{\pm}, \underline{k}_{\pm})$, in the coordinates $(x_{\pm}, y_{\pm}, z)$ defined by $(\underline{e}_x)_{\pm} = (\underline{k}_{\perp}/k_{\perp})_{\pm}$ and $(\underline{e}_y)_{\pm} = \underline{e}_z \times (\underline{e}_x)_{\pm}$, can be written as

$$\begin{bmatrix} -K_{xx} & 0 & -n_x n_{||} \\ 0 & n^2 - K_{yy} & -K_{yz} \\ -n_x n_{||} & K_{yz} & -K_{zz} \end{bmatrix}_{\pm} \cdot \tilde{E}_{\pm} = i \frac{4\pi}{\omega_{\pm}} J_{\pm}^{(2)} . \quad (2)$$

$n_{\pm} = (Ck/\omega)_{\pm}$ is the refractive index. The K's are the elements of the well-known dielectric tensor for a hot magnetized plasma due to Stix [4]. In the regime of interest here, they are approximately given by

$$K_{xx} \approx \omega_{pi}^2 / \Omega_i^2 ,$$

$$K_{yy} \approx \frac{\omega_{pi}^2}{\Omega_i^2} \left(1 - \frac{k_{\perp}^2 V_A^2}{\omega^2} \beta_{\perp i} \right) (1 + i\delta_1) ,$$

$$K_{zz} \approx (k_{||} \lambda_{De})^{-2} (1 + i\delta_2) ,$$

and $K_{yz} \approx -i(\omega_{pi}^2 k_{\perp} / \omega \Omega_i k_{||}) (1 + i\delta_3) . \quad (3)$

δ_1 , δ_2 , and δ_3 represent fractions of the electron collisional as well as collisionless dissipations. $J_{\pm}^{(2)}$ is the nonlinear contribution to the current of the sideband. The dominant

contribution comes from coupling between the ion polarization current induced by the pump and the density perturbation n_s of the (ω, k) mode; i.e.,

$$\frac{i4\pi}{\omega_{\pm}} J_{\pm}^{(2)} \approx \frac{n_s}{n_o} \frac{\omega_{pi}^2}{\Omega_i^2} (1 + \bar{\beta})^{-1} E_{o\perp}(\pm \omega_o) e_{\pm y} \quad (4)$$

Equation (4) into Eq. (2), we obtain

$$(DE_y)_{\pm} = - (1 + \bar{\beta})^{-1} (n_s/n_o) (E_{o\perp} \cos\theta)_{\pm} \quad (5)$$

where

$$D_{\pm} = 1 - (1 + \bar{\beta}) [kV_A/(\omega + i\Gamma)]_{\pm}^2 \quad (6)$$

and $\cos\theta_{\pm} = \mathbf{e}_{\pm x} \cdot (\mathbf{e}_{\pm x})_{\pm}$. Γ corresponds to the electron dissipation rate.

To calculate the response of the low-frequency ion-sound mode to the magnetosonic waves, we use the concept of ponderomotive potential [5]. Since $\omega \ll \Omega_j$ and $k_{\perp}^2 \rho_j^2 \ll 1$ ($j = e, i$), only dynamics parallel to B need to be retained. The ponderomotive potential ψ_j ($j = e, i$) corresponding to the dominant nonlinear parallel force $[(q/c)(\mathbf{v}_{\perp} \times \mathbf{b}_{\perp})_{\parallel}]_j$ then is

$$q_j \frac{\partial \psi_j}{\partial z} = - (q_j/c) (\underline{v}_{0\perp} \times \underline{b}_1^- + \underline{v}_1^- \times \underline{b}_{0\perp} + \underline{v}_{0\perp}^* \times \underline{b}_1^+ + \underline{v}_1^+ \times \underline{b}_{0\perp}^*)_j, \quad j = e, i. \quad (7)$$

We then substitute ψ_j along with the self-consistent potential ϕ into the j th-species one-dimensional (parallel to $\underline{B}$) Vlasov's equation

$$\frac{\partial f_j}{\partial t} + v_{||} \frac{\partial f_j}{\partial z} - \frac{q_j}{m_j} \frac{\partial}{\partial z} (\phi + \psi_j) \frac{\partial f_j}{\partial v_{||}} = 0. \quad (8)$$

Linearizing Eq. (8), we obtain the charge density perturbation ρ_j as

$$4\pi\rho_j(\omega, \underline{k}) = -k^2 \chi_j(\omega, \underline{k}) [\phi(\omega, \underline{k}) + \psi_j(\omega, \underline{k})], \quad (9)$$

where the linear susceptibility χ_j is given by

$$\chi_j = - (\omega_{pj}^2/k^2) \int dv_{||} \frac{d f_{0j}/dv_{||}}{v_{||} - \omega/k_{||}}. \quad (10)$$

Since $k^2 \lambda_{De}^2 \ll 1$, we can use the quasi-neutrality condition $\rho_e + \rho_i = 0$, which yields using Eq. (9)

$$\epsilon \phi = - \sum_j \chi_j \psi_j. \quad (11)$$

Here, $\epsilon = \chi_e + \chi_i$ is the low-frequency dielectric constant.

Substituting Eq. (11) back into Eq. (9), we can express n_s in terms of ψ_j as

$$\frac{n_s}{n_o} = - \frac{k^2}{4\pi n_o q_i} \frac{\chi_i \chi_e}{\epsilon} (\psi_i - \psi_e) . \quad (12)$$

$(\psi_i - \psi_e)$ can be calculated from Eq. (7). Let us note that

$$(\tilde{b}_\perp)_{o,+,-} \approx - (n_{||} E_y e)_{o,+,-} . \quad (13)$$

Carrying out the straightforward algebra, we find

$$\psi_i - \psi_e = - (1 + \bar{\beta})^{-1} \frac{C}{B\Omega_i} [E_{o\perp} (E_y \cos\theta)_- + E_{o\perp}^* (E_y \cos\theta)_+] . \quad (14)$$

The final dispersion relation can then be derived by combining Eqs. (5), (12), and (14). It is given as

$$\epsilon = - \chi_i \chi_e (1 + \bar{\beta})^{-2} \left| \frac{CkE_{o\perp}}{B\omega_{pi}} \right|^2 \left(\frac{\cos^2\theta_-}{D_-} + \frac{\cos^2\theta_+}{D_+} \right) . \quad (15)$$

Let us examine the above dispersion relation in more detail.

As mentioned earlier, for $T_e \lesssim 4T_i$ the ion-sound wave ($\omega \approx k_{||} C_s$)

is heavily damped by ion Landau damping; hence three-wave resonant decay is not allowed. However, there is the induced scattering (or quasimode) decay instability through the nonlinear ion Landau damping. For this decay process, we treat the lower sideband (ω_-, k_-) as a resonant mode but neglect the upper sideband (ω_+, k_+) as being off-resonant. Let $\omega = \omega_r + i\omega_i \sim k_{||} v_i$, $(\omega_1, k_1) = (\omega_0 - \omega, k_0 - k)$ and note $\chi_e \approx (k\lambda_{De})^{-2}$, Eq. (15) reduces to

$$\frac{\gamma_0}{\omega_1} \equiv \frac{\omega_i + \Gamma}{\omega_1} = \left| \frac{cE_{0\perp}}{BC_s} \right|^2 (1 + \bar{\beta})^{-2} \cos^2 \theta_1 \frac{T_i}{T_e} f(\xi), \quad (16)$$

where $\xi = (\omega_0 - \omega_1)/\sqrt{2}|k_0 - k_1|_{||} v_i$ and

$$f(\xi) = \frac{\sqrt{\pi} \xi \exp(-\xi^2)}{(1 + T_i/T_e + \xi Z_r)^2 + \pi \xi^2 \exp(-2\xi^2)}. \quad (17)$$

Z_r is the real part of the standard plasma dispersion function.

Maximum growth rate occurs at $\theta_1 = 0, \pi$ and $\xi \approx 0.7$ (i.e.,

$\omega_0 - \omega_1 \approx |k_0 - k_1|_{||} v_i$) where $f_M(\xi) \approx 0.6$ for $T_e \approx T_i$. The threshold condition ($\omega_i = 0$) is [4]

$$\left| \frac{cE_{0\perp}}{BC_s} \right|_{th}^2 \sim \frac{\Gamma}{\omega_1} \sim \beta^{3/2} \left(\frac{k_{\perp m_e}^2}{k_{|| m_i}^2} \right)^{1/2} \ll 1. \quad (18)$$

Equation (18) indicates that the threshold ion $E \times B$ drift speed due to the pump is much less than the ion-sound speed and,

therefore, this parametric instability can be readily excited in laboratory experiments.

III. NONLINEAR THEORY

In this section we consider the nonlinear evolution and eventual saturation of this decay process. We shall limit our treatment within the weak turbulence theory and adopt the cascading model [6]. In this model, the amplitudes of the magnetosonic waves excited by the pump grow in time and exceed the threshold values for destabilizing waves at lower frequencies; that is, the nonlinear evolution is characterized by further induced scattering of magnetosonic waves toward lower frequencies. Saturation is achieved when the net growth rate of each wave is balanced by the linear dissipation rate. Since the mechanism of nonlinear interactions among the magnetosonic waves is the same as that of the parametric excitation discussed in the preceding section, the corresponding wave kinetic equation describing the nonlinear evolution of the waves can be readily derived and is given by

$$\frac{1}{2} \frac{\partial N_k}{\partial t} = (\gamma_k^0 - \Gamma_k) N_k + \sum_{k'} M(k, k') N_{k'} N_k + S_k, \quad (17)$$

and

$$M(k, k') = \frac{4\pi C^2}{B^2 C_s^2} (1 + \bar{\beta})^{-2} \frac{T_i}{T_e} \omega_k \omega_{k'} \left| \frac{k_{\perp} \cdot k'_{\perp}}{k_{\perp} k'_{\perp}} \right|^2 f(\xi_{k, k'}) \quad (18)$$

Here, $N_{\tilde{k}} = |\tilde{E}_{\tilde{k}}|^2 / 4\pi\omega_{\tilde{k}}$ is the action, $\omega_{\tilde{k}}^2 = k^2 V_A^2 (1 + \bar{\beta})$, $\gamma_{\tilde{k}}^0$ is given in Eq. (16), $\Gamma_{\tilde{k}}$ is the linear dissipation rate mainly due to electrons and $\xi_{\tilde{k}, \tilde{k}'} = (\omega_{\tilde{k}'} - \omega_{\tilde{k}}) / \sqrt{2} |k_{\parallel}' - k_{\parallel}| v_i$. $S_{\tilde{k}}$ corresponds to the small spontaneous emission term. Note that $M(\tilde{k}, \tilde{k}') = -M(\tilde{k}', \tilde{k})$ is anti-symmetric and, hence $N_{\tilde{k}}$ is conserved in the scattering process. At saturation, we have with $S_{\tilde{k}} \rightarrow 0$

$$\sum_{\tilde{k}} \gamma_{\tilde{k}}^0 N_{\tilde{k}} = \sum_{\tilde{k}} \Gamma_{\tilde{k}} N_{\tilde{k}}; \quad (19)$$

that is, the total number of "plasmons" created by the pump equals to that destroyed by dissipation.

Wave kinetic equations of similar nature have been extensively studied both analytically and numerically for parametric instabilities of Langmuir waves [7] and lower-hybrid waves [8]. It, therefore, may be expected that the results would also be similar. We thus can estimate the effective collision frequency at saturation ν_f to be approximately equal to the linear parametric growth rate; i.e.,

$$\nu_f \approx \omega_0 |CE_{0\perp} / BC_S|^2. \quad (20)$$

Since the waves are mainly dissipated by electrons, the electron heating rate is estimated to be

$$\frac{dT_e}{dt} \approx \nu_f |E_{0\perp}|^2 / 4\pi \propto |E_{0\perp}|^4. \quad (21)$$

Let us also note that since during each scattering there is a net amount of energy ($\sim k_{\parallel} v_i N_k$) transfer to ions through nonlinear ion Landau damping, this process would also lead to ion heating. A detailed study of this wave kinetic equation will be reported in a future publication.

IV. CONCLUSIONS

In the previous sections we have shown that, in a plasma with $\beta < 1$ and $T_e \lesssim 4T_i$, parametric excitation of magnetosonic waves can occur through nonlinear ion Landau damping. The threshold condition and linear parametric growth rate are calculated. We then consider the nonlinear evolution and saturation of this decay instability using the weak turbulence theory and the cascading model. The nonlinear process is thus characterized by further induced scatterings of excited magnetosonic waves toward the lower frequencies. A corresponding wave kinetic equation is derived. By noting the similarities between this wave kinetic equation and those describing Langmuir and lower-hybrid turbulences, we then estimate the effective collision frequency and, consequently, the electron heating rate. We also note that this process also heats up ions through nonlinear ion Landau damping.

As in the cases of Langmuir and lower-hybrid turbulences, nonlinear saturation by the cascading process is justified if the "plasmons" are dissipated before they reach the lowest frequency

of the system; e.g., the electron plasma frequency and the lower-hybrid frequency. Otherwise, wave condensation and collapse may occur and the weak turbulence theory is no longer valid [9]. In the case studied here, the lowest frequency allowable is decided by the system dimension L ; i.e., $\omega_\ell \approx (2\pi V_A/L)(1 + \bar{\beta})^{1/2}$. Since $\omega_o/\omega_\ell \approx (k_o L/2\pi)$ can easily be of magnitude greater than unity, saturation by cascading appears to be justified for not-too-strong pumps.

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