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Temperature Screening Effect in Two-Ion Species Pfirsch-Schlüter Transport

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TEMPERATURE SCREENING EFFECT IN TWO-ION SPECIES
PFIRSCH-SCHLÜTER TRANSPORT

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ABSTRACT

We have solved the particle and heat flux problems in the collisional or Pfirsch-Schlüter regime for a two-ion species tokamak plasma using a kinetic approach with exact Fokker-Planck collision operators. These solutions are appropriate for analyzing the behavior of impurities in the low temperature plasma edge region where impurities first enter the plasma from the outside. We find that low charge states of low mass impurities can be screened effectively from further penetration of the plasma.

INTRODUCTION

It is well known that impurity species have undesirable effects in tokamak plasmas. Low mass impurities such as oxygen appear to be common to most tokamak experiments and on account of radiation from their low-charge states may play an important role in the plasma electron power balance. Such impurities enter the plasma from outside, and the plasma they first encounter is collisional because of its low temperature. Impurity transport in the collisional or Pfirsch-Schlüter regime in the low plasma-ion-to-impurity-ion mass ratio limit has had both a fluid treatment¹ and a kinetic theory treatment.² This limit is inappropriate for low mass impurities such as oxygen in a hydrogen or a deuterium

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plasma. A kinetic treatment using a model operator has been performed also.³ All of these treatments neglected the energy correlation of the cross-collision terms, which, as we see below, is responsible for a new temperature screening effect. By temperature screening⁴ we mean the reversal of the usual⁵ sense of the impurity particle flux in multiple ion species plasmas, i.e., impurities flow outward rather than inward.

ANALYSIS

The starting point of our analysis is the linearized drift kinetic equation much used in neoclassical transport theory⁶:

$$(v_{||}/Rq)\partial f_j/\partial\theta + v_{Drj}\partial F_j/\partial r = C_j(f_j) \quad (1)$$

in which the independent velocity variables are the total energy per unit mass $E_j \equiv v^2/2 + Z_j e \Phi(r)/M_j$ and the magnetic moment $\mu \equiv v_{\perp}^2/2B$, with $\Phi(r)$ the radial electric potential and $v_{\perp}$ the velocity component perpendicular to the magnetic field. The parallel velocity is given by $v_{||} = [2(E_j - \mu B - Z_j e \Phi/M_j)]^{1/2}$. We use the standard toroidal coordinate system (r, θ, ζ) , and represent the tokamak magnetic field as $\vec{B} = [\delta \hat{\theta}/q + \hat{\zeta}] B_0/h$, where $h = 1 + \delta \cos \theta$ and $\delta = r/R$ with q the safety factor and R the major toroidal radius. The radial drift velocity may be expressed as

$$v_{Drj} = (v_{||}/r)\partial/\partial\theta(v_{||}/\Omega_j)$$

where $\Omega_j = Z_j e B/M_j c$ is the particle gyrofrequency. The equilibrium distribution function F_j is assumed to be the local Maxwellian

$$F_j = N_j(r) \{M_j/[2\pi T(r)]\}^{3/2} \exp[-(M_j E_j - Z_j e\Phi)/T(r)],$$

and $f_j \equiv \psi_j F_j$ is the first order correction to F_j in the gyroradius expansion used to obtain Eq. (1). For the impurity ion mass comparable to the plasma ion mass, the temperature equilibration time is essentially a few collision times. This justifies our assumption that both species have the same temperature. The exact collision operator on the right-hand side of Eq. (1) may, therefore, be written in the linearized Landau form

$$C_j(f_j) = \sum_k C_{jk}(f_j, f_k), \quad (2)$$

where

$$C_{jk}(f_j, f_k) = 2\pi \ln \Lambda e^4 (Z_j Z_k)^2 M_j^{-1} \partial/\partial v_\beta \int d\vec{v}' F_j(\vec{v}) F_k(\vec{v}') \\ (M_j^{-1} \partial \psi_j / \partial v_\gamma - M_k^{-1} \partial \psi_k / \partial v'_\gamma) U_{\beta\gamma}$$

$$U_{\beta\gamma} \equiv u^{-3} (u^2 \delta_{\beta\gamma} - u_\beta u_\gamma) \text{ and } u_\beta = v_\beta - v'_\beta.$$

In the Pfirsch-Schlüter regime, Eq. (1) can readily be expanded again,⁷ in the inverse of the collision frequency, to yield in successive orders

$$C_j(f_j^{(-1)}) = 0 \quad (3)$$

$$C_j(f_j^{(0)}) = (v_{||}/Rq) \partial f_j^{(-1)} / \partial \theta \quad (4)$$

and

$$C_j(f_j^{(1)}) = (v_{||}/Rq)\partial f_j^{(0)}/\partial\theta + v_{Drj}\partial F_j/\partial r. \quad (5)$$

Equations (2) and (3) give a lowest order solution of the form

$$f_j^{(-1)} = \alpha_j M_j F_j + \beta M_j v_j^2 F_j / 2 \quad (6)$$

where α_j and β are determined later, and represent the density and temperature corrections. It is not difficult to verify that there is no velocity shift to this order. From momentum conservation of the collision operators C_j , and Eq. (4), we find the first relation between the α_j and β ,

$$\sum_j N_j \epsilon_j = 0,$$

where

$$\epsilon_j = (Rq)^{-1} \partial/\partial\theta (M_j \alpha_j + \frac{5}{2} T \beta).$$

The other two properties of C_j , viz. conservation of particles and energy, are obviously satisfied by Eq. (6). Upon substitution of Eq. (6) in Eq. (4), we obtain the equation for $f_j^{(0)}$,

$$C_j(f_j^{(0)}) = v_{||} \epsilon_j F_j + (v_{||}/Rq) \partial \beta / \partial \theta (\frac{1}{2} M_j v_j^2 - \frac{5}{2} T) F_j. \quad (7)$$

Equation (7) is closely related to the equation⁸ from which the classical simple plasma transport coefficients are obtained. In the analysis leading to that equation an expansion in the square root of the mass ratio was exploited in the cross-collision operator to uncouple the equations for f_j and f_k . Here we consider that the impurity ion mass is not necessarily much greater than the plasma ion mass and so do not use such an expansion.

Thus, f_j and f_k remain coupled through the cross-collision terms. We make use of the linear character of the problem to split Eq. (7) into two equations by defining

$$f_j \equiv h_j^{(1)} + h_j^{(2)},$$

so that

$$C_j(h_j^{(1)}) = v_{||} \epsilon_j F_j, \quad (8)$$

$$C_j(h_j^{(2)}) = (v_{||}/Rq) \partial \beta / \partial \theta (\frac{1}{2} M_j v^2 - \frac{5}{2} T) F_j. \quad (9)$$

To solve Eqs. (8) and (9), $h_j^{(1)}$ and $h_j^{(2)}$ are expanded in a series of generalized Laguerre (or Sonine) polynomials following the technique described by Chapman and Cowling,⁹

$$h_j^{(1)} = \tau_i \epsilon_j v_{||} \sum_{k=0}^{\infty} y_j^{(k)} L^{(3/2)}(x_j) F_j$$

$$h_j^{(2)} = \tau_i \frac{5}{2} (T/Rq) \partial \beta / \partial \theta v_{||} \sum_{k=1}^{\infty} z_j^{(k)} L^{(3/2)}(x_j) F_j$$

where $y_j^{(k)}$ and $z_j^{(k)}$ are coefficients to be determined, $\tau_i = 3M_i^{1/2} T^{3/2}/4\sqrt{\pi} (eZ_i)^4 N_i \ln \Lambda$ is the ion self-collision frequency, and $x_j \equiv M_j v^2/2T$.

We now focus on the problem with only one impurity species.

Utilizing the orthogonality properties of the polynomials we obtain from Eqs. (8) and (9) two sets of coupled equations for the coefficients

$$\sum_{\ell=0}^{\infty} \left[y_i^{(\ell)} (b_{k\ell} + \alpha c_{ik\ell}) - (M_i/M_1) (Z_1/Z_i)^2 y_1^{(\ell)} d_{ik\ell} \right] = -\delta_{k0}/\sqrt{2} \quad (10a)$$

$$- \sum_{\ell=0}^{\infty} \left\{ y_i^{(\ell)} d_{1k\ell} + y_1^{(\ell)} (Z_1/Z_i)^2 \left[(M_i/M_1)^{1/2} b_{k\ell} + (M_i/M_1) \alpha c_{1k\ell} \right] \right\} = -\delta_{k0}/\sqrt{2}\alpha \quad (10b)$$

where $k = 0, \dots, \infty$, and

$$\sum_{\ell=1}^{\infty} [z_i^{(\ell)} (b_{k\ell} + \alpha C_{ik\ell}) + \alpha (M_i/M_1) z_i^{(\ell)} d_{ik\ell}] = \delta_{k1}/\sqrt{2} \quad (11a)$$

$$\sum_{\ell=1}^{\infty} [z_i^{(\ell)} d_{1k\ell} + z_1^{\ell} (b_{k\ell} \alpha (M_i/M_1)^{1/2} + C_{1k\ell} M_i/M_1)] = (Z_i/Z_1)^2 \delta_{k1}/\sqrt{2} \quad (11b)$$

where $k = 1, \dots, \infty$, the subscripts i and 1 indicate the plasma ion and the impurity ion, respectively, $\alpha \equiv N_1 Z_1^2 / (N_i Z_i^2)$, and $\delta_{k\ell}$ is the Kronecker delta. The infinite matrices of coefficients $b_{k\ell}$, $C_{jk\ell}$ and $d_{jk\ell}$ are obtained from integrals evaluated by Braginskii⁸ and are given by

$$\sum_{k,\ell=0}^{\infty} b_{k\ell} \xi^k \eta^{\ell} = \sqrt{2} [1 - \frac{1}{2}(\xi + \eta)]^{-5/2} (1 - \xi\eta)^{-2} \xi\eta$$

$$[1 - \frac{1}{2}(\xi + \eta) - \frac{1}{8}\xi\eta + \frac{1}{4}\xi\eta(\xi + \eta) + \frac{3}{8}\xi^2\eta^2]$$

$$\sum_{k,\ell=0}^{\infty} C_{ik\ell} \xi^k \eta^{\ell} = (1 - \xi\eta)^{-2} [\lambda(1 - \xi\eta) + (1 - \xi)(1 - \eta)]^{-5/2}$$

$$\{(\lambda + 1)[\lambda(1 - \xi\eta) + (1 - \xi)(1 - \eta)](1 - \xi\eta)$$

$$+ 5\xi\eta(1 - \xi\eta)\lambda^2 + 2\lambda\xi\eta(1 - \xi)(1 - \eta)\}$$

$$\sum_{k,\ell=0}^{\infty} C_{1k\ell} \xi^k \eta^{\ell} = (1 - \xi\eta)^{-2} [1 - \xi\eta + \lambda(1 - \xi)(1 - \eta)]^{-5/2}$$

$$\cdot \{(\lambda + 1)[1 - \xi\eta + \lambda(1 - \xi)(1 - \eta)](1 - \xi\eta)$$

$$+ 5\xi\eta(1 - \xi\eta) + 2\lambda\xi\eta(1 - \xi)(1 - \eta)\}$$

$$\sum_{k, \ell=0}^{\infty} d_{ik\ell} \xi^k \eta^\ell = - [(1-\eta) \lambda + 1 - \xi]^{-5/2} \{ (1+\lambda)[\lambda(1-\eta) + 1 - \xi] + 3 \lambda \xi \eta \}$$

$$\sum_{k, \ell=0}^{\infty} d_{1k\ell} \xi^k \eta^\ell = - [1 - \eta + \lambda (1 - \xi)]^{-5/2} \{ (1+\lambda)[1 - \eta + \lambda (1 - \xi)] + 3 \lambda \xi \eta \}$$

where $\lambda \equiv M_i/M_1$. Eqs. (10) and (11) are to be solved numerically.

Before solving Eq. (10), we notice that the two $k = 0$ equations are identical. This redundancy reflects the physical fact that only the relative velocity of the two species can be determined, not the individual velocity of each species separately. We therefore choose a convenient frame of reference such that

$$y_1^{(0)} = 0$$

and Eqs. (10) reduce to

$$\sum_{\ell=0}^{\infty} y_i^{(\ell)} (b_{k\ell} + \alpha C_{ik\ell}) - (M_i/M_1)(Z_1/Z_i)^2 \sum_{\ell=1}^{\infty} y_1^{(\ell)} d_{ik\ell} = -\delta_{k0}/\sqrt{2} \quad (12a)$$

$k = 0, 1, \dots, \infty$, and

$$-\sum_{\ell=0}^{\infty} y_i^{(\ell)} d_{1k\ell} + (Z_1/Z_i)^2 \sum_{\ell=1}^{\infty} y_1^{(\ell)} [(M_i/M_1)^{1/2} b_{k\ell} + (M_i/M_1) \alpha C_{1k\ell}] = 0 \quad (12b)$$

$k = 1, \dots, \infty$.

It is not difficult to show that the final result is independent of the frame of reference chosen. Utilizing the next order equation, Eq. (5), we obtain the following solubility conditions which act as constraints to the solution for $f_j^{(0)}$,

$$\begin{aligned}
(Rq)^{-1} \partial / \partial \theta [B^{-1} \int d\vec{v} v_{\parallel} f_j^{(0)}] + B^{-1} \int d\vec{v} v_{Drj} \partial F_j / \partial r = 0 \\
(Rq)^{-1} \partial / \partial \theta \{B^{-1} \int d\vec{v} v_{\parallel} \frac{1}{2} v^2 [M_i f_i^{(0)} + M_1 f_1^{(0)}]\} \\
+ B^{-1} \int d\vec{v} \frac{1}{2} v^2 (M_i v_{Dri} \partial F_i / \partial r + M_1 v_{Dr1} \partial F_1 / \partial r) = 0
\end{aligned}$$

from which we get

$$\tau_i \epsilon_i = - (2q \cos \theta) [\partial \ln P_i / \partial r - (Z_i / Z_1) \partial \ln P_1 / \partial r] / \Omega_i y_i^{(0)}$$

and

$$\begin{aligned}
& \frac{5}{2} (Rq)^{-1} T \partial \beta / \partial \theta \tau_i [z_i^{(1)} + z_1^{(1)} N_1 M_i / (N_i M_1)] \\
& = (2q \cos \theta) \Omega_i^{-1} \{ [y_i^{(0)}]^{-1} [y_i^{(1)} - (M_i / M_1) y_1^{(1)}] \\
& \quad [\partial \ln P_i / \partial r - (Z_i / Z_1) \partial \ln P_1 / \partial r] + [1 + N_1 Z_i / (N_i Z_1)] \partial \ln T / \partial r \}
\end{aligned}$$

where $P_j \equiv N_j T$.

We have now the solution of Eq. (1) needed to evaluate the particle flux Γ_j and the heat flux Q_j given by

$$\Gamma_j = - Rq / (r \Omega_j) (2\pi)^{-1} \int_0^{2\pi} d\theta h \int d\vec{v} v_{\parallel} C_j(f_j)$$

and

$$Q_j = - Rq / (r \Omega_j) (2)^{-1} \int_0^{2\pi} d\theta h \int d\vec{v} \frac{1}{2} M_j v^2 C_j(f_j)$$

The resulting expressions are

$$\begin{aligned}
\Gamma_i = - q^2 \rho_i^{-2} \tau_i^{-1} N_i \{ C_1 [\partial \ln P_i / \partial r - (Z_i / Z_1) \partial \ln P_1 / \partial r] + \\
+ C_2 [1 + \alpha (Z_i / Z_1)^3] \partial \ln T / \partial r \}
\end{aligned} \tag{13}$$

$$\tilde{Q}_i = -q_i^2 \rho_i^2 \tau_{i1}^{-1} N_i T \{ C_2 [\partial \ln P_i / \partial r - (Z_i/Z_1) \partial \ln P_1 / \partial r] + C_3 [1 + \alpha (Z_i/Z_1)^3] \partial \ln T / \partial r \} \quad (14)$$

where $\tau_{i1} \equiv \tau_i / \sqrt{2\alpha}$

$$C_1 = -(\sqrt{2\alpha})^{-1} + \zeta_0 \zeta_1 [y_i^{(1)} - (M_i/M_1) y_1^{(1)}] / y_i^{(0)}$$

$$C_2 = -\zeta_0 \zeta_1$$

$$C_3 = \zeta_1 / \sqrt{2\alpha}$$

$$\zeta_0 = \sum_{k=1}^{\infty} [C_{iok} z_i^{(k)} + (M_i/M_1) d_{iok} z_1^{(k)}]$$

$$\zeta_1 = [z_i^{(1)} + \alpha (M_i/M_1) (Z_i/Z_1)^2 z_1^{(1)}]^{-1}$$

$$\rho_i^2 = 2T/M_i \Omega_{io}^2$$

and $\tilde{Q}_i = Q_i - \frac{5}{2} T \Gamma_i$. We also have from conservation of momentum that

$$\Gamma_1 = -Z_i \Gamma_i / Z_1 \quad (15)$$

and we find

$$\tilde{Q}_1 = \alpha (Z_i/Z_1)^3 \tilde{Q}_i. \quad (16)$$

It may be a bit surprising that apparently the Onsager relation does not have its usual form in Eqs. (13) and (14). However, if we evaluate the rate of entropy production $\dot{S}$ in this two-species system, defined by

$$\dot{S} = - (2\pi)^{-1} \int_0^{2\pi} d\theta \int d\vec{v} [\psi_i C_i(f_i) + \psi_1 C_1(f_1)]$$

then we soon find

$$\dot{S} = -\Gamma_i [\partial \ln P_i / \partial r - (Z_i/Z_1) \partial \ln P_1 / \partial r] - \tilde{Q}_i [1 + \alpha (Z_i/Z_1)^3] \partial \ln T / \partial r .$$

This implies that the driving forces for transport are $\partial \ln P_i / \partial r - (Z_i/Z_1) \partial \ln P_1 / \partial r$ and $[1 + \alpha (Z_i/Z_1)^3] \partial \ln T / \partial r$. Hence the Onsager relation, which is guaranteed by the self-adjointness of the collision operator, has exactly the form that appears in Eqs. (13, 14).

NUMERICAL RESULTS AND DISCUSSION

We have solved Eqs. (11) and (12) by matrix inversion numerically on a computer, increasing the maximum k and ℓ until the desired accuracy was obtained. In general, for maximum k , $\ell = 4$ the coefficients are obtained to within 2 percent for any mass or charge ratio. Selected cases were calculated with maximum k , $\ell = 40$ to look for divergences in the technique, but none were found. We illustrate the results for a hydrogen plasma with an oxygen impurity ($M_i/M_1 = 0.063$) in Figs. 1 through 3 where the coefficients of the pressure and temperature gradient factors of Eqs. (13) and (14) are plotted as functions of α for all the possible oxygen charge states. The range of α used spans a scale of oxygen concentration from highly dilute to highly concentrated. In Fig. 1 the curves for C_1 are given. For very large α , the curves for $Z \equiv Z_1/Z_i \geq 2$ are asymptotic to the curve for $Z = 1$. The results are qualitatively the same as Rutherford's,¹ which are plotted for comparison, but there are differences as large as 40%. This is not altogether surprising because of the mass ratio approximation he made which apparently also eliminates the charge ratio dependence. Samain's² coefficient corresponding to C_1 is unity,

independent of charge state, and is only applicable for $\alpha \gg 1$ where it is quantitatively in disagreement with our results and Rutherford's. No numerical evaluations were given in Ref. 3. The complete factor $[1 + \alpha (Z_i/Z_1)^3] C_2$ is plotted in Fig. 2 rather than C_2 to facilitate comparison with Rutherford's¹ and Samain's² results. For large α the curves for different $Z \geq 2$ are asymptotic to Z times the value for $Z = 1$ which becomes essentially constant for $\alpha \gtrsim 500$. This behavior is both qualitatively and quantitatively different from Rutherford's results which are again plotted for comparison. The most significant conclusion we draw from Figs. 1 and 2 is that for each charge state there is a different value of α such that for larger α the coefficient of the temperature gradient factor in Eq. (13) exceeds C_1 ; hence, since these coefficients have opposite signs whereas the gradients would normally have the same sign, the possibility of temperature screening occurs for large $\alpha \gtrsim 1$. This phenomenon is typically excluded by Rutherford's results. The equivalent coefficient of Samain has the value $-(3/2 - 1/Z)$ which has a maximum magnitude of $-3/2$, much smaller than our results but not entirely excluding the possibility of temperature screening.

Before discussing an example illustrating temperature screening, we consider Fig. 3 wherein is plotted the factor $[1 + \alpha(Z_i/Z_1)^3]C_3$, and, for comparison, Rutherford's equivalent factor. For large α , these curves also are asymptotic to Z times the value for $Z = 1$ which becomes essentially constant for $\alpha \gtrsim 500$. This behavior is again both qualitatively and quantitatively different from Rutherford's results; however, it leads to no new effect because the temperature gradient coefficient

in the non-convective heat flux, Eq. (14), is generally larger than the pressure gradient coefficient, cf. Figs. 2 and 3 and note that the values from Fig. 2 must be divided by $1 + \alpha_1(Z_i/Z_1)^3$ to obtain the pressure gradient coefficient C_2 . Samain does not calculate the heat flux.

Using the values from Figs. 1 through 3 in Eqs. (13) through (16) one can completely describe the particle and heat fluxes for a Pfirsch-Schlüter regime plasma containing hydrogen and any given oxygen ion impurity. Results for other impurity elements have been calculated and will be given in a later paper.

To illustrate temperature screening, we use the hydrogen-oxygen plasma results given in Figs. 1 and 2. We calculate for a plasma model in which the ion and impurity densities and temperatures have the same radial scale lengths. Equation (13) can then be written

$$\Gamma_i = -q^2 \rho_i^2 \tau_{i1}^{-1} N_i \partial \ln N_i / \partial r F_S \quad (17)$$

where

$$F_S = 2C_1(1 - Z_i/Z_1) + [1 + \alpha_1(Z_i/Z_1)^3]C_2.$$

In Fig. 4, F_S is plotted as a function of Z_1 times the fractional density of the impurity relative to the electron density for several oxygen charge states. (Note that the proton density relative to the electron density is the difference between unity and the abscissa.) It is seen from Eq. (17) that for the usual case in which density and temperature decrease with radius, i.e., $\partial \ln N_i / \partial r < 0$, when $F_S > 0$ the ion flux is positive and hence outward and from Eq. (15) the impurity flux is negative and hence inward. When $F_S < 0$, the senses of the

fluxes are reversed, i.e., temperature screening occurs. For this particular model of density and temperature dependence on plasma radius there is an accidental cancellation in the pressure gradient factor for $Z = 1$ so that temperature screening occurs for all α , even in Rutherford's and Samain's formulations. However, for $Z \geq 2$, our results continue to show temperature screening effects, whereas in Fig. 4 it is illustrated that Rutherford's do not. From the discussion above of Samain's coefficients, it is apparent that for $Z > 2$ his results do not predict temperature screening; however, for $Z = 2$, they do predict that the fluxes would vanish.

This temperature screening effect does not require unrealistically large temperature gradients for its operation, although it is seen from Fig. 4 that it does require relatively high concentrations of impurities with respect to the hydrogen concentration, especially for the large values of Z . Conditions for the occurrence of this effect may be present at the edges of many current tokamak plasmas where impurities may build up large concentrations. Dependent on the concentrations, there could be a recycling of the lower charge states between the plasma edge and the containing wall, but higher charge states might not be screened out so that impurities would still be able to pass into the plasma interior but at a reduced rate. For example, to explain recent results from the TFR tokamak, impurity recycling has been postulated.¹⁰

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FIGURE CAPTIONS

- Figure 1. Coefficient of the pressure gradient term in the expression for the ion particle flux for an oxygen impurity in a hydrogen plasma as a function of the impurity strength parameter $\alpha = N_1 Z_1^2 / N_i Z_i^2$, $M_i / M_1 = 6.3 \times 10^{-2}$, $1 \leq Z_1 / Z_i \leq 8$. The dashed line indicates Rutherford's results.
- Figure 2. Coefficient of the temperature gradient term in the expression for the ion particle flux for an oxygen impurity in a hydrogen plasma as a function of the impurity strength parameter $\alpha \equiv N_1 Z_1^2 / N_i Z_i^2$, $M_i / M_1 = 6.3 \times 10^{-2}$, $1 \leq Z_1 / Z_i \leq 8$. The dashed line indicates Rutherford's results.
- Figure 3. Coefficient of the temperature gradient term in the expression for the ion heat flux for an oxygen impurity in a hydrogen plasma as a function of the impurity strength parameter $\alpha = N_1 Z_1^2 / N_i Z_i^2$, $M_i / M_1 = 6.3 \times 10^{-2}$, $1 \leq Z_1 / Z_i \leq 8$. The dashed line indicates Rutherford's results.
- Figure 4. Illustration of "temperature screening" of impurities for plasma profiles in which particle densities and temperatures have the same functional dependence on plasma radius. As F_S changes from positive to negative, the direction of the plasma ion flux changes from out to in and the direction of the impurity flux from in to out. Hydrogen plasma with an oxygen impurity, $\frac{N_i}{N_e} = 1 - \frac{Z N_1}{N_e}$. The dashed curves were calculated using Rutherford's coefficients.

Figure 1

Figure 2

Figure 3

Figure 4

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