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CONRAD
HEAT CONDUCTION-RADIATION CODE
PART I

J.A. FILLO, R. BENENATI, AND J.R. POWELL

February 1976

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ABSTRACT

A computer code is developed for two-dimensional, non-steady heat conduction in heterogeneous, anisotropic solids with non-uniform volumetric internal heating. Thermal radiation and arbitrary heat flux boundary conditions are accommodated. Coolant tubes are taken into account by prescribing fluid temperatures and heat transfer coefficients. With regard to coolant tubes, we allow for: a) heat capacitance in the tube wall, coolant tube in contact with the structural material; b) zero heat capacitance in the tube wall, coolant tube in contact with the structural material; and c) zero heat capacitance in the tube wall, radiation gap between the wall and structure. The finite difference equations are solved by the standard explicit method. Stability as well as convergence of the iterative process are considered.

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NOMENCLATURE

A	cross sectional area
c	specific heat
D_1	height of cavity (Figs. 6,7)
D_2	width of cavity (Figs. 6,7)
e_b	emissive power (energy per length per unit time)
f	view factor
F	view Factor
h	heat transfer coefficient
i_b	blackbody intensity
k	thermal conductivity
K	inverse of thermal resistance
$\bar{x}$	half-length of node element
L_1	height of system (Figs. 6,7,8)
L_2	width of system (Figs. 6,7,8)
P_0	Bremsstrahlung loading
q	rate of heat flow
Q	volumetric heating rate
t	time
T	temperature
ΔV	elemental volume
x,y	cartesian co-ordinates defined in Figs. (6,7,8)

NOMENCLATURE (con't)

ϵ	emissivity
ϵ^*	convergence criteria
ρ	density
σ	Stefan-Boltzmann constant

1. INTRODUCTION

In order to evaluate the design potential of a new minimum activity blanket concept for fusion power reactors, it became evident that as a consequence of the designs (geometries) involved it was necessary to develop a heat transfer code to accommodate the new features. Rather than restrict the code to one or two particular designs, the analysis and code was developed for a wide range of geometries and boundary conditions. In addition the code may be applied in the analysis of other heat transfer problems where thermal radiation in enclosures is of importance and temperature distributions in solid structures is needed.

In essence the computer code developed is for two-dimensional, non-steady heat conduction in heterogeneous, anisotropic solids with non-uniform volumetric internal heating. Thermal radiation and arbitrary heat flux boundary conditions are accommodated. Coolant tubes are taken into account by prescribing fluid temperatures and heat transfer coefficients. With regard to coolant tubes, we allow for: a) heat capacitance in the tube wall, coolant tube in contact with the structural material; b) zero heat capacitance in the tube wall, coolant tube in contact with the structural material; and c) zero heat capacitance in the tube wall, radiation gap between the wall

and structure. The finite difference equations are solved by the standard explicit method.

In Part I we develop: a) the finite difference equations for internal nodes; b) detailed development of thermal radiant flux expressions with view-factor determination; c) thermal radiation boundary conditions coupled with conduction equation; and d) solution method. Part II is concerned with a detailed discussion of the computer program.

2. FINITE DIFFERENCE EQUATIONS

2.1 Difference Relationships for Interior Nodes

The energy equation for the interior nodes is derived by performing an energy balance on a small solid element of volume fixed in space. For a cartesian coordinate system, the shape of the element is a rectangular parallelepiped (Fig. 1). Since we assume an element of unit thickness, the heat conduction problem is two-dimensional with volumetric heat generation.

Applying the first law of thermodynamics, that is conservation of energy to the energy balance, we find that

$$\rho c \Delta V \frac{dT}{dt} = q_x|_{x-\Delta x/2} + q_x|_{x+\Delta x/2} + q_y|_{y-\Delta y/2} + q_y|_{y+\Delta y/2} + Q \Delta V \quad (2.1-1)$$

where c is the specific heat, ρ the density, t the time, T the temperature, q the rate of heat flow, Q the volumetric heat generation rate and ΔV the volume of the element $= \Delta x(\Delta y)(1)$.

The left-hand side of Eq. (2.1-1) represents the change in the internal energy of the element during the time increment Δt . For the case of solid elements the q -terms represent the net heat conducted into the element during Δt while the volumetric heat generation rate is $Q \Delta V$. We assume that the elements are in good thermal contact and that adjacent elements may be different materials. Both temperature and heat flux

must be continuous at an interface assumed to be in good thermal contact.

To evaluate the heat fluxes we consider a plane wall composed of layers of materials having different thicknesses and thermal conductivities. Fig. 2 illustrates a wall of two such layers. The rate of heat flow across the interface is found from Fourier's law, i.e.,

$$q_{int} = -kA \frac{dT}{dx} \quad (2.1-2)$$

which must be continuous. Applying Eq. (2.1-2) at the interface (2) we find

$$q_{int}/A = \frac{T_1 - T_2}{\Delta x_{12}/k_{12}} = \frac{T_2 - T_3}{\Delta x_{23}/k_{23}} \quad (2.1-3)$$

Solving for each of the temperature differences, one obtains

$$T_1 - T_2 = \frac{q_{int} \Delta x_{12}}{Ak_{12}} \quad (2.1-4a)$$

$$T_2 - T_3 = \frac{q_{int} \Delta x_{23}}{Ak_{23}} \quad (2.1-4b)$$

Adding Eqs. (2.1-4a) and (2.1-4b) the interface temperature, T_2 , is eliminated. The heat flow across the interface is obtained in terms of the centered temperature difference:

$$q_{int} = \frac{T_1 - T_3}{\left(\frac{\Delta x_{12}}{k_{12}A} + \frac{\Delta x_{23}}{k_{23}A} \right)} \quad (2.1-5)$$

As an alternate boundary condition at the interface we could impose the criterion of continuous flux and an operable differential equation but this is not considered in this report.

With respect to Eq. (2.1-1) and Figs. (1) and (3), the q 's may be expressed as

$$q_x)_{x+\Delta x/2} = \frac{T_3 - T_1}{\ell_{13}/k_{13} + \ell_{31}/k_{31} A_{31}} \quad (2.1-6a)$$

$$q_x)_{x-\Delta x/2} = \frac{T_5 - T_1}{\ell_{15}/k_{15} A_{15} + \ell_{51}/k_{51} A_{51}} \quad (2.1-6b)$$

$$q_y)_{y+\Delta y/2} = \frac{T_2 - T_1}{\ell_{12}/k_{12} A_{12} + \ell_{21}/k_{21} A_{21}} \quad (2.1-6c)$$

$$q_y)_{y-\Delta y/2} = \frac{T_4 - T_1}{\ell_{14}/k_{14} A_{14} + \ell_{41}/k_{41} A_{41}} \quad (2.1-6d)$$

Eq. (2.1-1) may now be written in a concise form as

$$\rho_j c_j \Delta V_j \frac{dT_j}{dt} = \sum_{i=1}^4 K_{ij} (T_i - T_j) + Q_j^* \Delta V_j \quad (2.1-7)$$

where

$$K_{ij} = \frac{1}{\ell_{ij}/k_{ij} A_{ij} + \ell_{ji}/k_{ji} A_{ji}}, \quad A_{ij} = A_{ji} \quad (2.1-8)$$

In the present notation i represents the nodes surrounding the node of interest j . Heterogeneous, anisotropic solids are admitted in the formulation; i.e., at each node thermal properties

may be temperature dependent and, in addition, the thermal conductivity may be orientation dependent.

In the computer code the thermal conductivity, density, and specific heat are allowed to vary linearly with temperature. The specific forms are given by

$$k_{ij} = A_{ij} + B_{ij} T_i \quad (2.1-9a)$$

$$k_{ji} = A_{ji} + B_{ji} T_j \quad (2.1-9b)$$

as well as

$$\rho_j = C_j + D_j T_j \quad (2.1-9c)$$

and

$$C_j = E_j + F_j T_j . \quad (2.1-9d)$$

From Eq. (2.1-7) it is seen that the heat capacity is lumped at the nodal points. Heat flows only along the resistors where the distributed resistance has been lumped. The temperature at any node represents the temperature at that point in the continuous region. Considering this temperature to be the average temperature of the element is acceptable in the interior of the region but can lead to errors when one is considering boundary elements.

2.2 Coolant and/or Heating Tubes

2.2a Heat Capacitance in Tube Wall

With a slight modification of the definition of K_{ij} , we can include the effects of coolant or heating tubes as depicted

in Fig. 4a. In other words the flow must be normal to the structure and tubes are assumed to be rectangular. If h_{31} is the heat transfer coefficient and T_3 the mean fluid temperature, then Eq. (2.1-1) becomes

$$\rho_1 c_1 \Delta V_1 \frac{dT_1}{dt} = K_{21} (T_2^n - T_1^n) + \frac{1}{\ell_{13}/k_{13}A_{13} + 1/h_{31}A_{31}} (T_3^n - T_1^n) + K_{41} (T_4^n - T_1^n) + K_{51} (T_5^n - T_1^n) + Q_1^n \Delta V_1 \quad (2.2-1)$$

where T_3 is the fixed fluid temperature, the mean or bulk temperature. Noting that

$$1/h_{31}A_{31} = \ell_{31}/\ell_{31}h_{31}A_{31}$$

and defining

$$k_{31}^* = \ell_{31}h_{31}, \quad 1/h_{31}A_{31} = \ell_{31}/k_{31}^*A_{31}$$

Eq. (2.2-1) becomes

$$\rho_1 c_1 \Delta V_1 \frac{dT_1}{dt} = K_{21} (T_2^n - T_1^n) + K_{31} (T_3^n - T_1^n) + K_{41} (T_4^n - T_1^n) + K_{51} (T_5^n - T_1^n) + Q_1^n \Delta V_1 \quad (2.2-2)$$

where K_{31} is defined in terms of k_{31}^* . In other words by properly defining k_{ij} the effect of coolant or heating tubes may be taken into account and we may determine the tube wall temperature as a function of time.

2.2b Neglect of Heat Capacitance in Tube Wall

With respect to Fig. 4b, the temperature at node 1 refers to the temperature in the structure other than the tube.

Assuming one-dimensional heat flow at the interface

$$q_{\text{int}}/A = \frac{T_1 - T_2^*}{\Delta x_{13}/k_{13}} = \frac{T_2^* - T_3^*}{\Delta/k_w} \quad (2.2-3)$$

Since we neglect the heat capacitance in the tube wall, q_{int} is also equal to

$$q_{\text{int}}/A = h_{31} (T_3^* - T_3) \quad (2.2-4)$$

Eliminating T_2^* and T_3^*

$$q_{\text{int}} = \frac{T_1 - T_3}{\left(\Delta x_{13}/k_{13}A + \Delta/k_wA + 1/h_{31}A \right)}$$

so that Eq. (2.1-1) may be written

$$\rho_1 c_1 \Delta V_1 \frac{dT_1}{dt} = K_{21} (T_2^n - T_1^n) + \frac{1}{\ell_{13}/k_{13}A_{13} + \Delta/k_wA_{31} + 1/h_{31}A_{31}} \cdot (T_3^n - T_1^n) + K_{41} (T_4^n - T_1^n) + K_{51} (T_5^n - T_1^n) + Q_1^n \Delta V_1 \quad (2.2-6)$$

where Δ refers to the tube wall thickness and k_w is the thermal conductivity of the tube wall.

Defining an equivalent k^* ,

$$\frac{1}{k_{31}^*} = \frac{1}{k_w \ell_{31}/\Delta} + \frac{1}{h_{31} \ell_{31}} \quad (2.2-7)$$

so that Eq. (2.2-6) becomes

$$\rho_1 c_1 \Delta V_1 \frac{dT_1}{dt} = K_{21} (T_2^n - T_1^n) + K_{31} (T_3^n - T_1^n) + K_{41} (T_4^n - T_1^n) + K_{51} (T_5^n - T_1^n) + Q_1^n \Delta V_1 \quad (2.2-8)$$

where

$$K_{31} = \frac{1}{\epsilon_{13}/k_{13}A_{13} + \epsilon_{31}^*/k_{31}A_{31}}$$

Eq. (2.2-8) is an equivalent "conduction" equation.

2.2c Radiation Gap Between Tube Wall and Structure

In this case we assume that a radiation gap exists between the coolant tube and solid structure. Referring to Fig. 4c, the heat flux across the right face of element "1" (heat flow out of the element) is

$$q_{int}/A = \frac{T_1 - T_{s1}}{\Delta x_{13}/k_{13}} \quad (2.2-9)$$

Assuming now a vacuum gap between T_{s1} and T_{s2} as well as one-dimensional heat flow

$$q_{int} = F_{1-2} \sigma (T_{s1}^4 - T_{s2}^4) A; \quad F_{1-2} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \quad (2.2-10a)$$

or in terms of a radiation heat transfer coefficient

$$q_{int} = h_r (T_{s1} - T_{s2}) A \quad (2.2-10b)$$

where

$$h_r = F_{1-2} \sigma (T_{s1}^2 + T_{s2}^2) (T_{s1} + T_{s2}) \quad (2.2-10c)$$

σ , Stefan-Boltzmann constant;

F_{1-2} , the overall radiant interchange factor between surfaces S_1 and S_2 ;

σ_1 , emissivity of surface 1;

σ_2 , emissivity of surface 2;

Neglecting the heat capacitance of the tube wall, the energy balance between nodes 1 and 3 results in

$$q_x)x + \Delta x/2 = \frac{T_3 - T_1}{\frac{\ell_{13}}{k_{13}A_{13}} + \frac{1}{h_r A_{13}} + \frac{\Delta}{k_w A_{31}} + \frac{1}{h_{31} A_{31}}} \quad (2.2-11)$$

T_3 is the fluid temperature and h_{31} the convective heat transfer coefficient. Introducing again an equivalent conductivity, k^* , i.e.,

$$\frac{1}{k^*} = \frac{1}{h_r \ell_{31}} + \frac{1}{k_w \ell_{31}/\Delta} + \frac{1}{h_{31} \ell_{31}} \quad (2.2-12)$$

The basic form of Eq. (2.1-7) is recovered. At this point, though, h_r is not known explicitly. On the other hand since the finite difference equations are to be solved by the explicit method, within a given time step, T_{s1} and T_{s2} can be solved by an iterative procedure which in turn determines h_r .

The manner in which we do this is as follows (all notations are with reference to Figs. 3 and 4c, positive heat flow is into element "1"):

$$a) \quad q = \frac{T_{s1} - T_1}{\ell_{13}/k_{13}A} = \frac{T_{s1} - T_1}{U_1} \quad (2.2-13)$$

where

$$U_1 = \ell_{13}/k_{13}A \quad (A = 2\ell_{12});$$

$$b) \quad q = h_r (T_{s2} - T_{s1})A = U_3 (T_{s2} - T_{s1}) \quad (2.2-14)$$

where $U_3 = h_r A$;

$$c) \quad q = \frac{T_3 - T_{s2}}{\Delta/k_w A + 1/h_f A} = \frac{T_3 - T_{s2}}{U_2} \quad (2.2-15)$$

where

$$U_2 = (\Delta/k_w + 1/h_f) \frac{1}{A} \quad (k_w = k_{31});$$

d) combining (a), (b) and (c)

$$q = \frac{T_3 - T_1}{U_1 + U_2 + 1/U_3}; \quad (2.2-16)$$

e) at this point we neglect U_3 in (d), i.e.,

$$q \approx \frac{T_3 - T_1}{U_1 + U_2} = U (T_3 - T_1) \quad \text{where } U = \frac{1}{U_1 + U_2} \quad (2.2-17)$$

so that we can obtain a first estimate of q since

$T_3 - T_1$ is known (at time n);

f) knowing q and combining (b) and (c), again neglecting

h_r , we find

$$T_{s1} = T_3 - qU_2 = T_3 + (T_1 - T_3) UU_2 \quad (2.2-18)$$

(note that T_{s1} could also be obtained from (a), i.e.,

$$T_{s1} = T_1 + qU_1 = T_1 + (T_3 - T_1) UU_1); \quad (2.2-19)$$

g) knowing T_{s1} , we estimate h_r by neglecting T_{s2} ;

$$h_r = 4F_{1-2} \sigma T_{s1}^3; \quad (2.2-20)$$

h) with a first estimate of h_r , we can now find an improved estimate of q from (e), i.e.,

$$U = \frac{1}{\frac{1}{U_1} + \frac{1}{U_2} + \frac{1}{U_3}}, \quad (2.2-21)$$

so that

$$q = U(T_3 - T_1) \quad (2.2-22)$$

i) from (a)

$$T_{s1} = T_1 + qU, \quad (2.2-23)$$

and from (c)

$$T_{s2} = T_3 - qU_2; \quad (2.2-24)$$

j) knowing T_{s1} and T_{s2} , h_r can be determined from

$$h_r = F_{1-2} \sigma (T_{s1}^2 + T_{s2}^2) (T_{s1} + T_{s2}); \quad (2.2-25)$$

k) q can now be updated (see (d)) and the process repeated until convergence of h_r is obtained.

3. DEVELOPMENT OF RADIANT FLUX EXPRESSIONS

Since we are interested in operating the blankets at relatively high temperatures, and cavities are introduced into structures, (see Figs. 6 and 7) it is necessary to develop a detailed analysis of the thermal radiation characteristics for such structures. In other words, we wish to study the effectiveness of such structures and ascertain whether or not the temperature limitations imposed by material considerations are adhered too. The analysis is not limited to blanket applications though.

Consider two "black" elements dA_j and $dA_{j'}$, as shown in Fig. 5a:

Figure 5a.

We wish to obtain a general expression for the energy exchange between these surfaces when they are maintained at different temperatures. The surfaces in question are two differential elements. The problem becomes essentially one of determining the amount of energy which leaves one line element and reaches the other.

The angles φ_j and φ_i , are measured between a normal to the element in question and the line r drawn between the elements. The projection of dA_j on the line between centers (Fig. 5b) is

$$dA_j \cos \varphi_j \quad (3-1)$$

Figure 5b. Elements for radiant flux expressions.

We assume that the surfaces are diffuse, i.e., that intensity of the radiation is the same in all directions. The intensity is the radiation emitted per unit length and per unit angle in a certain specified direction. In order to obtain the energy emitted by the element dA_j in a certain direction, we must multiply the

intensity by the projection of dA_j in the specified direction. Thus the energy leaving dA_j in the direction given by the angle φ_j is

$$i_b dA_j \cos \varphi_j \quad (3-2)$$

where i_b is the blackbody intensity. The radiation arriving at any element dA_n at a distance r from A_j would be

$$i_b dA_j \cos \varphi_j dA_n / r \quad (3-3)$$

where dA_n is constructed normal to the radius vector. The quantity dA_n / r represents the angle subtended by the element dA_n (see Fig. 5c)

Figure 5c.

The intensity may be obtained in terms of the emissive power by integrating Eq. 3-3 over a half-circle enclosing the element dA_j . In polar co-ordinates (Fig. 5d)

$$dA_n = r d\theta = l d\theta \quad (3-4)$$

Figure 5d.

assuming a circle of radius unity. Then

$$e_b dA_j = i_b dA_j \int_{-\pi/2}^{\pi/2} \cos \theta d\theta = i_b dA_j (2) \quad (3-5)$$

so that $e_b = 2i_b$ where e_b is the emissive power (energy per length per unit time). Referring to Fig. 5a, the element dA_n is given by

$$dA_n = \cos \varphi_j dA_j \quad (3-6)$$

The energy leaving dA_j which impinges on $dA_{j'}$ is

$$dq_{j \rightarrow j'} = e_{bj} \cos \varphi_j \cos \varphi_{j'} \frac{dA_j dA_{j'}}{2r} \quad (3-7)$$

Energy leaving $dA_{j'}$ and impinging on dA_j is

$$dq_{j' \rightarrow j} = e_{bj'} \cos \varphi_{j'} \cos \varphi_j \frac{dA_{j'} dA_j}{2r} \quad (3-8)$$

The net energy exchange between j and j' is

$$q_{net\ j-j'} = (e_{bj} - e_{bj'}) \int_{A_j} \int_{A_{j'}} \frac{\cos \varphi_j \cos \varphi_{j'} dA_j dA_{j'}}{2r} \quad (3-9)$$

while the total exchange between j and all j' is

$$q_{net\ j} = \sum_{j'}^{J'} (e_{bj} - e_{bj'}) \int_{A_j} \int_{A_{j'}} \frac{\cos \varphi_j \cos \varphi_{j'} dA_j dA_{j'}}{2r} \quad (3-10)$$

where J' is the total j' elements.

Since $e_{bj} = \sigma T_j^4$, $e_{bj'} = \sigma T_{j'}^4$ and approximating the integrals by assuming that φ_j , $\varphi_{j'}$ and r do not vary over A_j and $A_{j'}$ (small areas) Eq. (3-10) becomes

$$q_{\text{net}} = \sum_{j'}^{J'} \sigma (T_j^4 - T_{j'}^4) \frac{\cos \varphi_j \cos \varphi_{j'} \Delta A_j \Delta A_{j'}}{2r} \quad (3-11)$$

Eq. (3-11) forms the basis for detailed thermal radiation boundary conditions for the cavity regions.

4. BOUNDARY CONDITIONS

4.1 Difference Relationships for Surface Nodes

As noted previously when considering the boundary elements the boundary energy balances must take into account the appropriate boundary conditions. In particular we consider the geometries depicted in Figs. 6, 7 and 8, hereafter referred to as I, II and III, respectively. Several of the boundary conditions are common to all three geometries. As we shall note later, the physical structures for which the computer code is applicable need not be restricted to geometries, I, II and III along with the respective boundary conditions. The geometries and boundary conditions in Figs. 6, 7 and 8 serve as the building blocks for other structures.

Symmetry is assumed along the western boundary for I, II and III as well as the eastern for I and III. In either case the boundary conditions are given by

$$\frac{\partial T}{\partial x} = 0 \quad (4.1-1)$$

which is satisfied by setting $T_1 = T_5$ (western) or $T_1 = T_3$ (eastern). On a boundary where

$$\frac{\partial T}{\partial y} = 0 \quad (4.1-2)$$

the condition is satisfied by setting $T_1 = T_4$.

For all three geometries the northern boundary experiences radiative energy exchange with the surroundings. In differential form the boundary condition is given by

$$-kA \left. \frac{\partial T}{\partial y} \right|_{y=0} = \sigma F \epsilon A (T_s^4 - T^*{}^4) - P_o A \quad (4.1-3)$$

where F is the view factor between the element of area A and the sink area. In our case $F = 1.0$ since we assume parallel planes and ϵ , the emissivity of the sink area, is taken to be a certain percentage of the first wall area. T_s is the surface or boundary temperature of the blanket, and T^* is the sink temperature. The plasma is assumed to be perfectly transparent to thermal radiation. Bremsstrahlung radiation from the plasma to the blanket surface is accounted for via P_o . This mode of radiation is assumed isotropic. Note that P_o may be replaced by any other constant heat source.

We assume a zero capacitance surface node. *Performing an energy balance on the surface node, we find

$$q_{\text{surroundings} \rightarrow s} + q_{1 \rightarrow s} = 0. \quad (4.1-4)$$

or

$$\epsilon \sigma F (T^*{}^4 - T_s^4) A + P_o A + \frac{k_v A}{\Delta y / 2} (T_1 - T_s) = 0 \quad (4.1-5)$$

Rearranging

$$T_s = T_1 + \frac{\Delta y}{2k_v A} \left[\epsilon \sigma F A (T^*{}^4 - T_s^4) + P_o A \right], \quad (4.1-6)$$

* See Appendix A for an alternate formulation of the boundary conditions.

we note that T_s is a function of T_1 which, of course, is an unknown.

An energy balance on 1 results in

$$\begin{aligned} \epsilon \sigma F A \left(T_s^{*4} - T_1^4 \right) &= P_o A + Q_1 \Delta V_1 Q + K_{31} \left(T_3 - T_1 \right) \\ &+ K_{41} \left(T_4 - T_1 \right) + K_{51} \left(T_5 - T_1 \right) = \rho_1 c_1 \Delta V_1 \frac{dT_1}{dt} . \end{aligned} \quad (4.1-7)$$

Starting from the initial temperature distribution, the temperature T_1 at time $n+1$ can be determined assuming that at time zero $n=0$. From Eq. (4.1-6) T_s at time $n+1$ is determined since T_1 is known at the new time. Since $T > T^*$ and P_o is positive, we find that the terms in the brackets of Eq. (4.1-6) can be positive or negative, a function of the wall loading as well as the time.

4.2 Case I

For the internal cavity, black-body radiant-energy exchange is assumed between surfaces. For example referring to Fig. 6, there is radiant interchange between any surface A_j and all surfaces $A_{j'} = 1$ through $A_{j'}$ on the opposite side of the cavity. In addition there is interchange between all surfaces $A_{k=1}$ through A_K as well as $A_{m=1}$ through A_M . From Fig. 6 we see that

$$\cos \theta = \frac{D_2}{R} = \left(D_2^2 + y_{jj'} \right)^{1/2} \quad (4.2-1)$$

so that the total interchange between j and all j' elements on the opposite side of the slot is from Eq. (3-11).

$$q_{j \rightleftharpoons \Sigma j'} = \frac{\sigma}{2} D_2^2 A_j \sum_{j'=1}^{J'} \frac{(T_j^4 - T_{j'}^4) A_{j'}}{(D_2^2 + y_{jj'}^2)^{3/2}}, \quad (4.2-2)$$

which may also be written as

$$q_{j \rightleftharpoons \Sigma j'} = \sum_{j'=1}^{J'} f_{jj'} (T_j^4 - T_{j'}^4), \quad (4.2-2a)$$

where

$$f_{jj'} = \frac{\sigma}{2} \frac{D_2^2}{(D_2^2 + y_{jj'}^2)^{3/2}} A_j A_{j'}. \quad (4.2-2b)$$

In a similar development, the interchange between j and all k and m elements may be written as

$$q_{j \rightleftharpoons \Sigma k} = \sum_{k=1}^K f_{jk} (T_j^4 - T_k^4) \quad (4.2-3a)$$

$$q_{j \rightleftharpoons \Sigma m} = \sum_{m=1}^M f_{jm} (T_j^4 - T_m^4) \quad (4.2-4a)$$

where

$$f_{jk} = \frac{\sigma}{2} \frac{x_{jk} y_{jk}}{(x_{jk}^2 + y_{jk}^2)^{3/2}} A_j A_k \quad (4.2-3b)$$

$$f_{jm} = \frac{\sigma}{2} \frac{x_{jm} y_{jm}}{(x_{jm}^2 + y_{jm}^2)^{3/2}} A_{jm} \quad (4.2-4b)$$

The horizontal distances, x_{jk} and x_{jm} and vertical distances, y_{jk} and y_{jm} are measured from j to the node in question, k or m . Referring to Fig. 6 and letting node 1 represent the first node in from the boundary, an energy balance on node 1 yields ($j=1$ to J).

$$\begin{aligned} \sum_{j'=1}^{J'} f_{jj'} (T_{j'}^4 - T_j^4) + \sum_{k=1}^K f_{jk} (T_k^4 - T_j^4) + \sum_{m=1}^M f_{jm} (T_m^4 - T_j^4) \\ + K_{21} (T_2 - T_1) + K_{41} (T_4 - T_1) + K_{51} (T_5 - T_1) + Q_1 \Delta V_1 = \\ \rho_1 c_1 \Delta V_1 \frac{dT_1}{dt} \quad (4.2-5) \end{aligned}$$

The forward difference expression is introduced to discretize $\frac{dT_1}{dt}$ and all terms on the left hand side of Eq. (4.2.-5) are evaluated at time n .

Performing an energy balance on the surface node, we find

$$\begin{aligned} \sum_{j'=1}^{J'} f_{jj'} (T_{j'}^4 - T_j^4) + \sum_{k=1}^K f_{jk} (T_k^4 - T_j^4) + \sum_{m=1}^M f_{jm} (T_m^4 - T_j^4) \\ + \frac{k_h A}{\Delta x/2} (T_1 - T_j) = 0. \quad (4.2-6) \end{aligned}$$

T_j at time $n+1$ can be determined as T_s is determined in Eq. (4.1-6).

Energy balances for the remaining boundaries result in
m boundary:

$$\begin{aligned} \sum_{j=1}^J f_{mj} (T_j^4 - T_m^4) + \sum_{j'=1}^{J'} f_{mj'} (T_{j'}^4 - T_m^4) + \sum_{k=1}^K f_{mk} (T_k^4 - T_m^4) \\ + K_{21} (T_2 - T_1) + K_{31} (T_3 - T_1) + K_{51} (T_5 - T_1) + Q_1 \Delta V_1 = \\ \rho_1 c_1 \Delta V_1 \frac{dT}{dt} . \end{aligned} \quad (4.2-7)$$

k boundary:

$$\begin{aligned} \sum_{j=1}^J f_{kj} (T_j^4 - T_k^4) + \sum_{j'=1}^{J'} f_{kj'} (T_{j'}^4 - T_k^4) + \sum_{m=1}^M f_{km} (T_m^4 - T_k^4) \\ + K_{31} (T_3 - T_1) + K_{41} (T_4 - T_1) + K_{51} (T_5 - T_1) + Q_1 \Delta V_1 = \\ \rho_1 c_1 \Delta V_1 \frac{dT_1}{dt} \end{aligned} \quad (4.2-8)$$

where

m boundary:

$$f_{mj} = \frac{\sigma}{2} \frac{x_{mj}^2 y_{mj}}{(x_{mj}^2 + y_{mj}^2)^{3/2}} A_m A_j \quad (4.2-9)$$

$$f_{mj'} = \frac{\sigma}{2} \frac{x_{mj'} y_{mj'}}{(x_{mj'}^2 + y_{mj'}^2)^{3/2}} A_m A_{j'} \quad (4.2-10)$$

$$f_{mk} = \frac{\sigma}{2} \frac{D_1^2}{(x_{mk}^2 + D_1^2)^{3/2}} A_m A_k \quad (4.2-11)$$

k boundary:

$$f_{kj} = \frac{\sigma}{2} \frac{x_{kj} y_{kj}}{(x_{kj}^2 + y_{kj}^2)^{3/2}} A_k A_j \quad (4.2-12)$$

$$f_{kj'} = \frac{\sigma}{2} \frac{x_{kj'} y_{kj'}}{(x_{kj'}^2 + y_{kj'}^2)^{3/2}} A_k A_{j'} \quad (4.2-13)$$

$$f_{km} = \frac{\sigma}{2} \frac{D_1^2}{(x_{km}^2 + D_1^2)^{3/2}} A_k A_m \quad (4.2-14)$$

Appropriate energy balances on the surface nodes are also determined. Since the geometry is symmetrical about the center line through the cavity, only one-half of the geometry need be analyzed. Consequently no energy balance for j' elements are written. Note that coolant tubes are not depicted in Fig. 6.

4.3 Case II

As in the previous case black-body radiant energy exchange is assumed between surfaces. Referring to Fig. 7, there is radiant interchange between any surface A_j and all surfaces $A_{j+1} = 1$ through A_J , on the opposite side of the slotted blanket. In addition there is interchange between all surfaces $A_{k=1}$ through A_K and the view factors are the same as those given by Eqs. (4.2-2b, 4.2-3b).

The only remaining radiant interchange for the element A_j is with the plasma and the sink (T^*) boundary. Both are assumed to be treated by a source plane at $y=0$. Again, the plasma is perfectly transparent to thermal radiation and assumed to be isotropic. From Hottel and Sarofim¹ for the case of radiation between a finite area and an infinitesimal plane element we find for the view factor, F_{12} (see Fig. 10a).

Surface Element dA , and the Normal Through the Corner of a Rectangle, with dA , in a Plane Perpendicular to the Edge of the Rectangle

Figure 10a. Surface element dA , and the normal through the corner of a rectangle, with dA , in a plane perpendicular to the edge of the rectangle.

$$F_{12} = \frac{1}{2\pi} \left[\sin^{-1} \frac{Z}{(Z^2+Y^2)^{1/2}} - \frac{Y}{(X^2+Y^2)^{1/2}} \cdot \right. \\ \left. \cdot \sin^{-1} \frac{Z}{(X^2+Y^2+Z^2)^{1/2}} \right] \quad (4.3-1)$$

With respect to Figs. 7 and 10a and for one-half of the slot, we assume that $Z \rightarrow \infty$ (consistent with the 2-D conduction analysis) so that

$$F_{12} = \frac{1}{2\pi} \left[\sin^{-1}(1) - \frac{Y}{(X^2+Y^2)^{1/2}} \sin^{-1}(1) \right], \quad (4.3-2)$$

Hence,

$$F_{12} = \frac{1}{2\pi} \left(\frac{\pi}{2} \right) \left[1 - \frac{Y}{(X^2+Y^2)^{1/2}} \right] = \frac{1}{4} \left[1 - \frac{Y}{(X^2+Y^2)^{1/2}} \right] \quad (4.3-3)$$

For both halves of the slot

$$F_{12} = 2 \left(\frac{1}{4} \right) \left[1 - \frac{Y}{(X^2+Y^2)^{1/2}} \right] = \frac{1}{2} \left[1 - \frac{Y}{(X^2+Y^2)^{1/2}} \right] \quad (4.3-4)$$

Note that

$$\lim_{Z \rightarrow \infty} \sin^{-1} \frac{Z}{(Z^2+Y^2)^{1/2}} = 1 \quad \text{and} \quad \lim_{Z \rightarrow \infty} \sin^{-1} \frac{Z}{(X^2+Y^2+Z^2)^{1/2}} = 1$$

The radiant energy interchange between A_j and the plasma is

$$q_j \leftarrow \text{plasma} = F_j A_j P_o \quad (4.3-5)$$

and between A_j and the sink (T^*)

$$q_{j \rightarrow T^*} = \sigma F_j A_j [T_j^4 - T^{*4}] \quad (4.3-6)$$

where, from Eq. (4.3-4),

$$F_j = \frac{1}{2} \left[1 - \frac{y_j}{(D_2^2 + y_j^2)^{1/2}} \right] \quad (4.3-7)$$

Referring to Fig. 7 and again letting node 1 represent the first node in from the boundary, an energy balance on the surface node j ($j=1$ to J) yields

$$\sum_{j'=1}^{J'} f_{jj'} (T_{j'}^4 - T_j^4) + \sum_{k=1}^K f_{jk} (T_k^4 - T_j^4) + \sigma F_j A_j (T^{*4} - T_j^4) + F_j Po A_j + \frac{k_h A_j}{\Delta x / 2} (T_1 - T_j) = 0 \quad (4.3-8)$$

while the energy balance on node 1 results in

$$\sum_{j'=1}^{J'} f_{jj'} (T_{j'}^4 - T_1^4) + \sum_{k=1}^{K_1} f_{jk} (T_k^4 - T_1^4) + \sigma F_j A_j (T^{*4} - T_1^4) + F_j Po A_j + K_{21} (T_2 - T_1) + K_{41} (T_4 - T_1) + K_{51} (T_5 - T_1) + Q_1 \Delta V_1 = \rho_1 c_1 \Delta V_1 \frac{dT_1}{dt} \quad (4.3-9)$$

For the k boundary there is radiant interchange between any surface A_k and all surfaces $A_j=1$ through A_J as well as $A_{j,}=1$ through $A_{J,}$ on opposite sides of the slotted blanket. The view factors are the same as those given by Eqs. (4.2-12 and 4.2-13).

As with element A_j in the radiant interchange between A_k and the plasma and sink, radiant view factors must be determined. The plasma and sink are assumed to be treated as source planes at $y=0$. For the orientation of the finite area and infinitesimal plane element shown in Fig. 10b),

Figure 10b. Surface element dA , whose normal passes through the corner of a rectangle parallel to dA .

The view factor, F_{12} is given by [1]

$$F_{12} = \left[\frac{1}{2\pi} \frac{Z}{(Y^2+Z^2)^{1/2}} \sin^{-1} \frac{X}{(X^2+Y^2+Z^2)^{1/2}} + \frac{X}{(X^2+Y^2)^{1/2}} \sin^{-1} \frac{Z}{(X^2+Y^2+Z^2)^{1/2}} \right] \quad (4.3-10)$$

With respect to Figs. 7 and 10b and for one-half of the slot, we assume that $Z \rightarrow \infty$ so that

$$\lim_{Z \rightarrow \infty} \frac{Z}{(Y^2+Z^2)^{1/2}} \sin^{-1} \frac{X}{(X^2+Y^2+Z^2)^{1/2}} = 0 \quad (4.2-11a)$$

and

$$\lim_{Z \rightarrow \infty} \sin^{-1} \frac{Z}{(X^2+Y^2+Z^2)^{1/2}} = \pi/2 \quad (4.3-11b)$$

Hence,

$$F_{12} = \frac{1}{2\pi} \left[\frac{X}{(X^2+Y^2)^{1/2}} \times \frac{\pi}{2} \right] = \frac{1}{4} \frac{X}{(X^2+Y^2)^{1/2}} \quad (4.3-12)$$

For both halves of the slot

$$F_{12} = \frac{1}{2} \frac{X}{(X^2+Y^2)^{1/2}} \quad (4.3-13)$$

An energy balance on the surface node k ($k=1$ to K) yields

$$\sum_{j=1}^J f_{kj} (T_j^4 - T_k^4) + \sum_{j'=1}^{J'} f_{kj'} (T_{j'}^4 - T_k^4) +$$

$$\sigma F_k A_k (T_k^{*4} - T_k^4) + F_k PoA_k + \frac{k_v A_k}{\Delta y/2} (T_1 - T_k) = 0 \quad (4.3-14)$$

while the energy balance on node 1 results in

$$\sum_{j=1}^J f_{kj} (T_j^4 - T_k^4) + \sum_{j'=1}^{J'} f_{kj'} (T_{j'}^4 - T_k^4) +$$

$$\sigma F_k A_k (T_k^{*4} - T_k^4) + F_k PoA_k + K_{31} (T_3 - T_1) +$$

$$K_{41} (T_4 - T_1) + K_{51} (T_5 - T_1) + Q_1 \Delta V_1 = \rho_1 c_1 \Delta V_1 \frac{dT_1}{dt} \quad (4.3-15)$$

In this case, from Eq. (4.3-13),

$$F_K = \frac{1}{2} \frac{D_2^{-x_k}}{[(D_2^{-x_k})^2 + D_1^2]^{1/2}} \quad (4.3-16)$$

4.4 Corner Node

The corner node depicted in Fig. 11a requires special consideration.

Figure 11a. Corner node.

Performing energy balances on the surface nodes (S_T , S_R),

we find

$$\epsilon \sigma F_T (T^*{}^4 - T_{ST}^4) + P_o A_T + \frac{k_v A_T}{\Delta y / 2} (T_1 - T_{ST}) = 0 \quad (4.4-1)$$

$$\sigma F_R (T^*{}^4 - T_{SR}^4) + F_R P_o A_R + \frac{k_h A_R}{\Delta x / 2} (T_1 - T_{SR}) = 0 \quad (4.4-2)$$

while an energy balance on 1 results in

$$\begin{aligned} \epsilon \sigma F_T (T^*{}^4 - T_{ST}^4) + P_o A_T + \sigma F_R (T^*{}^4 - T_{SR}^4) + \\ F_R P_o A_R + Q_1 \Delta V_1 + K_{41} (T_4 - T_1) + K_{51} (T_5 - T_1) = \\ \rho_1 c_1 \Delta V_1 \frac{dT_1}{dt} \end{aligned} \quad (4.4-3)$$

The temperature T_{ST} and T_{SR} are determined from Eqs. (4.4-1 and 4.4-2) where $F_T=1$ while F_R is found from Eq. (4.3-7).

5. SOLUTION OF FINITE DIFFERENCE EQUATIONS

5.1 Explicit Method of Solution

For unsteady heat conduction the time derivative $\frac{dT_j}{dt}$ is made discrete by introducing the forward difference expression

$$\frac{DT_j}{dt} = \frac{T_j^{(NEW)} - T_j^{(OLD)}}{\Delta t} = \frac{T_j^{n+1} - T_j^n}{\Delta t} \quad (5.1-1)$$

In addition the right hand side of Eq. (2.1-7) as well as thermal properties are assumed to be evaluated at the "old" time (i.e., at time n). This results in the standard explicit method; the temperature at time $n+1$ is an explicit function of the known temperatures and internal heat generation at time n .

Solving for the new temperature, we find that

$$T_j^{n+1} = T_j^n + \frac{\Delta t}{\rho_l c_j \Delta V_j} \left[\sum_{i=1}^4 K_{ij} (T_i^n - T_j^n) + Q_j \Delta V_j \right] \quad (5.1-2)$$

Since in a number of applications we are concerned with steady-state operation of the blanket, that is, steady, periodic behavior, temperature solutions are generated from arbitrary initial conditions as the asymptotic solutions of the non-steady temperature build-up in the solid.

5.2 Stability Requirement

The calculation of T_j^{n+1} as a function of time as specified by Eq. (5.1-2) is straightforward so long as the Δt associated

with each iteration is kept sufficiently small. Thus, Δt is based on a stability criteria such that the numerical result utilizing Eq. (5.1-2) does not diverge from the correct solution nor introduce numerical instabilities. The criterion for Δt is [2].

$$\Delta t \leq \left(\frac{\rho_j c_j \Delta V_j}{\sum_{i=1}^m K_{ij}} \right) \min \text{ for all } j \quad (5.2-1)$$

The question which arises are the effects of volumetric heating as well as the thermal radiation boundary conditions on stability. Investigations have been made examining time step variations as well as node spacing. From all of the runs made to date and for our parameters, it has been found that the time step criterion used has been set by the equality in Eq. (5.2-1) and $M=4$. In other words the time step is based on the internal nodes.

5.3 Convergence Test for Iterative Method

As previously noted in a number of applications, we are interested in steady, periodic temperature distributions. This implies that a convergence test must be introduced to terminate the iterative process. In essence the criteria adopted is a comparison of the magnitude of the difference between the temperatures obtained from successive cycles, i.e.,

$$|T_j^{n+1} - T_j^n| \leq \epsilon \quad (5.3-1)$$

We have chosen $\epsilon=10^{-5}$ which was determined on a trial and error basis. In several cases we have checked the above criteria against an overall energy balance of the system studied and found that the criteria as given by Eq. (5.3-1) is satisfactory. In addition this criteria is employed for true steady state calculations.

6. DESCRIPTION OF COMPUTER CODE

In this section we give a brief description of the computer code and the manner in which a physical structure must be described for the present code to be applied. The computer program is written entirely in FORTRAN IV and consists of four principle routines to handle data input, radiant view factor determination, step-by-step solutions of the heat balance of all nodal points, and print-out of final results. Data input has been simplified to the maximum extent possible while maintaining a large degree of flexibility in the specifics of the structure analyzed. Eighteen node-types have been identified and constitute the basic building blocks used to describe the structure. In other words the code could be used for other heat transfer applications with thermal radiation boundary conditions so long as the physical structure can be described by some combinations of the following node types (all surfaces not explicitly mentioned are conduction faces):

<u>Type</u>	<u>Description</u>
1	an interior node;
2	a surface node with radiation at the upper surface;
3	a surface node with radiation at the right hand surface;

- 4 a surface node with radiation at the lower surface;
- 5 a surface node with radiation at the left hand surface;
- 6 a corner node with radiation at the upper and right hand surfaces;
- 7 a corner node with radiation at the lower and right hand surfaces;
- 8 a corner node with radiation at the lower and left hand surfaces;
- 9 a corner node with radiation at the upper and left hand surfaces;
- 10 a corner node with radiation at the upper surface and symmetry at left hand surfaces;
- 11 a corner node with radiation at the upper surface and symmetry at right hand surface;
- 12 a corner node with radiation at the lower surface and symmetry at right hand surface;
- 13 a corner node with radiation at the lower surface and symmetry at left hand surface;
- 14 an edge node with symmetry at right hand surface;
- 15 an edge node with symmetry at lower surface;
- 16 an edge node with symmetry at left hand surface;

- 17 a corner node with symmetry at
 the lower and right hand surfaces;
- 18 a corner node with symmetry at
 the lower and left hand surfaces.

All of the view factors previously determined are applicable to the node-type descriptions.

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APPENDIX A

In this section we reconsider the energy balances on the surface nodes. Whereas we treat zero capacitance surface nodes in the main text, Section 4 and in the computer program, here we assume that the capacitance is lumped at the surface, including a surface node.

The advantage of lumping the capacitance at the surface is that the routine for solving the surface temperature is the same as for an internal node. In other words, there is not an additional calculation as in Eq. (4.1-6). On the other hand, there are several disadvantages. Notably for unsteady conditions [3] in the case of a boundary condition of a constant surface temperature that is different from the initial temperature, all the surface capacitors would be charged instantly. In effect this capacitance is being neglected. This can result in serious errors in transient problems at small values of time. For our geometries and in general for prescribed flux, convective or radiative boundary conditions, lumping the capacitance at the surface results in a larger number of special boundary nodes, in particular, corner nodes. This will become evident in the analysis. Of course the way in which we treat the boundary nodes should not be based on simplicity of computer programming but rather on a physical basis, i.e., are we modeling the

physical phenomena correctly and generating "correct" temperature distributions.

Lastly, with regards to numerical stability, if a potential violation of the second law is to be avoided, the criterion for an internal node is:

$$M = \frac{\Delta^2}{\alpha \Delta t} > 4$$

where Δ =grid spacing ($\Delta=\Delta x=\Delta y$) and α , thermal diffusivity, = $K/\rho c$. The boundary elements may result in different criteria than the interior elements. In particular for a convective boundary condition, lumping the capacitance at the surface results in [3].

$$M \geq 4 + Bi$$

where Bi is the Biot modulus, $\frac{h\Delta}{K}$.

A large Biot modulus will require a small time step. For the zero capacitance surface node and convective boundary conditions, the criterion becomes

$$M \geq 3 + \frac{2Bi}{2+Bi}$$

so that the advantage of the zero capacitance case for large Bi modulus is evident.

From several numerical comparisons we have made with radiation boundary conditions, in general for small times

(several time steps from $t=0$) the surface temperatures do differ between the two methods but at large times differences are imperceptible. For completeness, though, we consider lumping the capacitance at the surface nodes for several conditions. Not all special cases are included here.

a) Northern Boundary

In all three cases, I, II and III an energy balance on the surface node (Fig. 12 results in)

$$\begin{aligned} \epsilon \sigma F (T_s^4 - T_{\infty}^4) + P_o A + Q_s \Delta V_s + K_{31} (T_3 - T_s) + \\ K_{41} (T_4 - T_s) + K_{51} (T_5 - T_s) = \rho_s c_s \Delta V_s \frac{dT_s}{dt} \end{aligned} \quad (A-1)$$

Note that the surface element is one-half of an internal element, i.e., the surface volume is 1/2 that of an internal element.

b) Corner Node

The volume of the corner node, Case II (Fig. 12), becomes one-fourth of an internal node. The energy balance is the same as Eq. (4.4-3) but with T_1 replaced by T_s , $T_{sT} = T_{sR}$ and ΔV_1 calculated accordingly.

c) Internal Cavity Corner Node

An internal cavity corner node, Case I, (Fig. 11b), is truly a special case. The surface nodes are compared in the figure.

Note that for the zero capacitance surface node case, no corner node arises. Performing an energy balance on S results in

$$\begin{aligned}
 & \sum_{j'=1}^{J'} f_{mj'} (T_{j'}^4 - T_S^4) + \sum_{k=1}^K f_{mk} (T_k^4 - T_S^4) + \\
 & \sum_{j'=1}^{J'} f_{jj'} (T_{j'}^4 - T_S^4) + \sum_{k=1}^K f_{jk} (T_k^4 - T_S^4) + \\
 & K_{21} (T_2 - T_S) + K_{31} (T_3 - T_S) + K_{41} (T_4 - T_S) + \\
 & K_{51} (T_5 - T_S) + Q_S \Delta V_S = \rho_S C_S \Delta V_S \frac{dT_S}{dt} \left(\begin{matrix} m=1 \\ j=1 \end{matrix} \right) (A-2)
 \end{aligned}
 \tag{F-2}$$

where $f_{mj'}$, f_{mk} , $f_{jj'}$, and f_{jk} are the same view factor expressions which have been previously determined.

Figure 1. Interior element-heat fluxes.

Figure 2. One-dimensional heat transfer through a composite wall.

Figure 3. Node configuration.

Figure 4a.

Figure 4b.

Figure 4c. Node configuration - coolant tubes.

Figure 8. Model geometry - Case III.

Figure 9. Surface node.

CAPACITANCE LUMPED AT SURFACE ZERO-CAPACITANCE SURFACE

Figure 11b. Internal corner node.

SURFACE NODE

CORNER NODE

Figure 12. Surface capacitance nodes.