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Proposal for the Construction of a High-Beta Tokamak at LASL

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PROPOSAL FOR THE CONSTRUCTION OF A HIGH-BETA TOKAMAK AT LASL

by

P. C. T. Van der Laan, J. P. Freidberg, and K. S. Thomas
with contributions from

P. R. Forman, J. P. Goedbloed, R. K. Linford, L. W. Mann,
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ABSTRACT

The large heating rate inherent to implosion heating allows the rapid generation of high-beta tokamak plasmas. A study of these plasmas in the proposed HBT machine can give information on how MHD equilibrium and stability limit β and q . Both a wide current profile and a moderate elongation of the minor cross section should help to raise the permissible peak β values in HBT to at least 20%. The longer term loss processes occurring in MHD-stable plasmas are to be investigated.

The main parameters of HBT are: $R = 0.30$ m, minor cross section a racetrack of width and height 0.24 m and 0.48 m, $B_z = 2$ T, $I_p \approx 750$ kA.

1. INTRODUCTION

An important advantage of tokamaks is that they provide a relatively easy method to form an interesting and well-behaved plasma in a simple magnetic field configuration. The reason for this is that the induced plasma current, while forming and heating the plasma, will soak in slowly and will relax by itself to a stable profile. The ohmic heating inherent to this formation process can produce very interesting plasma parameters, but the problem still remains how to raise the beta and the temperature of the resulting plasma to the values that are necessary for reactors. Of the various heating methods that could provide additional heating, especially neutral injection is being actively pursued at present.

In the machine described in this report we propose to use implosion heating both to heat and to form the plasma in a tokamak-like geometry. Implosion heating produces a peaked pressure profile and also high-beta values. A longitudinal bias field will be included and will be adjusted so that experiments can yield the limiting values of beta and of the safety factor q that are compatible with the requirements for equilibrium and stability.

The initial plasma density, before the implosion, will be chosen fairly high ($\approx 3 \times 10^{20} \text{ m}^{-3}$) to avoid too much anomalous heating and diffusion. This leaves only the kinetic energy of the imploding plasma as an energy input; the advantage is however, that it allows a better control over the resulting current profiles because the fluxes and q values built in during the short formation time are being conserved. A consequence of this is that the current profiles can be optimized by an appropriate choice of the time-histories of the two field components. A current profile wider than the pressure profile is favorable for equilibrium and stability; in this case force-free currents flow in the low-density plasma outside the central plasma column.

The choice of the initial plasma density leads to final densities in the compressed plasma of several times 10^{21} m^{-3} . This density is high for tokamak standards and this has consequences for the later stages of the discharge. Impurity radiation may cause rapid cooling; another process limiting the lifetime may be a shrinkage of the current channel, perhaps followed by a disruptive instability. The characteristic times for these and other processes are discussed in Sec. II.H.

The extrapolation of the proposed High-Beta Tokamak to a reactor and the problems of this extrapolation are briefly discussed in Sec. II.G. This section also deals with the relevance of the proposed machine itself for the existing Tokamak program. A list of reasons to propose the HBT machine are given below; of these, (1) through (5) are closely related to the Tokamak program.

(1) Making high β is important for the Tokamak program.

(2) HBT is a relatively inexpensive device to test how tokamak confinement can be improved by influencing j and p profiles and by a moderate elongation of the cross section.

(3) HBT can reach desirable profiles without much turbulence which might result in fewer impurities.

(4) High heating rates at high temperatures will result in flux and q conservation in standard tokamaks also. What happens and whether diffusion limits this conservation can be studied in HBT now.

(5) High-beta tokamak plasmas produced by pinch techniques can easily start at densities above the "high-density limit" of tokamaks, that is, above the empirical line in the $\log \bar{n}_e$ vs $\log B/R$ graph. It is of interest to study the evolution of such an "over-dense" plasma in time.

(6) Screw pinches and belt pinches have shown good behavior for many years. An experiment testing the potentials of these approaches further, especially to try to increase the lifetimes to more than 100 μs is worthwhile.

(7) The proposed HBT is a real pinch but also a tokamak and provides for LASL a useful link with the tokamak world.

(8) Implosion heating at low β and at the filling densities chosen requires high ring voltages. LASL has the high-voltage capability and know-how to incorporate these voltages in this compact machine.

(9) LASL has a strong theory program on the high-beta tokamak to support the experimental efforts.

II. SCIENTIFIC SCOPE

A. Motivation and Objectives

It is of considerable interest to produce a plasma in a tokamak-like configuration at the highest possible β value, a "Maximak" in Mukhovatov and Shafranov's¹ terminology. Of the various heating methods that can increase the β of a tokamak above what can be reached by ohmic heating, we choose implosion heating because it is straightforward, relatively inexpensive and uses LASL's expertise. An additional advantage of implosion heating is that it allows the programming-in of desirable current and field profiles, so that the discharge does not have to go through the turbulent phase of normal tokamak buildup. More particularly, the current distribution can be made wider than the implosion-produced pressure distribution. The force-free currents that then flow in the annular region around the central plasma, considerably increase the critical value of β , β_{crit} , at which a stable equilibrium is possible (see Sec. III.D). To increase β_{crit} even more, the minor cross section will be elongated by a factor 2, close to the optimum elongation predicted by theory (see Sec. III.E).

Thus combining features of tokamaks, screw pinches, and belt pinches, the experiment should be able to confine plasmas with a β of 20% to 25% for at least

100 μ s. The limits of β and of the safety factor q are to be investigated. Extensive theoretical calculations carried out in LASL Group CTR-6 are available to make the parameter study more meaningful. When the confining fields are maintained longer, as is planned in the second version of the machine, diffusion studies become possible. This would provide information which is not yet known at high values of β .

B. Implosion-Heated Tokamak

In implosion heating most of the magnetic field energy moves into the tube in a short time. This rapid buildup of the resulting tokamak configuration causes it to have a number of special features, as described in this section. A comparison with a "standard" high-beta tokamak is difficult; each heating method (neutral injection, turbulent heating, etc.) may introduce its own features. For implosion heating at such filling pressures that anomalous resistivity is not too important, we have basically two features, each with important consequences.

(1) The transition through the low-temperature resistive stage takes a very short time. Diffusion is therefore not as essential as it is in the formation stage of the standard tokamak. The field lines and the plasma therefore move much more together. Consequences of this are:

(a) The plasma tends to conserve the q value of the field lines as they move in. This is equivalent to the conservation of the pitch of the field lines in a cylindrical system. The reason is in both cases that the longitudinal and the azimuthal flux between adjacent magnetic surfaces can only be conserved if q , or the pitch, is conserved during the motion. The time history of q at the wall, especially during the risetime of the fields, is then important, since it allows a control over the q -profiles and thereby the current distribution in the plasma. The currents that help to conserve the q must flow parallel to B in the low-density plasma surrounding the central plasma. More will be said about these force-free currents in Sec. 11.C. Of course these currents can only flow if the field lines stay in the low-density plasma without intersecting the insulating wall. This leads to important differences between machines with or without conducting shell. If the beneficial effect of the force-free currents is to be used, a closely fitting metal shell, or an elaborate field programming system is essential.

(b) If a conducting shell is used, no separatrix can develop in the low-density plasma, since this would involve a change of q to infinity. Such a

change of topology is impossible¹ if the field lines around the plasma are initially well behaved, as they are immediately after the implosion, or in the case of other heating methods, before the heating has started.

(c) If a vertical field has to ensure the equilibrium, it should be pulsed, preferably at as late a time as possible. If applied too early, open field lines are built in, which will cause losses.

(d) The final equilibrium reached by the plasma depends strongly on the way the configuration is produced. This limits the applicability of equilibrium calculations, unless a time-dependent study is done.

(e) In the standard tokamak the current distribution, as given by the applied E-field and the temperature distribution, may not be compatible with a high poloidal β .² In a rapidly formed configuration the current distribution can adjust itself, provided the integral q values on the magnetic surfaces are conserved.

(2) The rapid formation excites a number of oscillatory modes.

(a) The implosion itself, which may be considered as an $m = 0$, $k = 0$ mode, moves the plasma away from the wall and allows it to find its equilibrium position, without the need for a limiter. The implosion is, as in all pinches, followed by damped radial oscillations.

(b) In a toroidal machine an $m = 1$, $k = 0$ mode is excited. The implosion brings the plasma more or less to the center of the tube. From there the plasma moves over a distance Δ to the eccentric equilibrium position. For higher β values the outward force is approximately given by the formula for the toroidal θ pinch, $F_{\text{out}} = NkT/R$, where N is the total number of particles per meter length. The kinetic energy in the outward motion follows from the work done by the outward force

$$F_{\text{out}} \Delta = 2 NkT \Delta/R, \quad (1)$$

which obviously is an appreciable fraction of the total thermal energy per meter length of the plasma column $3/2 NkT$.

The outward motion of the plasma can be stopped by the interaction of the toroidal plasma current I_ϕ and the image currents in the shell (or a vertical field in other machines). The toroidal current must be present and should be induced as soon as B_ϕ is applied to avoid the creation of a pure theta-pinch core that one could never stop.

The inward force produced by the shell can be written as $F_{in} = \frac{1}{2} I_\phi^2 \partial L_i / \partial \Delta$, where L_i is the inductance associated with the poloidal flux within the shell. Roughly speaking L_i decreases parabolically with Δ , which makes F_{in} linear in Δ . This would give an approximately harmonic oscillation around the equilibrium with an overshoot as large as the equilibrium shift. To avoid this undesirable situation, I_ϕ should still be rising during the first outward motion of the plasma.

The plasma will reach this most eccentric point at a time, roughly given by the drift time to the wall, for a toroidal θ pinch

$$t_d = \left\{ bR m_i / k (T_i + ZT_e) \right\}^{\frac{1}{2}}. \quad (2)$$

For a 300-eV deuterium plasma in a torus with $R = 30$ cm, and half-width of the tube $b = 12$ cm, this is only 1.1 μ s.

(c) Especially in machines with elongated cross sections an $m = 2$, $k = 0$ oscillation is excited. The amplitude of this oscillation depends on the eventual shape of the equilibrium. For the modest elongation of the proposed experiment an elliptic cross section of the plasma seems possible. The conducting shell and the force-free currents should both help³ to keep the central plasma elongated, even at larger compression ratios.

C. Force-Free Currents

Force-free currents are present in standard tokamaks also. Let us consider as an example a tokamak with a $\beta_{pol} = 1$, which means that only ϕ currents are flowing. A simple argument shows that, for this case, the components of the current density $j_\perp$ and $j_\parallel$, perpendicular and parallel to the B-field, have a ratio

$$j_\perp / j_\parallel = B_\theta / B_\phi = r/Rq. \quad (3)$$

This ratio is small which means that the angle between current and field is small (9° for $q = 2$ and $r/R = 3$, and less for smaller radii) and that the largest component of the current density is force-free. This component is, just as the total current, a result of the applied E field and the distribution of plasma resistivity, built up during the many milliseconds of the formation of the discharge.

In a pulsed discharge where most of the field lines are moving in with the plasma in a short time, not so much the applied E fields, but the motion of the conducting medium determines where the currents flow. Plasma inertia and pressure slow down the motion and thereby create gradients in the driving field and consequently, currents perpendicular to B. These currents are the only currents in pure theta or z pinches.

Whenever two field components are being pushed into a toroidal or cylindrical system, the plasma has to move in such a way that both fluxes, longitudinal and azimuthal, are conserved. In general, it can only do that by carrying a current which modifies the fluxes enough to let them both stay constant. To use an analogy: a steam turbine (field A) pushes a shorted dynamo rotor (the plasma) across a magnetic field (field B). The roles of fields A and B are interchangeable.

Perpendicular currents do of course contribute to the fluxes that have to be conserved, but the current component that can adjust its magnitude without the constraint of the equation of motion is the force-free component. Outside the central dense plasma column, the current has to be exclusively force-free because the inertia of and the pressure gradients in the low-density outside plasma are small.

The magnitude of the force-free currents can be found from the field configuration. In a cylindrical system one of the possible formulas for j_{θ} is

$$\mu_0 j_{\theta} = |B| \left(\frac{\partial \alpha}{\partial r} + \frac{\sin \alpha \cos \alpha}{r} \right), \quad (4)$$

where $|B|$ is the magnitude of the field and α is the angle between the field and the z axis. The field configuration at any given time depends on the implosion dynamics and on the time-history of the two field components at the wall. This last point is described best by giving the time-history of $|B|$ and the pitch (or q) at the wall, since the field lines conserve this last quantity after they cross the wall and become embedded in the low-density plasma.

Of the many possible time-histories of the pitch at the wall a simple one, both for this "pitch convection" model and in experimental practice, is the pitch applied constant in time. This time-history generates a configuration with a uniform pitch.^{4,5} We discuss this special case in the next section.

D. Constant-Pitch or Constant-q Configuration

This configuration is generated when $B_{\text{tor}}/B_{\text{pol}}$ is kept constant at the wall of the tube during the risetime of the fields. In cylindrical geometry the equations for B_z and B_θ in a force-free constant-pitch field are

$$B_z = \frac{C}{1 + \mu^2 r^2} \quad B_\theta = \frac{\mu r C}{1 + \mu^2 r^2}, \quad (5)$$

where μ is given by B_θ/rB_z . Using these cylindrical formulas to get approximate answers for a torus with circumference $2\pi R$, we find that to have $q > 1$ we should keep $\mu < 1/R$. This limits the argument μr for Eq. (5) to less than b/R , the inverse aspect ratio of the torus. For small r , perpendicular currents associated with plasma pressure and inertia of the central plasma modify the fields of Eq. (5).

The current density in the force-free plasma can be found from Eq. (5).

$$\mu_0 |j| = \frac{2\mu |B|}{1 + \mu^2 r^2}. \quad (6)$$

For small μr the ratio $\mu_0 |j|/|B|$, which is commonly called α , is approximately constant, so that the constant-pitch field then looks very much like the Lundquist field,⁶

$$B_z = AJ_0(\alpha r) \text{ and } B_\theta = AJ_1(\alpha r). \quad (7)$$

According to Taylor's calculations⁷ this field is the lowest energy state to which a plasma may relax through turbulence. In this case where the configuration is generated directly, no relaxation is necessary; the fact that the field is in a lowest energy state can be used as an argument for the increased stability of the constant-pitch configuration.

Compared to the reversed-field pinch configuration, only small currents are needed for the constant-pitch field with $q > 1$. The ratio of the α values for the two cases is

$$\alpha_{\text{cp}}/\alpha_{\text{rfp}} \approx \frac{2}{R} \frac{3.0}{B} = \frac{2}{3} \frac{b}{R}. \quad (8)$$

In both cases the currents go down to zero near the wall. The precise profile influences whether local instabilities are to be expected in the fall-off region. In screw pinches this falloff seems to be caused by turbulence⁸ close to the wall early in time.

The elongation of a factor 2 chosen for the proposed experiment and the toroidal curvature make the cylindrical equations less applicable. However, as an assumption for the equilibrium calculations, a constant α seems a reasonable choice. Irrespective of elongation and toroidal curvature the integral q value (or the rotational transform) has to be conserved.

A physical explanation of how the force-free currents improve the equilibrium and the stability (at least for long wavelengths) is the following (see Fig. II.1). When the plasma moves out, the poloidal field is being squeezed between the shell and the plasma column. Since the pitch has to be conserved, the angle between field line and axis increases at the outside and decreases on the inside. This reduces the j_z component at the outside and increases it at the inside. This produces a restoring force on the plasma similar to what the image currents on the shell do, but more effectively, because these currents are closer to the plasma.

A second effect of the increase of the j_z component on the inside is that this will counteract the tendency for the perpendicular currents on the inside to reverse.² This reversal of perpendicular currents occurs when diffuse profiles are considered, before a zero in the poloidal field is seen. For these diffuse profiles the parallel and perpendicular currents are intermixed so that a reversal may not take place or occur at higher β_{pol} .

E. Outline of Proposed Device

The shape and the dimensions of the toroidal discharge vessel are given in Fig. II.2. The factor 2 elongation is chosen in view of the theoretical predictions (Sec. III.E). Compared to an elliptical cross section, the racetrack shape has the advantage that sheet insulation can be used on the inside surface and that machining operations, such as grinding of the ceramic sectors and the fabrication of the coils, are easier.

The size of the minor cross section is chosen such that its circumference (1.234 m) does not lead to too large a ring voltage when a poloidal E field of 1 kV/cm is applied. In circumference and applied E field the device resembles the Implosion Heating Experiment; a difference however, is that in view of the

Fig. II.1.

Schematic view of the force-free currents in the outer region of a centered (a) and a displaced (b) plasma column.

Fig. II.2.

Shape and dimensions of the ceramic (99.5% Al_2O_3) discharge vessel.

toroidal geometry, two instead of four feedpoints are chosen. Note that the E fields mentioned here are the E fields in the presence of plasma, and are therefore a measure for the implosion velocity.

The aspect ratio is chosen to be 2.5, which is about the smallest value technically feasible. The force problems in the central part of the machine have to be considered in more detail. The number of volt seconds required in the transformer is approximately 0.12 (see the parameters given in Table VII). This number depends strongly on the current profile. To have sufficient flexibility, we plan to use an air-core transformer even though this leads to an increase of the size of the ϕ bank.

To have the beneficial effect of the force-free current a closely fitting shell is to be used. This shell keeps all primary poloidal flux outside the discharge region; the mutual inductance of the transformer is therefore the same as the primary inductance.

To increase the flexibility of the programming, two separate coil systems are planned. This allows independent changes in amplitude and timing of the two fields. If more detailed changes in the wave shape are to be made, this can be done in the smaller bank, the bank for the ϕ currents.

Parameters of the machine are listed in Table I.

TABLE I
MACHINE PARAMETERS

Major radius	R	0.30 m
Height	$2b_1$	0.48 m
Width	$2b_2$	0.24 m
Volume	V	194 dm^3
Minor circumference		1.234 m
Toroidal field at R = 0.30 m	B_ϕ	2.0 T
θ inductance, single turn, 1.5 cm dead space	L_θ	87 nH
Toroidal current estimate for q (wall) = 2	I_ϕ	750 kA
Filling pressure deuterium	p_0	0.67 Pa (5 micron)
Final plasma density	n_e	$5 \times 10^{21} \text{ m}^{-3}$
Final plasma temperature	T	300 eV

F. Comparison with Other Experiments

If we define a high-beta tokamak as a toroidal system in which the plasma is confined by helical field lines with q values larger than 1, and in which a heating method other than ohmic heating is used to raise the total β , screw pinches and belt pinches should be included. Neutral-injection heated tokamaks start from a standard tokamak discharge; a comparison with ORMAK Upgrade, the recently approved machine at Oak Ridge, is included at the end of Sec. II.F. Tokamaks with two-dimensional adiabatic compression (Tuman, Leningrad, and Tosca, Culham) do not increase β (in fact $\beta \propto B^{-1/3}$) and are therefore not high-beta tokamaks according to our definition.

Table II gives a list of parameters of existing machines with circular or elongated minor cross section.

Whether or not the machine is equipped with a conducting shell has important consequences. Without shell, many of the field lines do intersect the

TABLE II
PARAMETERS OF HIGH-BETA TOKAMAKS USING PINCH TECHNOLOGY
(INCLUDING SCREW PINCHES AND BELT PINCHES)

Circular Cross Section		R (m)	r (m)		B ₀ (T)	I _p (kA)	SHELL
SPICA	Jutphaas	0.6	0.2		1.5	300	yes
TEE	Jülich	0.25	0.095		0.45	25	partly
STP-2	Nagoya	0.25	0.1		0.93	70	yes + B _{vert}
TPE-1	Tanashi	0.4	0.05		2.0	20	yes
THOR	Maryland	0.5	0.2		1.0		coils
Elongated Cross Section			h	w			
BP1	Garching	0.23	0.9	0.14	0.7	200	partly
BP2	Garching	0.52	2.6	0.45	0.8	200	partly
HVBP	Garching	0.3	1.1	0.2	0.3		partly
TENQ	Jülich	0.5	1.25	0.4	0.7	200	no
	Lausanne	0.16	0.7	0.1	1.7	200	partly
Torus I	Columbia	0.225	0.45	0.15	1.2	160	no
TERP	Maryland	0.15	0.4	0.15	0.08		no

insulating wall. This means that the force-free currents induced along those field lines have to die out on a short time scale; some experiments show evidence of this initial fast decay. The decay of the currents means that only lower β plasmas can be kept in equilibrium. At these lower β values implosion heating becomes less efficient and most experimenters therefore tend to go to lower filling pressures where anomalous resistivity becomes important because of the higher drift velocities of the electrons.

The anomalous heating process is more efficient at the lower β values than the implosion heating and is therefore an easier method to obtain reasonably high temperatures. A drawback is however, that the resulting current and pressure profiles cannot be predicted too well. Some force-free currents may persist near the dense core and this may explain why β values of 5% to 10% are still reported.

In the five small screw-pinch machines studied in Jutphaas as predecessors of SPICA, a problem was always the rather rapid decay of the toroidal current. Likely causes for this are too low temperatures of the plasma and by a too strong outward Poynting flux after the crowbarring.⁹ In SPICA this problem is more or less solved; the higher temperature (60 eV) and the low-loss crowbar system (decay times 1.2 and 2.1 ms for the primary toroidal and poloidal circuits)

have resulted in a decay time for I_ϕ of 500 to 700 μ s, in a discharge where the plasma is stable for at least 100 μ s.⁸

Differences between the proposed machine (HBT) and SPICA are:

(1) HBT has higher voltages and therefore reaches higher temperatures.

Tentative figures for HBT: $E_{pol} \approx 1$ kV/cm, $T_{e,i} \approx 300$ eV;

for SPICA: $E_{pol} = 200$ V/cm, $T_{e,i} = 50$ eV.

(2) HBT has an elongated cross section and an aspect-ratio of 2.5 (SPICA has 3.0), and therefore β_{cr} for equilibrium and stability is higher (needs to be substantiated by calculations, but already seems evident).

(3) HBT has a higher value of B_ϕ/R : SPICA has 1.5 T/0.6 m = 2.5 T/m, whereas HBT has 2 T/0.3 m = 6.67 T/m. The parameter $B_\phi/R = 1/2 \mu_0 qj$ and is therefore a measure of the ohmic power input.

(4) Related to (3) and (2), I_ϕ is higher in HBT than in SPICA; estimated to be 750 kA compared to 300 kA in SPICA. The estimate depends on the q in the major part of the plasma (≈ 1.5) and on the width of the layer in which the force-free currents are not properly induced.

(5) In its first form HBT may have decay times of the fields that are shorter than those in SPICA. As described in the Engineering Description, the proposal includes additional circuitry to increase the decay times considerably.

The proposed machine can also be compared to ORMAK Upgrade.¹⁰ The latter machine can be maximized in terms of β_p (up to 2.9) or Γ , the ratio of beam energy to plasma energy (up to 0.39) at an injection power of 2 MW and a toroidal plasma current of 200 kA. At the full toroidal field of 4 T the average total β is then 5.2×10^{-3} ; a higher β of about 1.9% is possible at lower q (4) and a lower toroidal field (1.6 T).

The HBT machine will be operated to maximize the total β ; both the elongated shape and the force-free currents are favorable in this respect. Because implosion heating is used, high β values are easily obtained; in fact a bias field of appropriate magnitude has to be added to reduce β below the limits for MHD stability and equilibrium. Expected peak values of β are then still 20% to 25%; at a compression ratio of 3 this corresponds to an average β of 2.2% to 2.8%.

The rapid buildup of the configuration in the HBT makes it possible to influence the shape of the current profiles; in particular it allows the generation of the favorable uniform q profiles. In ORMAK Upgrade some additional current can be produced by injection but the relative amount of additional current is limited to several tens of percent.

The particular types of equilibria that occur in Flux-Conserving Tokamaks (FCT)¹¹ are seen when

$$\tau_H < \tau_E < \tau_{\text{skin}} , \quad (9)$$

where the symbols indicate the heating time, the energy confinement time, and the resistive diffusion time. In the HBT the implosion process heats the plasma in such a short time that condition (9) is easily met, which means that FCT effects are certain to show up. In ORMAK Upgrade the heating time is expected to be shorter than τ_E , but the actual ratio depends on the details of the transport processes.

The parameter β_p is not used in this comparison with ORMAK Upgrade because its value is ambiguous for the HBT with force-free currents. For instance the values obtained from the equation

$$\beta_p = 8\pi \langle p \rangle / \mu_0 I_\phi^2 \quad (10)$$

may be less than 1 if the averaging is done over the whole tube, but go up with approximately r^{-4} if $\langle p \rangle$ and I_ϕ are taken for more inside magnetic surfaces.

G. Extrapolations and Relevance to the Tokamak Program

In Sec. IV we describe briefly a feasibility experiment, based on the principles of the proposed HBT, with parameters extrapolated so that $n\tau$ values of 10^{20} sm^{-3} and a temperature of 5 keV can be reached.

The heating in this experiment is done in two stages. A fast initial rise of the B_ϕ field to 0.5 T heats the plasma to 1 keV; a slower rise to 5 T then heats the plasma adiabatically to 5 keV ($\propto B^{2/3}$). The β value that can be kept in stable equilibrium influences the effectiveness of the implosion heating, and depending on this, the value of E_{pol} at the wall has to be adapted; in the description in Sec. IV an E field of 2 kV/cm is assumed. For plasma densities of $5 \times 10^{21} \text{ m}^{-3}$ a confinement time of 20 ms would provide the $n\tau$ value of 10^{20} .

The extrapolation of a screw pinch towards a reactor is being studied by a group of physicists and engineers in the Netherlands. Table III is taken from a January 1976 status report of this group. The underlined quantities are chosen; the other parameters are calculated and roughly optimized. Tentative conclusions at this stage are that the ohmic losses in the copper compression coils

TABLE III
PARAMETERS OF A SCREW PINCH REACTOR

Aspect ratio	<u>A = 3</u>
Maximum beta (from $\beta \cdot A \sim 0.75$)	<u>$\beta = 0.25$</u>
Small radius	b = 4.20 m
Large radius	R = 12.6 m
Blanket thickness (gas cooled)	<u>$d_b = 0.80$ m</u>
Bias field	<u>$B_v = 0.22$ T</u>
Density at t = 0	$n_o = 0.31 \times 10^{20} \text{ m}^{-3}$
Rise time implosion field strength	$\tau = 2.9 \text{ } \mu\text{s}$
Maximum poloidal electric field at wall	<u>$E_v = 2$ kV/cm</u>
Magnetic field at end of implosion	$B_i = 0.41$ T
Implosion bank energy per cycle	$W_i = 271$ MJ
Compression field	$B_a = 5.5$ T
Adiabatic compression energy per cycle	$W_a = 98$ GJ
Burn temperature	$T_b = 16.3$ keV
Burn time ^a	$t_b = 9.3$ s
Cycle time	$t_c = 19.3$ s
Down time $t_c - t_b$	<u>$t_{cl} = 10$ s</u>
Duty cycle	<u>$t_b/t_c = 0.48$</u>
Burnup fraction ^a	$b_u = 39\%$
Efficiency magnetic energy transfer system	<u>$\eta_m = 0.95$</u>
Rate of losses by magnetic energy transfer	$P_m = 0.26$ GW
Thickness Cu coils (70% filling factor)	<u>$d_c = 1.5$ m</u>
Rate of Ohmic losses in coils	$P_\Omega = 0.50$ GW
Gross efficiency of conversion thermal energy to electric energy	<u>$\eta_{th} = 0.35$</u>
Gross thermal output	<u>$P_{th} = 4.18$ GW</u>
Wall loading	<u>$P_v = 1.67$ MW/m²</u>
Gross electric output	$P_{eg} = 1.46$ GW
Nett electric output after covering the losses	<u>$P_{en} = 700$ MW</u>
Nett efficiency P_{en}/P_{th}	<u>$\eta_n = 16.8\%$</u>

^aAssuming β can be kept constant below 0.25 during the burning phase.

are serious and that an increase of β would lead to large improvements. Belt-pinch-like geometries and sustained-field circuitry (one superconducting and one pulsed-coil system) are therefore to be considered.

Apart from the above extrapolations we also list a number of questions that are relevant to the tokamak program and that can be studied in the proposed device.

(1) Making high β is important for the tokamak program.

(2) HBT is a relatively cheap device to test how tokamak confinement can be improved by influencing j and p profiles and by a moderate elongation of the cross section.

(3) HBT can reach desirable profiles without much turbulence which might result in fewer impurities.

(4) High heating rates at high temperatures will result in flux and q conservation in standard tokamaks also.¹¹ The problem of what happens and whether diffusion limits this conservation can be studied in HBT now.

(5) High-beta tokamak plasmas produced by pinch techniques can easily start at densities above the "high-density limit" of tokamaks, that is above the empirical line in the $\log \bar{n}_e$ vs. $\log B/R$ graph. It is of interest to study the evolution of such an "over-dense" plasma in time.

Experimental and theoretical evidence exists and will hopefully be corroborated in HBT at increased temperatures that higher β values can be confined if the current profile is wider than the pressure profile (point 2). A question is whether this "blanket" of force-free currents could be produced in a standard tokamak. Two suggestions can be made at this stage.

(1) Some form of beam-injection could be used to generate the force-free currents. The force-free currents could be beam-induced in the direct sense, or could result from the heating of the edges of the plasma and the widening of the T profile. If neutrals are injected the low-trapping efficiency in the outer edges of the plasma would be a problem. Electron injection would require a magnetically shielded snout reaching into the outer edge of the plasma.

(2) In a large tokamak a rise of the two field components in a few hundred μs would permit the use of the "pitch-convection" technique which, if properly programmed, can result in the desirable force-free currents. To conserve the pitch of the inward-moving field lines, the magnetic Reynolds number $\mu_0 v L / \eta$ should be large compared to 1, where v is the inward velocity and L a characteristic length. Problems with this approach are the long time-constant of most metal liners and the fact that the increase of B leads to a decrease of β .

H. Characteristic Times for the HBT

In a discussion about the lifetime of the discharge in the HBT machine we list a number of characteristic times.

1. MHD Time. The growth time for the fastest growing MHD instabilities is approximately given by the radius of the plasma divided by the sound speed. For a 300-eV deuterium plasma of 4-cm radius this is only 0.2 μs . This is of course the most pessimistic case and when the plasma approaches stability the growth times become much longer. To find meaningful β and q limits for equilibrium and stability one would like to keep the plasma parameters reasonably constant during 50 to 100 μs .

2. Radiation-Cooling Time. After the rapid initial heating the plasma can cool down when energy lost by radiation exceeds the ohmic energy input. Impurity radiation can be quite bad in this respect, especially at the higher densities in the central region.

Calculations of the temperature as a function of time have been done with the program IMPRAD II (Sergio Ortolani and Hugh Karr). Results are shown in Fig. II.3. The initial ion and electron temperatures were assumed to be 500 eV

Fig. II.3.

Computer calculations of the decay of T_i and T_e for two plasma densities and for two impurity levels. The implosion at $t = 0$ heats the ions to 500 eV, the electrons to 100 eV. Equipartition brings T_i and T_e together at late times. A current of 500 A/cm² contributes some ohmic heating

and 100 eV, to account for the fact that most of the energy in the shock heating is deposited in the ions. The classical equipartition brings the electron temperature up so that the temperature at later times does not depend much on the initial ratio of T_i/T_e . A computer run where both T_e and T_i were set equal to 300 eV at $t = 0$ confirmed this.

In view of the recent work of H. Griem, the radiated power loss was increased by a factor of five over the values given by McWhirter.

Assuming that the curves of Fig. II.3 give an exponential decay of the average temperature, we calculate the $1/e$ cooling-down times as given in Table IV. To first-order, these times should be proportional to $1/n_{imp}$, since radiation losses are proportional to $n_e n_{imp}$ and the energy content of the plasma is proportional to n_e . As the table shows, the proportionality to $1/n_{imp}$ is not exact. Examination of the code runs indicates that this is caused by the different time-histories of the various ionized states of oxygen in this transiently heated plasma.

For the plasma densities considered, the ohmic heating is not very important. In the outside region we have about the same current density as in the center but fewer particles to heat. If the impurity concentration is uniform the impurity radiation is less. Both effects lead to higher temperatures in the outside plasma.

Table IV clearly implies that the impurity content of the plasma in HBT should be less than a few tenths of a percent. This of course requires a careful design of the vacuum system and a large number of precautions during installation and operation.

Experiments with theta pinches in a 99% Al_2O_3 tube in Scylla IV-1 have shown impurity levels as low as 0.065% oxygen (Ref. 12, numbers recalculated by

TABLE IV
RADIATION-COOLING TIMES

Density (m^{-3})	Oxygen Impurity	
	0.5%	0.1%
5×10^{21}	650 μs	4.2 ms
1×10^{22}	380 μs	2.6 ms

Franz Jahoda). Measurements in the ISAR II theta pinch in Garching¹³ showed impurity levels of 0.085% oxygen, 0.095% carbon, and 0.0025% nitrogen.

In terms of impurity control, the proposed HBT has a number of disadvantages compared to the linear theta pinches, such as the joints between the various segments and the limited conductance of the side ports used for pumping. Advantages of HBT over the linear theta pinches are: a 99.5% Al_2O_3 ceramic is now available, the wall contact at the ends of the coil, which brings in impurities in the linear machine is absent in the torus, and the surface/volume ratio is lower in HBT (12 m^{-1}) than in Scylla IV-1 (46 m^{-1}).

3. Characteristic Times for Diffusion and Heat Transport. At the expected plasma temperature and density, 300 eV and $5 \times 10^{21} \text{ m}^{-3}$, the mean free path of the particles is approximately 0.3 m. Since this is less than the length qr ($\approx 0.6 \text{ m}$) the Pfirsch-Schlüter description of diffusion and heat conduction is applicable.

To estimate the particle diffusion time τ_D we use the approximate expression

$$\tau_D \approx a^2 / 2 D p_0, \quad (11)$$

where a is the radius of the plasma, D is the diffusion coefficient, and p_0 is the peak plasma pressure. In the Pfirsch-Schlüter regime, D is given by

$$D = \eta(1 + q^2) / B^2, \quad (12)$$

so that the diffusion time can be written as

$$\tau_D \approx \frac{a^2 B^2}{2\eta(1 + q^2)p_0}. \quad (13)$$

If we take $a = 0.12 \text{ m}$, the half-width of the racetrack cross section, $B = 2 \text{ T}$, $q = 2$, and the plasma temperature and density as 300 eV and $5 \times 10^{21} \text{ m}^{-3}$, we find a τ_D of 93 ms.

More important is the heat conduction. The heat flux going through one m^2 is given by

$$Q = -\kappa \nabla T = -\frac{2n_i k^2 T}{m_i \omega_i^2 \tau_i} (1 + 1.6 q^2) \nabla T, \quad (14)$$

where κ is set equal to the classical ion heat conductivity, enhanced by the Pfirsch-Schluter factor, and where τ_i stands for the ion-ion collision time. To estimate the energy confinement time τ_E , we set the heat flux through a cylindrical surface during τ_E equal to the initial thermal energy of the plasma in the cylinder. We find then for a $Z = 1$ plasma

$$\tau_E = 3 a n k T / 2Q. \quad (15)$$

We approximate ∇T by T/a , use Eq. (14) for Q , and insert for τ_i the expression $11.4 \times 10^6 A^{\frac{1}{2}} T_i^{3/2} / n_i \ln \Lambda$. We then obtain

$$\tau_E = 9.5 \times 10^{18} \frac{a^2 B^2 T^{\frac{1}{2}}}{n A^{\frac{1}{2}} (1 + 1.6 q^2) \ln \Lambda}. \quad (16)$$

For $a = 0.12$ m, $B = 2$ T, $T = 3.48 \times 10^6$ K, $n = 5 \times 10^{21} \text{ m}^{-3}$, $A = 2$ (deuterium), and $q = 2$, we find $\tau_E = 1.5$ ms.

Clearly, the plasma loses its energy because of heat conduction before diffusion can have much influence. Although a τ_E of 1.5 ms seems reasonably long, we must realize that Eq. (16) gives only an estimate and that τ_E depends strongly on the temperature and density profiles. The low-density and fairly hot outside plasma is favorable in this respect. Anomalous electron heat loss can shorten τ_E .

4. Onset Disruptive Instability. Because the density in HBT will be above the "high-density limit" of tokamaks ($5 \times 10^{15} \text{ cm}^{-3}$, high-density limit at appropriate B_0/R is approximately $8 \times 10^{13} \text{ cm}^{-3}$) the development of a disruptive instability is possible. In fact, one of the objectives of the experiment is to study the processes limiting the lifetime of an MHD-stable plasma, and the disruptive instability could be the limiting process in this respect.

Estimates for characteristic times are, however, not easily made. The wide current channel and the larger ohmic heating input per particle in the low-density outside plasma are both factors that delay the shrinkage of the current channel.

The earlier mentioned loss processes, impurity radiation, and heat conduction are important for this evolution of the current profile. A third loss process is the influx of neutrals from the wall and the charge-exchange losses resulting from that; how large this influx is in pinches is not well known.

The wide current profile is essential for the equilibrium and stability of HBT. A shrinkage of the current channel can therefore lead to a loss of MHD equilibrium or stability. Whether this is completely analogous to the disruptive instability depends on the precise mechanism of the disruptive instability.

The general problem of the decay of the poloidal and toroidal currents is complicated and has only been studied for simplified models.¹⁴ The influence of the external circuits should be taken into account, especially if they are not power-crowbarred.⁹ The decay of the force-free currents close to the wall has little effect on equilibrium and stability.¹⁴

I. Diagnostics

The High-Beta Tokamak experiment, as proposed, is expected to incorporate three essential features that must be documented experimentally. These include: elongated shape, currents outside the main plasma, and high beta. We will discuss them in order.

1. Shape of Plasma and Toroidal Shift. An elongated shape of the discharge vessel and a flux-conserving shell are to be incorporated in this experiment. Careful measurements of the actual shape of the plasma that results from this arrangement will be required. With an expected central plasma density of $\sim 3 \times 10^{15} \text{ cm}^{-3}$ and an electron temperature of $> 100 \text{ eV}$, the self-luminosity of the plasma will be an extremely useful tool in determining the dimensions and position of the main plasma body. The assumption is that the measured radiation is the result of bremsstrahlung radiation resulting from electron encounters with ions. With an approximately constant electron temperature, the light is proportional to n_e^2 . Both streak cameras and luminosity measurements by photomultiplier arrays should be employed. The photomultiplier array scheme¹⁵ is readily calibrated, and hence should yield quantitative data. This type of measurement yields a relative plasma profile and must be normalized to a measurement across a major chord made with a He-Ne laser interferometer, which gives a direct measurement of the line integral of the electron density. This interferometric measurement should be in the plane of the torus to avoid any error that would be made by missing the plasma due to the toroidal equilibrium shift. A further check on the density of the plasma should be made by Thomson

scattering of ruby laser light. The method of mounting the return mirror for the interferometer is complicated by the insulation requirements of the main discharge windings (see Fig. IV.3). A dielectric interference mirror which is nonconducting will be obtained to reflect the laser beam allowing double-pass measurements in the plane of the torus.

The toroidal shift will also be measured by sine-cosine coils.¹⁶ Such measurements will be a natural extension of measurements made on existing experiments with two provisos. The distribution of coil winding density will not be sine and cosine as the experimental symmetry is no longer cylindrical. The signal resulting from such measurement will yield the position of the centroid of the toroidal current but this probably will not coincide with the equilibrium shift of the plasma. This is a result of a redistribution of the currents outside the main plasmas reducing the effect of a plasma displacement at the coils. This reduction in sensitivity of the sine-cosine coils will require that they be calibrated with the aid of streak cameras.

2. Force-Free Currents Outside the Main Plasma. The most obvious approach to measure these currents is to insert magnetic probes inside the plasma. This has two serious drawbacks: a) the probe may well disturb the plasma, and b) the plasma may destroy the probe. The high-beta tokamak, as described in this report, has a considerable advantage over a conventional tokamak in that there is no prolonged uncontrolled setup phase to the discharge. The pitch profile, whether correct or not, is programmed in rapidly from the very start. As a result, it should be possible to study at least the early stages of the discharge using magnetic probes inserted inside the discharge tube although the banks may have to be crowbarred to avoid destroying the probe. The probes should be removed whenever late stages of the discharge are being studied.

Electrical measurement of the voltage applied in the toroidal direction and the resulting current are expected to be of great aid in measuring the behavior of the external currents. For appropriate symmetry and from plots of $\int V_\phi dt / I_\phi$, plasma inductance and hence an effective radius of the current channel can be deduced. To facilitate such measurements appropriate voltage dividers and Rogowski coils will be installed.

Accurate Thomson scattering measurements of the electron temperature in the low-density ($n_e \sim 10^{14} \text{ cm}^{-3}$) plasma in the exterior region will have to be made. Electron temperature profiles will be used to predict the time evolution of the

external currents. This prediction can be correlated with the effective radius as measured with Rogowski coils. These measurements would be easier with a multichannel Thomson scattering apparatus to obtain radial temperature profiles on a single shot.

3. High Beta. Intimately connected with the subject of high beta is the question of stability. It will be the purpose of this work to vary the condition of the experiment and attempt to find upper limits on beta that are commensurate with plasma stability. To get a measure of beta, two approaches should be pursued.

As long as probes are inserted within the plasma, they should be used to estimate the plasma pressure by measuring the magnetic pressure due to B_θ and B_ϕ and assuming pressure balance. Such measurements have been used successfully in other experiments. The results are specified to within a constant. This constant is usually determined by requiring that the pressure is either zero at the discharge tube wall or is positive at all radii. In this experiment, with its emphasis on beta, these probe measurements could be supplemented by a direct measurement of the particle pressure at the wall with a piezo-electric pressure probe located at the wall.

Direct measurement of $nk (T_i + T_e)$ should be considered. Thomson scattering, as required for other measurements, will provide a measure of n_e and T_e . Collective scattering ($\alpha > 1$) could be implemented to attempt to determine T_i if time and manpower permit. This could either be accomplished by small-angle scattering of ruby light or larger angle scattering of infrared light. Plasma stability, or the lack of it, will be assessed by both streak photographs and sine-cosine coils.

4. Experimental Constraints. We now consider what constraints the diagnostics impose upon the design of the experiment. The luminosity measurements, be they streak camera or photomultiplier arrays, require a transparent section of the discharge tube and access through the conducting shell. It is expected that several locations will be available around the torus for such measurements both in and perpendicular to the plane of the torus. The transparent discharge vessel, or a transparent section, should be quartz if possible. The choice of quartz is two fold; it has proven its durability in many pinch experiments and it will pass the $3.4\text{-}\mu\text{m}$ radiation of a He-Ne laser, thus enabling a more sensitive interferometric measurement of the electron density. If we assume the plasma to have an elliptical cross section of 8×16 cm with an electron density

of $3 \times 10^{15} \text{ cm}^{-3}$, the single-pass fringe shift for an interferometer measuring in the plane of the torus will be 0.068 fringes for 0.633- μm light and 0.37 fringes for 3.39- μm He-Ne. In measuring the plasma density outside the main column interferometrically, an even longer wavelength laser would prove advantageous. With a path length of $\sim 40 \text{ cm}$ and $n_e \sim 10^{14} \text{ cm}^{-3}$, the fringe shift for 3.39 μm is 0.06 fringes, which is somewhat marginal considering that other shorter chords will be investigated, and a range of filling pressures employed that may well result in lower densities outside the main plasma column. To provide for this contingency, windows of other materials should be considered.

Thomson scattering measurements will require entrance and exit ports parallel to the axis of the torus if conventional scattering procedures are to be employed. If the rejection of stray light is to be done with a three-grating device,¹⁷ a nonconducting blue-glass absorber could be installed on the inside of the torus to provide some slight form of beam dump. The importance of measurements of the plasma outside the main plasma column is rather high in this experiment. This being the case, conventional techniques might well be preferable, as the scattered signal from the low-density plasma could be too small to permit accurate measurements.

Other measurements, which require a physical hole in the discharge vessel, include probe measurements and any spectroscopic measurements that deal with photons of wavelength less than 0.300 μm . Energy loss measurements due to radiation will likely be contemplated after the machine is operational. These holes might also double as pumping ports.

III. SCIENTIFIC SCOPE - THEORY

A. Introduction

A review is given of the current status of high-beta tokamak theory. The review describes predictions of equilibrium and stability theories as applied to the proposed experiment. In addition important problem areas which need further investigation are discussed.

The review begins with a section describing the basic theoretical concept of the high-beta tokamak and its relation to conventional tokamaks. A brief discussion is then presented on two methods which have been proposed to maximize the β in a tokamak; noncircularity and force-free currents. The problem of diffuse MHD equilibrium is discussed in the next section. The goal here is to

estimate maximum β values for realistic diffuse profiles with fixed values of the safety factor q . This section is followed by a discussion of MHD stability of high-beta tokamaks. The basic question addressed is that of determining the β limitations resulting from various MHD instabilities, with emphasis on external kink modes. Finally, in the last section, theoretical calculations of implosion heating in a high-beta tokamak are presented. The main purpose is to help determine appropriate experimental design parameters so that the properties of the plasma are in the regime of scientific interest.

B. The High-Beta Tokamak

In this section we describe the basic theoretical concept of the high-beta tokamak and discuss its relationship to conventional tokamaks. The basic physics and scaling laws are derived from the surface-current model because of its simplicity. However, it should be noted that the scaling laws derived here, hold for arbitrary profile configurations as well.

Consider an axisymmetric torus as illustrated in Fig. III.1. Here R is the major radius and the plasma cross section is assumed circular with radius a . In the plasma there is only a toroidal field

$$\underline{B} = \frac{B_i}{1 + (r/R)\cos\theta} \underline{e}_\phi,$$

and a constant pressure p ; B_i is the internal toroidal field at the mid-plane $\theta = \pi/2$.

In the vacuum there is a toroidal field as well as a poloidal field arising from toroidal currents flowing on the plasma surface,

$$\underline{B} = \frac{B_o}{1 + (r/R)\cos\theta} \underline{e}_\phi + \underline{B}_p,$$

where B_o is the external toroidal field at $\theta = \pi/2$ and $\underline{B}_p$ is the poloidal field.

Fig. III.1.
Geometry of the high-beta tokamak.

The exact equation for toroidal equilibrium is

$$\left[p + \frac{B^2}{2\mu_0} \right]_{r=a} = 0 ,$$

and can be written as

$$2\mu_0 p + \frac{B_i^2}{(1 + \epsilon \cos \theta)^2} = \frac{B_o^2}{(1 + \epsilon \cos \theta)^2} + B_p^2(\theta) ,$$

where $\epsilon = a/R$ is the inverse aspect ratio.

Quantities of physical interest are the total β and the poloidal beta β_p , given by

$$\beta = 2\mu_0 p / B_o^2 \quad \text{and} \quad \beta_p = 2\mu_0 p / \langle B_p^2(\theta) \rangle ,$$

and the safety factor q , which measures how far a magnetic line travels the long way around after having gone around the short way.

$$q = \frac{1}{2\pi} \int \frac{B_\phi d\ell}{\rho B_p} ,$$

where $d\ell$ is the poloidal arc length and $\rho = R [1 + \epsilon \cos \theta]$.

The pressure balance relation is exact. However, to most easily understand the differences between a conventional tokamak and a high-beta tokamak it is useful to consider the equilibrium in the context of an aspect ratio expansion. If we define the inverse aspect ratio as $\epsilon \equiv a/R$, then the physics of the conventional tokamak and the high-beta tokamak correspond to two different expansions in ϵ .

1. Conventional Tokamak Expansion. This is the expansion that applies to most existing tokamaks, ST, Ormak, T-3, etc. It probably applies to PLT, PDX, and Doublet III when only joule heating is used, but may break down when additional heating is applied, say from neutral beams.

The basic physics contained in the expansion is that the large toroidal field provides stability but does not support any plasma pressure; that is, the plasma pressure is supported entirely by the poloidal field, which in addition creates the necessary force to achieve toroidal force balance. In terms of our equilibrium relation this regime corresponds to the case where $B_j = B_0$ and pressure balance reduces to

$$2\mu_0 p = B_p^2 .$$

In the limit $\epsilon \ll 1$, it follows that the safety factor q is given by

$$q = \frac{aB_0}{RB_p} = \epsilon \frac{B_0}{B_p} \sim 1 .$$

Since stability requires that $q \geq 1$, this implies that B_p/B_0 must scale as

$$\frac{B_p}{B_0} \sim \epsilon .$$

The poloidal β , defined as $\beta_p \equiv 2\mu_0 p/B_p^2$ is given by

$$\beta_p = 1 .$$

in view of the pressure balance relationship consistent with the conventional tokamak expansion. Finally, the toroidal β , defined by $\beta_T = 2\mu_0 p/B_0^2$ can be written as

$$\beta_T = \frac{\epsilon^2 \beta_p}{q^2} \sim \epsilon^2 .$$

Summarizing, from the point of view of pinches, a conventional tokamak can be thought of as a toroidal Z pinch. Both plasma containment and toroidal equilibrium are provided by the toroidal current. There is a strong vacuum toroidal field solely for stability. Since only the poloidal field contains the

plasma pressure, the stability condition $q \gtrsim 1$ implies that the total β 's will be fairly low, $\beta \sim \epsilon^2$.

2. High-Beta Tokamak Expansion. The regime described by the high-beta tokamak expansion most likely corresponds to the operation of the existing and future large tokamaks with substantial external heating.

The basis for the high-beta tokamak expansion is the assumption that the jump in the toroidal field, $B_0 - B_1 \sim \epsilon$. This jump can support a plasma pressure $p \sim \epsilon$. Note that p is one order larger than in the conventional tokamak expansion. If we substitute the high-beta assumption into the pressure balance relation, we obtain to leading order in ϵ

$$2\mu_0 p + B_1^2 = B_0^2 ,$$

or

$$\beta_T = 2\mu_0 p / B_0^2 \sim \epsilon ,$$

which is clearly a θ -pinch pressure balance relation. Actually this condition should be written as $2\mu_0 p + B_1^2 - B_0^2 = \text{const}(\epsilon^2)$. The small constant left over is not very important in the leading order pressure balance but is vital in next order which determines the condition for toroidal force balance. To next order in pressure balance we have

$$B_p^2(\theta) = \langle B_p^2 \rangle + 2\beta_T \epsilon B_0^2 \cos\theta ,$$

where for obvious reasons we have defined the above mentioned constant as $\langle B_p^2 \rangle$.

Note that there is an equilibrium limit in order that $B_p^2(\theta)$ be real which is given by

$$\langle B_p^2 \rangle > 2\beta_T \epsilon B_0^2 .$$

By introducing the poloidal $\beta_p \equiv 2\mu_0 p / \langle B_p^2 \rangle$ this condition can be written in the more familiar form as

$$\beta_p < \frac{1}{2} \left(\frac{R}{a} \right) \sim \frac{1}{\epsilon} .$$

Violation of this criterion results in having the separatrix move onto the plasma surface.

In the limit of small ϵ , the safety factor q can be written as

$$q = \frac{aB_o}{R} \langle 1/B_p \rangle \sim 1 ,$$

which again must be of order unity for stability purposes. As in the conventional tokamak, the condition $q \geq 1$ implies that $B_p/B_o \sim \epsilon$.

Again, summarizing from the point of view of pinches, a high-beta tokamak can be thought of as a toroidal θ pinch; that is higher β 's are possible by using the toroidal field for both stability and plasma containment. The sole function of the poloidal field is to prevent the toroidal drift to the wall. In the high-beta tokamak the poloidal field is made as small as possible, consistent with equilibrium, to maximize the stability and thus the plasma tends to be shifted a finite amount from the geometric axis.

C. Maximizing Beta in a Tokamak

This section contains a qualitative discussion of the problem of optimizing the design of a tokamak so as to maximize the critical β with respect to MHD-stability. We make the implicit assumption that some form of heating is available which is capable of increasing the β above the experimentally observed joule heating limit $\beta_p \lesssim 1$. Such methods are RF heating, neutral-beam injection, turbulent heating and implosion heating. The very important question of heating itself is discussed in Sec. III.F. Assuming such heating exists, the question remains of how to maximize the critical β . Two methods for doing this have been proposed and each will be discussed separately.

1. Noncircularity. In recent years there has been a large effort aimed at maximizing the critical β in a tokamak by appropriately shaping the cross section. The culmination of this effort is the large Doublet III experiment, scheduled to come into operation several years from now.

The essence of the argument about the advantages of noncircularity is straightforward and is given as follows. In a noncircular tokamak, the safety factor q is given by

$$q \approx \frac{B_0}{2\pi R} \int \frac{dl}{B_p} ,$$

where the path of integration is in the poloidal direction and constrained to lie on a flux surface. Clearly, q scales as

$$q \approx \frac{aB_0}{RB_p} \left(\frac{c}{2\pi a} \right) ,$$

where a is the short radius of the noncircular plasma and c the circumference of the cross section. If we combine this with the definitions of β_T and β_p

$$\beta_T \approx \frac{2\mu_0 p}{B_0^2}$$

$$\beta_p \approx \frac{2\mu_0 p}{B_p^2} ,$$

we arrive at

$$\beta_T = \frac{\epsilon^2 \beta_p}{q^2} \left(\frac{c}{2\pi a} \right)^2 .$$

Consider now a configuration with fixed ϵ and β_p . If the critical q required for stability does not change very much as a function of elongation, then the critical β_T increases quadratically with elongation and this is the potential advantage of noncircular cross sections. However, theoretical calculations of gross kink instabilities indicate that the critical q does increase with elongation, thereby at least partially offsetting the gain in β_T (see Sec. III.E). The theory indicates that for elliptic cross sections there is an optimum elongation of about 2:1 which maximizes the critical β . This is why the proposed experiment is elongated, rather than circular, and

why the elongation has been chosen to be 2:1. Theoretically this increases the critical β by a factor of about 2 over the circular case.

4. Force-Free Currents. A second and equally important way to increase the critical β is to design the experiment so that there are substantial amounts of force-free currents flowing outside the main core of plasma. This fact was recognized experimentally by the Dutch group as early as 1966 in screw-pinch experiments and led to the construction of SPICA. In SPICA there are substantial force-free currents, and stable plasmas with peak $\beta \sim 20\%$ have been contained for about 100 μ s, the decay time of the plasma.

Force-free currents are relatively easy to create experimentally (in fact it may be difficult not to have them). A conducting shell is required to contain the flux surfaces and thus prevent the force-free currents from flowing out of the system. Separate programming capability on both toroidal and poloidal circuits is also required in order to control the shape of the pressure and current profile.

The force-free currents have several advantages with regard to plasma behavior. First, with force-free currents flowing outside the plasma core, toroidal equilibrium is much easier to achieve for a given value of q on axis. Secondly, stability calculations (discussed in Sec. III.E) indicate that by optimally choosing the proper amount of force-free current, the stability against global kink modes is greatly improved. At the optimum a gain in critical β by a factor of three over the vacuum case is obtained. This is associated with two possible effects. At the optimum condition, the pitch of the magnetic field lines is nearly constant. Thus, most perturbations will tend to increase the shear in the magnetic field, and this should be a stabilizing factor. The second feature is that with the force-free currents flowing on the outside, transverse plasma displacements will compress considerably more flux than in the vacuum case. Thus, force-free currents strongly enhance the effect of wall stabilization.

In the proposed experiment we intend to utilize a conducting shell and have separate field programming capabilities on the toroidal and poloidal circuits in order to fully exploit the advantages of force-free currents.

D. MHD Equilibrium

The problems of calculating MHD equilibria of high-beta tokamaks is a fairly straightforward numerical computation in most cases of interest. There are two main purposes for calculating diffuse MHD equilibria. First, by varying

the profiles it is possible to obtain estimates of the highest β values possible, and to make comparisons of the gain in β from conventional to high-beta operation, including the effects of force-free currents and noncircularity. Second, by using realistic pressure and current profiles it is possible to make rather good estimates of the coil inductance, total toroidal current, magnetic stresses, etc., which are of great value in the design of the experiment. A summary of the current status of our MHD equilibrium calculations is presented as well as a discussion of important problem areas which need further investigation.

1. Equilibrium Equation. The MHD equation describing axisymmetric toroidal equilibrium is the well-known "Grad-Shafranov" equation

$$\Delta^* \psi = - [\mu_0 \rho^2 p' + I I'] ,$$

where

$$\Delta^* = \rho \frac{\partial}{\partial \rho} \left(\frac{1}{\rho} \frac{\partial}{\partial \rho} \right) + \frac{\partial^2}{\partial z^2} ,$$

and p and I are two arbitrary functions of the flux ψ . The magnetic field is related to ψ as follows.

$$\underline{B} = \frac{1}{\rho} \left[I(\psi) \underline{e}_\phi + \underline{e}_\phi \times \nabla \psi \right] .$$

The equilibrium equation has been solved for a wide choice of the functions p and I . In the work summarized here the function $p(\psi)$ has been chosen to be

$$p(\psi) = p_0 [1 - \psi/\psi_b]^{\nu_p} ,$$

where ψ_b is the value of flux on the boundary, ν_p is a free parameter related to the width of the pressure profile, and p_0 is a free parameter proportional to the plasma β .

Since at high-beta operation we expect the pressure balance relation to approximate that of a θ pinch, we choose the function $I(\psi) = \rho B_\phi$ as follows.

$$F^2(\psi) = B_o^2 R^2 - 2\mu_o R^2 p_o [1 - \psi/\psi_b]^{\nu_p} + j_o B_o^2 R^2 [1 - \psi/\psi_b]^{\nu_j}.$$

The first term is the basic toroidal field. The second term represents the diamagnetic well in the toroidal field due to the plasma pressure. The last term is related to the toroidal current distribution. The parameter j_o sets the amplitude of this current while ν_j sets the width of the toroidal current profile. If $\nu_j < \nu_p$ the current profile is wider than the pressure profile indicating the existence of force-free currents outside the plasma core. If both ν_j and $\nu_p > 1$ then the total toroidal current and plasma pressure vanish on the boundary as can be seen by substituting into the flux equation

$$\Delta^* \psi = -\frac{j_o B_o R}{2\psi_b} \left[\nu_j \left(1 - \frac{\psi}{\psi_b}\right)^{\nu_j-1} + \frac{2\mu_o p_o}{j_o B_o^2} \frac{(\rho^2 - R^2)}{R^2} \nu_p \left(1 - \frac{\psi}{\psi_b}\right)^{\nu_p-1} \right].$$

The first set of results was computed for a circular boundary. The quantities of physical interest are defined as follows.

$$\text{Peak Toroidal } \beta \quad \beta_T = \frac{2\mu_o p(\psi_{\min})}{B_o^2}.$$

$$\text{Average Toroidal } \beta \quad \langle \beta_T \rangle = \frac{2\mu_o}{B_o^2} \frac{\int p(\psi) d\tau}{\int d\tau}.$$

$$\text{Average Plasma Radius} \quad \frac{\langle a^2 \rangle}{a^2} = \frac{\int p(\psi)/p(\psi_{\min}) d\tau}{\int d\tau} = \frac{\langle \beta_T \rangle}{\beta_T}.$$

$$q \text{ on axis} \quad q(0) = \frac{1}{2\pi} \int_{\psi_{\min}}^{\psi_b} \frac{B_\phi d\ell}{\rho B_p}.$$

Inverse of the Toroidal Current $q^* = \frac{aB_o}{R\langle B_p \rangle}$,

where $\langle B_p \rangle = \frac{\mu_o I}{2\pi a} = \frac{1}{2\pi} \int_0^{2\pi} B_p(a, \theta) d\theta$.

Poloidal β $\beta_p = \frac{2\mu_o}{\langle B_p \rangle^2} \frac{\int p(\psi) d\tau}{\int d\tau} = \frac{q^{*2}}{\epsilon^2} \langle \beta_T \rangle$.

2. Constraints. In order to compute the value of β_T uniquely, appropriate constraints must be applied. In an approximate sense we want to compute the maximum β_T , $\langle \beta_T \rangle$ for a "given shape" pressure profile and "fixed q."

The first question that arises is which "q" should be held fixed. In our calculations we hold $q(0)$ fixed at unity. This is a fairly strict requirement which in low-beta theory corresponds to the stability condition for both internal kink modes and localized interchange modes.

We point out that very little is known about the internal kink mode in the high-beta regime, for either circular or noncircular configurations. This is a very important problem area which needs further study, particularly in view of the stringent stability condition that it imposes.

In the computations the pressure profile is "held fixed" by setting $v_p = \text{const}$. The question remains as to what value to hold v_p fixed at. Broader profiles obviously give rise to higher values of $\langle \beta_T \rangle$. However, if the plasma is surrounded by a vacuum region, or a resistive plasma layer, and the current profiles do not vanish sufficiently rapidly at the plasma edge, then high-m number external kink modes are unstable. For instance consider the low- β straight cylinder limit where the toroidal current is given by

$$j = j_A [1 - r^2/a^2]^v .$$

The stability diagrams for long wavelength, global kink modes are shown in Fig. III.2 for various values of v , assuming the plasma is surrounded by a vacuum region extending to infinity. We see that for small v (broad profiles) there are instabilities for high values of q^* . As v increases (narrower

Fig. III.2.

Stability diagrams for long wavelength kink modes in the low-beta cylindrical limit for various values of ν .

profiles) the windows shrink, and at $\nu \geq 3$, all windows above $q^* = 1$ completely vanish. The value $\nu = 3$ approximately corresponds to $\nu_p \sim 3-4$.

Thus the dilemma in choosing an appropriate value of ν_p is as follows. If we maximize $\langle \beta_T \rangle$ by using broad profiles and setting $q(0) = 1$, we may be fooling ourselves since stabilization of higher m number, external kink modes may require a very high value of q^* . This value of q^* could be more restrictive than $q(0) = 1$. On the other hand, low-beta theory indicates that even though higher m modes are unstable, their nonlinear evolution is not catastrophic and in fact to a large extent can be disregarded. This is a question that needs further study in the high-beta regime.

Since there is no clear-cut way to choose ν_p , results are presented for several different values. The range of $\nu_p \sim 2-3$ is of particular interest since it corresponds approximately to the expected plasma width resulting from implosion heating and adiabatic compression in the proposed experiment.

3. Conventional Tokamak. In order to assess the advantages of the high-beta tokamak, we first present results for the conventional tokamak to serve as a reference point. We define a conventional tokamak as one in which the toroidal field is used only for stability, and not for plasma containment. This is equivalent to setting the function $I(\psi)$ equal to a constant implying that $B_\phi \sim 1/\rho$; that is B_ϕ is a vacuum field. For our profiles this is accomplished by setting $\nu_j = \nu_p$ and $j_o = 2\nu_o p_o / B_o^2$.

Equilibrium solutions were computed numerically for the conventional tokamak case for various values of ν_p . The results, summarized in Table V, correspond to $q(0) = 1$ and an inverse aspect ratio $\epsilon = 0.4$.

We note the following. The value of β_p depends only weakly on ν_p , the profile parameter. For the cases studied, $\beta_p \approx 1$ in the conventional tokamak regime.

TABLE V

CONVENTIONAL TOKAMAK PARAMETERS FOR $q(0) = 1$, $\varepsilon = 0.4$ AND VARIOUS v_p

v_p	β_p	β_T	$\langle\beta_T\rangle$	$\langle a^2 \rangle^{1/2}/a$	q^*	δ
2	1.04	0.0666	0.0174	0.511	3.09	0.246
3	1.06	0.0282	0.00401	0.377	6.51	0.261
4	1.08	0.0156	0.00107	0.281	12.71	0.290

The peak β_T and average $\langle\beta_T\rangle$ are strongly dependent on v_p , as might be expected. Even for the broadest profile, $v_p = 2$, β_T is less than 7% and $\langle\beta_T\rangle$ is less than 2%. For the case $v_p = 3$, β_T is only 2.8% and $\langle\beta_T\rangle$ is 0.4%. When $v_p = 4$, the β values are lower yet.

The average plasma radius shrinks rapidly with increasing v_p . When $v_p = 2$ we have a broad profile filling up about one quarter of the volume. When $v_p = 4$, only about 8% of the volume is occupied by plasma.

The value of q^* is closely associated with the condition for stability against external, global kink modes. Even for the broad profile $v_p = 2$, we see that $q^* > 3$, which we hope is sufficient for stability. If so $q(0) = 1$ is the more stringent condition.

The shift of the magnetic axis, normalized to the plasma radius a , is defined as δ . We see that the shifts are substantial for $\varepsilon = 0.4$ but not too sensitive to the value of v_p .

Summarizing, we see that a conventional tokamak, even with a tight aspect ratio, $\varepsilon = 0.4$, can support only relatively low β values when $q(0) = 1$ and the current vanishes on the boundary.

4. High-Beta Tokamak. In this section we attempt to maximize the value of β_T and $\langle\beta_T\rangle$ by optimizing the parameters to correspond to high-beta tokamak operation. This is done as follows. First, we consider the case where no appreciable force-free currents flow outside the plasma core; that is we set $v_j = v_p$ so the widths of the current and pressure profiles are about the same. We next vary the parameter $2\mu_0 p_0 / j_0 B_0^2$ until $\langle\beta_T\rangle$ reaches a maximum for $q(0) = 1$. This parameter turns out to be directly proportional to the shift δ .

One interesting feature of the equilibria is that for $q(0) = 1$, the maximum value of $\langle\beta_T\rangle$ does not occur for the maximum shift δ or the maximum value of β_p (i.e., the equilibrium limit). This can be seen in Fig. III.3. Here we plot $\langle\beta_T\rangle$, β_T , β_p , and $1/q^*$ vs δ for $v_p = 2$, $q(0) = 1$, and $\varepsilon = 0.4$. We see that to

Fig. III.3
Curves of $\langle \beta_T \rangle$, β_T , β_p , and $1/q^*$ vs the shift δ for $\nu_p = \nu_j = 2$, $q(0) = 1$, and $\epsilon = 0.4$.

maintain $q(0) = 1$, the total toroidal current $I \sim 1/q^*$ actually decreases with increasing shift. This is associated with two features. First as δ increases the center of the plasma shifts outward to a region of weaker toroidal field. Second, for larger shifts the toroidal current peak and magnetic axis move closer together. Both of these effects tend to decrease $q(0)$. Thus to maintain constant $q(0)$ the current must be decreased. At relatively small shifts when δ increases, β_p increases because both p and $q^* \sim 1/I$ increase. In this region $\langle \beta_T \rangle$ also increases because of the increase in p . For larger shifts, the current has decreased sufficiently so that it can no longer support the same total pressure in toroidal equilibrium. Above this point, p and hence $\langle \beta_T \rangle$ decrease even though δ is still increasing. However β_p continues to increase, the decrease in p more than being offset by the continuing decrease in q^* required to keep $q(0) = 1$. Note that even though $\langle \beta_T \rangle$ has a maximum, β_T does not. At large δ as I decreases the total plasma pressure decreases, but the peak plasma pressure actually increases slightly.

TABLE VI

COMPARISON OF β BETWEEN CONVENTIONAL AND HIGH-BETA TOKAMAK FOR $\epsilon = 0.4$, $q(0) = 1$

	$v_p = 2$		$v_p = 3$	
	Conventional Tokamak	High-Beta Tokamak	Conventional Tokamak	High-Beta Tokamak
β_p	1.04	1.79	1.06	2.00
β_T	0.0666	0.0763	0.0282	0.0352
$\langle\beta_T\rangle$	0.0174	0.0188	0.00401	0.00457
δ	0.246	0.319	0.261	0.362

Given in Table VI are the values of β_T , $\langle\beta_T\rangle$, and β_p for values of δ corresponding to maximum $\langle\beta_T\rangle$. Also given are the conventional tokamak parameters for comparison.

The most striking feature of the results is that despite substantial increases in β_p and δ , the gain in β_T and $\langle\beta_T\rangle$ in going from conventional to high-beta operation is quite small, approximately a factor 1.1. This is undoubtedly associated with the tight aspect ratio $\epsilon = 0.4$. Much larger relative gains are obtained by using smaller ϵ . However, the absolute values of β_T are uninterestingly low. In fact even when $\epsilon = 0.4$, the values of β_T and $\langle\beta_T\rangle$ are quite low indicating the need for new methods of increasing β_T . Two such methods are force-free currents and noncircularity.

5. Effect of Force-Free Currents. In this section we examine the effects of force-free currents on the critical β_T from the equilibrium point of view. This is done by computing equilibria for a fixed pressure profile (i.e., $v_p = \text{const}$) while broadening the width of the current profile (i.e., v_j decreasing).

Intuitively, we expect force-free currents flowing outside the plasma to help raise β_T ; that is, the addition of external force-free currents does not dramatically change the value of $q(0)$ which is determined by the internal current. However, the external currents should be able to support a higher total plasma pressure by providing more flux for the plasma to compress against the shell.

Results were obtained for a variety of v_j for the two cases $v_p = 2$ and 3. In the calculations v_j can get as small as unity. This corresponds to having a finite toroidal current flowing at the boundary. Such a current distribution may be a reasonable approximation to the proposed experiment. Whether or not such currents are desirable under reactor-like conditions is unclear at this point.

Fig. III.4.

Curves of $\langle \beta_T \rangle$, β_T , β_p , and $1/q^*$ vs the current profile factor ν_j for $q(0) = 1$, $\epsilon = 0.4$, and $\nu_p = 2$ and 3.

Illustrated in Fig. III.4 are plots of $\langle \beta_T \rangle$, β_T , $1/q^*$, and β_p vs the current profile factor ν_j for the cases $\epsilon = 0.4$, $\nu_p = 2$ and 3. Each point on the curve corresponds to the maximum $\langle \beta_T \rangle$ over a range of δ (i.e., the maximum point in Fig. III.3).

We see that as the current profile is broadened (ν_j decreasing), the average β , $\langle \beta_T \rangle$ increases monotonically. The gains in $\langle \beta_T \rangle$ are quite substantial, particularly when ν_j is allowed to approach unity. For the case $\nu_p = 3$, the gain in $\langle \beta_T \rangle$ is about a factor $5\frac{1}{2}$ from $\langle \beta_T \rangle = 0.0046$ at $\nu_j = 3$ to $\langle \beta_T \rangle = 0.0264$ at $\nu_j = 1$. For $\nu_p = 2$ the gain is also substantial, about a factor $2\frac{1}{2}$ from $\langle \beta_T \rangle = 0.019$ at $\nu_j = 2$ to $\langle \beta_T \rangle = 0.046$ at $\nu_j = 1$. Despite the significant gains in $\langle \beta_T \rangle$ the absolute values are still relatively low. Only the most optimistic case $\nu_p = 2$, $\nu_j = 1$, which corresponds to a broad pressure profile and substantial currents at the wall, gives rise to $\langle \beta_T \rangle$'s approaching 5%.

The same qualitative behavior is found for the peak β , β_T . The gains in β_T are somewhat lower than for $\langle\beta_T\rangle$. For $v_p = 3$ the gain in β_T is about a factor 3, while for $v_p = 2$ the gain is a factor of about 2. As $v_j \rightarrow 1$, the peak β 's lie between 10-15%, which are starting to become interesting from the point of view of implosion heating.

Finally, we note that β_p decreases and $I \sim 1/q^*$ increases as the profile broadens. In fact when v_j is sufficiently low, the value of β_p at maximum $\langle\beta_T\rangle$ is actually less than unity. This would appear to contradict the well-known conventional wisdom that $\langle\beta_T\rangle$ reaches a maximum when $\beta_p \sim R/a$. The explanation is straightforward. As more force-free currents are added outside the plasma core at fixed $q(0) = 1$, the total toroidal current must be increasing. This gives rise to the decrease in β_p . It also points out that the use of β_p as a figure of merit can be very misleading when significant force-free currents flow outside the plasma.

Summarizing, we see that the addition of force-free currents flowing outside the plasma core substantially increase the values of $\langle\beta_T\rangle$ for fixed $q(0) = 1$. At the most optimistic conditions the value of $\langle\beta_T\rangle$ approaches 5% which is an interesting value from reactor considerations. But there is little margin of safety.

6. Equilibrium in a Racetrack. The equilibrium and the localized MHD stability were studied with a computer program¹⁸ developed to not only calculate theoretical parameters of interest but also to calculate parameters which aid in the engineering design of the device. The code solves the Grad-Shafranov equation (Sec. III.D.1) for the racetrack cross section shown in Fig. II.2 with the following functions used for the source terms:

$$p = p_0 \left[1 - \frac{(\psi_{\max} - \psi)}{W_p} \right]^{v_p},$$

where ψ is the poloidal magnetic flux times $(2\pi)^{-1}$, p is the plasma pressure, p_0 is the peak plasma pressure, $\psi_{\max}$ is ψ at magnetic axis (ψ at boundary = 0), W_p is plasma width in units of ψ , and v_p is the plasma pressure profile shaping parameter

$$\mu_0 I = RB_0 - CRB_0 \left[1 - \frac{(\psi_{\max} - \psi)}{W_I} \right]^{\nu} P + \alpha \psi \frac{P_0 - P}{P_0} ,$$

where I is the poloidal current times $(2\pi)^{-1}$, R is the major radius, B_0 is the vacuum field at the geometric center, C is the parameter which controls the diamagnetic well in the toroidal magnetic field ($0 \leq C \leq 1$ allows the field dip to be scaled by the vacuum field), W_I controls the width of the diamagnetic well, and α is the parameter scaling the force-free currents.

The first term in this expression is the vacuum-field term; the second term provides the depression in the toroidal magnetic field which allows the equilibrium to have a high beta and still remain below the Kruskal-Shafranov limit; and the third gives the force-free currents which dominate at the edge of the plasma and in the pressureless plasma, and diminish to zero at the magnetic axis. Note: The pressure and the last two terms of the expression for I are not allowed to go negative.

The input parameters were adjusted so that the equilibrium satisfied the following criteria:

(1) High beta -- there are a multitude of ways of defining beta values. For our purposes here we define the toroidal β_T and the energy beta β_E ; $\beta_T = 1 - (B_T/B_V)^2$ evaluated at the magnetic axis (B_V is the undepressed toroidal field). The energy beta = total plasma energy/total field energy.

(2) The Kruskal-Shafranov limit should not be exceeded on any flux surface, i.e., $q > 1$ everywhere. Also SPICA experimental results dictate that q should be nearly constant across the device.

(3) Localized MHD calculations should show stability on all flux surfaces, i.e., Mercier $Q > 0$ everywhere.¹⁹

(4) Small toroidal shift and a sufficiently large compression to keep the bulk of the plasma away from the wall.

The input parameters used to achieve a sample equilibrium satisfying these constraints and the results are shown in Table VII.

TABLE VII
PARAMETERS FOR EQUILIBRIUM CALCULATION

INPUT						
p_o (MPa)	ν_p	$\frac{W_p}{p}$	B_o (T)	C	$\frac{W_I}{I}$	α (m ⁻¹)
0.55	2	$0.3 \psi_{\max}$	2.0	0.172	$0.3 \psi_{\max}$	6.0

OUTPUT							
β_T (%)	β_E (%)	Min q	Min Q Mercier	I_ϕ (kA)	Shift (cm)	L_i (nH)	L_f (nH)
31.4	10.4	1.15	5.4	934	3.56	60.2	126

$L_f = 2\pi\psi_{\max}/I_\phi$ is a measure of the inductance seen by the ϕ -capacitor bank. L_i = poloidal field energy/ $\frac{1}{2} I_\phi^2$, measures the plasma inductance in terms of the poloidal energy; this inductance is smaller than L_f by about a factor two.⁹ I_ϕ -- this toroidal plasma current can be smaller in the experiment if the force-free currents do not flow adjacent to the wall. The toroidal current density does not change sign; its minimum value is still 30% of the average j_ϕ . The equilibrium configuration of Table VII is illustrated in Fig. III.5.

Fig. III.5

The magnetic surfaces in the top half of the racetrack (left) and graphs of the variation of B_ϕ , B_p , p , and q on the midplane of the torus (right). The safety factor q is calculated on each magnetic surface and is therefore only plotted on one side of the magnetic axis. The main axis of the torus is at the left. The parameters for this equilibrium are shown in Table VII.

E. MHD Stability

A survey is presented of the MHD-stability theory of high-beta tokamaks. By and large, most of the theory which is applicable to the high-beta regime is concerned with the stability of gross external kink modes. These modes are very dangerous experimentally, almost always leading to a loss of plasma confinement. Consequently, an accurate knowledge of the threshold condition is important for the design of any experiment.

A significant amount of effort has been applied to studies of the gross kink mode using the surface-current model. Some work has been done with diffuse profiles. As of now the effects of elongation and force-free currents in the high-beta regime have only been studied with the surface-current model. However, the important calculation which simultaneously includes both effects has not yet been done, although the problem is actively being investigated.

The results of the analysis indicate that in an approximate sense, the stability of gross kink modes is determined by the value of $q^* \sim 1/I$, rather than $q(a)$, the safety factor evaluated at the plasma edge using Mercier's definition. In low-beta systems $q^* \approx q(a)$, so it is difficult to distinguish which is fundamental. At high-beta, the difference between q^* and $q(a)$ becomes more pronounced and it appears that q^* , inversely proportional to the toroidal current, is more important.

We have seen in the discussion of equilibrium that the condition $q(0) = 1$ imposes a fairly strict condition on the upper limit of $\langle \beta_T \rangle$. The condition $q(0) = 1$ is usually associated with internal kink modes in low-beta theory. Even though the external kink mode is probably more dangerous than the slower growing internal mode, the internal mode may in an absolute sense be sufficiently dangerous that experiments must be designed to avoid them. Considerable effort has gone into the study of internal kink modes at low beta, but almost no effort as yet has been applied to study the high-beta regime. This is an important problem, for both circular and noncircular cases, which needs further investigation, since the internal kink mode may give rise to the most stringent stability criterion.

Before beginning the survey, we comment briefly on the requirement of satisfying the Mercier stability criterion. Profiles which violate the Mercier criterion are unstable to highly localized interchange modes having low growth rates. The effects of such modes on confinement and transport properties of a plasma is not well established. Also, relatively small changes in the shape

and location of the flux surfaces near the magnetic axis can have a large effect on the Mercier criterion. In many high-beta tokamaks, the large shift of the interior flux surfaces helps in satisfying the criterion. This is another area where more research is needed, particularly in the case where significant force-free currents flow giving rise to a relatively low-shear system.

1. Circular Case. Early studies of kink instabilities in high-beta tokamaks deal with circular cross-section plasmas described by the surface-current model. In this model the plasma pressure is assumed constant out to a radius $r = a_s$. All currents flow only in the surface and the plasma is surrounded by a vacuum region which extends out to a perfectly conducting shell.

The surface-current model only contains information about external kink modes, which are relatively insensitive to the profile. Any information associated with the violation of Mercier's criterion or internal kink modes is excluded from the surface-current model.

The end result from such calculations is the prediction of a critical β and q required for stability. Because the profiles are quite simplified with respect to realistic diffuse experimental profiles, it is worthwhile to briefly discuss the method by which the results are interpreted. This clearly involves some judgment and intuition, and is thus open to debate. What is required is the relationship between the physical parameters of the sharp boundary and diffuse systems. We have chosen the following relationships.

We assume the radius a_s of the sharp boundary model is equal to the average radius of the equivalent diffuse system.

$$a_s^2 = \langle a^2 \rangle .$$

Recall that in a diffuse system, $\langle a^2 \rangle$ is defined as

$$\frac{\langle a^2 \rangle}{a^2} = \frac{\langle \beta_T \rangle}{\beta_T} .$$

A vacuum (or resistive plasma) extends from a_s to "a" the edge of the system. Here "a" has the same significance in sharp boundary and diffuse models. The constant-pressure profile is assumed to correspond to a diffuse parabolic profile extending to $r = a_0$ with peak pressure p_0 . The constants p_0 and a_0 are determined by requiring that both systems contain the same plasma energy and

have the same average radius. This gives $p_o = p_s$ (sharp boundary pressure) and $a_o = \sqrt{2} a_s$. If we give sharp boundary parameters a_s/a , $\epsilon_s = a_s/R$, and $\beta_{TS} = 2\mu_o p_s / B_o^2$, the equivalent diffuse profile parameters are given by

$$\epsilon = \frac{a}{a_s} \epsilon_s ,$$

$$\beta_T = \beta_{TS} , \text{ and}$$

$$\langle \beta_T \rangle = \left(\frac{a_s}{a} \right)^2 \beta_{TS} .$$

In the sharp boundary model, the concept of $q(0)$ has no significance since $B_p = 0$ everywhere inside the plasma. The safety factors usually considered are $q_s(a)$, the Mercier value just outside the plasma surface

$$q_s(a) = \frac{a_s B_o}{2\pi R} \int_0^{2\pi} \frac{d\theta}{B_p(a, \theta)} ,$$

and q_s^* , the inverse of the total toroidal plasma current

$$q_s^* = \frac{2\pi a_s^2 B_o}{\mu_o R I} ,$$

$$\mu_o I = a \int_0^{2\pi} B_p(a, \theta) d\theta .$$

Note that q_s^* is similar but not exactly equivalent to q^* in a diffuse system. The point to keep in mind is the following. In a diffuse system q^* measures the total toroidal current out to a radius a . However, q_s^* is related to the total toroidal current flowing in the plasma core $r = a_s$. In the surface current model it is easy to distinguish the core region, whereas it is somewhat vague in a diffuse system. This is why the values of q_s^* calculated here are higher than typical values of q^* in diffuse systems.

The results from the sharp boundary theory are shown in Fig. III.6. Plotted here is the curve of critical β_{TS}/ϵ_s vs $q_s(a)$. We see that the highest values of β_{TS} occur for values of $q_s(a) \rightarrow \infty$, the equilibrium limit where the separatrix moves onto the plasma surface. The highest value of β_{TS} possible is

$$\beta_{TS} = .21(a_s/R) .$$

A more useful way to illustrate the results is to plot the critical β_{TS}/ϵ_s vs q_s^* . In the high-beta ordering, even though $q_s(a) \rightarrow \infty$, the toroidal current ($\sim 1/q_s^*$) remains finite. It can be shown analytically that the limit $q_s(a) \rightarrow \infty$ is equivalent to the equilibrium condition $(\beta_{TS}/\epsilon_s)q_s^{*2} = \pi^2/16$. The stability curve is shown in Fig. III.7. The interesting feature here is to note that there is an optimum toroidal current for achieving the maximum critical β_{TS} . At higher currents, the stability condition becomes more severe. At lower

Fig. III.6 .

Stability curve of β_{TS}/ϵ_s vs $q_s(a)$.

Fig. III.7.

Stability curve of β_{TS}/ϵ_s vs q_s^* .

currents, the equilibrium condition determining the maximum pressure which can be contained becomes more severe. At the optimum, the value of q_s^* is given by

$$q_s^* = 1.71 \quad .$$

Using these results and our relationships between sharp boundary and diffuse parameters, we list in Table VIII the values of β_T and $\langle\beta_T\rangle$ for the cases $a/a_s = 2, 3$ and $a/R = 0.4$ at the optimum q_s^* . These results should be compared with Table VI in the high-beta tokamak equilibrium section. We see that the values of β_T , $\langle\beta_T\rangle$ are not too different. The sharp boundary model in all cases predicts more pessimistic values of $\langle\beta_T\rangle$, β_T than diffuse equilibrium studies with $a(0) = 1$. This is a consequence of the small coefficient "0.21" which appears in the stability criterion and may be indicative of the fact that stabilization of gross kink modes at high beta may be more difficult than originally anticipated.

TABLE VIII
OPTIMUM VALUES OF $\langle\beta_T\rangle$ AND β_T FOR GROSS KINK MODES

	$a/a_s = 2$	$a/a_s = 3$
$\langle\beta_T\rangle$	0.0105	0.0031
β_T	0.042	0.028

Some work has been done on the stability of high-beta tokamaks with diffuse profiles. The problem with these calculations has been alluded to earlier. A specific class of diffuse profiles [i.e., $p \sim \exp(-\psi)$] was studied. Almost certainly this was not the optimum set. Peaked pressure profiles could be made stable to external kink modes. However, this coupled to the condition $q(0) = 1$ gives rise to very low $\langle\beta_T\rangle$'s. Broader profiles have much higher critical β 's with $q(0) = 1$ but are susceptible to higher m-number instabilities. Ignoring these instabilities, which is a serious assumption, and treating very broad profiles $(\langle a^2 \rangle / a^2)^{1/2} \approx 0.6$, it was found that the critical $\langle\beta_T\rangle$ is approximately given by $\langle\beta_T\rangle \approx 0.1(a/R)$ or about 4% for $\epsilon = 0.4$. This is likely optimistic because of the very broad pressure profile.

Again we point out that even in the high-beta tokamak regime, the average $\langle\beta_T\rangle$ is quite small in absolute value indicating the need for such modifications as noncircularity and force-free currents.

2. Effect of Noncircularity. The sharp-boundary stability calculations described in the previous sections have been extended to include the ellipse as well as a variety of other noncircular cross sections.

In the noncircular case most of the definitions of the physical parameters go over in a straightforward manner from the circular case. The only quantity which needs to be more carefully defined is q_s^* . This is done in such a way that $q_s^* \rightarrow q_s(a)$ in the low β , $\epsilon \rightarrow 0$ limit.

$$q_s^* = \frac{c^2 B_0}{2\pi R \mu_0 I}$$

$$\mu_0 I = \oint B_p dl ,$$

where c is the circumference of the cross section.

The stability results for the sharp-boundary elliptical cross section indicate that the marginal stability curves, β_{TS}/ϵ_s vs $q_s(a)$ and β_{TS}/ϵ_s vs q_s^* have the same qualitative form as in the circular case for any given value of b/a (i.e., height to width ratio). In particular the maximum value of β_{TS} occurs at the equilibrium limit $q_s(a) \rightarrow \infty$.

Fig. III.8.

Critical β_{TS}/ϵ_s vs elongation, in the limit $q_s(a) \rightarrow \infty$.

One interesting feature of the results is that there is an optimum elongation for maximizing β_{TS} . This is shown in Fig. III.8. As the plasma is elongated in the horizontal direction the stability gets monotonically worse. In the vertical direction stability first improves and then becomes worse for very large elongations. This behavior can be explained as follows. As the cross section is elongated in either direction, the bad curvature associated with the poloidal field gets concentrated near the tips, producing an overall decrease in

stability. The toroidal field curvature is favorable on the inside and unfavorable on the outside. Vertical elongation is more stable than horizontal elongation because the inner and outer surfaces are more closely coupled, thereby prohibiting the perturbation from becoming localized in the bad curvature region. Eventually however, the bad curvature of the poloidal field dominates. This gives rise to an optimum elongation.

The stability results indicate that the optimum elongation is $b/a = 2.2$. At this value the critical q_s^* is

$$q_s^* = 2.2 \quad ,$$

which is slightly higher than for the circular case. The critical value of β_{TS} has increased substantially from $\beta_{TS}/\epsilon_s = 0.21$ to

$$\beta_{TS} = 0.37(a/R) \quad ,$$

a gain of almost a factor of two. Using the relationships between sharp and diffuse profile systems, the values of β_T and $\langle\beta_T\rangle$ are summarized in Table IX for the cases $\epsilon = 0.4$ and $a/a_s = 2, 3$.

Despite the gains, the absolute values are still low, showing the need for further improvement.

3. Effects of Force-Free Currents. The effect of force-free currents on gross kink instabilities has been investigated in two cases. First for a straight cylindrical pinch with arbitrary β and second for a toroidal pinch in the high-beta tokamak regime. Both cases considered circular cross-section plasmas.

TABLE IX
OPTIMUM VALUES OF $\langle\beta_T\rangle$ AND β_T FOR THE ELLIPTICAL CASE

	$a/a_s = 2$	$a/a_s = 3$
$\langle\beta_T\rangle$	0.0185	0.0055
β_T	0.074	0.049

In both cases the model used was an extension of the sharp-boundary surface-current model previously discussed. The plasma interior and plasma surface remain unchanged. The outer vacuum region is replaced by a region carrying force-free currents satisfying

$$\nabla \times \underline{B} = \alpha \underline{B} \quad ,$$

with α a constant. The assumption of a constant α is a theoretical idealization which greatly simplifies the analysis. It is probably a reasonable assumption with respect to experiment except very near the conducting shell where the currents decrease. In the theory, the outer region is treated in terms of the perturbed magnetic field $\underline{B}_1$ rather than the plasma displacement $\underline{\xi}$. Thus the stability thresholds include resistive as well as ideal MHD instabilities.

The stability behavior is again described in terms of β_{TS} , q_s^* , and $q_s(a)$. (Note that in the surface-current model q_s^* does not include the current flowing in the force-free region. Consequently, when such currents flow, q_s^* is significantly greater than q^* as calculated from a corresponding diffuse profile.)

The effects of the force-free currents are described by introducing a new parameter Γ_s given by

$$\Gamma_s = \alpha R q_s^* \quad .$$

The object of the stability calculations is to maximize β_{TS} by finding the optimum combination of q_s^* and Γ_s .

In the original work by the Dutch on the straight cylindrical pinch, the optimization was performed and applied to the SPICA experiment. From analytically derived formulae it was found that the maximum β was about 20% and was achieved when $\Gamma_s \approx 2$, $q_s^* \approx 1$.

Recent work has been aimed at exploring the toroidal high-beta effects of force-free currents. We have seen in previous sections that toroidal effects are destabilizing, with respect to external kink modes, leading to much stronger limitations on β than in the straight case.

A typical set of stability diagrams of β_{TS} vs q_s^* is illustrated in Fig. III.9 for four values of Γ_s . Each stability region is bounded from above by the equilibrium limit $\beta_{TS} q_s^{*2} \leq \pi^2/16$. The shaded regions correspond to stability. Note that β_{TS} and q_s^* are scaled with n , the toroidal mode number in

the appropriate way. In the previous sections, n was set equal to unity as it could easily be shown that this was the worst case.

We see that for $\Gamma_s \neq 0$, there are two regions of stability. The right-hand region is similar qualitatively to the vacuum case and in fact is identical when $\Gamma_s = 0$. The other region of stability occurs at low values of nq_s^* . In actuality this does not really correspond to stability. If a low value of q_s^* is chosen experimentally, modes with sufficiently high n can be found which lie in the unstable region. For values of q_s^* not too low, narrow windows of stability can be found in some cases. However we shall disregard these optimistic regions as being too sensitive to the model. Thus, our prescription for determining stability is to consider only the right-hand region, and in this case $n = 1$ is clearly the worst mode.

The maximum β_{TS} and q_s^* for a given Γ_s are plotted vs Γ_s in Fig. III.10. The region nearest the optimum β_{TS} is drawn in as a dashed line. The numerical computations actually show that in this region, stability is achieved for β_{TS}/ϵ_s arbitrarily large, a very optimistic result. Again, because of the narrowness of this high-stability region, it may well be spurious or pathological when viewed experimentally or theoretically with diffuse profiles. To obtain the maximum β_{TS} , we simply have connected the neighboring points with a smooth dashed line. Under these conditions we find that the optimum parameters are given by

$$q_s^* \approx 1 ,$$

$$\Gamma_s \approx 2 , \text{ and}$$

$$\beta_{TS} \approx .62 \epsilon_s .$$

As before, we convert these results into values of β_T and $\langle \beta_T \rangle$ and obtain the information given in Table X.

TABLE X.
OPTIMUM VALUES OF $\langle \beta_T \rangle$ AND β_T FOR THE FORCE-FREE
CURRENT CASE

	$a/a_s = 2$	$a/a_s = 3$
$\langle \beta_T \rangle$	0.031	0.00915
β_T	0.124	0.083

Fig. III.9.

Stability diagrams of $\beta_{TS}/\epsilon_s n^2$ vs nq_s^* for various values of Γ_s .

Fig. III.10.

Maximum β_{TS}/ϵ_s and q_s^* vs the force-free current parameter Γ_s .

We see that there are substantial gains, about a factor of three over the circular case with vacuum. Nevertheless the absolute values of $\langle \beta_T \rangle$, β_T are still somewhat low. Clearly the next step is to combine the effects of non-circularity and force-free currents. This problem is under active investigation.

F. Heating

Experimental evidence from existing tokamaks indicates that joule heating alone is not capable of creating plasmas in the high-beta regime. In our experiment we propose to heat the plasma by the technique of implosion-heating followed by adiabatic compression. This is a well-established technique supported by many years of experimental and theoretical experience in high-beta pinches. It is probably fair to say that now, implosion heating, adiabatic compression is the most reliable and thoroughly investigated method for producing high-density, high-temperature plasmas. This is not to imply that there are no problems. The scaling of implosion heating, adiabatic compression to future large experiments poses very serious technological problems. The best method for heating future large tokamaks has not yet been definitively settled, but for the sizes envisaged, other methods may be more desirable. Nevertheless, for the size experiment which we are considering, implosion heating, adiabatic compression appears to be within our experience and technological capability. We should be able to implement the heating without any major new technological developments. In contrast to other methods, the major problem from point of view of satisfying scientific requirements, is to create sufficiently low-beta plasmas.

Two types of calculations have been done to obtain estimates of the circuit parameters required to produce the desired experimental plasmas. The first of these are snow-plow calculations and the second, hybrid calculations (Vlasov ions fluid electrons). As of now the calculations applicable to the high-beta tokamak regime have been carried out for straight cylindrical plasmas with circular cross section. Also, the poloidal field has been neglected since in the regime of interest $B_p \ll B_T$. This is probably a reasonable approximation.

The hybrid calculations done so far have shown a good implosion, even if a sizable bias field is present. It is however, difficult to obtain final beta values because of problems with the thermalization of the ions and the long computer runs required. Since the final β has to be known for the parameter study, we report here only on the snow-plow calculations.

The snow-plow model is a simple model which, under certain conditions, gives reasonable estimates of several important global plasma parameters after implosion heating and adiabatic compression. Such parameters are the final total temperature $T_e + T_i$, compression ratio a/a_s , maximum electric field, etc. Other important features such as the density and temperature profiles, distribution of energy between electrons and ions, are not accounted for at all in the snow-plow model.

In an approximate sense, the main requirement to justify using the snow-plow model is that the experimental conditions closely resemble an implosion; that is, the thickness of the sheath, which is determined by whatever micro-instabilities are present, should always remain much narrower than the radius of the conducting shell. In practice this reduces to the condition

$$\frac{c}{\omega_{pi}} \ll a ,$$

where c is the velocity of light and ω_{pi} the ion plasma frequency. Thus "good" implosions occur for sufficiently high densities. The proposed experiment is designed to operate at a filling pressure of 5μ ($n = 3.3 \times 10^{14} \text{ cm}^{-3}$) which is equivalent to $c/\omega_{pi} \approx 2 \text{ cm}$. Since $a > 12 \text{ cm}$ the implosion condition is satisfied.

The basic equation describing a snow-plow implosion is given by

$$\frac{d}{dt} \left(M \frac{dR}{dt} \right) = - (F_o - F_i) ,$$

where $R(t)$ is the plasma radius,

$$M = \pi(a^2 - R^2)L\rho_o \quad \text{is the mass of plasma swept up,}$$

$$F_o = \frac{B_o^2(t)}{2\mu_o} 2\pi R(t)L \quad \text{is the external force, and}$$

$$F_i = \frac{B_i^2}{2\mu_o} 2\pi R(t)L \quad \text{is the internal force.}$$

We assume that the radius of the conducting shell is located at $R = a$, the length of the plasma is L , and the filling density is ρ_0 . The internal B_i is determined by the conservation of flux

$$B_i = B_{\text{bias}} \left(\frac{a}{R} \right)^2 .$$

$B(t)$ is the applied external field which is assumed to rise sinusoidally to its peak value B_0 in a time t_{rise} after which it remains constant. The snow-plow equation is solved numerically. A schematic illustration of $R(t)$ and $B(t)$ is given in Fig. III.11. The equation is integrated up till a time t_{imp} at which the plasma motion has stopped. At this time the work done on the plasma is given by

$$E = \int_{R_i}^a (F_o - F_i) dr ,$$

where R_i (determined numerically) is the radius at $t = t_{\text{imp}}$. The energy E is assumed to be redistributed according to the relation

$$E = \frac{3}{2} n_f (T_e + T_i)_f \pi R_f^2 L + 2\pi L \int_{R_i}^{R_f} p \, dR ,$$

where the subscript f refers to the time after thermalization. The first term is the plasma thermal energy and the second term, the work done expanding against the magnetic field. Combining this relation with pressure balance

$$2\mu_0 n_f (T_e + T_i)_f + B_{\text{bias}}^2 \left(\frac{a}{R_f} \right)^4 = B_0^2$$

enables us to solve for all the quantities after thermalization. The final values are obtained by letting the plasma undergo an adiabatic compression with $\gamma = 5/3$.

We present results for two different cases. In each case it is assumed that the adiabatic compression continues until the compression ratio a/a_s becomes equal to 3. The fields are normalized so that the maximum E field at the boundary is 1 kV/cm, and the initial filling density is 5μ ($n_0 = 3.3 \times 10^{14} \text{ cm}^{-3}$). Results are calculated for various B_{bias} . The cases differ as follows. For case A, t_{rise} is adjusted so that it exactly equals the implosion time ($t_{\text{rise}} = t_{\text{imp}}$) for each B_{bias} . In case B, we set $t_{\text{rise}} = t_{\text{imp}}/3$.

The results are illustrated in Fig. III.12. We observe two general features which agree with intuition. First, implosion heating gets more and more difficult as one tries to obtain low-beta values. This is a consequence of the fact that to get low beta, large bias fields are required, and most of the work done by the external field goes into compressing the bias flux rather than

Fig. III.11.

Schematic illustration of $R(t)$, $B(t)$ vs t for the snow-plow model.

Fig. III.12.

Curves of the total final temperature $T_e + T_i$, external field B_0 , and bias field B_{bias} vs β_T for $E = 1\text{kV/cm}$, $n_0 = 3.3 \times 10^{14} \text{ cm}^{-3}$, and $a/a_s = 3$.

heating the plasma. Implosion-heating to low betas leads to relatively lower temperatures and requires higher external and bias fields.

Second, implosion heating is improved by using a faster rise time on $B(t)$. For rather minor increases in the amplitude of B_0 , relatively large increases occur in $T_e + T_i$ with faster rise times.

In the proposed experiment, we hope to be able to vary the peak beta β_T from about 10% and up. The average beta $\langle\beta_T\rangle$ will depend on the compression ratio a/a_s which is expected to vary from about 2-3. A large part of parameter space should be accessible since the toroidal field B_0 will be variable up to about 2 T. The experiment is being designed to have a very fast rise time so that over much of parameter space, the total temperature $T_e + T_i$ should be on the order of 500-600 eV or higher.

We remark that although the parameters predicted here are reasonable intuitively, they should not be taken too literally because of the simplicity of the model.

G. Conclusions

(1) In a high-beta tokamak the plasma pressure is contained by a diamagnetic well in the toroidal magnetic field as opposed to containment by the poloidal field in a conventional tokamak. With the restriction $q \gtrsim 1$ the maximum β 's which can be contained in a high-beta tokamak are $\beta \sim \epsilon$. In a conventional tokamak, β 's are limited to $\beta \sim \epsilon^2$, indicating a gain on the order of $1/\epsilon$ in the high-beta regime.

(2) Given that some appropriate heating method exists to produce plasmas with any specified β value, we discussed two methods for optimizing the maximum β 's possible. The use of both force-free currents and noncircular cross sections can each significantly increase the maximum attainable value of β .

(3) Equilibrium studies of diffuse high-beta tokamaks with circular cross sections satisfying the condition $q(0) = 1$ indicate the following. A diffuse tokamak operated the conventional way has a very small value of $\langle\beta_T\rangle$ for reasonable pressure profiles even when $\epsilon = 0.4$. When little or no force-free currents are flowing outside the plasma core the gain in β going to the high-beta regime is almost negligible when $\epsilon = 0.4$. The reason is that $\epsilon = 0.4$ is not a small number. Much larger relative gains in β are possible if $\epsilon \ll 1$, but then the absolute values are uninterestingly low. The critical β 's for the equilibrium studies are substantially increased (i.e., a factor $2\frac{1}{2}$ -5) if force-free currents are allowed to flow. The gains in β associated with noncircularity have not yet

been fully explored. One complicating factor is that it is not clear what dependence on elongation the condition $q(0) = 1$ should take in a noncircular configuration.

(4) MHD-stability calculations of external kink modes also put strong limitations on β . In a circular high-beta tokamak without force-free currents, the critical β_T as predicted by the surface-current model is quite low. Making the cross-section noncircular (but still with no force-free currents) increases the critical β_T by a factor of about 2 at the optimum elongation $b/a \approx 2$. If force-free currents are allowed to flow in the circular case, then at optimum conditions the gain in β_T is about a factor 3. At the optimum conditions the fields outside the plasma have nearly a constant pitch. The calculation which includes simultaneously force-free currents and noncircularity is actively under investigation but is not yet finished.

(5) Preliminary studies indicate that implosion heating, adiabatic compression is a viable method of heating a high-beta tokamak of the size considered here. It should be possible to create plasmas with peak β 's on the order 10-30% at a total temperature $T_e + T_i$ of about 600 eV. Typically the external field required is about 2 T.

(6) The design of the proposed experiment is consistent with the guidelines suggested by theory. It operates at high density to insure a good implosion. It has a conducting shell surrounding the plasma so that force-free currents can be programmed over a reasonable range of parameters to maximize β_T . It has a noncircular cross section to further help maximize the critical β . We expect that a combination of force-free currents, noncircularity, and high-beta tokamak operation can lead to interesting values of β_T on the order of 5%; i.e., peak β_T 's on the order of 20%.

IV. ENGINEERING DESCRIPTION

As outlined in previous sections, the High-Beta Tokamak requires a fast rising magnetic field for implosion heating. This magnetic field must then be sustained for a time long enough to study plasma confinement and stability. The problems of providing this magnetic field wave form are similar to problems previously studied at LASL in the Staged Theta-Pinch experiment. The proposed LASL High-Beta Tokamak would be constructed in the same experimental area as the Staged Theta Pinch, using many of the same personnel and would take advantage of the experience gained on the Staged Theta Pinch.

The layout of the High-Beta Tokamak in the experimental area is shown in Figs. IV.1 and IV.2. Figure IV.1 shows an elevation view of the experiment. For simplicity, only the implosion capacitor bank for the toroidal field is shown. Figure IV.2 shows a plan view of the experiment. The purpose of the iron cores is explained below. Most of the steel support structure now used for the Staged Theta Pinch is retained. Use of this structure and of other components from the Staged Theta Pinch will reduce the time required to construct the experiment.

The proposed electrical circuit for the toroidal magnetic field is shown in Fig. IV.3. The high-voltage implosion capacitor bank and the low-voltage confinement capacitor bank are fired simultaneously. Iron cores are used to prevent the energy from the high-voltage bank from going into the low-voltage bank during the initial magnetic field rise. In order to insure a large inductance during the field rise, the iron cores must have sufficient volt-seconds so they will not go into saturation. They must also have a frequency response adequate to respond to the rapidly rising voltage of the implosion circuit. Iron cores constructed of 2-mil-thick laminations should meet these requirements.

The implosion capacitor bank will have an energy of 400 kJ and will raise the toroidal magnetic field to 1.7 T, 85% of its peak value, in 2 μ s. The

Fig. IV.1.
Elevation view of the layout of the High-Beta Tokamak
in the present Staged Theta Pinch experimental area.

Fig. IV.2.

Plan view of the layout of the High-Beta Tokamak. The iron cores provide isolation between the implosion and the slow compression banks.

crowbar decay time will be 100-150 μ s in the absence of the slow power-crowbar bank. This will allow the initial stability studies to be conducted using only the implosion bank. Raising the initial field to a large fraction of the maximum long-period field also greatly reduces the amount of energy from the low-voltage bank which must be dissipated in the high-voltage crowbars.

The expected magnetic field wave form is shown in Fig. IV.4. Figure IV.4(a) shows the wave form for the first 50 μ s and Fig. IV.4(b) shows the wave form for 2.5 ms. The fast-implosion magnetic field reaches a peak of 1.7 T in 2 μ s. A slight dip then occurs because the same number of volt-seconds is required to saturate the iron core as is needed to provide isolation between the two banks. A study is under way of methods to minimize the size of this dip.

The voltage on the low-voltage power-crowbar bank reaches zero in 60 μ s, at which time the bank is crowbarred. The magnetic field reaches its peak of 2.0 T in 0.4 ms. This is because some of the current from the low-voltage bank flows

initially through the high-voltage crowbars. After the peak field is reached the magnetic field then decays to $1/e$ of its peak value in 2 ms.

The current for the toroidal magnetic field will be fed to toroidal field coils using two conical feeds in order to increase the rate of the initial field rise. The geometry of the two feeds is shown in Fig. IV.5. The current feeds are on the inside of the torus. This provides easy access for diagnostics, makes

Fig. IV.3.

Proposed circuit for the High-Beta Tokamak. The iron cores isolate the slow 10-kV power-crowbar bank during the initial magnetic field rise which is produced by the fast 150-kV bank.

Fig. IV.4.

Wave form of the toroidal magnetic field in the High Beta Tokamak. (a) Time scale 0-50 μ s. (b) Time scale 0-2.5 ms.

Fig. IV.5.

Layout of the High-Beta Tokamak. The two feeds for the toroidal field are on the inside. This provides easy access for diagnostics, makes it possible to ground the outside shield, and reduces the field errors near the eccentric plasma.

it possible to ground the outside shield, and reduces the field errors near the eccentric plasma. Each feed slot will be connected to a 150-kV capacitor bank. Because of the geometry of the experiment, most of the source impedance of the implosion bank is in the region near the discharge chamber. The source impedance of the bank itself is less important. Therefore it is proposed to construct the bank of 1- μ F, 75-kV capacitors. Each of these units will be equipped with standard Scyllac start and crowbar switches. Similar units have operated satisfactorily at 75 kV on the Scylla 1B experiment. To get the required 150 kV for the implosion bank, two banks of these units would be used in series. The series connection of the banks would be made at or near the point where the cables from the bank connect to the parallel plate transmission lines which feed the toroidal field coils. This design creates some insulation problems but a similar problem was solved on the Staged Theta Pinch.

The low-voltage capacitor bank for the confinement magnetic field will be constructed of new high-energy-density 10-kV capacitors. Ignitrons will be used for start and crowbar switches on this bank.

To achieve the magnetic field values of Fig. IV.4, energies of 400 kJ and 2 MJ are required, respectively, for the high-voltage implosion and low-voltage confinement banks.

The current in the plasma column which generates the poloidal magnetic field will be provided by an air-core transformer. An air-core transformer is used for two reasons. First, because of the small major radius of the torus and the required magnetic flux, the magnetic field along the axis of the torus would exceed the saturation values of the ferromagnetic materials which might be used. Second, it is necessary in order to reach the objectives of the experiment to control the q profiles in the plasma and this can be done with more flexibility if an air-core transformer is used.

The primary of the transformer will be driven by a circuit similar to the one shown in Fig. IV.3. The circuit parameters will be varied to achieve the desired q profiles. The computer calculations given in Sec. III.E show that for the example of Fig. III.5, where $q(\text{wall}) = 1.83$ and $I_\phi = 934$ kA, an energy of about 55 kJ and a flux of 0.12 Vs are required in the secondary. Because an inefficient air-core transformer is used and because amplitude adjustments have to be done with the poloidal ϕ bank rather than the toroidal θ bank, the ϕ bank should be much larger than 55 kJ. Capacitor bank energies of 200 kJ and 1 MJ for the fast and slow banks are proposed.

The primary of the air-core transformer will consist of an end-fed or "Marshall" coil arrangement. This design is shown schematically in Fig. IV.6. The current to the primary is fed from the top by coaxial cables. These cables are connected to coils which form a cylinder whose axis is the major axis of the torus. The coils have a helical spiral so that the only net flux is along the major axis of the torus. This flux returns outside the shell around the plasma, providing the flux linkage which drives the current in the plasma. The design of the end-fed coil is shown in Fig. IV.7.

In order to drive a current in the plasma column, a "cut" is required in toroidal field coils. This will require the design of a complex insulation pattern to provide sufficient high-voltage insulation without introducing large magnetic field errors. Design of this insulation arrangement and the development of suitable iron cores to isolate the fast and slow capacitor banks are the two main problems in the construction of the LASL High-Beta Tokamak.

In order to control the plasma β , a preionized plasma with an embedded magnetic field will be required. This will be accomplished by preionizing the

Fig. IV.6.
Conceptual drawing of the High-Beta Tokamak. An end-fed or "Marshall" coil is used as a primary coil for the air-core transformer.

deuterium gas fill in the presence of a magnetic field. This field will be provided by a slow 10-kV capacitor bank which will be inductively isolated from the θ bank. Preionization will be by means of a toroidal current induced in the plasma.

The discharge chamber will be made of 99.5% alumina. It will be composed of 30 straight ceramic sections, each in the shape of a racetrack. These will be joined together into sectors using a ceramic or glass seal. The sectors will be connected with an O-ring seal of the type used successfully on several LASL experiments. The advantages of making the discharge chamber in a racetrack geometry are outlined in Sec. II.E.

A possible form for a future feasibility experiment based on this HBT approach is shown in Fig. IV.8. Because a peak field of 5 T is assumed and

Fig. IV.7.

Central part of the High-Beta Tokamak. The shell carries the θ currents. The pressurized vessel around the primary ϕ coil prestresses the θ coils to counter-balance the large inward force.

adiabatic compression from 0.5 T to 5 T only raises the plasma temperature by a factor of 5, a plasma temperature of 1 keV is needed at the end of the implosion in order to reach a final temperature of 5 keV.

To generate the large implosion E field, four feed points are needed with 150 kV at each feed in the presence of plasma. This probably requires banks with 300-kV charge voltage. As in the proposed HBT the theta coils serve as shells.

Fig. IV.8.

Possible form of a feasibility experiment based on the HBT approach. The staging system could use HETS (Homopolar Energy Transfer and Storage).

The straight cylindrical inner theta coil has slanted conductors so that it also induces the toroidal current in the plasma.

If more than four feed points would be needed, fractional-turn Marshall coils are possible. A combination of toroidal and poloidal coils is then difficult.

Approximately 30 MJ would be required to bring the toroidal field slowly up to 50 kG. The HETS technology (Homopolar Energy Transfer and Storage) could be used.

V. ESTIMATED COST AND MANPOWER REQUIREMENTS

A. Estimated Cost of HBT

<u>1. Major Procurement Budget</u>	<u>Cost (1976 Dollars)</u>
1. Theta Banks (Toroidal Field)	
Fast Bank	\$ 400 K
400 kJ @ 150 kV	
Slow Bank	200 K
2 MJ @ 10 kV	
2. ϕ Banks (Poloidal Field)	
Fast Bank	200 K
200 kJ @ 150 kV	
Slow Bank	100 K
1 MJ @ 10 kV	
3. Iron Cores	
Theta Circuit	240 K
ϕ Circuit	120 K
4. Collector Plates, Torus, Structure	250 K
	<u>\$1510 K</u>
<u>2. Construction Budget</u>	
Major Procurement	\$1510 K
Craft Labor 10%	150 K
Technician (E-2)	250 K
Design Engineering	440 K
Contingency 12%	285 K
	<u>\$2635 K</u>
<u>3. Capital Equipment Budget</u>	
1. Power Supplies	\$ 20 K
2. Screen Room	15 K
3. Computer (control and data analysis)	100 K
4. Scopes	50 K
5. Data Acquisition (A/D)	30 K
6. Digital Time Delay and Pulser (24 channels)	50 K
7. Diagnostic Equipment (lasers, etc.)	50 K
8. Miscellaneous	25 K
	<u>\$ 340 K</u>

B. Manpower Requirements for HBT

Estimates of the required manpower are:

in FY 1977	1 Scientific Staff
	2 Engineers
	3 Other Technical
in FY 1978	7 Scientific Staff
	2 Engineers
	11 Other Technical
in FY 1979	7 Scientific Staff
	2 Engineers
	11 Other Technical

C. Cost Estimate by Fiscal Year

	<u>FY 77</u>	<u>FY 78</u>	<u>FY 79</u>
Total Man Years	6	20	20
Scientific Man Years	1	7	7
Manpower Cost	\$325	\$1015	\$1100
Major Procurement	275	1235	100
Other Construction Costs		<u>1125</u>	
Total Cost	\$600 K	\$3375 K	\$1200 K

VI. MILESTONES

Preliminary estimates for the time (in months) required for various activities as of the date of approval (month 0) are:

Planning and component development	0 to 12
Build and test prototype	4 to 15
Detailed design HBT	4 to 10
Procurement	10 to approximately 21
Construction of HBT	13 to 25
Checkout and initial operation	25 to 31

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