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**PLASMA PHYSICS
LABORATORY**

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Anomalous Transport Due to Shear-Alfven Waves

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ABSTRACT

The behavior of shear-Alfven eigenmodes and the accompanied anomalous transport have been investigated. In the particle simulation, equilibrium thermal fluctuations associated with the eigenmodes have been observed to nullify the zeroth-order shear near the rational surface through the induced second-order eddy current, and, in turn, give rise to the formation of magnetic islands which cause rapid electron energy transport in the region. The theoretical verification of the observed behavior is discussed.

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Since the recent recognition that magnetic field fluctuations associated with low-frequency microinstabilities may be responsible for the anomalous electron energy transport in tokamaks,¹⁻³ various linear stability analyses have been performed³⁻⁵ to determine the onset conditions for the drift-Alfven waves, which are thought to be responsible for the experimental observation.⁶⁻⁸ It has been shown that, in a sheared slab, drift-Alfven eigenmodes which include the finite- β modified branch and the shear-Alfven branch, are always stable for a collisionless plasma.^{4,5} This prediction prompted us to study the problem using a 2 1/2-dimensional finite- β particle simulation code.⁹ The simulation results have indicated that the shear-Alfven eigenmodes indeed exist. The eigenmodes have the tearing-type parity of odd ϕ and even $A_{\parallel}$, and are localized near the rational surface ($k_{\parallel} = 0$). Although the modes are linearly damped, finite-amplitude fluctuations associated with the radial eigenmodes with $k_y \rho_s = 1$ have been observed. They are the equilibrium thermal fluctuations representing the balance between the Cherenkov emission and the damping rate. Mode coupling between different radial eigenmodes may also play a role here. These fluctuations, in turn, have produced a second-order eddy current which reduces shear near the rational surface. A theoretical analysis, based on the quasilinear treatment of the second-order drift kinetic equation, has verified its existence. The saturation takes place when the zeroth-order current as well as the shear are completely nullified in the region. Consequently, the fluctuations in $A_{\parallel}$ give rise to the formation of magnetic islands with several ρ_i 's in width, and

rapid electron energy transport across the rational surface follows. In this letter, we will present the simulation results in detail together with their theoretical interpretations.

Let us first describe the linear formalism of drift-Alfven waves relevant to our problem. For a low- β and collisionless plasma in a sheared slab where fluctuations are proportional to $\exp(ik_y y - i\omega t)$, the second order radial eigenmode equations for the electrostatic potential ϕ and the vector potential $A_{\parallel}$, obtained from quasineutrality condition and Ampere's law, can be written as ^{4,5,10}

$$\phi'' = (\sigma/x_A^2) (\psi/x - \phi) + b_S \phi, \quad (1)$$

$$\psi'' = (\sigma/x) (\psi/x - \phi) + b_S \psi. \quad (2)$$

Here the independent variable x is normalized by $\rho_s (= (T_e/m_i)^{1/2}/\omega_{ci})$,

$\psi = \omega A_{\parallel}/k'_{\parallel} c$, $k'_{\parallel} = k_y \rho_s/L_s$ where L_s is the shear length, $b_S = k_y^2 \rho_s^2$, $x_A = (1 + \omega^*/\tau\omega)^{1/2} \omega/k'_{\parallel} v_A$ where ω^* is the electron diamagnetic drift frequency, $v_A = c\omega_{ci}/\omega_{pi}$, $\tau = T_e/T_i$, $\sigma = -x_A^2 F$, $F = \{aZ'(\xi_e) + \tau Z'(\xi_i)\}/\{Z'(\xi_i)(x_A^2/x^2) + 2\xi_i Z(\xi_i)\}$, $a = (1 - \omega^*/\omega)/(1 + \omega^*/\tau\omega)$, $\xi_{\alpha} = \omega/\sqrt{2} k'_{\parallel} x v_{t\alpha}$ where α denotes species, $v_{t\alpha} = (T_{\alpha}/m_{\alpha})^{1/2}$ and Z is the usual plasma dispersion function. Equations (1) and (2) are valid in the limit of $k^2 \rho_i^2 \ll 1$ where $\rho_i = v_{ti}/\omega_{ci}$.

Because of the behavior of $\psi/x - \phi$ near the rational surface ($x = 0$), it is evident these equations are not amenable to numerical solutions. Since their WKB solutions indicate that both $\psi/x - \phi$ and $\phi - x\psi/x_A^2$ vanish for large x ,

Eqs. (1) and (2) can be rewritten, with a new variable $\phi = \phi - x\psi/x_A^2$, as

$$\phi'' - b_s \phi + (2/x_A^2) \psi' = 0 \quad (3)$$

$$\psi'' + Q\psi + (\sigma/x)\phi = 0 \quad (4)$$

where $Q = \sigma/x_A^2 - \sigma/x^2 - b_s$. These equations are easily solved by a standard fourth-order shooting code. Since we are only interested in the spatially evanescent solutions, the WKB starting conditions at large x are $(\phi, \phi', \psi, \psi') = (0, 0, 1, -\sqrt{Q})$ and $(-x/x_A^2, \sqrt{b_s}/x_A^2, 1, -\sqrt{b_s})$. The boundary conditions at $x = 0$ are $\phi = 0, \psi' = 0$. Furthermore, Eqs. (3) and (4) can be decoupled through a perturbative scheme and, to the lowest order, the eigenfrequencies of the system can be determined by $y'' + Qy = 0$,¹¹ where Q is defined in Eq. (4). These frequencies, which can be obtained easily with a WKB code,¹² have been used successfully as initial guesses for the shooting code. In the absence of inhomogeneity, we have found that only damped shear-Alfven eigenmodes exist in the sheared slab. However, they become less stable when β increases. The localization of the mode near the rational surface is due to the Larmor radius effects as well as ion Landau damping.

A 2 1/2-Dimensional (x, y, v_x, v_y, v_z) finite- β code⁹ has been used for the investigation. The code is based on the Darwin formulation of Maxwell's equations for $\omega \ll kc$, in which the transverse displacement current in

Ampere's law is neglected. The compression along the external magnetic field has also been ignored in the code, in accordance with the usual tokamak approximation for a low- β plasma. For the particle pushing, the exact dynamics for the ions are kept, whereas the electrons are treated as guiding center particles. With the additional assumptions of $\omega \ll \omega_{pe}$ and $k_{\parallel} \ll k_{\perp}$, the field equations used in the code are $\nabla^2 \phi = -4\pi\rho$ and $\nabla^2 A_{\parallel} = -(4\pi/c)J_{\parallel}$, where ρ is the charge density and $J_{\parallel}$ is the parallel current. The electric field can be calculated by $E = -\nabla\phi - (1/c) \partial A_{\parallel} / \partial t$ and the magnetic field by $B = \nabla \times A_{\perp}$.

The particle simulation has been carried out in a simple slab geometry with $L_x \times L_y = 64\Delta \times 32\Delta$, where Δ is the grid size. The external magnetic field is in the z direction. The potentials, ϕ and $A_{\parallel}$, are assumed to be periodic in y and vanish at the boundaries in x . The shear is generated self-consistently by a uniform current $u/v_{te} = 0.05$ in the z direction and, therefore, the rational surface is located at the middle of the system in x , i. e., $x = 0$. The other simulation parameters are: $m_e/m_i = 1/400$, $\tau = 4$, ave. no. density = $16/\Delta^2$, particle size/ $\Delta = 1.5$, $\lambda_{De}/\Delta = 1$, $\omega_{ci}/\omega_{pi} = 0.2$, $c/v_{te} = 4$, $\omega_{ci}\Delta t = 0.08$, $\beta = 0.4\%$, $L_s/\rho_s = 273$, $k_y\rho_s = 1.0m$, where $m = 0, \pm 1, \pm 2, \dots$, and $\omega^* = 0$, i.e., no spatial inhomogeneity. The simulation has been carried out for $\omega_{ci}t = 0 - 160$.

In the simulation, we have observed that $k_y\rho_s = 1.0$ modes are the dominant fluctuations. The measured frequency spectra for the corresponding ϕ and $A_{\parallel}$, taken at the positions where the amplitude is the

largest, are shown in Fig. 1, along with the spatial structures obtained through the use of spatial correlation function.¹³ Also shown in Fig. 1 is the maximum amplitude reached in the simulation for ϕ and $A_{\parallel}$. The time evolution of these modes indicates that what we have observed is not the growth of an instability. Instead, it is the result of the spatial redistribution of the field energy. The theoretical eigenfrequency for this case, obtained from Eqs. (3) and (4), is $\omega/\omega_{ci} = \pm .0045 - .012i$. The real part agrees rather well with the measurements. The deviation is due to the resolution of the numerical data as well as the nonlinear effects, which will be discussed later. The theoretical mode structures are plotted in Fig. 1. Again, there is a good agreement with the simulation results. Therefore, we can conclude that these fluctuations are indeed shear-Alfvén eigenmodes and their amplitude is apparently determined by the balance between the damping rate and the Cherenkov emission as well as mode coupling. When a larger system ($64\Delta \times 64\Delta$) is used in the simulation, $k_y \rho_s = 1$ modes are found to remain dominant. This is consistent with the results from the mode equations, which give no localized solutions for $k_y \rho_s = 0.5$ modes. It should be noted here that the existence of a small electron drift has negligible affect on the theoretical results.

In the simulations, we have also observed the formation of magnetic islands as well as the appearance of a y independent eddy current, $\Delta J_{\parallel}(x)$, near the rational surface, as shown in Fig. 2. The eddy current, which is nonlinear in nature and is in the direction opposite to the zeroth-order current, acts to nullify the shear near the rational surface. Consequently, magnetic islands emerge in the shear-free region as the result of finite amplitude oscillations of the eigenmodes. These islands, as we know, can cause rapid electron energy transport across the rational surface.

The existence of the eddy current can be verified theoretically. Using the second-order drift kinetic equation for the electron, we obtain, after some algebra, the quasilinear momentum equation as

$$s \partial v_{\parallel 0}^{(2)} / \partial t = \partial A_{\parallel 0}^{(2)} / \partial t + K'(x), \quad (5)$$

where $v_{\parallel 0}^{(2)}$ and $A_{\parallel 0}^{(2)}$ are the y -averaged second-order fluid velocity and vector potential, respectively, $s = (c/v_{te})(m_e/m_i)^{1/2}$

and

$$K(x) = 2\sqrt{b_s} \operatorname{Im}[-A_{\parallel} \phi^* + (v_A/v_{te})^2 (A_{\parallel}' - b_s A_{\parallel}) \phi^* \\ - 2\{\xi_e^2 - 1/Z'(\xi_e)\} (\phi'' - b_s \phi) A_{\parallel}^*] .$$

In this equation, length and time are in the units of ρ_s and ω_{ci} , respectively, and the linear potentials are normalized by T_e/e . In arriving at the expression for $K(x)$, Eqs. (1) and (2) in the limit of cold ions have been used. From Ampere's law, which gives $\delta^2(A_{\parallel 0}^{(2)})'' = s v_{\parallel 0}^{(2)}$ with $\delta = c/\omega_{pe} \rho_s$, the solution of Eq. (5) is

$$s \partial v_{\parallel 0}^{(2)} / \partial t = K'(x) - (1/2\delta) \int_{-\infty}^{\infty} d\zeta K'(\zeta) \exp(-|x - \zeta|/\delta) . \quad (6)$$

Substituting the results from Eqs. (3) and (4) into Eq. (6), we then obtain the second order eddy current due to the presence of linear eigenmodes, as shown in Fig. 2, where $J_{\parallel 0}^{(2)} = -en v_{\parallel 0}^{(2)}$. It compares very well with the

simulation results. Presumably, if the linear eigenmodes have been calculated self-consistently taking into account the quasilinear changes, Eq. (6) would have reproduced the exact simulation results. This is also the case for the linear results shown in Fig. 1. At $x = 0$, Eq. (6) can be approximated by

$$\partial \langle \phi_1^2 \rangle / \partial t = K'(0) = -p \operatorname{Im} [A_1(0) [\phi'(0)]^*] \quad \text{where } p = 2\sqrt{b_s} [1 + (v_A/v_{te})^2]$$

b_s . It shows that the time required for nullifying the zeroth-order drift is $\omega_{ci} t = 100$, when $|A_1(0)| = 7\%$ is used. This value is comparable to the simulation result as shown in Fig. 2.

When the spatial inhomogeneity is included, we have obtained similar simulation results which also agree with the theoretical predictions. With the presence of an electron temperature gradient in the simulation, rapid electron energy transport has been observed, which causes the plateau formation near the rational surface. In real tokamak geometry, the rational surfaces are closely packed. For the modes with $k_y \rho_i = 1$, they are only of the order of ρ_i apart. Therefore, the mechanism described here could be very detrimental to the tokamak confinement.

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References

1. J. D. Callen, Phys. Rev. Lett. 39 (1977) 1540.
2. K. Mølviq, S. P. Hirshman, and J. C. Whitson, Phys. Rev. Lett. 43 (1979) 582.
3. J. F. Drake, N. T. Gladd, C. S. Liu, and C. L. Chang, Phys. Rev. Lett. 44 (1980) 994.
4. K. T. Tsang, J. C. Whitson, J. D. Callen, P. J. Catto, and J. Smith, Phys. Rev. Lett. 41 (1978) 557.
5. Y. C. Lee and L. Chen, Phys. Rev. Lett. 42 (1979) 708.
6. E. Mazzucato, Phys. Rev. Lett. 36 (1976) 792.
7. C. M. Surko and R. E. Slusher, Phys. Rev. Lett. 37 (1976) 1747.
8. S. J. Zweben, C. R. Menyuk and R. J. Taylor, Phys. Rev. Lett. 42 (1979) 1270.
9. W. W. Lee and H. Okuda, private communication.
10. S. M. Mahajan, R. D. Hazeltine, H. R. Strauss and D. W. Ross, Phys. Rev. Lett. 41 (1978) 1375.
11. P. J. Morrison, Jr. and W. W. Lee, private communication.
12. R. B. White, J. Comput. Phys. 31 (1979) 409.
13. W. W. Lee, W. M. Nevins, H. Okuda, and R. B. White, Phys. Rev. Lett. 43 (1979) 347.

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Fig. 1. Mode structures and frequency spectra for the shear-Alfvén eigenmodes.

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Fig. 2. The second-order eddy current and the time evolution of magnetic flux surfaces due to the shear-Alfvén eigenmodes.