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TOROIDAL PLASMA TIME EVOLUTION
PART II:
OHMIC HEATING DURING START-UP

BY

MARIA STOENESCU

**PLASMA PHYSICS
LABORATORY**

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SUMMARY

The toroidal plasma time evolution is being analyzed based on a multiple circuit plasma model. The inductive coupling between plasma and external coils and between various plasma subsections is taken into account. The plasma is considered in quasistatic equilibrium in the magnetic field generated by external coils. More correct transport treatment than in Part I of the present report and consideration of the neutral particles lead to realistic time evolution of the radial skin current and temperature profiles when compared with experiments.

LIST OF SYMBOLS

A	plasma aspect ratio, R_p/a_p
$\bar{A}$	a matrix formed from terms in induction and equilibrium equations
a_j	minor radius of the outer boundary of toroidal plasma section j
a_p	minor radius of plasma
a_{sk}	an element of the matrix $\bar{A}$
a_ξ	the ratio of external (vertical field or divertor) coil current ξ to external coil current 1
$\bar{B}$	a vector formed from terms in induction and equilibrium equations
B_{VDz}	z -component of magnetic induction due to external (vertical field or divertor) coils at the center of the plasma
B_θ	θ -component of magnetic induction due to plasma current at the inner surface of the plasma: $R=R_p-a_p, z=0$
$B_{\theta j}$	the contribution to B_θ of the j th plasma section
c_s	an element of the vector $\bar{B}$
I_p	plasma current
I_{pj}	current in the j th plasma section
I_ξ	current in the ξ th external coil
k_j	$B_{\theta j}/I_{pj}$
L_{jj}	self-inductance of the j th plasma section
M_{cj}	mutual inductance between the j th plasma section and all ohmic heating coils
$M_{j,j'}$	mutual inductance between the j th and j' th plasma sections
$M_{\xi j}$	mutual inductance between the ξ th external (vertical field or divertor) coil and the j th plasma section
n	particle density
P_ξ	$B_{VDz} = \sum_\xi P_\xi I_\xi$
R	position coordinate with plasma torus axis as origin

LIST OF SYMBOLS (continued)

R_{ej}	electrical resistance of the j th plasma section
R_p	major radius of plasma torus
R_ξ	R -coordinate of the ξ th external (vertical or divertor) coil
T	temperature
$\bar{X}$	a vector of current time derivatives
x_k	an element of the vector $\bar{X}$
z_ξ	perpendicular distance from the ξ th external (vertical or divertor) coil to the midplane
z	position coordinate parallel to the plasma torus axis with origin at the midplane
β_θ	ratio of average plasma gas pressure to poloidal magnetic pressure at $R=R_p - a_p$, $z = 0$
θ	angular coordinate with pole at $R=R_p$, $z = 0$, and origin in the direction of $R=0$, $z=0$
μ	magnetic permeability
ϕ_c	poloidal magnetic flux due to ohmic heating coils which passes through the plasma torus

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LIST OF TERMS IN TRANSPORT EQUATIONS

Ion density conservation:

1. diffusion
2. injection source
3. fusion sink
4. compression

Alpha density conservation:

5. diffusion
6. fusion source
7. compression

Ion energy conservation:

8. diffusion and conduction
9. injection source
10. α - i collisions
11. fusion sink
12. e - i collisions
13. compression

Electron energy conservation:

14. diffusion and conduction
15. α - e collisions
16. i - e collisions
17. ohmic heating
18. radiation
19. radiation
20. compression

LIST OF TERMS IN THE INDUCTION EQUATION (1) (KIRCHOFF
VOLTAGE EQUATION) FOR ZERO COMPRESSION

1. Plasma self-induced electromotive force. (EMF)
2. EMF induced by the control coils
3. EMF induced by vertical and divertor coils
4. EMF induced by the rest of the plasma in the analyzed
plasma shell
5. Ohmic voltage drop

FORMULATION OF A MULTI-NODE TOROIDAL PLASMA HEATING

The induction equation written for the electric current of each plasma subsection, if no compression exists, is, according to equation (20) of Part I for $\dot{R}_p = 0$:

$$L_{jj} \dot{I}_{pj} = - M_{cj} \dot{I}_c - \sum_{\xi} M_{\xi j} \dot{I}_{\xi} - \sum_{j' \neq j} M_{jj'} \dot{I}_{pj'} - R_{ej} I_{pj} \quad (1)$$

and expresses the fact that a plasma subsection current varies in time due to the generation of electric fields induced by external coils and by the rest of the plasma subsections, and due to the ohmic resistive drop.

The vertical and divertor field coil currents are varied in time such as to maintain the plasma in a stable location, in accordance with the equilibrium condition formulated by J. Greene, J. Johnson and K. Weimer (Phys. Fluids 14, 3 (1971) 671):

$$B_{vDz} = - 10^{-7} \frac{\sum I_{pj}}{R_p} \left(\ln \left(\frac{8R_p}{a_p} \right) - \frac{5}{4} + \frac{2\mu n T}{(\sum_j k_j I_{pj})^2} \right) \quad (2)$$

where

$$k_j \equiv \frac{B_{\theta j}(r=a_p, \theta=0)}{I_{pj}} \equiv k_j(R, R_p, a_j, a_{j-1}) \quad (3)$$

$$\overline{nT} \equiv \frac{1}{a_p^2} \sum_j (nT)_j (a_j^2 - a_{j-1}^2) \quad (4)$$

and where

$$B_{VDz} = \sum_{\xi} P_{\xi} (R_{\xi}, z_{\xi}, R) I_{\xi} \equiv \sum_{\xi} P_{\xi} a_{\xi} I_1 \quad (5)$$

is the total magnetic field produced by the vertical and divertor coils. The ratios between various coil currents I_{ξ} and a reference coil current I_1 :

$$a_{\xi} \equiv \frac{I_{\xi}}{I_1} \quad (6)$$

are predetermined in order to realize a given shape of the magnetic field.

By differentiating versus time equation (2) under the assumption that the vertical and divertor coil currents vary such as to maintain in static equilibrium a plasma of variable current and kinetic pressure, one obtains the following equation (which corresponds to equation (11) of Part I for a multishell plasma and for $\dot{R} = 0$):

$$\sum_{\xi} p_{\xi} a_{\xi} \dot{i}_1 =$$

$$= - \frac{10^{-7}}{R_p} \dot{i}_p \left(\ln \frac{8R_p}{a} - \frac{5}{4} + \beta_{\theta} - \beta_{\theta} I_p \frac{2k_j}{B_{\theta}} \right) + I_p \beta_{\theta} \frac{\dot{nT}}{nT} \quad (7)$$

where:

$$I_p \equiv \sum_j I_{pj} \quad (8)$$

$$B_{\theta} \equiv \sum_j B_{\theta j} (r=a_p, \theta=0) =$$

$$= \sum_j k_j I_{pj} \quad (9)$$

$$\beta_{\theta} \equiv \frac{2\mu nT}{\left(\sum_j k_j I_{pj} \right)^2} \quad (10)$$

$$k_j \equiv k_j (R=R_p - a_p, R_p, a_j, a_{j-1}) \quad (11)$$

Equations (1) and (7) constitute a system of $(j_{\max} + 1)$ linear equations for the plasma shell currents time rate of change $\dot{i}_{pj}$ and for the reference coil current time rate of change $\dot{i}_1$, which summarizes as:

$$\bar{\bar{A}} \cdot \bar{X} = \bar{B}, \quad X \equiv (\dot{i}_1, \dot{i}_{pi}, \dots) \quad (12)$$

(See Tables 1 and 2 for matrixes $\bar{\bar{A}}$ and $\bar{B}$) and which may be solved by matrix inversion.

The particle density and temperatures are evaluated from the conservation equations (16) - (19) of Part I, reproduced below with the additional terms:

$$\frac{d}{dt} n_i = \text{div}(-D \text{ grad } n_i) + S_i - \frac{1}{2} (\sigma^- v) n_i^2 + (\delta n_i)_{\text{compr}} \quad (13)$$

$$\frac{d}{dt} n_\alpha = \text{div}(-D \text{ grad } n_\alpha) + \frac{1}{4} (\sigma^- v) n_i^2 + (\delta n)_{\text{compr}} \quad (14)$$

$$\begin{aligned} \frac{d}{dt} \left(\frac{3}{2} n_i T_i \right) = & \text{div} (K_e \text{ grad } T_e + \frac{3}{2} T_e D_1 \text{ grad } n_e) + \\ & + S_i W_i + \frac{1}{4} (\sigma^- v) U_{\alpha i} n_i^2 - \frac{1}{2} (\sigma v) \frac{3}{2} (n_i T_i) n_i - \\ & - C_1 \frac{(\frac{3}{2} n_i T_i) n_e}{T_e^{3/2}} \left(1 - \frac{T_e}{T_i} = W_{\text{compr}} \right) \end{aligned} \quad (15)$$

$$\begin{aligned} \frac{d}{dt} (K_e \text{ grad } T_e + \frac{3}{2} T_e D_1 \text{ grad } n_e) + \frac{1}{4} (\sigma^- v) U_{\alpha c} n_i^2 + \\ + C_1 \frac{(\frac{3}{2} n_i T_i) n_e}{T_e^{3/2}} \left(1 - \frac{T_e}{T_i} \right) + \eta j^2 - \\ - W_{\text{Brem}} - W_{\text{cycl}} + W_{\text{compr}} \end{aligned} \quad (16)$$

$$C_1 = 0.8 \times 10^{-18} \frac{\text{m}^3 \text{keV}^{3/2}}{\text{s}}$$

all units are MKS except temperature unit which is in keV

$$W_{\text{Brem}} = 3.0 \times 10^{-21} n_e T_e^{1/2} (n_i + 4n_\alpha + Z_I^2 n_I) \frac{\text{keV}}{\text{m}^3 \cdot \text{s}} \quad (17)$$

(T_e in keV)

$$W_{cycl} = 3.5 \times 10^3 \left(1 + 2 \frac{n}{n_i} + Z_I \frac{n_I}{n_e} \right) \times$$

$$\times n_i^{1/2} B_T^{5/2} (T_e^{11/4} + 0.0049 T_e^{15/4}) \frac{\text{keV}}{\text{m}^3 \cdot \text{s}} \quad (18)$$

(T_e in keV)

Resistivity expression used for the evaluation of plasmas
VI and VII:

$$\eta = \eta_s + \eta_n \quad (19)$$

$$\eta_s = 3.25 \times 10^{-9} \ln \Lambda Z_{eff} / T_e^{3/2} \text{ ohm} \cdot \text{m} \quad (20)$$

$$\eta_n = 7.5 \times 10^{14} \sqrt{T_e} Q_{en} n_n / n_e \text{ ohm} \cdot \text{m} \quad (21)$$

η in ohm·m

n in m^{-3}

T in keV

APPROXIMATIONS OF PRESENT FORMALISM

The integral form of Maxwell's equation for the induction electric field is:

$$\oint_{\Gamma} \vec{E} \cdot d\vec{s} = - \int_{S_{\Gamma}} \dot{\vec{B}} \cdot d\vec{s} \quad (22)$$

where S_{Γ} is a surface resting on the curve Γ , or:

$$\oint_{\Gamma} \vec{E} \cdot d\vec{s} = - \frac{d}{dt} \sum_{\substack{\Gamma' \neq \Gamma \\ \Gamma' = \Gamma}} \int_{S_{\Gamma'}} J_{\Gamma'} \delta\sigma \frac{d\vec{x} \times \vec{r}}{r^3} \cdot d\vec{s} \quad (23)$$

where $\delta\sigma$ is the cross section area of the circuit Γ' , over which the current density is constant.

If one chooses the circuit Γ' such as to correspond to a "current tube" defined as a closed surface which is tangent in every point to the current, then $J_{\Gamma'} \delta\sigma \equiv I_{\Gamma'}$ is constant along Γ' in equation (23). Summing over all current tubes Γ' one obtains the total induction electromotive force built in the circuit Γ .

If current tubes are inexistent or unknown, then the circuits Γ' may consist of any three dimensional subdivisions of the physical system, which are not necessarily identifiable with current tubes.

In toroidal coordinates the triple set of circuits consists of a set of tori Γ'_{ϕ} of curvilinear trapezoidal cross-section limited by $r = \text{const.}$, $\theta = \text{const.}$, $0 \leq \phi \leq 2\pi$, a set of poloidal annuli Γ'_{θ} limited by $r = \text{const.}$, $\phi = \text{const.}$, $0 \leq \theta \leq 2\pi$, and a

set of elements Γ_r' limited by $\theta = \text{const.}$, $\phi = \text{const.}$, $0 < r < \infty$.

For a physical system in which the current is defined through its toroidal components $J_{\Gamma_r'}$, $J_{\Gamma_\theta'}$, $J_{\Gamma_\phi'}$ the summation over Γ' in equation (23) is therefore identical with summation over Γ_r' , Γ_θ' , and Γ_ϕ' .

If axial symmetry is satisfied the toroidal component of the current is constant, and if Γ is a toroidal circuit the induced electromotive force becomes:

$$\begin{aligned} \oint_{\Gamma} \vec{E} \cdot d\vec{s} &= - \frac{d}{dt} \left[\sum_{\Gamma_r'} \iint J_{\Gamma_r'} \delta\sigma_r \frac{d\vec{\ell} \times \vec{r}}{r^3} \cdot d\vec{s} + \right. \\ &+ \sum_{\Gamma_\theta'} \iint J_{\Gamma_\theta'} \delta\sigma_\theta \frac{d\vec{\ell} \times \vec{r}}{r^3} \cdot d\vec{s} + \\ &+ \left. \sum_{\Gamma_\phi'} I_{\Gamma_\phi'} \iint \frac{d\vec{\ell} \times \vec{r}}{r^3} \cdot d\vec{s} \right] \approx \\ &\approx - \frac{d}{dt} \sum_{\substack{\Gamma_\phi' \neq \Gamma \\ \Gamma_\phi' = \Gamma}} I_{\Gamma_\phi'} M_{\Gamma_\phi' \Gamma} \end{aligned} \quad (24)$$

$$M_{\Gamma_\phi' \Gamma} \equiv \int_{S_\Gamma} \int_{\Gamma_\phi'} \frac{d\vec{\ell} \times \vec{r}}{r^3} \cdot d\vec{s} \quad (25)$$

is the mutual inductance coefficient of curves Γ_ϕ' and Γ .

The approximation introduced in the last part of equation (24) is valid as long as the magnetic field flux of the poloidal current through a toroidal current Γ is small compared with the magnetic field flux of any other toroidal current through the toroidal current Γ . A circular poloidal current generated a zero magnetic field flux through any toroidal curve Γ which lays in the median plane of the torus, due to symmetry reasons. The flux through any toroidal curve Γ off the median plane is small due to the sign variation of the magnetic field over the surface deliniated by the curve Γ .

QUALITATIVE DESCRIPTION OF
EXPECTED PLASMA PARAMETERS
TIME EVOLUTION:

a) Plasma current

The magnitude of one of the vertical coil current time rates of change $\dot{I}_1(I_\xi$ for $\xi=1$) and of the plasma currents time rate of change $\dot{I}_{pj}$ (j varies with the plasma section) are determined by the matrix equation:

$$\bar{A} \bar{X}_{k*} = \bar{B}_{k*} \quad (k* = 1, 2, \dots, (j_{\max} + 1)) \quad (26)$$

or its equivalent form:

$$\sum_k a_{sk} x_k = c_s \quad (k = 1, 2, \dots, (j_{\max} + 1)) \quad (27)$$

or

$$|A| x_{k*} = |B_{k*}| \quad (28)$$

where $k* = 1$ corresponds to the coil current I_1 , and $k* = 2, \dots, (j_{\max} + 1)$ corresponds to the plasma currents $I_{p1}, \dots, I_{pj_{\max}}$. The determinant $|A|$ is constructed with the elements of matrix $\bar{A}$ and the determinant $|B_{k*}|$ is identical with $|A|$ except for the elements of one column, different for each unknown x_{k*} , which are being substituted with the elements c_s .

The elements a_{sk} of matrix $\bar{A}$ are represented on Table 1.

The element a_{11} represents the ratio of the coil-produced magnetic field, and the electric current of one of the coils, chosen as reference coil.

The elements a_{1k} , for $k=2, \dots, (j_{\max} + 1)$ where $j_{\max}$ is the total number of plasma sections, may be expressed functionally as:

$$a_{1k} \equiv a_{1k}(A, \beta_{\theta}, k_j, k) \quad (29)$$

and therefore the variation of a_{1k} with respect to the plasma section j at any given moment is determined by $k_j \equiv B_{\theta j}/I_{pj}$.

The elements a_{sk} for $s > 1$ and $k = 1$ are linear combinations of the external coils - plasma sections mutual inductance coefficients, and the elements a_{sk} for $s > 1$ and $k > 1$ consist of the mutual inductance coefficients between various plasma sections. Consequently the elements a_{sk} for $s > 1$ and any k vary in time only during the compressional phase.

The elements c_s are expressed in Table 2 and may be expressed functionally as follows:

$$c_1 \equiv c_1\left(\frac{\dot{nT}}{nT}, \text{ plasma compression}\right) \quad (30)$$

$$c_s \equiv c_s(I_c, R_{ej} I_{pj}, \text{ plasma compression}) \quad (31) \\ (s > 1)$$

It results that the time evolution of the coil and plasma section currents is determined by: β_0 , the ratios k_j between the magnetic field and electric current of each plasma section, the ratio k of the overall plasma, the relative rate of plasma heating $\dot{nT}/nT$, the control coil applied voltage $\dot{\phi}_c$, the ohmic voltage drops $R_j I_{pj}$, the mutual inductance coefficients, and the plasma compression rates.

It is useful to express the ratio of two unknowns in the following way:

$$\begin{aligned} \frac{x_{k^*}}{x_{k^{*+1}}} &= \frac{|B_{k^*}|}{|B_{k^{*+1}}|} = \frac{\sum_s a_{s,k^{*+1}} M_{s,k^{*+1}}^{k^*}}{\sum_s a_{s,k^*} M_{s,k^*}^{k^{*+1}}} = \\ &= \frac{a_{1,k^{*+1}} M_{1,k^{*+1}}^{k^*} + \sum_{s>1} M_{s,k^*} M_{s,k^{*+1}}^{k^*}}{a_{1,k^*} M_{1,k^*}^{k^{*+1}} + \sum_{s>1} M_{s,k^*} M_{s,k^*}^{k^{*+1}}} \end{aligned} \quad (32)$$

where the $|B_{k^*}|$ determinant is expanded upon the elements of the column k^{*+1} and the determinant $|B_{k^{*+1}}|$ is expanded upon elements of column k^* . It is obvious that the two expansions contain identical sets of minors:

$$M_{s,k^{*+1}}^{k^*} = M_{s,k^*}^{k^{*+1}} \quad (33)$$

and differ only through the expansion coefficients, which are the elements of the columns k^{*+1} and k^* .

The expansion coefficients depend on plasma shell through k_j and the minors depend on plasma shell through $(\frac{\dot{nT}}{nT})_j$ and $R_{ej} I_{pj}$, therefore this radial distribution of the last two terms determines the time evolution of the skin current.

b) Density.

If compression is absent the ion density varies in time according to the net balance between the positive effect of the injection and the negative effect of the diffusion and fusion. In other words refueling has to account for the limited lifetime of the particles due to boundary loss and/or fusion.

For alpha particles the boundary loss competes with the fusion source.

The electron density is close to the ion density for a quasi-neutral plasma.

c) Temperature.

The electron heating is controlled by the processes represented by the terms 14-20 in the "List of Terms in the Transport Equations". The positive terms correspond to the alpha particle - electron collisions, ion-electron collisions (if the ion temperature is larger than the electron temperature), the ohmic heating and the compressional heating. For all cases presented in the figures the only heating mechanism is the ohmic heating and eventually the ion-electron heating and the predominant cooling effect is the cooling associated with boundary particle loss and conduction.

The ion heating is controlled by the processes represented by the terms 8-13 in the "List of Terms in transport equations".

During the initial phase they are collisionally heated by the electrons which are heated ohmically. Diffusion particle transport and conduction energy transport tend to homogenize radially the particle density and temperature and act therefore as either energy source or sink for any specific plasma element. The fusion sink radial distribution is determined at each instant by particle temperature radial distribution. The radial energy deposition by high energy ion beam penetration is a function of the injection mechanism and scrape-off and plasma geometry and structure.

RESULTS

Figures 1-28 present time evolution of plasma parameters, electric current density, temperature, particle density, magnetic properties, averaged over each concentric torus, of circular cross section and the time evolution of coil currents and induced voltage.

The present analysis is equivalent to a standard one dimensional (1D) treatment. The plasma shells are numbered from the magnetic axis outward. A set of values of any graphed parameter in all plasma shells, at any moment, indicates the momentary radial distribution.

According to the magnetic coil geometry (two cases are analyzed, the Princeton Reference Design, Figure 1, and the Princeton Large Torus), and plasma initial conditions, several cases are studied, identified as "plasma" I, ... "plasma" VII. They are summarized in Table 3.

All plasmas are heated ohmically and the first five are also heated through neutral injection (Table 4).

The plasma current time evolution is governed primarily by induction (mutual and self) phenomena, as the resistive effect is negligible at the considered temperatures. A radial current density peaked towards the exterior (plasmas I-V) is formed, variations towards the edge being produced by boundary conditions. The unrealistically long time persistence of the skin current for the first five plasmas is explained by the error in the evaluation of the resistivity due to the lack of neutrals in the in-

initial model. Electron and ion-neutral collisions alter considerably the resistivity as results from the time evolution of the plasma current density for plasmas VI and VII, for which an increasing towards the edge neutral concentration is taken into account.

Plasma VI simulates an initial start-up phase time evolution of the Princeton Large Torus, and the reversal of the radial current density profiles from peaked towards the exterior, to peaked in the center behaves very closely with previous Princeton experiments. Table 5 presents values of the induction equation terms at times 10 to 20 msec for plasma VI.

Comparing the plasma current time evolution of plasmas VI and VII (Fig. 16) one may conclude that geometric effects related to reactor size and coil distribution do influence the skin current time evolution.

The radial profile of the particle density time variation is controlled by the injected particle radial deposition rate (input to the code), being determined by the injected particle source and the integrated over each plasma shell boundary diffusion flux. The transport coefficients were assimilated from "A Computer Model of Radial Transport in Tokamaks", P. H. Rutherford and D. F. Duchs, Princeton Plasma Physica Laboratory MATT-1272 Report.

Plasmas VI and VII are described during a short time interval, before the neutral injection start, therefore the plasma density is determined exclusively by transport effects.

The ion temperature is presented on Fig. 3 (plasmas I - V) and Fig. 17 (plasmas VI, VII). The temperature increases due to high energy particle injection and due to electron-ion collisions for plasmas I, II and due to electron-ion collisions only for plasmas III, VII. The abrupt temperature decrease of plasma VI (Princeton Large Torus) is determined by a larger transport.

The radial ion temperature profile tends to be peaked towards the exterior initially as a consequence of the electron ohmic heating distribution peaked also toward the exterior during the skin current formation. The diffusion tends to reverse the ion temperature profile to a centrally peaked profile. For example, plasma III undergoes a faster profile inversion which can be attributed to the fact that transport effects increase more rapidly with plasma current than the ohmic heating.

The electron temperature radial profile time evolution (Figs. 4, 19, 20) is a result of the ohmic heating, transport and ion electron heating during the high energy particle injection phase. The ion and electron density profiles presented on Figs. 5 and 6 are determined by injection sources and diffusion.

Figure 22 represents experimental electron temperature profiles in the ST tokamak, which have the same trend with the evaluated electron temperature in the PLT tokamak shown on Fig. 20, plasma VI.

CONCLUSIONS

The results described in Part II of "Toroidal Plasma Time Evolution" describes the start-up evolution of plasma parameters and coil currents, for a restricted coil current evolution which ensures the overall stability of the plasma discharge (constant major radius).

The time rate of change of the radial distribution of the magnetic field and current density in the plasma are evaluated at the present level of knowledge of electric conduction, based on an arbitrary neutral density distribution.

The particle density and temperature evaluation are based on an arbitrary radial distribution of injected particle and energy deposition rates (Table 4).

The transport of particles, energy and radiation are properly considered, however based on a stipulated impurity concentration.

The present computer coded model represents a rapid simple tool for transitory phases analysis and for comparative results of various toroidal fusion reactors.

Comparisons with experiments are very satisfactory.

The continuation of the present stage of the work with sensitivity analysis of the results towards the three arbitrary assumptions (neutral density, impurity concentration, injection desposition rate) and substitution of these assumptions with correct treatments for neutrals, impurities and injected ions, modifications which represent a low level of additional computations, are recommended.

Figure 1 (from MAT T 1050, Princeton)

1

1

Figure 2

External coil currents (A) versus
time (sec)

Plasmas I, II

1

1

1

Figure 2 (cont)

Idem

Plasmas III, IV, V

Figure 3

Ion temperature (kev) versus time (sec)

Plasmas I, II

(Plasma section numbers increase outwards)

3

3

3

Figure 3 (cont.)

Idem

Plasmas III, IV, V

Figure 4
Electron temperature (kev) versus time (sec)
Plasmas I, II

4

4.1

4.2

Figure 4 (cont.)
Electron temperature
(keV) versus time (s)
Plasmas III, IV, V

5

5

Figure 5
Ion density (m^{-3}) versus time (sec)
Plasmas I, II

5

5

5

Figure 5 (cont.)
 Ion density (m^{-3}) versus
 time (s)
 Plasmas III, IV, V

6

6

Figure 6
Electron density (m^{-3}) versus time (sec)
Plasmas I, II

6

6

6

Figure 6 (cont.)
Electron density (m^{-3})
versus time (s)
Plasmas III, IV, V

7

7

Figure 7

Alpha particle density (m^{-3}) versus
time (sec.).

Plasmas I, II

Figure 8

Plasma current (A) versus time (sec).

Plasmas I, II

8

8

8

Figure 8 (cont.)

Plasma current (A) versus
time (sec).

Plasmas III, IV, V

96

96

96

Figure 9
Plasma current
density (A/m^2) versus time
(sec).

91

92

97

Figures (plasma I) :

10 Applied voltage (V)

11 q

12 Beta theta

91

92

97

98

Figures (plasma III) :

10 Applied voltage

11 q

PLT - Pictorial

Fig. 14. Cross Section of OH and SF Coils

96

Figure 15
Plasma VI (PLT)

59

Figure 16
Plasma VI (PLT)

96

Figure 15
Plasma VII

59

Figure 16
Plasma VII

3

Figure 17
Plasma VI (PLT)

53

Figure 18
Plasma VI (PLT)

3

Figure 17
Plasma VII

53

Figure 18
Plasma VII

4

Figure 19
Plasma VI (PLT)

54

Figure 20
Plasma VI (PLT)

4

Figure 19
Plasma VII

54

Figure 20
Plasma VII

Figure 21
Plasma VI (PLT)

91

Figure 21
Plasma VII

91

Fig. 22. Experimental ST Tokamak T_e profiles

Table 1
Elements of matrix $\bar{A}$

Coefficients of:

i_1 -----

$i_{p,(k-1)}$ -----

a_{s1} :

a_{sk} :

$\sum_{\xi} a_{\xi} p_{\xi}$ -----

$\frac{10^{-7}}{R_p} \left[\ln \left(\frac{8R_p}{a_p} \right) - \frac{5}{4} + \beta_{\theta} \right] - 2 \times 10^{-7} \frac{\beta_{\theta}}{R_p} \frac{I_p}{B_{\theta}} K_{k-1}$

$\sum_{\xi} M_{\xi 1} a_{\xi}$ -----

$M_{1,(k-1)}$ -----

$\sum_{\xi} M_{\xi 2} a_{\xi}$

$M_{2,(k-1)}$ -----

Table 2

Elements of vector $\vec{B}$

$$- I_1 \left(\sum_{\xi} P_{\xi} \dot{a}_{\xi} + \dot{R}_p \sum_{\xi} \frac{\partial P_{\xi}}{\partial R_p} a_{\xi} \right) +$$

$$+ 10^{-7} \frac{I_p}{R_p} \left[- \beta_{\theta} \frac{\dot{nT}}{nT} + \frac{\dot{a}_p}{a_p} + 2\beta_{\theta} \frac{\dot{K}}{K} + \left(\ln \frac{8R_p}{a_p} - \frac{9}{4} + \beta_{\theta} \right) \frac{\dot{R}_p}{R_p} \right]$$

$$- M_{c1} \dot{I}_c - I_c \dot{M}_{c1} - \sum_{\xi} M_{\xi 1} \dot{a}_{\xi} I_1 - \sum_{\xi} a_{\xi 1} \dot{M}_{\xi 1} - \sum_{\text{all } j} I_{pj} \dot{M}_{j1} - R_{e1} I_{p1}$$

$$- M_{c2} \dot{I}_c - I_c \dot{M}_{c2} - \text{-----}$$

|

Table 3
Initial Conditions

Plasma	Geometry	Initial conditions		
		T (eV)	n (cu^{-3})	I _p (A)
I	PRD *	10	4.6×10^{11}	10^5
II	"	10^3	4.6×10^{11}	10^5
III	"	10	4.6×10^{11}	10^5
IV	"	10	4.6×10^{11}	10^3
V	"	10	4.6×10^{11}	10^2
VI	PLT **	1	10^{13}	10^3
VII	PRD	1	10^{13}	3.1×10^4

* Princeton Reference Design

** Princeton Large Torus

Table 4

Neutral Injection

Plasma	Injected Particles	
	$\frac{\text{Rate}}{(\text{part/sec.cm}^3)}$	$\frac{\text{Energy}}{(\text{eV})}$
I	5.0×10^{10}	5×10^{14}
II	10^{12}	2.5×10^{14}
III	--	--
IV	--	--
V	--	--
VI	--	--
VII	--	--

Table 5

Relative magnitude of terms in the
induction equation.

Plasma VI (PLT)

TIME= 0.9970941E-02 ≈ 10 ms

KIRCHHOFF VOLTAGE EQUATION TERMS

1	0.7520474E 02	0.1125240E 03	0.4991971E 02	-0.8458661E 02	-0.2651985E 01
2	0.6992413E 02	0.1125240E 03	0.4991971E 02	-0.7606921E 02	-0.1645004E 02
3	0.2268063E 02	0.1125240E 03	0.4991971E 02	-0.1019430E 03	-0.3781984E 02
4	0.7023575E 01	0.1125240E 03	0.4991971E 02	-0.1048501E 03	-0.5450982E 02
5	-0.3515396E 00	0.1125240E 03	0.4991971E 02	-0.9670044E 02	-0.6609460E 02
6	-0.7730988E 00	0.1125240E 03	0.4991971E 02	-0.8830194E 02	-0.7491469E 02
7	-0.7447060E 00	0.1125240E 03	0.4991971E 02	-0.8107860E 02	-0.8210970E 02
8	-0.7304263E 00	0.1125240E 03	0.4991971E 02	-0.7497202E 02	-0.8820197E 02
9	-0.4887942E 00	0.1125240E 03	0.4991971E 02	-0.6951326E 02	-0.9338911E 02
10	0.7193957E 00	0.1125240E 03	0.4991971E 02	-0.6406848E 02	-0.9765570E 02

TIME= 0.1998271E-01 ≈ 20 ms

KIRCHHOFF VOLTAGE EQUATION TERMS

1	0.1047388E 03	0.1001995E 03	0.3488490E 02	-0.1849908E 02	-0.1184612E 02
2	0.2405142E 02	0.1001995E 03	0.3488490E 02	-0.7379774E 02	-0.3723491E 02
3	-0.1337440E 00	0.1001995E 03	0.3488490E 02	-0.8071039E 02	-0.5450746E 02
4	-0.2050312E 01	0.1001995E 03	0.3488490E 02	-0.7301521E 02	-0.6491924E 02
5	-0.2235675E 01	0.1001995E 03	0.3488490E 02	-0.6523335E 02	-0.7208650E 02
6	-0.1257446E 01	0.1001995E 03	0.3488490E 02	-0.5883095E 02	-0.7751070E 02
7	-0.1141052E 01	0.1001995E 03	0.3488490E 02	-0.5417041E 02	-0.8205484E 02
8	-0.1772097E 01	0.1001995E 03	0.3488490E 02	-0.5080905E 02	-0.8604807E 02
9	-0.6579561E 00	0.1001995E 03	0.3488490E 02	-0.4675172E 02	-0.6899048E 02
10	0.6373405E 01	0.1001995E 03	0.3488490E 02	-0.3900433E 02	-0.8970642E 02

(See List of Terms in the Induction Equation, page x)