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POLARIZED PARTICLE LEVITATION IN HEXAPOLE FIELD

Research Report #2

June 1976

by

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MASTER

prepared under
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to

Target Research and Fabrication Group
Laser Fusion Program
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Polarized Particle Levitation in Hexapole Field

I. Introduction

Proposed here is a novel electrostatic levitation scheme which uses the force exerted by a non-uniform electric field on a polarized particle. The scheme differs from conventional quadrupole levitation devices principally in that the levitated particle is uncharged. In order to provide the proper force required to achieve dynamic stabilization, a very intense non-uniform time-varying electric field produced by a three-dimensional hexapole electrode structure is utilized. The primary advantage of this levitation scheme might accrue in target fabrication operations where particle charge is undesirable or where reproducible charging of the particles themselves is difficult, due to high resistivity. The disadvantages of this scheme, as compared to charged particle levitation, are (i) a more complex electrode structure and (ii) significantly higher voltages.

The scheme has possible application to molecular mass spectrometry, in situations where un-ionized but strongly polar or polarizable molecules are to be trapped or confined for analysis.

II. Polarization Force Effects

A. Review of Polarization Forces

Polarization forces result when a body in which charge separation exists is subjected to a non-uniform electric field. Figure 1 is a free body diagram of such a polarized particle or dipole. Though the net charge is zero, the net force on the dipole is in general non-zero.

$$\vec{F}_{\text{total}} = q\vec{E}(\vec{r} + \vec{d}) - q\vec{E}(\vec{r}) \quad (1)$$

Figure 1 Free Body Diagram of Dipole
in Nonuniform Electric Field

Figure 2 Dielectric Sphere in
Nonuniform Electric Field

If the dipole's physical dimensions are small compared to the characteristic length of the non-uniform electric field, then a Taylor series expansion of $\vec{E}(\vec{r} + \vec{d})$ may be performed to reduce Eq. (1):

$$F_{\text{total}} \approx q\vec{d} \cdot \nabla \vec{E}. \quad (2)$$

In Eq. (2), only the first-order term has been retained. The quantity $q\vec{d}$ is recognized as the dipole moment $\vec{p}$, and so the polarization force expression is:

$$\vec{F} = (\vec{p} \cdot \nabla) \vec{E}. \quad (3)$$

It is convenient to categorize the various physical situations where this force may be present based on the type of polarization. A body may have either an *induced* or *permanent* dipole moment. For an induced dipole, $\vec{p}$ is a function of $\vec{E}$. Examples of induced dipoles are dielectric spheres, droplets, and polarizable molecules. Refer to Table I which gives expressions for $\vec{p}$ and $\vec{F}$ for induced spherical dipoles. Important quantities are defined in Figure 2. The assumptions inherent in the force expressions of Table I are that the spheres are *not* deformed by the field and that the radius of the sphere R is much smaller than the characteristic length of any electric field non-uniformities.

A permanent dipole exhibits polarization which is independent of the electric field. The best example of a permanent dipole is a polar molecule, such as water. One might also envision a permanently polarized electret sphere.

	dielectric sphere	conducting sphere
dipole moment $\vec{P}$	$\frac{4\pi R^3 \epsilon_1 (\epsilon_2 - \epsilon_1) \vec{E}}{\epsilon_2 + 2\epsilon_1}$	$4\pi R^3 \epsilon_1 \vec{E}$
dielectrophoretic force $\vec{F}$	$\frac{2\pi R^3 \epsilon_1 (\epsilon_2 - \epsilon_1)}{\epsilon_2 + 2\epsilon_1} \nabla E^2$	$2\pi R^3 \epsilon_1 \nabla E^2$

Table I. Expressions for Dipole Moment and Dielectrophoretic Force of Spherical Dielectric and Conducting Spheres.

B. Dielectrophoresis

The term "dielectrophoresis" was used by Pohl [1] to refer to the separation of materials of greater or lesser permittivity using strong non uniform electric fields. It is mentioned here in order to explain the distinctions from and similarities to the polarized particle levitation scheme described in this report. Particles of relatively lower dielectric permittivity ($\epsilon_2 < \epsilon_1$) can be levitated in a static electric field. From Table I it is seen that such particles ($\epsilon_2 < \epsilon_1$) should collect at electric field minima, while particles with $\epsilon_2 > \epsilon_1$ should collect at electric field maxima. It is easy to envision electrode structures with local static three-dimensional electric field minima, for example the three-dimensional quadrupole. However, local three-dimensional static electric field maxima can be shown not to exist [2] except at electrode surfaces. Therefore, bubbles or particles of relatively lower dielectric permittivity ($\epsilon_2 < \epsilon_1$) can be dielectrophoretically

levitated, while particles of relatively higher permittivity ($\epsilon_2 > \epsilon_1$) can not be levitated in this manner. Pictures of statically levitated gas bubbles in a dielectric insulating liquid are shown in Figure 3.

Dynamic levitation of $\epsilon_2 > \epsilon_1$ particles using dielectrophoretic forces has been considered, with the resulting conclusion that the required electric fields are too strong to make the scheme practical.

C. Elongation

Liquid droplets and bubbles of gas or vapor tend to elongate in the direction of an applied electric field. In certain target fabrication processes, where droplet deformations must be minimized, this elongation can be significant enough to limit the maximum allowed electric field. The elongation problem is thoroughly discussed elsewhere [3].

The effect of elongation on the force calculation can be accounted for by making the assumption that the initially spherical droplets or bubbles deform into prolate spheroids [4]. Refer to Figure 4, where the quantities used in the equations below are defined:

$$\bar{p} = \frac{4\pi}{3} ab^2 (\epsilon_2 - \epsilon_1) \bar{E}_{1z} . \quad (4)$$

E_{1z} is the uniform electric field inside the ellipsoidal bubble. From Stratton [5],

$$E_{1z} = \frac{E_{oz}}{1 - \frac{ab}{2\epsilon_1} (\epsilon_1 - \epsilon_2) A} \quad (5)$$

where E_{oz} is the uniform external imposed field and

(fluid: Corn Oil)

(a) Bubble Levitator

(fluid: Corn Oil)

(b) Bubble Positioner

Figure 3 Varicus Bubble Orientation Schemes (In both figures, gravity is downward).

Figure 4 Definition of the semi-major and semi-minor Axes of Liquid Droplet deformed in an Electric Field

$$A = \int_0^{\infty} \frac{ds}{(s+a^2)^{3/2}(s+b^2)} \quad (6)$$

$$= \frac{2}{3a^3} \left[1 + \frac{3q}{5a^2} + \frac{3q^2}{7a^4} + \dots \right], \quad (7)$$

with $q = a^2 - b^2$. When the interfacial tension of the droplet or bubble is small, the effect of elongation on the net dielectrophoretic force is significant.

III. Theory of Polarized Sphere Levitation

Figure 5 shows the electrode structure in sectional view. The four electrodes produce a three-dimensional hexapole electric field which, by trial and error, is found to be the simplest cylindrical Laplacian electric field type with the required gradient. The figure defines the important geometrical factors, and it also shows the high voltage excitation to the electrodes. Note that the two phase ac high voltage is applied alternately to adjacent electrodes, while the dc center grounded high voltage is applied between the top and bottom end caps, with the two ring electrodes at the mid-point dc potential. The resulting electric field thus has ac and dc components. Near the center of the electrode system where levitated particles will be located, the dc electric field is essentially uniform, while the ac hexapole electric field has a strong gradient and, precisely at the center, a zero field magnitude. The dc electric field serves to polarize the particle so that $\bar{p} = f(\bar{E}_{dc})$; however, because this dc electric field is essentially uniform, it exerts no force on the particle. The ac hexapole electric

Figure 5a Three-dimensional hexapole electrode structure (shown in cross-section) with high voltage circuit. V_{dc} supplies the polarizing field and V_{ac} supplies the dynamic stabilizing field.

Figure 5b. Geometrical Details of
the Three-dimensional
Hexapole Electrode Structure.

field is zero at the center where the particle resides and so this field component contributes very little to the induced particle polarization. At the same time, the ac hexapole field is highly non-uniform, and thus exerts a strong dynamic force on the dipole. Dynamic stabilization of the dc polarized particle is then possible. The polarization force can be represented approximately as follows:

$$\vec{F} \approx \left[\vec{p}(\vec{E}_{dc}) \cdot \nabla \right] \vec{E}_{ac} \quad (8)$$

The conditions under which Eq. (8) is valid are checked in the next section.

A. Electrostatic Field Solutions

The total electric field solution consists of the separate ac and dc components, which can be solved independently and then added.

1. AC Hexapole Field

The cylindrically symmetric three-dimensional hexapole field solution for the geometry of Figure 5 in spherical coordinates is [6]:

$$\Phi(r, \theta) = \frac{V_{ac}}{4h^3} (r) r^3 [5\cos^3\theta - 3\cos\theta] . \quad (9)$$

Figure 5b shows some of the geometric details of the electrode structure required to produce this electric field. The potential function may be rewritten in a form more convenient for the derivation of the electric field quantities.

$$\Phi(r_c, z) = \frac{V_{ac}}{4h^3} (r) (2z^3 - 3zr_c^2), \quad (10)$$

where z and r_c are the axial and radial displacements in cylindrical coordinates. Then the electric field components in cylindrical coordinates are

$$E_{r_c} = \frac{3V_{ac}(t)}{2h^3} z r_c, \quad (11)$$

$$E_z = \frac{3V_{ac}(t)}{4h^3} (r_c^2 - 2z^2), \quad (12)$$

$$E_\phi = 0. \quad (13)$$

2. DC Polarizing Field

The dc polarizing electric field is imposed by a center grounded dc high voltage supply with the plus and minus sides connected to the end cap electrodes, and the two ring electrodes dc coupled to ground. This is shown in Figure 5a. In order to solve for the dc electric field E_{dc} , near the center of the electrode structure, a numerical approach employing a relaxation solution of Laplace's equation was employed. The details of this approach are covered in Appendix A. The most important results for the present analysis are the dc electric field and its first axial derivative as a function of the applied dc voltage at or near the center of the electrode structure:

$$\bar{E}_{dc} \approx \frac{V_{dc}}{2h} \hat{z}, \quad (14)$$

$$\frac{\partial \bar{E}_{dc}}{\partial z} \approx 0.14 \frac{V_{dc}}{h} \hat{z} z. \quad (15)$$

$r_c, z \ll h$

The Eq. (14) and (15) allow calculation of the dipole moment $\vec{p}(\vec{E}_{dc})$ and the effect of the derivative of $\vec{E}_{dc}$ on the total polarization force.

Note that the net electric field experienced by the particle consists of the sum of the ac and dc components,

$$\vec{E} = \vec{E}_{ac} + \vec{E}_{dc} , \quad (16)$$

which near the center of the electrode structure are given by Eq. (11), (12), (13) and (14).

B. Equations of Motion

1. The Polarization Force

The equations of motion are written for a small droplet of dielectric liquid with radius R such that

$$r \ll h . \quad (17)$$

If the sphere stays near the center of the electrode structure,

$$|\vec{E}_{dc}| \gg |\vec{E}_{ac}| , \quad (18)$$

and the polarization will be principally due to the dc polarizing electric field. For a dielectric sphere with permittivity ϵ_2 in a medium of permittivity ϵ_1 ,

$$\vec{p} \approx \frac{4\pi R^3 \epsilon_1 (\epsilon_2 - \epsilon_1)}{\epsilon_2 + 2\epsilon_1} \vec{E}_{dc} . \quad (19)$$

If the dielectric sphere has a fairly short charge relaxation time, the dc electric field will tend to be excluded from the droplet [7], and the sphere will appear to be perfectly conducting to the dc field:

$$\bar{p} \rightarrow \bar{p}_0 = 4\pi R^3 \epsilon_1 \bar{E}_{dc} . \quad (20)$$

The effect of elongation on the dipole moment $\bar{p}$ is neglected here.

Assuming the dipole moment to be $\bar{p}_0$ from Eq. (20), the polarization force is:

$$\bar{F} \approx (\bar{p}_0 \cdot \nabla) (\bar{E}_{ac} + \bar{E}_{dc}) , \quad (21)$$

$$\approx p_0 \frac{\partial}{\partial z} \left[\bar{E}_{ac} + \bar{E}_{dc} \right] . \quad (22)$$

Using Eq. (11), (12), (13) and (14), this force expression can be reduced to:

$$\bar{F} = p_0 \left\{ \frac{3V_{ac}(t)}{2h^3} \hat{r}_c \hat{r}_c - \frac{3V_{ac}(t)}{h^3} z\hat{z} + .14 \frac{V_{dc}}{h^3} z\hat{z} \right\} . \quad (23)$$

The dc polarizing field must be kept as small as possible anyway to minimize elongation, thus

$$3V_{ac}(t) \gg 0.14 V_{dc} \quad (24)$$

is likely to be a good approximation. The force expression then indeed takes the approximate form of Eq. (8).

The equations of motion can now be written for small excursions of the particle from the center. Viscous drag is neglected. See Appendix A where its effect is considered.

$$m \frac{d^2 \bar{\xi}}{dt^2} = p_0 \left\{ \frac{3V_{ac}(t)}{2h^3} \xi_{rc} \hat{r}_c - \frac{3V_{ac}(t)}{h^3} \xi_z \hat{z} \right\}, \quad (25)^*$$

where $\bar{\xi} = \xi_{rc} \hat{r}_c + \xi_z \hat{z}$ is the vector displacement of the particle and m is the mass of the particle:

$$m = \frac{4}{3} \pi R^3 \rho_2. \quad (26)$$

Assuming a sinusoidal ac voltage, $V_{ac}(t) = V_{ac} \cos \omega t$, and employing Eq. (14), (20) and (26) in the vector equation of motion (25), we obtain:

$$\frac{d^2 \xi_{rc}}{dt^2} = \frac{9\epsilon_1 V_{dc} V_{ac}}{4\rho_2 h^4} \cos \omega t \xi_{rc}, \quad (27)$$

$$\frac{d^2 \xi_z}{dt^2} = - \frac{9\epsilon_1 V_{dc} V_{ac}}{2\rho_2 h^4} \cos \omega t \xi_z. \quad (28)$$

2. Mathieu's Equation and Stability

If the normalization

$$\omega t = 2\tau \quad (29)$$

is used, Eq. (27) and (28) reduce to

---

*The effective mass of the fluid displaced by the droplet plus viscous damping effects are considered in Appendix B.

$$\frac{d^2 \xi_{rc}}{d\tau^2} = \left[\frac{9\epsilon_1 V_{dc} V_{ac}}{\rho_2 \omega^2 h^4} \right] \cos 2\tau \xi_{rc}, \quad (30)$$

$$\frac{d^2 \xi_z}{d\tau^2} = \left[\frac{-18\epsilon_1 V_{dc} V_{ac}}{\rho_2 \omega^2 h^4} \right] \cos 2\tau \xi_z. \quad (31)$$

Eq. (30) and (31) fit the canonical form of Mathieu's equation [8]:

$$\frac{d^2 X}{d\tau^2} + (a - 2q \cos 2\tau)X = 0, \quad (32)$$

with $a = 0$ and

$$q_z = -2q_{rc} = -\frac{9\epsilon_1 V_{dc} V_{ac}}{\rho_2 \omega^2 h^4}. \quad (33)$$

Simultaneously stable solutions to both ξ_{rc} and ξ_z imply successful levitation, subject to certain limits on the maximum trajectory. For $a = 0$, stability of Eq. (32) is assured when $|q| < .908$ [9]. Thus, dynamically stable levitation of polarized particles in the hexapole electrode structure is achieved for

$$|q_z| = |2q_{rc}| < .908. \quad (34)$$

Eq. (34) reduces to

$$\frac{9\epsilon_1 V_{dc} V_{ac}}{\rho_2 \omega^2 h^4} < .908. \quad (35)$$

Eq. (35) describes the necessary and sufficient condition for a dynamically stable trajectory for a polarized particle. Unfortunately, this condition is not really sufficient to guarantee successful particle confinement. Other effects limiting the range of parameters for successful particle containment are considered in the next chapter.

IV. Feasibility Calculations

Based on the theory of the last chapter plus additional considerations, basic relationships among the various experimental parameters (frequency, voltage, droplet diameter, etc.) can be established. These relationships are developed here and then used to determine the useable range of operating conditions for successful containment of polarized (uncharged) particles.

A. Basic Relationships

1. Dynamic Stabilization

The inequality Eq. (35) assures the condition of dynamic stabilization required for successful levitation. Using Eq. (33) in (35) gives a conveniently expressed condition on the frequency of the ac electric field ($f = \frac{\omega}{2\pi}$):

$$f > \frac{.5}{h} \left(\frac{\epsilon_1}{\rho_2} \right) \left(V_{dc} V_{ac} \right)^{1/2}. \quad (36)$$

2. Effect of Gravity

Gravity tends to displace the center of motion of a levitated particle downward from the center of the electrode structure. It also produces a time-periodic motion about the displaced center of motion. Without some force to compensate for gravity, the combined static and

dynamic displacements must be smaller than the physical dimensions of the structure, for otherwise, the particle will strike the electrodes and be lost. While gravitational compensation is easily achieved for charged particle/quadrupole systems, there is no convenient means to accomplish it using polarization forces on charged particles. Therefore, any feasibility study of the hexapole dynamic levitation system must account for the effect of gravity on displacement of the particle's center of motion. The effect of the gravitational term on the stability condition Eq. (34) is itself small [9].

Approximate first harmonic analyses of Mathieu's equation have been performed to estimate the effect of the gravity term on the solutions [9,10]. The result for the displacement of the center of motion z_0 , which dominates over the amplitude of the dynamic motion, is

$$z_0 \approx \frac{8g}{2 \omega^2} \frac{1}{q_z}, \quad (37)$$

where g is the gravitational acceleration ($g = 9.81 \text{ m/sec}^2$). The condition that the particle not be lost is:

$$z_0 < h. \quad (38)$$

Using Eq. (26), (33), (37), and (38), another condition on the ac electric field frequency is derived:

$$f < \frac{.5 \epsilon_1}{\rho_2 g^{1/2} h^{7/2}} V_{dc} V_{ac} \quad (39)$$

3. Liquid Droplet Elongation

The elongation of liquid droplets in electric fields is considered in a previous report [3]. In the present case the finite dc electric field is principally responsible for droplet deformation. For incompressible conducting droplets deforming into ellipsoids, in the limit of small deformations [3]:

$$\gamma - 1 = \frac{9\epsilon_1 D_0}{16\epsilon} E_{dc}^2, \quad (40)$$

where $\gamma = a/b$ is the ratio of the semi-major axis to the semi-minor axis, defined in Figure 4. If the levitation of cryogenic liquid droplets (e.g., hydrogen) is attempted, the elongation may be excessive, due to the very low surface tension Σ of such liquids.

Even if some elongation can be tolerated, a limit on the maximum electric field E_{dc} is still imposed by the stability of the droplet [4]. For conducting incompressible droplets, the limit is

$$E_{dc} < E_{crit} = 2.15 \cdot 10^5 \left[\frac{\Sigma}{\epsilon_1 D_0 / \epsilon_0} \right]^{1/2}, \text{ mks units only} \quad (41)$$

If the levitated particles are liquid droplets, they are subject to elongation due to the dc electric field. Assuming that some elongation is tolerable, but that droplet instability is to be avoided, Eq. (14) and (41) yield:

$$V_{dc} < 4.3 \cdot 10^5 h \left[\frac{\Sigma}{\epsilon_1 D_0 / \epsilon_0} \right]^{1/2}, \text{ mks units only.} \quad (42)$$

B. Calculated Results

The Eq. (36), (39), and (41) may be used to determine the range of voltages and frequency over which the containment of droplets of liquid hydrogen can be achieved using the hexapole ac electric field. In Table II are found the relevant physical properties of n-H₂ liquid at its normal boiling point.

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relative dielectric constant	1.231
density (kg/m ³)	70.
surface tension (Nt/m)	.00194
normal boiling point (°K)	20.

Table II. Properties of Liquid Hydrogen at its Normal Boiling Point [11].

The inequalities (36) and (39) may be used to map out the range of values of frequency f versus the geometric mean voltage $(V_{dc} V_{ac})^{\frac{1}{2}}$ in which successful particle confinement can be obtained. Refer to Figure 6 which shows the region of stable operation for $h = .5$ cm. Note that the stability region does not depend on the particle size. A plot of a different type is that shown in Figure 7. This plot shows the limits imposed on the dc voltage for a 1.0 mm diameter liquid hydrogen droplet with various values of frequency.

Figure 7 shows the effect of imposing stringent limits on the allowable elongation of droplets in the dc electric field. If elongation must be kept at a minimum, then the dc electric field must be kept small and the ac electric field has to be very large. This problem is less serious for smaller droplet diameters, because smaller droplets do not deform so excessively. Electrical breakdown is apt to impose a further limit, probably on the ac electric field.

Figure 6 Graph of Stable Levitation
Conditions for Cryogenic Hydrogen

Lower (Geometric)
Stability Limit,
 $z_0 < h$

Upper (Mathieu
Stability) Limit,
 $q_2 < .908$

Figure 7. Ac Peak Voltage vs. dc Voltage with Corresponding Elongation for 1.0 mm Diam. Cryogenic Hydrogen Droplets in Vacuum.

An experiment designed to simulate the levitation of liquid hydrogen is presently being designed. The simulation involves the levitation of small conducting droplets in an immiscible dielectric liquid medium of almost neutral buoyancy. The analysis of this problem plus the results of the feasibility calculations are found in Appendix B.

An assumption inherent to these feasibility considerations is that bulk or surface charge relaxation has had time to progress to the point where the dc polarizing electric field is excluded from the interior of the droplet. With no electrical conductivity data for liquid hydrogen, this assumption is difficult to evaluate. The calculated results of Figures 6 and 7 do not apply for the insulating case. The results for the insulating case are easily obtained, however by using Eq. (19) for p_0 in Eq. (22) and

$$\gamma - 1 = \frac{\epsilon_1 D_0}{16 \epsilon} \left[\frac{\epsilon_1}{\epsilon_1 - \epsilon_2} - \frac{1}{3} \right]^{-2} E_{dc}^2$$

for the deformation expression [3].

V. Conclusion

The hexapole electrostatic levitation scheme considered here shows promise in the dynamic confinement of uncharged polarized particles. Application to the levitation of liquid droplets or spherical mandrels for laser fusion targets has been considered. The calculations show that, for the levitation of liquid droplets, the dc polarizing electric field must be limited to minimize elongation distortion or droplet instability. This problem becomes especially severe for larger droplets of liquid hydrogen or any liquid with very low surface tension. Elongation and possible droplet instability may very seriously limit the

application of the hexapole confinement system in liquid droplet levitation. In order to compensate for the limited useable dc polarizing electric field magnitude, an intense ac electric field is required. The ac field itself is not expected to contribute much to the elongation if the center of motion is not displaced too far from the center of the electrode structure.

In order to obtain some experimental experience with the levitation of uncharged liquid droplets, an experiment has been designed to simulate the low density and low surface tension of liquid hydrogen. The simulation is achieved with droplets of a relatively conducting liquid in an insulating dielectric liquid medium. The calculations are reviewed in Appendix B. The principal conclusion drawn from these calculations is that properly chosen liquid pairs can simulate all the properties of a liquid hydrogen sphere except for effective mass. The viscous damping does not appear to be important except of small droplets of diameter less than about 0.6 mm.

The stringent limits placed upon the dc and ac electric fields in levitating liquid droplets do not exist for solid particles because they do not elongate. Therefore, larger dc polarizing fields may be used, and the ac electric field requirement is easier to meet. Experimental simulation with solid, almost buoyant particles is presently being contemplated.

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APPENDIX A

Computation of the DC Polarizing Field Using the Relaxation Technique

The solution to the electric dc (polarizing) potential of the hexapole electrode configuration can be obtained using a numerical technique called the relaxation method. The electric potential (ϕ) satisfies Laplace's equation,

$$\nabla^2 \phi = 0, \quad (A1)$$

which is solved by the relaxation method to yield a solution. From such a solution one can numerically differentiate the potential to find the dc electric field ($\bar{E}_{dc}$) and its gradient ($-\frac{\partial \bar{E}_{dc}}{\partial z}$).

The relaxation method replaces continuous space by a finite grid or matrix. The values of ϕ at each point on the grid are required to satisfy a set of finite difference equations which replace Laplace's equation (Figure A1). Since our electrode structure possesses azimuthal symmetry, Laplace's equation is written in two dimensional cylindrical coordinates:

$$\begin{aligned} \nabla^2 \phi = 0 &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) + \frac{\partial^2 \phi}{\partial z^2}, \\ &= \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2}. \end{aligned} \quad (A2)$$

By using a Taylor series expansion about point 0 in Figure A1, one can approximate the derivatives in equation (A2) to give the following result:

$$\begin{aligned} \phi_0 &= \frac{\phi_1 + \phi_2 + \phi_3 + \phi_4}{4} + \frac{\phi_2 - \phi_4}{8n}, \quad n \neq 0 \\ &= \frac{\phi_1 + 4\phi_2 + \phi_3}{6}, \quad n = 0 \end{aligned} \quad (A3)$$

Figure A1 Notation for Finite
Grid Relaxation Calculation

where it is assumed that

$$\Delta r = \Delta z ,$$

and

$$r = n\Delta r .$$

Note that $\phi_2 = \phi_4$ for $n = 0$, due to symmetry.

The basic technique is to use an iterative process to proceed systematically through the matrix, applying equation (A3) to each field point thereby obtaining a better approximation for ϕ_0 . (Initially, each matrix point is assigned a value or guess to begin the process). The relaxation method, if used without "over-relaxation", is guaranteed to produce convergence upon the correct values for the potentials. When the change in ϕ_0 from one iteration to the next is less than a specified accuracy, the approximate solution to Laplace's equation has been found. Obviously, this method lends itself easily to computer applications.

The solution accuracy is sensitive to two main parameters: 1) matrix field size, and 2) boundary conditions. The choice of matrix or grid size is usually a compromise between greater accuracy (small Δr , Δz) and shortened computation time. This choice is best determined by experience. Unknown boundary conditions present another source of inaccuracy, but the problem can be alleviated by suitable approximations, and sometimes symmetry. For example, very often a boundary may be conveniently set at zero potential if it is situated far from the electrodes.

A Fortran IV relaxation program run on a CDC 6400 digital computer obtained a solution to the DC hexapole potential. The computer performed about 5000 iterations over a field of 10,000 grid points per quadrant in

approximately 1000 seconds. The potential values were relaxed until they were stable to five significant digits. The graphical solution and approximated boundary conditions are shown in Figure A2. Inspection of this figure reveals the close similarity of the equipotential lines along the z-axis to the potential between a parallel-plate capacitor.

$\bar{E}_{dc}$ is found by a numerical differentiation of the electric potential and $\frac{\partial \bar{E}_{dc}}{\partial z}$ by a second numerical differentiation of ϕ . The dc electric potential and field as plotted along the z-axis are shown in Figure A3. By empirical curve fitting, the field can be shown to be:

$$\bar{E}_{dc} \approx \left(0.48 \frac{V_{dc}}{h} + 0.07 \frac{V_{dc}}{h^3} z^2 \right) \hat{z} \quad (A4)$$

Near the center of the structure, $\bar{E}_{dc}$ can be accurately approximated by the field inside a parallel-plate capacitor:

$$\begin{aligned} \bar{E}_{dc} &= 0.48 \frac{V_{dc}}{h} \hat{z} \\ &\approx \frac{V_{dc}}{2h} \hat{z} \end{aligned} \quad (A5)$$

By differentiating equation (A4), an expression for $\frac{\partial \bar{E}_{dc}}{\partial z}$ can be obtained:

$$\frac{\partial \bar{E}_{dc}}{\partial z} = 0.14 \frac{V_{dc}}{h^3} z \hat{z} \quad (A6)$$

Figure A2. Equipotential Plot of Solution
for dc Polarizing Electric Field.

Figure A3 dc Electric Potential and Field Along
the z-Axis ($V_{dc} = 2000$ V)

APPENDIX B

Levitation of Simulated Laser Fusion Targets

Due to the expense of obtaining cryogenic hydrogen and the inconvenience of working with it in a laboratory, the motivation for a simulated levitation scheme is apparent. If levitation can be sustained using a simple but accurate simulation, then such a scheme may be feasible using cryogenic D-T pellets. Simulation using two immiscible dielectric liquids, one as the droplet fluid and the other as the medium fluid, is the subject of this analysis.

The selection of these liquids, of course, is based on imitation of the pertinent physical and electrical properties of cryogenic hydrogen in a vacuum. These properties appear in the stability and deformation equations and are used in certain simplifying assumptions. In all, the choice is based on five considerations:

1. The liquids must be mutually immiscible.
2. The surrounding medium liquid must be a good insulator ($\sigma_1 > \sim 10^{-9}$ mho/m) to prevent breakdown resulting from the strong electric fields inside the electrode structure.
3. The droplet liquid (sphere) must be a good conductor as compared with the medium liquid to maximize the induced dipole moment. This enables one to use the assumption $\epsilon_2 \rightarrow \infty$ and thus simplify the dipole moment expression.
4. The mass densities of the two liquids must be approximately equal to model the low mass density of cryogenic hydrogen.
5. To model droplet elongation due to the electric field, the fluid pair should possess a low interfacial tension to match that of cryogenic hydrogen.

These conditions are satisfactorily met by a droplet sphere of acetophenone surrounded in a medium of Dow Corning's 200 series 10 centistoke silicone oil. Table B1 presents a comparison of the pertinent properties of cryogenic hydrogen in a vacuum to acetophenone in 10 cs. silicone oil.

TABLE B1

Properties	H ₂ - Vacuum (T = 20°K)	Acetophenone - Silicone Oil (T = 25°C)
ρ_2, ρ_1	70., 0 $\frac{\text{kg}}{\text{m}^3}$	1027., 940. $\frac{\text{kg}}{\text{m}^3}$
$\epsilon_2/\epsilon_0, \epsilon_1/\epsilon_0$	1.231, 1.00	17.39, 2.64
Σ	$1.94 \cdot 10^{-3} \text{ Nt/m}$	$2.2 \cdot 10^{-3} \text{ Nt/m}$
σ_2, σ_1	?, 0	$6 \cdot 10^{-7}, 1 \cdot 10^{-13} \frac{\text{mho}}{\text{m}}$

All these values are known, tabulated quantities except for the interfacial tension between acetophenone and silicone oil. This value was experimentally determined by using the drop-weight method [12,13].

The liquid-liquid simulated levitation theory is basically the same as that outlined for cryogenic hydrogen in a vacuum except for two factors. One of these is the damping effect due to the viscous nature of the 10 cs. silicone oil medium. Such a lossy medium adds a velocity-dependent term to Mathieu's equation which only very slightly changes

the dynamic stability limit [14], Eq. (36), but which substantially alters the geometric stability limit, Eq. (39).

The second factor which affects the simulated levitation theory is the effective mass (m_{eff}) of the droplet sphere in the medium liquid. Dynamically speaking, the effect of the medium liquid pressure is equivalent to an addition to the inertia of the droplet, the increment being one-half of the liquid displaced [15]. In other words,

$$m_{eff} = m + \frac{1}{2}m' \quad (B1)$$

$$\begin{aligned} \text{where: } m &= \frac{4}{3}\pi R^3 \rho_2 \text{ (inertial mass of droplet),} \\ m' &= \frac{4}{3}\pi R^3 \rho_1 \text{ (mass of liquid displaced by droplet).} \end{aligned}$$

Unfortunately, this extra mass contribution does make a significant change in Mathieu's equation by making more stringent the requirements for successful particle containment. The Mathieu's stability limit for the simulation experiment becomes:

$$f > \frac{0.49}{h^2} \left(\frac{\epsilon_1 V_{dc} V_{ac}}{\rho_2 + \frac{1}{2}\rho_1} \right)^{\frac{1}{2}} \quad (B2)$$

$$\text{or} \quad V_{ac} < \frac{4.16(\rho_2 + \frac{1}{2}\rho_1)h^4 f^2}{\epsilon_1 V_{dc}} \quad (B3)$$

Thus the requirements for the simulated levitation scheme become more stringent than for cryogenic hydrogen droplets.

Both factors, viscous damping and effective mass, affect the geometric stability limit. The complete equation of motion in the z-dimension is as follows:

$$\ddot{\xi}_z + 2\dot{\xi}_z + (a - 2q \cos 2\tau) \xi_z = \frac{4(\rho_1 - \rho_2)g}{\omega^2(\rho_2 + \frac{1}{2}\rho_1)} \quad (B4)$$

where $\kappa = \frac{3\mu}{R^2 \omega(\rho_2 + \frac{1}{2}\rho_1)}$, $\left(\begin{array}{l} \text{The fluid sphere model has} \\ \text{been used for Stokes drag} \\ \text{force, i.e. } F_d = 4\pi\mu Rv. \end{array} \right)$

$$q = - \frac{8.64 \epsilon_1 V_{dc} V_{ac}}{\omega^2 h^4 (\rho_2 + \frac{1}{2}\rho_1)}$$

$$a = - \frac{0.81 \epsilon_1 V_{dc}^2}{\omega^2 h^4 (\rho_2 + \frac{1}{2}\rho_1)}$$

Assuming a solution of the form

$$\xi_z(\tau) = z_0 + C_1 \sin 2\tau + C_2 \cos 2\tau \quad (B5)$$

to equation (B4), an expression for z_0 , the displaced center of motion, can be obtained. Using the approximation that $a \ll q$ and neglecting the higher harmonics,

$$z_0 = - \frac{8(1+\kappa^2)(\rho_2 - \rho_1)g}{\omega^2(\rho_2 + \frac{1}{2}\rho_1)q} \quad (B6)$$

The net amplitude of the oscillatory motion is:

$$C = \sqrt{C_1^2 + C_2^2} = \frac{4(1+\kappa^2)^{\frac{1}{2}}(\rho_2 - \rho_1)g}{\omega^2(\rho_2 + \frac{1}{2}\rho_1)q} \quad (B7)$$

To avoid the loss of the droplet sphere from the electrode structure, the geometric requirement that $|C| + |z_0| < h$ must be made. For $q < 0.4$ (which is reasonable for the liquids considered), the net amplitude, C , becomes negligible compared to z_0 . Therefore $|z_0| < h$ is a sufficient criterion for stability. The simulated geometric stability limit is now:

$$f < \frac{0.49 \epsilon_1 V_{dc} V_{ac}}{[(1+\kappa^2)(\rho_2 - \rho_1)(\rho_2 + \frac{1}{2}\rho_1)h^7 g]^{1/2}} \quad (B8)$$

or

$$V_{ac} > \frac{2.06 [(1+\kappa^2)(\rho_2 - \rho_1)(\rho_2 + \frac{1}{2}\rho_1)gh^7]^{1/2} f}{\epsilon_1 V_{dc}} \quad (B9)$$

Substituting the physical and electrical properties of acetophenone and 10 cs. silicone oil into Equations (B2-3, B8-9), a range of frequencies and ac voltages for dynamic levitation can be written:

$$2.45 \cdot 10^{-3} (V_{dc} V_{ac})^{1/2} < f < 1.14 \cdot 10^{-6} V_{dc} V_{ac} \quad (B10)$$

$$\frac{8.80 \cdot 10^5 f}{V_{dc}} < V_{ac} < \frac{1.67 \cdot 10^5 f^2}{V_{dc}} \quad (B11)$$

Equations (B10, B11) are displayed graphically in Figures B1, B2 respectively. Droplet elongation due to the dc field is also shown in Figure B2. For $\gamma \leq 1.10$, the elongation for a 1 mm diameter acetophenone droplet sphere in 10 cs. silicone oil is:

$$\gamma - 1 = 6.05 \cdot 10^{-8} V_{dc}^2 \quad (B12)$$

Figure B1 Stable Levitation of Acetophenone
Droplets in 10 cs. Silicone Oil
($R > 300 \mu\text{m}$)

Upper (Mathieu
Stability) Limit,
 $q_2 < .908$

Figure B2. Stable Levitation of Acetophenone
Droplets in 10 cs. Silicone Oil
with Elongation Effects.
($R = 500 \mu m$)

The acetophenone droplet will become unstable and disrupt when $\gamma \approx 1.85$ or $V_{dc} = 1950$ volts [4]. Viscous damping and effective mass have virtually no effect on elongation.

Figure B3 shows the effect of viscous damping on the voltage criteria for geometric stability. Since $\kappa \propto 1/R^2$, this plot shows how damping can become significant for small droplet radii. Note the rapid increase in minimum $(V_{dc} V_{ac})^{1/2}$ requirements for $R < 300\mu$. By limiting droplet radii to $R > 300\mu$, the viscous damping contribution can be effectively neglected.

The fact that effective mass reduces the region of stable dynamic levitation in this simulation scheme may pose problems. Since dc voltages must be kept well below 2000 volts to prevent excessive droplet deformation, ac voltage requirements are exceedingly high. An ac high-voltage source with a minimum frequency capability of 40 Hz must have a peak output of at least 30,000 volts to keep the elongation below $\gamma \approx 1.10$. With such high ac electric fields, breakdown within the electrode structure using even the best insulating liquid may be unavoidable. These prospective problems could be eliminated by using a solid (hollow) undeformable sphere in an insulating fluid medium. Then higher dc voltages could be employed to reduce the ac peak voltage requirements. Simulated levitation of this type may be more feasible.

Figure B3. Effects of Viscous Damping on Minimum $(V_{dc} V_{ac})^{1/2}$ as Determined by Droplet Radius. For Acetophenone Droplets in 10 cs. Silicone Oil.

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NOMENCLATURE

- a - semi-major axis of ellipsoid, also coefficient in Mathieu eq.
- A - de-polarization factor
- b - semi-minor axis of ellipsoid
- C - coefficients
- $\vec{d}$ - charge separation vector distance
- D - diameter
- $\vec{E}$ - electric field
- f - frequency
- $\vec{F}$ - vector force
- g - gravitational acceleration (9.81 m/sec^2)
- h - hexapole electrode spacing parameter
- m - mass
- n - radius index for matrix
- $\vec{p}$ - dipole moment vector
- q - charge, also coefficient in Mathieu eq.
- r - radial coordinate in spherical coordinate system
- $\vec{r}$ - position vector
- R - droplet radius
- r_c - radial coordinate in cylindrical coordinate system
- s - integration variable
- t - time
- v - velocity
- V - voltage
- z - axial coordinate in cylindrical coordinate system

NOMENCLATURE (continued)

- γ - semi-major axis ratio
- ϵ - dielectric permittivity
- θ - polar coordinate in spherical coordinate system
- κ - damping coefficient
- μ - viscosity
- $\vec{\xi}$ - vector particle displacement
- ρ - mass density
- Γ - surface tension
- σ - electrical conductivity
- τ - normalized time
- Φ - potential function
- ϕ - azimuthal coordinate in cylindrical coordinate system
- ω - radian frequency