Three Simple Classes of Mixing Models*

MASTER

John F. Bolzan, William F. Palmer, and Stephen S. Pinsky

Department of Physics
The Ohio State University
Columbus, Ohio 43210

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Abstract

Three classes of Mixing Models are examined with respect to mass constraints and Okubo-Zweig-Iizuka Rule Violation.

*Research supported in part by the U.S. Energy Research and Development Administration under Contract AT(11-1)-1545.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

In the limit of ideal mixing and the Okubo Ansatz for trilinear couplings, ψ and ϕ mesons---and their partners in other J^P multiplets---are stable against strong decay into mesons composed only of u, d quarks. Duality diagrams are a graphic realization of the static notion of pure wave functions as well as the dynamical notion of continuous quark line interaction implicit in the Okubo Ansatz. This general picture for the new narrow resonances has been very persistent; we continue to refer to it, following Freund and Nambu, as the Okubo-Zweig-Iizuka (OZI) Rule.¹ In this limit, masses within the U(3) multiplets obey ideal mixing mass formula [or U(4) extensions] with ω degenerate with ρ and K^* the arithmetic mean of ϕ and ω .

An understanding, then, of the decay and production systematics of the new particles must involve theories of mixing and the breaking of the OZI Rule. In fact, numerous authors have written numerous papers on this subject, couched in various languages and based on various dynamical theories.²⁻¹³ Despite the number and variety, we find certain unifying properties. In this note we would like to make some simple observations concerning three classes of models (MI, MII, and MIII) which we believe span many approaches and which probably exhaust the simplest possibilities.

For simplicity, we restrict ourselves first to a U(3) scheme, with ϕ initially $s\bar{s}$ and ω initially $\frac{1}{2}(u\bar{u} + d\bar{d})$. Extension to U(4) is straightforward. (ω and ϕ refer generically to the quark species states without regard to spin-parity.) Common to all schemes, one has "bare" masses m_ω and m_ϕ [physical masses are denoted μ_ω and μ_ϕ , etc.] which satisfy the ideal mixing mass formulae

$$\begin{aligned} m_\omega^2 &= m_\rho^2 \\ m_\omega^2 + m_\phi^2 &= 2m_{K^*}^2 \end{aligned} \quad (1a)$$

In all schemes it is a dynamical assumption that the interaction which mixes the ideally mixed states contributes only to isosinglet masses. This is the case, for example, when the interaction is unitary flavor singlet gluon exchange¹⁴ or when the interaction is the Pomeron continued to timelike momentum transfer.^{15,16,17} Thus

$$\begin{aligned} m_{\omega}^2 &= \mu_{\rho}^2 \\ m_{\varphi}^2 &= 2\mu_*^2 - \mu_{\rho}^2 \end{aligned} \quad (1b)$$

and one has the luxury of knowing one's bare masses, which play a quite physical role in these models.

The remaining choice lies in the nature of the isosinglet mixing interaction and the size of the channel space in which it is to be realized.

In models of class MI the interactions is purely off-diagonal in the 2×2 system. In models of class II the interaction proceeds through a third, independent, "quarkless" channel or intermediary, again with only off-diagonal elements, in a 3×3 system. In models of class III the interaction remains in a 2×2 system, is rank unity, and possibly energy dependent. Apart from the strikingly different physical interpretations, models of class II are simply class I models in a higher dimension, whereas class III models, as will be seen, are closely related to those of class II, with the intermediary no longer an independent state.

A few recent examples in the literature of these classes of models are Harari³ (MI, pseudoscalar mixing), Freund and Nambu² (MII, vector mixing, with the "O"-meson or glue-ball) and the present authors⁹⁻¹³ (MII, vector, scalar, pseudoscalar, and tensor mixings; MIII, pseudoscalar mixings) as well as Chew and Rosensweig⁴ (MIII, cylinder corrections in various channels).

We discuss the models in a simple propagator-mass matrix formalism, in the channel space basis [ideally mixed $s\bar{s}$ and $\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$ states], with "bare" propagator

$$P = \begin{bmatrix} (s-m_\omega^2)^{-1} & 0 \\ 0 & (s-m_\phi^2)^{-1} \end{bmatrix} \quad (2)$$

MI. In models of class MI, the transition between ω and ϕ channel states is given by

$$H = \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix} \quad (3)$$

yielding a "renormalized" propagator

$$\pi = P \sum_{\pi} (HP)^n = (P^{-1} - H)^{-1} \quad (4)$$

$$\pi^{-1} = \begin{pmatrix} s-m_\omega^2 & -c \\ -c & s-m_\phi^2 \end{pmatrix} \quad (5)$$

Solving the eigenvalue problem we have

$$\pi = \sum_{\alpha=\omega,\phi} \frac{v_\alpha v_\alpha^\dagger}{s-\mu_\alpha^2} = \sum_{\alpha} \frac{\begin{bmatrix} (\mu_\alpha^2 - m_\omega^2)^{-1} & [(\mu_\alpha^2 - m_\omega^2)(\mu_\alpha^2 - m_\phi^2)]^{-\frac{1}{2}} \\ [(\mu_\alpha^2 - m_\omega^2)(\mu_\alpha^2 - m_\phi^2)]^{-\frac{1}{2}} & (\mu_\alpha^2 - m_\phi^2)^{-1} \end{bmatrix}}{(s-\mu_\alpha^2) [(\mu_\alpha^2 - m_\omega^2)^{-1} + (\mu_\alpha^2 - m_\phi^2)^{-1}]} \quad (6)$$

with μ_α the roots of

$$(\mu_\alpha^2 - m_\omega^2)(\mu_\alpha^2 - m_\phi^2) - c^2 = 0 \quad (7)$$

and

$$V_{\alpha} = \begin{pmatrix} \frac{c}{\mu_{\alpha}^2 - m_{\omega}^2} \\ 1 \end{pmatrix} \left[\frac{c^2}{(\mu_{\alpha}^2 - m_{\epsilon}^2)} + 1 \right]^{-\frac{1}{2}}. \quad (8)$$

Defining the mixing angle by

$$\begin{aligned} |\omega\rangle &= \cos\theta |\bar{\omega}\rangle + \sin\theta |\bar{\varphi}\rangle \\ |\varphi\rangle &= -\sin\theta |\bar{\omega}\rangle + \cos\theta |\bar{\varphi}\rangle \end{aligned} \quad (9)$$

where $|\bar{\omega}\rangle$, $|\bar{\varphi}\rangle$ represent channel states, we have

$$\tan\theta = \frac{\frac{\mu_{\omega}^2 - m_{\omega}^2}{2}}{\frac{\mu_{\omega}^2 - m_{\varphi}^2}{2}}.$$

For a respectable multiplets (vector, tensor, or pseudoscalars) the masses are all known and the mixing is completely determined, and one can calculate all OZI forbidden processes. The model, of course, can easily fail because the two physical masses must be fit by a single parameter c . This class of models can, however, be enlarged to allow c to be a function of s ; the complications encountered with this generalization will become clear when Models MII and MIII are discussed.

III. Let us now expand the system to include a third "quarkless" or "glue-like"¹⁸ channel,

$$P = \begin{pmatrix} (s-m_\omega^2)^{-1} & 0 & 0 \\ 0 & (s-m_\phi^2)^{-1} & 0 \\ 0 & 0 & h^{-1}(s) \end{pmatrix} \quad (10)$$

where $h^{-1}(s)$ is a linear function of s when the bare glue resonants, but we allow for more general behavior. If this third channel now interacts with ω and ϕ as an SU(3) singlet [this condition is appropriate to models in which glue or quarkless states (perhaps multiparticle) are unitary singlets - it may easily be relaxed] then

$$H = \begin{pmatrix} 0 & 0 & \sqrt{2} f \\ 0 & 0 & f \\ \sqrt{2} f^* & f^* & 0 \end{pmatrix} \quad (11)$$

and

$$\pi^{-1} = \begin{pmatrix} s-m_\omega^2 & 0 & -\sqrt{2} f \\ 0 & s-m_\phi^2 & -f \\ \sqrt{2} f^* & -f^* & h(s) \end{pmatrix} \quad (12)$$

Solving again the eigenvalue problem, we find

$$\pi = \sum_{\alpha} \frac{V_{\alpha}(s) V_{\alpha}^{\dagger}(s)}{s-M_{\alpha}^2(s)} \quad (13)$$

where the sum is over ω , ϕ , and glue channels, $M_{\alpha}^2(s)$ are the solutions of

$$(M^2 - m_w^2)(M^2 - m_\varphi^2)(M^2 + h(s) - s) - |f|^2(M^2 - m_w^2) - 2|f|^2(M^2 - m_\varphi^2) = 0 \quad (14)$$

with $V_\alpha(s)$ the associated eigenvectors

$$V_\alpha(s) = \begin{pmatrix} \sqrt{2} f (M_\alpha^2(s) - m_w^2)^{-1} \\ f (M_\alpha^2(s) - m_\varphi^2)^{-1} \\ 1 \end{pmatrix} \left[2f^2 (M_\alpha^2(s) - m_w^2)^{-2} + f^2 (M_\alpha^2(s) - m_\varphi^2)^{-2} + 1 \right]^{-\frac{1}{2}}. \quad (15)$$

For the "standard" case in which $h(s)$ is linear in s , $M_\alpha^2(s)$ and $V_\alpha(s)$ are independent of s ; otherwise they depend on s and the poles of the theory in that case are solutions of

$$\mu_\alpha^2 = M_\alpha^2(\mu_\alpha^2) \quad (16)$$

which may be multivalued (in which case, below, an extra summation is implied).

Then the propagator evaluated at the poles is

$$\pi \approx \sum_{\alpha} \frac{V_\alpha(\mu_\alpha^2) V_\alpha^\dagger(\mu_\alpha^2)}{(s - \mu_\alpha^2) \left[1 - \frac{dM_\alpha^2(\mu_\alpha^2)}{ds} \right]}. \quad (17)$$

To obtain a rank one normalized propagator residue matrix, the term $1 - \frac{dM_\alpha^2(\mu_\alpha^2)}{ds}$ must be either absorbed into a coupling constant renormalization

$$G_\alpha \rightarrow G_\alpha \left[1 - \frac{dM_\alpha^2(\mu_\alpha^2)}{ds} \right]^{-\frac{1}{2}} \quad (18)$$

or we may require that $\frac{dM_\alpha^2(\mu_\alpha^2)}{ds} \equiv 0$, implying the condition

$$\frac{dh(\alpha^2)}{ds} = 1$$

on $h(s)$, satisfied by the MII "standard" case. In either case the remaining propagator $\bar{\pi}$ has the form

$$\bar{\pi} = \sum_{\alpha} \left[\begin{array}{ccc} \frac{2}{(\mu_{\alpha}^2 - m_{\omega}^2)^2} & \frac{\sqrt{2}}{(\mu_{\alpha}^2 - m_{\omega}^2)(\mu_{\alpha}^2 - m_{\phi}^2)} & \frac{\sqrt{2} f^{-1}}{\mu_{\alpha}^2 - m_{\omega}^2} \\ \frac{\sqrt{2}}{(\mu_{\alpha}^2 - m_{\omega}^2)(\mu_{\alpha}^2 - m_{\phi}^2)} & \frac{1}{(\mu_{\alpha}^2 - m_{\phi}^2)^2} & \frac{f^{-1}}{\mu_{\alpha}^2 - m_{\phi}^2} \\ \frac{\sqrt{2} f^{*-1}}{\mu_{\alpha}^2 - m_{\omega}^2} & \frac{f^{*-1}}{\mu_{\alpha}^2 - m_{\phi}^2} & |f|^{-2} \end{array} \right] \frac{1}{(s - \mu_{\alpha}^2) \left[\frac{2}{(\mu_{\alpha}^2 - m_{\omega}^2)^2} + \frac{1}{(\mu_{\alpha}^2 - m_{\phi}^2)^2} + |f|^{-2} \right]} \quad (19)$$

In MII "standard" with $h(s) = s - m_0^2$ and m_0 the "bare" quarkless states' mass, the two free parameters f and m_0 are determined by the physical ϕ and ω masses, with μ_0 an output (to be compared, naturally, with masses of quarkless states determined in simple calculable models). It is remarkable that this model can fail: the determinantal equation may require $|f|^2 < 0$. This can not be overcome with a Hermitian interaction matrix. This problem arises for certain multiplets in the Freund and Nambu² scheme and has led us to consider models of class MIII.

MIII. Consider a reordering of the perturbation sum in MII, as follows

$$\pi = (I + PH)(P^{-1} - Q)^{-1}, \quad (20)$$

with

$$Q = HPH = \begin{bmatrix} \frac{2f^2}{h(s)} & \frac{\sqrt{2}f^2}{h(s)} & 0 \\ \frac{\sqrt{2}f^2}{h(s)} & \frac{f^2}{h(s)} & 0 \\ 0 & 0 & \frac{2f^2}{s-m_\omega^2} + \frac{f^2}{s-m_\phi^2} \end{bmatrix} \quad (21)$$

Now Q [thus $(P^{-1} - Q)^{-1}$] has the form of a direct sum, with block diagonal separation of the quark and quarkless channels, with all communication between these channel species in the PH term, which has only off-block-diagonal terms.

If one were persuaded, by experiment or prejudice, that an extra quarkless channel simply does not materialize (i.e., that completeness is realized within quark channels] then one is led to the following theory (MIII) which is a kind of projection or truncation of the 3×3 theory:

$$\pi = (P^{-1} - Q)^{-1} = P \sum_n (QP)^n \quad (22)$$

where P is as in MI and

$$Q = g(s) \begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix} \quad (23)$$

It is significant that Q is rank 1; the results which follow can easily be extended to any rank 1 interaction. As before we have

$$\pi = \sum_{\alpha} \frac{V_s(s) V_{\alpha}^{\alpha}}{s - M_{\alpha}^2(s)} \quad (24)$$

where now $M_{\alpha}(s)$ are solutions of

$$(M^2 - m_w^2)(M^2 - m_\varphi^2) - 2g(s)(M^2 - m_\varphi^2) - g(s)(M^2 - m_w^2) = 0 \quad (25)$$

and

$$V_\alpha(s) = \begin{bmatrix} \frac{\sqrt{2}}{M_\alpha^2(s) - m_w^2} \\ \frac{1}{M_\alpha^2(s) - m_\varphi^2} \end{bmatrix} \left[\frac{2}{(M_\alpha^2(s) - m_w^2)^2} + \frac{1}{(M_\alpha^2(s) - m_\varphi^2)^2} \right]^{-\frac{1}{2}} \quad (26)$$

with the physical particle poles solution of

$$\mu_\alpha^2 = M_\alpha^2(\mu_\alpha^2) \quad (27)$$

which again may be multivalued. As before, we have the propagator evaluated at the pole in the form Eq.(17), with a similar treatment for the $1 - \frac{dM_\alpha^2}{ds}$ term. Setting this term to unity puts the condition on $g(s)$,

$$\frac{dg}{ds}(\mu_\alpha^2) = 0.$$

Whether handled this way or through vertex renormalization, the remaining propagator for $\bar{\pi}$ is

$$\bar{\pi} = \sum_\alpha \frac{\begin{bmatrix} 2(\mu_\alpha^2 - m_w^2)^{-2} & \sqrt{2}(\mu_\alpha^2 - m_w^2)^{-1}(\mu_\alpha^2 - m_\varphi^2)^{-1} \\ \sqrt{2}(\mu_\alpha^2 - m_w^2)^{-1}(\mu_\alpha^2 - m_\varphi^2)^{-1} & (\mu_\alpha^2 - m_\varphi^2)^{-2} \end{bmatrix}}{(s - \mu_\alpha^2) \left[2(\mu_\alpha^2 - m_w^2)^{-2} + (\mu_\alpha^2 - m_\varphi^2)^{-2} \right]} \quad (28)$$

It is to be noted that, in contrast to MI, the eigenvectors here are not orthogonal, and physical states can not be represented as a single unitary transformation of the channel states. This is due to the s -dependence of the

interaction. It is clear that there is no a priori requirement that φ and ω be orthogonal in channel space (even ignoring mixing with other isosinglets); that would be necessary only if they were degenerate. Thus a single angle does not suffice here to describe the mixing.

In the particular case $\frac{d}{ds} g(s) \equiv 0$ (MIII "standard") the eigenvectors are orthogonal, and there is no coupling constant normalization. Interestingly enough, that simple one parameter theory [$g(s) = \text{constant}$] is distinct from the one parameter MI theory and leads to different mixing.

The specific SU(3) singlet nature of the interaction can be relaxed, keeping the rank 1 structure. This implies

$$Q = g(s) \begin{bmatrix} \text{ctn}^2 \beta & \text{ctn} \beta \\ \text{ctn}^2 \beta & 1 \end{bmatrix}. \quad (29)$$

Then the determinantal condition becomes

$$(M^2 - m_\omega^2)(M^2 - m_\varphi^2) - \text{ctn}^2 \beta g(s)(M^2 - m_\omega^2) = 0 \quad (30)$$

and the eigenvectors are

$$V_\alpha(s) = \begin{bmatrix} \frac{\text{ctn} \beta}{M_\alpha^2(s) - m_\omega^2} \\ 1 \\ \frac{1}{M_\alpha^2(s) - m_\varphi^2} \end{bmatrix} \left(\frac{\text{ctn}^2 \beta}{(M_\alpha^2(s) - m_\omega^2)^2} + \frac{1}{(M_\alpha^2(s) - m_\varphi^2)^2} \right)^{-\frac{1}{2}} \quad (31)$$

Generalizations to U(4) of MII and MIII are straightforward. We briefly mention some possibly useful results: for MII the generalization of Eq.(15) is

$$V_{\alpha} = \begin{bmatrix} \frac{\sqrt{2}}{M_{\alpha}^2(s) - m_{\omega}^2} \\ \frac{1}{M_{\alpha}^2(s) - m_{\phi}^2} \\ \frac{1}{M_{\alpha}^2(s) - m_{\psi}^2} \\ f^{-1} \end{bmatrix} \left[\frac{2}{(M_{\alpha}^2(s) - m_{\omega}^2)^2} + \frac{1}{(M_{\alpha}^2(s) - m_{\phi}^2)^2} + \frac{1}{(M_{\alpha}^2(s) - m_{\psi}^2)^2} + |f|^{-2} \right]^{-\frac{1}{2}} \quad (32)$$

and the generalization for the eigenvalue condition is

$$(M^2 - m_{\omega}^2)(M^2 - m_{\phi}^2)(M^2 - m_{\psi}^2)(M^2 + h(s) - s) - |f|^2(M^2 - m_{\omega}^2)(M^2 - m_{\phi}^2) \\ - |f|^2(M^2 - m_{\omega}^2)(M^2 - m_{\psi}^2) - 2|f|^2(M^2 - m_{\phi}^2)(M^2 - m_{\psi}^2) = 0. \quad (33)$$

For MIII, the corresponding results are

$$V_{\alpha}(s) = \begin{bmatrix} \frac{\sqrt{2}}{M_{\alpha}^2(s) - m_{\omega}^2} \\ \frac{1}{M_{\alpha}^2(s) - m_{\phi}^2} \\ \frac{1}{M_{\alpha}^2(s) - m_{\psi}^2} \end{bmatrix} \left[\frac{2}{(M_{\alpha}^2(s) - m_{\omega}^2)^2} + \frac{1}{(M_{\alpha}^2(s) - m_{\phi}^2)^2} + \frac{1}{(M_{\alpha}^2(s) - m_{\psi}^2)^2} \right]^{-\frac{1}{2}} \quad (34)$$

and

$$\begin{aligned}
& (M^2 - m_\omega^2)(M^2 - m_\phi^2)(M^2 - m_\psi^2) - g(s)(M^2 - m_\omega^2)(M^2 - m_\phi^2) \\
& - g(s)(M^2 - m_\omega^2)(M^2 - m_\psi^2) - 2g(s)(M^2 - m_\phi^2)(M^2 - m_\psi^2) = 0.
\end{aligned} \tag{35}$$

Finally, how does one choose among the classes of mixing models? If one possesses a fundamental theory, then a particular mixing is implied and there is no need to read on. Alternatively, one can test the various models to see if one is preferred in fitting OZI violation data, thus limiting the class of fundamental theories to those which lead to a correct mixing model.

In Ref.12, we studied the pseudoscalar mesons in the context of models II and III. At the present level of experimental data they both are acceptable, but improved data should shortly provide a definitive test. Model MI, we believe, clearly fails (but see also Ref.3).

For the vector mesons one can not, in models of class MII ("standard" variety) satisfy the mass constraints with $|f|^2 > 0$. We conclude therefore that the strict Freund and Nambu² model is wrong for the case to which it was originally applied.

For purposes of illustration, let us show how a corresponding model of class MIII fares within this multiplet. The rate $\phi \rightarrow \rho\pi$ is given by

$$\Gamma(\phi \rightarrow \rho\pi) = p_\rho^3 \frac{G_{\omega\rho\pi}^2}{4\pi} \mathcal{R}_{\omega\omega}^\phi \tag{36}$$

where $\mathcal{R}_{\omega\omega}^\phi$ is the residue of the ϕ pole in the ω channel,

$$\mathcal{R}_{\omega\omega}^\phi = \frac{2}{(\mu_\phi^2 - m_\omega^2)^2} \left/ \left[\frac{2}{(\mu_\phi^2 - m_\omega^2)^2} + \frac{1}{(\mu_\phi^2 - m_\phi^2)^2} + \frac{1}{(\mu_\phi^2 - m_\psi^2)^2} \right] \right. \tag{37}$$

and $G_{\omega\rho\pi}^2/4\pi$ is rather well determined from several other processes.¹² If a D^* or F^* meson were discovered, then m_ψ , the bare mass of the ψ would be known [through analogies of Eq.(1b)] and $\mathcal{R}_{\omega\psi}^\varphi$ would be completely determined. Since, however, the ψ is known to be extremely pure relative to φ , $\mu_\varphi^2 - m_\psi^2 \approx \mu_\varphi^2 - \mu_\psi^2$ is a very good approximation in $\mathcal{R}_{\omega\psi}^\varphi$, which depends insensitively on the bare ψ mass, with very sensitive dependence on the mass $m_\omega^2 = \mu_\rho^2$; we obtain the experimental width $\Gamma(\varphi \rightarrow \rho\pi) = 660$ KeV for $\mu_\rho = 760 \pm 2$ MeV (error reflecting uncertainty in $G_{\omega\rho\pi}^{12}$) while the experimental mass is 770 ± 10 . The simple MIII model appears to survive this test. Note no positivity requirements such as $|f|^2 > 0$ plague MIII. Alternatively, if one uses the energy independent rank 1 MIII interaction of Eq.(29), relaxing the SU(3) singlet nature of the interaction, one finds the (now quite simple) mass constraints imply $\tan^2 \beta = 1.77$ and $\Gamma(\varphi \rightarrow \rho\pi) = 660$ KeV for $\mu_\rho = 773 \pm 2$ MeV. Thus this simple form of MIII also fares quite well so far as $\varphi \rightarrow \rho\pi$ suppression is concerned, but there would appear to be difficulty with the non-SU(3) symmetric interaction [$\tan^2 \beta = \frac{1}{2}$ in SU(3) limit]. Finally, in the simple MI model we have $\Gamma(\varphi \rightarrow \rho\pi) = 660$ KeV for $\mu_\rho = 763 \pm 2$ MeV. It is clear that in the nearly ideally mixed vector multiplet, no one class of models seems sharply distinguished as the best. We take the point of view, however, that there should be a single class of models for all multiplets. Then, in view of MI problems with the pseudoscalars (see Refs.3 and 12) we believe that models of class II and III, with interactions suggested by the gluon exchange picture, are those more likely to succeed.

References

1. G. Zweig, CERN report No. 8419/TH 412, 1964; S. Okubo, Phys. Letters 5, 165 (1963); I. Iizuka, K. Okuda, and O. Shito, Progr. Theor. Phys. 35, 1061 (1966).
2. P. G. O. Freund and Y. Nambu, Phys. Rev. Letters 34, 1645 (1975).
3. H. Harari, Phys. Rev. Letters 20, 1385 (1968); P. G. O. Freund, Phys. Rev. Letters 20, 235 (1968).
4. C. Rosenzweig and G. F. Chew, Phys. Letters 58B, 93 (1975); G. F. Chew and C. Rosenzweig, Phys. Rev. D (to be published); C. Rosenzweig, PITT-156.
5. Chan Hong-Mo, Ken-ichi Konishi, J. Kwiecinski, and R. G. Roberts, RL-75-192/T.148, and P. Aurenche et al., RL-75-084/T.121. Rutherford Lab preprints.
6. A. Kazi, G. Kramer, and O. H. Schiller, DESY 75/11 and 75/10.
7. J. F. Willemssen, M.I.T. preprint CTP #491.
8. M. Chaichian and M. Hayashi, CERN TH 2082.
9. J. F. Bolzan, K. A. Geer, W. F. Palmer, S. S. Pinsky, Phys. Rev. Letters 35, 419 (1975).
10. J. F. Bolzan, K. A. Geer, W. F. Palmer, and S. S. Pinsky, Phys. Letters 59B, 351 (1975).
11. W. F. Palmer and S. S. Pinsky, Ohio State preprint COO-1545-171.
12. J. F. Bolzan, W. F. Palmer and S. S. Pinsky, Ohio State preprint COO-1545-179.
13. J. F. Bolzan, W. F. Palmer and S. S. Pinsky, Ohio State preprint COO-1545-180.
14. H. Fritzsch, CALT-68-524, to be published in Fortschritte der Physik.
15. R. Carlitz, M. B. Green, and A. Zee, Phys. Rev. D 4, 3439 (1971).
16. S. S. Pinsky and D. R. Snider, ANL-HEP-PR-75-37. Phys. Rev. D (to be published).
17. V. Barger, McGill Institute of Physics International Summer School, June 1975.
18. R. L. Jaffe and K. Johnson, M.I.T. preprint, CTP #508. For details of the M.I.T. Bag, see K. Johnson CTP #494, Lectures at the XVth Cracow School of Theoretical Physics, Zakopane, Poland, June 6-18, 1975.