High-Pressure, Flux-Conserving Tokamak Equilibria

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HIGH-PRESSURE, FLUX-CONSERVING TOKAMAK EQUILIBRIA

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Oak Ridge National Laboratory

THERMONUCLEAR DIVISION

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ABSTRACT

Magnetohydrodynamic (MHD) tokamak equilibria are found with values of β up to 20% and prescribed MHD safety factor values [e.g., $q(\text{axis}) = 1$ and $q(\text{edge}) = 4.8$] for tokamaks with aspect ratio $A = 4$ and D-shaped cross section. If such equilibria could be attained experimentally, they would be very attractive for decreasing the projected costs of tokamak power reactors substantially. In the flux-conserving tokamak (FCT) model [1], where rapid heating is applied to an already relatively hot plasma, these high β equilibria are achievable. We study the quasi-static evolution of FCT equilibria as β increases. An operating window is found in the pressure profile width w_p : for high β the values of w_p must lie between 0.40 and 0.55 of the plasma minor width. Within this window, plasma current and poloidal β increase monotonically with β . For fixed plasma boundary, significant poloidal surface currents are induced, but these can be eliminated by small increases in the plasma minor radius, the pressure profile width, and the vacuum toroidal field.

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1. INTRODUCTION

Plasma β in a tokamak fusion reactor is calculated by the following equation:

$$\beta = 8 \pi \bar{p} / B_{\phi 0}^2 ,$$

where $\bar{p}$ = volume average plasma pressure

$B_{\phi 0}$ = externally applied toroidal magnetic field at the geometric center ($R_0, \phi, 0$).

Analyses of tokamaks have normally assumed for the purposes of design conservation that the plasma β is in the neighborhood of 3%. Such a value for β is consistent with the choice of safety factors (for MHD instabilities) $q_0 = 1$ at the magnetic axis and $q_a \sim 3 - 5$ at the plasma edge. The "poloidal beta" is calculated from the following:

$$\beta_p = 8 \pi \bar{p} / \bar{B}_p^2 ,$$

where $\bar{B}_p = I_p / 2a$

I_p = toroidal plasma current

a = effective minor radius of the discharge.

Poloidal beta is usually assumed to be less than the toroidal aspect ratio $A \equiv R_0/a$ [2]. But as was shown in Ref. [3], this is a result of the analytical expansion calculations which assume $a/R_0 \ll 1$ and $\beta_p \sim 1$. The condition $\beta_p \gtrsim A$ is a signpost indicating significant distortion of the plasma away from the cylindrical approximation, rather than a limit on plasma equilibria.

The reactor designs which result from these assumptions appear to be of a size and cost that are very uncomfortable to both the funding research agencies and to the prospective users, the electrical utilities companies [4]. It is therefore of interest to find more acceptable

operating conditions by studying ways to avoid the "restriction" $\beta_p < A$. (One must of course pay careful attention to the neutron loading of the walls exposed to the hot plasma; plasma strategists are also seeking ways to increase the "credible" upper limits of wall loading from the present $0.5 - 2 \text{ MW/m}^3$.)

Here we ask two questions: are there equilibria with high β and high q_a (for stability reasons), and are they accessible? Briefly, we find that equilibria exist with β values above 20%, and that these equilibria are achievable if current diffusion is not too severe and MHD instabilities do not set in.

It has been observed by Clarke [5] that, as tokamaks penetrate further into the regime where plasma temperatures are a few kilovolts, and as minor radii increase to a substantial fraction of a meter, it may be expected that the plasmas will begin to behave as if magnetic fluxes are frozen into the plasma; provided, of course, that the plasma continues to exhibit no more than mildly anomalous current (skin) diffusion.

We examine the equilibria which would result from rapid heating of a hot plasma which has already attained such a flux-frozen or flux-conserving state (FCT, for flux conserving tokamak). Evaluation of classical skin times with temperatures of 2-20 keV and scale distance greater than 10-20 cm yield $1-10^3$ sec as times of interest. This is sufficiently long to make the study of FCT equilibria of great interest. As will be shown, our results indicate that a reasonable window exists for the attainment of high β equilibria in the flux-conserving fashion; this has important implications for the design and operation of a

tokamak reactor. Experiments and detailed transport analyses must of course be made to determine whether and how FCT equilibria can be attained.

Thus, for equilibrium studies we take the FCT condition: that the poloidal and toroidal magnetic fluxes (ψ and Φ , respectively) are frozen into the plasma. The differential ratio $d\Phi/d\psi$, known as the q safety factor, should then be invariant during the FCT heating phase. In a pure FCT model, where high conductivity is assumed throughout the plasma, the total poloidal and toroidal fluxes in the plasma would also be preserved. These invariants form the basis for the present results. However, independent of the applicability of the flux conservation model, the equilibria we present are interesting in their own right as "target" configurations of high β toward which experimenters may direct their efforts. Because the current profiles will be shown to remain broad with $\beta \leq 10\%$, suitable current density tailoring brought about by careful injection of particles and momentum could make them achievable even in the absence of flux-conservation. The outer regions of a more realistic plasma will probably remain cool before rapid heating takes place, so the periphery would be a non-FCT region. Analysis of this mixed situation is beyond the scope of this present study and must await the coupling of equilibria to a realistic time- and space-dependent transport code.

At the plasma boundary a sizable surface current would be induced by heating in an FCT unless steps were taken to prevent this. Experiments have shown that surface currents have lifetimes much shorter than the classical skin time in the bulk of the plasma [6]. Thus, avoidance of

large surface currents is an important aspect of the study. It is shown that the surface current can be eliminated by increases in plasma size, pressure profile width, and vacuum toroidal field.

Another area of further research is the stability of the high equilibria. In the present work, we have assumed that maintaining an initially quiescent q profile during rapid heating is an adequate precaution. One expects a critical value in β above which instabilities will result; but recent observations on belt pinches [7] give strong indications that the critical β may be above 50%, higher than the 10-20% values discussed here. The MHD instability analysis codes should be focused on the high β equilibria obtained here to provide guidance on this question.

It should be emphasized that the present approach to high β is different from the doublet approach [8], in that strong noncircularity of the plasma cross section is not required. Ideal local stability analyses [9] have indicated that a D-shaped plasma of mild elongation and triangularity may have comparable potential in achieving stable high β configurations. Therefore, in this work only the D-shaped plasmas are examined in detail.

In a subsequent paper, results will be presented showing how high beta equilibria can be obtained using a finite number of lumped field shaping coils rather than the conducting shell assumed in this work. For the equilibria having no shell, it is of course necessary to provide for control of axial and radial excursions [13]; it remains to be determined whether passive or active control will be required at high beta values.

The formulae describing the high β FCT equilibria will be presented in Section 2. Methods of solution of cylindrical and toroidal cases will be discussed in Section 3. Numerical results will be presented in Section 4, followed by discussion in Section 5.

2. FORMULAE

MHD equilibria in axisymmetric tokamaks can be described by the poloidal flux $\psi(R, Z)$ that satisfies the equation [10, 3]

$$R^2 \operatorname{div}(R^{-2} \operatorname{grad} \psi) = -4\pi R^2 p' - FF', \quad (1)$$

in cylindrical coordinates (R, ϕ, Z) . Here the plasma pressure $p(\psi)$ and the toroidal flux $F(\psi)$ are functions of ψ and the primes indicate derivatives with respect to ψ . Eq. (1) has been solved [3, 9, 10], with the expressions of $p'(\psi)$ and $FF'(\psi)$ being given as linear or nonlinear functions in ψ .

The safety factor $q(\psi)$ is related to F by

$$q(\psi) = \frac{1}{4\pi^2} FV' \langle R^{-2} \rangle, \quad (2)$$

where

$$V' = 2\pi \oint \frac{d\ell}{B_p}, \quad V' \langle X \rangle = 2\pi \oint \frac{d\ell X}{B_p},$$

$B_p (= |\nabla\psi|/R)$ is the poloidal field, and $\oint d\ell$ is taken along a surface of constant ψ . For FCT equilibria, $q(\psi)$ is a given function: namely the q profile of the reference state where flux-conserving heating begins. Since $V' \langle R^{-2} \rangle$ can be evaluated exactly only when $\psi(R, Z)$ is known, Eq. (2) does not provide an explicit form of $F(\psi)$ for use in the right-hand side of Eq. (1). Previous numerical solutions of ψ with different values of $\beta_p [= 8\pi\bar{p}/(\langle B_p^2 \rangle / \langle B_p \rangle^2) \propto \bar{p}/I_p^2]$, using nonlinear forms of $p(\psi)$ and $FF'(\psi)$ to produce smooth toroidal current profiles, have shown that $q(\psi)$ steepens as β_p increases [3, 9].

A relationship between $FF'(\psi)$, $q(\psi)$, and $p'(\psi)$ can be obtained from the flux surface average of Eq. (1):

$$\frac{1}{V'\langle R^{-2} \rangle} (V'\langle B_p^2 \rangle)' = - \frac{1}{\langle R^{-2} \rangle} 4\pi p' - FF' . \quad (3)$$

Combining the left-hand side of Eq. (3) with Eq. (2) to eliminate V' , one obtains an ordinary differential equation (ODE) reminiscent of the "queer differential equation" discussed by Grad and Stevens [11]:

$$FF' \left(\frac{\langle B^2 \rangle}{\langle B_\phi^2 \rangle} \right) + F^2 \left[\left(\frac{\langle B_p^2 \rangle}{\langle B_\phi^2 \rangle} \right)' + \frac{q'}{q} \frac{\langle B_p^2 \rangle}{\langle B_\phi^2 \rangle} \right] + \frac{4\pi p'}{\langle R^{-2} \rangle} = 0, \quad (4)$$

where

$$\langle B^2 \rangle = \langle B_p^2 \rangle + \langle B_\phi^2 \rangle, \quad \langle B_\phi^2 \rangle = F^2 \langle R^{-2} \rangle.$$

Eq. (4) is one of several equivalent forms that relate FF' to q and p . This particular form has a close analogy in the cylindrical case, where a numerical scheme is easily developed (see Section 3).

Provided that $\langle B_p^2 \rangle$, $\langle B_\phi^2 \rangle$, $\langle R^{-2} \rangle$ and p' are known, the q term in Eq. (4) plays a dominant role in determining the form of $FF'(\psi)$.

The boundary condition required for Eq. (1) with a given RHS is that $\psi = \psi_b$ is specified at a given plasma boundary. The boundary condition required for the ODE (4) is that a value of $F^2(\psi_a)$ be specified by Eq. (2), where $\psi_b \geq \psi_a \geq \psi_m$ and $\psi = \psi_m$ at the magnetic axis. The conservation of the plasma poloidal flux,

$$(\psi_b - \psi_m) = \text{fixed}, \quad (5)$$

is achieved after an equilibrium is obtained, through scaling (ψ, P, F) to $(\alpha\psi, \alpha^2 P, \alpha F)$ using an appropriate constant α .

3. METHODS OF NUMERICAL SOLUTION

For given $q(\psi)$ and $p'(\psi)$, it is not immediately clear that Eqs. (1)-(5) have solutions. We study a cylindrical case to help answer this question, to provide some physical insight, and to find numerical schemes applicable to the toroidal case.

3.1 CYLINDRICAL CASE

In this case, Eq. (1) becomes

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d\psi}{dr} \right) = -4\pi p'(\psi) - B_z(\psi) B_z'(\psi), \quad B_\theta = -\frac{d\psi}{dr}, \quad (6)$$

where B_z is the "toroidal" field and r is the minor radius in the cylindrical coordinates (r, θ, z) . Also, the variable q used in the toroidal case can be replaced by the field line pitch μ given by

$$\mu = r B_z / B_\theta. \quad (7)$$

3.2 PLASMA SIZE EXPANSION (CONSTANT μ AND p')

The simplified case with constant μ and p' can be solved analytically according to the following equation:

$$r(r^2 + \mu^2) \frac{d B_\theta}{dr} + (r^2 - \mu^2) B_\theta - 4\pi p' r^3 = 0, \quad (8)$$

where we have substituted Eq. (7) into Eq. (6). The solutions for fixed $\psi(r=0)$ and $\psi(a) = 0$ can be written as

$$P = \pi h^2 p_0'^2 (a^2 - r^2) - C \pi h p_0' \ln \left(\frac{a^2 + \mu^2}{r^2 + \mu^2} \right), \quad (9)$$

$$B_z = 2\pi h p_0' \mu - 2\pi C / (r^2 + \mu^2), \quad (10)$$

$$C = (h - 1) a^2 p_0' / \ln \left(1 + \frac{a^2}{\mu^2} \right), \quad (11)$$

where $h (= p'/p_o')$ is a heating parameter and a is the plasma minor radius. Since $p_o' < 0$, Eq. (1) shows that for fixed a , $B_z(r=a)$ has to increase as h increases. This is expected; $B_z(r=0)$ has to decrease with respect to $B_z(r=a)$ as the plasma becomes progressively diamagnetic, and therefore $B_z(r=a)$ has to increase to preserve the total toroidal flux.

Here one sees two obvious ways to avoid a poloidal surface current: the vacuum B_z field could be increased, or for fixed vacuum B_z the plasma radius could be increased. The first of these may be difficult if large changes in B_z are required in times less or comparable to the energy containment time (~ 1 sec).

For the second case, Eqs. (10) and (11) give

$$h = (c_h - \frac{a_o^2}{\mu^2}) / (c_h - \frac{a^2}{\mu^2}), \quad c_h = (1 + \frac{a^2}{\mu^2}) \ln(1 + \frac{a^2}{\mu^2}), \quad (12)$$

where $a = a_o$ when $h = 1$. Now, based on Eq. (9), the ratio of the volume-averaged pressure can be written as

$$\eta = \frac{\bar{p}}{p_o} = \frac{a^2}{a_o^2} h^2 - 2h(h-1) \frac{a^2 + \mu^2}{a^2} [1 - \frac{\mu^2}{a^2} \ln(1 + \frac{a^2}{\mu^2})]. \quad (13)$$

Eqs. (13) and (14) are plotted in Fig. 1, which shows that a relatively small (10%) increase in minor radius corresponds to a large (four-fold) increase in the averaged pressure without inducing a surface current.

3.3 NUMERICAL PROCEDURES (NONCONSTANT μ AND p')

Use of Eq. (7) to eliminate B_z in the right-hand side of Eq. (6) gives

$$r(r^2 + \mu^2) \frac{d^2\psi}{dr^2} = -4\pi r^3 p' - (r^2 - \mu^2) \frac{d\psi}{dr} - r\mu' \left(\frac{d\psi}{dr}\right)^2. \quad (14)$$

This equation is readily solved by a successive over relaxation procedure if $p'(\psi)$ and $\mu(\psi)$ are prescribed [12]. However, the toroidal equations cannot be reduced to a form equivalent to Eq. (14).

Another approach is indicated if Eq. (7) is used in the left-hand side of Eq. (6) to eliminate $d\psi/dr$. One obtains

$$B_z B_z' \left(1 + \frac{r^2}{\mu^2}\right) + B_z^2 \left(\frac{2rr'}{\mu^2} - \frac{\mu' r^2}{\mu^3}\right) + 4\pi p' = 0, \quad (15)$$

where r is considered as a function of ψ . This and Eq. (6) suggest the use of a numerical scheme similar to that discussed by Grad and Stevens [11]. Eqs. (6) and (15) are solved alternatively, with the solution to one equation used in resolving the other equation.

The procedure may be symbolized as

$$[\psi(r)]_n \xrightarrow{\text{step 1}} [r(\psi)]_n \xrightarrow{\text{step 2}} [B_z B_z'(\psi)]_n \xrightarrow{\text{step 3}} [\psi(r)]_{n+1}, \quad (16)$$

where the subscript n denotes the n th time a corrected $B_z B_z'$ is obtained. In the above, step 1 involves an inversion of the solution $\psi(r)$, step 2 then solves Eq. (15) with the prescribed $\mu_T(\psi)$, and step 3 solves Eq. (6) using the $B_z B_z'(\psi)$ function just obtained to produce an improved $\psi(r)$, etc.

With 21 mesh points in r , and a given nonlinear function $p'(\psi)$, $\mu_T(\psi)$ is readily reproduced with an averaged derivation

$(\Delta\mu/\mu = |\mu_T - \mu|/\mu)$ of less than 0.1% after more than ten-fold

increases in plasma β . Typical results with fixed plasma radius are shown in Fig. 2. It is seen that, in addition to the well-digging by the pressure, the longitudinal current density J_z has to increase where ∇p increases. Also, the value of $B_z(r = a)$ immediately within the plasma boundary is seen to increase by nearly 10% when β increases from 1.6% to 17.3%. At a fixed vacuum B_z of 50 kG, this discontinuity corresponds to a poloidal surface current of over 0.4 MA/m. If $A = 4$ and $a = 1$ m, the total poloidal surface current is above 10 MA, which is 3 times the longitudinal plasma current.

It is clear that the discontinuity in B_z at the plasma surface should be avoided. In the cylindrical case this can be achieved by expanding the plasma minor radius or by increasing the vacuum B_z during the plasma heating. It will be shown in the next section that for the toroidal case the discontinuity shows an additional strong dependence on the width of the pressure profile.

Another numerical procedure is worth mentioning. In view of Eq. (7), one can formally express $B_z B_z'$ in terms of a polynomial in $\mu(\psi)$. Further approximation then leads to the proportionality

$$B_z B_z' \propto \mu^N, \quad (17)$$

where N is an integer ranging from 1 to 8. With Eq. (17), step 2 of the numerical procedure then becomes

$$[B_z B_z']_{n+1} = [B_z B_z']_n \mu_T^N / [\mu]_n^N. \quad (18)$$

This procedure is found to be less efficient than, but just as accurate as, the previous procedure.

3.4 TOROIDAL CASE

Procedures analagous to Eqs. (16) and (18) can be applied to Eqs. (1), (2), and (4). Here, we should replace $\psi(r)$ by $\psi(R, Z)$, B_z by F , μ by q , and we should replace step 1 in Eq. (16) by numerical calculations of the surface variables $\langle B_p^2 \rangle$, $\langle B_\phi^2 \rangle$, q , V' , and $\langle R^{-2} \rangle$.

For a given $q(\psi)$, $P(\psi)$ is the only other profile function to be prescribed. We choose the form

$$P(\psi) = h \left[C_1 (X - 1) + \frac{C_2}{2} (X^2 - 1) + \frac{C_3}{(L+1)} (X^{L+1} - 1) \right], \quad (19)$$

where

$$X = (\psi - \psi_m) / (\psi_b - \psi_m), \quad C_3 = -C_1 - C_2, \quad C_1 < 0.$$

Thus the pressure and current density are zero at ψ_b . For a given $q(\psi)$, it is seen that the size of (C_2/C_1) determines the pressure profile half-width w_p and the values of $V' \langle R^{-2} \rangle$, which are functions of ψ . In view of Eq. (2), the value of $F(\psi_b)$ depends on $V' \langle R^{-2} \rangle$ and q at ψ_b . Thus, for fixed values of $q(\psi_b)$, h , and C_1 , the value of $F(\psi_b)$ can be modified by C_2 and is in turn modified by w_p .

4. FCT EQUILIBRIUM RESULTS

Using a 40×33 grid and 30 equal divisions in ψ , we have regularly obtained high β ($> 10\%$) equilibrium for a D-shaped tokamak of aspect ratio $A = 4$. Using a prescribed $q(\psi)$ profile obtained from a low β equilibrium, (e.g., see Fig. 3), the high β equilibria regularly reproduce the $q(\psi)$ profile to within 0.1%. However, this corresponds to an accuracy in determining $F(\psi)$ only within $\sim 1\%$. By using a 60×50 grid and 40 divisions in ψ , accuracies in determining $q(\psi)$ and $F(\psi)$ can be improved to 0.03% and 0.3%, respectively. By a further increase to 72×60 , an accuracy for $F(\psi_b)$ in the neighborhood of 0.1% can be obtained. This level of exactness in $F(\psi)$ is necessary to ensure that the total poloidal surface current is calculated with an accuracy within a few percent of the toroidal plasma current.

For the $q(\psi)$ given in Fig. 3 and with fixed plasma boundary, the dependence of $F(\psi_b)$, I_p , β , and β_p on the pressure half-width w_p (normalized to the minor width of the plasma) is exemplified by Fig. 4(a-c) for various h values. It is seen that for a given h , the values of $F(\psi_b)$, I_p , β , and β_p increase with w_p over a range roughly from 0.40 to 0.55. Beyond this range, numerical procedures have resulted in solutions with finite ΔQ values that do not decrease with increased grid size. This seems to indicate that the FCT equilibria with respect to the reference safety factor of Fig. 3 exist only within the w_p range, and that outside of this range only non-FCT equilibria exist.

This window for access to high β FCT equilibria is depicted in Fig. 5. It indicates that for fixed β the discontinuity ΔB_ϕ decreases with increasing w_p . It is seen that within the available range of w_p , $\Delta F(\psi_b)$ can be reduced by more than half, with roughly a 25% increase in w_p . This reduction in B_ϕ is believed to be similar to that due to the expansion of the plasma boundary, except that it now occurs at the inner flux surfaces.

At $\beta = 10\%$ and with $w_p \approx 0.55$, there is a discontinuity of $\sim 1\%$ in B_ϕ at the fixed plasma boundary. The total poloidal surface current corresponds to $1/3$ of the plasma current. This situation motivated us to examine how a small increase in plasma size influences ΔB_ϕ . Calculations similar to those represented by Figs. 4 and 5 were performed after a 1.7% increase in the plasma minor radius. The results are summarized in Fig. 6, which shows that with this increase of plasma size, plasma heating to 15-20% can be achieved without inducing poloidal surface current.

Typical ψ plots, and the current and pressure profiles for these equilibria are given in Figs. 7 and 8, respectively. We find that only negligible reversed current occurs for the cases with larger w_p and then only when $\beta > 10\%$. Also, the current peaks progressively outward as β increases, and it has a half-width near $1/3$ of the plasma minor width when $\beta = 10\%$. As β approaches 20%, the current width approaches $1/5$ of the plasma minor width. It is believed that values of β higher than 20% [13] can be obtained by improving the accuracy of the numerical calculations. However, at higher β the current profile would become increasingly more narrow, move further toward

the plasma edge, and make the assumption of flux-conservation less reasonable.

On the other hand, without assuming flux-conservation, the equilibrium at a 10% β seems sufficiently attractive to suggest experiments that actively modify the current profile to produce high β , high q_a equilibria.

The dependences of I_p , β_p and ℓ_i [$= (\text{volume av. } \bar{B}_p^2) / \bar{B}_p^2 \text{ at edge}$] on β are plotted in Fig. 9, which shows nearly 60% and 100% increases in I_p as β increases from 1% to 10% and 20%, respectively. The values of β_p reach 4.7 and 5.9 at the latter values of β . Both I_p and β_p show no signs of saturation for the calculated range of β , while ℓ_i is seen to decrease slowly with β . These results are in qualitative agreement with the theoretical results obtained by Clarke [5] and by Clarke and Sigmar [14], who use a high β , circular plasma model. The I_p can be considered as being inductively driven by outward shifts of inner flux surfaces that result from heating.

We have also obtained FCT equilibria for reduced values of q_a and for circular plasmas. For the same aspect ratio of 4, β values up to 28% have been obtained with $q_a = 2.1$ in the D-shaped plasma (see Fig. 10). In the case of a circular plasma with $q_a = 2.9$, values up to 12% have been obtained.

5. DISCUSSION AND CONCLUSION

We have obtained axisymmetric, high β MHD equilibria with the prescribed safety factor $q(\psi)$ and pressure $p(\psi)$. If the total poloidal flux $(\psi_b - \psi_m)$ is also required to be fixed, these equilibria then apply to a flux-conserving tokamak (FCT), where plasma heating occurs in a time shorter than the plasma resistive skin time. With $q_a = 4.8$ and $q_o = 1$, β values up to 20% have been obtained in a D-shaped plasma. In a circular plasma, values of β up to 12% have been obtained with $q(\psi_b) = 2.9$. By reducing q_a to 2.1 in a D-shaped plasma, values of β up to 28% have been calculated.

We have found that the toroidal plasma current progressively peaks toward the outer region of the pressure profile as β increases. Thus active current profile modification (e.g., by neutral beam injection) becomes necessary to produce or maintain high β current configuration beyond the plasma resistive time. The plasma current I_p and β_p are found to increase with β . Thus, OH power supplies that permit such current rises should be used when rapid, non-ohmic plasma heating is applied.

The toroidal field immediately within a constant plasma boundary is found to increase by a few percent when β is increased in an FCT fashion. For a fixed vacuum field, the changes would induce poloidal surface currents of the same order of the toroidal plasma current. This process is close to the inverse of the theta pinch experiments [6], where skinned currents of a much larger relative magnitude are intentionally produced for the purpose of plasma heating.

Since surface currents have been observed to introduce relatively rapid flux diffusion across the boundary, thus reducing the accessibility of high β FCT equilibria, methods by which such surface currents can be eliminated are discussed. These methods include increasing the vacuum toroidal field (by $\sim 5\%$), the plasma minor radius (by $\sim 2\%$), and the pressure profile width (by $\sim 25\%$) for β values up to 20%. It is assumed that an appropriate poloidal field system is used to center and shape the plasma as rapid heating occurs.

Rapid adjustment to the vacuum toroidal field is essentially limited by the relatively large reactive power supply required. Rapid changes in the plasma minor cross section seems feasible only in plasmas bounded by magnetic separatrixes. Also, the plasma expansion would have to displace toroidal flux and thus is inductively coupled to the toroidal field coils. The characteristics of the toroidal field power-supply circuit therefore play an important role in determining the plasma size during heating. Of equal importance are modifications in the pressure profile width by intensive neutral beams with appropriately chosen beam energy, or energies, and adjustable energy deposition profiles. This together with the demonstrated heating capability [15], seems to suggest the neutral beam as a primary method to reach high β in tokamaks. These mechanisms appear practical according to present day technologies and suggest themselves as important experimental aspects of a high β , flux-conserving tokamak.

We conclude that equilibria with β from 10-20% and $q_a \sim 5$ exist. They are most likely to be achieved when large and rapid heating is applied to a relatively hot tokamak plasma where the flux is

conserved. Several other investigations now become interesting.

These include the possible limits on β set by stability requirements and those on w_p set by energy balance requirements.

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Fig. 1 The dependences of the heating ratio $\eta (= \bar{p}/\bar{p}_0)$ and the heating parameter h on plasma radius (a/a_0) in a cylindrical FCT plasma with uniform μ and p' and zero surface current.

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Fig. 2 Typical behavior of pressure p , longitudinal field B_z , and longitudinal current J_z before ($\beta = 1.6\%$) and after ($\beta = 17.3\%$) flux conserving heating in a cylindrical plasma where $\mu(\psi)$ and $p'(\psi)$ are not uniform.

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Fig. 3 Typical profile of $q(\psi)$ for a D-shaped FCT with $A = 4$.

Fig. 4 The dependences of β , β_p , I_p , and $\Delta B_\phi/B_\phi$ (at the plasma edge) on the normalized half-width w_p of the pressure profile in a D-shaped plasma. The safety factor $q(\ )$ is given in Fig. 3 and (a) $h = 5$, (b) $h = 10$, (c) $h = 15.4$. FCT equilibria are found only within the range covered by solid lines.

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Fig. 5 The window for access to high β FCT equilibria with fixed plasma radius heated from the low β (1.2%) equilibrium of Fig. 9a.

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Fig. 6 The window for access to high β FCT equilibria with a 1.7% increase in the plasma minor radius, heated from the low β (1.2%) equilibrium of Fig. 9a.

Fig. 7 Typical flux surfaces for FCT equilibria at different β and β_p values. Equilibria with $\beta_p > A$ are readily obtained. Significant deformation of the inner flux surfaces is seen.

Fig. 8 Typical evolution of (a) the pressure and (b) the current profiles during FCT heating. The magnetic axis shifts by a $1/4$ of the horizontal plasma width when β reaches near 20%.

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Fig. 9 The variation of I_p , β_p and l_i with β during FCT heating as indicated by Figs. 4-6. Each continuous curve represents the results of a given h [Eq. (19)] but increasing w_p and β . For a given β , the influence of w_p on I_p and β_p is negligibly small, while that on l_i is significant.

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Fig. 10 (a) The flux surfaces and (b) the profiles of a typical high β equilibrium with $\beta = 28\%$, $\beta_p = 3.75$, $q_0 = 1$, and $q_a = 2.1$ in a tokamak with $A = 4$.

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