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APPROXIMATE CALCULATIONS FOR THE REACTIVITY EFFECTS OF VOIDS IN NUCLEAR REACTORS

GLEN E. PUTNAM

PREPARED BY AEROJET NUCLEAR COMPANY FOR
ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

IDAHO OPERATIONS OFFICE UNDER CONTRACT E(10-1) -1375

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**APPROXIMATE CALCULATIONS
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ABSTRACT

In this report, approximate methods are derived that enable the effects of certain kinds of void regions in nuclear reactors to be adequately modeled at greatly reduced costs. This reduction in costs is accomplished by: (a) specifying void boundary conditions so as to reduce the problem to a simple albedo type; (b) reducing the number of spatial dimensions; and (c) using diffusion theory, where possible, rather than transport theory or Monte Carlo methods.

SUMMARY

The problem of computing the reactivity effect of a void region in a nuclear reactor can be solved if, as at the outer reactor boundaries, suitable boundary conditions are imposed on the void boundaries (or surfaces). Conditions are described that enable the void boundary conditions to be reduced to a simple albedo (α) type for transport theory or Monte Carlo applications. In many cases in which the reactor is uniform in a direction parallel to a void that extends completely through the reactor along one of the dimensions, an α averaged over that dimension can be used. In such cases, the spatial variable of that dimension can be eliminated and an order of magnitude reduction in the difficulty and expense of solving the problem can be effected. In some cases, two dimensions can be eliminated in this manner. Another order of magnitude reduction in the difficulty and cost of solving the problem can be effected in those cases in which diffusion theory could be used except for the presence of the void. In most of these cases, a diffusion theory solution is again enabled by the application of a current-to-flux ratio ($Q = J/\phi$) boundary condition that is approximately equivalent to the albedo condition.

In cases in which the variation of α or Q on the void surface must be handled directly (thus prohibiting application of an average over an entire dimension), methods are described for finding α or Q values to be applied to the different segments of the void surface. Approximate methods suitable for most cases are derived for the calculation of the albedos and the current-to-flux ratios that are needed for application at the void boundaries.

For diffusion theory programs that do not allow the Q -type boundary condition on internal surfaces, methods are presented for the calculation of constants that can be used in the void region as the regular absorption cross section Σ_a and the diffusion coefficient D that will force the $J = Q\phi$ boundary condition in one-dimensional cases and approximately force it in two- and three-dimensional cases when the void is shaped as a right cylinder or a slab. For the right cylindrical void extending through the reactor, curves are given for the average α , Q , Σ_a , and D values (versus the reactor height-to-void diameter ratio, H/d) for those cases in which the reactor can be assumed uniform in the direction parallel to the axis of the cylinder.

In a final section, the integrals necessary for the calculation of the α s and Q s are reduced to forms easily integrable by numerical methods. In some of the more difficult cases, the numerical methods are derived and presented in detail.

CONTENTS

ABSTRACT	ii
SUMMARY	iii
I. INTRODUCTION	1
II. METHODS AND ASSUMPTIONS	2
1. DESCRIPTION OF THE PROBLEM	2
2. REDUCTION OF THE NUMBER OF SPATIAL VARIABLES	3
3. SOLUTION OF PROBLEMS WITH DIFFUSION THEORY AND THE HANDLING OF INSEPARABLE PROBLEMS	6
4. THE CALCULATION OF THE ALBEDOS	9
5. FORCING THE BOUNDARY CONDITION $J = Q\phi$	11
5.1 Basic Assumptions	11
5.2 The Cylindrical Void Case	11
5.3 The Slab Void Case	23
5.4 Mesh Spacing Considerations When Forcing the $J = Q\phi$ Boundary Conditions with D_3 and $\Sigma_a s$ in the Void Region	26
6. NUMERICAL INTEGRATION OF THE ALBEDO INTEGRALS	29
6.1 The Cylindrical Void	29
6.2 The Slab Void	36

FIGURES

1. Average albedo, α , versus H/d	16
2. Q versus H/d	17
3. $\Sigma_a H$ versus H/d [for use with $B^2 = (\Pi/H)^2$ in the void — Option C.2.a; D/H used from Figure 4]	18
4. D/H versus H/d [for use with $\Sigma_a H$ for Option C.2.a with $B^2 = (\Pi/H)^2$ in the void or for use with $\Sigma_a H$ for Option C.2.b with $B^2 = 0$ in the void]	19
5. $\Sigma_a H$ versus H/d (for use with $B^2 = 0$ in the void for Option C.2.b)	21

6.	D/H versus H/d [for use with $\Sigma_a = 0$ and $B^2 = (\Pi/H)^2$ in the void for Option C.2.c]	22
7.	Coordinate system for the cylindrical void	31
8.	Coordinate system for the slab void	37

TABLE

I.	Average α , Q, $\Sigma_a H$, and D/H Values Versus H/d for a Cylindrical Void	35
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APPROXIMATE CALCULATIONS FOR THE REACTIVITY EFFECTS OF VOIDS IN NUCLEAR REACTORS

I. INTRODUCTION

The occurrence of a void region (or a very low-density region) in a reactor allows neutrons leaving one point on the region surface to proceed by direct flight to a distant point of greater or lesser neutron importance. Since this phenomenon can be rigorously represented only with transport theory or Monte Carlo methods, the cost of adequately predicting the effect of the void on reactivity or flux distributions is often prohibitive. In some cases, three-dimensional (3-D) or 2-D transport theory solutions of the problems may be economically feasible, but there is a great incentive to reduce the problems, if possible, to 2-D or 1-D cases. This incentive exists because the solution cost is reduced by about an order of magnitude by the elimination of a space variable. Solution costs can be reduced by an additional order of magnitude if the effects of the void can somehow be satisfactorily approximated by employing a diffusion theory code instead of a transport theory or Monte Carlo code.

In this report, approximate methods are derived that enable the effects of certain kinds of void regions to be adequately predicted by solving the problem with fewer spatial dimensions than ordinarily required and by using diffusion theory rather than transport theory or Monte Carlo methods.

II. METHODS AND ASSUMPTIONS

1. DESCRIPTION OF THE PROBLEM

The effect of a void region or a low-density region^[a] in a reactor can be handled rigorously if the number and angular distribution of the neutrons entering the surface from the void at each point on the surface of the region are known and imposed as a boundary condition. Such a boundary condition can be imposed in terms of a pointwise albedo $\alpha(\bar{r}, \bar{\Omega}_i)$ defined as

$$\alpha(\bar{r}, \bar{\Omega}_i) = \frac{j_i(\bar{r})}{j_o(\bar{r})} f(\bar{r}, \bar{\Omega}_i) \quad (1)$$

where the subscripts i and o denote neutron directions into the surface (from the void) and out of the surface (into the void), respectively.

The definitions of the functions in Equation (1) are

$$j_o(\bar{r}) \equiv \int_{\Omega_o} N(\bar{r}, \bar{\Omega}_o) \bar{n} \cdot \bar{\Omega}_o d\Omega \quad (\text{the current out of the surface into the void}) \quad (2)$$

$$j_i(\bar{r}) \equiv - \int_{\Omega_i} N(\bar{r}, \bar{\Omega}_i) \bar{n} \cdot \bar{\Omega}_i d\Omega \quad (\text{the current from the void into the surface}) \quad (3)$$

$$f(\bar{r}, \bar{\Omega}_i) = -N(\bar{r}, \bar{\Omega}_i) / \int_{\Omega_i} N(\bar{r}, \bar{\Omega}_i) \bar{n} \cdot \bar{\Omega}_i d\Omega = N(\bar{r}, \bar{\Omega}_i) / j_i(\bar{r}) \quad (4)$$

hence

$$\int_{\Omega_i} f(\bar{r}, \bar{\Omega}_i) \bar{n} \cdot \bar{\Omega}_i d\Omega = -1 \quad (5)$$

where

$\bar{n} \equiv$ the unit vector normal to the surface of the void at point $\bar{r}$ (pointing into the void)

$\bar{\Omega} \equiv$ the unit vector denoting neutron direction

[a] A low-density region is one in which the neutron mean free path is so large that effectively all neutrons can cross it without a collision.

$N(\bar{r}, \bar{\Omega}) \equiv$ the angular flux; i.e., the number of neutrons emitted per unit time per unit area ($\perp$ to $\bar{\Omega}$) per unit solid angle

$\Omega_0 \equiv$ the solid angle in which $\bar{\Omega}_0$ is directed into the void

$\Omega_i \equiv$ the solid angle in which $\bar{\Omega}_i$ is directed into the surface from the void.

In these equations, the energy variable has been dropped for simplicity. All the energy-dependent quantities are either functions of the same energy or belong to the same energy group.

The boundary condition imposed at each point on the void surface can be written

$$N(\bar{r}, \bar{\Omega}_i) = \alpha(\bar{r}, \bar{\Omega}_i) j_0(\bar{r}). \quad (6)$$

Solving the problem using these boundary conditions is probably more difficult than representing the hole explicitly, because of the added problem of finding the $\alpha(\bar{r}, \bar{\Omega}_i)$ functions. However, in many cases, some simplifying assumptions can be justified that allow an approximate form of this boundary condition problem to be reduced in difficulty by eliminating the need to consider one or two of the spatial variables or by allowing the use of diffusion theory instead of transport theory.

2. REDUCTION OF THE NUMBER OF SPATIAL VARIABLES

The effect of the void on the reactivity of the reactor is assumed to be the primary concern in the problem so that the methods used to reduce the number of spatial variables of the problem will be based on preserving the effect of the void on reactivity. The reactivity effect of the void is easily shown to be

$$\Delta\rho = \frac{1}{U} \left[\int_S \int_{\Omega_0} \psi^*(\bar{r}, \bar{\Omega}_0) N(\bar{r}, \bar{\Omega}_0) \bar{n} \cdot \bar{\Omega}_0 d\Omega ds \right. \\ \left. + \int_S \int_{\Omega_i} \psi^*(\bar{r}, \bar{\Omega}_i) f(\bar{r}, \bar{\Omega}_i) j_i(\bar{r}) \bar{n} \cdot \bar{\Omega}_i d\Omega ds \right] \quad (7)$$

where $\psi^*(\bar{r}, \bar{\Omega})$ is the neutron importance (or adjoint) function^[a], U is the integral of the importance weighted fission neutron source, and the integrals shown are over the void surface, S . This equation expresses the fact that the reactivity effect of placing the void in the reactor is the importance weighted average (over the void surface) of the new neutron current into the void. The first term is the weighted average current into the void, and the second term is the weighted average current from the void.

To simplify Equation (7), the neutron importance function is assumed to vary little with $\bar{\Omega}$ so that it can be replaced by the average importance ϕ^* , where

$$\phi^* \equiv \frac{1}{4\pi} \int_{\Omega} \psi^*(\bar{r}, \bar{\Omega}) d\Omega . \quad (8)$$

This substitution makes possible the reduction of Equation (7) to

$$\Delta\rho = \frac{1}{U} \left[\int_S \phi^*(\bar{r}) j_o(\bar{r}) ds - \int_S \phi^*(\bar{r}) j_i(\bar{r}) ds \right] \quad (9)$$

or

$$\Delta\rho = \frac{1}{U} \left[\int_S \phi^*(\bar{r}) j_o(\bar{r}) ds - \int_S \phi^*(\bar{r}) \alpha(\bar{r}) j_o(\bar{r}) ds \right] \quad (10)$$

or

$$\Delta\rho = \frac{1}{U} \int_S \phi^*(\bar{r}) J(\bar{r}) ds \quad (11)$$

where $J(\bar{r})$ is the net current defined as

$$J(\bar{r}) \equiv j_o(\bar{r}) - j_i(\bar{r}) \quad (12)$$

and $\alpha(\bar{r})$ is defined as

$$\alpha(\bar{r}) \equiv - \int_{\Omega_i} \alpha(\bar{r}, \bar{\Omega}_i) \bar{n} \cdot \bar{\Omega}_i d\Omega = j_i(\bar{r}) / j_o(\bar{r}) . \quad (13)$$

[a] The adjoint can be that of either the perturbed (void present) reactor or the unperturbed reactor so long as the neutron flux is that of the unperturbed or the perturbed, respectively. However, the fluxes computed hereafter are for the case with the void present so that the adjoint is required to be that for the reactor without the void.

Now, a reactor problem is considered in which the solution for the neutron fluxes and currents can be adequately represented by a product of functions of the spatial variables when the void is not present. That is, the scalar flux, for example, can be expressed as a separable function of the form $f(r,\theta)g(x)$ or $f(y,z)g(x)$. Next, consider a void extending from one end ($-X$) of the reactor to the other (X) — a void that has uniform shape, cross sectional area, and position in the r,θ or z,y plane regardless of x — such as a right cylinder with its axis parallel to x . When this void is placed in the reactor, the solutions may no longer be well represented as separable functions because the void allows a redistribution of the neutron densities on the void surface in the x direction so that $J(\bar{r})$, $\phi(\bar{r})$ (the scalar flux), $j_0(\bar{r})$, and $j_1(\bar{r})$ are not proportional to the same function of x . However, the problem can be restored to a type again allowing separable solutions if $\alpha(\bar{r})$ is constant with regard to x ; hence, $j_1(\bar{r})$, $j_0(\bar{r})$, $J(\bar{r})$, and $\phi(\bar{r})$ on the void surface are each again represented by a function of the other two variables times the same function of x .

The void in the new problem will cause the same reactivity change provided an average α is defined so that

$$\int_{-X}^X \phi^* j_0 (1-\alpha) dx = \int_{-X}^X \phi^* j_0 dx - \int_{-X}^X \phi^* j_1 dx \quad (14)$$

where the dependence of ϕ^* , j_0 , and j_1 on the r,θ or z,y variables has been omitted for the sake of simplicity. This equation can be solved for α to obtain the obvious result

$$\alpha = \int_{-X}^X \phi^* j_1 dx / \int_{-X}^X \phi^* j_0 dx. \quad (15)$$

Since the solution to the problem using the average α is assumed separable, the reactivity effect of the void can be calculated without consideration of the x variable. This elimination of the x variable reduces a three-dimensional (3-D) problem to a 2-D one or a 2-D problem to a 1-D one. Furthermore, if the solution allows separability in both the x and y directions, if the void extends from one end ($-Y$) or the reactor to the other end (Y), and if the void also satisfies the same conditions on constancy of shape, cross sectional area, and z position as was imposed for the void in the x direction, the same methods can be used to eliminate the y variable from the void reactivity problem. That is, the importance weighted average value of the albedo α becomes

$$\alpha = \int_{-Y}^Y \int_{-X}^X \phi^* j_1 dx dy / \int_{-Y}^Y \int_{-X}^X \phi^* j_0 dx dy. \quad (16)$$

Finally, the use of either the average albedo α or a $J = Q\phi$ type of boundary condition in this reduced 1-D problem completely decouples the reactor into two independent regions – one on either side of the void. Hence a further simplification is effected if interest is in the solution for only one of the regions.

3. SOLUTION OF PROBLEMS WITH DIFFUSION THEORY AND THE HANDLING OF INSEPARABLE PROBLEMS

The solution of the problem with the average α requires transport theory or Monte Carlo methods because j_0 and j_1 are not defined in diffusion theory. However, the fact that the ratio of $J(\bar{r})$ to $\phi(\bar{r})$ is constant with regard to x in the new problem does allow a diffusion theory solution with a boundary condition of the type

$$J(\bar{r}) = Q\phi(\bar{r}) \quad (17)$$

imposed at the void surface. Again, with the reactivity effect of the void conserved, the value of Q can be computed from the equation

$$\int_{-X}^X \phi^* Q \phi dx = \int_{-X}^X \phi^* J dx \quad (18)$$

Therefore, Q can be computed from

$$Q = \frac{\int_{-X}^X \phi^* (j_0 - j_1) dx}{\int_{-X}^X \phi^* \phi dx} \quad (19)$$

or

$$Q = (1-\alpha) / \left[\int_{-X}^X \phi^* \phi dx / \int_{-X}^X \phi^* j_0 dx \right] \quad (20)$$

The reduction of the problem to one with one less spatial variable requires a knowledge of the quantities

$$\int_{-X}^X \phi^* j_i dx \quad \text{and} \quad \int_{-X}^X \phi^* j_0 dx$$

to find α for a transport theory solution, and the additional knowledge of $\int_{-X}^X \phi^* \phi dx$ is required to make possible a diffusion theory solution with Q applied at the void boundary.

To find $\int_{-X}^X \phi^* \phi dx$, it will be assumed here that the relationship of ϕ , J , j_0 , and j_i can be derived from the often used assumption that $N(\bar{r}, \bar{\Omega})$ is expressible as a $P_1(u)$ function at the void surface (where $u = \bar{n} \cdot \bar{\Omega}$). That is

$$N(\bar{r}, \bar{\Omega}) = \frac{\phi}{4\pi} + \frac{3J}{4\pi} \bar{n} \cdot \bar{\Omega} \quad (21)$$

from which is readily derived

$$j_0 = \frac{\phi}{4} + \frac{J}{2} \quad (22)$$

and

$$j_i = \frac{\phi}{4} - \frac{J}{2} \quad (23)$$

If Equations (22) and (23) are used to solve for ϕ , the following is obtained

$$\phi = 2(j_0 + j_i) \quad (24)$$

and Equation (20) for Q can be immediately reduced to

$$Q = (1-\alpha)/[2(1+\alpha)] \quad (25)$$

Hence, the solution of the problem with either transport or diffusion theory centers on finding a suitable method to compute the average albedo α .

In cases in which the solution to the problem is not separable or the conformation of the void may not meet the conditions required in Section II-2, the effects of the void can be treated by the application of a pointwise variable $\alpha(\bar{r})$ or a pointwise variable boundary condition of the form

$$J(\bar{r}) = Q(\bar{r})\phi(\bar{r}) \quad (26)$$

on the surface of the void. In this case, the variable $Q(\bar{r})$ is again approximated by Equation (25) from the pointwise variable albedo $\alpha(\bar{r})$ computed from

$$\alpha(\bar{r}) = j_i(\bar{r})/j_o(\bar{r}) \quad (27)$$

The application of pointwise variable boundary conditions may be impractical, but it should be sufficient to treat the problem by partitioning the void surface into a number of surface segments S_j and applying a constant average value, α_j or Q_j , on each segment. The average Q_j is computed from the average albedo α_j as

$$Q_j = (1-\alpha_j)/2(1+\alpha_j) \quad (28)$$

where

$$\alpha_j = \int_{S_j} \phi^*(\bar{r}) j_i(\bar{r}) ds / \int_{S_j} \phi^*(\bar{r}) j_o(\bar{r}) ds \quad (29)$$

In cases in which $\phi^*(\bar{r})$ varies little over the surface element S_j , it is, of course, sufficient to compute

$$\alpha_j = \int_{S_j} j_i(\bar{r}) ds / \int_{S_j} j_o(\bar{r}) ds \quad (30)$$

4. THE CALCULATION OF THE ALBEDOS

A rigorous calculation of the average albedos from Equations (15), (16), (29), and (30) poses a problem as difficult to solve as the original reactor problem, for which transport theory or Monte Carlo methods are required.

The calculation of $j_i(\vec{r})$ and $j_o(\vec{r})$ requires the knowledge of $N(\vec{n}, \vec{\Omega}_0)$, since, by definition

$$j_o(\vec{r}) \equiv \int_{\Omega_0} N(\vec{r}, \vec{\Omega}_0) \vec{n} \cdot \vec{\Omega}_0 d\Omega \quad (31)$$

and

$$j_i(\vec{r}) = \int_S \frac{N(\vec{r}', \vec{\Omega}_0) \cos \gamma' \cos \gamma ds'}{\rho^2} \quad (32)$$

where the quantities $\cos \gamma'$, $\cos \gamma$, and ρ are defined in terms of the vectors shown on the following sketch.

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In the sketch

$$\bar{\rho} = \bar{r} - \bar{r}' = \bar{\Omega}_0 \rho \quad (\text{where } \rho = |\bar{\rho}|) \quad (33)$$

$\bar{\Omega}_0$ = the unit vector denoting the neutron direction along $\bar{\rho}$ from the surface element ds' to surface element ds

$\bar{n}'$ = the unit vector normal to ds' and directed into the void between ds' and ds

$\bar{n}$ = the similar unit vector normal to ds

$$\cos \gamma' = \bar{\Omega}_0 \cdot \bar{n}' = \bar{\rho} \cdot \bar{n}' / \rho \quad (34)$$

$$\cos \gamma = -\bar{\rho} \cdot \bar{n} / \rho. \quad (35)$$

In order to obtain an approximate value for $j_i(\bar{r})$ from Equation (32) without exact knowledge of $N(\bar{r}', \bar{\Omega}_0)$, it will now be assumed that $N(\bar{r}', \bar{\Omega}_0)$ can be approximated by an isotropic function.

In this case

$$j_0(\bar{r}) = \int_{\Omega_0} N(\bar{r}', \bar{\Omega}_0) \bar{\Omega}_0 \cdot \bar{n}' d\Omega = \pi N(\bar{r}', \bar{\Omega}_0) \quad (36)$$

so that Equation (32) can be written

$$j_i(\bar{r}) = \frac{1}{\pi} \int_S \frac{j_0(\bar{r}) \cos \gamma' \cos \gamma ds'}{\rho^2}. \quad (37)$$

The equation for the average albedo for any segment S_j of the surface S is, therefore

$$\alpha_j = \frac{1}{\pi} \int_{S_j} \phi^*(\bar{r}) \int_S \frac{j_0(\bar{r}) \cos \gamma' \cos \gamma ds'}{\rho^2} ds' ds \quad / \quad \int_{S_j} \phi^* j_0(\bar{r}) ds. \quad (38)$$

The integrals in this expression for α_j can usually be evaluated (numerically, if necessary) for the given void configuration if suitable approximations for $j_0(\vec{r})$ and $\phi^*(\vec{r})$ can be found. In many cases, $j_0(\vec{r})$ can be assumed proportional to the scalar flux $\phi(\vec{r})$ as computed for the case without the void perturbation. The function $\phi^*(\vec{r})$ should correspond to the one for the case unperturbed by the void, but, in many cases, in which it may be inconvenient to compute $\phi^*(\vec{r})$ for the unperturbed reactor case it is not unreasonable to assume that $\phi^*(\vec{r})$ is equal to the scalar flux; that is, $\phi(\vec{r})$.

5. FORCING THE BOUNDARY CONDITION $J = Q\phi$

5.1 Basic Assumptions

Not all diffusion theory codes allow the boundary condition $J = Q\phi$ to be imposed directly on a boundary surface. When such codes are used, the boundary condition can be more or less closely forced by choosing appropriate values for the diffusion theory constants, D and Σ_a , to use in the void volume adjacent to the boundary surface.

Only two cases will be considered: (a) a cylindrical void of diameter d with its axis parallel to the x direction and perpendicular to the r, θ or z, y plane, and (b) a slab-shaped void of width W , with the slab surfaces at $z = -W/2$ and $z = W/2$, where z is perpendicular to the x and y directions.

It will be assumed that the flux on the surface of the cylindrical void varies only with position x along the axis of the cylinder (and not with an azimuthal angle perpendicular to x and radius r) and that the flux on the slab void surface is the same at $z = -W/2$ and $z = W/2$ at a given x, y point.

It will further be assumed that the solution for the fluxes in the void region (with the chosen D and Σ_a values in the void) are "piecewise" separable. That is, the flux in the cylindrical void region can be expressed in the form $\phi(r, x) = f(r)g(x)$, and the flux in the slab can be expressed in the form $f(z)g(x)h(y)$ in each region opposite a given rectangular area on the x, y surface.

5.2 The Cylindrical Void Case

5.2.1 The General Expressions for Σ_a and D to Force $J = Q\phi$. Under the condition that $\phi(r, x)$ is separable, the diffusion equation in the void can be written

$$-g(x) \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial f(r)}{\partial r} - f(r) \frac{\partial^2 g(x)}{\partial x^2} + \frac{\Sigma_a}{D} f(r) g(x) = 0. \quad (39)$$

Since Σ_a and D do not vary with x in the given x interval, Equation (39) can be written

$$-\frac{1}{f(r)} \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial f(r)}{\partial r} + \frac{\Sigma_a}{D} = \frac{\partial^2 g(x)}{g(x) \partial x^2} = -B_x^2 \quad (40)$$

The equation for $f(r)$ is, therefore,

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial f(r)}{\partial r} + \frac{\Sigma_a + DB_x^2}{D} f(r) = 0 \quad (41)$$

for which the solution is

$$f(r) = c I_0(Kr) \quad (42)$$

where I_0 is the Modified Bessel Function of the First Kind of order zero, c is some constant, and

$$K = \sqrt{(\Sigma_a + DB_x^2)/D} \quad (43)$$

At the void surface at $r = \text{radius } a$, the desired boundary conditions is

$$J(a, x) = Q\phi(a, x) = Qc I_0(Ka) g(x) \quad (44)$$

where J is the net current into the void region, as represented in diffusion theory by

$$J(a, x) = D \left. \frac{\partial f(r)}{\partial r} \right|_{r=a} g(x) = Dc K I_1(Ka) g(x) \quad (45)$$

By substituting $J(a, x)$ from Equation (45) in the right side of Equation (44) the following is obtained

$$DK I_1(Ka) = Q I_0(Ka) \quad (46)$$

after dividing c and $g(x)$ from both sides. After some manipulation, Equation (46) can be rewritten

$$DK^2 = \frac{4Q}{d} R(Ka) \quad (47)$$

where

$$R(Ka) = \frac{Ka}{2} \frac{I_0(Ka)}{I_1(Ka)} \quad (48)$$

Since $DK^2 = \Sigma_a + DB_x^2$, the boundary condition Equation (44) is forced, according to Equation (47), if the total effective cross section in the void, $\Sigma_a + DB_x^2$, is

$$\Sigma_a + DB_x^2 = \frac{4Q}{d} R(Ka) \quad (49)$$

and if the diffusion coefficient D is

$$D = 4QR(Ka) / (dK^2) \quad (50)$$

Some special cases can now be considered.

5.2.2 Case "Cylindrical.1" (C.1) Approximately Forcing the $J = Q\phi$ Boundary Condition in Piecewise Fashion on Intervals of x . In this case, some assumption must be made concerning the value of DB_x^2 in the given x interval in order to apply Equation (49) to determine Σ_a . In the absence of any means of estimation of DB_x^2 , it is suggested that a fairly large value of K^2 be chosen (since K^2 is arbitrary) to make D quite small. A small D tends to justify an assumption either that

$$DB_x^2 \ll \Sigma_a \quad (51)$$

or that DB_x^2 is small compared to the neutron removal cross sections in the materials adjacent to the void. That is, if both Σ_a and DB_x^2 are small enough, there is little loss or gain of neutrons in the void region and the inequality of Equation (51) is irrelevant.

For reasons to be discussed later, it is desirable to keep the value of $|K^2 a^2|$ from becoming much larger than unity, perhaps 1.0 or 2.0. If this is done, the function $R(Ka)$ is not far from unity. For $|K^2 a^2| \leq 2$, an approximation for $R(Ka)$ that is less than 4% in error

is $R(Ka) \doteq 1 + K^2 a^2 / 8$. Hence, $R(Ka) \doteq 1 \pm \sim 0.25$ for $|K^2 a^2| \leq 2$. If $|K^2 a^2|$ is chosen in this range, an approximate expression for D can then be derived from Equation (50) as

$$D \doteq d\bar{Q}/K^2 a^2 (\pm \sim 25\%). \quad (52)$$

For the given Q and $d (=2a)$ values the user can then quickly determine from Equation (52) what value of $|K^2 a^2|$ to choose in order to make D reasonably small, perhaps, 0.1 to 0.5, and thereby tend to justify neglecting DB_x^2 . With DB_x^2 assumed to be zero, the formula for Σ_a becomes simply

$$\Sigma_a = \frac{4QR(Ka)}{d} \quad (53)$$

and D can be computed from this Σ_a as

$$D = \Sigma_a / K^2 \quad (54)$$

where, of course, K^2 is obtained from the chosen value of $K^2 a^2$ as

$$K^2 = 4(K^2 a^2) / d^2. \quad (55)$$

5.2.3. Case C.2 Forcing the $J = Q\phi$ Boundary Condition and Eliminating the Spatial Variable x . When the right cylindrical void runs completely through the reactor (from $-X$ to $+X$) and the reactor is critical and uniform in the axial (x) direction, then the diffusion theory solutions for the fluxes and currents are separable and the variable x can be eliminated as described in Section II-2 by employing an average Q at every point on the surface of the void in the boundary condition $J = Q\phi$. If this boundary condition is forced by employing average Σ_a and D in the void region, the Σ_a and D values can be found by means of Equations (49) and (50), as derived earlier in this section. In this case, however, B_x^2 is known because $g(x) = \cos \frac{\pi x}{H}$, where $H = 2X$ is the total (effective) length of the reactor in the x direction. In this case

$$B_x^2 \equiv \frac{-\partial^2 g(x)}{g(x) \partial x^2} = \left(\frac{\pi}{H} \right)^2 \quad (56)$$

and Σ_a and D are, therefore

$$\Sigma_a = \frac{4QR(Ka)}{d} - D(\pi/H)^2 \quad (57)$$

and

$$D = \frac{4QR(Ka)}{4K^2} \quad (58)$$

or, to make D a function of Q, d, and Ka, only

$$D = dQR(Ka)/(Ka)^2 \quad (59)$$

In this case, the axial neutron leakage in the eliminated x direction in the reactor is handled, as usual, by adding a DB_x^2 term to the absorption cross section in every region of the reactor, including the void region. The desired boundary condition is then forced at the void surface when this DB_x^2 is added to the Σ_a from Equation (57) so that Equation (49) is satisfied.

If the diffusion theory solution is performed with $B_x^2 = (\pi/2x)^2$ in every region of the reactor, including the void, and Σ_z and D are computed from Equations (57) and (58) or (59), this case will be called Option C.2.a. As is shown in Section II-6.1, Q is a function only of the ratio H/d. For Case C.2, curves showing α and Q versus H/d are presented in Figures 1 and 2, respectively. Since the values of $\Sigma_a H$ and D/H can also be expressed as functions of H/d, curves of these functions can be drawn, and the user can then find Σ_a and D from these curves and the given H and H/d values.

(1) Option C.2.a. For this option, $\Sigma_a H$ and D/H are derived from Equations (57) and (59) as

$$\Sigma_a H = QR(Ka)[4 - \pi^2(d/H)^2/(Ka)^2](H/d) \quad (60)$$

$$D/H = QR(Ka)(d/H)/(Ka)^2 \quad (61)$$

The buckling $B_x^2 = (\pi/H)^2$ is used in the void, as elsewhere in the reactor. These values are plotted in Figures 3 and 4 for a chosen $Ka = 0.25$ [for which $R(Ka) = 1.0078$].

(2) Option C.2.b. In some cases it may be more convenient to solve the reactor problem with no B_x^2 input for the void region. In this instance $B_x^2 = 0$, and the absorption cross section in the void must be increased from the Option C.2.a value by an amount DB_x^2 to compensate.

Fig. 1 Average albedo, α , versus H/d .

Fig. 2 Q versus H/d .

Fig. 3 $\Sigma_a H$ versus H/d [for use with $B^2 = (\Pi/H)^2$ in the void – Option C.2.a; D/H used from Figure 4].

Fig. 4 D/H versus H/d [for use with $\Sigma_a H$ for Option C.2.a with $B^2 = (\pi/H)^2$ in the void or for use with $\Sigma_a H$ for Option C.2.b with $B^2 = 0$ in the void].

The equations for $\Sigma_a H$ and D/H now simplify to

$$\Sigma_a H = 4QR(Ka)(H/d) \quad (62)$$

and, again

$$D/H = QR(Ka)(d/H)/(Ka)^2. \quad (63)$$

The values of these quantities are plotted on Figures 5 and 4, respectively. Again, the chosen Ka is 0.25.

(3) Option C.2.c. A formula for D/H can be derived to use in evaluating a D to use in the void region when it is desired to input no absorption cross section, as such, ($\Sigma_a=0$), but to have the effective absorption cross section be comprised entirely of DB_x^2 , where $B_x^2 = \pi/H^2$.

To force the desired boundary condition, in this case

$$DB_x^2 = D \left(\frac{\pi}{H} \right)^2 = \frac{4QR(Ka)}{d} \quad (64)$$

and

$$D = dQR(Ka)/(Ka)^2. \quad (65)$$

But now Ka is no longer arbitrary because Equations (64) and (65) demand that

$$(Ka)^2 = (\pi^2/4)(d/H)^2. \quad (66)$$

From Equations (64) and (65)

$$\frac{D}{H} = \frac{4}{\pi^2} QR\left(\frac{\pi d}{2H}\right)(H/d). \quad (67)$$

Values for D/H are plotted in Figure 6.

Fig. 5 $\Sigma_a H$ versus H/d (for use with $B^2 = 0$ in the void for Option C.2.b).

Fig. 6 D/H versus H/d [for use with $\Sigma_a = 0$ and $B^2 = (\Pi/H)^2$ in the void for Option C.2.c].

5.3 The Slab Void Case

5.3.1 The General Expressions for Σ_a and D to Force $J = Q\phi$. If $\phi(x,y,z)$ is separable in the void for the given rectangle on the x,y plane, the diffusion equation in the void region is

$$-g(x)h(y) \frac{\partial^2 f(z)}{\partial z^2} - f(z)h(y) \frac{\partial^2 g(x)}{\partial x^2} - f(z)g(x) \frac{\partial^2 h(y)}{\partial y^2} + \frac{\Sigma_a}{D} f(z)g(x)h(y) = 0. \quad (68)$$

The ratio Σ_a/D does not vary with x or y in the region; so Equation (68) can be written

$$-\frac{1}{f(z)} \frac{\partial^2 f(z)}{\partial z^2} + \frac{\Sigma_a}{D} = \frac{1}{g(x)} \frac{\partial^2 g(x)}{\partial x^2} + \frac{1}{h(y)} \frac{\partial^2 h(y)}{\partial y^2} = -B_x^2 - B_y^2. \quad (69)$$

The equation for f(z) is, therefore,

$$\frac{-\partial^2 f(z)}{\partial z^2} + \frac{(\Sigma_a + DB_x^2 + DB_y^2)}{D} f(z) = 0 \quad (70)$$

and the solution is

$$f(z) = c \cosh(Kz) \quad (71)$$

where c is some constant and

$$K = \sqrt{(\Sigma_a + DB_x^2 + DB_y^2)/D}. \quad (72)$$

Since the net current into the void region at the surface $z = \frac{W}{2}$ is

$$J\left(\frac{W}{2}, x, y\right) = D \frac{\partial f(z)}{\partial z} \Big|_{z=W/2} = cDK \sinh(KW/2) \quad (73)$$

the desired boundary condition is

$$cDK \sinh(KW/2) = Qc \cosh(KW/2) \quad (74)$$

or

$$DK^2 = \frac{2QR(KW/2)}{W} \quad (75)$$

where

$$R(KW/2) \equiv \left(\frac{KW}{2} \right) \cosh(KW/2) / \sinh(KW/2) . \quad (76)$$

Equation (74) for the desired boundary condition is satisfied if Σ_a and D are computed from

$$\Sigma_a + DB_x^2 + DB_y^2 = \frac{2QR(KW/2)}{W} \quad (77)$$

and

$$D = \frac{2QR(KW/2)}{WK^2} . \quad (78)$$

5.3.2 Case "Slab.1" (S.1), Approximately Forcing the $J = Q\phi$ Boundary Condition on Piecewise Rectangles of the x,y, Surface of the Void. As in the cylindrical void case, lack of knowledge of the buckling squared, $B^2 = B_x^2 + B_y^2$, leaves no alternative but to try to justify neglecting the DB^2 term by choosing K^2 large enough to make D small. If the DB^2 term is negligible, the expressions for Σ_a and D are simply

$$\Sigma_a = \frac{2QR(KW/2)}{W} \quad (79)$$

and

$$D = \frac{2QR(KW/2)}{WK^2} = \frac{\Sigma_a}{K^2} . \quad (80)$$

The user can easily determine the approximate value of D that will result from a given choice of $KW/2$ by making use of the fact that for $|(KW/2)^2| \leq 1$, $R(KW/2)$ is well-approximated by

$$R(KW/2) = 1 + \left(\frac{KW}{2} \right)^2 / 3 \quad (\text{to within } \pm 4\%). \quad (81)$$

Hence

$$D = \frac{WQ}{2(KW/2)^2} \pm 4\% \text{ for } \left| \left(\frac{KW}{2} \right)^2 \right| \leq 1. \quad (82)$$

5.3.3 Case S.2, Approximately Forcing the $J = Q\phi$ Boundary Condition Piecewise on Intervals of y and Eliminating the Spatial Variable x . When the slab void runs completely through a critical reactor in the x direction and the reactor is uniform in the x direction, the x variable can be eliminated provided the void meets the conditions specified in Section II-2 on the shape, cross-sectional area, and z position. In this case, the average α for the j th interval on y is

$$\alpha_j = \int_{\Delta y_j} \int_{-X}^X \phi^*(x,y) j_j(x,y) dx dy / \int_{\Delta y_j} \int_{-X}^X \phi^*(x,y) j_0(x,y) dx dy. \quad (83)$$

If the assumptions of II-5.1 are valid for the void, the boundary condition $J_j(y) = Q_j \phi_j(y)$ can be approximately forced on the j th y interval if Σ_{aj} and D_j values are found from Equations (77) and (78) with Q_j replacing Q . The Q_j is, as defined before, equal to $(1-\alpha_j)/[2(1+\alpha_j)]$. The flux shape in the x direction is a constant times $\cos \frac{\pi x}{2X}$, so that B_x^2 is equal to $(\pi/2X)^2$. As in Section II-5.3.2, the DB_y^2 term in Equation (77) remains unknown; hence, $KW/2$ should again be chosen large enough to make the DB_y^2 small and justify neglecting it. By neglecting the DB_y^2 term the equations for D_j and Σ_{aj} are

$$D_j = \frac{2Q_j R(KW/2)}{WK^2} \quad (84)$$

and either

$$\Sigma_{aj} = 2Q_j R(KW/2)/W - D_j (\pi/2X)^2 \quad (85)$$

or

$$\Sigma_{aj} = 2Q_j R(KW/2)/W \quad (86)$$

depending on whether the buckling squared B_x^2 is input for the void region as $(\pi/2X)^2$ or zero, respectively.

5.3.4 Case S.3 Forcing the Boundary Condition $J = Q\phi$ in the Slab Void and Eliminating the x and y Spatial Variables. If the slab void extends completely from $-X$ to $+X$ in the x direction and from $-Y$ to $+Y$ in the y direction, both the x and y variables can be

eliminated from the diffusion theory problem if the conditions and assumptions of Sections II-2 and II-5.1 are satisfied. The average albedo α is computed from Equation (16), and the average $Q = (1-\alpha)/[2(1+\alpha)]$ is applied at $z = -W/2$ and $z = +W/2$ in the one-dimensional problem. The total buckling squared is

$$B^2 = B_x^2 + B_y^2 = (\pi/2X)^2 + (\pi/2Y)^2 \quad (87)$$

so that the D and Σ_a values in the region can be computed from Equation (75) as

$$D = \frac{2QR(KW/2)}{WK^2} = \frac{WQR(KW/2)}{(KW/2)^2} \quad (88)$$

and either

$$\Sigma_a = \frac{2QR(KW/2)}{W} - D[(\pi/2X)^2 + (\pi/2Y)^2] \quad (89)$$

or

$$\Sigma_a = \frac{2QR(KW/2)}{W} \quad (90)$$

depending on whether the buckling squared B^2 is input for the void as $[(\pi/2X)^2 + (\pi/2Y)^2]$ or zero.

5.4 Mesh Spacing Considerations When Forcing the $J = Q\phi$ Boundary Conditions with D s and Σ_a s in the Void Region

As has been pointed out, the argument of the R function of Equations (48) and (76) is arbitrary. This arbitrariness makes possible a choice of a fairly large value when it is desired to justify the neglect of DB^2 terms in computing Σ_a for piecewise application or a choice of an argument of $Hd/2H$ in the cylindrical void case for Option C.2.c of Section II-5.2.3. In most of the computer programs used to solve diffusion theory problems, difference equations are solved for the mesh points which are defined by the choice of mesh spacings along the different spatial directions of the problem. Since the validity of the void representation methods depends on the fact that the diffusion theory solution in the void is some constant times $I_0(Kr)$ for the cylindrical void and $\cosh(Kz)$ for the slab void, the mesh spacing on r or z in the void must be small enough to cause the difference equation solutions to be good approximations of these functions.

An adequately small mesh can be chosen on the basis of the fact that the relative neutron flux varies from unity at the center of the void to $I_0(Ka)$ or $\cosh(\frac{KW}{2})$ at the void surface for the cylindrical and slab void, respectively. If the mesh is chosen so that the variation of the I_0 or $\cosh$ function is not excessive from one mesh point to the next, especially near the void boundary, then the difference equation solution will closely approximate the correct function.

Another type of restriction must be imposed on the mesh spacings in the void when Q is negative. A negative Q constitutes a homogeneous source (one proportional to the neutron flux). This source is forced by means of the negative K^2 and $\Sigma_a + DB^2$ values obtained from the respective equations for the void when D is assumed positive and the imaginary argument of the R function is limited to values less than the first root of $I_0(ix)$ and $\cosh(ix)$ for the cylindrical and slab voids, respectively^[a]. The negative $(\Sigma_a + DB^2)$ values reduce the magnitude of the diagonal elements in the matrix expression of the difference equations that are solved by the diffusion theory codes. This reduction eliminates the strict diagonal dominance that usually guarantees a successful solution by the code; and, if the user chooses the argument of the R function too large, the diagonal elements can be reduced to zero or even made large and of opposite sign. In the latter case, negative flux values can occur and the validity of the solution becomes questionable. If the $|K^2|$ and, therefore, the argument of the R function are chosen too large in the void region, the problem can become one for which no physical solution exists.

In order to minimize the chances for an invalid or nonexistent solution when Q is negative, it is advisable to choose the argument of the R function quite small (compared to the first root value) and impose restrictions on the mesh spacings that will guarantee, at least, that the diagonal matrix elements are not changed in sign. Usually, an imaginary R argument of $0.5i$ or less should suffice, and mesh spacing restrictions can be derived as follows (for almost all of the commonly used diffusion theory codes):

(1) Cylindrical void, 1-D cylindrical reactor model

The diagonal elements (here, assumed positive) of the matrix are the sum of two terms, the most negative of which is

$$\left[\frac{-(2r + \frac{3h}{2}) \frac{h}{2} k^2}{2} + \frac{(r + \frac{h}{2})}{h} \right] \quad (91)$$

[a] The first roots (zeros) of $I_0(ix)$ and $\cosh(ix)$ occur at $x=2.4$ and $x=\pi/2$, respectively; and, for values of x less than these, the function R is always positive. Hence if D is positive, $\Sigma_a + DB^2$ and K^2 must have the same sign as Q . When Q is negative, it is necessary to limit x to values less than the first root values because the diffusion equation $-\nabla \cdot \nabla \phi - K^2 \phi = 0$ in the void becomes critical for these combinations of $|K^2|$ and the void dimensions. That is, the void is a critical reactor by itself, regardless of what is outside the void. For larger values of $|K^2|$ or the void dimensions, no physical solution exists.

where $k^2 = K^2$ and $K^2 < 0$.

This term will be positive if

$$h^2 < \frac{(r + \frac{h}{2})^4}{(2r + \frac{3h}{2}) k^2} \quad (92)$$

or

$$h < \sqrt{\frac{r + \frac{h}{2}}{2r + \frac{3h}{2}}} \frac{d}{ka} \quad (93)$$

where a is the radius of the void and $d = 2a$.

The right-hand sides of these inequalities are smallest for large values of r so that a mesh spacing of

$$h < \frac{d}{\sqrt{2} ka} \quad (94)$$

assures positive diagonal elements.

Since ika is the argument of the R function, the h can be larger if smaller arguments are chosen for the arbitrary ka values.

(2) Slab void, 1-D slab reactor model

Each (positive) component of a diagonal element has the form

$$\left[-\frac{hk^2}{2} + \frac{1}{h} \right] \quad (95)$$

where $k^2 = K^2$ and $K^2 < 0$.

This term will be positive if

$$h < \frac{\sqrt{2}}{k} = \frac{W}{\sqrt{2}(kW/2)} \quad (96)$$

where $\frac{ikW}{2}$ is the R argument and W is the slab void width.

(3) Slab void, 2-D Z-Y reactor model

Each (positive) component of a diagonal element has the form

$$\left[\frac{-h_1 h_2 k^2}{4} + \frac{h_2}{2h_1} + \frac{h_1}{2h_2} \right] \quad (97)$$

where $k^2 = |K^2|$, $K^2 < 0$, and h_1 and h_2 are the mesh spacings for a mesh rectangle.

This term will be positive if

$$\frac{h_1 h_2}{\sqrt{h_1^2 + h_2^2}} < \frac{\sqrt{2}}{k} = \frac{W}{\sqrt{2} (kW/2)} \quad (98)$$

Since the left-hand term is largest when $h_1 \gg h_2$ or $h_2 \gg h_1$, a safe value is assured if the larger of h_1 and h_2 is

$$h < \frac{\sqrt{2}}{k} = \frac{W}{\sqrt{2} (kW/2)} \quad (99)$$

where $\frac{ikW}{2}$ is the R argument and W is the slab void width.

6. NUMERICAL INTEGRATION OF THE ALBEDO INTEGRALS

6.1 The Cylindrical Void

For a cylindrical void that satisfies the basic assumptions of Section II-5.1, Equation (37) for $j_i(\vec{r})$ can be written

$$j_i(\vec{r}) = \frac{a}{\pi} \int_{-X}^X \int_0^{2\pi} \frac{j_o(\vec{r}') \cos \gamma' \cos \gamma}{\rho'^2} d\phi' dx' \quad (100)$$

since $ds = a d\phi' dx'$.

The void is considered to extend from $-X$ to $+X$ and any contribution to $j_1(\vec{r})$ from the circular areas at the ends of the void has been assumed negligible.

From the diagram of the coordinate system on Figure 7, $\vec{r}'$, $\vec{r}$, $\vec{\rho}$, $\vec{n}'$, and $\vec{n}$, can be expressed in terms of the z,y, and x components as

$$\vec{r}' = \begin{Bmatrix} a \cos\phi' \\ a \sin\phi' \\ x' \end{Bmatrix} \quad (101)$$

$$\vec{r} = \begin{Bmatrix} a \cos\phi \\ a \sin\phi \\ x \end{Bmatrix} \quad (102)$$

$$\vec{\rho} = \begin{Bmatrix} a(\cos\phi - \cos\phi') \\ a(\sin\phi - \sin\phi') \\ x - x' \end{Bmatrix} \quad (103)$$

$$\vec{n}' = \begin{Bmatrix} -\cos\phi' \\ -\sin\phi' \\ 0 \end{Bmatrix} \quad (104)$$

and

$$\vec{n} = \begin{Bmatrix} -\cos\phi \\ -\sin\phi \\ 0 \end{Bmatrix} \quad (105)$$

where $x = |\vec{r}|$ and $x' = |\vec{r}'|$.

Therefore

$$\cos\gamma' = \frac{\vec{\rho} \cdot \vec{n}'}{\rho} = \frac{a(\cos\phi - \cos\phi')(-\cos\phi') + a(\sin\phi - \sin\phi')(-\sin\phi') + (x - x')(0)}{\rho} \quad (106)$$

$$\cos\gamma = \frac{\vec{\rho} \cdot \vec{n}}{\rho} = \frac{-[a(\cos\phi - \cos\phi')(-\cos\phi) + a(\sin\phi - \sin\phi')(-\sin\phi) + (x - x')(0)]}{\rho} \quad (107)$$

and

$$\rho^2 = a^2(\cos\phi - \cos\phi')^2 + a^2(\sin\phi - \sin\phi')^2 + (x - x')^2. \quad (108)$$

ANC-A-9549

Fig. 7 Coordinate system for the cylindrical void.

These expressions can be reduced to the more convenient forms

$$\cos \gamma' = \frac{d}{2\rho} [1 - \cos(\phi - \phi')] \quad (109)$$

$$\cos \gamma = \frac{d}{2\rho} [1 - \cos(\phi - \phi')] \quad (110)$$

and

$$\rho^2 = \frac{d^2}{2} [1 - \cos(\phi - \phi')] + (x - x')^2 \quad (111)$$

where $d = 2a$ is the void diameter.

If the interest is only in the average of $j_i(\vec{r})$ with respect to azimuthal angle ϕ , this average is

$$\bar{j}_i(x) = \frac{1}{2\pi} \int_0^{2\pi} j_i(x, \phi) d\phi \quad (112)$$

and with Equations (109), (110), and (111) the average can be written as

$$\bar{j}_i(x) = \frac{1}{2\pi^2 d} \int_{-x}^x \int_0^{2\pi} j_0(x', \phi') \int_{\phi'}^{\phi' + \pi} \frac{[1 + \cos(\phi - \phi')]^2 d\phi d\phi' dx'}{\{2[(x - x')/d]^2 + 1 - \cos(\phi - \phi')\}^2} \quad (113)$$

Use has been made of the fact that

$$\int_0^{2\pi} j_i(x, \phi) d\phi = 2 \int_{\phi'}^{\phi' + \pi} j_i(x, \phi) d\phi. \quad (114)$$

The integration with respect to ϕ in Equation (113) is facilitated by the substitution of a variable corresponding to

$$(1 - \omega^2)/(1 + \omega^2) = \cos(\phi - \phi') \quad (115)$$

so that

$$d\phi = 2d\omega/(1+\omega^2) \quad (116)$$

$$\omega = 0 \text{ at } \phi = \phi' \quad (117)$$

and

$$\omega = \infty \text{ at } \phi = \phi' + \pi. \quad (118)$$

In terms of the new variable, Equation (113) becomes

$$\bar{j}_i(x) = \frac{1}{\pi^2 d \{[(x-x')/d]^2 + 1\}^2} \int_{-x}^x \int_0^{2\pi} j_o(x', \phi') \int_0^\infty \frac{\omega^4 d\omega d\phi' dx'}{(\beta^2 + \omega^2)^2 (1 + \omega^2)} \quad (119)$$

where

$$\beta^2 = \left(\frac{x-x'}{d}\right)^2 / \left[\left(\frac{x-x'}{d}\right)^2 + 1\right]. \quad (120)$$

If

$$\frac{\omega^4}{(\beta^2 + \omega^2)^2 (1 + \omega^2)} = \frac{1}{(1 - \beta^2)^2 (1 + \omega^2)} + \frac{\beta^4}{(1 - \beta^2)(\beta^2 + \omega^2)^2} + \frac{\beta^2(\beta^2 - 2)}{(1 - \beta^2)^2 (\beta^2 + \omega^2)} \quad (121)$$

is expressed in Equation (119), the integration with respect to ω is easily carried out with the result

$$\bar{j}_i(x) = \frac{1}{2\pi d} \int_{-x}^x \int_0^{2\pi} j_o(x', \phi') (1 + \beta^3/2 - 3\beta/2) d\phi' dx'. \quad (122)$$

Now, an average $j_o(x')$ is defined with respect to ϕ' as

$$\bar{j}_o(x') \equiv \frac{1}{2\pi} \int_0^{2\pi} j_o(x', \phi') d\phi' \quad (123)$$

and $\bar{j}_i(x)$ can be written

$$\bar{j}_i(x) = \frac{1}{d} \int_{-X}^X \bar{j}_0(x') \left\{ 1 + \frac{1}{2} \left[\frac{\left(\frac{x-x'}{d}\right)^2}{\left(\frac{x-x'}{d}\right)^2 + 1} \right]^{3/2} - \frac{3}{2} \left[\frac{\left(\frac{x-x'}{d}\right)^2}{\left(\frac{x-x'}{d}\right)^2 + 1} \right]^{1/2} \right\} dx' \quad (124)$$

If there is interest in applying an average albedo or $J = Q\phi$ type of boundary condition in a piecewise fashion on intervals of x (as in Section II-5.2.2), then, on the j th segment of the surface, Equation (38) becomes

$$\alpha_j = \frac{1}{d} \int_{x_j}^{x_j + \Delta_j} \phi^*(x) \int_{-X}^X \bar{j}_0(x') [1 + \beta^3(x, x')/2 - 3\beta(x, x')/2] dx' dx / \int_{x_j}^{x_j + \Delta_j} \phi^*(x) \bar{j}_0(x) dx \quad (125)$$

where

$$\beta(x, x') = \left(\frac{x-x'}{d} \right)^2 / \left[\left(\frac{x-x'}{d} \right)^2 + 1 \right] \quad (126)$$

As stated before, the average Q for the segment is

$$Q_j = (1 - \alpha_j) / [2(1 + \alpha_j)] \quad (127)$$

The expression for α_j in Equation (125) is easily evaluated numerically for any given functions $\bar{j}_0(x')$ and $\phi^*(x)$.

As can be shown, α_j can be expressed as a function of the single parameter H/d , where $H=2X$, for any given $\bar{j}_0$ and ϕ^* functions.

When Δ_j extends from $-X$ to $+X$ and the average α or Q is for the entire length of the void, a single curve can be plotted for α or Q versus H/d , given any assumed shapes for the $\bar{j}_0$ and ϕ^* functions.

For this case, Table I shows the variation with H/d of the α and Q values as well as the various $\Sigma_a H$ and D/H values corresponding to the three different options C.2.a, C.2.b, and C.2.c discussed in Section II-5.2.3. For H/d values ranging from 0.25 to 20, the quantities shown on Table I were evaluated numerically using the assumptions that $\bar{j}_0(x) = \cos \frac{\pi x}{X}$, the same as ϕ^* for the axially uniform case.

The α values are plotted in Figure 1 versus H/d .

The Q values are plotted in Figure 2 versus H/d .

TABLE I

AVERAGE α , Q, ΣH , AND D/H VALUES VERSUS H/d FOR A CYLINDRICAL VOID

Height to Diameter Ratio, H/d	Average Albedo, α	Current to Flux Ratio, Q	Option C.2.a $B^2=(\Pi/H)^2$ in Void ΣH a	Options C.2.a or C.2.b, $\Sigma a \neq 0$ in Void D/H	Option C.2.b, $B^2=0$ in Void ΣH a	Option C.2.c, $\Sigma a = 0$ and $B^2=(\Pi/H)^2$ in Void D/H
0.0	0.0	0.5	---	---	0.0	---
0.25	0.18381	3.4473-1 ^[a]	-2.1910+2	2.2235+1	0.34741	1.1972-1
0.50	0.33188	2.5082-1	-7.9328+1	8.0888	0.50555	9.7372-2
0.75	0.44900	1.9013-1	-3.9770+1	4.0877	0.57484	8.4919-2
1.00	0.54106	1.4891-1	-2.3098+1	2.4011	0.60026	7.7302-2
1.25	0.61357	1.1974-1	-1.4642+1	1.5446	0.60338	7.1919-2
1.50	0.67112	9.8401-2	-9.8450	1.0578	0.59501	6.7670-2
1.75	0.71724	8.2331-2	-6.9064	7.5861-1	0.58081	6.4086-2
2.00	0.75459	6.9932-2	-5.0009	5.6382-1	0.56382	6.0948-2
2.50	0.81050	5.2332-2	-2.8040	3.3754-1	0.52740	5.5599-2
3.00	0.84950	4.0686-2	-1.6663	2.1868-1	0.49204	5.1144-2
3.50	0.87768	3.2572-2	-1.0215	1.5006-1	0.45956	4.7356-2
4.00	0.89867	2.6685-2	-6.3141-1	1.0757-1	0.43029	4.4089-2
4.50	0.91470	2.2276-2	-3.8371-1	7.9820-2	0.40409	4.1242-2
5.00	0.92721	1.8884-2	-2.2043-1	6.0900-2	0.38062	3.8737-2
6.00	0.94522	1.4082-2	-3.2910-2	3.7844-2	0.34059	3.4535-2
7.00	0.95729	1.0909-2	5.9818-2	2.5130-2	0.30785	3.1145-2
8.00	0.96579	8.7010-3	1.0751-1	1.7538-2	0.28060	2.8347-2
9.00	0.97200	7.0993-3	1.3202-1	1.2719-2	0.25757	2.5994-2
10.00	0.97668	5.8990-3	1.4392-1	9.5119-3	0.23780	2.3981-2
12.00	0.98316	4.2467-3	1.4911-1	5.7063-3	0.20543	2.0697-2
14.00	0.98734	3.1843-3	1.4351-1	3.6675-3	0.17971	1.8096-2
16.00	0.99023	2.4552-3	1.3394-1	2.4743-3	0.15836	1.5940-2
18.00	0.99232	1.9282-3	1.2287-1	1.7273-3	0.13992	1.4080-2
20.00	0.99390	1.5305-3	1.1122-1	1.2340-3	0.12340	1.2416-2

[a] The -1 means times 10^{-1}

The $\Sigma_a H$ values for Option C.2.a, when $B_x^2 = (\Pi/H)^2$ is used in the void, are plotted versus H/d in Figure 3. The D/H values for use with this option or for Option C.2.b are plotted in Figure 4 versus H/d .

When $B_x^2 = 0$ in the void, the $\Sigma_a H$ values should be taken from Figure 5 for Option C.2.b, and the D/H values should be taken from Figure 4.

When $\Sigma_a = 0$ in the void but $B_x^2 = (\Pi/H)^2$, the D/H values plotted versus H/d in Figure 6 should be used (Option C.2.c).

6.2 The Slab Void

6.2.1 Slab Void Running From $x = -X$ to $+X$ And From $y = 0$ to Y With a Surface at $y = 0$. The coordinate system diagramed in Figure 8 shows that the current $j_1'(\eta, \mu, \omega)$ entering the void surface at $\bar{\eta}$, $\bar{\mu}$, and $\bar{\omega}$ is the sum of two integrals if there is no contribution from the surfaces at $\eta = |\bar{\eta}| = -X$, $\eta = +X$, or $\mu = |\bar{\mu}| = Y$ but there are contributions from the surfaces at $\omega = |\bar{\omega}| = -W/2$ and $\mu = 0$. Therefore

$$j_1'(\eta, \mu, \omega) = \frac{1}{\Pi} \int_{-X}^X \int_0^Y j_0'(\eta_1, \mu_1, \omega_1) \frac{\cos \gamma_1 \cos \gamma}{\rho_1^2} d\mu d\eta + \frac{1}{\Pi} \int_{-X}^X \int_{-W/2}^{W/2} j_0'(\eta_2, 0, \omega_2) \frac{\cos \gamma_2 \cos \gamma'}{\rho_2^2} d\omega d\eta \quad (128)$$

where

$$\rho_1 = |\bar{\rho}_1| = \sqrt{(\eta - \eta_1)^2 + (\mu - \mu_1)^2 + W^2} \quad (129)$$

$$\rho_2 = |\bar{\rho}_2| = \sqrt{(\eta - \eta_2)^2 + (\mu - \mu_2)^2 + (\omega - \omega_2)^2} \quad (130)$$

$$\cos \gamma = -\bar{\eta} \cdot \bar{\rho}_1 / \rho_1 = \frac{\bar{\omega}}{\omega} \cdot \frac{\bar{\rho}_1}{\rho_1} = \frac{\bar{\omega}}{\omega} \cdot \frac{\bar{\omega} - \bar{\omega}_1}{\rho_1} = W / \rho_1 \quad (131)$$

$$\cos \gamma' = -\bar{\eta} \cdot \bar{\rho}_2 / \rho_2 = \frac{\bar{\omega}}{\omega} \cdot \frac{\bar{\omega} - \bar{\omega}_2}{\rho_2} = (W/2 - \omega_2) / \rho_2 \quad (132)$$

$$\cos \gamma_1 = \bar{\eta}_1 \cdot \bar{\rho}_1 / \rho_1 = \frac{\bar{\omega}}{\omega} \cdot \frac{\bar{\omega} - \bar{\omega}_1}{\rho_1} = W / \rho_1 \quad (133)$$

and

Fig. 8 Coordinate system for the slab void.

$$\cos \gamma_2 = \bar{\eta}_2 \cdot \bar{\rho}_2 / \rho_2 = \frac{\bar{\mu}}{\mu} \cdot \frac{\bar{\mu}}{\rho} = \mu / \rho_2 \quad (134)$$

Hence, it is possible to write

$$\cos \gamma_1 \cos \gamma / \rho_1^2 = W^2 / [(\eta - \eta_1)^2 + (\mu - \mu_1)^2 + W^2]^2 \quad (135)$$

and

$$\cos \gamma_2 \cos \gamma' / \rho_2^2 = (W/2 - \omega_2) \mu / [(\eta - \eta_2)^2 + \mu^2 + (W/2 - \omega_2)^2]^2. \quad (136)$$

The following transformations of variables are now made. In the first integral,

$$\begin{aligned} x &= \eta / W \\ x' &= \eta_1 / W \\ y &= \mu / W \end{aligned} \quad (137)$$

and

$$y' = \mu_1 / W$$

and in the second integral

$$x' = \eta_2 / W \quad (138)$$

and

$$z' = (W/2 - \omega_2) / W.$$

In terms of these variables, Equation (128) becomes

$$\begin{aligned} j_i(x, y) &= \frac{1}{\pi} \int_{-a}^a \int_0^b j_0(x', y') dy' dx' / [(x - x')^2 + (y - y')^2 + 1]^2 \\ &+ \frac{1}{\pi} \int_{-a}^a \int_0^1 j'_0(x', z') z' y dz' dx' / [(x - x')^2 + y^2 + (z')^2]^2 \end{aligned} \quad (139)$$

where

$$a = X/W,$$

$$b = Y/W,$$

$$j_i(x, y) = j_i'(\eta, \mu, \omega)$$

$$j_0(x', y') = j_0'(\eta_1, \mu_1, \omega_1)$$

and

$$j_0'(x', z') = j_0'(\eta_2, 0, \omega_2). \quad (140)$$

The contribution to the current $j_1'(\eta_2, 0, \omega_2)$ produced by neutrons from the surface at $\omega = +W/2$ is

$$\beta_1'(\eta_2, 0, \omega_2) = \frac{1}{\pi} \int_{-X}^X \int_0^X j_0'(\eta, \mu, \omega) \frac{\cos \gamma_2 \cos \gamma'}{\rho_2^2} d\mu d\eta. \quad (141)$$

With a transformation of variables, this equation can be written as

$$\beta_1(x, z) = \frac{1}{\pi} \int_{-a}^a \int_0^b j_0(x', y') y' z dy' dx' / [(x-x')^2 + (y')^2 + z^2]^2 \quad (142)$$

where

$$\beta_1(x, z) = \beta_1'(\eta_2, 0, \omega_2)$$

$$j_0(x', y') = j_0'(\eta, \mu, \omega)$$

$$x' = \eta/W$$

$$y' = \mu/W$$

$$z = (W/2 - \omega_2)/W$$

$$x = \eta_2/W$$

$$a = X/W$$

and

$$b = Y/W.$$

(143)

If the currents j_0 and the importance functions ϕ^* on the surfaces at $\omega = +W/2$ and $\omega = -W/2$ can be assumed to be the same for a given x and y , then the average albedo on the j th segment of the surface at $\omega = +W/2$ or at $\omega = -W/2$ can be written

$$\begin{aligned} \alpha_j = & \int_{x_j}^{x_j + \Delta x_j} \int_{y_j}^{y_j + \Delta y_j} \phi^*(x, y) \left\{ \frac{1}{\pi} \int_{-a}^a \int_0^b j_0(x', y') dy' dx' / [(x-x')^2 + (y-y')^2 + 1]^2 \right. \\ & \left. + \frac{1}{\pi} \int_{-a}^a \int_0^1 j_0(x', z') z' y dz' dx' / [(x-x')^2 + y^2 + (z')^2]^2 \right\} dy dx / \\ & \int_{x_j}^{x_j + \Delta x_j} \int_{y_j}^{y_j + \Delta y_j} \phi^*(x, y) j_0(x, y) dy dx. \end{aligned} \quad (144)$$

With the same assumptions on j_0 and ϕ^* , the fact that $j_i(x, z) = \beta_1(x, z) + \beta_1(x, 1-z)$ can be used to write the average albedo on a surface segment at $\mu = 0$ as

$$\alpha_{j'} = \frac{\int_{x_{j'}}^{x_{j'} + \Delta x_{j'}} \int_{z_{j'}}^{z_{j'} + \Delta z_{j'}} \phi^*(x, z) [\beta_1(x, z) + \beta_1(x, 1-z)] dz dx}{\int_{x_{j'}}^{x_{j'} + \Delta x_{j'}} \int_{z_{j'}}^{z_{j'} + \Delta z_{j'}} \phi^*(x, z) j_0(x, a) dz dx} \quad (145)$$

provided $\phi^*(x, z) = \phi^*(x, 1-z)$.

The average albedo for the entire void surface, under the same assumptions, is

$$\alpha = \frac{2 \int_{-a}^a \int_0^b \phi^*(x, y) j_i(x, y) dy dx + 2 \int_{-a}^a \int_0^1 \phi^*(x, z) \beta_1(x, z) dz dx}{2 \int_{-a}^a \int_0^b \phi^*(x, y) j_0(x, y) dy dx + \int_{-a}^a \int_0^1 \phi^*(x, z) j_0(x, z) dz dx} \quad (146)$$

where $j_i(x, y)$ and $\beta_1(x, z)$ are as defined in Equations (139) and (142), respectively, and use has been made of the fact that

$$\int_0^1 \phi^*(x, z) [\beta_1(x, z) + \beta_1(x, 1-z)] dz = 2 \int_0^1 \phi^*(x, z) \beta_1(x, z) dz. \quad (147)$$

The numerical integration of the integrals in Equations (139) and (142) must be done with careful regard to numerical error. The first integral of (139) can be evaluated to within an error of a few percent if the ranges of x and y are uniformly partitioned into intervals, Δx_i and Δy_j , of 0.2 or less. The integral can then be approximated by

$$h_1(x, y) = \sum_{i=1}^I \sum_{j=1}^J \bar{j}_0(x_i, y_j) \frac{\Delta x_i \Delta y_j}{\pi} [(x-x_i)^2 + (y-y_j)^2 + 1]^2 \quad (148)$$

where $\bar{j}_0(x_i, y_j)$ is regarded as the average for the mesh rectangle that goes from $x_i = -a + (i-1)\Delta x_i$ to $x_{i+1} = -a + i\Delta x_i$ and from $y_j = (j-1)\Delta y_j$ to $y_{j+1} = j\Delta y_j$. For example, $\bar{j}_0(x_i, y_j)$ is usually taken as the value at the midpoint of the rectangle or the simple arithmetic average of the values at the corners of the rectangle.

The second integral of Equation (139) can be numerically integrated successfully by again assuming that $j_0(x', z')$ is constant at some average value over the mesh rectangle that goes from $x_i = -a + (i-1)\Delta x_i$ to $x_{i+1} = -a + i\Delta x_i$ and from $z_j = (j-1)\Delta z_j$ to $z_{j+1} = j\Delta z_j$. Then

$$\begin{aligned} & \frac{1}{\pi} \int_{-a}^a \int_{z_j}^{z_{j+1}} j_0(x', z') z' y dx' dz' / [(x-x')^2 + y^2 + (z')^2]^2 \\ & \doteq \frac{1}{2\pi} \int_{-a}^a \bar{j}_0(x', z_j) y \left\{ \frac{1}{[(x-x')^2 + y^2 + z_j^2]} - \frac{1}{[(x-x')^2 + y^2 + z_{j+1}^2]} \right\} dx'. \end{aligned} \quad (149)$$

Integration of Equation (149) over an interval on x results in

$$\begin{aligned} & \frac{1}{\pi} \int_{x_i}^{x_{i+1}} \int_{z_j}^{z_{j+1}} j_0(x', z') z' y dz' dx' / [(x-x')^2 + y^2 + (z')^2]^2 \\ & \doteq f(x, y)_{i,j} \doteq \frac{\bar{j}_0(x_i, z_j)}{2\pi} \left\{ \frac{y}{\sqrt{y^2 + z_j^2}} \left[\tan^{-1} \left(\frac{x_{i+1} - x}{\sqrt{y^2 + z_j^2}} \right) - \tan^{-1} \left(\frac{x_i - x}{\sqrt{y^2 + z_j^2}} \right) \right] \right. \\ & \quad \left. - \frac{y}{\sqrt{y^2 + z_{j+1}^2}} \left[\tan^{-1} \left(\frac{x_{i+1} - x}{\sqrt{y^2 + z_{j+1}^2}} \right) - \tan^{-1} \left(\frac{x_i - x}{\sqrt{y^2 + z_{j+1}^2}} \right) \right] \right\} \text{ for } y > 0. \end{aligned} \quad (150)$$

If $y = z_1 = 0$

$$f(x,0)_{i,1} = j_0(x,0)/2 \quad (151)$$

for $x_i \leq x < x_{i+1}$ and $f(x,0)_{i,1} = 0$ otherwise.

The total integral is, therefore, the double sum

$$h_2(x,y) = \sum_{i=1}^I \sum_{j=1}^J f(x,y)_{i,j} \quad \text{for } y > 0 \quad (152)$$

or

$$h_2(x,0) = j_0(x,0)/2 \quad \text{if } y = z_1 = 0. \quad (153)$$

In many cases, $j_0(x',z')$ can be assumed to be a function of x' , alone, and only a single sum must be evaluated. That is, the integral reduces to

$$h_2(x,y) = \sum_{i=1}^I \frac{j'_0(x_i)}{2\pi} \left\{ \left[\tan^{-1} \left(\frac{x_{i+1}-x}{y} \right) - \tan^{-1} \left(\frac{x_i-x}{y} \right) \right] \right. \\ \left. - \frac{y}{\sqrt{y^2+1}} \left[\tan^{-1} \frac{x_{i+1}-x}{\sqrt{y^2+1}} - \tan^{-1} \left(\frac{x_i-x}{\sqrt{y^2+1}} \right) \right] \right\} \quad \text{for } y > 0 \quad (154)$$

and to

$$h_2(x,0) = j'_0(x)/2 \quad \text{for } y = 0. \quad (155)$$

The integral in Equation (142) can be similarly evaluated as

$$\beta_1(x, z) = \sum_{i=1}^I \sum_{j=1}^J g(x, z)_{i,j} = \sum_{i=1}^I \sum_{j=1}^J \frac{j_0(x_i, y_j)}{2\pi} \left\{ \frac{z}{\sqrt{z^2 + y_j^2}} \left[\tan^{-1} \left(\frac{x_{i+1} - x}{\sqrt{z^2 + y_j^2}} \right) - \tan^{-1} \left(\frac{x_i - x}{\sqrt{z^2 + y_j^2}} \right) \right] \right. \\ \left. - \frac{z}{\sqrt{z^2 + y_{j+1}^2}} \left[\tan^{-1} \left(\frac{x_{i+1} - x}{\sqrt{z^2 + y_{j+1}^2}} \right) - \tan^{-1} \left(\frac{x_i - x}{\sqrt{z^2 + y_{j+1}^2}} \right) \right] \right\} \text{ for } z > 0. \quad (156)$$

If $z = y_1 = 0$

$$g(x, 0)_{i,1} = \frac{j_0(x, 0)}{2} \quad (157)$$

for $x_i < x < x_{i+1}$ and $g(x, 0)_{i,1} = 0$ otherwise; hence

$$\beta_1(x, 0) = \frac{j_0(x, 0)}{2} \text{ for } z = y_1 = 0. \quad (158)$$

To evaluate the albedos of Equations (144), (145), or (146) a much coarser mesh can usually be defined on the surfaces when performing the final numerical integrations because $h_1(x, y)$, $h_2(x, y)$, and $\beta_1(x, z)$ vary more slowly than the functions integrated to obtain them.

6.2.2 Slab Void Running from $x = -X$ to $+X$ and from $y = -Y$ to $+Y$. When the slab void runs completely through the reactor in both the x and y directions, the current $j_i(x, y)$ and the albedos can be obtained simply from a reduced form of Equations (139), (144), and (145). That is, the current into the surface is

$$j_i(x, y) = \frac{1}{\pi} \int_{-a}^a \int_{-b}^b j_0(x', y') dy' dx' / [(x-x')^2 + (y-y')^2 + 1]^2 \quad (159)$$

The average albedo on the jth segment of the surface is

$$\alpha_j = \frac{\int_{x_j}^{x_j+\Delta x_j} \int_{z_j}^{y_j+\Delta y_j} \phi^*(x,y) \left\{ \frac{1}{\pi} \int_{-a}^a \int_{-b}^b j_0(x',y') dy' dx' / [(x-x')^2 + (y-y')^2 + 1]^2 \right\}}{\int_{x_j}^{x_j+\Delta x_j} \int_{y_j}^{y_j+\Delta y_j} \phi^*(x,y) j_0(x,y) dy dx} \quad (160)$$

and the albedo averaged over the total surface is

$$\alpha = \frac{\int_{-a}^a \int_{-b}^b \phi^*(x,y) \left\{ \frac{1}{\pi} \int_{-a}^a \int_{-b}^b j_0(x',y') dy' dx' / [(x-x')^2 + (y-y')^2 + 1]^2 \right\}}{\int_{-a}^a \int_{-b}^b \phi^*(x,y) j_0(x,y) dy dx} \quad (161)$$

The numerical integration involved in obtaining $j_i(x,y)$ can be performed as in Equation (148); that is

$$j_i(x,y) \doteq \sum_{i=1}^I \sum_{j=1}^J \bar{j}_0(x_i, y_j) \frac{\Delta x_i \Delta y_j}{\pi} / [(x-x_i)^2 + (y-y_j)^2 + 1]^2. \quad (162)$$

Again, the assumption is made that the j_0 and ϕ^* functions are the same on either side of the void ($z = 0$ and $z = 1$) for a given x and y .

When $j_0(x,y)$ and $\phi^*(x,y)$ are even functions of x and y (such as $\cos \frac{\pi x}{X} \cos \frac{\pi y}{Y}$), α can be expressed as

$$\alpha = \int_0^a \int_0^b \phi^*(x,y) j_i(x,y) dy dx / \int_0^a \int_0^b \phi^*(x,y) j_0(x,y) dy dx \quad (163)$$

by virtue of symmetry. Performing the final integration over x and y from only 0 to a and 0 to b , respectively, reduces the work required for the numerical integration by a factor of four.

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