SIMULATION OF A MAJOR TOKAMAK
DISRUPTION

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Simulation of a Major Tokamak Disruption*

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It is known that the internal tokamak disruption leads to a current profile which is flattened inside the surface where the safety factor equals unity. It is shown that such a profile can lead to $m = 2$ magnetic islands which grow to fill a substantial part of the tokamak cross section in a time consistent with the observations of the major disruption.

The involvement of the tearing mode in the major tokamak disruption has been suspected for some time.¹ We have developed a numerical method for examining the full nonlinear behavior of tearing modes of a single helicity.² The essential approximation in this analysis is the use of the Tokamak ordering $B_z \gg B_\theta$ to expand the magnetohydrodynamic fluid equations to lowest order in the inverse aspect ratio. In this approximation modes of different helicities are uncoupled and the full nonlinear development of an initial perturbation consisting of a single helicity can be described in two dimensions: the full description of the mode then recovered by helical extension.

Consider an initial perturbation of the form $f(r) \exp [i(m\theta + kz)]$ with $k = n/R$, where m, n are the poloidal and toroidal mode numbers. Defining a helical ribbon through $\vec{h} = \hat{z} - (kr/m) \hat{\theta}$ and $\vec{r}$ and writing $\vec{B} = \nabla\psi \times \hat{z} + B_z \hat{h}$, ψ satisfies $\vec{B} \cdot \nabla\psi = 0$ and is equal to the magnetic flux through the helical ribbon. A cylindrical equilibrium is specified by the safety factor $q(r)$, which in terms of this helical flux is given by $q = (m/n)[1 + (mR\psi'/nB_z r)]^{-1}$ and a mode rational surface occurs where $\psi' = \partial\psi/\partial r = 0$.

Derivation of the reduced set of magnetohydrodynamic fluid equations is carried out in Ref. 2. By substituting $\vec{B} = \nabla\psi \times \hat{z} + B_z \hat{h}$ and $\vec{J} = \nabla \times \vec{B}$ into the equation of motion, eliminating the unknown pressure by operating with $\hat{z} \cdot \nabla$, and keeping only lowest order in inverse aspect ratio, but including finite conductivity in Ohm's law, we find a closed set of equations. The resulting two-dimensional equations are, in the scaled variables of Ref. 2

$$D\psi/Dt = -\eta J/S + E \quad (1)$$

$$D/Dt \nabla(\rho \nabla A) = z \cdot (\nabla\psi \times \nabla J) - \nabla\rho \times \nabla(v^2/2) \quad (2)$$

$$D\rho/Dt = 0 \quad (3)$$

$$J = -\nabla^2\psi - 2 \quad (4)$$

$$\hat{v} = -\vec{z} \times \vec{\nabla} A$$

where D/Dt is the convective derivative, J the toroidal current density, η the resistivity, E the electric field at the tokamak wall, ρ the plasma density, and v the plasma velocity. The parameter S equals τ_R/τ_A , where τ_R and τ_A are the resistive and toroidal Alfvén times respectively. In present tokamak discharges $S \approx 10^6$ at the plasma center and decreases as $\eta^{-1} \rho^{-1/2}$.

The numerical code developed directly advances Eqs. (1) - (5) and has been described previously.³

Analysis of the results of this code has led us to the construction of an analytical model for the observed saturation of the mode at a relatively narrow island width.⁴ When this saturation occurs, it can be understood quite simply. The tearing mode is driven by the difference in magnetic field energy between the initial state (no island) and the final (an island of width w). This transition requires the tearing and reconnection of magnetic lines of force in a resistive layer near the mode rational surface and the growth rate is thus a hybrid between the magnetohydrodynamic and the resistive time scales. The essential feature of the saturation is that this magnetic driving energy

decreases to zero as the island width increases; the way in which it does so being dependent on the amount of shear present in the equilibrium.

Physically, an increased shear inhibits large island formation by making the island configuration less energetically favorable.

There are cases for which saturation does not occur, that is the island grows until it fills all the area available to it. We have found three such cases thus far:

CASE 1. A perturbation with poloidal mode number $m=1$. In this case the growth is fast ($\gamma \sim S^{2/3} / \tau_R$), and the mode exponentiates with no sign of slowing or saturating until the island essentially occupies the entire region inside the $q=1$ surface. This case is associated with the Internal Disruption.⁵

CASE 2. The interaction of two islands of the same helicity. This occurs when the current profile is hollow, as often happens in the early stages of a tokamak discharge, and there are two $q=2$ surfaces. The mode is fast, and grows until the entire region between the two singular surfaces has been filled, and may give rise to anomalously rapid current penetration.⁶

CASE 3. The subject of this letter, a flattened current profile. In this case the growth is relatively slow, ($\gamma \sim S^{2/5} / \tau_R$). However, if the $q=2$ surface falls within a narrow window, the saturation theory predicts an island width which is too large to be within the validity of the theory. Also, such a current profile is subject to highly nonlinear kink modes if there exists a vacuum region,² and some kink-like behavior might be expected with typical density and resistivity profiles. This case, as we will show,

can give rise to the destruction of plasma confinement on a time scale consistent with that observed for the Major Tokamak Disruption.

Theoretical calculations⁶ and experimental results show that the $m=1$ internal disruption can give rise to a flattening of the current profile out to and beyond the $q=1$ surface. In Fig. 1 are shown current profiles and safety factor profiles typical before (a), and just after (b), the occurrence of an internal disruption. J_a is the so called flat current profile,⁷

$$q(r) = c [1 + (r/r_0)^8]^{1/4}.$$

In Fig. 2 is shown the predictions from the saturation theory for these cases. Although for large widths this theory is probably only of qualitative use, two points are worth mentioning. Firstly, note that for fixed position of the $m=2$ singular surface, r_2 there is a very sudden onset of a large $m=2$ island as r_0 , a measure of the current profile thickness, increases, due to the steepness of the saturation curve near the right-hand cutoff point. Beyond this point, given approximately by $r_2 = 1.4 r_0$, the mode is stable. Thus, r_0 cannot be too much smaller than r_2 . On the other hand, one finds that if r_0 is too much larger than this value, the value of q at the origin becomes significantly larger than unity, which is not likely experimentally. Thus there is a narrow window of possible values of r_0 which allow the rapid growth of a $m=2$ island to a large width. In the history of a discharge we thus expect the internal disruption to occur in general many times before the resulting flattened current profile happens to fit this narrow window.

Secondly, a flattening of the current profile, leading to J_b of Fig. 1 can produce a significant change in the saturation results, curve w_b of Fig. 2. In particular changes of the current profile at the plasma center can change the mode at r_2 from stable to unstable.

In Fig. 3 are shown the helical flux contours for the flattened profile, case b of Figs. 1 and 2, $r_0 = 0.6$, $r_2 = 0.7$. This profile agrees reasonably with that observed by N. Sauthoff and S. Von Goeler⁸ immediately preceding a major disruption. They observe the flattening to extend well beyond the $q=1$ surface.

We have taken S to be 10^4 at the $q=2$ surface. The time is given in units of τ_A at the plasma center, which is less than one microsecond for typical discharges. The mode is observed to continue exponential growth with no slowing down up to the final width shown. With this value of S the tearing width $w_t \approx 10^{-2}$. The abatement of the growth for $w \gg w_t$ described by Rutherford⁹ does not occur.

In Fig. 4 are shown the associated plasma density contours. We have taken the plasma density to be a gaussian function of r with $\rho(0)/\rho(r_2) \approx 10$, but the results are insensitive to the exact form. The same case has been computed with a uniform density and does not essentially differ. One notes, however, in the last frame of Fig. 4, the onset of a "vacuum" bubble, modified in shape from the results of Ref. 2 by the presence of shear and the imperfect vacuum-plasma boundary.

The result in Figs. 3 and 4 is perhaps an extreme example, with q at origin of 1.37. For more likely values ($q \sim 1.1$) the width still grows to a rather large value ($w \approx 0.4$) whereas a corresponding peaked current profile with the same total current saturates at about half this width.

The boundary condition of constant electric field at the tokamak wall which we impose does not allow us to examine the question of the negative voltage spike observed toward the end of the major disruption. Moreover, as has been previously suggested,¹⁰ this effect may be the result of the expanding plasma making contact with the limiter or wall,

We have repeated the same case with a version of the code which advances the plasma temperature and includes a more realistic model for the resistivity through $\eta = T^{-3/2}$. Again we find essentially the same growth and final width.

A version of the code has also been developed to treat finite Larmor radius effects through the addition of the Hall term to Ohm's law and including gyroviscous terms in the equation of motion. Preliminary results indicate that although there is a nonlinear shift of the real frequency of the mode, the saturation results are unaffected. Results of these calculations will be included in a subsequent publication.

A number of effects may contribute to the major disruptions seen in tokamaks in addition to those which we are able to treat in our code, which is limited to a single helicity. Nonlinear coupling between modes may be important and can lead to large regions of ergodic field lines, as can toroidal effects acting on a single large island.

In this note we show that when the current profile internal to $q=1$ is very flat and is consequently very steep in the neighborhood of $q=2$ the $q=2$ island evolves to a very large size. This is a more general phenomenon in the sense that a profile with large flattening at $q=1$ and $q=2$ would be violently unstable at $q=3/2$, etc. Hence, one might expect that in the final stages of shrinking, flattening, and disruption many modes would be involved.

Nonetheless, we have shown that for a profile flattened by $m=1$ internal disruptions which is not unlike those seen experimentally one may expect the $m=2$ mode to grow to a size which by itself would almost certainly initiate a drastic loss of confinement were such factors as ergodicity and interaction with the limiter taken into account. Moreover, we note that the time scale ($100 \mu \text{ sec}$) is similar to that seen experimentally, and that a relatively small evolution of the zeroth order profile such as could occur in a few msec via sawtoothing is enough to change the $m=2$ mode from linearly stable to the "disruptive" case we have shown. Thus, it is probably fair to say that even if the simple large $m=2$ mode is not a complete explanation of the disruption, it would suffice, in the sense that a flattened profile of the type seen experimentally would be disrupted by a single $m=2$ mode, if more complicated events did not lead to a slightly earlier disruption.

REFERENCES

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¹ B. B. Kadomtsev, Fiz. Plazmy 1, 710 (1975); Sov. J. Plasma Phys. 1, 389 (1975).

² M. N. Rosenbluth, D. A. Monticello, H. Strauss, and R. B. White, Phys. Fluids 19, 1987 (1976).

³ B. V. Waddell, D. A. Monticello, M. N. Rosenbluth, and R. B. White, Proc. of the 7th Conf. on Numerical Simulation in Plasmas, Courant Institute, 1975.

⁴ R. B. White, D. A. Monticello, M. N. Rosenbluth, and B. V. Waddell, Phys. Fluids 20, 800 (1977).

⁵ B. V. Waddell, D. A. Monticello, M. N. Rosenbluth, and R. B. White, Nucl. Fusion 16, 528 (1976).

⁶ R. B. White, D. A. Monticello, M. N. Rosenbluth, and B. V. Waddell, Plasma Physics and Controlled Nuclear Fusion (International Atomic Energy Agency, Vienna, 1977) Vol. I.

⁷ H. P. Furth, P. H. Rutherford, H. Selberg, Phys. Fluids 16, 1054 (1973).

⁸ S. Von Goeler and N. Sauthoff, private communication.

⁹ P. H. Rutherford, Phys. Fluids 16, 1903 (1973).

¹⁰ B. B. Kadomtzev and O. P. Pogutse, Zh. Eksp. Teor. Fiz. 65,
575 (1973); [Sov. Phys. JETP 38, 283 (1974)] .

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Fig. 1. Current profiles before (a) and after (b) a $m=1$ internal disruption. Also shown are the corresponding safety factor profiles. The current profiles J_b , J_c are the flat current profile of Ref. 7 with $r_0 = 0.6$, 0.45 . J_a is obtained by modifying J_b for $r < r_1$.

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Fig. 2. Predicted saturation widths for the profiles of Fig. 1.

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Fig. 3. Helical flux contours during the growth of a $m=2$ mode using the current profile J_b of Fig. 1. Here the resistivity is modelled as $\eta \sim 1/J$, $S=10^4$ at $r=r_2$. The units of time are the central Alfvén time τ_A .

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Fig. 4. Density contours for the case of Fig. 3. Note the development of a "vacuum" bubble.