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ESTIMATE OF THE EFFECT OF ANOMALOUS LENGTH OF
ELECTRON BUNCH ON QUANTUM LIFETIME

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MASTER

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ABSTRACT

Electron bunches are observed to have a length larger than that value determined by the balance between radiation damping and quantum fluctuations. The question then arises as to whether this bunch lengthening and its associated energy widening significantly affect the quantum lifetime. As a first step, an order of magnitude model is presented here. The conclusions are that the effect could be observable at SPEAR II. Furthermore, it appears that PEP and PETRA operating at high energy could have drastically reduced lifetimes for particles near the outside rim of the bunch. It is suggested that experiments might be performed at SPEAR II to test the proposition that bunch lengthening and a reduced quantum lifetime are correlated.

Quantum Lifetime: Synchrotron Radiation Dominant

If the dominant effect in the motion of particles in an electron beam is the synchrotron radiation then the bunch characteristics are determined entirely by a balance between the average damping and the stochastic fluctuations arising from the emitted radiation.¹

In this case the e-folding quantum lifetime, τ_q , is given by

$$\tau_q = \frac{1}{2} \tau_r e^{\xi/\xi}$$

where τ_r is the radiation damping time constant,

$$\tau_r = E/U_o f_o ,$$

with E the electron energy, f_o the revolution frequency, and U_o the radiated energy loss per turn; while

$$\xi = \frac{\epsilon_{\max}^2}{2\sigma_E^2} ,$$

with σ_E the rms natural energy spread due to the radiation equilibrium balance; and $\epsilon_{\max}$ represents the energy stability limit, essentially the distance in energy units from the central energy to the rf bucket, the boundary between stable and unstable particle trajectories. We need expressions for U_o , σ_E and $\epsilon_{\max}$. These can be estimated from the gross beam and machine parameters:

$$U_o = \frac{C_Y E^4}{\rho} ,$$

$$\sigma_E/E = \gamma (C_q/J_E \rho)^{\frac{1}{2}} ,$$

and

$$\epsilon_{\max} = \left[\frac{U_o E f(q)}{\pi \eta h} \right]^{\frac{1}{2}} ;$$

1. M. Sands, SLAC Report, SLAC-121 (1970). All the simple expressions for the various parameters can be obtained in this report.

where

γ is the electron energy in units of the electron rest energy,

ρ is the magnetic radius of curvature,

J_E is the energy partition constant, which for our purposes we take approximately, $J_E = 2$,

q is the "overvoltage,"

$$q = V/U_0 ,$$

V is the rf peak voltage,

h is the rf harmonic number ($f_{rf} = hf_0$),

η is the frequency slip factor,

$$\eta = - \frac{pdf_0}{f_0 dp} = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} ,$$

p is the electron momentum, and

γ_t is the machine transition energy in electron rest energy units.

The quantum constants, C_Y and C_q are

$$C_Y = 8.85 \times 10^{-5} \text{ m} \cdot \text{GeV}^{-3} ,$$

$$C_q = 3.84 \times 10^{-13} \text{ m} .$$

F is the overvoltage stability function

$$F(q) = 2\{(q^2 - 1)^{\frac{1}{2}} - \cos^{-1}(1/q)\} ,$$

or

$$F(q) \rightarrow 2q - \pi, \text{ large } q .$$

The natural energy spread is connected to the natural bunch length, σ_{rms} , because of the synchrotron motion. Writing the length in radians, θ_{rms} ,

$$\theta_{rms} = \sigma_{rms}/R ,$$

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with R the average machine radius, we have

$$\frac{\sigma_E}{E} = \frac{v_s}{\eta} \theta_{rms},$$

where v_s is the synchrotron wave number ($f_{synch} = v_s f_o$),

$$v_s = \left(\frac{\eta h V \cos \varphi_s}{2\pi E} \right)^{\frac{1}{2}}.$$

The stable phase angle, φ_s , is a function of the overvoltage, q ,

$$\cos \varphi_s = \left(1 - \frac{1}{q^2} \right)^{\frac{1}{2}}.$$

Thus, we can write ξ in the simple form,

$$\xi = \frac{F(q)}{q \theta_{rms}^2 h^2 \cos \varphi_s}.$$

Review of Equations

$$\tau_q = \frac{1}{2} \tau_r e^{\xi/\xi}$$

$$\tau_r = E/U_o f_o$$

$$U_o = C_v E^4 / \rho$$

$$\xi = F(q) / [q \theta_{rms}^2 h^2 \cos \varphi_s].$$

(1)

Effect of Unstable Equilibrium State on Quantum Lifetime

We would like to get a first rough estimate of the effect of an increase in bunch size, both length and energy width, on the quantum lifetime. Specifically, since the interaction of the bunch with the surrounding medium results in an increase in the longitudinal phase space area of the bunch "core," we would like to know what if any impact this has on the lifetime. We suggest the following simple

picture of the process of bunch lengthening: Divide the bunch into a discrete sequence of phase space rings, each with its own natural bunch size measured with respect to the phase space radius of the ring. But now each ring is a different distance from the unstable boundary. The outer rings therefore have a reduced lifetime.

Let θ be the distance of a given ring from the bunch center. Then we propose that the quantum lifetime can be roughly estimated from Eq. (1), but with ξ , written $\bar{\xi}$, now a function of θ which we take simply as

$$\bar{\xi} = (\sqrt{\xi} - r)^2 ,$$

where we have written

$$r = \theta / \theta_{\text{rms}} .$$

Thus, for the central "ring," the quantum lifetime is unaffected. However, for the outer ring, represented by the anomalous equilibrium bunch length, $\theta = \theta_{\text{EQ}}$, the lifetime is reduced. Of course, the model is clearly incorrect for small ξ . In other words, the basic assumption that the stability boundary be well out into the tail of the distribution becomes violated. Thus, we take this model to be an order-of-magnitude estimate. It should, however, provide a signal telling whether or not the design parameters of a given machine provide for a reasonable lifetime.

Modified Equation for $\bar{\xi}$ (Lifetime for Outer Ring, θ_{EQ})

$$\bar{\xi} = (\sqrt{\xi} - r_{\text{EQ}})^2 ,$$

$$r_{\text{EQ}} = \theta_{\text{EQ}} / \theta_{\text{rms}} \text{ (bunch lengthening ratio),}$$

$$\theta_{\text{EQ}} \text{ is the rms anomalous bunch length (radians).}$$

To keep the expression monotonic as $r_{\text{EQ}} \rightarrow \xi$ (perhaps, an unrealistic case), we write, roughly,

$$\tau_q = \frac{1}{2} \tau_r e^{\bar{\xi}/\xi} = \frac{1}{2} \tau_r e^{(\sqrt{\xi} - r_{\text{EQ}})^2 / \xi} .$$

Quantum Lifetime for SPEAR II Outer Ring

SPEAR II Example: The SPEAR II parameters are given in Table I.

Table I. SPEAR Parameters and Results

Energy, E	3.0 GeV
Peak Voltage, V	3.4 MV
Magnetic Radius of Curvature, ρ	12.7 m
Energy Loss, U_o	564.4 keV/turn
Overvoltage, q	6.02
Overvoltage Stability Function F(q)	9.07
Average Radius, R	37.45 m
Revolution Frequency, f_o	1.275 MHz
Radiation Damping Time, τ_r	4.17 msec
Stable Phase, $\cos \varphi_s$	0.986
γ	6000
rms Energy Spread, σ_E/E	7.38×10^{-4}
Transition Energy, γ_t	4.94
Frequency Slip Factor, η	0.041
rf Harmonic Number	280
rf Frequency	357.0 MHz
Synchrotron Wave Number, ν_s	0.045
Synchrotron Period, τ_{synch}	17.4 μsec
rms Natural Bunch Length, θ_{rms}	0.672 mrad
ξ	43.16
Bunch Lengthening Factor, $r = \theta_{\text{EQ}}/\theta_{\text{rms}}$	2.68
$\bar{\xi}$, ξ Factor for Outer Ring	15.14
τ_q (Central "Ring")	∞
τ_q (Outer Ring)	3.03 minutes

From Table I, we see that the quantum lifetime for outer ring particles in the case we have chosen is reduced from many years for central "ring" particles to a few minutes. Such an effect could be experimentally observable.

Quantum Lifetime for Outer Ring Particles in PEP and PETRA

The parameters and results for PEP and PETRA are given in Table II.

Table II. PEP and PETRA Parameters and Results

<u>Parameter</u>	<u>Units</u>	<u>PEP</u>	<u>PETRA</u>
Energy, E	GeV	15.0	15.0
Peak Voltage, V	MV	44.0	34.3
Average Radius, R	m	344.9	366.7
γ	--	3×10^4	3×10^4
Magnetic Radius of Curvature, ρ	m	169.9	192.05
Energy Loss, U_0	MeV/Turn	26.4	23.3
Overvoltage, q	--	1.67	1.47
Overvoltage Stability Function, F(q)	--	0.82	0.51
Revolution Frequency, f_0	kHz	138.4	130.2
Radiation Damping Time, τ_r	msec	4.11	4.95
Stable Phase, $\cos \varphi_s$	--	0.80	0.73
rms Energy Spread, σ_E/E	--	1.01×10^{-3}	0.95×10^{-3}
Transition Energy, γ_t	--	14.82	17.30
Frequency Slip Factor, η	--	4.55×10^{-3}	3.34×10^{-3}
rf Harmonic Number, h	--	2589	2304
rf Frequency	MHz	358.3	300.0
Synchrotron Wave Number, ν_s	--	0.066	0.045
Synchrotron Period, τ_{synch}	μsec	108.9	169.9
rms Natural Bunch Length, θ_{rms}	mrad	0.0696	0.0705
rms Natural Bunch Length, σ_{rms}	cm	2.40	2.59
ξ	--	18.90	18.01
Quantum Lifetime τ_q (Central "Ring")	hours	4.88	2.53
ξ (Real Lattice Computation)	--	21.4	21.2
τ_q (Central "Ring")-Lattice Comp.	hours	52.5	52.2
Bunch Lengthening Factor, r	--	4.5	4.0
$\bar{\xi}$, ξ Factor for Outer Ring	--	0.016	0.365
τ_q (Outer Ring)	--	Very Fast Loss	
Bunch Lengthening Factor, r	--	1.8	1.6
$\bar{\xi}$	--	7.99	9.03
τ_q (Outer Ring)	sec	0.28	0.97

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The simple computation illustrated in Table II for PEP and PETRA parameters indicates that even a small amount of bunch lengthening will not be tolerable from the point of view of decreased quantum lifetime. It is therefore important to test whether or not the bunch lengthening phenomenon is, in fact, related to the quantum lifetime in the manner suggested here. Experiments could be done at SPEAR to provide information on this point.