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Energy Storage and Transfer with Homopolar Machine for a Linear Theta-Pinch Hybrid Reactor

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ENERGY STORAGE AND TRANSFER WITH HOMOPOLAR MACHINE FOR A LINEAR THETA-PINCH HYBRID REACTOR

by

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ABSTRACT

This report describes the energy storage and transfer system for the compression coil system of a linear theta-pinch hybrid reactor (LTPHR). High efficiency and low cost are the principal requirements for the energy storage and transfer of 25 MJ in or 25 GJ for a 1-km LTPHR. The circuit efficiency must be $\sim 90\%$, and the cost for the circuit 5-6 ϵ /J. Scaling laws and simple relationships between circuit efficiency and cost per unit energy as a function of the half cycle time are presented.

An important consideration concerns the pulse repetition rate of 2.25 pulses per second, 70×10^6 shots/yr, or 1.7×10^9 shots over the 25-yr plant life. Current interruption to initiate energy transfer is not feasible at this rate. We consider, therefore, a simple ringing circuit with contactors to make and break at the periodically occurring zero-current instances. Even this simple operation will require considerable development effort for an inexpensive and reliable contactor that may be replaced during annual plant maintenance.

We consider capacitors and homopolar machines as energy storage elements with both functioning basically as capacitors. The advantage of the homopolar machine in this application is its relatively low cost, whereas that of capacitors is better efficiency.

The drum-type homopolar machine is best for endurance considerations. It is being developed under sponsorship of the Electric Power Research Institute (EPRI) for the Reference Theta-Pinch Reactor (RTPR). The EPRI-RTPR homopolar machine is designed for 80×10^6 shots, which is insufficient for our LTPHR application. The necessary improvement in endurance is achieved for the LTPHR by eliminating the spokes and inner bearings of the EPRI-RTPR machine and by letting the fiber-reinforced drum spin in a concentric bearing housing. Gas is required as the bearing lubricant because of the relatively large bearing diameter as compared with inner bearings supporting the drum with spokes, and because of the required low loss.

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I. INTRODUCTION

The energy storage and transfer system for a fusion hybrid reactor is characterized by two somewhat conflicting requirements—high efficiency and low cost. The former results from the prerequisite positive energy balance and the importance of minimizing energy loss; the latter reflects the practical point of view that the investment per kilowatt plant output not exceed a conventional amount. Both requirements are stringent for a linear theta-pinch hybrid reactor (LTPHR) because of the large magnitude of energy to be transferred in and out of the magnetic confinement coil systems, compared with the part of the magnetic energy actually interacting with the plasma. In this report, the focus is on problems concerning energy transfer and storage for the compression coils, such as efficiency, cost, and reliability.

Incentives exist for solving these problems, because the hybrid reactor concept offers the advantage that the plasma confinement time and density criterion of 1×10^{14} sec/cm³ neutrons need not be satisfied as in a pure fusion reactor. This principle is workable as long as the neutron flux interacting with the ²³²Th breeding blanket is sufficient each cycle so that the potential energy of the bred fuel, ²³³U, from the ²³²Th + n → ²³³U reaction and of the thermal output (likewise deposited in the breeding blanket) overbalances positive energy with the sum of all losses, and as long as the cost criterion is satisfied. Many additional criteria, such as operating reliability, feasibility of materials handling, and environmental compatibility, must also be met.

A. System Discussion

A single-coil energy storage and transfer system might be used in a LTPHR for three purposes:¹ for implosion heating and staging of the plasma and for adiabatic compression during plasma burn time, which immediately follows the implosion heating and staging. Implosion heating requires a 0.1-μs field rise to reach 1.5 T and staging by crowbar action to maintain the magnetic field until it becomes enhanced by the adiabatic compression field. Approximately 400 kV for a single-turn coil or 200 kV for a split implosion coil arrangement are needed for the fast field.

The joule loss in the leads becomes excessive if the implosion coils are also used for compression. The loss is excessive because the magnetic field requirement is high (25 T), which implies an excitation current of ~4.6 MA, and the energy transfer time is short (~1.5 ms), resulting in excessive eddy losses in

the coil leads. The dual-purpose coil concept has therefore been abandoned, and a separate multiple-turn compression coil has been designed. The implosion coil flux returns in a 4-cm-thick ferrite cylinder, which separates the compression coil from the implosion coil. In this design the energy requirement for the compression coil is a factor 1.8 greater than that of the dual-purpose coil concept. The magnetic field energy per unit reactor length is 25 MJ/m, which corresponds to a total magnetic energy for a 1-km LTPHR of 25 GJ.

An important consideration in this study concerns the pulse repetition rate, which is 2.25 pulses/s, corresponding to 70×10^6 shots/yr, or 1.75×10^9 shots over a 25-yr plant life. Magnetic energy storage and transfer by current interruption clearly is not feasible in this application. The proposed circuit is the L-C ringing circuit, shown in Fig. 1, in which the switch closes and opens as close as possible to the times at which the current goes through zero. This operation will require considerable switch development so that an inexpensive and reliable contactor becomes available that can be replaced during annual plant maintenance. The proposed cycle is also shown in Fig. 1.

B. Efficiency and Cost Requirements

Figure 2 shows a system energy flow diagram, where P_H is the power fed back into the magnetic field coil system (implosion staging and compression coils), $P_1 = \eta P_H$ is the small and therefore neglected fraction of coil energy feeding into the plasma, f_{blk} is the joule loss in the coils that may be recovered as part of blanket heat, and f_e is the joule loss in the external electric circuits, such as power supply loss and capacitor loss. $\eta_e P_H$ is that part of the magnetic field energy restored in the external electric circuit, E_1 is the energy equivalent of bred ²³³U, and P_{th} is thermal energy removed from the breeding blanket. The circuit efficiency should be roughly $1 - f_e f_1 > 0.9$.

Typically, a 1-km LTPHR has an 18-metric-ton loading of ²³³U with a doubling time of 11 yrs, which corresponds to an electrical power output from the integrated system of uranium burner and converted heat from the blanket of 7.4 GWe. The total plant cost is not to exceed 1000 \$/kWe or 7.4 GS. If 90% of this cost is needed for plant components other than the compression coil capacitor bank, then the capacitor bank should not exceed a cost limit per unit stored energy of 5.3 \$/J.

These numbers may serve as guidelines, but need exists for an efficiency-vs-cost relationship that can be used for optimizing the overall LTPHR system.

Fig. 1.
Compression coil circuit.

Fig. 2.
LTPHR energy flow diagram.

This report addresses this need, and the desired cost-efficiency-transfer time relationship is developed in Sec. IV.

C. Circuit Alternatives

For the given pulse rate, we consider only the simplest type circuit requiring no current interruption. This circuit causes voltage reversal on the energy storage capacitor. It seems unlikely that capacitors designed for this operation and for ac service life can be developed so that the installed cost does not exceed the cited cost limit per unit stored energy. However, we will show that homopolar machines can be built for this operation and meet the cost criterion, although the efficiency criterion seems only marginally satisfied. For capacitors, we assume $1 - f_L L_{\infty} = 0.94$ and 14 ¢/J . Overall system analysis is to determine whether homopolar machines or capacitors best serve the requirements of the LTPHR.

II. COMPRESSION COIL DESIGN

The compression coil design is determined by the following parameters.

Inside radius: 14 cm (i.e., inner wall radius = 10 + 4 cm implosion heating coil and flux return).

Magnetic flux density: 25 T.

Minimum coil height requirement to obtain minimum magnetic field energy and minimum neutron absorption in the coil.

Half-cycle time $\tau_{1/2} \approx 3 \text{ ms}$ (i.e., energy transfer time capacitor-to-coil and back).

Joule loss of the coil leads through the blanket (1-m length) and out to the homopolar machine (additional 1-m length) should be as small as possible consistent with other requirements.

Voltage between leads to be less than 30 kV to keep the insulation requirements on the leads and the lead inductance small.

Adequate structural endurance characteristics.

One design seems to meet the above requirements. It is described below and dimensioned in Fig. 3.

Number of turns per coil: 10

Coil current corresponding to 25 T: 0.448 MA

Number of fully transposed conductor strands: 50

Coil inductance: 55.5 μH

Pulse resistance: 368 $\mu\Omega$

dc resistance: 368 $\mu\Omega$

Conductor length for 10-turn coil only: 12.0 m;
for 2 leads: 4 m.

The coil leads are fastened to plugs, shown in Fig. 4, which allow removal of the blanket with coils and vacuum chamber in 1-m sections.

Each of the 50 conductor strands consists of magnet wire 1.9 by 9 mm² clad with 0.4-mm-thick stainless steel for increasing the endurance

Fig. 3.

LTPHR magnetic confinement coils.

strength.² The steel cladding is oxydized to provide negligible interstrand conductance.

III. HOMOPOLAR MACHINE DESIGN

The operating principle is explained and the design equations are developed in the following subsections on the homopolar machine and its subsystems and for the L-C circuit with compression coils.

A. Electrodynamic System

The electrodynamic system treated here includes the equations for the L-C circuit.

1. **Fundamentals of the Homopolar Machine.** A voltage is induced in a closed circuit (i.e., a conductor) moving with respect to a homogeneous magnetic field, as shown in Fig. 5. This effect is used in eddy current brakes, where the emf induced in the conductor shown in Fig. 5 will produce eddy currents, which in turn interact with the magnetic field, thus causing the motion to be resisted in accordance with Lenz's law.

If the circuit is closed externally and no current path exists, then an emf is induced, and no mechanical interaction with the magnetic field can occur (i.e., no "braking force" exists). This is true for the configuration shown in Fig. 6 and demonstrates the operating principle for avelic, or homopolar, machines. The former name already implies that current and voltage are dc when the angular motion is constant. The latter indicates that the poles are not arranged in pairs as in conventional dc machines, but as if the magnetic pole were inside the spinning cylinder, with the pole of opposite polarity at infinity. In other words, the magnetic flux returns are not germane to the operating principle and could be omitted if sufficiently strong magnetic monopoles existed.

Several conceivable configurations, all operating on the same principle, will be discussed in Sec. B. For deriving the electrodynamics, we refer to Fig. 7, which shows the homopolar machine version commonly referred to as a "spool type." Here, voltage is induced in the drum and in the disk-like ends, all in one integral structure. The voltage induced in the spool is collected by the two sets of stationary current collecting brushes (a and b) connected to the rest of the circuit. The internal machine inductance is minimized by the return conductor serving as current return concentric with the rotor.

The flux intercepting the rotor is derived in Appendix A as

$$\Phi = 2\pi r_B A_z, \quad (1)$$

where A_z is twice* the vector potential at the brush with $A_z = 0$ being defined on the axis, and with r_B being the radius of the current collector surface. The voltage between brushes a and b is derived from Faraday's law in Appendix A, i.e.,

$$V_{ab} = d\Phi/dt, \quad (2)$$

*Twice because the potential $\frac{1}{2} A_z$ exists at brush a, and $-\frac{1}{2} A_z$ at brush b, and 0 potential on the axis, in Fig. 7.

Fig. 4.

LTPHR compression coil assembly.

Fig. 5.
Conductor with internally closed circuit moving in a homogeneous magnetic field.

which indicates that the potential is azimuthally directed, and where $\phi = 2\pi A_z$.

Because of the rotor's motion, Eq. (1) may be written as a partial differential equation with time and spatial derivatives i.e.,

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + \omega \frac{\partial\phi}{\partial\psi} \quad (3)$$

where

$$\omega = \frac{d\psi}{dt} \quad (4)$$

For steady-state angular motion, we have $\partial\phi/\partial\psi = \phi/2\pi$. The first expression on the right side of Eq. (3) is zero for dc excitation $\phi = \text{constant}$. Equation (2) may thus be rewritten as follows for dc excitation

$$V_{ab} = -\frac{\omega\phi}{2\pi} \quad (5)$$

Equation (5) is the well-known operating equation for the homopolar machine. Substituting Eq. (1) in Eq. (5), i.e.,

$$V_{ab} = -\omega(r_B A_\psi) \quad (6)$$

shows that the voltage is a function of only the angular speed ω and the radius-corrected vector potential ($r_B A_\psi$) at the collector. Therefore, both

NOTE- IF CIRCULATING CURRENT i_{fig} EXISTED, THEN AZIMUTHAL SYMMETRY REQUIRES THAT IT EXISTED SIMILARLY EVERYWHERE IN THE CYLINDER, THUS CAUSING SELF-CANCELLATION.

Fig. 6.
Conductor with external circuit return moving in a homogeneous magnetic field.

Fig. 7.
Homopolar spool-type machine.

should be chosen as large as possible to obtain maximum output voltage V_{out} , which is a requirement for fast-discharge machines. Maximum radius-corrected vector potential $(r_0 A_z)$ is obtained if the flux density

$$\vec{B} = \frac{1}{r} \left[\vec{a}_r \frac{\partial(rA_z)}{\partial r} - \vec{a}_\theta \frac{\partial(rA_z)}{\partial \theta} \right] \quad (7)$$

at $r = r_0$ and the collector radius r_0 are both chosen as large as possible. Disks are unique in this respect because the collector surfaces may be placed at maximum potential.

The design Eq. (5) or (6) can easily be verified if we allow current in the external circuit to interact with the magnetic field. The force between this current circulating in the direction $\hat{z}$ and the flux density vector $\vec{B}$ is given by

$$\vec{F} = I \oint d\vec{s} \times \vec{B}, \quad (8)$$

where the integral is to enclose the entire flux between current collectors, a and b, i.e.,

$$\oint d\vec{s} \times \vec{B} = \frac{I}{2\pi r_0} \hat{z}. \quad (9)$$

If from Eqs. (8) and (9) the mechanical power $F\bar{v}$ is computed and compared with the electric power output VI from Eq. (5), an identity is obtained, which serves as verification.

2. Selection of a Design for the Fast-Discharge Machine. The spool-type homopolar machine offers the basic advantage of optimum "flux cutting," i.e., $(r_0 A_z) = \text{maximum}$. A severe material fatigue problem exists, however, which will be discussed in Sec. C. This endurance question does not exist in the same severity for the drum rotor, in which the brushes slide on a cylindrical drum, as shown in Fig. 8. Here the drum, carrying virtually uniform current density, interacts with the magnetic field whose flux density B varies little in the axial direction and not at all in the azimuthal direction. The stress to be transmitted in the drum is therefore small, and no stress concentration exists such as the one in the drum-to-disk transition of the spool machine.

We will show in Sec. A.3 that high speed and low moment of inertia are desired for the rotor of a fast-discharge machine. Low-density materials such as aluminum should therefore be chosen for the drum. If the drum is wrapped with a high-strength material such as the graphite-filament/epoxy composite dis-

cussed in Sec. C, then the speed can be further increased with the inertia increasing little.

One of the major considerations in selecting a suitable design is brush friction loss. It is advantageous to let most of the required contact pressure be provided by electrodynamic interaction between brush current and the magnetic leakage field between brushes, drum, and return conductor. The resulting electrodynamic force is pointed in the right direction; the contact load increases with the square of the current. The brushes are located as shown in Fig. 8.

For a given geometry the flux increases with the power 2.33, as defined in Sec. 4, of the brush radius, i.e.,

$$\Phi = r_B^{2.33}, \quad r_B < (r_1)_{\text{coil}}. \quad (10)$$

This is a good reason for placing the current return inside the drum, thus eliminating the need to increase the distance between the collector surface at $r = r_0$ and the inner coil surface at $r = (r_1)_{\text{coil}}$.

We will show in Sec. IV that, for short discharge times, many compression coils must be connected in parallel to a multisection homopolar machine, which may result in several megamperes of discharge current. Double compensation will then be required to limit the radial electrodynamic load between return conductor and drum. More than half the current will return through the inner return conductor and the balance will return through the additional coaxial path in the space between coil and drum as shown in Fig. 9.

Fig. 8.
Homopolar drum-type machine.

3. Electromechanical Design Equations. We write the design equations with notations for the linear dimensions, resistances, and inductances as defined in Fig. 9 for the machine with drum rotor. (The electrodynamic forces on the rotor will be treated in Sec. C.) The iron shown in Fig. 9 minimizes reluctance of that path of the magnetic flux that intersects the rotor. The outer iron shell is only to confine the flux for environmental safety. Equations whose derivation is trivial are given in the following list without derivation.

Return conductor resistance:

$$R_{12} = \rho_{12} \frac{\ell}{r_2^2 - r_1^2} \frac{1}{\pi}, \quad (11)$$

where

$$\rho_{12} = \rho_{Cu} \frac{234.5 + T}{254.5},$$

with T = operating temperature ($^{\circ}\text{C}$), $\rho_{Cu} = 10^{-4}/54 \Omega \text{ cm}$;

Drum resistance:

$$R_{34} = \rho_{34} \frac{\ell}{r_4^2 - r_3^2} \frac{1}{\pi} \quad (12)$$

Fig. 9.

Definition of dimensions for the drum-type machine with double compensation (i.e., inner and outer current return).

with

$$\rho_{34} = \rho_{Al} \frac{234.5 + T}{254.5},$$

$$\rho_{Al} = 10^{-4}/30.5 \Omega \text{ cm};$$

Return conductor inductance:

$$L_{12} = \frac{\mu_0 \ell}{2\pi(r_2^2 - r_1^2)} \left(\frac{r_1^4}{r_2^2 - r_1^2} \ln \left| \frac{r_2}{r_1} \right| - \frac{3r_1^2 - r_2^2}{4} \right), \quad (13)$$

where

$$\mu_0 = 0.4\pi \cdot 10^{-6} \text{ H/m};$$

Inductance of the return conductor-to-drum interspace:

$$L_{23} = \frac{\mu_0 \ell}{2\pi} \ln \left| \frac{r_3}{r_2} \right|. \quad (14)$$

Similarly, for L_{45}

Drum inductance:

$$L_{34} = \frac{\mu_0 \ell}{2\pi(r_4^2 - r_3^2)} \left(\frac{r_4^4}{r_4^2 - r_3^2} \ln \left| \frac{r_4}{r_3} \right| - \frac{3r_4^2 - r_3^2}{4} \right); \quad (15)$$

similarly for L_{56}

Moments of inertia:

Drum:

$$\Theta_{34} = \frac{\pi}{2} \gamma_{34} \ell (r_4^4 - r_3^4), \quad (16)$$

where

$$\gamma_{34} = 2.71 \text{ g/cm}^3;$$

Fiberwrap:

$$\Theta_{45} = \frac{\pi}{2} \gamma_{45} \ell (r_5^4 - r_4^4), \quad (17)$$

where

$$\gamma_{45} = 1.66 \text{ g/cm}^3 ;$$

For the entire drum:

$$\theta = \theta_{34} + \theta_{45} . \quad (18)$$

4. Flux and Excitation Scaling. The proportions of coil and iron have empirically been optimized to within a few percent of an apparent optimum, with the aid of the magnetic field design code POISSON. The coil's inner radius was kept constant at $r_6 = 118$ cm, whereas the ratio of iron length to coil length and the total sector length l were varied. We assumed that the coil was enclosed in a toroidal dewar with 2.5-cm wall thickness, 2.0 cm for the super insulation, and 0.5 cm for structural elements such as stainless steel walls. A flux plot for this configuration is shown in Fig. 10.

The optimized set of linear proportions (in cm) as shown in Fig. 9 follows.

$$\begin{aligned} r_6 &= 118 \\ W &= 18 \\ l_6 &= 113 \\ r_7 &= 136 \\ r_7^* &= 170 \\ r_7 &= 109 \\ l &= 228 \text{ cm} \end{aligned}$$

The variable in the scaling problem for system optimization is the linear scaling factor λ operating on all linear dimensions, except the radial thickness w of the excitation coil.

We denote the reference dimensions in the following by the subscript 0 and define

$$\begin{aligned} \phi &= \text{flux effective at collector radius,} \\ (\text{IN}) &= \text{excitation ampere-turns,} \\ J &= \text{excitation coil average current density,} \\ w &= \text{excitation coil radial width, and} \\ B &= \text{maximum flux density in the coil.} \end{aligned}$$

By skipping trivial derivations, we can write the three fundamental scaling relationships

$$\begin{aligned} \frac{\phi}{\phi_0} &= \lambda \frac{(\text{IN})}{(\text{IN})_0} \\ \frac{B}{B_0} &= \frac{1}{\lambda} \frac{(\text{IN})}{(\text{IN})_0} \\ \frac{(\text{IN})}{(\text{IN})_0} &= \lambda \frac{w}{w_0} \frac{J}{J_0} \end{aligned} \quad (19)$$

where two scale variables may be eliminated in Eq. (19) as follows.

First, we substitute in Eq. (19) the J -vs- B relationship defining the limit for stable operation in the superconducting state for a stabilized superconductor. This may be expressed as

$$\frac{J}{J_0} = \exp \frac{B_0 - B}{B} \quad (20)$$

for a NbTi conductor³ with copper area to superconductor area ratio 2.9, where

$$\begin{aligned} J_0 &= 75 \text{ A/mm}^2, \\ B_0 &= 8 \text{ T,} \\ B &= 4.4 \text{ T,} \end{aligned}$$

and where J_0 has been multiplied by an assumed coil packing factor of 0.4.

Second, we scale the coil's radial width w with the linear scaling factor λ by setting in Eq. (19)

$$\lambda \frac{w}{w_0} = 1 \quad (21)$$

and inserting Eqs. (20) and (21) in Eq. (19) to obtain the set of two equations

$$\frac{B}{B_0} = 1 - \frac{B}{B_0} \ln \left| \frac{1}{\lambda} \frac{B}{B_0} \right| \text{ and} \quad (22)$$

$$\frac{\phi}{\phi_0} = \lambda^2 \frac{B}{B_0} .$$

For a given scaling factor λ , the flux density B is now uniquely determined by the first part of Eq. (22), and ϕ by the second part. The design code DRUMTYP uses Newton's method to solve the transcendental Eq. (22) for B .

The flux ϕ is yet to be multiplied by a correction factor to account for the fact that the space for the dewar wall must be kept constant and that there is some, although small, saturation effect. The former increases the flux density with increasing scaling factor; the latter decreases it because flux density increases with increasing λ according to the first part of Eq. (22). This small correction was determined empirically from POISSON computations and may be written

$$f = 1.0373 - 0.0373 \frac{B}{B_0} \quad (23)$$

Fig. 10.
 Flux plot for standard machine proportions
 and plot of flux at axial position $z = 15$ cm vs
 radius.

The flux $\phi_{ref} = f \cdot \phi$ obtained from Eqs. (22) and (23) exists with respect to the linearly scaled reference radius $r_{ref} = \lambda r_{30}$, where $r_{30} = 104$ cm. Therefore, we must know the flux as a function of radius for the given set of linear proportions. This is shown in Fig. 10 as determined from POISSON calculations, and may be expressed as

$$\frac{\phi(r)}{\phi(r_{ref})} = \left(\frac{r}{r_{ref}} \right)^{2.328} \quad (24)$$

5. L-C Circuit Equations. The L-C energy transfer circuit is shown in Fig. 11, where V_0 is the initial output voltage per Eqs. (5) and (24), multiplied by the number of rotors n connected in series, i.e.,

$$V_0 = n \frac{\omega}{2\pi} \lambda^2 f \phi_0 \left(\frac{r}{\lambda r_{30}} \right)^{2.328}, \quad (25)$$

where $\phi_0 = 36.3$ Wb as obtained from POISSON.

Let ΔV_B be the sum of $2n$ brush voltage drops, each one equal to ~ 0.8 V as shown in Sec. E. Let L_{tot} and R_{tot} be the sum of all inductances and resistances including those of the compression coil and the leads. For the purpose of circuit optimization, 10% was added to the sum of the internal resistances of Eqs. (11) - (15) to account for lead loss in the homopolar machine, and 10% was added to the compression coil inductance and resistance to account for its lead loss.

The equivalent capacitance of the machine is obtained from the stored energy correspondence

C = HOMOPOLAR MACHINE EQUIVILANT CAPACITANCE

ΔV_B = BRUSH VOLTAGE DROP (~ 0.8 V PER BRUSH)

R_{tot} = SUM OF ALL CIRCUIT RESISTANCES

L_{tot} = SUM OF ALL CIRCUIT INDUCTANCES

Fig. 11.

L-C energy transfer circuit.

$$\frac{1}{2} \theta \omega^2 = \frac{1}{2} C V_0^2$$

and its time derivative

$$\theta \omega \frac{d\omega}{dt} = C V_0 \frac{dV_0}{dt},$$

which show that the following relationship holds:

$$C = \theta \left(\frac{\omega}{V_0} \right)^2 \quad (26)$$

N compression coils are connected in parallel to the same homopolar machine so that the current $I = NI_{coil}$, the coil inductance $L_C = 1.1 L_{coil}/N$, and the coil resistance $R_C = 1.1 R_{coil}/N$; the factors 1.1 are to include the leads.

The circuit equation may be written immediately in the Laplace domain, i.e.,

$$0 = \frac{-\Delta V_B}{s} + \frac{I(s)}{sC} + \frac{V_0}{s} + I(s) (R_{tot} + sL_{tot}), \quad (27)$$

where s is the Laplace operator and where $-\Delta V_B/s$ and V_0/s are the initial value terms, assuming that the initial voltage generated is $V_0 - \Delta V_B$.

The current is thus obtained from inverse transformation of Eq. (27), i.e.,

$$i = \frac{V_0 - \Delta V_B}{\Omega L} e^{-\delta t} \sin \Omega t,$$

$$\text{where } \Omega^2 = \Omega_0^2 - \delta^2,$$

$$\Omega_0^2 = 1/(L_{tot}C), \quad (28)$$

$$\delta = R_{tot}/(2L_{tot}).$$

The maximum current occurs for $\Omega t = \arctan(\Omega/\delta)$ for which Eq. (28) yields

$$I_{max} = \frac{V_0 - \Delta V_B}{Z_0} \exp \left[-\frac{\delta}{\Omega} \arctan \left(\frac{\Omega}{\delta} \right) \right], \quad (29)$$

$$\text{where } Z_0 = \sqrt{L/C}$$

The machine voltage $v_c(t) = 1/C \int i dt$ is now easily obtained, i.e.,

$$v_c = (V_0 - \Delta V_B) \left(\cos \Omega t - \frac{\delta}{\Omega} \sin \Omega t \right) e^{-\delta t}. \quad (30)$$

At time $t = \pi/\Omega$, we have $i = 0$, and from Eq. (29)

$$\frac{v_c(\pi)}{v_o} = \frac{\Delta v_B}{v_o} - \left(1 - \frac{\Delta v_B}{v_o}\right) \exp(-\delta\pi/\Omega).$$

The circuit efficiency is given by

$$\eta_{\text{circuit}} = \left(\frac{v_c(\pi)}{v_o}\right)^2,$$

and the loss per half-cycle may be written

$$\frac{1}{2} (1 - \eta_{\text{circuit}}) C v_o^2.$$

We later add to this loss the coasting losses, which consist of brush friction loss, bearing loss, refrigeration loss, and charging loss, to yield the total loss per pulse cycle.

B. Machine Topology Considerations

Figure 12 illustrates the topological configurations that have been considered for fast-discharging homopolar machines. These alternatives are discussed in some detail in this section.

Figure 12a shows the single disk, which is mechanically the simplest configuration. The shaft can be simply supported in two radial bearings, resulting in the highest natural frequencies of any of the designs. However, it pays for its mechanical simplicity in electrical complexity. To avoid extremely high axial thrusts during discharge, equal currents should be collected from terminals at each end of the machine. Also, the simple disk does not economically lend itself to large installations, because once the practical size limit is reached for a single disk, the entire system must be duplicated.

Another drawback of the single disk approach has been discussed in detail previously.⁴ Briefly stated, for a given energy requirement, the discharge time is inversely proportional to the equivalent capacitance of the homopolar machine(s). For a given energy storage, multiple rotors are capable of producing higher voltages and thereby lower capacitances and shorter discharge times. This is true because for a given rotor material and operating stress level, the energy stored in the rotor is proportional to the radius cubed, whereas the voltage is directly proportional to the radius.

The final limitation of a single disk machine that should be mentioned is the fact that as discharge times become small and peak decelerating torques consequently become high, the machine structure

and foundation must be capable of sustaining the large forces involved.

The counterrotating disk configuration in Fig. 12b eliminates this last problem because the net angular momentum of the two rotors is always constant (zero). In addition the voltage of such a machine is greater for the same energy, or a machine storing twice the energy (two rotors) can be built using approximately the same field coil and stator assembly required by the simple disk. The counterrotating system is also electrically more convenient because all power is available from a single set of terminals, but it is mechanically more complicated because the rotors are cantilevered by the shafts. This tends to lower the natural frequencies of the shaft-rotor-bearing system and results in the possibility of interference with rotational speeds encountered during the operating cycle. Although conveniently doubling the energy available in a single disk system, this system still is not suitable for large, multiple rotor machines necessary for multigigajoule installations.

Figure 12c illustrates a logical extension of the configuration shown in Fig. 12b in which the cantilevered rotors of adjacent machines are placed on common shafts. This "spool" configuration does lend itself to multiple rotor installations and also eliminates one set of sliding contacts (the shaft brushes) per rotor, thereby providing a decrease in brush losses and a corresponding increase in machine efficiency. Unfortunately, the return conductor routing becomes complex if the machine's internal inductance is to be minimized.

The inside return conductor spool machine, shown in Fig. 12d, significantly simplifies the return conductor configuration at the expense of a slight penalty in internal inductance.

Until now, the discussion of topological alternatives for homopolar machines centered around disk-type devices, that is, devices using axial magnetic fields and radial current flow. A similar family of devices, referred to as drum-type machines, uses radial magnetic fields and axial current flow. The similarities and differences between disk and drum devices have been covered in detail in Ref. 4, and the outside and inside return conductor equivalents are shown in Fig. 12e and f. The outside return conductor drum machine allows conventional, small-diameter rotor bearings at the expense of increased flux leakage because the return conductor in this configuration occupies the area of highest flux density. The inside return conductor drum configuration allows greater flux linkage and consequently higher voltage for the same rotor but it requires the development of large-diameter, high-speed bearings for the outside of the drum. However, this seems to be an ideal application for gas bearings, as shown in Sec. D.

Fig. 12.
Alternative topological configurations for fast-discharging homopolar machines.

The relative advantages of the spool and drum configurations can best be summarized as follows.

- The drum configuration has the potential of lower internal impedance and therefore higher discharge efficiency.

- The disk and spool configurations have the potential for maximum flux linkage and therefore higher voltage and lower equivalent capacitance, resulting in lower potential discharge times for the spool configurations.

C. Rotor Mechanical Considerations

The two types of homopolar machine configurations, the disk and drum types, are again shown in Fig. 13.

1. Disk Machine. The disk configuration was successfully designed, built, and tested at the University of Texas at Austin. The configuration is mechanically simple, consisting essentially of a disk that is shrunk fit onto a solid shaft.

When conceptual applications began requiring more than one disk, and particularly in fusion studies where losses are critical, the spool design emerged (Fig. 14).

Because a hole in the shaft for an internal return conductor would be required in our design, the effect of the hole on the fatigue life of the machine must be considered.

Let us assume that a disk and a solid shaft are to be shrunk fit together and spun with angular velocity ω . The maximum tangential stress at the inner surface of the disk is

$$\sigma_t = \frac{\rho\omega^2}{8} \left[(3+\mu)c^2 - (1+3\mu)b^2 \right] + \frac{E\delta_{in}}{2} \left(1 + \frac{b^2}{c^2} \right) \quad (31)$$

where μ = Poisson's ratio for the material

ρ = density of material

c = outer radius of disk

b = inner radius of disk

a = inner radius of shaft

E = Young's modulus of elasticity

δ_{in} = initial shrink fit interference per unit interface radius.

If a hole is put in the center of the shaft, the tangential stress at the inner radius of the disk becomes

DISK TYPE

DRUM TYPE

Fig. 13.

Disk shrunk on shaft and composite reinforced drum.

$$\sigma_t = \frac{\rho\omega^2}{4} (3+\mu)c^2 \left[\frac{1}{2} + \frac{b^2}{c^2} \left(\frac{1-\mu}{3+\mu} - \frac{1}{2} \right) + \frac{a^2}{2} \left(\frac{1}{b^2} + \frac{1}{c^2} \right) \right] + \frac{E\delta_{in}}{2} \frac{(b^2-a^2)(c^2+b^2)}{b^2(c^2-a^2)} \quad (32)$$

The maximum alternating stress

$$\sigma_{alt} = \sigma_t(\omega = \omega_{max}) - \sigma_t(\omega = 0) \quad (33)$$

SPOOL TYPE

Fig. 14.

Spool consisting of disks shrunk on hollow shaft.

is less if Eq. (31) is substituted in Eq. (33), than with Eq. (32) substituted. This indicates that the fatigue life of the hollow shaft design is inferior to that with solid shaft. Therefore, the speed must be reduced if a hollow shaft is used. This is to be expected because any hole, regardless of size, prevents a radial restraining force between two points 180° apart on its periphery.

2. Drum Machine. Consider two concentric drums, one the rotor material and the other the composite wrap (Fig. 13b), rotating at some angular velocity ω . Assume that the composite is not pre-tensioned. Each material tries to grow radially outward due to centrifugal force. However, the amounts of growth differ because of the materials' different stiffnesses. The composite is much stiffer and stronger than the rotor material. Therefore, the rotor material tries to grow more than the composite and is thus constrained by the composite by an apparent interference

$$\Delta = U_c - U_r \text{ (at interface radius)}$$

$$\Delta = \frac{\omega^2}{4} \left\{ \frac{\rho_c}{E_c} \left[(3+\mu_c) r_s^2 + (1-\mu_c) r_o^2 \right] - \frac{\rho_r}{E_r} \left[(3+\mu_r) r_s^2 + (1-\mu_r) r_o^2 \right] \right\} \quad (34)$$

where Δ = apparent interference per radius unit
 U_c = radial growth of composite per radius unit

- U_r = radial growth of rotor material per radius unit
- ρ_c = density of composite
- ρ_r = density of rotor material
- E_c = Young's Modulus of Elasticity of composite
- E_r = Young's Modulus of Elasticity of rotor material
- μ_c = Poisson's ratio of composite
- μ_r = Poisson's ratio of rotor material

This apparent interference causes an interface pressure, P .

$$P = - \frac{\Delta}{\frac{1}{E_c} \left(\frac{r_o^2 + r_s^2}{r_s^2 - r_o^2} + \mu_c \right) + \frac{1}{E_r} \left(\frac{r_o^2 + r_s^2}{r_o^2 - r_s^2} - \mu_r \right)} \quad (35)$$

Note that Δ will be a negative value because the rotor material tries to grow more than the composite.

With P determined, one can then determine the stresses. The largest stress for each material will be at its respective inner radius and can be calculated using the following equations:

$$\sigma_{total} = \sigma_{pin} \pm \sigma_{pressure}$$

σ_{tr} = tangential stress of rotor material at inner radius r_3

$$\sigma_{tr} = \rho_r \omega^2 \frac{3+\mu_r}{8} \left[2r_o^2 + r_s^2 \left(1 - \frac{1+3\mu_r}{3+\mu_r} \right) \right] - 2P \frac{r_o^2}{r_s^2 - r_o^2} \quad (36)$$

σ_{tc} = tangential stress of the composite at inner radius r_4

$$\sigma_{tc} = \rho_c \omega^2 \frac{3+\mu_c}{8} \left[2r_s^2 + r_o^2 \left(1 - \frac{1+3\mu_c}{3+\mu_c} \right) \right] + P \frac{r_s^2 + r_o^2}{r_s^2 - r_o^2} \quad (37)$$

Using the following values and previously stated equations, we calculated the maximum tangential stresses for the composite and rotor material.

Rotor material — aluminum (6063-T6)

$$\begin{aligned}\rho_r &= 2.71 \times 10^3 \text{ kg/m}^3 \\ E_r &= 7.58 \times 10^{10} \text{ N/m}^2 \\ \mu_r &= 0.33\end{aligned}$$

Composite-graphite fibers in an epoxy matrix (Celion GY-70)

$$\begin{aligned}\rho_c &= 1.66 \times 10^3 \text{ kg/m}^3 \\ E_c &= 28.93 \times 10^{10} \text{ N/m}^2 \\ \mu_c &= 0.25\end{aligned}$$

$$\begin{aligned}r_3 &= 0.967 \text{ m} \\ r_4 &= 0.970 \text{ m} \\ r_5 &= 0.973 \text{ m} \\ \omega &= 264 \text{ rad/s}\end{aligned}$$

$$\begin{aligned}\Delta &= -1.953 \times 10^{-3} \\ P &= 3.870 \times 10^5 \text{ N/m}^2 \\ \sigma_{ir} &= 6.180 \times 10^7 \text{ N/m}^2 \\ \sigma_{ic} &= 2.347 \times 10^8 \text{ N/m}^2\end{aligned}$$

These stresses are well under the allowable stresses for each material for a 25-yr life, based on Goodman diagrams.

To account for the electrodynamic forces, we let I_1 be the current through the inner return conductor, I_2 be the outer return current, and $I_0 = I_1 + I_2$ be the rotor current. The corresponding field energy

$$W = \frac{1}{2} \left[(L_{12} + L_{23}) I_1^2 + L_{34} I_0^2 + (L_{46} + L_{66}) I_2^2 \right],$$

where the inductances were defined in Eqs. (14) and (15). The radial electrodynamic pressure on the drum

$$\begin{aligned}P_r &= \frac{1}{4\pi r_1^2} \left[I_1^2 \frac{dL_{12}}{dr_1} + I_1^2 \left(\frac{\partial L_{12}}{\partial r_1} + \frac{\partial L_{13}}{\partial r_1} \frac{dr_1}{dr_1} \right) \right. \\ &\quad \left. + I_2^2 \frac{\partial L_{46}}{\partial r_1} \frac{dr_1}{dr_1} \right]\end{aligned}$$

where the derivatives of L_{12} and L_{66} vanished because of their r_1 -independence and where

$$\frac{dr_1}{dr_1} = 1 - 2\nu \frac{r_4 - r_1}{(1+\nu)r_4 + (1-\nu)r_1}.$$

The tangential stress resulting from the internal magnetic pressure

$$\sigma_{\text{magnetic}} = P_m (r_4^2 + r_1^2) / (r_4^2 - r_1^2).$$

Remembering that the spin stress is proportional to ω^2 and the magnetic stress to i^2 , we recall the relationships Eqs. (26) and (28). Neglecting the exponential term in Eq. (28), we write for the combined stress

$$\sigma = \sigma_{\text{spin}} - (\sigma_{\text{spin}} - \sigma_{\text{magnetic}}) \sin^2 \Omega t.$$

The current returned via the outer return conductor, i.e., the ratio I_1/I_2 is chosen so $\sigma_{\text{spin}} - \sigma_{\text{magnetic}} > 0$. The desired current ratio is easily achieved in practice by connecting the proper number of coils to the outer return conductor, and the rest to the inner one.

D. Gas Bearing System

A gas bearing is suited for designs involving high relative velocities and light loads typical in the L-C circuit applications discussed in this report. In the case of the 1.025-m-radius rotor listed in Table I, for example, the mass of the rotor is only 415 kg. The bearing pressure from this gravity load is 875 N/m^2 (0.13 psi). At a loading this light, the rotor could actually slide in the bearing, with no lubricant, to a significant speed without excessive wear.⁵

Following is a list of the assumptions used in calculating the loss due to the radial bearings and a short discussion of the errors involved in their use.

a. *Bearing Type.* A hydrodynamic gas bearing* is well suited for this application. However, we assume a quarter station film-compensated hydrostatic gas bearing, as shown in Fig. 15. In this bearing, pressurized gas is pumped through orifices into the clearance between the bearing and rotor. The reason for this bearing choice is its advantage of greater flexibility in avoiding critical frequency problems over that typical for hydrodynamic bearings. During detailed design, the hydrodynamic bearing should not, however, be disregarded, because its much lower loss is attractive. In other words, the quarter station film-compensated bearing was chosen because it is the most conservative in terms of stability and loss.

b. *Bearing Clearance.* The bearing clearance c is assumed to be equal to 0.001 R. This is relatively large for a gas bearing and may have to be reduced for stability reasons.⁶ As we will show later, the bearing losses are dominated by the compressor power. Thus, any reduction in the clearance will tend to

*A hydrodynamic bearing achieves pressurization by spinning.

Fig. 15.

Quarter station, film-compensated gas bearing.

reduce the bearing loss due to the resulting reduction in gas flow. The main reasons for choosing such a large clearance are assembly consideration and rotor growth, where the latter is on the same order as the bearing clearance. The clearance at rest and zero current is obviously larger than the operating clearance. In other words, a large clearance is desirable to reduce the relative magnitude of the spin and electrodynamic rotor growths.

c. *Gas.* The gas is assumed to be hydrogen at an ambient pressure of 1 atm. Because of cooling considerations, this pressure must be increased, however. This results in a decreased kinematic viscosity, making operation in the laminar regime impossible for the given range in clearance.

d. *Flow Regime.* Our calculations for laminar gas flow in the bearing were carried out for two reasons. (1) Initial evaluation of turbulent flow indicates significant reduction in compressor power accompanied by an increase in drag on the rotor. Because bearing loss is dominated by compressor pumping power, a net reduction in bearing loss may result for turbulent analysis as compared with laminar flow analysis. (2) The bearing with 1 atm hydrogen operates in the laminar flow regime, and the calculations for it are simple, whereas numerical

evaluation of the turbulent hydrostatic bearing complicates parameter space mapping.

Following is the algorithm used in calculating the loss associated with the quarter station laminar flow system. The equations cited are used in the design program DRUMTYP described in Sec. IV.

The first step is to calculate the required bearing stiffness K_b , to insure that the first bearing-rotor critical speed is above $\sqrt{1.5}$ times the operating speed, i.e.,

$$K_b = 1.5 \omega^2 m,$$

where m is the rotor mass.

Next, the difference between the supply pressure P_s and the ambient pressure inside the machine P_a is calculated using the equation⁵

$$K_b = \frac{C_L D L_B (P_s - P_a)}{0.5c},$$

where C_L = load coefficient = 0.17,

D = bearing diameter, and

L_B = bearing length.

Next, the gas mass flow rate G required to maintain this pressure drop is calculated using⁵

$$G = \frac{\pi R C^3 (P_1^2 - P_a^2)}{6 \mu l_2 R T},$$

where P_1 = bearing pocket pressure = $P_s + P_a/2$

μ = gas viscosity = 1×10^{-5} Nsm⁻² for H₂

$l_2 = L_B/4$

R = gas constant = 4121 J kg⁻¹ K⁻¹, and

T = ambient temperature = 343 K.

The compressor power H to supply the gas can now be calculated using⁵

$$H = \frac{G R T k}{(k-1)\eta} \left[\left(\frac{P_s}{P_a} \right)^{\frac{k-1}{k}} - 1 \right],$$

where k = ratio of specific heats = 1.4

η = compressor efficiency ≈ 0.85 .

The laminar shear loss H_s for the bearing can now be calculated using⁵

$$H_s = \frac{2 \pi \mu \omega^2 R^3 L_B}{c}.$$

Applying turbulent theory⁷ to this case predicts an increase in drag by a factor of about 3.

The total loss for the bearings without considering the benefits of turbulence on the compressor loss varies between 42 and 95 kW time-averaged over the 0.4-s pulse period, or 0.21 - 0.47% of the energy stored initially for the examples listed in Tables I and II of Sec. IV.

The major effects of discharge on the rotor-bearing system occur in two areas: gas dynamics in the bearing and electrodynamic forces on the rotor due to deviations from ideal symmetry in the actual design. The gas dynamics problem is related to the turbulence induced by the sudden starting and stopping of the cylinder. Because the flow regime is turbulent during steady state, this additional turbulence is not considered significant. The electrodynamic forces on the rotor are due to deviations from the ideal, perfectly aligned case because they originate from misalignment of the electrical and mechanical axes and from uneven current distribution in the rotor. The study of these phenomena has begun at the University of Texas at Austin. Initial calculations show that these forces do not cause failure of the 1.025-m rotor radius reference machine, partly because of the rotor displacements that can be tolerated in the gas bearing. Axial (thrust) gas bearings of conventional design to withstand axial forces generated from misalignment can be used. In this case the bearing loss is dominated by the radial bearing.

E. Current Collecting Brushes

The sliding electrical contacts and their actuating mechanism will require further development. The operating conditions described here are based on data collected by the Energy Storage Group at the University of Texas at Austin⁸. At speeds from 80 to 170 m/s and a current density of 6.7 A/mm², coefficients of friction from 0.15 to over 0.35 were measured. Their 5-MJ homopolar machine generates 0.7-s, 560-kA pulses with brush current densities of 24.9 A/mm². Data were collected using sintered copper-graphite brushes sliding on AISI 4340 steel slip rings.

Controlled atmosphere tests are now being run to investigate the performance of similar brushes on specially coated aluminum alloy slip rings. We expect preliminary results by mid 1976.

Because of the numerous pulses required in this application, the high repetition rate, and the very high $J \times B$ forces in the brush area, conventional sliding brush mechanisms are unacceptable. The brushes must be raised between discharges because of the brush and rotor heating considerations, the efficiency, and brush wear requirements. The brushes

can be mounted on cantilevered beams, which serve as current carrying and brush locating functions (Fig. 16). The University of Texas 5-MJ homopolar machine uses similar beams, which deflect elastically to raise and lower the brushes and plastically to compensate for wear. Such devices can withstand large $J \times B$ forces and still require low actuation force. They also are susceptible to fatigue from the numerous cycles anticipated (assuming 1-yr brush life), but they are easily replaced in units with the brushes.

This suggests the possibility of using individual brushes polymerically bonded into circumferential rings (Fig. 16b). The brush rings could then be deflected onto the slip rings by an actuating fluid, and the polymer could supply the restoring spring force. The direct action of the fluid on the brushes and the required brush system response suggest that acoustic shock waves or a resonant fluidic system be used to control the up-down cycle of the brushes. Work is under way at the University of Texas on systems using this principle.

Fig. 16.
Current collecting brush mechanism.

It is assumed in the optimization computations that the brush contact load averaged over the 0.4-s pulse cycle is 45 mbar. For the $\tau_{12} = 3$ ms case, this is equivalent to a static load of 1 bar over a 9-ms interval during which the brushes are lowered to contact with the drum, plus an electrodynamic brush load ($\propto i^2$) with 8-bar crest. The friction coefficient is assumed to be 0.22 as shown in Table II of Sec. IV, and the brush voltage drop is assumed $\Delta V_B = 0.8V$ per brush.

F. Thermal Considerations

Most of the joule losses occur in the rotor during energy transfer to and from the machine. Additional losses are caused by friction and contact resistance at the brush-rotor interfaces. The joule heating is uniform in the aluminum conductor of the rotor. On the inner surface at both ends of the rotor, a surface heat flux will occur because of the contact resistance and frictional heating of the brushes.

In the 1.025-m motor discussed in Sec. IV, the rotor cooling system must remove 1.6 MW (5.4×10^6 BTU/h) per rotor. Because pulse time τ_{12} is small compared with the 0.4-s pulse cycle, the generated heat appears primarily in the rotor and will be evidenced by a rise in temperature. Most of the active cooling must then be essentially accomplished between pulses. Assuming the aluminum conductor material weighs 382 kg (840 lb) and the specific heat is $879 \text{ J/kg}^\circ\text{K}^{-1}$ (0.21 BTU/lb $^\circ\text{F}$), we find that for a single pulse the temperature rise is 2°C (3°F). This periodic rise in temperature coincides with the transfer of energy from the machine and is superimposed on the steady-state operating temperature of 60°C (140°F).

The aluminum conductor is surrounded by a composite of graphite fibers in an epoxy matrix. The rotor spins in an annular space formed by the gas bearing and field coils on the outside and the return conductor on the inside. Cooling electric generators by flowing hydrogen gas axially in the annular gap between stationary and rotating parts is common. Assuming that the homopolar machine is cooled by hydrogen gas, an expression for the steady-state heat removal can be written

$$\dot{Q}_T = \dot{m}_I C_p \Delta T_{gi} + U_I A_I (T_R - T_C) + \dot{m}_O C_p \Delta T_{go} + U_O A_O (T_R - T_B) \quad (38)$$

where

$\dot{Q}_T$ = average heat rate

$\dot{m}_I$ = gas flow rate—inner gap

$\dot{m}_O$ = gas flow rate—outer gap

ΔT_{gi} = temperature rise of the gas—inner gap

ΔT_{go} = temperature rise of the gas—outer gap

C_p = specific heat of the gas

U_I = overall heat transfer coefficient across inner gap

U_O = overall heat transfer coefficient across composite and outer gap

A_I = area of cylinder at inner radius

A_O = area of cylinder at outer radius

T_R = temperature of rotor

T_C = temperature of return conductor

T_B = temperature of bearing

Equation (38) requires that the average heat generation rate equal the heat rate that can be transferred to the hydrogen gas flowing in the inner gap and across that gap to the return conductor. It also includes heat transferred to the hydrogen gas used by the bearing (in the outer gap) and across that gap to the bearing surface. The heat removed across the gap is assumed to be removed by a higher density fluid.

Heat transfer coefficients for flow modes encountered with axial flow between rotating concentric cylinders is not well described in the literature for the inner gap of the proposed configuration. However, heat transfer estimates can be obtained from Kays and Leung⁹ for axial flow between stationary walls; these heat transfer rates would be considered conservative for our configuration because the swirling flow induced by rotation tends to improve heat transfer. For the outer gap, Becker and Kaye¹⁰ have reported data for "diabatic flow;" however, our case does not fall in the range of their data for axial flow. Therefore, a zero axial flow correlation was used for our estimates.

Operation of the gas bearing requires a hydrogen gas flow of 600 kg/h (1350 lb/h). This flow offers advantages for heat removal by means of its heat capacity [with $\Delta T_{gi} = 39^\circ\text{C}$ (70°F)] and convective transfer to the bearing surface [cooled to an ambient temperature of 10°C (50°F)]. These advantages would be significant in that 20% of the total heat generated would be removed. The remaining heat removal would require ~ 5400 kg/h (12000 lb/h) of hydrogen with $\Delta T_{gi} = 39^\circ\text{C}$ at 5 atm. Active cooling of the gas bearing and return conductor would require $51 \text{ m}^3/\text{h}$ (225 gpm) of water at $\Delta T_w = 11^\circ\text{C}$ (20°F). We believe that careful evaluation of such effects as rotor surface roughness, unsteady motion of the rotor, and swirling flows would lower the required hydrogen flow rate and operating pressure. In such a case more heat would be transferred across the gap; however, a higher water flow rate would also

*Diabatic is synonymous to nonadiabatic.

be necessary. These flow rate estimates for cooling suggest that care must be exercised in designing such a system.

G. Cost Equations

The conceptual design work for the Reference Theta-Pinch Reactor (RTPR) is under contract with the Electric Power Research Institute (EPRI).¹¹ Costing of the 1.3-GJ, 60-ms discharge machine for the (RTPR) is done with the following cost formula.

$$\$_{tot} = 1.2n (M_{Fe} \$_{Fe} + M_{SC} \$_{SC} + M_C \$_{C} + A_B \$_{B}), (39)$$

where M_{Fe} = iron weight

M_{SC} = superconductor weight including copper

M_C = weight of drum conductors and return conductors

A_B = brush-to-collector contact area.

The coefficients in the cost equation are

$$\$_{Fe} = 1.59 \text{ \$/kg}$$

$$\$_{SC} = 63.8 \text{ \$/kg}$$

$$\$_{C} = 9.10 \text{ \$/kg}$$

$$\$_{B} = 4.81 \text{ \$/cm}^2.$$

The set of constants, Eq. (40), has been derived on the premise of large production quantities with considerable cost reduction from learning curves, assuming that three RTPR reactors each with fifty 1.3-GJ homopolar machines have already been completed.

For our purpose, we do not consider large production quantities or small laboratory quantities, and the following cost formula appears to be realistic.

$$\begin{aligned} \$_{tot} = 1.2 \left[nM_{Fe} \$_{Fe} + nM_C \$_{C} + nA_B \$_{B} \right. \\ \left. + nM_{coil} \$_{coil} + nV_{He} \$_{He} + nL_n (IN) \$_{SC} \right. \\ \left. + (nP_{ref})^{0.7} \$_{ref} + n\lambda^2 \$_{dewar} \right] \end{aligned} \quad (41)$$

where M_{Fe} = iron weight

M_C = weight of drums and return conductors

A_B = brush-to-collector contact area

M_{coil} = weight of stabilized superconductor (i.e. $0.4 \times \text{coil volume} \times 8.9 \text{ g/cm}^3$)

V_{He} = volume of liquid helium inventory (i.e. $0.6 \times \text{coil volume}$)

(IN) = excitation ampere-turns

l_w = mean turn length

P_{ref} = refrigeration power

λ = scaling factor

$\$_{Fe} = 1.59 \text{ \$/kg}$

$\$_{C} = 9.0 \text{ \$/kg}$

$\$_{B} = 4.80 \text{ \$/cm}^2$

$\$_{coil} = 17.5 \text{ \$/kg}$

$\$_{He} = 2.0 \text{ \$/l}$

$\$_{SC} = 0.00165 \text{ \$ A}^{-1} \text{ m}^{-1} (0.5 \text{ mills/A-ft})$

$\$_{ref} = 8400 \text{ \$ kW}^{-0.7}$

$\$_{dewar} = 8400 \text{ \$}$

The factor 1.2 in Eq. (41) accounts for the assembly cost, which should be 1.25 in those cases where double compensation is required. Cost computed for a machine designed for the LTPHR using Eq. (41) is usually 1.23-1.31 times that obtained from Eq. (39) with the constants, Eq. (40).

The refrigeration power is given¹²

$$P_{refr} = \left(\frac{T_{op}}{4.0} - 1 \right) \frac{P_{loss}}{\eta_{compr}}, \quad (42)$$

where T_{op} = operating ambient machine temperature $\approx 353 \text{ K}$

$$P_{loss} = P_{surface} + P_{lead} + P_{rel}$$

$$P_{surface} = p (T_{op} - 4.0) \cdot S$$

$$p = 1.25 \text{ m W K}^{-1} \text{ m}^{-2}$$

$$S = \text{dewar surface}$$

$$P_{lead} = 0.25 \text{ W (lead loss)}$$

$$P_{rel} = 0.1 \text{ W (relief valve loss)}$$

IV. OPTIMIZATION

DRUMTYP is a FORTRAN program that maps the parameter space for L-C-R circuits with the homopolar machine and carries out point design. Figure 17 shows a simplified flow diagram of DRUMTYP.

When mapping the parameter space, the following quantities are used.

Constants—Compression coil L_{coil} , R_{coil} , I_{coil}

Parameters— N = number of compression coils connected in parallel

n = number of rotors connected in series

r_1 = rotor radius

$t_{12} = \pi/\Omega$

Independent variables — t_{drum} = drum conductor thickness

$v = \omega r_3$ = drum surface speed

Fig. 17.
Program DRUMTYP simplified flow diagram.

Dependent variables — $\eta = 1 - f_{L,ex} + f_{b,k} =$
efficiency

$$c = 100 \$_{tot} / 1/2 CV_0^2 = \text{cost rate (\$/J)}$$

Figure 18 is a plot of η vs c for $N = 32, 36$, and 40 , $\tau_{1/2} = 3$ ms, where the parameter $n = 32$ and 36 . The plot shows that efficiency and cost rate increase with increasing radius.

The curves in Fig. 18 move up and to the left, indicating decreasing cost and increasing efficiency with increasing number of rotors (for a given number of parallel coils) to an optimum beyond which the trend reverses. Hence, with each N , there is associated an optimum n , and for this set of L_{coil} , R_{coil} , and L_{coil} values, the optimum

$$n_{opt} \approx N.$$

The figure shows that a substantial number of coils must be paralleled for short discharge times, which may result in large currents requiring double compensation.

A useful optimization procedure is therefore as follows, for given $\tau_{1/2}$.

(1) Guess initial N .

(2) Guess initial n .

(3) Increase n ; if the η vs c characteristic improves, continue; if not, decrease n ; continue until optimum n_{opt} is found.

(4) Change N to $N = n_{opt}$, and verify that n_{opt} is still the same, or go back to 3.

With each $\tau_{1/2}$ value a set (N, n) is associated where $N \approx n = n_{opt}$ exists. This optimum characteristic fits within an error $|\pm \epsilon| < 0.5\%$ and, neglecting the 4% cost increase discussed in Sec. G, for double compensation, we find the following equation empirically

$$\eta = H_0 \left[1 - \exp(-c/K_0) \right]. \quad (43)$$

The parameters H_0 and K_0 in Eq. (43) have been plotted in Fig. 19. The variable c is constrained to

$$1.5 \text{ to } 3.5 < c/K_0 < \sim 1.9 \text{ to } 15 \begin{cases} 1.9 \text{ for } \tau_{1/2} = 1.5 \text{ ms} \\ 3 \text{ for } \tau_{1/2} = 2.0 \text{ ms} \\ 5 \text{ for } \tau_{1/2} = 3.0 \text{ ms} \\ 6 \text{ for } \tau_{1/2} = 5.0 \text{ ms} \\ 9 \text{ for } \tau_{1/2} = 10.0 \text{ ms} \\ 13 \text{ for } \tau_{1/2} = 30.0 \text{ ms} \end{cases}$$

where the upper limit is set by the flux model. In other words, if c/K_0 increases beyond 2 to 15, then the flux density, which is scaled in accordance with the first part of Eq. (22), will exceed our arbitrary but practical limit of 8 T, as a result of $\lambda > 1.2$. The lower limit in the c/K_0 constraint corresponds to the

minimum rotor radius for which structural stability in the drum-composite structure exists, with a 1.5 factor of safety. Optimization could be carried to $\lambda > 1.2$ if B is kept constant at 8 T; however, this was not done.

Some uncertainty exists in the magnetic flux value ϕ_0 , for which the lower limit of the computed values $36.3^{+1.9}_{-0}$ has been chosen. This results from the uncertain width of the collector area, which is a design variable. If the computations leading to the curves plotted in Fig. 18 are repeated for the upper limit, i.e., for 38.2 Wb, we obtain an increase of 0.5% in η , leading to $\epsilon = 1.0\%$, which together with the fitting error results in the error bars in Fig. 19. The normalization is treated in Appendix B.

The question arose as to how much better the efficiency-vs-cost characteristic of a spool machine might be compared with the drum type. A split coil configuration was computed for this purpose, in which the disks extend into the region where the vector potential maximum exists, i.e., in the space between coil halves when each coil is split in its flux-normal plane $z = 0$. The reference flux increases for this configuration to numbers from 40 - 42 Wb if the maximum field in the coils is maintained. This results in an improved η -vs- c characteristic, plotted in Fig. 18, and amounts to an efficiency increase of 1 - 3%, which makes possible $\eta = 91\%$ machine for $c = 4.4$ $\$/J$ as compared with 5.6 $\$/J$ for a drum machine with the same efficiency for $\tau_{1/2} = 3$ ms.

Table I gives the characteristic design numbers for $\tau_{1/2} = 3$ ms, corresponding to $N = 36$ and $n = 32$ in Fig. 18. Tabulated are all major problem constants used in the computations leading to the plots of Figs. 18 and 19. The loss and cost elements of one of the tabulated designs, the 13th line ($f_{cycle} = 0.9$, $c = 4.84$ $\$/J$), are listed in Table II.

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Fig. 18.

Efficiency $\eta = 1 - f_{ex}f_L + f_{blk}$ vs cost rate (ϕ/J) for number of parallel coils $N = 32, 36, 40$.

Fig. 19.
Optimization characteristics
 η = number of machine sections in series
 H_o = efficiency limit
 K_o = logarithmic cost decrement vs discharge time $\tau_{1/2}$.

TABLE I

36 PARALL. COMPR. COILS, 7 MM ALLOWANCE FOR EXTERNAL RETURN

$\tau_{12} = .00300000$ $I_{MAX} = 16135000$
 $N_{SECTIONS} = 32$ $L_{COIL} = 1.696E-05$ H. $R_{COIL} = 1.124E-05$ O,
 $OPERATG. TEMP = 60.0$ DEG C,
 MECH. SAFETY FACT, $R_{TOP} = 1.5$ MIN. BEARG. CLEAR. $C = 2.0 + 0.0005 \cdot R_5$,
 REFERENCE FLUX $\Phi_{10} = 36.30$ MB. AVE. RADIAL BRUSH LOAD = .0450 BAR,
 $\gamma_{34} = 2.71$ G/CM³, $\gamma_{12} = 8.90$ G/CM³, $\gamma_{45} = 1.66$ G/CM³,
 $\text{COND}_{34} = 30.50$ S M/MM², $\text{COND}_{12} = 54.00$ S M/MM², AT 20 DEG C
 $\text{BRUSH DROP} = .8$ V FRICT. COEFF = .22, CURRENT DENSITY MAX = 23.00 A/MM²
 $\text{PULSE PERIOD} = .40$ SEC, CLEAR BTW. FIBERWRAP AND COIL = 3.4 CM²
 RETURN COND. THICKNESS = 2.0 CM, RADIAL CLEAR RETURN COND-DRUM = .40 CM,
 FRACTION OF TIME AT CONSTANT SPEED = 0.00, BEARG LENGTH = 32.0 CM

r_4	ωr_4	η_{cycle}^b	c^c	$\frac{1}{2} cv_o^2$	ω	$v(\pi)/v_o$	λ^d	Thickness AL	Fiber- glass	f_{blk}
R4 [cm]	SPEED [cm/s]	FCYCL [1]	C/J [C/J]	WCO [J]	OMEGA B [sec ⁻¹] [T]	VCPIV [1]	LAMB [1]	TDRUM [cm]	TFIBR [cm]	FBLK [1]
N = 32										
72.50	31122	.767	2.534	2.75E+08	431.3	6.82	-.881	.652	.337	.998 .016
75.00	29473	.807	2.720	2.69E+08	395.3	6.90	-.903	.672	.436	.878 .016
77.50	28086	.828	2.889	2.67E+08	364.8	6.98	-.914	.692	.513	.792 .016
80.00	26848	.844	3.058	2.65E+08	338.1	7.06	-.922	.713	.584	.716 .017
82.50	25721	.855	3.230	2.64E+08	314.2	7.14	-.928	.733	.650	.647 .017
85.00	24687	.864	3.407	2.63E+08	292.9	7.21	-.932	.754	.713	.584 .017
87.50	23734	.873	3.590	2.62E+08	273.7	7.29	-.936	.775	.782	.507 .017
90.00	22847	.879	3.779	2.62E+08	256.2	7.36	-.939	.795	.838	.454 .017
92.50	22021	.884	3.975	2.61E+08	240.4	7.43	-.942	.816	.891	.404 .017
95.00	21248	.889	4.178	2.61E+08	225.9	7.50	-.944	.837	.941	.358 .017
97.50	20523	.893	4.389	2.61E+08	212.6	7.57	-.945	.858	.989	.314 .017
100.00	19842	.896	4.608	2.61E+08	200.5	7.64	-.947	.879	1.036	.274 .017
102.50	19202	.899	4.835	2.61E+08	189.3	7.70	-.948	.899	1.080	.235 .017
105.00	18598	.902	5.070	2.60E+08	179.0	7.77	-.950	.920	1.123	.199 .017
107.50	18027	.905	5.314	2.60E+08	169.5	7.83	-.951	.941	1.165	.165 .017
110.00	17488	.907	5.567	2.60E+08	160.7	7.89	-.952	.962	1.205	.133 .017

^aAllowing 0.7 cm for the outer return conductor.

^b $\eta_{\text{cycle}} = 1 - f_{\text{exL}} + f_{\text{blk}}$.

^cTo be increased by 4% for double compensation.

^dSection length = 228λ cm.

TABLE II

**LOSS AND COST SUMMARY ON THE DESIGN PER TABLE I.
FOR $r_1 \approx 102.5$ cm, $\lambda \approx 0.899$**

Losses in Per Cent of $\frac{1}{2} CV_0^2$ for 0.4-s Pulse

Period

Drum joule loss	5.8%
Return conductor joule loss	0.2
Brush joule loss (voltage drop)	0.6
Brush friction loss ^a	1.3
Bearing loss = windage loss	0.4
Refrigeration loss	0.02
Power supply loss (charging loss)	0.2
Compression coil loss	<u>1.7</u>
Total loss incl. compression coil	10.2%

Costs in Per Cent of Total Machine Cost

Iron ($nM_{Fe} S_{Fe}/S_{tot}$) =	12.3%
Supercond. coil [$n(m_{cu}) S_{coil} +$ $V_{HE} S_{He} + I_m I N S_{sc}] / S_{tot}$ =	44.4
Refrigeration [$(nP_{ref})^{0.75} S_{ref} +$ $n\lambda^2 S_{Stewart}] / S_{tot}$ =	3.4
Return conductor, drum ($n M_c S_c / S_{tot}$) =	6.2
Current collection ($n A_B S_B / S_{tot}$) =	17.4
Assembly ($1 \cdot 1/1.2$) =	<u>16.4</u>
Total machine cost $S_{tot} = 12.4$ M\$	100%

^a Assuming that the brushes make contact for 9 ms at an average pressure of 0.7 bar superposed with an electrodynamic load peaking at 8 bar.

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APPENDIX A

ELECTROMAGNETIC EXCITATION

Let us write Faraday's law and Stoke's Theorem

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad (\text{A-1})$$

$$\oint_C \vec{E} \cdot d\vec{s} = - \int_F (\vec{\nabla} \times \vec{E}) \cdot d\vec{f} \quad (\text{A-2})$$

The potential vector $\vec{A}$ is defined by

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad (\text{A-3})$$

The curls drop out if Eq. (A-3) is substituted in (A-1), which yields

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = 0 \quad (\text{A-4})$$

Integrating Eq. (A-4) with Stoke's theorem leads to

$$\oint_C \vec{E} \cdot d\vec{s} = - \frac{\partial}{\partial t} \int_F \vec{\nabla} \times \vec{A} \cdot d\vec{f} \quad (\text{A-5})$$

For cylindrical coordinates, the right side of Eq. (A-5) may be written

$$\vec{\nabla} \times \vec{A} \cdot d\vec{f} = - \left(\frac{\vec{a}_r}{r} \frac{\partial(rA_\phi)}{\partial z} + \frac{\vec{a}_z}{r} \frac{\partial(rA_\phi)}{\partial r} \right) \cdot d\vec{f}, \quad (A-6) \quad \oint \vec{v} \cdot d\vec{s} = v = - \frac{d\phi}{dt} \quad (A-9)$$

where

$$d\vec{f} = \vec{a}_r \cdot 2\pi r \, dz + \vec{a}_z \cdot 2\pi r \, dr. \quad (A-7) \quad \frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + v \frac{\partial\phi}{\partial x}, \quad (A-10)$$

Carrying out the inner product multiplication in Eq. (A-6) yields

$$\int_F \vec{\nabla} \times \vec{A} \cdot d\vec{f} = 2\pi \int_F d(rA_\phi) = \phi, \quad (A-8)$$

where ϕ is the flux through the circle with radius r . Substituting Eq. (A-8) in Eq. (A-5) yields

The total derivative on the right side of Eq. (A-9) is

where the partial time derivative is zero in this dc application, and where v denotes velocity. The velocity with which the flux linkage in Fig. 10 is changed is $v = dx/dt$. In other words, the flux lines in Fig. 10 are lines of constant vector potential, and the conductor position moves to constantly changing values in this potential field.

APPENDIX B

NORMALIZATION OF THE EFFICIENCY-VS-DISCHARGE TIME PLOTS

Let the reference configuration be given by N_o parallel coils with n_o turns each and with reference total current I_o , reference coil field B_o , and reference effective coil dimensions r_o and l_o .

We want to know the relationships to be met if a different number of coils N are paralleled and the total current $I = I_o$ and total inductance $L = L_o$ are to remain the same. We can write

$$1 = \frac{L}{L_o} = \frac{N_o}{N} \left(\frac{n}{n_o} \right)^2 \left(\frac{r}{r_o} \right)^2 \left(\frac{l_o}{l} \right) \quad (B-1)$$

and

$$1 = \frac{I}{I_o} = \frac{N}{N_o} \left(\frac{B}{B_o} \right) \left(\frac{n_o}{n} \right) \left(\frac{l_o}{l} \right). \quad (B-2)$$

Eqs. (B-1) and (B-2) require that

$$\frac{B}{B_o} \left(\frac{r}{r_o} \right)^2 \frac{n}{n_o} = 1; \quad (B-3)$$

in other words, conservation of $n_o r_o \pi B_o = 25 \text{ Wh-turns}$ is the premise for this optimization, where $n_o = 10$, $r_o = 0.178 \text{ m}$, $B_o = 25 \text{ T}$.