INVESTIGATIONS OF THE STRUCTURE AND ELECTROMAGNETIC
INTERACTIONS OF FEW-BODY SYSTEMS

DOE/ER/40270--36

DE92 040011

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Progress Report

1 August 1991 to 31 July 1992

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July 1992

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY
UNDER GRANT NUMBER
DE-FG05-86-ER40270

MASTER

AUG 25 1992

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INTRODUCTION

This progress report will summarize the work of The George Washington University (GWU) nuclear theory group during the last year under DOE Grant No. DE-FG05-86-ER40270. During this period, the faculty working under this grant were D.R. Lehman (P.I.), H. Haberzettl, W.C. Parke, and L.C. Maximon. One postdoctoral research scientist, Hiroshi Ito, has been working with the group since 1 July 1991. One graduate student, John Woloschek, is working under the direction of Professor Lehman, but other than computer time, supplies, and travel expenses to relevant meetings, he is not directly supported by the grant. In addition, three new graduate students, all supported by the grant, joined the effort as of 1 June 1992 through a supplemental addition of funds from DOE and tuition cost-sharing by GWU. Outside collaborators with the group during the period indicated were B.F. Gibson of Los Alamos National Laboratory, A. Eiro and A. Fonseca of the U. of Lisbon, Y. Koike of Hosei University, W. Sandhas of the U. of Bonn, and E. Tomusiak of the U. of Saskatchewan.

The format of the Progress Report is as follows:

1. Papers published or in press, submitted for publication, and in preparation;
2. Invited talks at conferences and meetings;
3. Invited talks at universities and laboratories;
4. Contributed papers or abstracts at conferences;
5. Visitors to the group;
6. Research progress by topic.

PAPERS PUBLISHED

A.C. Fonseca and D.R. Lehman, "Exact Three-Body Calculation of Polarization Observables in $^1\text{H}(\vec{d}, \gamma)^3\text{He}$ ", Phys. Lett. B 267, 159(1991).

W.C. Parke, Y. Koike, D.R. Lehman, and L.C. Maximon, "Quality of the Three-Nucleon Bound-State Wave Function from a Two-Nucleon Separable Expansion Method", Few-Body Systems 11, 89(1991).

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A.C. Fonseca and D.R. Lehman, "NN P-Wave Effects in Polarized Deuteron Capture on Hydrogen: An Exact Faddeev Calculation", Few-Body Systems, Suppl. 6, 279(1992).

H. Ito, W.W. Buck, and F. Gross, "Electromagnetic Properties of the Pion as a Composite Nambu Goldstone Boson", Phys. Rev. C45, 1918(1992).

L.C. Maximon, "Evaluation of the Spherical Bessel Transform of a Whittaker Function: An Application of a Difference Equation Method", J. Math. Phys. 33, 2005(1992).

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V. Punjabi, C.F. Perdrisat, E. Cheung, J. Yonnet, M. Boivin, E. Tomasi-Gustafsson, R. Siebert, R. Frascaria, E. Warde, S. Belostotsky, O. Miklucho, V. Sulimov, R. Abegg, and D.R. Lehman, " T_{20} in the inclusive breakup of 4.5 GeV polarized ^6Li ", Phys. Rev C (in press).

H. Ito, W.W. Buck, and F. Gross, "The Axial Anomaly and the Dynamical Breaking of Chiral Symmetry in the $\gamma^*\pi^0 \rightarrow \gamma$ reaction", Phys. Lett. B (in press), and CEBAF preprints #CEBAF-TH-91-12 & 13.

H. Habermatzl, "Cluster-dynamical Approach to N-body Scattering", Phys. Rev. C, (in press).

W.C. Parke, "Methods in Few-Body Nuclear Physics", Proceedings of HUGS at CEBAF, June 1991, (in press).

PAPERS SUBMITTED

H. Ito and F. Gross, "Gauge Invariant, Dynamical Theory of the Compton Scattering from Relativistic Composite Systems", submitted to Phys. Rev. C.

Th. Januschke, T.N. Frank, W. Sandhas, and H. Habermatzl, "Neutron-deuteron Scattering Calculations with Realistic NN Interactions Using the W-matrix Representation of the Two-body Input", submitted to Phys. Rev. C.

T.N. Frank, Th. Januschke, W. Sandhas, and H. Habermatzl, "Neutron-deuteron Breakup Calculations with Realistic NN Interactions Using the W-Matrix Representation of the Two-body Input", submitted to Phys. Rev. C.

H. Habermatzl, "Relativistic Cluster Dynamics of Nucleons and Mesons", submitted to Phys. Rev. C.

PAPERS IN PREPARATION

A.S. Raskin, E.L. Tomusiak, and D.R. Lehman, "A Study of the $^4\text{He}(\bar{\gamma}, d)d$ Reaction", to be submitted to J. Phys. G.

D.R. Lehman and B.F. Gibson, "Formalism Underlying the $A = 4$ Λ Hypernuclei 1^+ Equations", to be submitted to Phys. Rev. C.

A.C. Fonseca, J. Haidenbauer, D.R. Lehman, and W.C. Parke, "Ten-Channel Three-Nucleon Bound-State Wave Functions from a Separable Expansion Method", to be submitted to Few-Body Systems.

INVITED TALKS AT CONFERENCES AND MEETINGS

D.R. Lehman, "Faddeev Calculation of Polarization Observables in $^1\text{H}(\bar{d}, \gamma)^3\text{He}$ ", Invited Talk, 13th International Conference on Few-Body Problems in Physics, Adelaide, Australia, 5-11 January 1992, Adelaide, Australia, 5-11 January 1992; associated abstract published with A.C. Fonseca, Proceedings of the XIIIth International Conference on Few-Body Problems in Physics (Flinders University, Adelaide, Australia, 5-11 January 1992), edited by I.R. Afnan and R.T. Cahill, (Institute for Atomic Studies, Report FIAS-R-216, 1992) p. 244.

D.R. Lehman, "NN P-Wave Effects in Polarized Deuteron Capture on Hydrogen: An Exact Faddeev Calculation", Invited Talk, 13th International Conference on Few-Body Problems in Physics, Adelaide, Australia, 5-11 January 1992; associated abstract published with A.C. Fonseca, Proceedings of the XIIIth International Conference on Few-Body Problems in Physics (Flinders University, Adelaide, Australia, 5-11 January 1992), edited by I.R. Afnan and R.T. Cahill, (Institute for Atomic Studies, Report FIAS-R-216, 1992) p. 246.

L.C. Maximon, "Precision Three-Nucleon Bound-State Wave Functions from Separable Expansions of Realistic Two-Nucleon Potentials", Invited Talk, 13th International Conference on Few-Body Problems in Physics, Adelaide, Australia, 5-11 January 1992; associated abstract published with co-authors J. Haidenbauer, Y. Koike, D.R. Lehman, and W.C. Parke, Proceedings of the XIIIth International Conference on Few-Body Problems in Physics (Flinders University, Adelaide, Australia, 5-11 January 1992), edited by I.R. Afnan and R.T. Cahill, (Institute for Atomic Studies, Report FIAS-R-216, 1992) p. 302.

H. Haberzettl, "The Efimov Effect in Neutron-Deuteron Scattering", Spring Meeting of the American Physical Society, Washington, DC, April 23, 1992, Bull. Am. Phys. Soc. 37,2007(1992).

INVITED TALKS AT UNIVERSITIES AND LABORATORIES

H. Ito, "Electromagnetic Gauge Invariance in Relativistic Few-Body Problems", at the Department of Physics, Carnegie-Mellon University, Pittsburgh, PA, 18 October 1991.

H. Ito, "Electromagnetic Gauge Invariance in Few-Body Problems", at the Department of Physics, University of Maryland, College Park, MD, 30 October 1991.

H. Haberzettl, "Relativistic Cluster Dynamics of Nucleons and Mesons", Department of Physics, University of Maryland, College Park, MD, 4 November 1991.

H. Haberzettl, "Relativistic Cluster Dynamics of Nucleons and Mesons", Theory Division, T-5, Los Alamos National Laboratory, Los Alamos, NM, 28 January 1992.

H. Haberzettl, "Cluster-dynamical Approach to the Scattering of Mesons and Nucleons", Theory Division, Argonne National Laboratory, Argonne, IL. 16 April 1992.

CONTRIBUTED PAPERS AND ABSTRACTS AT CONFERENCES

K. Gebhardt, W. Jäger, E. Finckh, T.N. Frank, Th. Januschke, W. Sandhas, and H. Haberzettl, "Determination of the Neutron-neutron Scattering Length a_{nn} ", Proceedings of the XIIIth International Conference on Few-Body Problems in Physics (Flinders University, Adelaide, Australia, 5-11 January 1992), edited by I.R. Afnan and R.T. Cahill, (Institute for Atomic Studies, Report FIAS-R-216, 1992) p. 152.

T. N. Frank, H. Haberzettl, and W. Sandhas, "Neutron-deuteron Break-up Collisions, Proceedings of the XIIIth International Conference on Few-Body Problems in Physics (Flinders University, Adelaide, Australia, 5-11 January 1992), edited by I.R. Afnan and R.T. Cahill, (Institute for Atomic Studies, Report FIAS-R-216, 1992) p. 320.

K. Gebhardt, W. Jäger, E. Finckh, T.N. Frank, Th. Januschke, W. Sandhas, and H. Haberzettl, "Determination of the Neutron-neutron Scattering Length a_{nn} ", Proceedings of the Spring Meeting of the Nuclear Physics Section of the German Physical Society (Salzburg, Austria, 24-28 February 1992), Verhandl. DPG (VI) 27, p. 121.

T.N. Frank, H. Haberzettl, and W. Sandhas (presenter), "Three-nucleon Break-up Processes", Proceedings of the XIIIth European Conference on Few-Body Physics (Marciana Marina, Isola d'Elba, Italy, 9-14 September 1991), edited by C. Ciofi degli Atti, E. Pace, G. Salmè, and S. Simula, (Few-Body Systems, Suppl. 6, 1992) p. 270.

V.B. Belyaev (presenter), M. Decker, H. Haberzettl, L.J. Khaskilevitch, and W. Sandhas, "Muonic Molecules of Light Nuclei, Proceedings of the XIIIth European Conference on Few-Body Physics (Marciana Marina, Isola d'Elba, Italy, 9-14 September 1991), edited by C. Ciofi degli Atti, E. Pace, G. Salmè, and S. Simula, (Few-Body Systems, Suppl. 6, 1992) p. 332.

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RESEARCH PROGRESS BY TOPIC

In order to make it easy for the reader to see the specific research carried out and the progress made, the following report of progress is done by topic. Each item has a format layout of Topic, Investigators, Objective, Significance, and Description of Progress, followed at the end by the relevant references. As is clear from the topics listed, the emphasis of the GWU theory group has been on the structure and electromagnetic interactions of few-body nuclei. Both low- and intermediate-energy electromagnetic disintegration of these nuclei is considered. When the excitation energy of the target nucleus is low, the aim has been to handle the continuum part of the theoretical work numerically with no approximations, that is, by means of full three- or four-body dynamics. When structure questions are the issue, numerically accurate calculations are always carried through, limited only by the underlying two-body or three-body interactions used as input. Implicit in our work is the question of how far one can go within the traditional nuclear physics framework, i.e., nucleons and mesons in a nonrelativistic setting. Our central goal is to carry through state-of-the-art few-body calculations that will serve as a means of determining at what point standard nuclear physics requires quark degrees of freedom in order to understand the phenomena in question. So far, in the problems considered, there has been no evidence of the necessity to go beyond the traditional approach, though we always keep in mind that possibility. As our work is involved with questions in the intermediate-energy realm, moving from a nonrelativistic framework to a relativistic one is always a consideration. Currently, for the problems that have been pursued in this domain of energy (see publication list), the issues concern far more the mechanisms of the reactions and structural questions than the need to move to relativistic dynamics.

A significant guiding factor in the work of the GWU theory group is the current experimental effort and capabilities in the areas of interest. The work at low excitation energies is motivated by the beautiful radiative capture experiments with polarized beams being carried out at TUNL, Wisconsin, IUCF, and SIN. At intermediate energies, the capabilities of coincidence experiments at NIKHEF has been a source of inspiration for carrying out detailed three-body calculations. More recently, the prospect of a source of polarized gammas at the Saskatchewan Accelerator Laboratory has lead to a study to examine what can be learned about the D-state of ^4He by disintegration into two deuterons by linearly polarized gammas. At the same time, much of the impressive data that has already been obtained at the new higher-duty accelerators, and data which will be forthcoming at the new facilities like CEBAF, depend critically on a full understanding of the QED processes inherent to electron scattering, i.e., the ability to calculate and account accurately for these effects. One member of our group (LCM) contributes significantly in this area. Finally, all of us interact and collaborate with the GWU nuclear experimentalists. In fact, the recent contribution of Prof. Haberzettl concerning the cluster dynamics of nucleons and mesons (see papers submitted), a subject in which he had a long-standing interest, came to final fruition following theoretical discussions about work that was an outgrowth of a meeting between the GWU experimentalists and theorists.

PROGRESS

TOPIC: Determination of the Neutron-Neutron Scattering Length a_{nn}

INVESTIGATORS: H. Haberzettl, K. Gebhardt^a, K. Jäger^a, E. Finckh^a, T.N. Frank^b, Th. Januschke^b, and W. Sandhas^b

OBJECTIVE: To extract the neutron-neutron scattering length from the data of the kinematically complete neutron-deuteron breakup experiment performed at Erlangen.

SIGNIFICANCE: Assuming charge symmetry, one would expect the scattering lengths for proton-proton (pp) and neutron-neutron (nn) scattering to be identical. Experimentally, however, the (Coulomb-corrected) value for the pp scattering length is found to be¹

$$a_{pp} = -17.3 \pm 0.3 \text{ fm} ,$$

while the nn values scatter between²

$$a_{nn} = -16.4 \pm 1.2 \text{ fm}$$

and

$$a_{nn} = -18.8 \pm 1.0 \text{ fm} .$$

Clearly, the current situation with respect to charge-symmetry breaking is inconclusive. Therefore, a reliable determination of the nn scattering length is important — not only with respect to the question of charge-symmetry breaking, but also since a_{nn} is a very crucial parameter for realistic nucleon-nucleon interactions used in few-body calculations. Its accurate value, therefore, is of significance for all theoretical investigations in few-nucleon systems.

DESCRIPTION OF PROGRESS: In order to determine the nn scattering length from $nd \rightarrow nnp$ breakup data, one needs to investigate the situation in which the two final neutrons leave the interaction volume with a vanishing relative momentum. This final-state interaction (FSI) of the two neutrons is dominated by the two-body 1S_0 partial wave. By performing a full three-body breakup Faddeev calculation and fitting the measured FSI peak with theoretical results due to various input values of a_{nn} for the 1S_0 subsystem channel, one obtains a best fit which thus determines the scattering length. In principle, this procedure is straightforward. In practice, however, it is a nontrivial task in view of the enormous numerical requirements.

In the Erlangen nd breakup experiment, one measures the kinematic variables of the two final neutrons. This is very difficult to do and requires great care in analyzing the experimental raw data. Ideally the measured values should sit on a curve in the plane spanned by the two neutron energies which is defined as the kinematic locus of neutron energies satisfying three-body energy conservation. In practice, however, due to the finite resolutions of the neutron detectors, the experimental data scatter quite a bit around this ideal curve. In order to extract cross sections, which usually are given as functions of the arc length along the kinematic curve, one therefore needs to project the measured values onto appropriate positions on the kinematic locus.

Previously, this projection was done by performing a Monte Carlo simulation of the

experiment. It was found, however, that this procedure was inadequate to provide the desired accuracy. We therefore performed a numerical simulation based on complete solutions of the Faddeev-type AGS equations for the breakup problem; as input we used the separable W-matrix representation³ of the Paris potential. Since each cross section is characterized by the energy of the incident neutron, the scattering angles θ_1 and θ_2 of the two final neutrons and the difference $\Delta\phi$ of their azimuthal angles, an adequate description of the finite detector resolutions requires variations of all four kinematic variables. We found that we had to calculate up to 375 different kinematic configurations to analyze one experimental cross section, i.e., we used 3 different energies and five different angles for each of the three angle variables (i.e., $375=3 \times 5 \times 5 \times 5$). This minimum requirement for a reliable analysis translates into quite a formidable numerical task which, however, was made manageable by employing the W-matrix formalism. Details and results for the latter for rearrangement and breakup neutron-deuteron scattering calculations with the Paris potential are given in Ref. 4.

In order to extract the nn scattering length from the data, we performed several sets of 375 calculations for an experimental situation dominated by the final-state interaction of the two neutrons, with each set employing different input values for the nn scattering length a_{nn} . Subjecting then the theoretical results to the same procedure usually used for the experimental data and performing a best fit for the FSI peak of the cross section, we find a preliminary value of

$$a_{nn} = -17.0 \pm 1.0 \text{ fm}$$

for the nn scattering length, which is compatible with the pp scattering length. Our analysis regarding the scattering length is almost completed and we do not expect our final results to be markedly different from this value.

Preliminary results of our analysis were reported at conferences at Adelaide and Salzburg⁵; a final publication detailing our findings is in preparation⁶.

^aUniversität Erlangen, Germany.

^bUniversität Bonn, Germany.

¹see R. Machleidt, Adv. Nucl. Phys. 19,189(1989), and references therein.

²G.F. Téramond and B. Gabioud, Phys. Rev. C36,691(1987); N. Koori et al., Book of Contributions to the 11th International IUPAP Conference on Few Body Systems in Particle and Nuclear Physics (Tokyo and Sendai, Japan, August 24-30, 1986), edited by T. Sasakawa et al., Suppl. to Research Report of the Laboratory of Nuclear Science, Vol 19, p. 406 (Tohoku Univ., 1986); I. Slaus, Few Body Systems, Suppl. 1,160(1987); K. Bodek et al., Few Body Systems 8,23(1990), and references therein.

³E.A. Bartnik, H. Haberzettl, and W. Sandhas, Phys. Rev. C34,1520(1986); E.A. Bartnik, H. Haberzettl, Th. Januschke, U. Kerwath, and W. Sandhas, Phys. Rev. C36, 1678 (1987); T.N. Frank, H. Haberzettl, Th. Januschke, U. Kerwath, and W. Sandhas, Phys. Rev. C38,1112(1988).

⁴Th. Januschke, T.N. Frank, W. Sandhas, and H. Haberzettl, Neutron-deuteron scattering calculations with realistic NN interactions using the W-matrix representation of the two-body input, submitted to Phys. Rev. C; T.N. Frank, Th. Januschke, W. Sandhas, and H. Haberzettl, Neutron-deuteron breakup calculations

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with realistic NN interactions using the W-matrix representation of the two-body input, submitted to Phys. Rev. C.

⁵K. Gebhardt, W. Jäger, E. Finckh, T.N. Frank, Th. Januschke, W. Sandhas, and H. Haberzettl, in Proceedings of the XIIIth International Conference on Few-Body Problems in Physics (Flinders University, Adelaide, Australia, 5-11 January 1992), edited by I.R. Afnan and R.T. Cahill, p. 152; in Proceedings of the Spring Meeting of the Nuclear Physics Section of the German Physical Society (Salzburg, Austria, 24-28 February 1992), Verhandl. DPG (VI) 27, p. 121.

⁶K. Gebhardt, W. Jäger, E. Finckh, T.N. Frank, Th. Januschke, W. Sandhas, and H. Haberzettl, in preparation.

PROGRESS

TOPIC: Cluster-Dynamical Approach to N-body Scattering

INVESTIGATOR: H. Haberzettl

OBJECTIVE: To formulate the nonrelativistic quantum-mechanical N-body problem in terms of clusters rather than individual particles.

SIGNIFICANCE: Practically all existing detailed formulations of the nonrelativistic N-body problem are based on individual particles. However, if the number of particles is appreciably larger than four, such formulations are very complicated. Thus, in practical applications, where one is usually faced with the necessity of having to make approximations of some sort, such detailed approaches tend to have serious drawbacks. It seems desirable, therefore, to have a description of N-body scattering processes where one deals with clusters rather than individual particles. From a practical point of view, such an approach should have the structural simplicity of the three-body problem, allow for a straightforward implementation of the Pauli principle and provide a detailed description of the underlying reaction mechanisms in order to facilitate approximations.

DESCRIPTION OF PROGRESS: A formulation of the quantum-mechanical nonrelativistic N-body problem was derived using as guiding principle the dynamical evolution of two-cluster partitions of the N particles during the scattering process. This concept was realized by first making a certain choice for the basic subcluster-exchange potentials and then requiring consistency at all subsystem levels to generate the detailed structure. Then, a decoupling was performed which divides the two-cluster partitions into primary and secondary ones, thus leading to a generalized optical potential-type formulation which makes the description of the reaction mechanisms very detailed and transparent and allows for a convenient graphical visualization. The resulting effective two-body equations of Lippmann-Schwinger form take full account of the Pauli principle and involve as input only physical subsystem transition amplitudes. This approach may be derived via an off-shell transformation of the corresponding Alt-Grassberger-Sandhas N-body equations which eliminates some off-shell dependence of the latter. As a direct consequence of the compound nature of spectator clusters, the final equations turn out to be four-dimensional, involving one off-shell energy and one vector momentum as integration variables. This finding is of considerable importance for the relativistic generalization described elsewhere in this report.

A publication detailing the formalism is in press¹.

¹H. Haberzettl, "Cluster-Dynamical Approach to N-body Scattering", Phys. Rev. C (in press).

PROGRESS

TOPIC: Relativistic Cluster Dynamics of Nucleons and Mesons

INVESTIGATOR: H. Haberzettl

OBJECTIVE: To formulate a relativistic N-particle scattering theory of nucleons and mesons

SIGNIFICANCE: Apart from the perturbative approach of old-fashioned perturbation theory, with its rather limited applicability, to date there exists no consistent dynamical formulation of the relativistic scattering problem. It is evident, therefore, that such a formulation is of considerable significance from both theoretical and practical points of view.

DESCRIPTION OF PROGRESS: A time-ordered relativistic scattering theory for nucleons and mesons based on clusters rather than individual particles was derived. The S-matrix-type approach provides a recursive hierarchy of Lippmann-Schwinger equations, each describing the dynamical evolution of the two-cluster configurations at different levels of the many-body problem. In a first manuscript submitted for publication¹, the presentation is restricted to a fixed number of particles; particle absorption and creation is treated in a subsequent paper which is in preparation². In Ref. 1, the main concern is the kinematic aspect of the problem. It is shown there that the on-shell results of the formalism are covariant, without requiring anti-particle contributions. This was achieved by constructing off-shell T matrices as invariants under a set of energy and momentum transformations which reduce to Lorentz transformations for on-shell energies. Spin degrees of freedom are dealt with by introducing cluster spinors based recursively on the fundamental spin-1/2 Dirac and the spin-1 vector spinors, thus generalizing the Rarita-Schwinger construction. Apart from the treatment of the spin, this relativistic formulation is formally equivalent to the non-relativistic N-body approach described elsewhere in this report. This equivalence may be of considerable importance from a practical point of view since it easily allows for a mixed treatment of situations in which some constituents must be treated relativistically while others may be incorporated nonrelativistically.

In its full implementation², including particle absorption and creation, the present approach leads to a recursive hierarchy of nonlinear scattering problems, where each hierarchical level is determined by the (conserved) number of baryons with the nonlinearity entailing contributions from infinitely many virtual mesons. The basic problem to be solved is pion-nucleon scattering; it is at this lowest level of the hierarchy that contact with field-theoretic approaches is made and that one needs to renormalize vertices in the usual way. At higher levels, renormalization will not be necessary.

¹H. Haberzettl, Relativistic cluster dynamics of nucleons and mesons. I: Kinematics and covariance, submitted to Phys. Rev. C.

²H. Haberzettl, Relativistic cluster dynamics of nucleons and mesons. II: Formalism and examples, in preparation.

PROGRESS

TOPIC: Calculation of Pion-Nucleon Scattering within the Relativistic Cluster-Dynamical Approach

INVESTIGATOR: H. Haberzettl

OBJECTIVE: To solve the pion-nucleon problem within the cluster dynamical framework of the relativistic N-body problem described elsewhere in this report and to represent the resulting pion-nucleon amplitudes such that they may serve as input in other problems with explicit mesonic degrees of freedom.

SIGNIFICANCE: As is described elsewhere in this progress report, pion-nucleon scattering is the basic problem to be solved for all applications of the relativistic cluster-dynamical (RCD) approach; it is at the same level of importance as the two-nucleon problem for the nonrelativistic N-body problem. Without its solution, applications of the RCD approach to higher-order processes, like nucleon-nucleon and pion-deuteron scattering, cannot be done consistently.

DESCRIPTION OF PROGRESS: The cluster-dynamical approach may be implemented at various levels of sophistication by choosing how many explicit mesons may appear internally in addition to the single external pion. Since the pure πN -problem, with only one pion in all intermediate states, is trivial, we want to formulate the problem such that we allow for the explicit appearance of two pions in intermediate states. Within the RCD approach, this immediately entails that the problem becomes nonlinear, with the nonlinearity leading to infinitely many implicit pions in intermediate states. Among other things, one must then also consider, for example, $\pi\pi$ interactions which introduce explicit ρ meson degrees of freedom. The πN scattering problem thus becomes rather complicated. We have therefore undertaken a comprehensive study of the existing literature in order to assess how many of and to what level of sophistication such intermediate configurations should be incorporated for an adequate description. In addition, we have clarified some theoretical questions regarding the inclusion of spin degrees of freedom.

We expect to complete the theoretical ground work within the next few months and then start writing a computer code.

PROGRESS

TOPIC: Exact Three-Body Calculation of Polarization Observables in ${}^3\text{H}(\vec{d}, \gamma){}^3\text{He}$

INVESTIGATORS: D.R. Lehman and A.C. Fonseca (U. of Lisbon)

OBJECTIVE: The ultimate scope of this program is to generate theoretical predictions for the key observables in $\vec{d} + p(n) \rightarrow \gamma + {}^3\text{He}({}^3\text{H})$ from 'realistic' nucleon-nucleon interactions like the Paris potential. Specifically, exact Faddeev calculations (bound-state and continuum) will be performed including the E1 and E2 operators and all components of the ground-state wave function connected through these operators to the allowable continuum states.

SIGNIFICANCE: Attainment of the above objective will permit us to elucidate the mechanism of the E2 rescattering effect and to examine D-state effects in the disintegration process through the data available on the polarization observables.

DESCRIPTION OF PROGRESS: During the period of this progress report five major accomplishments have been achieved:

1. A paper has been written¹ and published in **FEW-BODY SYSTEMS** that describes our work on generating the ${}^3\text{H}$ wave function with the EST expansion²⁻⁴ of the Paris potential in the five-channel framework. The results are in excellent agreement with the results of the Los Alamos group for the Paris potential, thus giving us complete confidence in our ${}^3\text{H}$ wave function so obtained.
2. The work on calculation of the ${}^3\text{H}$ wave function mentioned in the previous paragraph is based on the ${}^3\text{S}_1$ - ${}^3\text{D}_1$ and ${}^1\text{S}_0$ components of the two-nucleon interaction. As explained in our original proposal of four years ago, it will be necessary to include the P-wave components of the two-nucleon interaction in order to check the role played by the D-wave component in the ground-state wave function that is made up of a pair of nucleons in a P-wave with the spectator nucleon moving in a P-wave relative to the pair. As a consequence, two independent codes have been written (one in Lisbon and one at GWU) that allow us to include not only the above stated partial waves of the two-nucleon interaction, but the ${}^1\text{P}_1$, ${}^3\text{P}_0$, ${}^3\text{P}_1$, and ${}^3\text{P}_2$ components as well. The codes have been tested against each other for rank-1 interactions and complete agreement has now been attained. Both codes have been written to handle higher than rank-1 interactions and are being tested against each other to assure correctness of the results including calculation of the wave-function normalization. This work is almost complete and everything appears to check. In addition, the GWU code is being extended to include the ${}^1\text{D}_2$, ${}^3\text{D}_2$, and the ${}^3\text{P}_2$ - ${}^3\text{F}_2$ (coupled) partial waves of the nucleon-nucleon interaction in order to be able to generate a full 18-channel three-body wave function from 'realistic interactions'. Further details about this work can be found in the text of our renewal proposal for the period 1 February 1992 to 31 January 1995.
3. We now have a working code that calculates all the capture (photodisintegration) observables assuming the underlying two-nucleon interaction is rank one and includes only the nucleon-nucleon partial waves ${}^1\text{S}_0$ and ${}^3\text{S}_1$ - ${}^3\text{D}_1$, while the electromagnetic operator is pure E1. This is a fully consistent Faddeev calculation for both the bound and continuum states with the gauge invariant form of the E1 operator. We have

calculated cross sections plus vector and tensor analyzing powers for the radiative capture of polarized deuterons. Early results were presented as a poster at the Gordon Research Conference on Photonuclear Reactions during the summer of 1990 and a paper has been published in Physics Letters B that describes the main results of these calculations.⁵ Some of the major findings are the following: 1.) Exact three-body dynamics in the initial state is essential for interpreting the magnitude of the measured polarization observables at low energies (~ 20 MeV deuterons), i.e., neglecting the interaction between the incident deuteron and the target proton is a poor approximation; 2.) Of all components making up the ^3He wave function, it is the D-state that is totally responsible for the character of the tensor analyzing powers, even though there is little sensitivity to the percentage D-state in the deuteron generated by the nucleon-nucleon interaction models considered; 3.) As one moves upward in energy (~ 100 MeV deuterons), it becomes clear from the angular distributions of the cross sections and vector analyzing powers that at least the E2 operator will have to be included to interpret the data, while the deterioration of the agreement with the tensor analyzing powers indicates a role for the contribution from the E2 operator and/or the two-nucleon P-wave contributions to the D-state of ^3He ; 4.) Though one naively expects the initial-state rescattering effects to diminish with increasing incident deuteron energy, substantiated by the fact that this actually occurs for the differential cross section as the deuteron energy is increased, initial-state rescattering effects do not, in general, diminish in importance with increasing energy for the tensor analyzing powers. For example, $T_{20}(90^\circ)$, at an incident deuteron energy of 19.6 MeV, changes by a factor of two in including the full initial-state rescattering relative to neglecting the rescattering, and at 74 MeV, it not only changes magnitude by more than a factor of two, but it also changes sign; 5.) A hierarchical approach to various approximations made in treating the initial-state rescattering by numerous investigators is used to show that each one successively moves the theoretical predictions for T_{20} further from the data, clearly indicating their inadequacy.

4. A code has been written to extend the capture (photodisintegration) calculations to the case where the electromagnetic operator is $E1$, but the underlying two-nucleon interactions now include the 1S_0 , 3S_1 , 3D_1 , 1P_1 , 3P_0 , 3P_1 , and 3P_2 partial waves at the rank-1 level. This code has been checked and is operational. Early results from these calculations were presented in invited talks by Fonseca at the Elba (Italy) XIIIth European Few-Body Conference and at the April meeting of the American Physical Society (Washington, DC). It is found that the contributions from the NN P-wave interactions to the ^3He D-state have no more than a 3% effect on the tensor analyzing powers (TAP) at $E_d = 19.6$ MeV. However, the NN P-wave interactions in the initial-state continuum have a large effect (20 to 60% for $19.6 \leq E_d \leq 95$ MeV) on the magnitude of the TAP.

5. The code developed under item #4 directly above has been generalized to handle arbitrary-rank separable interactions. This code is currently being carefully checked using the EST expansions of the Paris interaction mentioned above. It is already apparent that the convergence of the TAP with the rank of the separable expansion is very rapid. As a consequence, results are already available for the Paris interaction at $E_d = 29$ and 95 MeV, both with and without the NN P-wave interactions. Calculations are currently underway for $E_d = 10$ MeV to compare with the new data available from the U. of Wisconsin⁶.

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PROGRESS

TOPIC: The Quadrupole Moment and the Ratio of the D-wave to S-wave asymptotic normalization constants of ${}^6\text{Li}$

INVESTIGATORS: J.P. Woloschek (Ph.D. Student) and D.R. Lehman

OBJECTIVE: To explain the quadrupole moment of ${}^6\text{Li}$ within the framework of three-body models.

SIGNIFICANCE: The quadrupole moment of ${}^6\text{Li}$ is one of its most difficult properties to predict. Its small negative value seems to imply, at least within the context of effective two-body (α -d) models, a subtle interplay between the S- and D-wave components of the ${}^6\text{Li}$ wave function. Successful prediction of this observable should lead to a deeper understanding of the D-wave component of the ${}^6\text{Li}$ wave function.

DESCRIPTION OF PROGRESS: One aspect of our previous work¹ on the elastic electromagnetic form factors of ${}^6\text{Li}$ was to extract the quadrupole moment from the slope of the quadrupole form factor as the electron momentum transfer goes to zero. Unfortunately, this turned out to be a numerically difficult problem due to the fact that the expressions for the form factors are five-fold integrals. Nevertheless, it appeared that the three-body models predict a positive quadrupole moment of $\approx 0.5 \text{ e-fm}^2$ in contrast to the experimental value of $-0.0644 \pm 0.0007 \text{ e-fm}^2$. To get a better grasp of this result, the alpha-deuteron-component contribution to the quadrupole moment was extracted from the three-body model. When only the alpha-deuteron contribution is present, the ${}^6\text{Li}$ quadrupole moment essentially originates from two terms: 1.) The intrinsic quadrupole moment of the deuteron reduced by the fraction of alpha-deuteron component in the three-body wave function; 2.) An interference contribution that originates from the possibility of either s-wave or d-wave relative motion between the alpha-particle and deuteron. In the three-body models, both these terms are positive! However, it has been argued recently,^{2,3} on the basis of alpha-deuteron cluster models, that the interference contribution must be negative. For this alpha-deuteron-projected term to be negative in the three-body models, the s-wave and d-wave effective alpha-deuteron wave functions must have opposite signs, especially at large distances. This is not the case; in particular, the s-wave and d-wave alpha-deuteron asymptotic normalization constants are of the same sign. As a consequence, it emphasizes the importance of having a reliable experimental determination of the d-wave alpha-deuteron asymptotic norm, relative to the s-wave, to check this prediction and indicate the plausibility of the alpha-deuteron cluster-model explanations of the ${}^6\text{Li}$ quadrupole moment.⁵ Nevertheless, one must keep in mind that the three-body wave functions also have a contribution coming from the alpha-(correlated np pair in the continuum) projection that makes up 35-40% of the norm.

As a consequence of the last point in the previous paragraph, we first carried out a standard calculation of the ${}^6\text{Li}$ quadrupole moment, i.e., directly from the quadrupole operator expectation value with the original three-body ground-state wave functions. The aim was to confirm the results extracted from the quadrupole form factor extrapolations. We found that the previous results are confirmed, i.e., the quadrupole moment does turn out to be positive with magnitude $\approx 0.4 \text{ e-fm}^2$. Next, we improved the representation of the ${}^3\text{S}_1$ - ${}^3\text{D}_1$ np interaction used in generating the three-body ground-state wave function. The np interaction underlying the original three-body ${}^6\text{Li}$ wave function is that of Yamaguchi-Yamaguchi⁴ which is known to give a poor

representation of the mixing parameter ϵ_1 and the wrong sign for the barred D-wave phase shift. Our improvement was to allow for an arbitrary-rank separable interaction so that an interaction that gives a reasonable representation of ϵ_1 can be used. Interestingly, even with an excellent representation of the np interaction, the EST expansion of the Paris potential,⁶ the quadrupole moment value is predicted to be positive with a magnitude of approximately ≈ 0.53 e-fm². In addition, the s-wave and d-wave alpha-deuteron asymptotic normalization constants retain the same sign though their values change somewhat compared to the original models. Thus, the inadequacy of existing three-body models in explaining the quadrupole moment of ⁶Li cannot be attributed to the simplicity of the original np interaction. On further reflection and backed up by our new calculations, this should not be surprising. Our new calculations with the Paris interaction indicate that the bulk of the quadrupole-moment value is already given by the alpha-deuteron-projected piece, i.e., approximately 80%. Thus, since the main component of the effective interaction between the alpha and deuteron comes from the alpha-nucleon interaction folded with the deuteron wave function, it immediately becomes clear that the np interaction enters at a secondary level.

Based on the above conclusions, we have looked at the question of inelasticity effects in the alpha-nucleon interaction. On the basis of a set of separable, but coupled-channel, alpha-nucleon interactions originally developed by Miyagawa, *et al.*,⁷ we have extended the three-body model to handle these more sophisticated interactions. The new alpha-nucleon interactions couple to a single hybrid deuteron-triton channel with a threshold at 18.4 MeV. With such an approach, the alpha-nucleon phase shifts and inelasticities are reproduced up to nucleon energies of approximately 60 MeV; whereas, our original alpha-nucleon interactions reproduced the phase shifts up to 20 MeV and ignored the inelasticities since the alpha-particle was considered to be elementary. Moreover, in the new interactions, some of the partial-wave couplings are intriguing from the viewpoint of the quadrupole moment and the angular momenta involved. For example, the alpha-nucleon ²S_{1/2} partial wave couples to the deuteron-triton ⁴D_{1/2} partial wave. In addition, we have now included the ²D_{3/2}, ²D_{5/2}, ²F_{5/2}, and ²F_{7/2} partial waves of the alpha-nucleon interaction besides the original ²S_{1/2}, ²P_{1/2}, and ²P_{3/2} partial waves. Such an approach recognizes the fact that the alpha-particle is not an elementary particle. We find that when the underlying NN interaction is that of the rank-1 Yamaguchi-Yamaguchi case, the D- to S-wave asymptotic-normalization ratio is already negative with only the S- and P-wave alpha-nucleon interactions present. Addition of the ²D_{3/2} partial wave of the alpha-nucleon interaction to this framework leads to the most negative value of the asymptotic-norm ratio. As the other alpha-nucleon partial waves are added, the asymptotic-norm becomes less negative. However, upon changing the NN interaction to the rank-6 EST expansion of the Paris potential, the asymptotic-norm ratio becomes positive for all alpha-nucleon partial-wave cases considered above. Even when the asymptotic-norm ratio is negative, the alpha-deuteron contribution to the ⁶Li quadrupole moment is *positive*.

The above model gives hope, but is limited in the sense that once the alpha-nucleon system makes the transition to the "deuteron-triton" channel, there are no interactions between the 'spectator' nucleon and either the deuteron or triton. As a result, three-body-force effects are not generated. The so-called 'dispersive' effect of the channel coupling is present, but no three-body-force component can be generated. As a

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consequence, we have constructed a rank-1, coupled-channel interaction where the alpha-nucleon system can make a transition to a pseudo-alpha-particle state that we call alpha* with the same threshold as indicated above. Within this framework, we once again have the dispersive effect of the channel coupling and we are checking carefully to assure that the rank-1 form that we have chosen for the interaction does not preclude the generation of three-body-force contributions. While that investigation is underway, results have already been obtained for the asymptotic norm ratio. The results are similar to the "deuteron-triton" channel coupling model. Effort is currently being focussed on understanding this outcome.

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PROGRESS

TOPIC: Sensitivity to the D-State of ^4He with Linearly Polarized Photons in the Reaction $^4\text{He}(\gamma, d)^2\text{H}$

INVESTIGATORS: D.R. Lehman, A.S. Raskin (Postdoc at U. of Saskatchewan), E.L. Tomusiak (U. of Saskatchewan), and H.R. Weller (Duke University)

OBJECTIVE: To examine the feasibility of using the $^4\text{He}(\vec{\gamma}, d)^2\text{H}$ reaction to provide information on the $L=2$ component of the ^4He ground state.

SIGNIFICANCE: The properties of the $^5\text{D}_0$ component of the ^4He ground state are theoretically of great interest, but have been difficult to determine experimentally. ^4He has no quadrupole moment to provide an indication as to the nature of the D-state. Instead, direct evidence for the ^4He D-state comes only from the ratio (called η_α) of the D-wave to S-wave asymptotic normalization constants in the $^4\text{He} \rightarrow d + d$ channel. To date, there has been no theoretical calculation of η_α and the experimental situation remains inconclusive.¹ There already has been an effort to learn about the ^4He D-state with the reactions $\gamma + ^4\text{He} \rightarrow d + d$ with unpolarized particles, but the theoretical interpretation of the data is difficult owing to the likely need to use exact four-body dynamics for the low energies involved. Almost all of the work alluded to is concerned with the capture process. However, the advent of new continuous wave electron facilities and their associated photon-tagging capabilities make the photodisintegration reaction amenable to detailed experimental study -- even with linearly polarized photons. Our purpose in this work is to investigate, within the context of a simple model of the ^4He nucleus, the sensitivity of cross sections and polarization observables to the D-state component of the ^4He wave function.

DESCRIPTION OF PROGRESS: This project has had two facets: 1.) Development of a general formalism for a linearly polarized gamma-ray capture particle-out reaction which allows us to generate the expression for the cross section as a function of the detected particle angle and the photon polarization angle in terms of the reduced matrix elements for any multiplicities and any particle spins. 2.) Examining the feasibility of using the $^4\text{He}(\vec{\gamma}, d)^2\text{H}$ reaction to provide information on the structure of the ^4He D-state. The first part of the project has been completed and has led to a publication² in *Atomic Data and Nuclear Data Tables* that contains tables of angular distribution coefficients for (γ, x) reactions with linearly polarized photons. Specific examples are given in this publication, one of which is the $^4\text{He}(\vec{\gamma}, d)^2\text{H}$ reaction. The second part of the project involved a detailed study of the $^4\text{He}(\vec{\gamma}, d)^2\text{H}$ reaction with a simple model for ^4He . From this work, there appears to be considerable sensitivity to the D-state component of the ^4He wave function in both the unpolarized and polarized responses associated with $^4\text{He}(\vec{\gamma}, d)^2\text{H}$, although it is clear that the interpretation of the data will require the use of models in which current conservation is treated carefully. A manuscript³ on this second phase of the work is almost completed and will be submitted to *J. Phys. G*.

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PROGRESS

TOPIC: Long-range behavior of the effective nucleon-deuteron interaction for the three-nucleon system

INVESTIGATORS: D.R. Lehman, L.C. Maximon, Walid Younes (former undergraduate physics major at GWU, now graduate student at Rutgers U.), and B.F. Gibson (Los Alamos National Laboratory)

OBJECTIVE: The aim of this work is to derive rigorously the asymptotic behavior of the effective interaction between a nucleon and deuteron within the bound three-nucleon system.

SIGNIFICANCE: One goal in attempts to understand the low-energy properties of the three-nucleon system is to determine whether these properties can be understood within the framework of two-body models with effective interactions. Until recently, such two-body models have met with no success. However, Tomio, Delfino, and Adhikari,^{1,2} recently published an analysis of the low-energy properties of the three-nucleon system based on an ad-hoc effective two-body interaction that has an attractive $e^{-\mu r}/r^2$ long-range tail which is essential for the success of the description (r is the separation distance between the center-of-mass of the deuteron and the neutron). In particular, their effective, three-parameter, potential is found to predict correctly the value of the doublet p-d scattering length obtained from exact three-body calculations and to reproduce the correlation between the doublet scattering length and the three-nucleon binding energy found in exact three-body calculations for both the n-d and p-d systems, after having been fit to the experimental values for the ^3H and ^3He binding energies and the n-d doublet scattering length. In an attempt to justify the long-range behavior of the n-d effective interaction used by Tomio *et al.*^{1,2}, Delfino, Frederico, and Tomio³ have derived the asymptotic behavior starting from a zero-range representation of the three-nucleon bound-state wave function. The essence of their approach is to project the bound-state wave function with a nucleon-deuteron state (freely moving relative to each other) and then substitute this resulting two-body wave function into the two-body Schrödinger equation to derive an effective potential that would lead to the assumed form of the effective n-d wave function within ^3H . They find by this approach that the effective n-d potential at large separation distances behaves as $-e^{-\lambda r}/r^{3/2}$. Though their result differs from that of Tomio *et al.*^{1,2}, they argue that it is similar in that both contain the qualitative feature of depending on a range associated with the size of the deuteron (through μ and λ). They also argue that their result must be valid for any short-range interaction since it is based on minimal assumptions about the three-nucleon wave function, i.e., a zero-range form. However, it is particularly this assumption about the form of the three-nucleon wave function which casts doubt on the reliability of their conclusions. It is well known that in zero-range approximation, the three-nucleon system has infinite binding energy.⁴ Historically, it was this fact that led the early workers on the two-nucleon interaction to the conclusion that the two-nucleon interaction has a short, but finite, range.⁵ Therefore, it is important to attempt another approach towards deriving the asymptotic behavior of the effective n-d potential in the three-nucleon system.

It should be noted, however, that Tomio, Frederico, and Delfino⁶ do extend their approach to a more general assumed form for the three-nucleon bound-state wave

function. They extend their zero-range model to include a parameter that, in some sense, accounts for the finite-range of the two-nucleon interaction. This parameter is set by requiring reproduction of the triton binding energy and the doublet n-d scattering length with the deuteron binding energy set at its experimental value. This time, the problem of finding the effective n-d potential must be approached numerically. They argue that their numerical results justify the phenomenological potential of Tomio, Delfino, and Adhikari.^{1,2}

DESCRIPTION OF PROGRESS: The approach taken in the current work is to begin from the three-body Schrödinger equation for the bound-state wave function without making specific assumptions about the two-nucleon interactions (possible three-nucleon interactions are neglected). The equation is projected with a bra state composed of a nucleon plus deuteron moving freely relative to each other (no detailed form for the deuteron wave function is assumed). The aim is to obtain an equation for this effective n-d wave function within ${}^3\text{H}$, i.e., for the overlap amplitude of the n-d state and the three-nucleon bound state. Except for the two terms containing the potentials in the coordinates that involve the 'external' nucleon and one of the nucleons in the deuteron, this falls out directly. In the latter 'permuted-potential' terms, a complete set of nucleon-correlated-pair states is inserted to affect the sought after equation. (The correlated-pair terms are the deuteron and an n-p scattering state.) It is then immediately apparent that we must deal with a coupled set of effective two-body equations, one corresponding to the effective nucleon-deuteron wave function and the other to the effective nucleon-n-p-continuum wave function. By iterative substitution, the effective nucleon-n-p-continuum wave function can be eliminated from the equation for the effective n-d wave function at the expense of having an infinite series of terms containing successively increasing powers of the two-nucleon interaction. In the effective n-d wave function equation, the leading term of the effective interaction is local (assuming the two-nucleon interactions are local), while the succeeding terms are nonlocal. These latter terms involve, in the simplest case, a Green's function with the two-nucleon interaction on either side. Initially, we completed a thorough study of the local part of the effective interaction.

Our purpose in investigating the local part of the n-d effective potential was to determine its long-range behavior. The local part of the effective potential is constructed from a matrix element of the 'permuted-potential' terms between deuteron wave functions, sometimes called a 'folding potential'. We started with specific potentials that were analytically tractable (s-waves only): 1. Zero-range; 2. Square-well; and 3. Hulthén. We found that the square-well and Hulthén potentials lead to an asymptotic behavior of $-(\text{constant}) e^{-4\gamma r}/r^2$, where the binding energy of the deuteron defines γ , i.e., $B_d = \gamma^2/M$. Moreover, the 'constant' multiplying the asymptotic form is given in terms of the potential parameters. Based on these analyses, it was then decided to assume an arbitrary short-range two-nucleon interaction, e.g., with Yukawa behavior at large separation distances, to see if the result can be made potential independent. The answer is that it can. The final expression has the above form with the constant being determined by a particular integral over the potential. The general result has been checked with the specific cases listed above and the previously obtained results have been reproduced. Interestingly, when we start with the assumption of a nonlocal two-nucleon interaction given by the standard Yamaguchi⁷ s-

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wave form, we find an asymptotic behavior of $-(\text{constant}) e^{-(2\gamma + \alpha)r}/r$, where $B_t - B_d = 3\alpha^2/4$, in general agreement with the conclusions of Fonseca, Redish, and Shanley concerning a Yukawa form.⁸ At least at the level of the local part of the n-d effective potential, there appears to be a difference in the asymptotic behavior depending on the nature, local or nonlocal, of the underlying two-nucleon interaction.

What has been learned from this investigation so far? Firstly, and most importantly, we found for the zero-range interaction that the local-part of the effective interaction is identically zero. Since it is expected, from our initial study so far, that the nonlocal contribution to the effective potential falls off differently and possibly more rapidly asymptotically than the local piece, and since the authors of Ref. 3 neglected the nonlocal contribution, this demonstrates that the *ansatz* used by these authors is invalid. Secondly, if our tentative conclusion about the asymptotic behavior of the nonlocal piece is correct, our result explicitly substantiates the adhoc assumption of Tomio et al.^{1,2} as to the asymptotic behavior of the effective n-d potential. Moreover, the precise parameter to be used in the exponential is obtained. It differs in numerical value significantly (approximately a factor of 4 larger) from that found by the fits in Refs. 1 and 2. The magnitude of the potential in the asymptotic region is explicitly given by the multiplicative constant. Thirdly, our asymptotic result and the exact expression for the potential can be used in combination to find at what separation distance the asymptotic behavior begins to dominate. This has been done numerically.

In addition to the above, we have been investigating, so far without a definitive conclusion, the question of the Efimov limit in the above framework. The Efimov limit is the situation where γ approaches zero and r is large but less than $1/\gamma$. In this limit, we would expect to find that the interaction approaches a $1/r^2$ behavior -- essential for the explanation of the Efimov effect.⁹ Our work on this question and the nature of the succeeding nonlocal corrections to the local part of the effective n-d interaction was minimal this past year, but we expect significant effort on this problem during the coming year.

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PROGRESS

TOPIC: Compton scattering of polarized photons from polarized protons

INVESTIGATOR: L.C. Maximon

OBJECTIVE: To provide an accurate, analytic, expression for the cross section for Compton scattering of both linearly and circularly polarized photons from polarized protons below pion threshold, including the effects of π^0 exchange.

SIGNIFICANCE: Recent measurements of the polarizabilities of the proton and of the neutron, as well as theoretical chiral model predictions for the polarizabilities have been the subject of several sessions at the Workshop on Hadron Structure from Photo-reactions at Intermediate Energies and the Baryons '92 conference, held in June and July of 1992. In particular, the measurements of Compton scattering on the proton, made with photons in the energy range 32-72 MeV by the group at Illinois¹, and with photons of energies 98 MeV and 132 MeV by the group at Mainz², have provided consistent results for the polarizabilities for the first time, but are inconsistent with earlier results from Moscow³. Additional measurements using circularly polarized photons are envisaged at facilities having tagged laser backscattered gamma rays, such as LEGS. While nucleon resonances must be taken into account at higher photon energies, it is important to have the cross section correct to order $(\omega/M)^2$, since this provides the major part of the cross section at the energies of these experiments. (Here ω is the incident photon energy, M is the proton mass.) This includes the effects of the proton charge, anomalous magnetic moment, and the electric and magnetic dipole polarizabilities. Although the contribution from t -channel exchange of a single neutral pion is of order $(\omega/M)^4$, it makes a rather important contribution and should be included⁴.

DESCRIPTION OF PROGRESS: The basis for this calculation was given in an earlier work on the scattering of linearly polarized photons from unpolarized protons⁵. Furthermore, this work did not include the effects of π^0 exchange. The calculation was thus considerably simpler than that now envisaged. The necessary matrix elements will now have to be programmed using symbolic manipulation programs such as REDUCE or FORM, and this work has just begun.

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PROGRESS

TOPIC: Kinematics of the scattering of two particles into three particles

INVESTIGATOR: L.C. Maximon

OBJECTIVE: To describe the kinematics of the scattering of two particles into a three-particle final state with the same pedagogical simplicity as that commonly used to describe the scattering into a two-particle final state.

SIGNIFICANCE: The current and planned high duty-factor electron accelerators facilitate coincidence measurements, and thus increase interest in the analysis of reactions with three-particle final states. Although a number of papers on the kinematics of these reactions have appeared, they lack, in general, the simplicity of the analysis of two-particle final state reactions and certainly do not have the clear geometric picture that is presented in the case of two-particle final states. The pedagogical presentation that is envisaged would be of particular use to both experimentalists and students new to the field.

DESCRIPTION OF PROGRESS: An analysis of the surfaces described by the three final-state particles in various possible frames of reference has been undertaken. The transformation of these surfaces to the frames generally of interest (center of mass, laboratory) is being examined.

PROGRESS

TOPIC: User's Guide to Radiative Corrections

INVESTIGATOR: L.C. Maximon

OBJECTIVE: This work was requested by B. Mecking and J. Mougey, group leaders at CEBAF, who view it as essential theoretical support for their experimental programs. The aim of the User's Guide to Radiative Corrections is to present, in an easily accessible form, a review of the state of the art expressions for the various radiative corrections which must be applied to high energy electron scattering data if one is to extract accurate nuclear information, at the one percent level. The review will include a list of relevant references for each of the corrections, accompanied by commentary indicating what approximations have been made, what would be the next most important correction to investigate, and an estimate of the accuracy of the formula presented. The radiative corrections to be considered fall under the general headings of Landau straggling, thick-target (external) bremsstrahlung, and the Schwinger radiative correction. Each one of these general topics will have subsections. For example, under the heading Schwinger radiative correction, we will consider recoil corrections, exponentiation, Coulomb corrections, multiple photon emission, photon emission by the target nucleus, and vacuum polarization. Moreover, there will be further subheadings indicating whether the expression for the cross section in question refers to polarized or unpolarized beams, polarized or unpolarized targets, and, very importantly, whether the corrections apply to single arm or coincidence experiments.

SIGNIFICANCE: The analysis of high-accuracy electron scattering measurements at high energies, such as those projected at CEBAF, requires a level of accuracy not previously demanded for the radiative corrections which must be applied to the experimental data, if the full potential of new accelerators and detectors is to be realized. The uncertainties in the theoretical expressions for the radiative corrections often constitute the limiting factor in obtaining accurate nuclear information. The computer programs for the analysis of the scattering data used in the past at most electron accelerator laboratories make use of theoretical work which was carried out almost twenty five years ago, and these computer programs often had inadequate accompanying documentation. Moreover, most of that work referred primarily to single-arm experiments; coincidence measurements, which are generally feasible only with the high duty cycle accelerators available within the last few years or currently under construction, were not generally of concern then. The User's Guide will provide a basis for the design of computer programs by making available in a readily accessible form, the most accurate expressions for each of these corrections, together with relevant documentation, comparison with earlier work, and an indication of the accuracy of the expressions given. It will include theoretical work which has appeared only within the last few years, often in unpublished reports.

DESCRIPTION OF PROGRESS: The task of assembling a library on radiative corrections is close to completion. This includes not only published articles, but, for most of the important work, material that has appeared only in unpublished reports and

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conference proceedings. Detailed discussions have been held with the two groups of theoreticians working actively in the field of radiative corrections to electron scattering, one headed by H. Navelet, at Saclay, the other headed by D.Yu. Bardin, presently at CERN. Their current work is being studied and put in a form readily accessible to experimentalists. Simplified expressions of their work, applicable to CEBAF energies, have been derived, and these have been compared with the standard expressions of Mo and Tsai. A second draft of the User's Guide is being written and will be sent to CEBAF for consideration and further input from the experimental community.

PROGRESS

TOPIC: Evaluation of the Spherical Bessel Transform of a Whittaker Function; An Application of a Difference Equation Method

INVESTIGATOR: L.C. Maximon

OBJECTIVES: To obtain a simple analytic expression for the spherical Bessel transform of the bound state Coulomb wave function for the ℓ -th partial wave, expressed as a Whittaker function. To derive a method for treating a wide class of integrals of similar form, based only on the existence of differential-difference equations for the functions involved, and the difference and differential equations satisfied by the integral.

SIGNIFICANCE: Integral transforms (Bessel, Hankel, Fourier, etc.) of functions which are the solution of wave equations (in general, second order differential equations) occur in almost any analysis of quantum mechanical problems. The method developed provides both analytic expressions for such transforms and recurrence relations which lend themselves readily to evaluation by computers. Since it relies essentially only on the differential and difference equations satisfied by the functions involved, it is applicable to a wide class of problems. This work was undertaken in support of the experimental effort at TUNL, conducted by E. Ludwig of UNC and R. Das at Duke. They used a low energy (7-10 MeV) deuteron beam on a heavy target, investigating the pickup reactions $d + n \rightarrow {}^3\text{H}$, $d + p \rightarrow {}^3\text{He}$, and $d + d \rightarrow {}^4\text{He}$ to obtain cross sections and analyzing powers, providing an experimental test of model wave functions. The method developed for obtaining integral transforms was applied in particular to the spherical Bessel transform of the bound state Coulomb wave function for the ℓ -th partial wave, expressed as a Whittaker function. This result was needed for the analysis of the experiments, providing an analytic expression for the effective n-d and p-d wave functions in momentum space, including the Coulomb interaction, for small momenta. This was required, in particular, for $\ell = 0$ and $\ell = 2$.

DESCRIPTION OF PROGRESS: This project has been completed; the work was published in the Journal of Mathematical Physics 33,2005(1992).

PROGRESS

TOPIC: Computer Program for Landau Straggling

INVESTIGATORS: L.C. Maximon and D.P. Heddle (Christopher Newport College and CEBAF)

OBJECTIVE: To write and make available for use by the community engaged in high and medium energy electron scattering experiments, a fast and accurate code for the computation of Landau straggling. This code will incorporate the best expressions available for the electron-electron interaction (Møller scattering) within the basic formulation given originally by Landau. The documentation accompanying this code will include a detailed consideration of the approximations involved in this formulation with references to relevant articles, together with a discussion of possible modifications for an improved version.

SIGNIFICANCE: The accuracy of the nuclear information obtained from high energy electron scattering measurements carried out with the new generation of electron accelerators depends on and is often limited by the theoretical uncertainties in the radiative corrections which must be applied to the experimental data. If useful nuclear information is to be obtained from high energy coincidence measurements, then the radiative corrections must be known theoretically at the one percent level. Landau straggling is one of these radiative corrections; this project is one element in the overall program of providing state of the art expressions for all of the radiative corrections.

DESCRIPTION OF PROGRESS: This project was completed; the work was published in Computer Physics Communications 70,77(1992).

PROGRESS

TOPIC: Precision triton and helium-3 bound-state wave functions

INVESTIGATORS: W.C. Parke, Y. Koike (Hosei U., Tokyo), D.R. Lehman, and L.C. Maximon

OBJECTIVE: To generate precise three-body bound-state momentum-space wave functions for ^3H and ^3He , using the best available NN interaction potentials represented by converged separable expansions, for later use in exact three-body photo- and electro-disintegration calculations.

SIGNIFICANCE: High quality wave functions for the three-nucleon system which can serve as reliable and practical input to photodisintegration (radiative capture) and electrodisintegration calculations, and which can be obtained easily, are of considerable current importance. For example, such wave functions are needed to properly interpret recently completed electrodisintegration experiments at NIKHEF and MIT, and future experiments at CEBAF and elsewhere. By working with separable representations of the two-nucleon interactions, it is possible to perform the full disintegration calculation on presently available supercomputers with fully consistent dynamics in the bound state and in the final state. Experiments will increasingly involve polarized beams and targets at low and intermediate energies as a probe of the detailed structure of ^3H and ^3He , such as their D-wave structure. This structure, in turn, reflects the underlying two-nucleon interaction. Thus, we should have a sensitive test of current models of the nucleon-nucleon interaction, including the need for explicit quark dynamics and relativity.

DESCRIPTION OF PROGRESS: We have shown^{1,2} in the first stage of our program that the PEST³ separable-potential representation of the Paris potential used in the three-nucleon bound state with a 'five-channel' calculation (including the $^1\text{S}_0$ and $^3\text{S}_1$ - $^3\text{D}_1$ partial waves of the NN interaction) gives results essentially indistinguishable from local-potential calculations for the binding energy, wave function components, and the S- and D-wave asymptotic normalization constants of ^3H . However, from a practical standpoint, our integral equations are in a single variable, while the local-potential calculations require equations (differential or integral) in two variables. As the separable-potential-rank index for the converged case is no more than five in the $^1\text{S}_0$ and six in the $^3\text{S}_1$ - $^3\text{D}_1$ two-body channels, while the angular integration to generate the kernels of the integral equations does not require more than 24 mesh points and the semi-infinite momentum integration for the integral equation no more than 40 points, we gain a factor of 6 or 7 in computation time, together with a simpler formalism, relative to a local-potential calculation.

We now have confidence that these wave functions can be used in a three-body calculation of the photodisintegration of ^3H or ^3He , with the dynamics taken from realistic two-nucleon forces and used consistently throughout the process, i.e. both in the bound state and in the all intermediate two and three-body rescattering states. This calculation is in progress and reported elsewhere in this document.

¹W.C. Parke, Y. Koike, D.R. Lehman and L.C. Maximon, Few Body Systems 11,89(1991),

²W.C. Parke, D.R. Lehman, Y. Koike, L.C. Maximon, and J. Haidenbauer, "Precision Three-Nucleon Bound-State Wave Functions from Separable Expansions of Realistic

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Two-Nucleon Potentials", published in Proceedings of the XIIIth International Conference on Few-Body Problems in Physics (Flinders University, Adelaide, Australia, January 5-11, 1992), edited by I.R. Afnan and R.T. Cahill, (Institute for Atomic Studies, Report FIAS-R-216, 1992) p. 302.

³D.J. Ernst, C.M. Shakin, and R.M. Thaler, Phys. Rev. C8,46(1973); *ibid*, C9,1780(1974); J. Haidenbauer and W. Plessas, Phys. Rev. C30,1822(1984); *ibid*, C32,1424(1985); J. Haidenbauer and Y. Koike, Phys. Rev. C34,1187(1986); J. Haidenbauer, Y. Koike, and W. Plessas, Phys. Rev. C33,439(1986).

PROGRESS

TOPIC: $\alpha + d \rightarrow {}^6\text{Li} + \gamma$ Scattering Cross Section and Polarization Observables at Low Energies

INVESTIGATORS: W.C. Parke and A. Eiro (U. of Lisbon)

OBJECTIVE: To calculate the cross section and polarization observables for the reaction $\alpha + d \rightarrow {}^6\text{Li} + \gamma$ using consistent three-body dynamics in both the initial and final states.

SIGNIFICANCE: Understanding the radiative capture of alpha particles on deuterons (or deuterons on alpha particles) has importance in astrophysics, nuclear physics, and nuclear technology. The capture reaction ${}^2\text{H}(\alpha, \gamma){}^6\text{Li}$ at low incident energies has been a focus of interest by astrophysicists and nuclear physicists.¹ The value of the cross section below center-of-mass energies of about 2 MeV is important in determining the abundance of ${}^6\text{Li}$ from the standard big-bang model and from cosmic ray generation. Most recently, it has been emphasized by Rolfs and Barnes in their review of astrophysically important radiative capture reactions that the reaction mechanism remains uncertain.² With polarized deuterons, or for the inverse photodisintegration process, polarized photons, the reaction may uncover information about the internal structure of the ${}^6\text{Li}$ nucleus. The reaction is also important in a controlled-fusion plasma. The analysis of the produced gammas can be used to determine the plasma temperature.³

Measurements at center-of-mass energies from 1 to 8 MeV have been made, showing a cross section between 4 and 100 nanobarns which seems to be dominated by an E2 direct capture mechanism⁴, although some admixture of E1 and possibly M1 may be present. Another series of measurements closer to the reaction threshold is nearly ready for data taking at TUNL.⁵

Besides the cross-section measurement, there has been recent interest in using radiative capture of polarized deuterons on the alpha particle^{5,6} to get a handle on the D-state component in ${}^6\text{Li}$, and in particular, on the ratio of the D to S asymptotic normalization constants for ${}^6\text{Li} \rightarrow \alpha + d$, as well as the difficult-to-predict electric quadrupole moment. The experiments pending at Duke (TUNL) will study the reaction ${}^4\text{He}(\vec{d}, \gamma){}^6\text{Li}$.⁵

As yet, there is no fundamental and fully consistent calculation of the radiative capture reaction, although several effective two-body calculations have been published.⁶ A three-body (alpha-particle, neutron, and proton) calculation for both the incoming and outgoing nuclear wave functions, using the same two-body interactions in the three-body scattering state and in the bound state, would satisfy self-consistency, in that the same fundamental dynamics would be used for all parts of the calculation. This approach has already been successful⁶ in the case of the calculation of the two-body photodisintegration of ${}^3\text{H}$ and ${}^3\text{He}$, where effective two-body models have not worked. Moreover, using a three-body wave function in the initial state would automatically include the ${}^6\text{Li}(3^+)$ resonance state in the α -d continuum. This state enhances the reaction cross section by almost two orders of magnitude near threshold (at about 700 keV in the center-of-mass). Such an approach should provide the sound footing necessary to delineate the reaction mechanism of this radiative capture reaction, including the role of off-shell two-body rescattering and intermediate three-states.

DESCRIPTION OF PROGRESS: We are currently engaged in a derivation of the full three-body radiative capture amplitude, with the bound-state and incoming scattering states both determined by the same three-body Hamiltonian. Our aim is to carry out a fully consistent effective two-body calculation within the three-body framework. We will then be prepared to compare with the contributions of initial-state rescattering, within a fully consistent effective two-body framework (no effective two-body calculations to date satisfy such consistency), and with the full three-body calculation to see the effects of three-body dynamics. Our effective two-body Born term results already show that the contribution to the reaction cross section can be sensitive to the quadrupole moment of the deuteron, particularly at center of mass energies (near 30 MeV) which probe the radial node in the α -d wave function (this node coming from the underlying Pauli exclusion principle for the nucleons in the two clusters). We have also calculated the tensor analyzing power T_{20} . As expected, this polarization observable is controlled by the D-state components in the α -d system, with the deuteron quadrupole term being the dominant contributor near 90° , even at center-of-mass energies of 4 MeV. We will be able to reduce the integral equations for the full three-body amplitudes which include rescattering corrections to effective two-body amplitudes. This will give us a clean way to test the contribution of continuum NN states in the intermediate rescattering process. At the beginning of this summer, we met in Portugal to map out the details of the calculation, including the full angular reduction necessary for programming the amplitude.

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PROGRESS

TOPIC: Ten-Channel Precision triton and helium-3 bound-state wave functions

INVESTIGATORS: W.C. Parke, D.R. Lehman, A. Fonseca (Lisbon), and J. Haidenbauer (Adelaide)

OBJECTIVE: To generate precise three-body bound-state momentum-space wave functions for ^3H and ^3He , using the best available NN interaction potentials for the S, D, and P-wave interactions, each represented by converged separable expansions.

SIGNIFICANCE: High quality wave functions for the three-nucleon system which can serve as reliable and practical input to photodisintegration (radiative capture) and electrodisintegration calculations. This work extends the applications described earlier under the topic "Precision triton and helium-3 bound-state wave functions". Including the P-wave NN interactions is necessary to account properly for the tensor analyzing powers in the polarization measurements for the capture process.

DESCRIPTION OF PROGRESS: We have generated the bound-state wave functions in parallel codes developed independently and then compared in their prediction of binding energy and percent wave-function components of definite symmetry type and total orbital angular momentum. We have included the $^1\text{S}_0$, the $^3\text{S}_1 - ^3\text{D}_1$, the $^1\text{P}_0$, and the three ^3P wave NN interactions. We have previously found that using PEST separable potentials of rank 4 and 5 for the $^1\text{S}_0$, the $^3\text{S}_1 - ^3\text{D}_1$, respectively, gave excellent convergence to the Paris potential configuration-space calculations. Keeping these, we tested the P-wave PEST representations with increasing rank. Convergence in the three-body wave-function components and binding energy occurs when the rank of the P-waves reaches 4. However, the binding energy (7.09 MeV) now differs from the configuration-space calculation¹ (7.11 MeV) by 0.3%. We plan on having a paper ready for publication by the end of the summer which will include these results and those of the S and D-wave asymptotic normalization constants.

¹B.F. Gibson (LANL), private communication.

TOPIC: D-State Structure of ${}^4\text{He}$

INVESTIGATORS: Hiroshi Ito (Postdoctoral Research Scientist), D.R. Lehman, and W.C. Parke

OBJECTIVE: To solve the four-nucleon bound-state equations by Faddeev methods with realistic two-nucleon interactions in order to investigate the D-state properties of ${}^4\text{He}$.

SIGNIFICANCE: Solution of the four-nucleon bound-state problem is intrinsically difficult. So far, it has been solved with the tensor force present by Faddeev methods only for the separable rank-1 interactions of Yamaguchi,¹ whereas, with realistic two-nucleon interactions, it has been calculated by variational methods,² variational Monte Carlo techniques,³ Green's function Monte Carlo methods,⁴ and by the ATMS (Amalgamation of Two-body correlations into Multiple Scattering processes).⁵ Whereas the Faddeev solutions should be numerically reliable to investigate the D-state properties of the ${}^4\text{He}$ nucleus, e.g., the ${}^4\text{He} \rightarrow d + d$ asymptotic normalization constants, the underlying interactions have a significant deficit in their representation of the two-nucleon tensor force. On the other hand, the solutions that have been obtained with realistic two-nucleon interactions tend not to be numerically well determined in the asymptotic region⁶ where one needs convergence to extract the D-state parameters like the asymptotic normalization constants and the Distorted Wave Born Approximation parameter, D_2 . Thus, wave functions obtained for the ${}^4\text{He}$ ground state by Faddeev methods with realistic two-nucleon interactions should permit us to predict reliable values for the asymptotic normalization constants and the DWBA parameter D_2 . As can be seen in the review by Lehman,⁷ the situation with regard to the D-state properties of ${}^4\text{He}$ is very unsettled both theoretically and experimentally, so the proposed theoretical work should help to clarify the situation. Moreover, the availability of such wave functions will permit us to calculate momentum distributions for ${}^4\text{He} \rightarrow n + {}^3\text{He}$, $\rightarrow p + {}^3\text{H}$, and $\rightarrow d + d$, all relevant to coincidence electron scattering experiments already carried out at MIT and NIKHEF, and planned for CEBAF.

DESCRIPTION OF PROGRESS: In solving four-body, bound-state problems, it is more transparent to begin with Schrödinger's equation and to solve the integral equations for the spectator functions defined below. This is our general direction⁸. It is quite different from the conventional t -matrix works of the original Yakubovsky work⁹ and subsequent extensions^{10,11}, where an approximation of the Bateman or Hilbert-Schmidt expansion type is used in the kernel of the equations. Once the realistic NN interaction is expressed in the separable form, our integral equations for the spectator functions are exactly soluble by using today's super-computer facilities. Before reporting the current progress, we describe the method⁸ by taking as example the 4-boson problem with a rank-one separable interaction.

The Schrödinger equation for the four-body bound state is given by

$$(H_0 + B)\Psi = - \sum_{i < j}^4 V(ij)\Psi, \quad (1)$$

where H_0 is the kinetic energy, and B is the binding energy. In this work, we assume that the two-body NN force, $V(ij)$, plays the dominant role in nuclear dynamics and that three- and four-body forces are absent or perturbatively treated later. With the

two kinds of possible cluster-partitions; (31) $\sim \psi(ab, c; d)$ and (22) $\sim \psi(ab; cd)$, the total wave function (Ψ) is expressed as a sum of 18 Faddeev components given by the possible distributions

$$\Psi = \sum^{12} \psi(ab, c; d) + \sum^6 \psi(ab; cd). \quad (2)$$

Here, the summations are taken for the possible distributions of particle numbers in the set {abcd}. The rank-one separable potential as a nonlocal operator in momentum space is given by,

$$V(ij) = -\alpha g(k'_{ij}) \int d^3 k_{ij} g(k_{ij}), \quad (3)$$

where k_{ij} is the relative momentum of the i -th and j -th particles. After suitable rearrangements of the Faddeev components, Eq. (1) can be reduced to a pair of coupled integral equations, where we have only two independent functions; the spectator functions $Q(p, q)$ and $R(t, s)$ defined by

$$\psi(12, 3; 4) = g(k) Q(p, q)/(H_0 + B) \quad (4a)$$

and

$$\psi(12; 34) = g(k) R(t, s)/(H_0 + B). \quad (4b)$$

Here, k, p, q, t , and s are the Jacobi momenta of the four particles, so that the other Faddeev components can be obtained by permuting the particle numbers in $Q(p, q)$ and $R(t, s)$. After the partial wave expansions, we have

$$Q(p, q) = 4\pi \sum_l \sqrt{2l+1} Q_l(p, q) [Y^{(l)}(\hat{p}) \otimes Y^{(l)}(\hat{q})]^0$$

and with a similar expression for $R(t, s)$. Now, the pair of coupled integral equations is given by

$$Q_L(p, q) = \int_0^\infty dk k^2 d\Omega_k d\Omega_q \times \sum_l \left\{ X_L(B; p, |k + \frac{1}{3}q|) Q_l(|q + \frac{1}{3}k|, k) P_L(\hat{q} \cdot [\hat{k} + \frac{1}{3}\hat{q}]) P_l(\hat{k} \cdot [\hat{q} + \frac{1}{3}\hat{k}]) \right. \\ \left. + X_L(B; p, |k - \frac{2}{3}q|) R_l(|q - \frac{1}{2}k|, k) P_L(\hat{q} \cdot [\hat{k} - \frac{2}{3}\hat{q}]) P_l(\hat{k} \cdot [\hat{q} - \frac{1}{3}\hat{k}]) \right\}, \quad (5a)$$

and

$$R_L(t, s) = \int_0^\infty dk k^2 d\Omega_k d\Omega_s \times \sum_l \left\{ Y_L^1(B; p, |k + \frac{1}{2}s|) Q_l(|s + \frac{2}{3}k|, k) P_L(\hat{s} \cdot [\hat{k} + \frac{1}{2}\hat{s}]) P_l(\hat{k} \cdot [-\hat{s} - \frac{2}{3}\hat{k}]) \right. \\ \left. + Y_L^2(B; p, |k - \frac{1}{2}s|) Q_l(|s - \frac{2}{3}k|, k) P_L(\hat{s} \cdot [-\hat{k} + \frac{1}{2}\hat{s}]) P_l(\hat{k} \cdot [s - \frac{2}{3}k]) \right\}. \quad (5b)$$

Here, $P_L(\hat{a} \cdot \hat{b})$ is the L -th order Legendre polynomial, and X_L and Y_L^n are the kernel

functions easily obtained by solving inhomogeneous integral equations for the L -th partial wave. Throughout the entire project, we deal with the same type of coupled integral equations as Eqs. (5a and 5b). We solve these integral equations to find the eigenvalue (B) and the solutions ($Q_L(p, q)$ and $R_L(t, s)$) by the power method on a CRAY.

We point out that Eqs. (5a and 5b) are the coupled integral equations involving the 4-dimensional integral: $dk, d\theta_k, d\theta_{q(s)}$ and $d\phi$ with $\phi = \phi_k - \phi_{q(s)}$. As a consequence, we have to interpolate the spectator functions in the integrand; for example, $Q_l(|q + \frac{1}{3}k|, k)$ must be interpolated from a few reference values of $Q_l(p_i, k)$ at the grid points p_i near by $|q + \frac{1}{3}k|$. This is a computational difficulty in our exact algorithm. In order to accomplish our objectives, we have divided the work into several levels of development.

- [Level-1] Solve the 4-boson problem of Eqs. (5a and 5b) including a few partial waves ($l = 0, 1, 2$) with rank-one separable force to check partial-wave convergence.
- [Level-2] Solve the 4-fermion problem with spin-dependent central forces for $l = 0$:
 - (i) derive the integral equations with the spin degrees of freedom;
 - (ii) solve the equations in (i) with spin-dependent interactions $V(^1S_0)$ and $V(^3S_1)$;
 - (iii) take $V(^1S_0) = V(^3S_1)$ limit in (i) and (ii) to check correctness of code;
 - (iv) normalize the wave function and calculate the S-wave asymptotic normalization constants.
- [Level-3] Repeat Level-2 with $l = 0, 1, 2$ to check convergence with partial waves.
- [Level-4] Solve the 4-fermion problem including the tensor and central forces:
 - (i) derive the integral equations with these forces present;
 - (ii) solve the equations from (i) and obtain the normalized wave function;
 - (iii) obtain the D-state probability and the asymptotic S- and D-wave asymptotic normalization constants.
- [Level-5] Repeat Level-4 with Paris NN interaction through use of separable-expansion methods as given by Refs. 12-14, for example.

Currently, we are at the the stage of Level-2(iii). In the work of Level-1, we first investigated the convergence of the solution method by changing the number of grid points in the quadrature for each of the four integrals. After several computer runs, we conclude that the grid points of $N_k = 16$ (dk -integral with Gegenbauer quadrature), $N_\theta = 10$ ($d\theta_k$ and $d\theta_{q(s)}$ integrals with Gauss-Legendre quadrature) and $N_\phi = 10$ ($d\phi$ integral with Simpson's method) provide convergence of the solution. Very stable values of binding energy and spectral functions are obtained with these grids. We tried the computations with two different methods of interpolation: 3-point and 4-point formulas of the Lagrange interpolation method. We have included $l = 0, 1$ and 2 partial waves and found that the effect of the $l = 1$ and 2 waves is less than 1%, which is reasonably small for the case of just central forces. As an important test of the present algorithm, which includes the 4-dimensional integral and the numerical interpolation, we repeated the calculation with $l = 0$ only. All the results obtained with 5 different sets of parameters led to precise agreement (differences are within 0.05%) with the published results⁸, where the integral equations involve only a 2-fold numerical integration for this special case. We regard this agreement as *major* progress in the entire project, because the computational aspects are the biggest obstacle in solving four-body problems.

PROGRESS

For Level-2, the integral equation is modified because of the additional degrees of freedom that are present with spin-dependent interactions. In the formal expressions, we have taken a special limit: $V(^1S_0) = V(^3S_1)$. The resulting equations agree with the ones for the 4-boson problem (Level-1), and we learned the physics of spin statistics given in the coupled integral equations. We are now developing the computer program for the case, $V(^1S_0) \neq V(^3S_1)$. We expect a manuscript to be prepared at the end of the summer that includes the numerical results of the wave function structure and the S-wave asymptotic normalization constants.

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PROGRESS

TOPIC: Gauge Invariant Compton Amplitudes from Relativistic Composite Systems.

INVESTIGATOR: Hiroshi Ito (Postdoctoral Research Scientist)

OBJECTIVE: To develop a theoretical basis for the gauge invariance of inclusive electron scattering from relativistic composite systems and to solve numerically the problem with tractable models for few-body systems.

SIGNIFICANCE: A structure function of inclusive electron scattering from a composite system is related to the virtual Compton amplitude. To meet the gauge invariance in the amplitude, a consistent dynamical theory is necessary with regard to the bound-state wave function, final-state interactions and two-body current operators. Within the Bethe-Salpeter formalism, we develop a dynamical theory of gauge invariant Compton scattering from relativistic composite systems. An explicit solution to the problem will be obtained by using covariant separable interactions. This work is expected to shed light on the relativistic dynamics in inclusive electron scattering. For example, the physics of y -scaling will be studied with this consistent relativistic approach.

DESCRIPTION OF PROGRESS: Inclusive electron scattering from a relativistic composite system is of great interest in association with today's high-energy-high-efficiency experimental facilities such as the Continuous Electron Beam Accelerator Facility (CEBAF). Interesting scaling phenomena are observed in this process; for example, x -scaling for the parton model in hadron physics and y -scaling^{1,2} in nuclear physics.

The response tensor of inclusive electron scattering can be expressed as an integral over the expectation value of the current commutator

$$W^{\mu\nu} = \int d^4x e^{iqx} \langle gs | [J^\mu(x), J^\nu(0)] | gs \rangle, \quad (1)$$

where $|gs\rangle$ is the ground-state wave function of the composite system. The response tensor, $W^{\mu\nu}$, can be expressed in terms of the virtual Compton amplitude, $T^{\mu\nu}$, i.e. the elastic-scattering of a virtual photon from the composite systems:

$$W^{\mu\nu} \propto \text{Im } T^{\mu\nu} \quad (2)$$

As a very significant theoretical point, the Compton amplitude must satisfy gauge invariance,

$$q_{1\mu} T^{\mu\nu} = q_{2\nu} T^{\mu\nu} = 0, \quad (3)$$

where $q_1(q_2)$ is the four-momentum of the virtual photon 1(2). The gauge invariance will be satisfied only with a dynamically consistent treatment of (a) the bound-state wave function, (b) the final state interactions, and (c) the electromagnetic two-body current operators.

Gauge invariance of the Compton scattering from a composite system has been extensively investigated in the framework of nonrelativistic dynamics³, but very few⁴ of the relativistic studies have ever been explicitly developed. Recently, a very challenging approach has been performed⁵ with the use of a covariant separable

interaction in the inclusive electron scattering from a relativistic composite system. Though the bound-state wave function and the final-state interactions are explicitly obtained in this approach, the total amplitude is not gauge invariant⁵ due to the use of a nonlocal interaction. This indicates the necessity of the systematic study of the dynamical aspect of gauge invariance in the Compton scattering. In the previous work^{6,7}, the investigator and his collaborator developed a theory of gauge invariance in the observables of a one-photon process such as the exclusive electromagnetic form factor. A constraint on the relativistic two-body (interaction) current, J_{2body}^μ , was obtained on a very general basis, where the one-body Ward-Takahashi identity and Bethe-Salpeter equation are applied to the current conservation, $q_\mu (J_{1body}^\mu + J_{2body}^\mu) = 0$. Furthermore, the explicit form of the two-body current was analytically derived⁶ by applying the minimal substitution method⁸ to a covariant separable interaction. With the bound-state solution of the Bethe-Salpeter equations, the matrix element of the current operator was explicitly calculated.

With these motivation and experiences, we propose to develop a theory of gauge invariant Compton scattering from relativistic few-body systems. Within the Bethe-Salpeter formalism, we follow a similar approach to that taken in Ref. 6. Firstly, we apply the one-body Ward-Takahashi identity to the current conservation of the Compton amplitude, Eq. (3). In Eq. (3), $T^{\mu\nu}$ must include the impulse amplitude, intermediate-state rescattering amplitude, and the contribution from two-photon interaction currents, all in a general form. Secondly, we introduce the Bethe-Salpeter wave equation into the above expression to obtain a general constraint on the two-photon interaction current. This constraint equation can be expressed in terms of the two-body force in the nuclear dynamics. Next, by using a covariant separable interaction or other models of nonlocal force, we derive the analytical expression for the interaction currents. The result must satisfy the above constraint. Finally, the total amplitude is proven to be gauge invariant, and we calculate the Compton amplitude by using the bound state wave function obtained with the separable interaction.

The Bethe-Salpeter equation has been solved for the two- and three-nucleon bound systems⁹ and for the scattering matrix¹⁰ with covariant separable interactions. Meanwhile, a relativistic generalization¹¹ of the EST-expansion¹², which is an approximate separable expansion of a local potential, has been published. With this successful development of the methods for solving relativistic few-body systems, we expect that the proposed project can be developed on the realistic basis of nuclear physics; for example, the use of realistic One-Boson-Exchange interactions should be possible. Nevertheless, within the proposed time scale, we focus on the basic theoretical development and the application to a simple model of two- or three-body systems in order to clarify the physical significance in the dynamical aspect of the problem.

The investigator has been collaborating with Prof. F. Gross (CEBAF and College of William & Mary) on this subject since July 1991. Considerable progress has been made during the period, and the conclusions are promising to open a new approach in the electromagnetic processes for few-body problems, as listed below.

[1] We found the essential difference between local and nonlocal forces in the processes of electrodisintegration and Compton scattering: In spite of the fact that the nuclear force is nonlocal due to the spatially extended structure of meson and nucleon, a crucial

distinction of the nonlocality has never been observed. For example, a nonlocal force through the two-pion exchange process with baryon resonance¹³ can be well approximated by using the local model of σ -exchange force. In the presence of the electromagnetic field, however, we found that the nonlocal interaction generates a new type of two-body current: Within the nonlocal distance $d = |x'_1 - x_1| \sim |x'_2 - x_2|$ in a nonlocal force, $V(x'_1, x'_2; x_1, x_2)$, the photon field may couple to the charged constituents participating in the interaction. For example, the photon couples to the excited baryon or pions, which virtually propagate to induce the nonlocality, in the nonlocal model of Ref.13. This is the physical origin of the new process associated with the nonlocal nuclear force. Subsequently, the conventional calculation of electrodisintegration including the impulse and final state interaction processes must be modified, since the nuclear dynamics is nonlocal at the short-range/large momentum transfer region. The theoretical development has been completed for the electrodisintegration of the deuteron.

[2] In the Compton Scattering, we must take into account another type of interaction current that involves the couplings of two photons with the nonlocal interaction: This conclusion can be naturally explained by extending the reason mentioned above into the case involving two photons. In the Compton scattering, we must include this two-photon interaction current, in order for the amplitude to be gauge invariant. The theoretical formulation of Compton scattering including all the necessary dynamical elements has been completed. A general constraint on the current operator, i.e., the two-body Ward-Takahashi identity, has been derived.

[3] We have derived the explicit form of the one- and two-photon interaction current operators from typical models of a nonlocal nuclear potential: The two-body Ward-Takahashi identity is merely a necessary condition which all the interaction currents must satisfy. We must prove the existence of interaction currents associated with nonlocal interactions. This can be sufficiently implemented by deriving the current operator themselves from simple models of the interaction. We have derived the one- and two-photon interaction current operators from (a) Two-pion exchange interaction with baryon resonance and (b) Covariant separable potential.

[4] Preparation for journal publications has been finished:

H. Ito and F. Gross, "Gauge Invariant, Dynamical Theory of the Compton Scattering from Relativistic Composite Systems", to be submitted to Phys. Rev. C.

H. Ito and F. Gross, "Gauge Invariant, Dynamical Model for the Compton Scattering from Relativistic Composite Systems", to be submitted to Phys. Rev. C.

An oral presentation of the results is planned at the Division of Nuclear Physics, Annual Fall Meeting of the American Physical Society at Santa Fe, NM, October 1992.

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10/9/92