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**IMPAC2—A Lumped-Mass Nonlinear Spring Computer
Program to Analyze Type B Shipping
Container Impact Problems**

by

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IMPAC2-- A LUMPED-MASS NONLINEAR
SPRING COMPUTER PROGRAM TO ANALYZE
TYPE B SHIPPING CONTAINER
IMPACT PROBLEMS

by

J. Counts and J. B. Payne

ABSTRACT

Information is provided for using the computer program, IMPAC2, which is designed to analyze the dynamic response of metallic shipping containers impacting an unyielding surface. The method of analysis, a description of the program, and an explanation of the input and output variables are presented.

I. INTRODUCTION

This manual presents a complete description of the IMPAC2 input and output parameters and provides the user with information to employ this program as a tool in the dynamic design of shipping containers. The program solves the equations of motion for a one-dimensional, lumped-mass, nonlinear spring mathematical container model. The solution procedure uses an iterative technique for determining the displacement, velocity, and acceleration of the masses at the end of each time step.

The input is specifically written to accommodate shipping container geometry and requires only overall dimensions, rather than the dimensions of each mass and spring. The container may consist of several hollow concentric cylinders, each of a different material and length. The user inputs the type of material, height, the inner and outer radii of the cylinders, and the angle at which the container is to strike the rigid horizontal target surface. With this input and the specification of the desired number of lumped masses, the program will compute the proper lumped masses. The spring characteristics

associated with the lumped masses are computed at each time step during execution. The program is written in FORTRAN and designed to run on CDC computers. The code is available from the Argonne National Laboratory Code Center.

II. BACKGROUND

IMPAC2 is based on a Los Alamos Scientific Laboratory (LASL) computer code originally developed by Donham.¹ Input statements that are convenient for shipping container geometry, several built-in material property descriptions, and a slicing feature that automatically generates a lumped-mass mathematical container model were incorporated with Donham's work by Kern and Merson.² A theoretical discussion of their work, which is referred to as IMPAC1, is given by Fuka.³

The IMPAC1 code was subsequently made more efficient and used to correlate with Oak Ridge National Laboratory (ORNL) experiments. That work is described in Ref. 4 and shows that the code gave reasonable estimates of measured permanent lead deformations.

IMPAC2, the present form of the LASL code, was developed explicitly to accommodate impact limiters and to correlate with strain and acceleration-time histories measured in ORNL and Battelle Columbus Laboratory (BCL) tests. The results of the correlation attempts are discussed in Ref. 5.

III. THEORY

The dynamic response of a container is simulated by a lumped-mass spring model, where the masses represent a series of slices taken parallel to the impact surface and connected by nonlinear springs. The forces exerted by the springs on the masses are determined from the stress-strain characteristics of the container materials. The mass of each slice is assumed to be at the upper boundary of that slice, and the spring length is the thickness of the slice (Fig. 1).

The displacement-, velocity-, and acceleration-time histories of the lumped masses are determined by dividing the time interval into sufficiently small increments and assuming a linear variation of acceleration across each increment. That is, if $\ddot{x}(t_m)$ is the acceleration at time t_m , then at any time, t , within the interval $t_m \leq t \leq t_{m+1}$, the acceleration will be:

Fig. 1. Conceptual shipping container and its lumped-mass mathematical model.

$$\ddot{x}(t) = \ddot{x}(t_m) + \frac{\ddot{x}(t_{m+1}) - \ddot{x}(t_m)}{t_{m+1} - t_m} (t - t_m). \quad (1)$$

Integrating the above equation from t_m to t yields the velocity:

$$\dot{x}(t) = \ddot{x}(t_m) (t - t_m) + \frac{[\ddot{x}(t_{m+1}) - \ddot{x}(t_m)]}{2(t_{m+1} - t_m)} (t - t_m)^2 + \dot{x}(t_m). \quad (2)$$

A second integration yields the displacement:

$$x(t) = \frac{\ddot{x}(t_m)(t - t_m)^2}{2} + \frac{[\ddot{x}(t_{m+1}) - \ddot{x}(t_m)](t - t_m)^3}{6(t_{m+1} - t_m)} + \dot{x}(t_m)(t - t_m) + x(t_m). \quad (3)$$

If the displacements of the masses (M_1) at time t_{m+1} are known, then the spring forces, and hence the forces F_1 on the lumped masses, can be determined at time t_{m+1} by:

$$F_1(t_{m+1}) = M_1 \ddot{x}(t_{m+1}). \quad (4)$$

An iteration technique (Newmark-Beta) is used to solve the above system of equations for the displacement, velocity, and acceleration at time t_{m+1} . The technique used in this program is to guess $\ddot{x}(t_{m+1})$ to be either 0.0 at $t=0.0$ or to be equal to the acceleration at the previous time step. With the velocity and displacement known, the spring force can be computed from the stress-strain relationship. The acceleration is then computed from Eq. (4) and compared with the guessed acceleration. If the difference between the assumed and calculated acceleration is larger than a given tolerance, the assumed acceleration is set equal to the calculated acceleration and another iteration through Eqs. (2-4) is performed.

The iterative technique just described must be used simultaneously for all the lumped masses, since the forces and displacements are dependent on each other. This procedure is continued until the assumed and calculated accelerations are within an assigned tolerance of one another.

IV. PROGRAM DESCRIPTION

A. Modeling

Most engineers will find modeling a shipping container for IMPAC2 analysis similar to a uniaxial deformation problem in mechanics of materials. However, their decisions will be made easier if the limitations and dynamic implications of the assumptions used in the code are made clear. This can be best accomplished by examining how the deformation models in the code are constructed and how they relate to the shipping container components.

The maximum number of shipping container components that can be included in the IMPAC2 one-dimensional impact analysis is shown in Fig. 2a. After the initial geometry and the number of slices are defined by the user, the code transforms the slices, which are taken parallel to the impact surface (Fig. 2b), into equivalent circular cylinders. Each ring in the slice is transformed on an equal volume basis into a circular ring as shown in Fig. 2c. The heights of the original and volume-equivalent rings are the same, but the horizontal areas are different unless the impact orientation is end-on (90°). There are two deformation models available in the code, but both models assume that the equivalent slices deform uniaxially along a common vertical, regardless of the container orientation. This assumption is natural for an end-on impact and reasonable for a corner - c.g. impact. An impact at any other angle results in a greater degree of approximation, because in a real container only local deformation would occur near the impact point before the container begins to rotate.

The uniaxial motion of the slices also implies that parallel plane sections originally perpendicular to the vertical remain plane and parallel. This means that the rings in a given slice must experience the same vertical deformation and strain. The vertical stress in each ring is then computed from the stress-strain law for that ring material. The stress in each ring is multiplied by the equivalent area of the ring to obtain the resultant vertical ring force. The forces in the individual rings are then summed to determine the resultant vertical force on that slice. The masses of the slices are considered to exist at slice interfaces so that the resultant force acting on that mass (Fig. 2d) is produced by the forces on adjacent slices.

Fig. 2. Container modeling elements.

The assumptions involved in this deformation model imply that welded contact exists between the adjacent material cylinders in the container, because all horizontal planes experience the same deformation. Completely welded contact is very difficult to attain in practice, and the assumption of its existence in this deformation model prevents consideration of either the two- or three-dimensional stress state in a real container. If the shielding were a relatively stiff material, such as depleted uranium, welded contact might be a desirable assumption. However, the other deformation model available in the code is better suited to lead, which is a more common shielding material and has relatively small stiffness.

The second deformation model is used by specifying clad-lead material. The lead shielding is considered to be clad with the outer shell, which is assumed to be massless. The outer shell is assumed to behave as a cylindrical membrane by neglecting bending, axial, and shear resistance of the shell, as well as its radial inertia.

The pressure (P) produced between the inner surface of the outer shell and the outer surface of a lead slice (Fig. 2e) is related to the circumferential (hoop) stress (σ_c) in the shell by the mechanics of materials formula

$$\sigma_c = P \frac{R}{H},$$

where H is the shell thickness and R is its mean radius. The assumption is made that the lead retains constant volume during deformation and has the effect of requiring the radial and hoop strains in a slice to be the negative one-half of its axial strain. Thus σ_c can be computed from the axial strain by using the constitutive equation for the outer shell material. (The code uses only an elastic perfectly plastic stress-strain curve for the outer shell material.) These assumptions, made first by Donham,¹ attempt to account for the restraining effect of the outer shell on the radial motion of the lead by adding the axial-strain dependent compressive stress (P) as a bulk stress to the axial stress in the lead. The latter stress (P') is computed from the nonlinear axial stress-strain relationship for the lead by itself. The code accounts properly for cladding only when the strain in the cladding is increasing. The clad model effectively increases the lead stiffness.

If welded contact is assumed for the cylinders constituting the container, then modeling is relatively straightforward. The sample problems presented later in the manual will help clarify the modeling technique for the clad-lead deformation model.

B. Material Properties

Densities and stress-strain characteristics are the only material property data needed. Density is used to define the mass, while the stress-strain characteristics are used to determine spring forces. The strains are determined from the change in length of the springs. Unloading from the nonlinear portion of the curve is assumed to occur along a defined modulus, and the maximum stress at a material interface is assumed to be that of the weaker material. The program is designed to handle loading and unloading.

The material properties for clad lead are a special case because of the lack of empirical data. The stiffening effects of the outer steel cladding on a lead ring are superimposed on the lead properties by assuming the cladding acts as a thin-wall pressure vessel restraining the outward flow of lead. The pressure resulting from the lead acting on the vessel wall is added to the axial stress found for the lead acting alone. The value of this pressure is limited to a maximum based on the yield stress assumed for the outer steel shell.

C. Input

The standard FORTRAN namelist procedures with the label DATA are used to input problem parameters. This feature permits the input of character strings consisting of names and values without a format specification. The first input card is a program identification card with format 8A10. The first data card immediately follows the program identification card and starts with a \$ in column two, followed immediately by the name DATA with no embedded blanks. Following DATA, the input parameters, separated by commas, are read until another \$ is encountered. All input on cards following the first data card must start after a blank column 1 and again be separated by commas. A set of default values included in the program will be used for the parameters the user does not specify. All parameter names beginning with I, J, K, L, M, or N

are input as integers and the rest are input as real numbers. The data items, separated by commas, may be in any of four forms:

- (1) $v = c$
- (2) $a = d_1, \dots, d_j$
- (3) $a(n) = d_1, \dots, d_m$
- (4) $a(m,n) = d_1, \dots, d_\ell$,

where v is a variable name, c is a constant, a is an array name, m and n are integer constant subscripts, and the d values are simple or repeated constants of the form $k*c$, where k is the repetition factor.

The input parameters can be grouped into three categories:

- (1) Parameters specifying computation and output options,
- (2) Parameters specifying container geometry, and
- (3) Parameters specifying materials.

The following is a summary of possible inputs to the program.

1. Computation and Output Control Parameters. If the consistent system of units used for input parameters is other than the inch-pound-second system, then the length, stress, and mass density conversion factors (C1, C2, and C3) are required. This is necessary since certain constants built into the program are in English units.

C1: The factor used to convert from inches to a different unit of length. If the International System of Units (SI) is used, then $C1 = 2.54 \times 10^{-2}$ is punched on the data card. If this equation is not included in the data cards (defaulted), all lengths will be in inches.

C2: The factor used to convert stress and elastic moduli to a different system of units. The SI factor is input as $C2 = 6.894757 \times 10^3$. If C2 is defaulted, stresses and elastic moduli will be in psi.

C3: The factor used to convert mass density. The SI factor requires $C3 = 1.06869 \times 10^7$, and if C3 is defaulted, the units of mass density will be $\text{lb} - \text{sec}^2/\text{in}^4$.

TDELTA: The time interval in seconds at which the calculations are made. The default value is 10^{-5} seconds.

TPRT: The time lapse in seconds between printouts. The default value is 0.1 seconds.

VELOCT: The initial velocity of the container. The default value is 527.2 in./sec, which corresponds to a 30-foot (9.14-m) drop.

ITP: The number of the slice for which a stress-time printout is desired. If ITP is not included in the data cards, no printout is produced. The variable printed is STRSAV(5,I), where I is the element number.

T1T: The maximum permissible strain permitted for any material. Default will result in a value of 0.35.

TEND: The maximum real time at which the calculations stop. The default value is 0.1 seconds. The calculations will stop when the total permanent deformation in all the springs reaches a maximum or the real time reaches TEND. If TEND is defaulted, the calculations are terminated when the rate of increase of total deformation (elastic and plastic) of the element farthest from the impact point has decreased to a certain value. Specifically, this is when the difference between the existing and previous deformation is less than or equal to 0.1% of the previous value. This termination time is arbitrary but has been found to insure that the important part of the impact period has been included. The user could modify the program to terminate when, say, a certain percentage of the kinetic energy has been absorbed. A smaller value of energy absorbed would decrease the running time.

2. Geometry. If the container is composed of axisymmetric concentric cylinders (Fig. 3), the geometry will be calculated by an automatic slicing routine. The user can specify the radii of the cylinders, their heights, and the impact angle. The program will determine the radii and thicknesses of the concentric equal-thickness rings sliced parallel to the impact plane. The automatic slicing routine also assumes the container is symmetric about a midplane parallel to its base. If a distribution of lumps is used that does not have this symmetry, the slices must be removed from the top of the container. This is accomplished by using the variable NELEM2 to redefine the total number of slices.

NFLS: The number of slices to be used. This value may not exceed 100, but the code could be modified to accommodate more slices.

Fig. 3. Input variables for slicing concentric cylinders.

CYL RAD: An array of at most five radii of concentric right circular cylinders to be used by the automatic slicing routine. The outermost cylinder is always number five.

HEIGHT: An array containing the heights of the concentric cylinders associated with CYL RAD to be used by the automatic slicing routine.

ANGLE: The angle between the longitudinal axis of the container and the impact surface (degrees). The default value is 90° (end-on).

The results of the automatic slicing routine can be overridden by entering the radii and thicknesses of the concentric rings in each slice. These parameters are defined as follows:

NELEM2: The new number of elements (slices) where NELEM2 is less than the previously input value for NFLS. Elements can be deleted from the problem by redefining the number of slices with this variable. An example of its use with an impact limiter is shown in the sample problems.

THICK: An array of at most 100 values containing the thickness of those layers for which the user specifies the thickness. See Fig. 4.

RADIUS: An array of dimension 5 (rings) by 100 (slices) containing the radii of the five rings and NFLS masses.

3. Material. The present version of the program is written for a maximum of 10 different materials. The first seven material locations have been given the following designations:

- 1 - steel
- 2 - clad lead
- 3 - uranium
- 4 - pine
- 5 - lead
- 6 - balza
- 7 - marine plywood

The stress-strain characteristics for material types one, three, four, and six are approximated by a bilinear curve fit. This fit requires as input the modulus of elasticity, the stress at the intersection of the two straight lines, and the slope of the second line. The stress-strain characteristics

Typical container

Sliced container

Fig. 4. Input variables for sliced disks.

for type seven are approximated by a polynomial fit. The constitutive equation for clad lead and lead is of the form:

$$\sigma = P_1 [1 + P_2 (\dot{\epsilon})^{P_3}] (\epsilon)^{P_4} ,$$

where

σ = true dynamic axial stress (P' in Fig. 2e),
 ϵ = engineering strain, and
 $\dot{\epsilon}$ = engineering strain rate.

This form was used by Slater et al.⁶ to correlate with dynamic compression tests of short, pure lead cylinders. They obtained the mean values

$$\begin{aligned} P_1 &= 4.04 \times 10^3 \text{ psi} = 2.79 \times 10^7 \text{ Pa}, \\ P_2 &= 0.004, \\ P_3 &= 0.66, \text{ and} \\ P_4 &= 0.44, \end{aligned}$$

and concluded that at ambient temperature the material was not strain-rate sensitive up to 250 sec^{-1} . Donham¹ obtained

$$\begin{aligned} P_1 &= 6.75 \times 10^3 \text{ psi} = 4.65 \times 10^7 \text{ Pa}, \\ P_2 &= 0.0004, \\ P_3 &= 2.0, \text{ and} \\ P_4 &= 0.44, \end{aligned}$$

from 9.14-m drop tests of cylinders 0.305 m to 0.914 m in length. Kern and Merson² used the latter values except for P_2 , which was 0.004. They noted that calculated and measured deformations agreed to within a few per cent, but predicted stresses seemed inordinately large. For strain rates of about 100 sec^{-1} , stress would be much larger for $P_3 = 2.0$ instead of 0.66.

The calculations in this manual were made with

$$\begin{aligned} P_1 &= 7000 \text{ psi} = 4.83 \times 10^7 \text{ Pa}, \\ P_2 &= 0.004, \end{aligned}$$

$P_3 = 0.66$, and

$P_4 = 0.44$.

which gave reasonable correlation with an ORNL experiment.⁵

The coefficients in the constitutive equations are input to the program as

P1: default = 7000 psi,

P2: default = 0.004,

P3: default = 0.66, and

P4: default = 0.44,

which makes it easy for the user to change values.

MATNO: An array of dimension 100 (slices) by 5 (rings) containing the type of material in each slice of each ring. MATNO is always required.

EMOD: The modulus of elasticity for each of the material types used. This is required for each material type because unloading in the plastic range is assumed to occur along this slope. The default values for the seven different types are given in Table I.

EMOD2: The slope of the second line in the bilinear stress-strain curve fit for each material type. The default values are zero, which implies that the material is elastic perfectly plastic.

STRS12: The stress at the knee of a bilinear stress-strain curve for each material type. STRS12 and STRSMX are identical for an elastic perfectly plastic material model. The default values are zero.

STRSMX: The maximum compressive stress for each type of material. The seven default values are given in Table I. STRSMX is the stress on which the "small" stress cutoff discussed in Section VI is based.

DENS: The mass density for each material type. The seven default values are given in Table I.

Material two (clad lead) requires the thickness, modulus of elasticity, and the tensile yield stress of the cladding. The following parameters are used for this purpose.

STLCLT: The cladding thickness of each slice. The default values are equal to zero.

ESTCLD: The modulus of elasticity of the cladding. The default value is that for steel (30×10^6 psi = 2.07×10^{11} Pa).

YSTCL: The tensile yield stress for the steel cladding. The default value is equal to 80 000 psi = 5.16×10^8 Pa.

TABLE I
DEFAULT VALUES FOR MATERIAL PROPERTIES

<u>Material</u>	<u>Type</u>	<u>Modulus of Elasticity psi</u>	<u>Maximum Stress psi</u>	<u>Mass Density lb-sec²/in.⁴</u>
Steel	1	30×10^6	5.5×10^4	7.32×10^{-4}
Clad Lead	2	1.2×10^6	5.0×10^3	1.06×10^{-3}
Uranium	3	26×10^6	48.3×10^3	1.76×10^{-3}
Pine Wood	4	9.7×10^5	2.4×10^3	3.8×10^{-5}
Lead	5	1.2×10^6	5.0×10^3	1.06×10^{-3}
Balsa Wood	6	4.2×10^5	1.25×10^3	1.2×10^{-5}
Marine Plywood	7	1×10^6	1.57×10^3	4.6×10^{-5}

D. Output

Most of the output from the program appears in tabular form for a given value of time. The time lapse between printouts is controlled by the input parameter TPRT. The table of solution results contains the following information:

- current spring length (based on constant volume),
- current radius of outermost cylinder (based on constant volume),
- mass of the element,
- force in the spring,
- strain rate,
- acceleration in g's (positive downward),
- velocity (positive downward),
- permanent deformation or set (positive for compression),
- remaining energy,
- total strain, and
- stress in the outermost cylinder (negative for compression).

The force, stress, mass, length, and energy units for these quantities will depend on the system chosen by the user.

When clad-lead material is used, the compressive stress between the lead and the outer shell is printed on a separate page under the heading PRESSURE.

V. SAMPLE PROBLEMS

Three sample problems are presented to illustrate how the program is used. The problems are described and the input and output for each problem are presented. The printout is limited to the initial geometry, all calculated variables at the time where the total permanent deformation reached a maximum, and a plot of the stress-time history for the element of the container nearest the impact surface. The input and output variables are all in the SI system.

The first sample problem represents a 9.14-m drop of a 205 211-newton steel-jacketed lead container striking the impact surface at a 90° angle (Fig. 5). The container has an inside radius of 0.137 m, an outside radius of 0.391 m, an inside height of 3.810 m, and an outside height of 4.318 m. The material is clad lead (type 2) with a steel thickness of 0.0127 m and a tensile yield stress of 5.516×10^8 pascals. The container was divided into 38 elements and the geometry calculated by the automatic slicing routine. The time interval was 5×10^{-6} seconds. Figure 6 shows the input for this problem and Figs. 7 and 8 are the output. Figure 13 shows the stress-vs-time plot for element one. This problem took 29 seconds of CDC 7600 time.

The second sample problem represents the same container described in problem one but with a balsa wood impact limiter (Fig. 5). The limiter is 0.568 m thick and is divided into 5 elements. The container and limiter were divided into 43 elements and the geometry was calculated by the automatic slicing routine. The number of elements was redefined to 43 elements by the variable NELEM2 so that the top impact limiter was eliminated from the problem. Figure 6 shows the input for this problem and Figs. 9 and 10 are the output. Figure 13 shows the stress vs. time plot for element seven, the second element of the container. This problem required 60 seconds of CDC 7600 computer time.

The last sample problem is identical to sample problem 2 except that the impact limiter was lumped into a single element (Fig. 5). The problem was

Fig. 5. Steel-jacketed lead cylinder with and without impact limiter.

```

SAMPLE PROBLEM 1, END ON DROP,NO IMPACT LIMITER,
$DATA
NFLS=38,CYL RAD(4)=0.136525,0.390525,
HEIGHT(4)=3.01,0.3179875,
MATNO(1,5)=38*2,STLCLT(1)=38*8.0127,
YSTCL=5.5158E8,C1=2.54E-2,C2=6899.757,
C3=1.06669E7,TDELTA=5.E-6,ITP=13

```

```

SAMPLE PROBLEM 2, END ON DROP WITH IMPACT LIMITER,
$DATA
NFLS=48,CYL RAD(4)=0.136525,0.390525,
HEIGHT(4)=3.01,5.4503,
ITP=6,
MATNO(1,5)=5*6,MATNO(6,5)=38*2,MATNO(40,5)=5*6,
STLCLT(6)=38*8.0127,YSTCL=5.5158E8,STRS12(6)=66184446.25,
NELEP2=43,C1=2.54E-2,C2=6899.757,
C3=1.06669E7,TDELTA=5.E-6

```

```

SAMPLE PROBLEM 3, END ON DROP,ONE SLICE FOR IMPACT LIMITER,
$DATA
NFLS=80,CYL RAD(4)=.136525,0.390525,
HEIGHT(4)=3.01,0.50525,
MATNO(1,5)=6,MATNO(2,5)=38*2,MATNO(40,5)=6,
STLCLT(2)=38*8.0127,YSTCL=5.5158E8,STRS12(6)=66184446.25,
C1=2.54E-2,C2=6899.757,C3=1.06669E7,
TDELTA=5.E-6,ITP=3,
THICK(1)=.56015625,NELEM2=398

```

Fig. 6. Input data for sample problems.

SLICE	HEIGHT	VOLUME	AREA	RADIUS	HEIGHT	MASS	RING 1	RING 2	RING 3	RING 4	CLADDING
1	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
2	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
3	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
4	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
5	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
6	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
7	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
8	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
9	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
10	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
11	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
12	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
13	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
14	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
15	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
16	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
17	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
18	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
19	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
20	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
21	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
22	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
23	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
24	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
25	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
26	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
27	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
28	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
29	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
30	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
31	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
32	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
33	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
34	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
35	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
36	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
37	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
38	.114	.054	.070	.300	6053.043	616.741	0	0	0	2	.01270
TOTAL HEIGHT =		4.31700									
TOTAL VOLUME =		1.04575									
TOTAL WEIGHT =		285211.20160									
1 = STEEL											
2 = CLAD LEAD											
3 = URANIUM											
4 = PONDICHERRY PINE											
5 = LEAD											
6 = RALSA											
7 = MARINE PLYWOOD											

Fig. 7. Initial geometry and masses
for sample problem 1.

SAMPLE PROBLEM 1. END ON DROP, NO IMPACT LIMITER.

SLICE	TIME	DR	MASS	FORCE	STRAIN	RATE	ACCELERATION	VELOCITY	PERM SET	ENERGY	STRAIN	STRESS
1	.1001	.0070	410.7411	-13120162.30	.01	.04	.01	.00532	.00	.000022110	.25000160	
2	.1004	.0035	410.7411	-12552507.13	.02	.06	.00	.007100	.00	.000310130	.24530340	
3	.1006	.1000	550.0075	-12152010.00	.04	.10	.05	.0000471	.50	.070033002	.25040600	
4	.1001	.1021	501.1057	-127101170.30	.00	.04	.05	.0000007	.50	.000030700	.27000152	
5	.1007	.1020	501.1057	-12070200.35	1.00	.107	.04	.0000007	.05	.000071113	.27200012	
6	.1003	.1010	501.1057	-11097753.71	1.5	.130	.21	.000375	11.00	.005500032	.25510370	
7	.1003	.1010	501.1057	-10005039.51	.00	.50	.21	.000362	12.03	.005270103	.25025700	
8	.1003	.1010	501.1057	-100003722.30	.01	.50	.20	.0000007	10.01	.005073205	.25277003	
9	.1003	.1010	501.1057	-100000705.20	.00	.50	.20	.0000007	22.00	.000000000	.25100215	
10	.1004	.1017	501.1057	-100000000.00	.01	.50	.20	.0000007	21.00	.000000000	.25250000	
11	.1000	.1000	501.1057	-100000000.00	.03	.50	.31	.0000007	25.70	.000000000	.25210000	
12	.1003	.1013	501.1057	-100000000.00	.00	.50	.37	.0000007	37.30	.000000000	.25200000	
13	.1007	.1000	501.1057	-100000000.00	.00	.50	.33	.0000007	20.00	.000000000	.25200000	
14	.1005	.1000	501.1057	-100000000.00	.05	.50	.20	.0000007	21.00	.000000000	.25200000	
15	.1000	.1000	501.1057	-100000000.00	.05	.50	.33	.0000007	20.17	.000000000	.25200000	
16	.1000	.1000	501.1057	-100000000.00	.07	.50	.15	.0000007	32.01	.000000000	.25200000	
17	.1005	.1000	501.1057	-100000000.00	.02	.29	.27	.0000007	20.23	.000000000	.25200000	
18	.1002	.1000	501.1057	-100000000.00	.03	.02	.20	.0000007	16.00	.000000000	.25200000	
19	.1000	.1000	501.1057	-100000000.00	.00	.70	.29	.0000007	23.05	.000000000	.25200000	
20	.1003	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	01.01	.000000000	.25200000	
21	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	35.05	.000000000	.25200000	
22	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	50.30	.000000000	.25200000	
23	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	00.00	.000000000	.25200000	
24	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	30.00	.000000000	.25200000	
25	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	00.00	.000000000	.25200000	
26	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	00.00	.000000000	.25200000	
27	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	50.51	.000000000	.25200000	
28	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	00.17	.000000000	.25200000	
29	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	50.77	.000000000	.25200000	
30	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	70.37	.000000000	.25200000	
31	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	05.77	.000000000	.25200000	
32	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	110.00	.000000000	.25200000	
33	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	120.00	.000000000	.25200000	
34	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	100.00	.000000000	.25200000	
35	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	100.00	.000000000	.25200000	
36	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	210.07	.000000000	.25200000	
37	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	230.07	.000000000	.25200000	
38	.1000	.1000	501.1057	-100000000.00	.00	.50	.30	.0000007	200.00	.000000000	.25200000	

ENERGY REMAINING/ENERGY INPUT = .00110
TOTAL PERMANENT SET = .10500
TOTAL NO. OF ITERATIONS = 0130
TIME SLICE MAXIMUM STRAIN
.0020300 1 .00000151

Fig. 8. Output of calculated variables for sample problem 1.

SLICE	WEIGHT	VOLUME	AREA	RADIUS	WEIGHT	MASS	RING 1	RING 2	RING 3	RING 4	CLADDING
1	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
2	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
3	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
4	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
5	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
6	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
7	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
8	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
9	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
10	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
11	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
12	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
13	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
14	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
15	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
16	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
17	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
18	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
19	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
20	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
21	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
22	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
23	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
24	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
25	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
26	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
27	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
28	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
29	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
30	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
31	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
32	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
33	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
34	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
35	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
36	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
37	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
38	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
39	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
40	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
41	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
42	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
43	.054	.054	.079	.398	6053.043	6.982	0	0	0	0	0.0000
TOTAL WEIGHT	2.0014										
TOTAL VOLUME	2.1197										
TOTAL WEIGHT	205553.0072										

1 = STEEL
 2 = CLAD LEAD
 3 = URANIUM
 4 = PONDEROSA PINE
 5 = LEAD
 6 = Balsa
 7 = MARINE PLYWOOD

Fig. 9. Initial geometry and masses for sample problem 2.

SAMPLE PROBLEM 2. END ON DROP WITH IMPACT LIMITER.

TIME = .0070500										
INPUT ENERGY = 1877773.23										
SLICE	THICK	D.R.	MASS	FORCE	STRAIN	RATE	ACCELERATION	VELOCITY	PERM SET	ENERGY
1	.0509	.5026	6.4820	-5088852.95	.6	.39	-.83	.050781	.00	.483900900
2	.0509	.5023	6.4820	-5085391.99	.9	.182	-.80	.054729	.02	.483539282
3	.0500	.5021	6.4820	-5072924.99	1.1	.03	-.15	.054680	.08	.483108402
4	.0500	.5019	6.4820	-5066579.37	1.8	.187	-.23	.054638	.19	.482658142
5	.0501	.5016	6.4820	-5079001.70	.5	-.1	-.26	.050577	.24	.482199886
6	.0501	.5013	6.167411	-5079331.09	.2	0	-.27	.054508	.23	.481358243
7	.1134	.3928	6.167411	-5081232.67	.3	.9	-.31	.081188	.29	.482938163
8	.1134	.3721	559.0975	-5047111.00	.3	4	-.34	.064210	.32	.483207921
9	.1134	.3662	541.3657	-5071833.02	.5	.5	-.39	.060219	.42	.483336575
10	.1134	.3662	541.3657	-5045122.96	.3	.8	-.43	.060209	.58	.483282228
11	.1134	.3662	541.3657	-5043500.20	.4	.5	-.47	.060202	.60	.483226791
12	.1134	.3661	541.3657	-5016537.80	.4	.10	-.51	.060196	.71	.483168842
13	.1134	.3661	541.3657	-4964412.86	.3	5	-.55	.060191	.81	.483114920
14	.1134	.3661	541.3657	-4909176.44	.5	.13	-.59	.060186	.93	.483067595
15	.1135	.3661	541.3657	-4910500.55	.3	.2	-.62	.060180	105	.482999123
16	.1135	.3661	541.3657	-4909128.51	.3	.3	-.66	.060177	117	.482967761
17	.1135	.3661	541.3657	-4894476.32	.3	.11	-.70	.060173	132	.482936473
18	.1135	.3661	541.3657	-4835025.35	.4	.2	-.71	.060169	138	.482875918
19	.1135	.3661	541.3657	-4821060.60	.1	1	-.76	.060162	158	.482812515
20	.1135	.3661	541.3657	-4830383.39	.2	1	-.79	.060156	168	.482761847
21	.1135	.3661	541.3657	-4834013.62	.1	.11	-.81	.060150	175	.482710296
22	.1135	.3661	541.3657	-4778292.63	.2	.0	-.85	.060144	184	.482637925
23	.1135	.3660	541.3657	-4776620.03	.1	.10	-.83	.060138	188	.482581639
24	.1135	.3660	541.3657	-4767472.19	.2	.9	-.85	.060131	197	.482507108
25	.1135	.3660	541.3657	-46657825.51	.2	.9	-.87	.060126	206	.482447770
26	.1135	.3660	541.3657	-4617564.33	.3	.15	-.91	.060121	221	.482386165
27	.1135	.3660	541.3657	-4512013.79	.1	.20	-.92	.060115	227	.482317449
28	.1135	.3660	541.3657	-4423732.61	.1	.27	-.92	.060110	238	.482233983
29	.1135	.3660	541.3657	-4282680.97	.1	.03	-.91	.060104	225	.482149179
30	.1135	.3660	541.3657	-4056280.69	.2	.61	-.89	.060099	215	.482036766
31	.1135	.3660	541.3657	-3729010.31	.5	.78	-.84	.060094	190	.481900980
32	.1135	.3660	541.3657	-3316070.96	.5	.68	-.79	.060089	167	.481773994
33	.1135	.3660	541.3657	-2997978.23	.5	.82	-.73	.060082	148	.481674725
34	.1135	.3660	541.3657	-2581360.26	.8	.59	-.68	.060081	124	.481536593
35	.1135	.3660	541.3657	-2251508.79	.4	.71	-.63	.060089	100	.481402929
36	.1135	.3661	541.3657	-1872174.37	.2	.44	-.61	.060151	99	.481386810
37	.1135	.3661	541.3657	-1683577.30	.5	.47	-.55	.060141	82	.4811714602
38	.1135	.3660	541.3657	-1386369.48	.1	.49	-.54	.060132	79	.4811556331
39	.1135	.3660	541.3657	-1125291.98	.4	.44	-.50	.060107	66	.4811269048
40	.1136	.3659	541.3657	-843288.06	.1	.48	-.48	.060076	63	.480928831
41	.1136	.3719	559.0975	-683486.67	.8	.39	-.48	.060081	52	.480729208
42	.1136	.3995	616.7411	-469810.99	.8	.45	-.43	.060086	57	.480666890
43	.1137	.3984	616.7411	-195667.87	.2	.32	-.41	.060050	51	.480389891

.332831887

ENERGY REMAINING/ENERGY INPUT = .00249

TOTAL PERMANENT SET = .332831

TOTAL NO. OF ITERATIONS = 23722

TIME SLICE MAXIMUM STRAIN

.0433058 1 .48586798

Fig. 10. Output of calculated variables for sample problem 2.

SLICE	HEIGHT	VOLUME	AREA	RADIUS	WEIGHT	MASS	RING 1	RING 2	RING 3	RING 4	CLADDING
1	.568	.272	.479	.390	345.895	34.710	0	0	0	0	0.00000
2	.114	.054	.479	.390	685.893	616.741	0	0	0	0	.01270
3	.114	.054	.479	.390	685.893	616.741	0	0	0	0	.01270
4	.114	.049	.430	.372	540.726	559.097	0	0	0	0	.01270
5	.114	.046	.421	.366	531.266	541.366	0	0	0	0	.01270
6	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
7	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
8	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
9	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
10	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
11	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
12	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
13	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
14	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
15	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
16	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
17	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
18	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
19	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
20	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
21	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
22	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
23	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
24	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
25	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
26	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
27	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
28	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
29	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
30	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
31	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
32	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
33	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
34	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
35	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
36	.114	.048	.421	.366	531.266	541.366	0	0	0	0	.01270
37	.114	.049	.430	.372	540.726	559.097	0	0	0	0	.01270
38	.114	.054	.479	.390	685.893	616.741	0	0	0	0	.01270
39	.114	.054	.479	.390	685.893	616.741	0	0	0	0	.01270
TOTAL WEIGHT		5.06614									
TOTAL VOLUME		2.1797									
TOTAL WEIGHT		285553.98691									

1 = STEEL
 2 = CLAD LEAD
 3 = URANIUM
 4 = PONDOROSA PINE
 5 = LEAD
 6 = Balsa
 7 = MARINE PLYWOOD

Fig. 11. Initial geometry and masses
for sample problem 3.

SAMPLE PROBLEM 3, END ON DROP, ONE SLICE FOR IMPACT LIMITER.
 TIME = .0072000
 INPUT ENERGY = 1000
 SLICE THICK 0.0 MASS FORCE
 1 .000 5021 30.0000 -491203.00
 2 .001 5013 416.7411 -4923303.51
 3 .002 5004 416.7411 -4978091.92
 4 .003 3721 591.0975 -4992762.34
 5 .004 3662 501.3657 -4977046.46
 6 .005 3662 501.3657 -4958042.94
 7 .006 3662 501.3657 -4940133.57
 8 .007 3661 501.3657 -4918913.08
 9 .008 3661 501.3657 -4901906.00
 10 .009 3661 501.3657 -4872761.49
 11 .010 3661 501.3657 -4825808.99
 12 .011 3661 501.3657 -4746716.05
 13 .012 3661 501.3657 -4702251.48
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 15 .014 3661 501.3657 -4693982.91
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 17 .016 3661 501.3657 -4635175.65
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 25 .024 3660 501.3657 -4272147.87
 26 .025 3660 501.3657 -4267781.95
 27 .026 3660 501.3657 -3801302.89
 28 .027 3660 501.3657 -3555025.61
 29 .028 3660 501.3657 -3220028.18
 30 .029 3660 501.3657 -2820028.18
 31 .030 3660 501.3657 -2595753.72
 32 .031 3660 501.3657 -1952867.46
 33 .032 3660 501.3657 -1652504.33
 34 .033 3660 501.3657 -1245746.77
 35 .034 3660 501.3657 -9960114.37
 36 .035 3718 591.0975 -667127.29
 37 .036 3904 616.7411 -499148.42
 38 .037 3904 616.7411 -190198.00
 39 .038 3904 616.7411 -190198.00

ENERGY REMAINING/ENERGY INPUT = .00274
 TOTAL PERMANENT SET = .332608
 TOTAL NO. OF ITERATIONS = 19098
 TIME SLICE MAXIMUM STRAIN
 .0030050 1 .42416419

PERM SET	VELOCITY	ACCELERATION	STRAIN RATE	ENERGY	STRAIN	STRESS
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.050547	-.27	32	.3	22.75	.003331500	-5317319
.000189	-.30	2	.3	27.00	.003690707	-10115149
.000249	-.30	3	.3	31.62	.004200114	-11206220
.000213	-.30	7	.3	40.02	.004270223	-11576081
.000207	-.30	2	.3	49.29	.004321030	-11640581
.000203	-.30	6	.3	57.09	.0043175101	-11589012
.000197	-.30	3	.3	67.19	.0043118307	-11437200
.000192	-.30	7	.3	76.62	.0043065234	-11394392
.000187	-.30	3	.3	87.72	.0043012592	-11359970
.000181	-.30	4	.3	98.11	.0042960255	-11328495
.000178	-.30	8	.3	117.30	.00429091399	-11298751
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.000157	-.30	4	.3	169.11	.0042719246	-11178751
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.000139	-.30	16	.3	203.92	.0042576970	-11088751
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.0000

Fig. 13. Stress-time history in the container element nearest the impact target for the sample problems.

divided into 40 elements, the number of elements was redefined to 39, and the thickness of the first element (the impact limiter) was redefined to 0.568 m (Fig. 6). Figures 11 and 12 are the output for this problem, and Fig. 13 shows the stress-vs -time plot for the second element of the container. The stress-vs -time plots for the corresponding container elements in sample problems two and three are essentially identical. This problem required 50 seconds of CDC 7600 computer time.

VI. TEMPORAL AND SPATIAL CONVERGENCE

A. Temporal Convergence

The 10^{-5} s default value for the time increment TDELTA has been found to produce temporal convergence for most problems. A few trial runs will enable the user to ascertain the effect of different values of TDELTA on a particular problem. It is important to realize that temporal convergence is not dependent on only the value of the time increment. For example, the difference between the element accelerations computed at the end of each iteration and the acceleration values assumed for that iteration must be less than or equal to 1% of the computed values for the program to converge. If this criterion is not satisfied after ten iterations, the acceleration values computed in the tenth iteration are assumed to be the solution for that time. The code indicates when convergence has not been attained after ten iterations by printing a message showing the results for the element that had the largest disparity. Some difficulty was encountered when the code tried to converge to very small values of acceleration occurring at elements remote from the impact end. The small values had little effect on the problem solution but did require more running time because of the increased number of iterations. This situation has been partially relieved by setting the two forces acting on an element equal to zero whenever a computed stress is less than a certain percentage of the preassigned maximum (STRSMX) stress for that material. The smallest stress allowed is presently 1% of STRSMX. The user can modify the code to have any percentage for any material.

B. Spatial Convergence

The lumped-mass mathematical model used in IMPAC2 is a discrete model formulation of a continuous system. The minimum number of discrete elements,

or lumps, necessary to accurately model the continuous system must be ascertained. The only way to determine that number for a given problem is to examine the spatial distribution of the dependent variables at various values of time. Spatial convergence, and hence an accurate model, will have been attained when an increase in the number of lumps does not produce any sensible change in the spatial distribution of the values of the dependent variables.

It is not usually necessary to examine all the dependent variables, because the spatial convergence of a particular variable will imply convergence of the others. Some impact analyses of shipping containers have used acceleration as the dependent variable on which to base spatial convergence. This was done because it was intended to use the maximum acceleration values to determine a pseudo-static inertial loading for a more accurate analysis of the outer shell. However, there is no theoretical maximum acceleration for these impact problems.

The point on a container that first contacts the rigid target is immediately brought to rest. This instantaneous change in velocity requires an infinite acceleration that will propagate along the length of the container. The code, however, will compute a finite value for the acceleration at the wavefront. The calculated magnitude of that acceleration will grow without bound as the thickness of the slices decreases by increasing the number of elements. Therefore, as the number of discrete elements is increased, the acceleration values occurring at the acceleration wavefront will grow without bound, and there is no meaning to the idea of a spatial distribution of maximum accelerations. Even a deformable target and the part of the container that first strikes it must instantaneously acquire the same velocity, producing an infinite jump in acceleration. A real container will experience a very high, but finite acceleration peak, which will be attenuated as it propagates. However, IMPAC2 does not contain these attenuating mechanisms.

The spatial distribution of the axial stress in the lead has been found to be the best dependent variable for ascertaining spatial convergence. Figure 14 shows the spatial distribution of axial stress in the lead 0.00101 s after initial end-on impact of a sample container. The container is 304.8 cm long by 62.0 cm in diameter with 1.02-cm-thick steel cladding. Spatial

Fig. 14. Spatial stress distribution
 at $t=1.01 \times 10^{-3}$ s.

convergence has essentially been attained with 100 lumps, but 30 lumps gives a reasonable solution.

The stress distribution shown in Fig. 14 has all the salient features this one-dimensional nonlinear wave propagation problem should have.⁷ These features can be checked by considering the axial stress-strain relationship for the lead. Neglecting strain rate, which has little effect on the values in Fig. 14, the axial stress-strain relationship for the lead is

$$\sigma = \frac{EH}{2R} \cdot \epsilon + 48.3 \times 10^6 \cdot \epsilon^{0.44} \text{ (Pa) } ,$$

where

σ = axial lead stress,
 ϵ = axial lead strain,
 E = modulus of elasticity of steel,
 H = cladding thickness, and
 R = mean radius of the cladding.

For this problem, $E = 20.7 \times 10^{10}$ Pa, $H = 1.02$ cm, and $R = 31.0$ cm. Substitution of these values in the above equation gives

$$\sigma = 34.1 \times 10^8 \epsilon + 48.3 \times 10^6 \epsilon^{0.44}$$

The leading stress amplitude propagates at a speed c_1 , corresponding to $c_1^2 = E_1/\rho$, where E_1 is the linear modulus in the $\sigma - \epsilon$ equation, and ρ , the mass-density of the lead, is 1.13×10^4 kg/m³. Hence,

$$c_1^2 = 34.1 \times 10^8 / 1.13 \times 10^4 , \text{ and} \\ c_1 = 550.0 \text{ m/s} .$$

At the time, $t = 1.01 \times 10^{-3}$ s (corresponding to Fig. 14) the leading wave-front should be

$$(550.0 \times 100)(\text{cm/s}) \times (1.01 \times 10^{-3} \text{ s}) = 55.6 \text{ cm}$$

above the impact surface. Figure 14 shows this wavefront to be 53.3 cm above the impact surface, which is good agreement.

The trailing stress plateau level of 34.7×10^6 Pa should propagate at a speed corresponding to the slope of the nonlinear part of the stress-strain curve at the stress difference above the leading wave. From Fig. 14, this stress difference is $34.7 \times 10^6 - 23.0 \times 10^6 = 11.7 \times 10^6$ Pa. From the nonlinear term in the constitutive relationship for lead, $11.7 \times 10^6 = 48.3 \times 10^6 \times \epsilon^{0.44}$, which gives $\epsilon = 0.0398$. The slope (E_t) of the nonlinear term at this strain is

$$E_t = \frac{d}{d\epsilon} \left\{ 48.3 \times 10^6 \times \epsilon^{0.44} \right\} \bigg|_{\epsilon = 0.0398} ,$$

which gives

$$E_t = 13.0 \times 10^7 \text{ Pa} .$$

The plastic wave speed (C_p) corresponding to $C_p^2 = E_t / \rho$ is 107.0 m/s. At $t = 1.01 \times 10^{-3}$ s, the plastic wavefront should therefore be $c_p \cdot t = 10.8$ cm above the impact surface. From Fig. 14, the value is 9.14 cm, which is good agreement. Based on these considerations, it is concluded that IMPAC2 gives a very good mathematical solution for the clad-lead model.

The pressure created between the lead and the outer shell can be used to analyze and design the shell and the end plates. The code prints the maximum value of these radial pressures in the column headed PRESSURE for each slice during a run. These maximum pressures can then be used as a radial loading for a pseudo-static analysis of the outer shell and its end plates. A dynamic shell analysis can be performed if the user wants to modify the IMPAC2 output in order to give the time history of the radial pressure at each slice and then use those pressures as temporal excitation for the shell.

VII. SUMMARY

A shipping container impact problem is a complex, nonlinear, three-dimensional situation. IMPAC2 represents an attempt to identify the most important features of such problems and cast them into a one-dimensional analysis. The accuracy of the results obtained with the code can be no better than the one-dimensional mathematical model.

The most important limitation of the mathematical model is the treatment of the outer shell as a cylindrical membrane. This model precludes direct calculation of the time-dependent bending and axial strains. Hence, there is no way to compare experimental strain gauge measurements with calculated values in order to substantiate the analysis. This situation is typical of some of the features not included in the one-dimensional analysis.

A new and more accurate mathematical model is being developed for these problems, but IMPAC2 is presently the only inexpensive and relatively simple analysis tool available. Every effort will be made to retain the low cost and simplicity features of IMPAC2 in the new mathematical model.

Even though IMPAC2 has limitations, estimates can be made of quantities pertinent to preventing the occurrence of the principal container failure modes. First, the thickness of the lead after impact is obviously important for shielding considerations. IMPAC2 can be used to give fairly accurate estimates of permanent lead deformation.

Another failure mode is the possibility of an opening occurring in the outer shell. In a fire, the lead could melt and escape resulting in loss of shielding. The stretching of the outer shell computed with IMPAC2 does give some indication as to whether or not the shell is thick enough to prevent tearing of the shell itself or welds. The pressure between the lead and the outer shell, which is computed in IMPAC2, can be used as a loading for a separate analysis of the shell and its end plates. However, there is presently no analytical or experimental evidence of how accurate this procedure would be.

It was not felt that an attempt should be made to present detailed design procedures for use with the approximate analysis represented by this code. This report has attempted to present the assumptions, limitations, and construction of IMPAC2 clearly enough that an engineer can make rational use of the results.

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