SUPERCONDUCTING COIL ENERGY
LOSS MEASUREMENT PROGRAM

Final Report
for Period October 14, 1975-April 13, 1976

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William F. B. Punchard
Paul M. Margosian

MAGNETIC CORPORATION OF AMERICA
179 Bear Hill Road
Waltham, Massachusetts 02154

June 1978

Prepared for
THE U.S. DEPARTMENT OF ENERGY
UNDER CONTRACT NO. E(11-1)-2813

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FOREWORD

This report was prepared by Magnetic Corporation of America, 179 Bear Hill Road, Waltham, Massachusetts 02154 under U.S. Energy Research and Development Administration Contract No. E(11-1)-2813, October 14, 1975.

The Work was administered under the direction of the Department of Energy, Chicago Operations Office. Mr. M. Chomiak and Mr. L. Parys served as Contract Specialists for this program. Mr. D. Beard of the Office of Magnetic Fusion Energy, Magnetic Systems Branch, Washington, D. C. served as Technical Monitor.

The report describes work performed during the period October 1975 to April 1976 by E. J. Lucas, W. F. B. Punchard and P. M. Margosian of Magnetic Corporation of America.

ABSTRACT

A description is given of an apparatus that makes use of an electronic technique for measuring losses in rapidly pulsed superconducting coils. Data taken with this apparatus on a coil having an energy storage capability of approximately 7 kJ is presented.

The apparatus described is essentially an electronic wattmeter. It makes use of magnetically coupled coil to cancel out as nearly as possible the inductive portion of the coil voltage during charge and discharge, thus leaving a voltage which is due almost entirely to the coil energy losses.

This voltage is then integrated and displayed along the vertical axis of an X-Y oscilloscope, the horizontal axis being the coil current. The area of the figure so formed is proportional to the energy dissipated during the cycle.

Two different techniques for sensing the inductive component of the coil voltage are described together with data taken by each technique. Data are presented for full cycle hysteresis loops at frequencies of 11.7, 16.2, and 26.2 Hz and for half cycle hysteresis loop at discharge rates of 54 and 348 amperes per second and several charge rates. These data show the coil losses to be higher than those predicted by a simplified theory taking into account magnetization losses only.

The specific apparatus described in this report is useful in making measurements over a limited range of operations and coil sizes, however, the technique described can be utilized for coils of virtually any size and over virtually any operational range.

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1. INTRODUCTION

Many applications of superconducting materials involve situations where the superconductor is subjected to transient magnetic fields. Under such conditions heat is generated in the superconductor which may adversely affect the performance of the device. A.C. losses, therefore, have great impact on the design of any superconducting magnet system involving transient magnetic fields and it is of the utmost importance to be able to predict the magnitude of these losses. It is presently not possible to predict these magnitudes to better than about a factor of two for any but the simplest of conductor geometries. Until such time, therefore, that A.C. loss theory becomes much more accurate, it is important that there be a means of accurately measuring A.C. losses. The predicted A.C. losses can then be checked by making measurements on small quantities of conductors before full-scale production begins.

The prime objective then of the program which this report describes was to develop a technique capable of accurately measuring A. C. losses in superconducting coils subject to transient fields. This development program was to include complimentary theoretical and experimental studies; a device was built and tested and the results of the tests compared with the results of some A.C. loss theory.

The most straightforward method of measuring A.C. losses is to measure the rate of boiloff of helium as the coil is charged and discharged. While this method is in principle very simple, it is in practice restricted to fairly long-term (typically minutes) repetitive experiments. An alternative technique, which is capable of measuring the losses during a single charge-discharge cycle of the coil, consists of displaying the hysteresis loop for the superconducting coil on an oscilloscope by means of appropriate electronic equipment. The area inside this loop is proportional to the energy loss in the coil.

This electronic method, first reported on by Wilson¹, was developed by Magnetic Corporation of America for use in measuring losses on high voltage superconducting inductive energy storage magnets. Such a device was used in a U.S.A.F. contract performed by Magnetic Corporation of America for the development of light-weight pulsed superconducting energy storage systems.^{2,3} During that program, reasonably good correlation was obtained between the measured losses and those predicted theoretically. However, complications such as the presence of a metal dewar precluded the possibility of any comprehensive testing.

Of the two methods for measuring losses, the electronic technique is more practical for experimental purposes than the boiloff method because it makes possible the evaluation of a coil by means of a number of single cycles instead of by repetitive operation.

II. THE ELECTRONIC TECHNIQUE AND APPARATUS

A. THEORY

To understand the electronic technique for measuring coil losses, it is perhaps simplest to begin by noting that the voltage appearing across the terminals of the coil consists of an inductive part, $L di/dt$, and a resistive part, Ri . The product of the inductive component of the voltage and the current is the rate at which energy is being stored or removed from the magnetic field of the coil. The product of the resistive component of voltage and the current is the rate at which energy is being dissipated by whatever loss mechanisms are present. If the resistive loss is integrated over the time required for one charge-discharge cycle of the coil, the result must be the energy dissipated during that cycle, which is the desired quantity.

In principle, integration of the total signal (resistive and inductive components) over a cycle gives the net energy dissipation since the inductive component integrates out to zero leaving only the resistive component. However, in practice it is helpful to remove the inductive component of voltage before performing the integration because it is 1,000 to 10,000 times larger than the resistive voltage. This, of course, greatly reduces the range over which the integrator must operate and also reduces the risk to equipment and personnel of high voltages. This can be achieved by producing a purely inductive voltage by means of a secondary winding on the coil or by means of a Rogowski coil (See Appendix G) around a current lead to the coil and subtracting it from an appropriate fraction of the voltage on the terminals of the coil. If the inductive component of the coil voltage is nulled in this fashion, the remaining voltage is purely resistive. If this voltage is integrated and fed to the vertical input of an oscilloscope while a voltage proportional to the coil current is fed to the horizontal input of the oscilloscope, the result is a figure which bounds an area that is proportional to the energy dissipated in the coil during a cycle. A mathematical derivation of the theory of this technique is presented in Appendix A. It shows that the area bounded by the figure on the oscilloscope is proportional to the energy loss in the coil. A schematic of an appropriate experimental setup is given in Figure II-1.

The basic components of the loss measurement device then are: a bucking coil of some kind, a resistive divider, a current shunt, and an integrator.

B. LIMITATIONS ON THE COMPONENTS

In order to perform a reliable loss measurement by using the technique discussed above, it is necessary to examine the specifications that must be met by these components of the basic electric circuit (shown in Figure II-1). In the analysis of component errors presented in Appendix B, it is shown that the error in the loss loop measurement depends upon integrator drift and on the product of the uncancelled inductive voltage with the phase shift error of the integrator. It is necessary that both the equivalent drift voltage (input dc voltage that would produce the observed drift with an ideal integrator) and the product of the uncancelled inductive voltage with phase

shift error of the integrator be small compared with the loss voltage which can, in some cases, be as small as one millivolt. It should be noted that it is also necessary to have an accurate current measurement which is free of phase shift errors.

Because the inductive voltage is typically 10^4 times larger than the loss voltage, it is necessary that the resistive divider pictured in Figure II-1 have a resolution of at least one part in 10^4 in order to produce a useful loss loop. If it is assumed that the uncanceled inductive voltage is about the same magnitude as the loss voltage, then it is necessary that the phase shift error of the integrator be no greater than 10^{-2} radian in order to make the phase shift contribution to the measurement error negligible. This restriction on the phase error of the integrator requires that the operational amplifier contained in the integrator (Figure II-2) have an upper frequency limit at least an order of magnitude above the highest operating frequency contemplated for the experiment (about 2 kHz in our application), an open loop gain of at least 10^4 to allow an integration time of ten seconds with a time constant of 33 milliseconds and a maximum phase shift of 10^{-2} radians, and have an input impedance large compared to the input resistor of 330 k Ω (Figure II-2). The performance requirements of the current shunt, the voltage divider, the integrator, and the overall circuitry layout are summarized in Table II-1.

C. THE CURRENT SHUNT

The design of the current shunt, which is constrained by the requirements for low inductance and for resistance stability as a function of frequency and current, is discussed in detail in Appendix C. A schematic is given in Figure II-3 and a summary of its electrical characteristics is given in Table II-2. From Table II-2 it can be seen that this current shunt fulfills the requirements summarized in Table II-1.

D. THE VOLTAGE DIVIDER

The design of the resistive divider, which is constrained by the requirements for high resolution, compatibility with the input impedance of the integrator, and resistance stability as a function of coil output voltage, is discussed in detail in Appendix D. A schematic of the version of this divider that was used with the 7 kJ coil is given in Figure II-4. The overall divider resistance of 59,000 ohms was determined by a trade-off between compatibility with the integrator and thermal stability; the selection of a large number of discrete components was dictated by the high voltage requirement as is explained in Appendix D. With this two-stage design, a maximum resolution of one part in 10^5 is available (one part in 10^4 from the resistor string and one part in 10^3 from the ten turn pot). It is clear that this fulfills the resolution requirements set forth in Table II-1.

E. THE INTEGRATOR

The requirements that must be satisfied by the integrator to ensure an accurate loss measurement led to the selection of a circuit with the minimum number of components, as shown in Figure II-5. Protection against high voltage transients, such as might occur during a quench (e.g., the coupling ratio between the storage coil and the sense coil might change thus disturbing the null of inductive voltage by a large amount), was accomplished by the use of a set of zener diodes. On examination of the entire instrumentation circuit (Figure II-2) it is clear that the maximum current that can flow through the protection circuit is limited by the resistance of

Block Diagram of Loss Measurement Circuit

Figure 11-2

TABLE II-1

COMPONENT REQUIREMENTS FOR ACCURATE LOSS MEASUREMENT

Current Shunt

Must have phase shift of less than 10^{-2} radians up through 2 kHz. Resistance must be independent of current up to 1,000 amps (i.e., no significant heating).

Voltage Divider

Must be able to stand up to 25 kV (pulsed) without arcing. Must have resolution of at least one part in 10^4 , one part in 10^5 preferred.

Integrator

Must be protected against input voltage transients. Must have phase shift error of less than 10^{-2} radian for integration times from 10 seconds to 0.5 ms; this requires an open loop gain greater than 10^4 and a frequency response greater than 20 kHz (at least 100 kHz preferred). Must have equivalent dc input voltage (drift) no greater than a few tens of microvolts. Must have high input impedance.

Circuitry

Isolation techniques selected must preclude any fraction of the voltage drop in the cables between the coil and the current shunt from entering the integrator.

Dimensions in Inches

Schematic Illustrating Noninductive Current Shunt

Figure II-3

TABLE II-2

CURRENT SHUNT CHARACTERISTICS

Material.....	321 Stainless Steel
Dimensions.....	Inner Cylinder-2.625" OD, 0.120" Wall Length 5" Outer Cylinder-3.000" OD, 0.120" Wall
DC Resistance.....	$R_{dc} = 0.28 \text{ m}\Omega$
DC Voltage.....	$V_{dc} = 560 \text{ mV}$ at 2,000 A
Skin Depth.....	$\delta = 0.78 \text{ inch}$ for 2 ms discharge $\delta = 0.25 \text{ inch}$ for 0.2 ms discharge
High Frequency Resistance.....	$\frac{R_{hf}}{R_{dc}} - 1 = 5.1 \times 10^{-5}$ (0.0051%) for 2.0 ms discharge $\frac{R_{hf}}{R_{dc}} - 1 = 5.1 \times 10^{-3}$ (0.51%) for 0.2 ms discharge

InductanceLow Frequency ($\omega = 0$)

$$L = 3.09 \times 10^{-9} \text{ H}$$

High Frequency ($\omega = \infty$)

$$L = 1.42 \times 10^{-9} \text{ H}$$

Temperature Rise - $\Delta T = 18 \text{ K}$ and $\frac{\Delta R}{R} = 1.9\%$ for 20 s charge

Voltage Divider to Provide Accurate Null
of the Inductive Voltage. The Voltage
Applied to the 10-Turn Pot is 1.5 Percent
of the Total Inductive Voltage

Figure II-4

Circuit Diagram of Integrator

Figure II-5

the voltage divider; for a 60 kV transient, the maximum current through the 59,000 ohm divider (Figure II-4 is one amp, a level which is well within the capabilities of the components selected. It was determined by experiment (obtaining a loss loop with and without the protection circuit installed) that the insertion of these zener diodes did not introduce measurable phase shift or amplitude errors into the integrator circuit. Resetting of the integrator was accomplished by means of a single pole, single throw mechanical (toggle) switch which controlled a Reed relay switch. In order to minimize drift, polycarbonate capacitors were used; these are among the lowest leakage ($< 10^{-10}$ amps) and most stable types commercially available. Temperature stable resistors and low noise cable were also used. In addition, a two-stage design was selected for the amplifier balance circuit which provides a resolution of at least one part in 10^6 . The input resistance of 330 k Ω was the smallest value that could be used with the largest readily available polycarbonate capacitors (3 microfarads) to produce an integrating time constant of one second (the largest needed for our experiments). It is necessary that this input resistor be as small as possible in order to minimize the long-term drift of the integrator.

Previous experience with the electronic loss measurement technique has shown the integrator to be the most troublesome system component. In particular, slow dc drifts of the amplifier were the most difficult to compensate. During these previous experiments, adequate performance was finally obtained by using an elaborate compensation network with an unusually stable, high impedance operational amplifier. To avoid these complications during this program, a chopper stabilized amplifier was used. The characteristics of this operational amplifier (Harris #2905-5) are given in Figure II-6. This device has an open loop gain of 75,000 or more, an input impedance greater than 10^{12} ohms, a frequency limit of 500 kHz and low leakage currents. The integrator fulfills the phase shift, frequency response, drift, and gain requirements of Table II-1. Originally, it was planned that this device be temperature-stabilized to ± 1 K by placing it in a component oven. However, this was found to be unnecessary.

The final set of considerations that affect the loss measurement is the physical arrangement of circuit elements. It is essential that the voltage entering the integrator contain no resistive elements other than those from the coil itself. Because of this requirement, the voltage across the coil is picked off by a pair of wires attached directly to the conductor where it emerges from the coil; this avoids measuring any portion of the power loss in the nonsuperconducting leads attached to the coil.

F. THE ISOLATOR CIRCUIT

It is necessary to isolate the current shunt by means of a differential amplifier with a very high common mode rejection ratio, or by some equivalent technique, in order to avoid picking up a significant fraction of the voltage drop on the power lead between the coil and the shunt. (See Figure II-7 which is a schematic showing the physical relationship between the circuit components.) During rapid coil discharge the situation is aggravated by any cable inductance. In this case the cable resistive drop was one volt and the inductive drop four volts at 1,000 amperes and a discharge time of 1 millisecond.

- Offset Voltage Drift..... $0.2\mu\text{V}/^{\circ}\text{C}$
- Offset Current Drift..... $1\text{pA}/^{\circ}\text{C}$
- Open Loop Gain..... 5×10^8
- Bandwidth..... 3MHz
- Slew Rate..... $2.5\text{V}/\mu\text{S}$
- True Differential Inputs

Characteristics of Operational
Amplifier Harris 2905-5

Figure II-6

Schematic of the Isolator Circuit

Figure 11-7

If isolation is to be achieved using a differential amplifier then the common mode rejection ratio must be very high because the A.C. loss signal voltages can be as low as one millivolt; even a very small fraction of the four volts would be enough to severely disturb the experiment. Similarly, because of the great sensitivity of this measurement to small error voltages the oscilloscope and the integrator were both operated from isolation transformers, and great care was taken to avoid ground loops.

During a previous series of experiments³ isolation was provided by this means, but the arrangement was not entirely satisfactory. Therefore, an alternative, more reliable means using optical isolators was developed during the course of this program.

The basic requirements for the optical isolator are high speed and linearity. A survey of available devices led to the selection of the Monsanto MCD-2 and MCD-4 (same device, different packages), data sheets for which are included in Appendix E. These devices are usable to frequencies above one megahertz, have a maximum nonlinearity of ± 5 percent over a wide operating range, and are readily available (especially MCD-4). Their primary disadvantage is that the characteristics are quite sensitive to temperature.

It was decided to improve the device linearity by means of a feedback technique. If one of these devices is placed inside the feedback loop of an operational amplifier its output will be forced to be a linear function of the input. If two well-matched devices are driven by the same current and if one is feedback stabilized, the other will also be stabilized because of its similar properties. The remaining nonlinearities will be a measure of the difference in the device characteristics and in their temperatures.

The measured characteristics of the three devices purchased are given in Figure II-8. As can be seen, two of these devices are closely matched although the fact that they are in different packages (MCD-2 and MCD-4) means that they will operate at slightly different temperatures. The two closely matched devices were used in the stabilizing circuit, a schematic of which is shown in Figure II-9. The operational amplifier used, Raytheon #4136, is described by a data sheet included in Appendix E. A schematic of the two ± 15 -volt power supplies used is given in Figure II-10. The regulators used, Motorola #MC1468, are also described by a data sheet included in Appendix E. The output current of the stabilized optical isolator (MCD-2) is shown in Figure II-11. The deviation of this curve from a straight line is about 0.5 percent over the input range corresponding to $\pm 1,000$ amps. For the complete circuit (Figure II-9) this output range corresponds to ± 6 volts. After a five-minute warmup period (room temperature), the thermal drift is about 0.1 volt per half-hour. This thermal drift could be eliminated by immersion in an ice water bath or by use of a component oven, although this was deemed unnecessary for the purposes of the experiments run under this program.

Transfer Functions of Optical Isolators, Monsanto MCD-2, MCD-4

Figure II-8

Stabilized Optical Isolator Circuit to be Used for Current Measurement. Output is ± 6 volts for $\pm 1,000$ amps input. Linearity is 0.5%. Output thermal drift is 0.1 volts per half hour. The separate ± 15 -volt power supplies are denoted by $\pm V_1$ and $\pm V_2$.

Figure II-9

Schematic of Isolated ± 15 -volt Power Supplies Used for Optical Isolator Circuit

Figure II-10

Figure II-11. - Output Current of Stabilized Optical Isolator Circuit

The response of this circuit to a square wave input is 20 microseconds (both rise and fall times). This limitation, imposed by the operational amplifier selected, corresponds to a maximum frequency of 12 kHz. A higher frequency version of this circuit would require the use of a higher speed operational amplifier (e.g., Fairchild μA 715) and a faster drive transistor to replace the D4101. Because the Rogowski coil had already imposed a more severe frequency limit of about a few kilohertz on these experiments, the circuit shown in Figure II-8 was used.

G. INDUCTIVE VOLTAGE BUCKING COILS

As previously explained, it is desirable to remove as much of the inductive component of voltage as possible before integrating. There are two relatively easy ways that this can be achieved; first by winding a secondary coil around the outside of the test coil and second by placing a Rogowski coil around one current lead.

At first sight, it is much easier to obtain a signal large enough to handle by using the secondary coil wound around the main coil since the coupling between the two coils is high. However, as can be seen by reading Appendix F, Analysis of Electronic Measurement of Losses by Use of Secondary Sense Coil, in reality the loss voltage signal is less than would be at first expected. Furthermore, this appendix shows that the interpretation of the hysteresis loops is much more complex than the interpretation of those obtained using a Rogowski coil, and that a detailed knowledge of the field distribution in the windings of the coils is also required. In practice, too, there are difficulties since the test coil changes shape because of electromagnetic forces acting on it during a charge and discharge cycle. This change in shape causes a change in the mutual inductance between the main coil and the secondary coil and leads to distortion of the hysteresis loop. (Both of these effects were observed during the experimental phase of the program.)

The Rogowski coil does not suffer from either of these major disadvantages. The theory associated with the interpretation of the oscilloscope traces is not complicated and in practice, the hysteresis loops obtained are free from distortion due to changing mutual inductances (the relative geometry of the Rogowski coil and the current lead does not change during the course of a cycle). Use of a Rogowski coil also leads to a more versatile instrument since, in principle, it is easier to fit a Rogowski coil around the current lead of a magnet whose A.C. losses are to be measured than to wind a secondary coil around it. However, there are difficulties associated with Rogowski coils; these difficulties involve obtaining a large enough A.C. loss voltage signal and at the same time avoiding phase shift, extraneous noise, etc.

Experiments were conducted, using both of these bucking coil methods (described in Section IV of this report). The secondary sense coil was already in existence and has been described elsewhere³. However, for the convenience of the reader, the pertinent details are summarized in Appendix H. (The "7" kJ Energy Storage Coil and its Associated Secondary Sense Coil.) A suitable Rogowski coil, however, was developed and built during the course of the program described here. The Rogowski coil is constrained by the same limitations on sensitivity, phase shift, and freedom from extraneous signals as all other components of the system. For long

(tenths of a second) switching times, a Rogowski coil with many turns (high sensitivity) is required; for short switching times (milliseconds) a coil with fewer turns is required, so as to avoid dangerously high voltages. Furthermore, the coil must be as near as possible to a perfect torus so as to minimize pickup of stray signals. Finally, there is a frequency beyond which phase shift, due to distributed capacitance, becomes significant. (This frequency limit can generally be avoided if the total length of wire in the coil is less than a quarter of a wavelength of the highest frequency to be encountered.) This particularly important sensing element requires tailoring for particular needs which, in our case, included maximum sensitivity consistent with a frequency response of at least 20 kHz with a minimum of spurious signal pickup. High sensitivity requires a large number of turns which, in turn, limit frequency response (because of distributed capacitance) and enhance stray signal pickup. Two somewhat different approaches were considered. These involved air core and ferrite core Rogowski coil designs.

Two prototype designs were built and tested, one being an air core design and the other a ferrite core design. After they were evaluated, a final air core version was built and tested. The prototype air core coil of 4,000 turns was wound on a toroidal form with inside diameter of 2.5", outside diameter of 6", and height of 4". This exhibited a sensitivity of one volt for a rate change of current of 11,000 amps per second; the spurious noise level was about 20 millivolts. An electrostatic shield which reduced the noise level by more than a factor of ten was added. The output signal level was increased by a factor of ten by passing the high current conductor around the coil 13 times. This technique caused no significant changes in the rest of the (experimental) circuit because the inductance added by the Rogowski coil primary (4 microhenries) was much less than the inductance of the coil to be tested (37 millihenries). The frequency response of this coil was at least 20 kHz. Thus, the primary disadvantage of an air core design, poor coupling, can be overcome by passing the primary current lead around the core several times (i.e., by making the Rogowski coil into a small transformer). The design details of the prototype air core Rogowski coil are summarized in Figure II-12.

Because it is desirable to use the smallest possible number of turns (to reduce spurious noise) consistent with good sensitivity, a ferrite core design was worked out. Good quality ferrite materials do not exhibit significant losses until well past 20 kHz. However, their properties, especially permeability, vary with temperature and with core magnetization. The simplest method for linearizing the core properties is to use an air gap of appropriate size. For example, if a core had a permeability of 2500 which varied by 30 percent over a range of temperatures and fields, then adding a sufficiently large (linear) air gap to reduce the average permeability to 25 would reduce the nonlinearity to 0.3 percent or less. A limit on ferrite cores is imposed by the condition that saturation not be approached at the highest current. A commercial ferrite core was obtained and fitted with a 1,000 turn secondary and a single turn primary. The characteristics of this coil were measured and are summarized in Figure II-13. The primary disadvantage of the ferrite core design, nonlinearity of the core material with respect to both temperature and primary current, can be overcome by inserting an air gap of appropriate size. The very small (one square inch core cross section) ferrite core coil that was tested remained linear out to about 200 amperes (see Figure II-14, a level at which the core approaches saturation. We decided that the final version should be an air core model and proceeded to design and build. However, we did give some thought as to

AIR CORE ROGOWSKI COIL--EXPERIMENTAL

- (a) Secondary Winding Details: I.D. = 2.5", O.D. = 6", Height = 4",
 N = 4000 Turns, R = 625 Ohms, L = .306 Henries. Primary,
 N = 13 Turns, R = 4.5 Milliohms, L = 4.63 Microhenries,
 Mutual Inductance, M = .95 Millihenries.
 Secondary Winding Capacitance = 86 pF, Self Resonant Frequency = 31 kHz.

- (b) Overall Winding Details: Shield attenuates electrostatic noise by 20:1;
 secondary winding geometry attenuates magnetic pickup by 40:1.

Figure II-12

Ferrite Core Rogowski Coil--Experimental

Ferrite Core Details: Air gap selected to provide effective μ of 280. Maximum flux density about 5 kilogauss. Self inductance of 1000 turn secondary, 718 millihenries; self inductance of single turn primary, about 1 microhenry, Mutual inductance, $M = 0.7$ millihenry. Self resonates at 17 kHz; secondary capacitance = 115 pf. Maximum primary current about 300 amperes.

To increase current capacity increase air gap length and core cross section by same ratio. For example, to get a current capacity of 1200 amperes increase the air gaps to .060" and the core cross section to 4 square inches. Four of these cores in parallel would be a practical implementation.

Figure 11-13

Ferrite Core Coil--Variation in μ With Current

Figure II-14

what such a ferrite core coil would look like. The most likely candidate for such a design would be a set of four of the same cores that we tested (Figure II-13) placed side by side to give a core cross section of four square inches. The air gaps would be increased to .060 inch. The primary (input) would be a cable (e.g., 3/0 A.W.G.) passed through the core window. The output winding would be about 700 turns of #28 A.W.G. wire. A ferrite core coil would require electrostatic and magnetic shielding of the same quality as that needed for the air core design.

The results of our thinking concerning the design of Rogowski coils are summarized in Appendix G, Rogowski Coil Design Considerations.

The final version of the air core Rogowski coil was very similar to the prototype Rogowski coil described above. The main difference was that the central hole was enlarged to accommodate a larger number of primary turns. The important characteristics are summarized in Table II-3. Figure II-15 is a schematic (roughly to scale) showing the Rogowski coil in the experimental steel drum. This drum served the dual purpose of providing magnetic shielding and containing liquid nitrogen. The liquid nitrogen was not an essential component of the system but was convenient because it reduced the resistance of the Rogowski coil primary from 14 m Ω to 1.88 m Ω . This reduced the ohmic heating and eliminated the need to wait between runs while the Rogowski coil primary cooled.

TABLE II-3

DETAILS OF FINAL VERSION OF AIR CORE ROGOWSKI COIL

Secondary Winding

Inner Diameter.....	= 3.0 inch
Outer Diameter.....	= 6.0 inch
Height.....	= 4.0 inch
Number of Turns.....	= 4,000 (#28 AWG wire heavy armored poly-thermaleze)
Resistance (Room Temperature).....	= 248.4 Ω
Self-Inductance.....	= 252 mH
Self-Resonant Frequency.....	= 31 KHz (electrostatic shield grounded)
Self-Resonant Frequency.....	= 26 KHz (electrostatic shield grounded)

Primary Winding

Inner Diameter.....	= 2.0 inch
Outer Diameter.....	= 7.0 inch
Height.....	= 6.0 inch
Number of Turns.....	= 40 (6 x #10 AWG wire)
Resistance (Room Temperature).....	= 14 m Ω
Resistance (LN ₂ Temperature).....	= 1.88 m Ω
Self-Inductance.....	80 μ H

Mutual Inductance Secondary/Primary = 2.16 mH

Schematic Showing Rogowski Coil in Electrostatic and Magnetic Shields

Figure II-15

III. OTHER EXPERIMENTAL APPARATUS

A. NON-CONDUCTING DEWAR

1. Introduction

One drawback associated with measuring losses electronically is that the losses measured by tapping the voltage across the coil terminals are those due not only to the coil itself, but also to any dissipative mechanism coupled to the coil. Consequently, it is not possible to accurately measure the losses due to a superconducting coil if it is tested in a closely coupled metal dewar at frequencies high enough to cause significant eddy current losses in the dewar walls. During a previous program^{2,3}, the primary obstacle to the performance of an accurate electronic loss measurement on the 7 kJ energy storage coil was the presence of the metallic dewar which behaved like a shorted single turn secondary winding of a transformer. There are added complications too, especially if the means of bucking out the inductive voltage is a secondary coil wound around the test coil. This is because the dewar wall acts like a magnetic shield, causing the self inductances of the test coil and of the sense coil (and hence their mutual inductance) to vary with frequency.

Since one object of this program was to compare measured and theoretical energy losses as a function of frequency, it was decided to completely eliminate the complications described above by using a wholly non-conducting dewar. The dewar used in the program is described below.

2. Design

Enquiries were made concerning the commercial availability of non-conducting dewars and it appeared that such dewars could indeed be procured. However, further investigation concerning the method of manufacture of such dewars indicated that the dewar structure is not completely non-conducting. A radiation shield was incorporated which was made from "coil-foil". "Coil-foil" is the trade name for mylar sheet which has numerous thin copper strips attached to it. Each strip is about 6×10^{-3} inches thick and 62×10^{-3} inches wide. Strips run parallel to each other and are separated by a distance of about 15×10^{-3} inches. The "coil-foil" is in the vacuum space and is attached to the inner wall of the dewar near the dewar top. The "coil-foil" cools by conduction down the copper strips and thereby provides a radiation shield.

Although there is metal in this type of non-conducting dewar, it is dispersed in such a way that the losses due to eddy currents circulating in it are minimized. However, in a program the purpose of which is to develop a technique capable of making accurate loss measurements, it is extremely important that the number of possible disturbing influences on loss measurements be kept to a minimum. Therefore, alternative means of obtaining a cryogenic container were investigated.

Eventually a very simple construction which is completely non-conducting was chosen and the dewar was built in-house. It consists of two concentric G10 tubes with appropriate G10 end plates. Joints were made by using epoxy resin. The space between the tubes was evacuated and the inner tube became the liquid helium container.

To reduce radiation losses a second non-conducting container was built to hold liquid nitrogen. The inner and outer walls of this liquid nitrogen container were made of thin G10 sheet. The thermal insulation between them was a closed cell polyurethane foam which was foamed in place. The end plates of the vessel were made from linen phenolic. The helium vessel sits inside the nitrogen vessel as shown in the schematic in Figure III-1.

Since the surfaces of the G10 are not highly polished, the surface emissivities are high (approaching that of a black body in fact) and from a heat leak standpoint it is an inefficient design. However calculations indicated that, assuming an emissivity of unity, the radiation heat leak would be equivalent to between two and three litres per hour of liquid helium with an additional one and a half litres per hour due to conduction. Thus it was expected that the total heat leak due to radiation and conduction down the dewar walls would be about 3.5 to 4.5 litres per hour. This was higher than desirable but was acceptable in view of the absence of disturbing influences on loss measurements.

3. Performance

It was found that the outer foam-insulated liquid nitrogen container held liquid nitrogen satisfactorily. One or two cold spots were observed on the outer surface, but they were not big enough to cause any problems.

The inner G10 walled vacuum vessel was first tested at room temperature and was found to be vacuum tight. The lowest pressure reached after a few hours of pumping was less than 5 μ Hg. This residual pressure was attributed not to leaks but to outgassing from the vessel walls. A mass spectrometer on the dewar pump-out line gave no indication of a helium leak in the epoxy joints nor of helium permeation through the G10 walls when helium gas was directed to the inner and outer surface of the vessel.

The system was then tested at liquid nitrogen temperature. The empty inner container (liquid helium vessel) was placed in the outer container (liquid nitrogen vessel) and liquid nitrogen was poured between the two. There was no indication of any leaks and the pressure in the vacuum space of the inner container remained at less than 1 μ Hg. The nitrogen was emptied from the outer container and the inner container was filled with liquid nitrogen. Again there was no indication of leaks. Finally both the inner and outer vessels were filled with liquid nitrogen without the vacuum integrity being impaired. Helium gas was bubbled through the liquid nitrogen in the inner container. A mass spectrometer connected to the vacuum pump out port on the inner vessel gave no indication of helium diffusing through the walls.

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Schematic Illustrating Construction of Non-Conducting Dewar

Figure III-1

The only difficulty encountered during these tests was the tendency of the inner vessel to float in the liquid nitrogen contained by the outer vessel. This was remedied by securely anchoring the inner vessel to the outer by means of three stainless steel cables attached to the flanges of the vessels. Twenty-four hours before loss measurements tests were conducted the coil and dewar system were precooled to liquid nitrogen temperature by filling both inner and outer vessels with liquid nitrogen. Just prior to filling with liquid helium on the day of the tests, the liquid nitrogen in the inner vessel was removed by pressurizing the gas over the surface of the liquid nitrogen. This is an important step for two reasons. First, if any liquid nitrogen remains in the bottom it will be cooled by the liquid helium being transferred into the dewar. Since solid nitrogen has a high specific heat, this would result in higher than necessary liquid helium usage. Second, the solid nitrogen so formed might clog the cooling channels in the coil under test. This would impair the operation of the coil and might lead to premature quenching.

Having removed the liquid nitrogen, liquid helium transfer into the dewar was started. Approximately 36 liters of liquid helium were used in obtaining a level of liquid helium about 4-1/2 inches over the coil. It was estimated that of this about 13 liters were used to cool the dewar and coil.

Soon after transferring liquid helium the boil-off from the dewar was measured using an integrating gas meter. A rate of about 5.5 to 6.5 liters of liquid helium per hour was recorded. This is rather high but at this stage the upper parts of the dewar were still being cooled down. (Because of the high specific heat, low thermal conductivity and thick walls, the non-conducting dewar has a longer thermal time constant than a similarly sized metallic dewar.) Subsequently, after thermal equilibrium had been reached, boil-off rates in the range of 3.5 to 4.5 liters per hour were recorded.

B. ELECTRICAL POWER CIRCUITRY

The remainder of the electric circuitry consisted of items concerned with charging the coil (i.e., power supplies and items concerned with breaking the current once established). The items described below were used at one time or another during the course of the experiments.

At any time either a rectifier or a set of submarine batteries could be used to charge the coil. The rectifier was continuously variable in voltage from 0 to 20V and could supply a maximum of 1,000 A at 20V. Eighteen submarine batteries (short circuit current 20,000 A) were available to give up to 36 volts total in increments of 2 volts. From the point of view of convenience, the rectifier with the adjustable output voltage was better to use. However, diodes in rectifiers are very susceptible to damage from high voltage transients whereas batteries are virtually impervious to such disturbances.

A standard vacuum interrupter (GE Power/Vac Vacuum Interrupter Model PV03G-12-15A18) was used as the main switch. This was powered by a custom built hydraulic actuator. A contactor set to open automatically about 200 ms after the vacuum switch opened provided backup in case of a failure by the vacuum switch.

Many of the experiments were done at discharge voltages less than that necessary to maintain a vacuum arc and consequently no arc quench circuitry was needed. However, for the fast discharges, a triggered spark gap (EG&G GP-22B or GP-12B, depending on the required breakdown voltage) was used as a closing switch to fire a counter-pulse current through the vacuum interrupter. Further details concerning the vacuum interrupter, hydraulic power supply, contactor, spark gap and the method of operation of the arc quench circuit can be found in reference 3.

IV. EXPERIMENTS PERFORMED USING THE ELECTRONIC LOSS MEASUREMENT DEVICE

A. INTRODUCTION AND SETTING THE DIVIDER RATIO

The basic circuit was assembled as shown in Figure IV-1. Variations of this circuit were used in all the experiments described in this section. Table IV-1 summarizes the kinds of experiments carried out during the tests.

Before any loss measurements were made it was necessary to adjust the voltage divider to obtain as near a minimum signal as possible; i.e., to as nearly as possible buck out the inductive component of voltage from the "7" kJ coil by the inductive component from the secondary of the Rogowski coil. This was achieved using the circuit shown in Figure IV-2.

For a constant ac voltage (V_{in}) across the coil terminals, the divider ratio was adjusted by moving the 10-turn potentiometer until a minimum residual voltage ($V_{residual}$) was obtained on the CRO. This minimum signal represents a loss voltage due to magnetization in the coil plus left over unbalanced inductive voltage from the coils. The results were as follows:

$$\begin{array}{ll} V_{in} & = 97.5 \text{ V} \\ V_{divider} & = 5.70 \text{ V} \\ V_{residual} & = 0.010 \text{ V} \\ V_{divider} & = 5.70 \text{ V} \end{array} \quad \begin{array}{l} \text{giving divider ratio } f = 17.1 \\ \\ \frac{\text{residual signal}}{\text{unbalanced signal}} = 0.175\% \end{array}$$

B. SLOW CHARGE AND SLOW DISCHARGE EXPERIMENTS

1. Air core Rogowski Coil

The first series of experiments (Table IV-1 Line 1-1) was performed using the circuit shown in Figure IV-3. These tests consisted of charging the coil to various maximum currents for several different rates of change of current, each time discharging the coil at a fixed rate of change of current. Half cycle hysteresis loops were recorded using the Rogowski coil.

The power supply voltage was preset, the contactor was closed, and the vacuum interrupter was closed. The coil then charged linearly with respect to time. After a preset time had elapsed, the vacuum interrupter was opened. In all experiments the voltage applied to the coil was less than that necessary to maintain an arc so no arc quench circuitry was required. To further aid in the breaking of the arc a large capacitor (1.5 F at 36V) was placed across the vacuum interrupter. This slowed down the rate of rise of voltage and prevented any voltage transients from disturbing the measurements in the rest of the circuit. Once the arc was out the current decayed through the diodes which served as the load. The voltage drop across these diodes (2 V) is independent of current for a wide range of currents so the current decayed linearly.

The charge rates used were approximately 37 A/s, 86 A/s, 165 A/s, 248 A/s, 301 A/s and 357 A/s. For each of these charge rates the coil was allowed to charge for six different times; 0.5 s, 1.0 s, 1.5 s, 2.0 s,

Schematic of Experimental Layout. Integrator uses chopper stabilized operational amplifier which is held at constant temperature.

Figure IV-1

TABLE IV-1

SUMMARY OF TYPES OF EXPERIMENTS PERFORMED

1. Charge Coil in Seconds, Discharge in Seconds

For various maximum currents and various charge rates obtain half cycle hysteresis loops using:

- (1) Air core Rogowski coil
- (2) Sense coil wound around 7 kJ coil

2. Ring Coil with Various Capacitors at Various Maximum Currents

- (1) Obtain full cycle hysteresis loops using air core Rogowski coil
- (2) Obtain traces of decay of current oscillation versus time

3. Charge Coil in Seconds, Discharge in Progressively Shorter Times

- (1) For various maximum coil currents obtain half cycle hysteresis loops using air core Rogowski coil

Schematic of Circuit Used to Find Divider Ratio which Gives Best Balance Between the Inductive Voltages of the "7" kJ Test Coil and the Rogowski Coil

Figure IV-2

Schematic Showing Circuit Used in Slow Charge and Discharge Experiments
(Table IV-1, Line 1-1)

Figure IV-3

2.5 s, and 3.0 s. The maximum coil currents so obtained covered the range ~ 20 A to ~ 750 A. The discharge rate which was constant for all tests at about 60 A/s.

For each test a half cycle hysteresis loop was obtained. Half cycle hysteresis loops are obtained when the current is cycled thus: $I = 0$ to $I = I_{\max}$ to $I = 0$. Figure IV-4 is a schematic showing the general form of full and half cycle hysteresis loops. Typical half cycle hysteresis loops for low and high currents respectively would follow paths such as j, b, j and j, b, f, j. Figure IV-5 is a reproduction of an oscilloscope trace taken during this first series of experiments. It is a composite picture of a number of half cycle hysteresis loops taken at increasing currents using a storage oscilloscope. Note that on each of the six tests the magnetization during the increasing current part of the cycle very closely follows the path taken by the previous lower current test. The areas enclosed by the curves of $\int V dt$ versus I are proportional to the energy dissipated. The actual formula is as follows:

$$E = f \times A \times \hat{V} \times \hat{I} \times \tau$$

where f = divider ratio

τ = integrator time constant (seconds)

A = area enclosed by loop (cm^2)

$\hat{V}$ = oscilloscope factor (vertical) (V/cm)

$\hat{I}$ = oscilloscope factor (horizontal) (A/cm)

The energy associated with each of the hysteresis loops recorded was calculated and the energy so obtained divided by two. The energy obtained thus is equivalent to the average energy dissipated during a charge or discharge. The results are shown as the lower curve in Figure IV-6 which is a plot of energy versus maximum coil current.

2. Secondary Sense Coil

Following this set of experiments the secondary of the Rogowski coil was disconnected from the circuit and in its place was substituted the sense coil wound around a form separate from but surrounding the "7" kJ coil.³ The circuit is shown in Figure IV-7. A balancing procedure was carried out by adjusting the voltage divider ratio as previously described and exactly the same set of experiments was performed as were described above. The voltage divider ratio was again found to be 17.1. This set of experiments is line 1-2 in Table IV-1. Half cycle hysteresis loops were obtained as previously described. The shapes of the loops are quite different from those previously described and furthermore the energies associated with the loops are very much smaller (by about an order of magnitude) than the equivalent loops obtained using the Rogowski coil. This is illustrated in Figure IV-8 which shows two sets of hysteresis loops obtained under identical conditions using the air core Rogowski coil and the secondary sense coil. The distorted shapes are attributed to mechanical deformation of the energy storage coil. The energy storage coil as it is charged expands under the influence of the electromagnetic forces. The sense coil however which does not experience these forces and which is not attached to the energy storage coil does not deform. Thus the relative dimensions of the two coils change.

Schematic Showing General Form of Full and Half Cycle Hysteresis Loops

Figure IV-4

A Series of Half Cycle Hysteresis Loops

Figure IV-5

The vertical axis is proportional to the magnetization of the superconductor, and the horizontal axis is proportional to the applied field. This picture was obtained using apparatus designed and built by MCA for measuring ac losses in superconductors. The area enclosed by successive loops is proportional to the energy dissipated in an energy storage test coil when charged through successively larger cycles of current 0 to $+I_{\max}$ to 0.

Figure IV-6. Energy Dissipated (Average of Charge and Discharge) in a "7" kJ Energy Storage Coil Vs. Maximum Coil Current

Schematic Showing Circuit Used in Slow Charge and Discharge Experiments (Table IV-1, Line 1-2) When the Sense Coil Was in Use

Figure IV-7

Rogowski Coil

Secondary Sense Coil

Comparison of Hysteresis Loops Obtained Under Identical Conditions Using the Air Core Rogowski Coil (Top) and the Secondary Sense Coil (Bottom).

Figure IV-8

This causes a change in the mutual inductance between the two which changes the bucking voltage and which results in a distorted hysteresis loop. It can be seen from the lower trace in Figure IV-8 that the distortion is greater the greater the coil current, which is consistent with the above hypothesis since the mechanical distortion would be proportional to the square of the current.

It is thought that the smaller loop area is due in an involved way to the pick up coil being affected directly by the diamagnetism of the windings of the energy storage coil. A preliminary theoretical model (see Appendix F, Analysis of Electronic Measurement of Losses by Use of a Secondary Sense Coil) suggests some partial cancellation of signals which leads to a reduced voltage output from the sense coil.

C. HIGH VOLTAGE RINGING EXPERIMENTS USING CAPACITORS

Following this set of experiments the sense coil was disconnected and replaced by the Rogowski coil. The adjustable power supply was removed from the circuit and was replaced by various capacitors which could be charged to high voltage by means of a high voltage supply which could subsequently be completely removed from the circuit. This set of experiments constitutes Lines 2-1 and 2-2 of Table IV-1. A schematic of the circuit used is shown in Figure IV-9.

With the contactor closed and the vacuum switch open the capacitor was charged to the desired voltage using the high voltage power supply. The high voltage disconnects were then opened. The ringing circuit was completed by closing the vacuum switch. Initially the current shunt and optical isolator were used to record the current but because of ground loop problems, the current shunt was abandoned in favor of a Pearson Model 301 current transformer.

Two kinds of measurement were made: (1) full cycle hysteresis loops (Table IV-1, Line 2-1), and (2) rate of decay of current oscillations with respect to time (Table IV-1, Line 2-2).

Figure IV-10 is a reproduction of an oscilloscope trace obtained during this set of experiments. It is a series of full cycle hysteresis loops obtained using a 2,500 μ F capacitor charged to 2 kV. The maximum current is about 520 A and the frequency is about 16.5 Hz.

Full cycle hysteresis loops are obtained by cycling the current $I = 0$ to $+I_{max}$ to 0 to $-I_{max}$ to 0, a path such as a, f, j, m, a on Figure IV-4. As energy is dissipated, the maximum current in the oscillation diminishes and so the size of the hysteresis loop also diminishes. Again referring to Figure IV-4, as the maximum current decreases paths such as a, e, g, j, l, n, a will be followed. The first few such loops can be seen in Figure IV-10.

Again by measuring the area enclosed by the loops the energy dissipated in a cycle can be calculated. The energies calculated from the full cycle loops were divided by 4 to obtain the average energy dissipated in a charge and a discharge and were plotted as the upper line in Figure IV-6.

Schematic Showing Circuit Used in Ringing Experiments (Table IV-1, Lines 2-1 and 2-2)

Figure IV-9

Full Cycle Hysteresis Loop

Figure IV-10

The vertical axis is proportional to the magnetization of the superconductor, and the horizontal axis is proportional to the applied field. This picture was obtained using apparatus designed and built by MCA for measuring ac losses in superconductors. The area enclosed by the outer perimeter of the figure is proportional to the energy dissipated in an energy storage test coil when charged through a cycle of current 0 to $+I_{\max}$ to 0 to $-I_{\max}$ to 0.

The average energy on charge and discharge being higher for full cycle loops than for half cycle loops is expected on theoretical grounds. It can be seen qualitatively by examining loops shown in Figure 19. Clearly the area enclosed by a full cycle loop a, f, j, m, a is more than twice the area of a half cycle loop j, b, f, j.

Oscilloscope traces of hysteresis loops were obtained for capacitors in the range 1,000 μF , 2,500 μF , and 5,000 μF at voltages ranging from 0.5 kV to 2 kV. These capacitors correspond to frequencies of 26.2 Hz, 16.6 Hz, and 11.7 Hz, respectively.

Following this set of experiments oscilloscope traces were obtained which showed the decay of current oscillations with respect to time, (Table IV-1, Line 2-2). The rate of decay depends upon the energy dissipated in the circuit due to hysteresis losses in the coil and due to ohmic losses in the rest of the circuit. The total loss per cycle can be inferred from the rate of current decay. If the losses in the rest of the circuit are calculated, then by subtraction the losses in the coil can be inferred. Estimates were made of the resistance and therefore loss of the leads, and the total losses calculated for some of the oscilloscope traces. When the difference between the two is taken to obtain the loss due to the coil, values higher than those obtained using the loop area method are obtained. These additional losses were attributed to dissipative losses in the capacitors. The range of capacitors used in this set of experiments was (in μF) 0.25; 1.0; 6.0; 100; 500; 1,000; 2,500; and 5,000 covering frequencies (in Hz) 1,660; 827; 338; 82.7; 37.0; 26.2; 16.6; and 11.7.

D. SLOW CHARGE, FAST DISCHARGE EXPERIMENTS

The final set of experiments involved charging the coil up in a few seconds, breaking the current using the vacuum interrupter and arc quench circuit and allowing the coil to discharge rapidly through a resistor (see Table IV-1, Line 3-1). The circuit used for this is shown in Figure IV-11. This operating sequence produces half cycle hysteresis loops. Loops were obtained at various maximum currents for load resistances of 0.6 Ω and 2.3 Ω corresponding to discharge time constants of 62 ms and 16 ms, respectively. The losses derived from this data are plotted in Figure IV-6 in the lower curve. The indications are that the losses measured at these fast discharges are not significantly different from those measured for the slow (seconds) charges and discharges.

Schematic Showing Circuit Used in Slow Charge and Fast Discharge Experiments (Table IV-1, Line 3-1)

Figure IV-11

V. A.C. LOSSES

A. INTRODUCTION

The theoretical work concerning magnetization losses in single core superconducting wire carried out during the course of this program is a continuation of the theory developed during a previous program concerning pulsed energy storage coils. This previous work is reported on in detail in Appendix I of Reference 3. The most serious limitation of the previous work is that it is applicable only to situations where the superconductor is fully penetrated by the magnetic flux. Full penetration occurs only if the applied field strength is above a certain value, determined by the superconductor properties and the diameter of the wire. The smaller the diameter of the superconductor then the smaller is the applied field necessary for complete penetration. Multifilament conductors contain strands of diameters of the order of tens of microns and such strands are penetrated at very low fields. Under such circumstances the fully penetrated theory is adequate. However, single core conductors rarely have superconductor diameters less than about two or three thousandths of an inch and under these conditions for niobium titanium full penetration does not occur until the applied field is of the order of 0.5 to 1.0 Wb/m². The use of formulae which assume full field penetration can result in significant errors even when applied to magnets where peak fields are cycled over ranges very much greater than 1.0 Wb/m². This is because during a field cycle some of the magnet is at low fields all the time and all of the magnet is at low fields some of the time. The previously derived theoretical model which assumes full penetration all the time, therefore overestimates the magnetization loss of the core as a whole. Consequently, the first task in the present program was to develop a theory which would be applicable at low fields where the superconductor is in a state of incomplete penetration.

B. WORK PERFORMED

It was assumed that the form of this j_c -B relationship was hyperbolic, thus:

$$j_c(B+B_0) = \alpha$$

where α and B_0 are constants characteristic of the particular superconductor used. Experiments were performed to determine these constants for the conductor used in the "7" kJ test coil. (See Appendix I, Experimental Evaluation of the j_c -B characteristics of the Conductor Used to Build the "7" kJ Coil).

Using this relationship a number of A.C. loss formulae were derived. These derivations are dealt with in more detail in Appendix K, Derivation of A.C. Loss Formulae. They are summarized in Figure V-1. These formulae were derived according to the following assumptions:

(1) The critical current density j_c is constant through the conductor from the surface to the depth of maximum flux penetration and is characterized by the value at the surface of the conductor (determined by the value of the applied field at the conductor surface).

Energy Losses Due to Superconductor Magnetization*

Critical Current Density
Versus Field Relationship

Flux Penetration Profiles into
Slab of Conductor of Thickness $2a$

$$(B_1 = 2 \mu_0 j_c a)$$

A. For isolated wire experiencing uniform field along its length (l)

B goes from 0 to B
where $B < B_1$

$$\frac{\Delta E_1}{l} = \frac{a B_0^3}{6 \mu_0^2 \alpha} \left\{ 1 + \frac{3B}{4B_0} \right\}$$

B goes from 0 to B_1

$$\frac{\Delta E_2}{l} = \frac{a B_0^3 B_1^3}{6 \mu_0^2 \alpha} \left\{ 1 + \frac{3B_1}{4B_0} \right\}$$

B goes from 0 to B
where $B > B_1$

$$\frac{\Delta E_3}{l} = \frac{\Delta E_2}{l} + 2a^3 \alpha \ln \left[\frac{B+B_0}{B_1+B_0} \right]$$

B. For length of wire which has a field varying linearly from 0 at one end to a maximum (B_m) at the other i.e. approximating conditions in a coil

B goes from 0 to B_m
where $B_m < B_1$

$$\frac{\Delta E_4}{l} = \frac{a B_0^3 B_m^3}{24 \mu_0^2 \alpha} \left\{ 1 + \frac{3B_m}{5B_0} \right\}$$

B_1 goes from 0 to B_1

$$\frac{\Delta E_5}{l} = \frac{a B_0^3 B_1^3}{24 \mu_0^2 \alpha} \left\{ 1 + \frac{3B_1}{5B_0} \right\}$$

B goes from 0 to B_m where $B_m > B_1$

$$\frac{\Delta E_6}{l} = \frac{f \Delta E_5}{l} + (1-f) \frac{\Delta E_2}{l} + (1-f) 2a^3 \alpha \left\{ \left(\frac{B_m+B_0}{B_1+B_0} \right) \ln \left[\frac{B_m+B_0}{B_1+B_0} \right] - 1 \right\} \quad \text{where } f = \frac{B_1}{B_m}$$

See Appendix K for deviations.

*Influence of transport current density not included.

Figure V-1

(2) The conductor has already been charged through one cycle $B = 0$ to $B_{\max}$ to 0. (This determines the locked in flux profile at the beginning of the cycle.) Furthermore, it was assumed that the wire cross-section was rectangular until after all the integrations had been carried out. A very approximate geometry factor was then calculated to convert from rectangular to round geometry. These loss formulae apply to the charge part of a half cycle hysteresis loop only.

Figure V-1 shows formulae applicable to lengths of wire experiencing a constant field along their length and also to lengths of wire experiencing a field varying linearly from zero at one end to a maximum at the other. This latter condition approximates the situation in a solenoid.

C. COMPARISON OF THEORY AND EXPERIMENT

The formula ΔE_6 in Figure V-1 was evaluated for the parameters relevant to the "7" kJ coil and is plotted in Figure V-2 which also shows the experimental data obtained using the electronic loss measurement device and the air core Rogowski coil. The upper line of data points represents the average of the energy dissipated on charge and on discharge when the coil is cycled from a current of zero to a maximum and back to zero again. The theoretical curve is only for the cycle of current for zero to a maximum and therefore the magnitudes of the theoretical and experimental curves are not directly comparable. However, it is easily seen that both curves follow very similar power laws. The theory predicts a power law such that the losses in the incomplete penetration region are proportional to the cube of the current and in the complete penetration region proportional to the first power of the current. Inspection of the experimental data indicates that at lower currents the variation is approximately cubic and at the higher currents the variation is tending to linear. The experimental and theoretical curves differ in magnitude by a factor of about 3 at the low current and to about 1.5 at the high current end. The power law similarity is reassuring and it is expected that if the formula for the other half of the cycle were to be calculated (i.e. $B_{\max}$ to $B = 0$) the agreement would be closer. Complete agreement, however, would not be expected even if this were achieved because of inadequacies in the model. For more realistic modeling the following would have to be taken into account:

- (1) The variation of j_c across the superconductor.
- (2) The actual distribution of field along the length of conductor in the "7" kJ coil.
- (3) The influence of the transport current on the field and critical current density distributions in the superconductor.
- (4) The circular shape of the wire. (This strictly should be taken into account before performing the integrations over the cross section of the wire.)

Formulae should also be derived which would be applicable to full cycle hysteresis loops.

Figure V-2 Energy Dissipated (Avg. of Charge and Discharge) in a "7" kJ Energy Storage Coil vs. Maximum Coil Current.

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D. SOME COMMENTS ON COIL PERFORMANCE AND A.C. LOSSES

The above formulae tacitly assume that the temperature of the conductor does not change during the course of a pulse. This is implicit in the assumption of the critical current density being a function of the applied field only. (The theoretical curve shown in Figure V-2 assumes the j_c -B curve for 4.2K.) In reality, during a charge/discharge cycle the losses cause the conductor to rise in temperature. Provided that the rate of heat generation is small enough steady state cooling exists and the conductor indeed does remain near the bath temperature. This would be the situation for long charge and discharge times. However at shorter charge and discharge times the power dissipation rises and the temperature of the conductor also rises. This has the effect of reducing the critical current density of the superconductor. Since the magnetization losses are directly proportional to the critical current density this causes the measured magnetization losses to be smaller than those predicted for 4.2 K superconductor properties. As the charge and discharge times become even shorter the temperature in the higher field regions of the windings (where the magnetization losses are highest) rises to the point at which it exceeds the critical temperature. These regions then go normal and instead of experiencing a magnetization loss they experience a transport current loss. This tends to increase the measured loss over the predicted loss based on magnetization alone.

Ideally, then, it would be expected that at long charge and discharge times the measured losses would agree with the theoretical losses but that as the pulse times were shortened and the conductor temperature rose the measured losses would be less than the losses predicted on 4.2 K superconductor properties. Furthermore, at still shorter pulse times (when parts of the coil would be starting to go normal) this trend might be reversed and the measured losses might rise again as transport losses become significant.

By observing the A.C. losses over a range of pulse times then, it might be possible to gain some insight into the conditions existing within the windings of a coil during transient operation. Such information would be of value in the design of pulsed coils.

Experiments were carried out over pulse times ranging from a few seconds to a few tens of milliseconds. Inspection of Figure V-2 shows that there was no significant difference between the losses measured at the two extremes. This indicates that the cooling is sufficiently good that at pulse times of the order of tens of milliseconds there is no significant rise in the temperature of the windings. This result was not unexpected since the coil on which the experiments were performed was designed to be operated in a pulsed mode, every layer being served by its own cooling channel. It has previously been operated at pulse repetition rates of four per second (~220 ms charge, 2 ms discharge, 30 ms dwell) for pulse train lengths of minutes without signs of quenching.

Unfortunately it was not possible to extend the experimental data to shorter discharge times so as to find the limitations of the "7" kJ coil because of equipment failure (a high voltage transient severely damaged key apparatus).

VI. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

With the rapidly increasing interest in applications of superconductivity involving the use of magnets exposed to transient field conditions, it is of great importance to be able to both measure and predict A.C. losses in superconductors.

During the course of this contract, Magnetic Corporation of America successfully developed apparatus for electronically measuring A.C. losses and derived formulae which more accurately predict magnetization losses in coils made from monofilament superconductors.

Key electronic equipment used consisted of an integrator, a Rogowski coil, a high resolution voltage divider, a non-inductive current shunt and an optical isolator. To make the A.C. loss measurements more meaningful a special completely non-conducting dewar was built.

Formulae were derived which allowed the prediction of the A.C. losses incurred in a superconducting coil during the charge half of a cycle of current from zero to a maximum and back to zero.

A series of experiments was conducted in which full cycle and half cycle hysteresis loops were obtained for various maximum coil currents for pulse lengths varying between a few seconds and a few tens of milliseconds. Such data was obtained using both an air core Rogowski coil and a sense coil wound round the test coil. Equipment failure prevented data from being extended to as short a discharge time as was desired.

The half cycle hysteresis loop experimental data obtained using the air core Rogowski coil and the theoretical curve followed closely similar power laws. The energy dissipated in going from $I = 0$ to $I_{\max}$ is different from that dissipated in going from $I = I_{\max}$ to 0 and the half cycle hysteresis loop measures only the sum of the two. Since the theory developed predicts the energy dissipated in going only from $I = 0$ to $I_{\max}$ a direct comparison of magnitudes of theory and experiment was not possible.

Half cycle hysteresis loop data taken using the sense coil wound round the test coil gave apparent loss energies about an order of magnitude lower than those obtained under the same conditions using the air core Rogowski coil. This behaviour was qualitatively expected from theoretical analyses performed during the course of the program.

Ringdown experiments were conducted by allowing capacitors of various sizes to discharge into the superconducting coil. The rate of decay of the current oscillations was observed. The purpose of this was to provide an independent check on the A.C. losses. The data obtained from these experiments has not been successfully interpreted because the capacitors apparently have a higher effective resistance than anticipated. Successful interpretation of this data cannot be achieved unless subsidiary experiments are carried out to measure the resistances of the capacitors. Experiments were ended because a high voltage transient damaged key apparatus.

The following is recommended for successful conclusion of the program:

1. Repair power equipment and instrumentation.
2. Devise a method of protection to avoid similar damage from recurring.
3. Conduct further experiments to extend the data to shorter discharge times.
4. Conduct subsidiary experiments over a range of frequencies to measure the resistance of the capacitors and any other circuit components.
5. Further develop the loss theory to take into account: the variation of the critical current density across the width of the wire; loss on charge and on discharge for full or half-cycle loops; transport current effects; actual field distribution along the length of the wire; and the round wire geometry.

REFERENCES

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3. Lucas, E.J.; Punchard, W.F.B.; Margosian, P.M.G., "Design, Construction and Testing of a Pulsed High Energy Inductive Superconducting Energy Storage System", Technical Report AFAPL-TR-75-60, September 1975.

APPENDIX A

DERIVATION OF RELATIONSHIP BETWEEN HYSTERESIS LOOP AND LOSS DURING CYCLE

The voltage which appears across the coil terminals consists of an inductive component associated with the rate of change of field in the energy storage coil and dissipative component associated with the loss mechanism.

If V_L = inductive component = $L \frac{di}{dt}$

V_R = dissipative component

$V_C = V_L + V_R$ = total voltage measured across coil terminals

A fraction f of the terminal voltage is tapped from a voltage divider R placed across the terminals and from this fraction fV_C is subtracted a voltage $M \frac{di}{dt}$ produced by the mutual inductance between an auxiliary coil and a current lead. The resulting voltage is fed to the input of an integrator.

$$\begin{aligned}\text{Thus input voltage to integrator} &= fV_C - M \frac{di}{dt} \\ &= fV_R + fV_L - M \frac{di}{dt} \\ &= fV_R + (fL - M) \frac{di}{dt}\end{aligned}$$

By suitably adjusting f and M , $fL - M$ can be made as small as is desirable. It should be noted that (as the following analysis will show) it is not essential for the method to be valid that $fL - M$ be made exactly zero. Making it small merely allows the range over which the integrator must work to be greatly reduced.

The output from the integrator is:

$$\frac{1}{\tau} \int_0^t [fV_R + (fL - M) \frac{di}{dt}] dt$$

where τ is the time constant of the integrator.

This signal is fed to the Y axis of an oscilloscope. A voltage proportional to the current through the coil is fed to the X axis. This signal is derived from the non-inductive shunt described in the text of this report and in Appendix C.

The combination of signals produces a hysteresis loop whose area is given:

$$A = \int_0^i \left\{ \int_0^t \left[fV_L + (fL - M) \frac{di}{dt} \right] \frac{dt}{\tau} \right\} di$$

$$A = \int_0^i \left\{ \int_0^t \frac{fV_L}{\tau} dt \right\} di + \int_0^i \left\{ \int_0^t \frac{(fL-M)}{\tau} \frac{di}{dt} dt \right\} di$$

$$A = \frac{f}{\tau} \int_0^i \left\{ \int_0^t V_L dt \right\} di + \frac{(fL-M)}{\tau} \int_0^i \left\{ \int_0^t \frac{di}{dt} dt \right\} di$$

$$\text{at } t=0 \quad i=0$$

$$t=t \quad i=i$$

$$A = \frac{f}{\tau} \int_0^i \left\{ \int_0^t V_L dt \right\} di + \frac{(fL-M)}{\tau} \int_0^i \left\{ \int_{i=0}^i di \right\} di$$

$$A = \frac{f}{\tau} \int_0^i \left\{ \int_0^t V_L dt \right\} di + \frac{(fL-M)}{\tau} \int_0^i i di$$

$$A = \frac{f}{\tau} \int_0^i \left\{ \int_0^t V_L dt \right\} di + \frac{(fL-M)}{\tau} \frac{i^2}{2}$$

$$\text{let } \int_0^t V_L dt = x$$

$$V_L dt = dx$$

$$A = \frac{f}{\tau} \int_0^i x di + \frac{(fL-M)}{2\tau} i^2$$

integrate by parts

$$\int_{i=0}^i \frac{1}{A} = \frac{f}{\tau} \left\{ x i - \int_0^i i dx \right\} + \frac{(fL-M)}{2\tau} i^2$$

$$\int_{i=0}^i \frac{1}{A} = \frac{f}{\tau} \left\{ i \int_0^t v_L dt - \int_0^i i v_L dt \right\} + \frac{(fL-M)}{2\tau} i^2$$

$$\int_{i=0}^i \frac{1}{A} = \underbrace{\frac{fi}{\tau} \int_0^t v_L dt}_{[1]} + \underbrace{\frac{(fL-M)}{2\tau} i^2}_{[2]} - \underbrace{\frac{f}{\tau} \int_0^i i v_L dt}_{[3]}$$

If we take cycle $i = 0$ to $i_{\max}$ to $i = 0$

$t = 0$ to $t = t_{\text{cycle}}$

then terms [1] and [2] = 0 and $A_{\text{cycle}} = -\frac{f}{\tau} \oint i v_L dt$.

but $i v_L$ is power loss = P and $A_{\text{cycle}} = -\frac{f}{\tau} \oint P_L dt = -\frac{f}{\tau} E_{\text{cycle}}$

where E_{cycle} is the energy dissipated in a cycle. Thus $E_{\text{cycle}} = -\frac{\tau}{f} A_{\text{cycle}}$.

APPENDIX B

EFFECT OF PHASE SHIFT AND DRIFT VOLTAGE ON LOSS LOOP MEASUREMENT

We measure the losses by taking:

$$E_A = \left[\frac{1}{\tau_I} \int dI \int dt \left(fV - M \frac{dI}{dt} \right) \right] \left(\frac{\tau_I}{f} \right)$$

where:

E_A is the apparent energy described by the loss loop

τ_I is the integrator time constant

$\int dI$ means to calculate the area inside the loss loop

f is the divider fraction

M is the mutual inductance to the sense coil

V and I are voltage and current, respectively.

For the sake of simplicity, we use sinusoidal quantities:

$$I = I_0 \cos \omega t$$

$$\begin{aligned} V &= RI + L \frac{dI}{dt} \\ &= I_0 \left[R \cos \omega t - \omega L \sin \omega t \right] \end{aligned}$$

where:

R represents the superconductor loss at a given current and frequency

L is the self inductance of the coil

Assuming the existence of a DC error voltage, v , and a phase shift error, ϕ , it is possible to write:

$$E_A = \left\{ \frac{1}{\tau_I} \int dI \int dt \left[\left(fV - M \frac{dI}{dt} \right) + v \right] \right\} \left(\frac{\tau_I}{f} \right)$$

$$E_A = \frac{1}{\omega \tau_I} \int dI \left\{ \cos \phi \left[a \sin \omega t - b \omega \cos \omega t \right] \right. \\ \left. + \sin \phi \left[a \cos \omega t + b \omega \sin \omega t \right] + V \omega t \right\} \frac{\tau_I}{f}$$

where:

$$a = fR$$

$$b = M - fL$$

The observed energy is calculated by noting that

$$dI = -I_0 \omega \sin \omega t dt$$

and integrating over a complete cycle. The result of this calculation is:

$$E_A = \frac{1}{2} I_0 \left(\frac{2\pi}{\omega} \right) \left[fRI_0 \cos \phi - 2v + I_0 (\omega M - fL) \sin \phi \right] \left(\frac{\tau_I}{f} \right)$$

Typical Values for Actual System:

Loss Voltage (fRI_0): 1mv or greater

DC Error Voltage (v): 30 microvolts or less

Phase shift, ϕ : less than 10^{-3} radian

Resolution of Null Circuit: less than 1 part in 10^5

APPENDIX C

NON-INDUCTIVE CURRENT SHUNT

For the electronic loss measurement technique to work properly, the coil current must be monitored accurately. A current shunt essentially is a resistor whose current is accurately known so that when the voltage developed across it is measured the current may be calculated. As long as the voltage measured is due only to the resistance of the shunt an accurate reading of the current may be obtained. However, strictly the voltage measured across the shunt is due to the sum of a voltage due to the shunt resistance and a voltage due to the shunt inductance. The former is proportional to the current and the latter is proportional to the rate of change of current. As long as the rate of change of current is small the inductive component of voltage will be small and the measured voltage will be due only to the resistance of the shunt. This situation occurs at long coil charge and discharge times. However, at short charge and discharge times unless special precautions are taken the inductive component becomes large and erroneous readings of current are obtained. Therefore, a shunt must be designed in such a way that its inductance is low enough so that at the shortest charge and discharge times likely to be encountered, the inductive component of voltage is small compared with the resistive component of voltage. This is achieved by designing a shunt which consists of two concentric cylinders as shown schematically in Figure II-3. The current flows down one cylinder and back up the other one. The cylinders are made so that they are as close together as is practically possible. In this way the current paths are as close together as possible but in opposite directions. The magnetic fields due to each therefore almost cancel and the inductance is much reduced.

Other considerations also enter into the design. The above coaxial arrangement takes care of the inductive problems but changes in resistance during a cycle must also be eliminated. These changes are due to two effects. Firstly, there may be a resistance change due to a temperature rise caused by ohmic heating and secondly at fast discharge times the resistance may change due to a skin effect.

The former may be eliminated by making the shunt sufficiently massive so that changes in temperature are small and have a negligible effect on the resistivity of the material.

The latter must be eliminated by suitably sizing the thickness of the coaxial cylinders. The skin effect is caused by a rapidly changing magnetic field causing an induced voltage which alters the current distribution in the cylinders. Under conditions of fast discharge the current tends to flow preferentially in the outer layers of the cylinder. The faster the discharge the more current "crowds" to the outside of the cylinders. This means that the material on the inside is not available for carrying current and therefore the resistance of the cylinders increases. Calculations indicated that at the discharge times to be encountered a wall thickness of 0.12 inch would be thin enough to eliminate any problems due to the skin effect. Table II-2 is a summary of the results of the analysis carried out to design the shunt.

APPENDIX D

HIGH RESOLUTION VOLTAGE DIVIDER

The primary specification for the voltage divider is high resolution. This requirement is met by means of the two-stage design shown in Figure II-4. Because the divider fraction must match the voltage ratio between the energy storage coil and a sense coil it is sufficient to measure this ratio to one percent accuracy and set up a resistor string such that the difference in divider fraction across the terminals of a single resistor brackets the measured coupling ratio to the sense coil. In other words, the divider fraction can be adjusted to within one percent of the desired value by making one straightforward measurement and constructing a string of discrete resistors accordingly. If a ten-turn potentiometer which has a resolution of one part in 1,000 is placed across the terminals of the specified resistor (as in Figure II-4), then the total resolution available in the divider fraction is one part in 10^5 .

The other requirements that must be fulfilled by the voltage divider are: (1) ability to withstand 60 kV without arcing, (2) resistance high enough that the components do not suffer excessive heating during a high voltage pulse, and (3) resistance low enough to preclude disturbance of the divider ratio by connection of an integrator with realistic input impedance (300 k Ω in our case). The first condition led to the selection of a string of about 60 two-watt resistors (each capable of supporting 1,000 volts) which is mounted on a sheet of plastic (18" x 20"). The terminals of this resistor string are separated by approximately 15-inches. The best compromise between the second and third conditions led to the selection of an overall resistance for the divider of about 60,000 ohms. This is five times smaller than the input impedance of the integrator. A practical upper limit on the input impedance of the integrator is set by the drift requirements as is discussed elsewhere. The heating of this resistor string can be checked by noting that the energy dissipated in the diver string, E_s , is given by:

$$E_s = \left(\frac{1}{2} L I_C^2 \right) \frac{R_l}{R_d} \quad D-1$$

where L is the coil inductance, I_C is the coil current, R_d is resistance of the divider, and R_l is the load resistance for the coil and is specified by:

$$R_l = \frac{V_{\max}}{I_C} \quad D-2$$

With the help of equation D-2, the expression for energy lost in the divider string can be converted to the form:

$$E_s = \left(\frac{1}{2} L I_C \right) \frac{V_{\max}}{R_d} \quad D-3$$

from which it is clear that the worst case (greatest loss) occurs for the highest values of $V_{\max}$ (60,000 volts) and of I_C (1,000 amps) for which R_1 is 60 ohms. If the coil is charged to its rated energy (10 kJ), the energy dissipated in the divider is given by equation D-1:

$$E_s = (10,000) (60/60,000) = 10 \text{ Joules}$$

which is about one Joule per resistor. Under these conditions, the maximum increase in divider temperature is only a few tenths of a Kelvin, a negligible amount.

APPENDIX E.

DATA SHEETS FOR THE FOLLOWING ELECTRONIC DEVICES:

1. Optical Isolator, Monsanto MCD-2, MCD-4
2. Quad 741 Operational Amplifier, Raytheon 4136
3. Voltage Regulator, ± 15 volts, Motorola #1468

Monsanto

PHOTODIODE
OPTO-ISOLATOR

MCD2

PRODUCT DESCRIPTION

The MCD2 is a diffused planar silicon PIN photodiode optically coupled to a diffused planar gallium arsenide light-emitting diode. It is mounted in a six-lead plastic DIP package.

PACKAGE DIMENSIONS

FEATURES & APPLICATIONS

- Ultra fast switching time — typically 50 nanoseconds
- Very high isolation resistance - $10^{11} \Omega$
- 1500 volt isolation emitter to detector
- Plastic dual-in-line package
- Long life — solid state reliability
- Excellent performance in linear and digital circuits
- Compact, rugged, light weight
- Low coupling capacitance 1.0pF

The MCD2 is intended for applications where a high degree of input to output isolation is required to provide unilateral signal transfer with ultra fast speed of response such as in:

- High speed isolated amplifiers
- High speed pulse transformers
- High frequency opto-electronic feedback circuits
- High speed isolated logic switch

ABSOLUTE MAXIMUM RATINGS

Storage temperature -55°C to 150°C
 Operating temperature -55°C to 100°C
 Lead soldering time @ 260°C — 7.0 seconds

LED (GaAs Diode)

Power dissipation @ 25°C ambient 150mW
 Derate linearly from 25°C 2.0mW/ $^{\circ}\text{C}$
 Continuous forward current 100mA
 Reverse voltage 3.0 volts
 Peak forward current (1 μsec pulse, 300 pps) 3.0 Amp

DETECTOR (Silicon PIN Photo Diode)

Power dissipation @ 25°C ambient 100mW
 Derate linearly from 25°C 3.33mW/ $^{\circ}\text{C}$
 Reverse voltage 50 volts
 ISOLATION VOLTAGE 1500 VDC

ELECTRO-OPTICAL CHARACTERISTICS (25°C Free Air Temperature Unless Otherwise Specified)

CHARACTERISTICS	MIN.	TYP.	MAX	UNIT	TEST CONDITIONS
Emitter					
Forward voltage		1.3	1.5	V	$I_F = 100 \text{ mA}$
Reverse current		0.15	10	μA	$V_R = 3.0 \text{ V}$
Capacitance		150		pF	$V = 0$
Detector					
Reverse breakdown voltage	50	75		V	$I_R = 10 \mu\text{A}$
Dark current (I_R)		15	50	nA	$V_R = 20 \text{ V}$
Capacitance		10		pF	$V_R = 20 \text{ V}$
Capacitance		40		pF	$V = 0$
Coupled					
DC current transfer ratio	0.15	0.2		%	$I_F = 50 \text{ mA}, V_R = 20 \text{ V}$
Breakdown voltage	1500			VDC	
	850			V _{RMS}	60 Hz
Resistance	10^{11}	10^{12}		Ω	$V_{ED} = 500 \text{ V}$
Capacitance		1.0		pF	
Bandwidth (See Figure 6)		5.0		MHz	Note 2
Rise and fall time		20		ns	$R_L = 50 \Omega$

TYPICAL ELECTRO-OPTICAL CHARACTERISTIC CURVES (25°C Free Air Temperature Unless Otherwise Specified)

Fig. 1. Steady-State AC Voltage Limit of Isolation Dielectric

Fig. 2. Output Current vs. Voltage

Fig. 3. Output Current vs. Input Current

Fig. 4. Photo Current vs. Temperature

Fig. 5. Dark Current vs. Temperature

Fig. 6. Typical Operating Schematics

Fig. 7. Output vs. Frequency

Fig. 8. Modulation Circuit Used to Obtain Figure 6

NOTES

1. The current transfer ratio (I_2/I_1) is the ratio of detector output current @ $V_R = 20$ volts, to the emitter input current. This ratio is linear to within $\pm 5\%$ for an emitter input from 20 mA to 80 mA.
2. The frequency at which the time-averaged sinusoidal current ratio is half the DC current value.
3. For design information send for Applications Handbook.
4. See Application Note AN514.

Monsanto

PHOTODIODE OPTO-ISOLATOR

MCD4

PRODUCT DESCRIPTION

The MCD4 is a standard four-lead, TO18 package containing a gallium arsenide light emitting diode optically coupled to a silicon planar PIN photodiode. It is hermetically sealed for Military Applications.

PACKAGE DIMENSIONS

FEATURES & APPLICATIONS

- Ultra fast switching time
- Very high isolation resistance - $10^{11} \Omega$
- 1500 volt isolation emitter to detector
- Hermetic package
- Long life - solid state reliability
- Excellent performance in linear and digital circuits
- Compact, rugged, light weight
- Low coupling capacitance 1.8 pF

The MCD4 is intended for applications where a high degree of input to output isolation is required to provide unilateral signal transfer with ultra fast speed of response such as in:

- High speed isolated amplifier
- High speed pulse transfers
- High frequency opto electronic feedback circuits
- High speed isolated logic switch

ABSOLUTE MAXIMUM RATINGS

Storage temperature -65°C to 150°C
Operating temperature -55°C to 125°C
Lead soldering time @260°C - 10.0 seconds

LED (GaAs Diode)

Power dissipation @ 25°C ambient 150 mW
Derate Linearly from 25°C 1.5 mW/°C
Continuous forward current 100 mA
Reverse voltage 3.0 V
Peak forward current (1 μ s pulse, 300 pps) . . 3.0 A

DETECTOR (PIN photodiode)

Power dissipation @25°C ambient 100 mW
Derate linearly from 25°C 1.0 mW/°C
Reverse voltage 50 V
ISOLATION VOLTAGE 1000 VDC

ELECTRO-OPTICAL CHARACTERISTICS

(25°C Free Air Temperature Unless Otherwise Specified)

CHARACTERISTICS	MIN.	TYP.	MAX.	UNITS	TEST CONDITIONS
Emitter					
Forward voltage		1.3	1.5	V	$I_F = 40 \text{ mA}$
Reverse current		0.15	10	μA	$V_R = 3.0 \text{ V}$
Capacitance		150		pF	$V = 0$
		400		pF	$V = 1 \text{ V}$
Detector					
Reverse breakdown voltage	50	75		V	$I_R = 10 \mu\text{A}$
Dark current		5	25	nA	$V_R = 20 \text{ V}$
Capacitance		10		pF	$V_R = 20 \text{ V}$
Capacitance		40		pF	$V = 0$
Coupled					
DC current transfer ratio	0.10	0.15		%	$V_R = 20 \text{ V}$
Breakdown voltage	1000			V	
Resistance emitter-detector		10^{11}		Ω	$V_{E-D} = 500 \text{ V}$
Capacitance LED to detector		1.8		pF	
Bandwidth (see figure 6)		5.0		MHz	note 2
Rise time and fall time		20		ns	50 Ω system

TYPICAL ELECTRO-OPTICAL CHARACTERISTIC CURVES (25°C Free Air Temperature Unless Otherwise Specified)

Figure 1 Output Current vs. Voltage

Figure 2 Output Current vs. Input Current

Figure 3 Current vs. Temperature

Figure 4 Dark Current vs. Temperature

Figure 5 Output vs. Frequency

Figure 6 Typical Operating Schematics

Figure 7 Modulation Circuit Used to Obtain Figure 6

NOTES

1. The current transfer ratio (I_2/I_1) is the ratio of detector output current @ $V_R = 20$ volts, to the emitter input current. This ratio is linear to within $\pm 5\%$ for an emitter input from 20 mA to 60 mA.
2. The frequency at which the time-averaged sinusoidal current ratio is half the DC current value.
3. For design information send for Application Notes Handbook.

GENERAL DESCRIPTION

The RM4136 and RC4136 include four independent high gain operational amplifiers internally compensated and constructed on a single silicon chip using the planar epitaxial process.

These amplifiers meet or exceed all specifications for 741 type amplifiers. Excellent channel separation allows the use of the 4136 quad amplifiers in all 741 operational amplifier applications providing the highest possible packaging density.

The specially designed low noise input transistors allow the 4136 to be used in low noise signal processing applications such as audio preamplifiers and signal conditioners.

DESIGN FEATURES

- Unity Gain Bandwidth: 3MHz
- Continuous Short Circuit Protection
- No Frequency Compensation Required
- No Latch-up
- Large Common Mode and Differential Voltage Ranges
- Low Power Consumption
- Parameter Tracking Over Temperature Range
- Gain and Phase Match Between Amplifiers

SCHEMATIC DIAGRAM

CONNECTION INFORMATION

ABSOLUTE MAXIMUM RATINGS

Supply Voltage	RM4136 ±12V TV4136 RC4136 ±18V	Storage Temperature Range	-65°C to +150°C
Internal Power Dissipation (Note 1)	800mW	Operating Temperature Range	RM4136 -55°C to +125°C RC4136 0°C to +70°C
Differential Input Voltage	±30V	Lead Temperature (Soldering 60s)	RV4136 -40°C to +85°C
Input Voltage (Note 2)	±15V	Output Short Circuit Duration (Note 3)	300°C Indefinite

ELECTRICAL CHARACTERISTICS (V_{CC} = ±15V, T_A = +25°C unless otherwise noted)

PARAMETER	CONDITIONS	RM4136			RV4136 RC4136			UNITS
		MIN	TYP	MAX	MIN	TYP	MAX	
Input Offset Voltage	R _S ≤ 10 kΩ		0.5	5.0		0.5	6.0	mV
Input Offset Current			5.0	200		5.0	200	nA
Input Bias Current			40	500		40	500	nA
Input Resistance		0.3	5.0		0.3	5.0		MΩ
Large Signal Voltage Gain	R _L ≥ 2 kΩ V _{OUT} = ±10V	50,000	100,000		20,000	100,000		V/V
Output Voltage Swing	R _L ≥ 10 kΩ	±12	±14		±12	±14		V
	R _L ≥ 2 kΩ	±10	±13		±10	±13		V
Input Voltage Range		±12	±14		±12	±14		V
Common Mode Rejection Ratio	R _S ≤ 10 kΩ	70	100		70	100		dB
Supply Voltage Rejection Ratio	R _S ≤ 10 kΩ		10	150		10	150	pV/V
Power Consumption	P _{LE} = All Outputs		210	340		210	340	mW
Transient Response (slew rate)	V _{IN} = 20 mV R _L = 2 kΩ C _L ≤ 100 pF							V/μs
Rise time			0.13			0.13		μs
Overhoot			5.0			5.0		%
Unity Gain Bandwidth			3.0			3.0		MHz
Slew Rate (unity gain)	R _L ≥ 2 kΩ		1.5			1.0		V/μs
Channel Separation	f = 10 kHz							dB
Open loop	R _S = 1 kΩ		105			105		dB
	f = 10 kHz R _S = 1 kΩ		105			105		dB

The following specifications apply for -55°C ≤ T_A ≤ +125°C for RM4136, 0°C ≤ T_A ≤ +70°C for RC4136.

Input Offset Voltage	R _S ≤ 10 kΩ		6.0			7.5		mV
Input Offset Current			500			300		nA
Input Bias Current			1500			800		nA
Large Signal Voltage Gain	R _L ≥ 2 kΩ V _{OUT} = ±10V	25,000			15,000			V/V
Output Voltage Swing	R _L ≥ 2 kΩ	±10			±10			V
Power Consumption	T _A = High		185	300		180	300	mW
	T _A = Low		245	400		240	400	mW

NOTES

1. Storage limits for lead temperatures are +250°C absolute for up to 4 hours for temperatures above +250°C.
2. For supply voltages less than ±15V, the standard maximum input voltage is equal to the supply voltage.
3. Short circuit may be no load or one amplifier only. I_{CC} = 45mA typical.

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MAXIMUM RATINGS (T_C = +25°C unless otherwise noted)

Rating	Symbol	Value			Unit	
Input Voltage	$V_{CC}, V_{EE} $	30			Vdc	
Peak Load Current	I_{PK}	100			mA	
Power Dissipation and Thermal Characteristics $T_A = +25^{\circ}\text{C}$ Derate above $T_A = +25^{\circ}\text{C}$ Thermal Resistance, Junction to Air $T_C = +25^{\circ}\text{C}$ Derate above $T_C = +25^{\circ}\text{C}$ Thermal Resistance, Junction to Case	P_D $1/\theta_{JA}$	G Package	R Package	L Package	Watts $\text{mW}/^{\circ}\text{C}$ $^{\circ}\text{C}/\text{W}$	
		0.8 5.4	2.4 16	1.0 6.7		
	θ_{JA} P_D $1/\theta_{JC}$	165 2.1 14	62 9.0 61	150 2.5 20	$^{\circ}\text{C}/\text{W}$ Watts $\text{mW}/^{\circ}\text{C}$ $^{\circ}\text{C}/\text{W}$	
		θ_{JC} 70	17	50		
	Storage Junction Temperature Range	T_J, T_{stg}	-65 to +175			$^{\circ}\text{C}$
	Minimum Short Circuit Resistance	$R_{SC(\text{min})}$	4.0			Ohms

OPERATING TEMPERATURE RANGE

Ambient Temperature	T _A	°C
MC1468 MC1568	0 to +75 -55 to +125	

ELECTRICAL CHARACTERISTICS (V_{CC} = +20 V, V_{EE} = -20 V, C₁ = C₂ = 1500 pF, C₃ = C₄ = 1.0 μF, R_{SC}⁺ = R_{SC}⁻ = 4.0 Ω, I_L⁺ = I_L⁻ = 0, T_C = +25°C unless otherwise noted) (See Figure 1)

Characteristic	Symbol*	MC1568			MC1468			Unit
		Min	Typ	Max	Min	Typ	Max	
Output Voltage	V _O	±14.8	±15	±15.2	±14.5	±15	±15.5	V _{dc}
Input Voltage	V _{in}	-	-	±30	-	-	±30	V _{dc}
Input Output Voltage Differential	V _{in} - V _O	2.0	-	-	2.0	-	-	V _{dc}
Output Voltage Balance	V _{Bal}	-	±50	±150	-	±50	±300	mV
Line Regulation Voltage (V _{in} = 18 V to 30 V) I _{Tlow} ^① to I _{Thigh} ^②	Reg _{in}	-	-	10	-	-	10	mV
		-	-	20	-	-	20	
Load Regulation Voltage (I _L = 0 to 50 mA, T _J = constant) (T _A = T _{low} to T _{high})	Reg _L	-	-	10	-	-	10	mV
		-	-	30	-	-	30	
Output Voltage Range L Package (See Figure 4) R and G Packages (See Figures 2 and 1J)	V _{OR}	±8.0	-	±20	±8.0	-	±20	V _{dc}
		±14.5	-	±20	±14.5	-	±20	
Ripple Rejection (f = 120 Hz)	RR	-	75	-	-	75	-	dB
Output Voltage Temperature Stability (T _{low} to T _{high})	TSV _O	-	0.3	1.0	-	0.3	1.0	%
Short Circuit Current Limit (R _{SC} = 10 ohms)	I _{SC}	-	60	-	-	60	-	mA
Output Noise Voltage (BW = 100 Hz - 10 kHz)	V _N	-	100	-	-	100	-	μV(RMS)
Positive Standby Current (V _{in} = +30 V)	I _B ⁺	-	2.4	4.0	-	2.4	4.0	mA
Negative Standby Current (V _{in} = -30 V)	I _B ⁻	-	1.0	3.0	-	1.0	3.0	mA
Long Term Stability	ΔV _O /Δt	-	0.2	-	-	0.2	-	%/k Hr

① T_{low} = 0°C for MC1468
= -55°C for MC1568

② T_{high} = +75°C for MC1468
= +125°C for MC1568

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$$N_{\text{L}} = 10^4, \quad N_{\text{H}} = 10^5$$

FIGURE 2 - VOLTAGE ADJUST AND BALANCE ADJUST CIRCUIT

Balance adjust available in F4C 85681, MC 14681, requires dual as line package only

FIGURE 4 – OUTPUT VOLTAGE ADJUSTMENT
FOR $8.0\text{ V} \leq |V_O| \leq 14.5\text{ V}$
(Ceramic-Packaged Devices Only, $L \geq 10\mu\text{H}$)

Year	2012	2013	2014	2015
1	1.2%	0.5%	1.0%	1.0%
2	1.0%	0.2%	0.5%	0.5%
3	0.5%	0.0%	0.0%	0.0%
4	0.0%	0.0%	0.0%	0.0%
5	0.0%	0.0%	0.0%	0.0%

$V_{\text{max}} = 2.5 \text{ A}$
 $V_{\text{max}} = 2.5 \text{ V}$
 $V_{\text{max}} = 2.5 \text{ V}$

is believed to be entirely reliable. However, no responsibility is assumed for inaccuracies. Furthermore, such information does not convey to the purchaser of the semiconductor devices described any license under the patent rights of Motorola Inc. or others.

$V_{CC} = +20\text{ V}$, $V_{EE} = -20\text{ V}$, $V_O = \pm 15\text{ V}$, $T_A = +25^\circ\text{C}$ unless otherwise noted

$V_{CC} = +20\text{ V}$, $V_{EE} = -20\text{ V}$, $V_O = \pm 15\text{ V}$, $T_A = +25^\circ\text{C}$ unless otherwise noted

Figure 1 is a line graph showing Output Voltage Regulation (mV) on the y-axis (0 to 7.0) versus Percent on the x-axis (0 to 100). The graph displays two sets of curves for negative and positive output voltages. The top set of curves is labeled 'NEGATIVE OUTPUT' and the bottom set is labeled 'POSITIVE OUTPUT'. Each set contains two curves for the temperature range $55^{\circ}\text{C} < T_A < +125^{\circ}\text{C}$ and $+35^{\circ}\text{C} < T_A < +125^{\circ}\text{C}$. The parameters are $R_{EC} = 4.0 \text{ OHMS}$ and $T_J = T_A$.

Graph showing Minimum Input Output Voltage Differential (Volts) versus Load Current (mA) for Positive and Negative Regulators. The graph includes the following parameters:

- $R_{SC} = 4.0 \text{ OHMS}$
- $\Delta V_O \leq 100 \text{ mV}$

The Y-axis represents the Minimum Input Output Voltage Differential (Volts), ranging from 0 to 4.0. The X-axis represents the Load Current (mA), ranging from 0 to 100. Two curves are plotted:

- POSITIVE REGULATOR**: This curve starts at approximately 2.8V at 0mA and decreases linearly to about 1.8V at 100mA.
- NEGATIVE REGULATOR**: This curve starts at approximately 1.8V at 0mA and decreases linearly to about 0.8V at 100mA.

Load Current (mA)	Positive Regulator Voltage (V)	Negative Regulator Voltage (V)
0	2.8	1.8
20	2.5	1.6
40	2.2	1.4
60	1.9	1.2
80	1.7	1.0
100	1.5	0.8

The graph shows the relationship between Total Load Current (mA) and Ambient Temperature (°C) for different packages and heat sink conditions. The y-axis ranges from 0 to 200 mA, and the x-axis ranges from -65 to +125 °C. The legend indicates:

- NO HEATSINK (solid line)
- INFINITE HEATSINK (dashed line)
- G PACKAGE (solid line with triangles)
- L PACKAGE (solid line with circles)
- R PACKAGE (solid line with squares)

 The graph also specifies $V_{in} = V_O = 3.0V$ and $V_{CE} = 1V$. The R PACKAGE curve is labeled with $P_{(1558)}$ and $P_{(1458)}$ at different temperatures.

The graph shows the relationship between the input-output voltage differential and the total load current for three different packages (1, 6, and 8) under two conditions: no heat sink and infinite heat sink. The y-axis represents the total load current in milliamperes (mA), ranging from 0 to 200. The x-axis represents the input-output voltage differential in volts ($V_{in} - V_{out}$), ranging from 0 to 16. The curves for the 1 and 6 packages show a decreasing trend in current as the voltage differential increases, while the 8 package curve shows an increasing trend. The 'NO HEATSINK' curves are generally higher than the 'INFINITE HEATSINK' curves for the 1 and 6 packages.

$V_{in} - V_{out}$ (V)	1 PACKAGE (NO HEATSINK)	1 PACKAGE (INFINITE HEATSINK)	6 PACKAGE (NO HEATSINK)	6 PACKAGE (INFINITE HEATSINK)	8 PACKAGE (NO HEATSINK)	8 PACKAGE (INFINITE HEATSINK)
0	180	160	140	120	100	80
2	170	150	130	110	90	70
4	160	140	120	100	80	60
6	150	130	110	90	70	50
8	140	120	100	80	60	40
10	130	110	90	70	50	30
12	120	100	80	60	40	20
14	110	90	70	50	30	10
16	100	80	60	40	20	0

Graph 1: I_{SC} vs R_{SC} for 10mA

R_{SC} (OHMS)	I_{SC} (mA)
40	100
80	75
160	40
240	25

The graph shows the limiting current (mA) on the y-axis (0 to 100) versus junction temperature (°C) on the x-axis (-75 to +125). Two curves are plotted: RSC-20 (0.1A IS) and RSC-10 (0.1A IS). Both curves show a linear increase in limiting current with temperature, with RSC-20 having a higher limiting current than RSC-10 across the entire temperature range.

Junction Temperature (°C)	RSC-20 Limiting Current (mA)	RSC-10 Limiting Current (mA)
-75	~15	~10
-50	~20	~15
-25	~25	~20
0	~30	~25
25	~35	~30
50	~40	~35
75	~45	~40
100	~50	~45
125	~55	~50

TYPICAL CHARACTERISTICS (continued)
 $V_{CC} = +20\text{ V}$, $V_{EE} = -20\text{ V}$, $V_O = \pm 15\text{ V}$, $T_A = +25^\circ\text{C}$ unless otherwise noted

FIGURE 11 - STANDBY CURRENT DRAIN

FIGURE 12 - STANDBY CURRENT DRAIN

FIGURE 13 - TEMPERATURE COEFFICIENT OF OUTPUT VOLTAGE

FIGURE 14 - LOAD TRANSIENT RESPONSE

FIGURE 15 - LINE TRANSIENT RESPONSE

FIGURE 16 - RIPPLE REJECTION

TYPICAL CHARACTERISTICS (continued)

$V_{CC} = +20\text{ V}$, $V_{EE} = -20\text{ V}$, $V_O = \pm 15\text{ V}$, $T_A = +25^\circ\text{C}$ unless otherwise noted.)

FIGURE 17 — OUTPUT IMPEDANCE

OUTLINE DIMENSIONS

CASE 614 METAL PACKAGE R SUFFIX

DIM	MILLIMETERS		INCHES	
	MIN	MAX	MIN	MAX
A	—	31.80	—	1.252
B	11.94	12.70	0.470	0.500
C	6.35	8.64	0.250	0.340
D	0.71	0.81	0.028	0.032
E	1.27	1.90	0.050	0.075
F	36° BSC	36° BSC	—	—
G	8.26 BSC	8.26 BSC	—	—
H	24.33	24.43	0.958	0.962
J	12.17	12.22	0.479	0.481
K	9.14	—	0.360	—
Q	3.61	3.86	0.142	0.152
R	—	17.78	—	0.700

Weight = 6.315 grams

CASE 603 METAL PACKAGE TO-100 G SUFFIX

Weight = 0.918 gram

DIM	MILLIMETERS		INCHES	
	MIN	MAX	MIN	MAX
A	8.51	9.39	0.335	0.370
B	7.75	8.51	0.305	0.335
C	4.19	4.70	0.165	0.185
D	0.407	0.533	0.016	0.021
E	—	1.02	—	0.040
F	0.406	0.463	0.016	0.019
G	5.84 BSC	—	0.230 BSC	—
H	0.712	0.864	0.028	0.034
J	0.737	1.14	0.029	0.045
K	12.70	—	0.500	—
L	6.35	12.70	0.250	0.500
M	36° BSC	—	36° BSC	—
N	—	1.27	—	0.050
Q	3.56	4.06	0.140	0.160
R	0.254	1.02	0.010	0.040

All JEDEC dimensions and notes apply.

CASE 632 CERAMIC PACKAGE TO-116 L SUFFIX

DIM	MILLIMETERS		INCHES	
	MIN	MAX	MIN	MAX
A	16.8	19.9	0.660	0.785
B	5.50	7.11	0.220	0.280
C	—	5.08	—	0.200
D	0.381	0.508	0.015	0.020
E	0.77	1.77	0.030	0.070
F	2.54 BSC	—	0.100 BSC	—
G	0.203	0.281	0.008	0.011
H	2.54	—	0.100	—
J	7.62 BSC	—	0.300 BSC	—
K	—	15°	—	15°
L	0.51	0.76	0.020	0.030
M	—	8.25	—	0.325

All JEDEC dimensions and notes apply.

Weight = 1.954 grams

NOTE:
DIMENSION "L" TO CENTER OF
LEADS WHEN FORMED PARALLEL.

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APPENDIX F

ANALYSIS OF THE ELECTRONIC MEASUREMENT OF LOSSES BY USE OF A SECONDARY SENSE COIL WOUND ON THE MAIN SUPERCONDUCTING COIL

The purpose of this analysis is to derive an expression for the signal voltage supplied to the input of the integrator (see Appendix A, Derivation of Relationship Between Hysteresis Loop and Loss During Cycle) in terms of the magnetization of the superconductor. The derivation generally follows the method given by Wilson¹ but is modified to take into account the influence of the magnetization on a secondary sense coil wound in close proximity to the coil on which losses are being measured.

The circuit schematic on which the analysis is based is shown in Figure F1. The significance of the circuit elements is described below.

L_1 is the superconducting coil on which the magnetization measurements are being made and V_1 is the voltage across its terminals. L_3 is the secondary sense coil wound around the superconducting coil. L_2 is a fictitious coil which gives the hysteretic effect and V_2 is a voltage element whose voltage is such a function of the superconducting coil current and field as to ensure that the hysteresis is of the correct value for the particular material. It is not necessary for this analysis to know the variation of V_2 . L_{mn} are the mutual inductances between the three circuits. R is the high resistance voltage divider; f is the fraction of the voltage divider used in the loss experiment and V_s is the signal voltage which is fed to the integrator.

The circuit equations which describe these circuits are as follows:

$$V_1 + L_1 \frac{di_1}{dt} + L_{21} \frac{di_2}{dt} + L_{31} \frac{di_3}{dt} = 0 \quad (1)$$

$$V_2 + L_2 \frac{di_2}{dt} + L_{12} \frac{di_1}{dt} + L_{32} \frac{di_3}{dt} = 0 \quad (2)$$

$$V_s + fV_1 + L_3 \frac{di_3}{dt} + L_{13} \frac{di_1}{dt} + L_{23} \frac{di_2}{dt} = 0 \quad (3)$$

If R is very large the current in circuit 3 (i_3) will be very small and can, therefore, be neglected. The equations then reduce to:

$$V_1 + L_1 \frac{di_1}{dt} + L_{21} \frac{di_2}{dt} = 0 \quad (1)$$

$$V_2 + L_2 \frac{di_2}{dt} + L_{12} \frac{di_1}{dt} = 0 \quad (2)$$

$$V_s + fV_1 + L_{13} \frac{di_1}{dt} + L_{23} \frac{di_2}{dt} = 0 \quad (3)$$

Manipulating equations 1 and 3 to eliminate V_1 gives:

$$V_s = fL_1 \frac{di_1}{dt} - L_{13} \frac{di_1}{dt} + fL_{21} \frac{di_2}{dt} - L_{23} \frac{di_2}{dt}$$

Writing the currents associated with circuit 2 in terms of fluxes this becomes:

$$V_s = (fL_1 - L_{13}) \frac{di_1}{dt} + f \frac{d\phi_{21}}{dt} - \frac{d\phi_{23}}{dt}$$

where ϕ_{21} is the flux due to circuit 2 linking circuit 1 and ϕ_{23} is the flux due to circuit 2 linking circuit 3. As indicated in Appendix A, $fL_1 - L_{13}$ can be made as small as is desired by suitable selection of f and L_{13} . It should be noted that as shown in Appendix A it is not necessary for the validity of the method to make $fL_1 - L_{13}$ exactly equal to zero. However, for simplicity, in this analysis it is assumed that this term is exactly zero.

The signal voltage then becomes:

$$V_s = f \frac{d\phi_{21}}{dt} - \frac{d\phi_{23}}{dt}$$

The first term is the influence of the magnetization on the voltage across the main coil. The second term is the influence of the magnetization on the voltage across the secondary sense coil. If the pickup coil were on a current lead far from the region influenced by the magnetization of the superconducting coil then the second term would disappear (i.e., $L_{23} \rightarrow 0$) and we would be left with the expression for the loss voltage previously derived. It only remains to derive expressions for the flux linkage in terms of the magnetization (see also reference 1).

ϕ_{21} - The Flux Due to Circuit 2 Linking Circuit 1

Consider an elemental dipole loop at a point p in the magnet occupying an elemental volume Δv_p as shown in Figure F2.

The flux due to this loop linking the whole of circuit 1 is ϕ_{p1} and:

$$\phi_{p1} = i_p L_{p1}$$

where L_{p1} is the mutual inductance between the loop and the winding 1.

Conversely, the flux due to the whole of winding 1 linking this elemental loop is ϕ_{1p} and:

$$\phi_{1p} = i_1 L_{1p} = B_{1p} A_p$$

Figure F-2. Elemental Dipole Loop at Point p in Magnet.
 A_p is the Area of the Loop and i_p is the Current.

Figure F-3. Load Lines of Arbitrary Points in
 Superconducting Coil Winding.

where i_1 is the current in circuit 1:

L_{1p} is the mutual inductance between the winding and the loop;

B_{1p} is the flux density due to the winding at the loop at point p ; and

A_p is the area of the loop.

$$\text{therefore, } L_{1p} = \frac{B_{1p}}{i_1} A_p$$

$$\text{but, } \text{but } L_{1p} = L_{p1}$$

$$\text{therefore, } L_{p1} = \frac{B_{1p}}{i_1} A_p$$

The magnetization of the volume containing the elemental loop is:

$$M_p = \frac{\mu_0 i_p A_p}{\Delta v_p}$$

$$\text{so, } A_p = \frac{M_p \Delta v_p}{\mu_0 i_p}$$

$$\text{and therefore, } L_{p1} = \frac{B_{1p}}{i_1} \frac{M_p \Delta v_p}{\mu_0 i_p}$$

$$\text{since, } i_p L_{p1} = \phi_{p1}$$

$$\text{then, } \phi_{p1} = \frac{B_{1p}}{i_1} \frac{M_p \Delta v_p}{\mu_0}$$

The total flux due to circuit 2 linking circuit 1 then is:

$$\phi_{21} = \sum_p \frac{B_{1p}}{i_1} \frac{M_p \Delta v_p}{\mu_0} = \sum_{\text{volume of coil}} \frac{B_{1p}}{i_1} \frac{M_p \Delta v_p}{\mu_0}$$

However, $\frac{B_{1p}}{i_1}$ is merely the slope of the load line of the superconducting coil at point p and is, therefore, constant for all i_1 . We can write the following:

$$\left(\frac{B_1}{i_1} \right)_p = c_{1p} \left(\frac{B_m}{i_m} \right)_1$$

where C_{1p} is a constant depending on the position of the point in the magnet and $\left(\frac{B_m}{i_m}\right)_1$ is the load line of the peak field position. See Figure F3.

$$\text{therefore, } \phi_{21} = \frac{1}{\mu_0} \left(\frac{B_m}{i_m}\right)_1 \sum_{\substack{\text{coil} \\ \text{volume}}} C_{1p} M_p \Delta v_p$$

ϕ_{23} - The Flux Due to Circuit 2 Linking Circuit 3

An exactly similar analysis can be carried out for the flux linkage ϕ_{23} (where subscript 1 appears above substitute subscript 3). The result is:

$$\phi_{23} = \frac{1}{\mu_0} \left(\frac{B_m}{i_m}\right)_3 \sum_{\substack{\text{coil} \\ \text{volume}}} C_{3p} M_p \Delta v_p$$

$\left(\frac{B_m}{i_m}\right)_3$ is the load line of the peak field position of the secondary sense coil and C_{3p} is a constant relating to the load line of the secondary coil at a point p within the superconducting coil windings.

Recalling from earlier in the analysis that the signal voltage

$$V_s = f \frac{d\phi_{21}}{dt} - \frac{d\phi_{23}}{dt}$$

and substituting the expressions for ϕ_{21} and ϕ_{23} leads to:

$$V_s = f \frac{d}{dt} \left\{ \frac{1}{\mu_0} \left(\frac{B_m}{i_m}\right)_1 \sum_{\text{vol}} C_{1p} M_p \Delta v_p \right\} - \frac{d}{dt} \left\{ \frac{1}{\mu_0} \left(\frac{B_m}{i_m}\right)_3 \sum_{\text{vol}} C_{3p} M_p \Delta v_p \right\}$$

$$V_s = \frac{1}{\mu_0} \sum_{\text{vol}} \left\{ \left[\underset{\textcircled{1}}{\left(\frac{B_m}{i_m}\right)_1} C_{1p} - \underset{\textcircled{2}}{\left(\frac{B_m}{i_m}\right)_3} C_{3p} \right] \frac{dM_p}{dt} \Delta v_p \right\}$$

Terms 1 and 2 may be calculated readily for a given geometry and principle having calculated them, the energy loss signal may be interpreted in terms of the coil magnetization. However, it should be noted that terms 1 and 2 are of opposite signs, thereby resulting in some measure of cancellation and leading to the possibility of a greatly reduced signal voltage. During this program this procedure was not performed quantitatively. However, the effect may be seen qualitatively by referring to Figure IV-8 which shows two oscilloscope traces taken under very similar coil operating conditions. The upper trace was taken using a signal derived from the air core

Rogowski coil and the lower using a signal from the secondary coil wound on the outside of the superconducting coil. It can be seen from the trace areas and trace current and voltage sensitivities that the integrated signal from the lower trace is much smaller than that from the upper suggesting that the partial signal cancellation described above is occurring in the measurement made with the closely coupled secondary coil.

APPENDIX G

ROGOWSKI COIL DESIGN CONSIDERATIONS

A. Introduction

Figure G-1 is a schematic representation of what is known as a Rogowski loop or coil.¹ With the use of Maxwell's equations in integral form, it can be shown that the voltage V is proportional to the change in the current I linked by this loop. First observe that

$$\oint_S \vec{B} \cdot d\vec{S} = \mu_0 \int \vec{j} \cdot d\vec{A} = \mu_0 I \quad G-1$$

where $\vec{B}$ is the magnetic induction, $\vec{S}$ is the arc length along the loop, $\vec{A}$ the area of the loop, $\vec{j}$ the current density and I is the total current passing thru the loop.

In addition,

$$\int \vec{E} \cdot d\vec{s} = - \int \frac{\partial B}{\partial t} \cdot d\vec{a} \quad G-2$$

where $\vec{E}$ is the electric field, $\vec{s}$ is the arc length along the coil and $\vec{a}$ is the cross sectional area enclosed by the winding.

Then, for unit arc length of $\vec{S}$, Eq. G-2 can be integrated so that

$$\epsilon = na \frac{dB_S}{dt} \quad G-3$$

Where ϵ is the E.M.F. induced in the coil per unit length along $\vec{S}$, n is the number of turns per unit length along $\vec{S}$ and B_S is the component of $\vec{B}$ along $\vec{S}$. Then, from G-3

$$B_S = \frac{1}{na} \int \epsilon dt \quad G-4$$

so that

$$\mu_0 I = \int \vec{B} \cdot d\vec{S} = \int B_S dS = \frac{1}{na} \int v dt \quad G-5$$

where $v = \epsilon dS$ is the total voltage induced across the Rogowski loop.

If an integrating circuit such as that shown in G-1 is used, then it is well known that the output of this circuit, V , is given by

$$V = \frac{1}{RC} \int v dt \text{ for } RC \gg \tau$$

Schematic Representation of the Rogowski Coil

Figure G-1

where τ is the time over which v is to be integrated. Hence, finally

$$I = \frac{1}{\mu_0} \frac{RC}{na} v \quad G-6$$

Note that this result is independent of the shape of the Rogowski loop and depends only on the choice of RC and na . Since n is the number of turns per unit length, this result is independent of the size (length) of the Rogowski loop. Also, since it is independent of the current distribution thru the loop, it can be easily and accurately calibrated by simply placing a conductor anywhere thru the loop and passing a known current thru it. Care must be exercised when Rogowski coils are used to prevent sensitivity of the coil to changes in the magnetic field passing through the plane of the coil. This can be done by connecting one end of the coil to a conductor passing through the center or along the outside of the coil or by winding the coil in an even number of layers so that both ends of the winding terminate at the same end of the loop.

Two different coil designs were investigated for use in the instrumentation system for this program. The first made use of air core coil and the second a ferrite core coil. The following are discussions of the design considerations that were made for each of these coils.

B. Air Core

1. The following parameters are specified:

- a) Maximum Current--this will specify the conductor size required if you include the duty cycle and longest pulse time.
- b) Mutual Inductance

$$e_{out} = M \frac{di_{in}}{dt}$$

where

$$M = \left(\frac{\mu_0 N_1 N_2 A}{\ell} \right) k$$

The primary and secondary numbers of turns are N_1 and N_2 , core cross sectional area is A , mean core length is ℓ , permeability of free space is μ_0 , coupling factor, k , is near one.

- c) Maximum operating frequency, amplitude distortion, phase shift
If the self resonant frequency of the coil is 5 times greater than the maximum operating frequency, then the maximum amplitude distortion is about 4 percent.

In general:

$$\text{Distortion Fraction} = \left(\frac{V_{\text{out}}}{V_{\text{nom}}} - 1 \right) = \frac{1}{\sqrt{\left[\left(\frac{f}{f_0} \right)^2 - 1 \right]^2 + \left(\frac{f}{f_0} \frac{R}{2\pi f_0 L} \right)^2}} - 1$$

where f_0 is the self-resonant frequency, R is the resistance of the output winding of inductance L , f is the operating frequency.

The phase shift angle, θ (referred to the undistorted output voltage, V_{nom}), is given by:

$$\tan \theta = \left\{ \frac{\frac{f}{f_0} \frac{R}{2\pi f_0 L}}{1 - \left(\frac{f}{f_0} \right)^2} \right\}$$

The relations for distortion fraction and phase shift (both specified) can be solved for the parameters:

$$\left(\frac{f}{f_0} \right) \quad \text{and} \quad \left(\frac{R}{2\pi f_0 L} \right)$$

Because the maximum operating frequency, f , is specified, the resonant frequency f_0 can now be found. The resonant frequency is given by

$$f_0 = \frac{1}{2\pi \sqrt{LC}}$$

where the winding capacitance, C , is usually in the range of 100 pF to 200 pF for coils of a few thousand turns on a moderate size (characteristic dimension about 6 inches) coil. This relation specifies the self inductance, L , of the coil, where

$$L = \frac{\mu_0 N^2 A}{l}$$

Then the coil resistance, R , can be found.

- d) Number of turns on the Primary, N_1 (usually one, but need not be).
2. The following parameters can then be calculated:
 - a) Coil self-resonant frequency, f_0 , coil self inductance, L , maximum coil resistance, R , as shown in 1 c) above.
 - b) The turns ratio,

$$\frac{N_2}{N_1} = \frac{L}{Mk} \quad (k \text{ is near unity}),$$

which then gives the secondary number of turns, N_2 . Given N_2 and R , the minimum wire size is specified.

- c) Ratio of coil cross-sectional area to coil length,

$$\frac{A}{l} = \frac{L}{\mu_0 N^2}$$

- d) Final values of A and l are determined by the space requirements of the output winding (fully specified) maximum frequency, amplitude distortion, phase shift, and mutual inductance of the input winding (fully specified by max. current, pulse length, and type of cooling), and of the electrostatic shield between primary and secondary.

C. Ferrite Core

1. The following parameters are specified.

- a) Maximum current, mutual inductance, maximum operating frequency, maximum amplitude distortion, maximum phase shift, number of turns on the primary, N_1 . All these are exactly as for the air core case except that the permeability, μ , is much greater than that for air, μ_0 .
- b) Maximum core flux density, B_{max} . This is limited by available materials. A typical value is 5 kilogauss; above this, the core properties (especially μ) change very rapidly leading to severe amplitude distortion. The non-linear properties of a ferrite core are much reduced in practice by inserting an air gap which produces a core with an effective μ of:

$$\frac{\mu_{eff}}{\mu_0} = \frac{1 + \frac{l_I}{l_g}}{1 + \frac{\mu_0}{\mu_I} \frac{l_I}{l_g}}$$

where l_I and l_g are the length of the iron and the length of the air gap, respectively, and μ_I is the permeability of the iron.

2. The following parameters can then be calculated:

- a) The resonant frequency, f_0 , the resistance, R, and the self inductance, L, of the secondary winding exactly as in 1 c) and 2 a) for the air core case, assuming a capacitance of 150 pF to 200 pF.

b) You now have the relations:

$$B_{\max} = \frac{\mu N_1 i_{\max}}{l}$$

$$M = \left(\frac{\mu N_1 N_2 A}{l} \right) k$$

$$L = \left(\frac{\mu N_2^2 A}{l} \right)$$

where $B_{\max}$, N_1 , $L_{\max}$, M , k , and L are specified.

The ratio of L to M gives N_2 , i.e.

$$N_2 = N_1 (L/M)$$

The ratio of $\frac{\mu}{l}$ is given by

$$\frac{\mu}{l} = \frac{B_{\max}}{N_1 i_{\max}}$$

The core cross section A , is now completely specified:

$$A = \frac{L N_1 i_{\max}}{N_2 B_{\max}}$$

The core length, l , is determined by the now specified conductor sizes for primary and secondary windings and the known core cross section.

The desired permeability, μ , is specified once the core length is selected. A practical upper bound on this number is set by core linearity requirements; it should not be more than about 10 percent of the permeability of the iron.

Given the desired permeability and the total core length, the air gap length is given by the relation in B 1 b).

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1. R. M. Patrick and A. M. Schneiderman, Axial Current Distribution in the Exhaust of the Magnetic Annular Arc, Avco Everett Research Laboratory Research Report 231, July 1966.

APPENDIX H

THE "7" kJ ENERGY STORAGE COIL AND ITS ASSOCIATED SECONDARY SENSE COIL

The specifications of the "7" kJ test coil are summarized in Table H - 1, and Figure H - 1 is a schematic drawn to scale showing its dimensions. The conductor used was a flattened tubular braid consisting of 48 individually insulated 4.5×10^{-3} inch diameter strands of single-core niobium-titanium/copper composite wire (copper to superconductor ratio 1.4:1). However, to obtain the desired operating current, the coil was wound with three of the 48-strand braids in parallel. The strands were insulated with polybondex approximately 0.25×10^{-3} inches thick. Each braid measured approximately 0.10 inches wide by 0.020 inches thick.

The coil mandrel and end flanges are of linen phenolic. The layers of windings are separated by strips of paper phenolic 0.031 inch thick by 0.12 inch wide placed on 0.50 inch centers. This provides 0.37 inch by 0.031 inch cooling passages. The interlayer insulation is mylar double wrapped and extends 0.50 inch beyond the ends of the windings to prevent voltage breakdown around the ends of layers. The space between the ends of the windings and the flanges is filled with .02 inch thick G10 to give support to the ends of the windings. The flanges have large sections cut away to ensure good helium ventilation. The windings are impregnated with epoxy to prevent premature quench caused by wire movement.

Figure H - 2 is a photograph illustrating the winding technique. After a layer of conductor was wound epoxy resin was brushed onto the exposed surface. Care was taken to ensure that there was sufficient epoxy resin to saturate the windings but that none dropped through the braid to block a cooling channel. While the epoxy resin was still wet, layers of mylar electrical insulation were wrapped around the windings. Figure H - 3 is a photograph of the completely wound coil. The three parallel braids can be clearly seen entering the coil at 120° intervals. The operating current, central field and peak field in Table VIII - 1 correspond to a stored energy of 7 kJ. These values are 0.65 times the values occurring when the coil is at the critical state. When the coil is in this situation, the current is 932 A, the central field is 3.12 Wb/m^2 , the peak field is 3.96 Wb/m^2 and the stored energy is 16.5 kJ.

Figure H - 4 is a schematic drawn to scale depicting the secondary sense coil in relation to the "7" kJ coil. The secondary coil consists of a 33-turn single layer winding mounted on a frame concentric with the "7" kJ coil but physically separate from the windings of the "7" kJ coil. Its self-inductance was found to be about 0.36 mH and the mutual inductance between it and the 7kJ were found to be about 2.4 mH. The voltage divider ratio measured when using the secondary coil as the inductive bucking coil during the loss measurement experiments was found to be .06.

TABLE H-1
SPECIFICATIONS OF "7" KJ ENERGY STORAGE COIL

Inner winding radius	6.35×10^{-2} m (2.54 inch)
Outer winding radius	12.2×10^{-2} m (4.81 inch)
Winding length	9.19×10^{-2} m (3.62 inch)
Operating current	616 A
Stored energy at operating current	7.0 kJ
Central flux density at operating current	1.93 Wb/m^2
Peak flux density at operating current	2.46 Wb/m^2
Inductance	36.9 mH
Number of Layers	39
Number of turns per layer	13
Total number of turns in coil	508

Conductor

Winding configuration	3 braids wound 3 in hand
Approximately braid dimensions	2.4×10^{-3} m x 5.1×10^{-4} m (0.094 inch x 0.020 inch)
Braid configuration	48 strands per braid
Total number of strands	144
Strand Diameter	1.14×10^{-4} m (4.5×10^{-3} inch)
Strand insulation material	Polybondex
Strand material	Nb/Ti in Cu Matrix
Copper to superconductor ratio	1.4
Number of filaments of Nb/Ti per strand	1

Voltage Insulation

Material	Mylar film
Configuration	4 layers between each layer of windings
Film thickness	1.3×10^{-5} m (0.5×10^{-3} inch)

Support Structure

None

Cooling Configuration

Cooling channels formed by axial slats 0.13 inch wide by 0.031 inch thick on 0.5 inch centers.

Dimensions in Inches

Figure H-1

Schematic Illustrating Dimensions of "7" kJ Energy Storage Coil.

Figure H-2

7 kilojoule model coil during winding. Upper illustration shows trifilar winding technique. Lower illustration shows application of epoxy resin to layer of conductor.

Figure H-3

7 kJ model energy storage coil. The coil is wound trifilar with three braids each consisting of 1×48 strands. Each strand is a single core niobium titanium copper composite with a copper to superconductor ratio of 1.4:1.

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Figure H-4. Sketch of 7kJ coil with sense coil added.

APPENDIX I

EXPERIMENTAL EVALUATION OF j_c -B CHARACTERISTICS OF CONDUCTOR USED TO BUILD THE "7" kJ COIL

To successfully predict the losses due to transient operation of superconductors it is of fundamental importance to know the form of the critical current density versus magnetic field strength relationship. Short sample measurements previously made on the conductor from which the "7" kJ coil was constructed indicated a linear relationship between the flux density (B) and critical current density (j_c) of the form:

$$j_c = j_0 \left(1 - \frac{B}{B_0} \right)$$

where j_0 and B_0 are constants. Previously derived loss formulae³ had used this relationship.

However, this is at variance with most published information concerning the j_c vs B relationship. It was concluded that this discrepancy was due to dubious experimental values of j_c at low fields. The wire under investigation has a low copper to superconductor ratio (1.4:1) and consequently is not very stable at low fields. This low field instability led to low values of j_c being recorded at low fields.

To remedy this situation further short sample tests were carried out on the conductor. To prevent premature quenching of the short sample ten 4.5×10^{-3} inch diameter strands of the conductor were stripped of insulation and soldered along their length to a piece of copper measuring 0.25 inch by 0.031 inch, thereby stabilizing the superconductor. The results are plotted in Figure I - 1 in the form of B versus $1/I$. It can be seen that this relationship is linear. This corresponds to a form

$$j_c(B + B_0) = j_0 B_0$$

i.e. hyperbolic which corresponds to the form generally accepted. A least squares fit to the data shown in Figure I - 1 gives $j_0 = 1.39 \times 10^{10}$ A/m², $B_0 = 0.559$ Wb/m².

Figure 1-1

Short Sample Data and Least Squares Fit for Conductor Used in Construction of "7" kJ Coil.

APPENDIX J

ANALYSIS OF THE RELATIONSHIP BETWEEN ENERGY LOSS AND DECAY OF COIL CURRENT DURING RINGING EXPERIMENTS

If a capacitor is discharged into a coil the current in the system will oscillate and the amplitude of the oscillations will decay as the energy in the circuit is dissipated. Provided the coil remains superconducting this dissipation will be due to the hysteresis losses in the superconductor and the ohmic losses in parts of the circuit external to the superconducting coil. Under certain circumstances, by a suitable choice of parameters, the ohmic losses can be made small in comparison with the hysteresis losses and under those circumstances the hysteresis losses can be inferred from the rate of decay of the oscillations.

The following analysis is an example of how the hysteresis losses may be inferred. The example is specific as far as the form of the hysteretic loss is concerned, i.e., $\Delta E \propto (k_1 i^3 + k_2 i^4)$ which applies at lower field before the superconductor is fully penetrated. However, similar analyses could be carried out for other forms of the energy loss.

Because the losses in a superconducting coil are a function of the peak current attained in a cycle, the decay of current in a ringing experiment must be faster than exponential. A good approximation to the predicted energy loss per cycle for the coil to be tested is:

$$\Delta E = 4 (k_1 i^3 + k_2 i^4)$$

where i is the peak current provided that the peak field of the coil is less than that which fully penetrates the conductor. The energy dissipated in a resistor, R , during a (sinusoidal) cycle is:

$$\Delta E = 1/2 i^2 R \tau$$

where τ is the time for one cycle. We can define an equivalent resistor, R , by equating these energy losses, ΔE , and using the fact that for a weakly damped series resonant circuit with inductance L and capacitance C ,

$$\begin{aligned} i &= \frac{V}{Z} \\ Z &= \sqrt{\frac{L}{C}} \\ &= 2\pi \sqrt{LC} \end{aligned}$$

which leads to the solution:

$$R = \frac{4}{\pi} k_1 \frac{V}{L} \left(1 + \frac{k_2}{k_1} V \right)$$

where V is the peak voltage during the cycle. This effective resistance, estimated from our loss analyses and the parameters of the conductor and coil used in our experiments, is plotted as a function of peak voltage (Figure J - 1). As long as the effective resistance, R , is significantly

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Figure J-1. Effective Ohmic Resistance to Simulate 7 "kJ" Coil Losses

less than the characteristic impedance of the circuit, $\sqrt{\frac{L}{C}}$, the oscillation will remain nearly sinusoidal at constant frequency during the decay of current in the circuit. However, the envelope will be different from exponential. We use the approximate expression:

$$V(t) = V_0 e^{-\frac{t}{2L} R}$$

where

$$R = R_w + \frac{4}{\pi} k_1 \frac{V(t)}{L} \left(1 + \frac{k_2}{k_1} V(t) \right)$$

and R_w , the wire resistance, is a few milliohms at low frequencies. If this equation is solved for $V(t)$ for a wire resistance of .05 ohm and parameters suitable to our coil

$$\left(\frac{4k_1}{\pi L} = 4.34 \times 10^{-6}, \quad \frac{k_2}{k_1} = 4.29 \times 10^{-3} \right),$$

the decay envelopes shown in Figure J - 2 are obtained. The equation can be rewritten:

$$\frac{2L}{t} \ln \frac{V_0}{V(t)} = R_w + \frac{4}{\pi L} k_1 V(t) + \frac{4}{\pi L} k_2 V^2(t)$$

If the initial voltage, V_0 and three points on the decay envelope are determined experimentally, it is possible to solve for the three constants R_w , k_1 , and k_2 , thereby determining an approximate curve fit to the coil loss curve.

Figure J-2. Decay Envelopes for Ringing Experiments on Coil ("7" kJ)

Solution to:

$$\frac{V_m(t)}{V_p} = e^{-\frac{R}{2L} t}$$

where $R = R_w + R_c$

$$R_w = .05 \, \Omega$$

$$R_c = 4.354 \times 10^{-6} V_m (1 + 4.293 \times 10^{-3} V_m)$$

APPENDIX K

DERIVATION OF A.C. LOSS FORMULAE

The treatment of the magnetization losses followed in this appendix is similar to that followed in Appendix I of Reference 3, some of which is summarized here for the convenience of the reader. The present treatment differs most importantly in that it takes into account the incomplete penetration of flux into the superconductor at low fields.

The instantaneous power dissipated per unit volume (p) in a superconductor is given by the product of the electric field (E) and the critical current density (j_c): -

$$p = E \cdot j_c$$

The energy losses for the whole coil are found by calculating this local power dissipation as a function of time and position and suitably integrating over the volume of the coil during the cycle.

In this derivation, for simplicity, a slab model of the superconductor has been chosen. The results derived thus will not be directly applicable to round wires. However, an approximate geometry factor to convert from rectangular to cylindrical geometry can be readily calculated. The geometry of the model used is shown in Figure K - 1 which illustrates a layer of rectangular superconductors immersed in a background field B which is parallel to the layer.

Furthermore, the treatment is for the case of zero transport current. The effect of ignoring this is to underestimate the losses. However, the correction factor is of the form $1 + (j_t/j_c)^2$ and so it becomes a significant effect only in the high field regions of the coil and then only during the part of the cycle when the transport current approaches the critical current. Overall this is a relatively small effect.

As the applied field is raised from 0 to B , shielding currents flow starting at the surface of the superconductor as shown schematically in Figure K - 2. These shielding currents screen the interior of the conductor from the applied field. Therefore, there is a variation in the local flux density in going from the surface to the interior of the superconductor. According to the critical state model the magnitude of the critical current density is a function of the local applied field. Therefore, strictly, the magnitude of the critical current density also varies across the width of the superconductor. However, in this derivation the simplifying assumption is made that the variation of B across the specimen is small and that the critical current density in the interior is determined by the value of the applied field at the surface.

As the applied field is raised from zero the shielding currents penetrate deeper into the interior. At first they do not extend to the center of the wire; this is the incomplete penetration situation. Eventually they extend to the center of the conductor and the conductor is fully penetrated. Figure K - 3 illustrates a typical $B - j_c$ curve for a superconductor. The field penetration and current density profile corresponding to the numbered points can be seen schematically in Figure K - 4.

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Figure K-1 Schematic Showing a Layer of Rectangular Superconductors Immersed in a Background Field B which is Parallel to the Layer.

Figure K-2 Schematic Illustrating the Shielding Currents Induced on the Surfaces of a Superconductor Immersed in a Field which Rises from Zero to B .

Figure K-3 Schematic Illustrating a Typical $B - j_c$ Curve for a Superconductor. The field penetration and current density profiles in a slab of superconductor corresponding to the points indicated can be seen in Figure K-4.

Figure K-4 Schematic Illustrating the Field Penetration Profiles and Critical Current Density Distributions in a Flat Slab of Superconductor as the Field is Raised from Zero to the Critical Field. See also Figure K-3 for points on the $B - j_c$ curve corresponding to the reference numbers above.

The flux profile can be calculated from the Maxwell relation $\nabla \times B = -\mu_0 j_c$. For the one dimensional case illustrated in Figure K - 5 this reduces to

$$\frac{dB}{dx} = -\mu_0 j_c$$

assuming j_c is independent of x then:-

$$B(x) = -\mu_0 j_c x + C$$

$$\text{for } -a < x < 0 \quad j_c = j_c^+$$

if $B = B(-a)$ at $x = -a$ then:-

$$B(x) = B(-a) - \mu_0 j_c^+ (x+a)$$

A similar expression can be derived for the region $0 < x < a$. The flux density profile expression is valid from the surface to the point x^* which is the electric center. The electric center is the point where the electric field vector is zero; it is also the point where there is a discontinuity in the critical current density.

Line 1 in Figure K - 5 illustrates the situation the first time the external field is raised from zero. Under these circumstances the position of the electric center is given by setting $B = 0$ at $x = x^*$.

$$\text{Then for } -a < x < 0 \quad x^* = \frac{B(-a)}{\mu_0 j_c^+} - a$$

This represents the situation for incomplete penetration. This is valid until complete penetration is reached at which time $x^* = 0$ and $B(-a) = \mu_0 j_c^+ a$. (Line 2 in Figure K - 5). After this point while the applied field is still being increased the electric center (x^*) remains at $x = 0$. (Lines 3 and 4 in Figure K - 5).

Figure K - 6 illustrates the situation when the applied field is decreased. Flux penetrates again starting at the surface. (This time the flux change is of the opposite sign.) The critical current density too begins to extend in from the surface (with the opposite sign to that during charge). In an analogous way the electric center migrates from the surface of the wire to the center. Flux profile 8 shows the locked-in flux profile when the external applied field has been reduced to zero. It will be noted that flux profile 4 had such an applied field that when the applied field was reduced to zero the electric center (x^*) just came to the center of the wire ($x = 0$).

The next time the external field is raised the situation is as in Figure K - 7. The flux and the critical current density extend in from the surface as the applied field is increased as was the situation the first time the applied field was raised from zero. However, this time the position of the electric center is different. Instead of being given by the intersection of the slope of the flux density profile with the $B = 0$ line, it is now given by the intersection with the locked-in flux line.

Distribution of Magnetic Field in a Conductor
as the External Field is Initially Increased

Figure K-5

Distribution of Magnetic Field in a Conductor
as the External Field is Decreased

Figure K-6

Distribution of Magnetic Field in a Conductor as the External Field is Increased for the Second Time

Figure K-7

The locked-in flux profile line is given:-

$$B(x) = -\mu_0 j_c^- x + C$$

$$B = 0 \text{ at } x = -a \quad \therefore \quad C = -\mu_0 j_c^- x$$

$$B(x) = -\mu_0 j_c^- (x+a) = \mu_0 j_c^+ (x+a) \quad (1)$$

The profile of flux during charge is:-

$$B(x) = B(-a) - \mu_0 j_c^+ (x+a) \quad (2)$$

The intersection of these lines at $x = x^*$ is given by the condition that $B(x)$ from (1) equals $B(x)$ from (2).

$$\text{Thus} \quad \mu_0 j_c^+ (x^*+a) = B(-a) - \mu_0 j_c^+ (x+a)$$

$$x^* = \frac{B(-a)}{2\mu_0 j_c^+} - a$$

This is valid for $x^* = -a$, ($B(-a) = 0$) to $x^* = 0$, ($B(-a) = 2\mu_0 j_c^+ a$). This is the range of external applied field for which incomplete penetration occurs for cycles subsequent to the first cycle. For $B(-a) > 2\mu_0 j_c^+ a$ the electric center (x^*) is zero and the conductor is fully penetrated.

We are now in a position to calculate the local power densities due to magnetization loss.

The electric field can be calculated using the Maxwell relation $\nabla \times \mathbf{E} = -\dot{\mathbf{B}}$ which in our simplified one-dimensional model becomes $\frac{dE}{dx} = -\dot{B}$

Then assuming that the variation of B across the conductor is small, the variation of $\dot{B}$ across the conductor is also small. Thus for $\dot{B}$ independent of x

$$E(x) = -\dot{B} x + C$$

But by definition $E(x) = 0$ at $x = x^*$ (the electric center).

$$\text{Therefore } E(x) = -\dot{B}(x-x^*)$$

The power dissipated locally per unit volume $\frac{p(x)}{\rho h dx} :-$

$$\begin{aligned} \frac{p(x)}{\rho h dx} &= E \cdot j_c \\ &= -\dot{B} (x-x^*) j_c \end{aligned}$$

The total power dissipated in the region $x = -a$ to x^* is given

$$\begin{aligned} \frac{P}{\ell h} &= \int_{-a}^{x^*} p(x) dx = -\dot{B} j_c \int_{-a}^{x^*} (x - x^*) dx = -\dot{B} j_c \left[\frac{x^2}{2} - x x^* \right]_{-a}^{x^*} \\ &= \frac{\dot{B} j_c}{2} (x^* + a)^2 \end{aligned}$$

By symmetry the power dissipated in the wire in the region $x = 0$ to a is the same as that dissipated from $-a$ to 0 . Hence the total power dissipated per unit length in the wire is:-

$$\frac{P}{\ell} = \dot{B} j_c h (x^* + a)^2$$

To proceed further we must distinguish between the situation where the wire is incompletely penetrated and the situation where full penetration exists.

For incomplete penetration $x^* = \frac{B(-a)}{2\mu_0 j_c^+} - a$ and this is valid for

$B(-a) = 0$ to $B(-a) = 2\mu_0 j_c^+ a$. For complete penetration $x^* = 0$ and this is

valid for $B(-a) > 2\mu_0 j_c^+ a$. It should be remembered also that these expressions were derived for the situation where the applied field is rising from zero to a maximum after previously being cycled $B = 0$ to $B_{\max}$ to 0 .

First we will consider the incomplete penetration situation.

$$P/\ell = \dot{B} j_c h (x^* + a)^2 \quad \text{where } x^* = \frac{B(-a)}{2\mu_0 j_c^+} - a$$

$$P/\ell = \dot{B} j_c h \left(\frac{B(-a)}{2\mu_0 j_c^+} \right)^2$$

(Note henceforth it is understood that B is the value of the field at the conductor surface, i.e. $B(-a)$)

The energy dissipated per unit length in going from zero to B is given:-

$$E/\ell = \int_0^B \frac{dB}{dt} \frac{j_c h B^2}{4\mu_0^2 j_c^2} dt = \frac{h}{4\mu_0^2} \int_0^B \frac{B^2}{j_c} dB$$

We assume the hyperbolic relationship between j_c and B (see Appendix I):-

$$j_c (B + B_0) = \alpha$$

$$\text{Then } E/l = \frac{h}{4\mu_0^2 \alpha} \int_0^B (B+B_0)^2 dB$$

$$E/l = \frac{h}{4\mu_0^2 \alpha} \left[\frac{B^4}{4} + \frac{B_0 B^3}{3} \right]$$

$$E/l = \frac{h B_0^3}{12\mu_0^2 \alpha} \left[1 + \frac{3B}{4B_0} \right]$$

For a rectangular wire of side $h = 2a$

$$E/l = \frac{a B_0^3}{6\mu_0^2 \alpha} \left[1 + \frac{3B}{4B_0} \right] = \Delta E_1/l$$

This is valid for $B \leq B_1$ where $B_1 = 2\mu_0 j_c^+ a$

$$\text{At } B = B_1 \quad E/l = \frac{a B_0 B_1^3}{6\mu_0^2 \alpha} \left[1 + \frac{3B_1}{4B_0} \right] = \Delta E_2/l$$

Now for the complete penetration case where B goes from 0 to B where $B > B_1$ the energy loss per unit length is given by the sum of $\Delta E_2/l$ (as defined above) and a second term applicable from $B = B_1$ to B . This second term will now be calculated.

The general term for the power dissipation is:-

$$P/l = \dot{B} j_c h (x^* + a)^2$$

For complete penetration $x^* = 0$

$$\text{Therefore } P/l = \dot{B} j_c h a^2$$

$$E/l = \int_{B_1}^B \frac{dB}{dt} \frac{\alpha h a^2}{(B+B_0)} dt$$

$$E/l = \alpha h a^2 \int_{B_1}^B \frac{dB}{(B+B_0)} = \alpha h a^2 \ln \left(\frac{B+B_0}{B_1+B_0} \right)$$

For a rectangular conductor of side $h = 2a$

$$E/\ell = 2\alpha a^3 \ln\left(\frac{B+B_0}{B_1+B_0}\right)$$

Thus for a conductor which experiences a field change from $B = 0$ to B where $B > B_1$ the loss per unit length is:-

$$\Delta E_3/\ell = \Delta E_2/\ell + 2\alpha a^3 \ln\left(\frac{B+B_0}{B_1+B_0}\right)$$

The formulae derived above are applicable to conductors which have no variation in field along their length at any given time. Since it is of interest to be able to estimate the losses in a coil these formulae must be modified to take into account the variation of field along conductor in a coil at any instant in time.

Consider Figure K - 8 - a. The lines in this figure represent the variation of field with time at discrete points in a coil. Thus Line 1 represents the field at the peak field position as a function of time and Line 5 represents the field at the zero field position. Hence at $t = 0$ $B = 0$ for both Lines (1) and (5) but at $t = T$ $B = B_m$ for Line 1 but $B = 0$ still for Line 5.

This figure can be divided into three zones as shown in Figure K- 8-b to simplify the calculation of the energy loss for the coil as a whole. There is a dividing line at $B = B_1$ where B_1 is the field at which full penetration occurs.

Above this line conductor is fully penetrated. Below the line only partial penetration exists. Region 1 contains conductor which always experiences incomplete penetration. In this region, which contains a fraction $f = \frac{B_1}{B_m}$ of the conductor (B_m being the maximum field to which the coil is

subjected), all the conductor starts off at field $B = 0$ at $t = 0$ but ends up with fields distributed between 0 and B_1 . The energy dissipated by the

conductor in this region is $E = \int \frac{E}{\ell_1} d\ell$ where $E/\ell_1 = \frac{aB_0 B^3}{6\mu_0 \alpha} \left(1 + \frac{3B}{4B_0}\right)$

If we assume that the field experienced by points in the coil varies linearly along the length of the conductor from $B = 0$ at $\ell = 0$ to $B = B_m$ at $\ell = L$ then in Region 1, where the field varies linearly from $B = 0$ at $\ell_1 = 0$ to $B = B_1$ at $\ell_1 = L_1$.

$$B = \frac{\ell_1 B_1}{L_1}$$

$$dB = \frac{B_1 d\ell_1}{L_1}$$

Figure K-8A Diagram illustrating fields at various points within windings of a coil as a function of time throughout a pulse. T is the charge time at which point the peak field position in the coil experiences a field B_m (line 1). Other points in the coil experience fields represented by lines 2 through 5.

Figure K-8B Diagram illustrating the regions in the coil over which the various derived ac loss formulae are applicable. See text for full explanation.

And in Region 1
$$E = \frac{L_1 a B_0}{6 B_1 \mu_0^2 \alpha} \int_0^{B_1} B^3 \left(1 + \frac{3B}{4B_0} \right) dB$$

$$E_1 = \frac{L_1 a B_0}{6 B_1 \mu_0^2 \alpha} \left[\frac{B_1^4}{4} + \frac{3B_1^5}{20B_0} \right] = \frac{L_1 a B_0 B_1^3}{24 \mu_0^2 \alpha} \left[1 + \frac{3B_1}{5B_0} \right]$$

But $L_1 = fL$ where $f = \frac{B_m}{B_1}$

Hence
$$E_1 = f \frac{LaB_0 B_1^3}{24 \mu_0^2 \alpha} \left[1 + \frac{3B_1}{5B_0} \right]$$

In Region 3 it can be seen that all the conductor in that region starts off at 0 and all of it ends at $B = B_1$. Therefore, no integration over length is required. The conductor is only partially penetrated and the energy dissipated by the conductor in this region is given by:-

Region 3
$$E = \frac{L_3 a B_0 B_1^3}{6 \mu_0^2 \alpha} \left[1 + \frac{3B_1}{4B_0} \right]$$
 where L_3 is length of conductor in this region.

And since $L_3 = (1 - f) L$

$$E_3 = (1 - f) \frac{LaB_0 B_1^3}{6 \mu_0^2 \alpha} \left[1 + \frac{3B_1}{4B_0} \right]$$

In Region 2 all the conductor is fully penetrated. At the beginning all the conductor is at field B_1 but at the end there is a variation of field along the length from $B = B_1$ at one end to $B = B_m$ at the other. To obtain the energy dissipated by conductor in this region it is necessary to suitably integrate over the length the expression for the energy dissipated under fully penetrated conditions.

Thus in Region 2
$$E = \int_{B_1}^{B_m} E/l_2 \, dl$$

Where
$$E/l_2 = 2\alpha a^3 \ln \left(\frac{B+B_0}{B_1+B_0} \right)$$

Again assuming a linear variation of field along the length of wire in Region 3 from $B = B_1$ at $z = 0$ to $B = B_m$ at $z = L_3$

$$B = B_1 + \frac{(B_m - B_1)}{L_3} z$$

$$dB = \frac{(B_m - B_1)}{L_3} dz$$

$$\begin{aligned} \text{And } E_2 &= \frac{2\alpha a^3 L_3}{(B_m - B_1)} \int_{B_1}^{B_m} \ln\left(\frac{B+B_0}{B_1+B_0}\right) dB \\ &= \frac{2\alpha a^3 L_3}{(B_m - B_1)} \left[(B+B_0) \ln(B+B_0) - B \ln(B_1+B_0) \right]_{B_1}^{B_m} \\ &= \frac{2\alpha a^3 L_3}{(B_m - B_1)} \left[(B_m+B_0) \ln\left(\frac{B_m+B_0}{B_1+B_0}\right) - (B_m - B_1) \right] \\ E_2 &= 2\alpha a^3 L_3 \left[\frac{(B_m+B_0)}{(B_m - B_1)} \ln\left(\frac{B_m+B_0}{B_1+B_0}\right) - 1 \right] \end{aligned}$$

$$\text{But } L_3 = (1-f) L \text{ where } f = \frac{B_1}{B_m}$$

$$\text{Therefore } E_2 = (1-f) L 2 \alpha a^3 \left[\frac{(B_m+B_0)}{(B_m - B_1)} \ln\left(\frac{B_m+B_0}{B_1+B_0}\right) - 1 \right]$$

Therefore, to summarize; the energy dissipated by a coil whose peak field goes from $B = 0$ to $B = B_m$ is:

$$\begin{aligned} E_{\text{total}} &= fL \frac{aB_0 B_1^3}{24\mu_0^2 \alpha} \left[1 + \frac{3B_1}{5B_0} \right] \\ &+ (1-f)L \frac{aB_0 B_1^3}{6\mu_0^2 \alpha} \left[1 + \frac{3B_1}{4B_0} \right] \\ &+ (1-f)L 2\alpha a^3 \left[\frac{(B_m+B_0)}{(B_m - B_1)} \ln\left(\frac{B_m+B_0}{B_1+B_0}\right) - 1 \right] \end{aligned}$$

$$\text{Where } f = \frac{B_1}{B_m}$$

Thus we have derived an expression for the energy dissipated during the charge part ($B = 0$ to B_{max}) of a cycle which goes $B = 0$ to B_{max} to 0 . For completeness the energy dissipated in the discharge part ($B = B_{max}$ to 0) should be calculated. Furthermore, similar analyses should be carried out for full cycle hysteresis loops.

The next stage in the sophistication of the model should be to take into account the variation of j_c across the superconductor. Following that the geometry effects concerning round wires should be incorporated at an early stage in the analysis and, finally, the effects of transport current density should be included.