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MODIFICATION OF THE ELECTROSTATIC SHEATH BY
SECONDARY EMISSION OF ELECTRONS

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MODIFICATION OF THE ELECTROSTATIC SHEATH
BY SECONDARY EMISSION OF ELECTRONS

L. S. Hall and I. Bernstein

ABSTRACT

The potential distribution at the surface of a rectilinear one-dimensional plasma in the presence of wall-emitting secondary electrons is computed as a function of the secondary emission coefficient under a variety of plasma conditions.

INTRODUCTION

The electrostatic sheath formed at the wall that terminates a one-dimensional plasma acts to preserve charge neutrality, reducing the flux of leaking electrons by bringing it into equality with the charge of the ions. As such, the sheath also acts as a thermal barrier by reducing the flow of heat via the electron channel below the heat flow that would be associated with a free-streaming loss of electrons. This thermal barrier is important for the containment of energy in a mirror-confined plasma, but recently the effects of secondary electron emission from end walls have been cited as a factor that reduces the ability of the sheath to act as a thermal insulator.¹ We shall examine this question here.

For simplicity, we will consider only those secondary electrons resulting from electron impact on the wall surface. It is a straightforward matter to include ion-induced secondaries or to add the effects of thermionic or photoelectric emission but these complexities are unnecessary for our discussion.

Monoenergetic electrons impinging at normal incidence give rise to a countercurrent of three kinds of electrons that add up to δ returning electrons for each incident electron.^{2,3} Some of these electrons are backscattered, some enter the wall and rediffuse out by multiple scattering, and some are emitted as true secondaries. The energy distribution of the secondary electrons is essentially independent of the energy of the incident electron. We model the process by assuming that the backscattered and rediffused electrons are emitted, on the average, with the same distribution as the incident electrons. There is a total of r reflected electrons for every electron striking the wall. Thus,

$$\delta = \delta_t + r,$$

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(1)

where δ_t is the "true" secondary emission coefficient. According to Kollath,³ the energy distribution of true secondaries is approximately Maxwellian at a temperature, $kT_s = 1.75$ eV.

CALCULATIONS

We suppose that, at the boundary between the plasma and the sheath, electrons move toward the wall in a Maxwellian distribution of velocities with temperature T_p . The potential variation between the wall and the sheath edge is shown schematically in Fig. 1 as a function of position, z . Three different configurations are allowed:

- The potential decreases monotonically from the plasma to the wall (Curve I).
- A minimum of electrostatic potential, Φ_o , appears between the wall and the plasma (Curve II).
- The potential increases monotonically from the plasma to the wall (Curve III).

If deep inside the plasma $\Phi = 0$, then the potential minimum Φ_o will occur at the wall in case I [$\Phi_o(I) = \Phi_w$] and at the sheath edge in case III [$\Phi_o(III) = 0$].

At any point to the right of $\Phi = \Phi_o$, the density of electrons is

$$\begin{aligned}
 n_e^+ = & (1/2)(1+r)N_p \int_{e(\Phi_o - \Phi)}^{\infty} dE \left[2m_e (E + e\Phi - e\Phi_o) \right]^{-1/2} f_p(E - e\Phi_o) \\
 & + (1/2)(1-r)N_p \int_{e(\Phi_o - \Phi)}^0 dE \left[2m_e (E + e\Phi - e\Phi_o) \right]^{-1/2} f_p(E - e\Phi_o) \\
 & + (1/2)N_s \int_0^{\infty} dE \left[2m_e (E + e\Phi - e\Phi_o) \right]^{-1/2} f_s(E + e\Phi_w - e\Phi_o), \quad (2)
 \end{aligned}$$

where

$$\begin{aligned}
 f_p(E) &= \left(2m_e / \pi kT_p \right)^{1/2} \exp(-E/kT_p), \\
 f_s(E) &= \left(2m_e / \pi kT_s \right)^{1/2} \exp(-E/kT_s). \quad (3)
 \end{aligned}$$

Fig. 1. Schematic diagram of the possible modes of potential variation in the sheath.

To the left of the potential minimum,

$$\begin{aligned}
n_e^< &= (1/2)(1+r)N_p \int_0^\infty dE \left[2m_e (E + e\Phi - e\Phi_0) \right]^{-1/2} f_p(E - e\Phi_0) \\
&+ (1/2)N_s \int_{e(\Phi_0 - \Phi)}^\infty dE \left[2m_e (E + e\Phi - e\Phi_0) \right]^{-1/2} f_s(E + e\Phi_w - e\Phi_0) \\
&+ (1/2)N_s \int_{e(\Phi_0 - \Phi)}^0 dE \left[2m_e (E + e\Phi - e\Phi_0) \right]^{-1/2} f_s(E + e\Phi_w - e\Phi_0) .
\end{aligned} \tag{4}$$

The total flux of primary electrons moving to the left at $\Phi = \Phi_0$ is

$$J_{po} = (1/2)N_p \int_0^\infty m_e^{-1} dE f_p(E - e\Phi_0) , \tag{5}$$

and the flux of secondaries moving to the right at $\Phi = \Phi_0$ is

$$J_{so} = (1/2)N_s \int_0^\infty m_e^{-1} dE f_s(E + e\Phi_w - e\Phi_0) . \tag{6}$$

Ions are assumed to be monoenergetic, arriving at the sheath edge with kinetic energy E_i and velocity directed toward the wall. Hence, at any point within the sheath, the ion density n_i and flux J_i are expressed as

$$\begin{aligned}
n_i &= N_i (1 - e\Phi/E_i)^{-1/2} , \\
J_i &= N_i (2E_i/m_i)^{1/2} .
\end{aligned} \tag{7}$$

The parameters of the distributions are related by quasi-neutrality,

$$\lim_{\Phi \rightarrow 0} (n_i - n_e) = 0 , \tag{8}$$

and by current balance,

$$J_i = (1-r)J_{po} - J_{so} , \tag{9}$$

with the secondary emission condition at the wall,

$$(1/2)N_s \int_0^\infty m_e^{-1} dE f_s(E) = \delta_t J_{po} . \tag{10}$$

The potential is given by Poisson's equation,

$$d^2\Phi/dz^2 = 4\pi e(n_i - n_e) , \quad (11)$$

with the boundary condition,

$$\lim_{\Phi \rightarrow 0} (d\Phi/dz) = 0 . \quad (12)$$

We define dimensionless parameters as follows:

$$\begin{aligned} \eta &= -e\Phi/kT_p, \\ s &= \left(4\pi n_p e^2/kT_p\right)^{1/2} z, \\ \alpha &= m_i/4\pi m_e, \\ b &= kT_p/E_1, \\ \theta &= T_s/T_p, \\ \lambda &= J_{so}/J_{po} = \delta_t \exp\left[-e(\Phi_w - \Phi_o)/kT_s\right]. \end{aligned} \quad (13)$$

We also set

$$\begin{aligned} T(x) &= \pi^{-1/2} \int_0^\infty dt e^{-t} (x+t)^{-1/2} \\ &= \sum_{j=0}^\infty (j!)^{-1} - 2^{j+1} (x/\pi)^{1/2} / (2j+1)!! x^j . \end{aligned} \quad (14)$$

A convenient set of tables and a computer program for $T(x)$ is given in Ref. (4).

To the right of $\eta = \eta_o$,

$$\begin{aligned} d^2\eta/ds^2 &= \left(N_i/N_p\right)(1+b\eta)^{-1/2} - e^{-\eta_o} \\ &+ (1/2)e^{-\eta_o} \left\{ (1-r)T(\eta_o - \eta) - \lambda\theta^{-1/2} T(\eta_o - \eta)/\theta \right\} \end{aligned} \quad (15)$$

and to the left,

$$\begin{aligned} d^2\eta/ds^2 &= \left(N_i/N_p\right)(1+b\eta)^{-1/2} - \lambda\theta^{-1/2} \exp[-\eta_o + (\eta_o - \eta)/\theta] \\ &+ 1/2e^{-\eta_o} \left\{ \lambda\theta^{-1/2} T[(\eta_o - \eta)/\theta] - (1+r)T(\eta_o - \eta) \right\} , \end{aligned} \quad (16)$$

where

$$\begin{aligned} N_i/N_p &= 1 - (1/2)e^{-\eta_o} \left[(1-r)T(\eta_o) - \lambda\theta^{-1/2} T(\eta_o/\theta) \right] \\ &= (1-r-\lambda)e^{-\eta_o} (\alpha b)^{1/2} . \end{aligned} \quad (17)$$

As $\eta \rightarrow 0$ on the right of $\eta = \eta_0$,

$$d^2\eta/ds^2 \approx \left\{ 1 - (1/2)b \left[1 + 1/2(1-r)e^{-\eta_0} T'(\eta_0) - (1/2)\lambda\theta^{-3/2} e^{-\eta_0} T'(\eta_0/\theta) \right] \right\} \eta, \quad (18)$$

where

$$T'(x) = dT/dx = T(x) - (\pi x)^{-1/2}. \quad (19)$$

Hence, if the sheath is to exist,^{5,6} then $\eta^{-1}d^2\eta/ds^2 \geq 0$ so that

$$b \leq b_{\max} = 2 \left[1 + (1/2)(1-r)e^{-\eta_0} T'(\eta_0) - (1/2)\lambda\theta^{-1/2} e^{-\eta_0} T'(\eta_0/\theta) \right]^{-1}. \quad (20)$$

Equation (20) reduces to the usual Bohm condition $b \leq 2$, i.e., $E_i \geq (1/2)kT_e$ when η_0 is large. Moreover, Eq. (17) requires that $2 < 1-r$ and, since $\lambda \geq 0$, there is also a maximum value of η_0 . Notice that when r is small compared to unity, this last critical condition is unchanged. The remaining relations are also essentially unaffected if we replace δ_t by $\delta_t/(1-r)$ in the definition of λ and elsewhere set $r = 0$.

To illustrate the behavior of the sheath equations, we take as an example $r = 0$, $\theta = 0.02$ ($kT_p \approx 90$ eV), and $\alpha = 292$ (deuterium ions). Then, the combination of the quasi-neutrality and current balance conditions (Eq. 17) gives the relationship between η_0 , λ , and b (see Fig. 2). In Fig. 3, $b_{\max}(\lambda, \eta_0)$ is plotted from Eq. (20) and the resulting limiting values for $b = b_{\max}$ are cross-plotted back into Fig. 2. For our example, $\eta_0(b, \lambda) \leq \eta_0(b_{\max}, \lambda) \leq 3.14$. We show both $b = b_{\max}$ and $\eta_0(b_{\max}, \lambda)$ in Fig. 4.

From Eq. (19), an immediate integral of Eq. (15) may be obtained so that, to the right of $\eta = \eta_0$,

$$\begin{aligned} (1/2)(d\eta/ds)^2 &= \left(2N_i/bN_p \right) \left[(1+b\eta)^{1/2} - 1 \right] - (1 - e^{-\eta}) \\ &+ e^{-\eta_0} \left\{ (1-\lambda)\pi^{-1/2} \left[\eta_0^{1/2} - (\eta_0 - \eta)^{1/2} \right] \right. \\ &- (1/2) \left[T(\eta_0 - \eta) - T(\eta_0) \right] + (1/2)\lambda\theta^{1/2} \\ &\times \left. \left[T(\theta^{-1}\eta_0 - \theta^{-1}\eta) - T(\theta^{-1}\eta_0) \right] \right\}. \end{aligned} \quad (21)$$

Fig. 2. Plot of $\eta_0(\lambda, b)$ from Eq. (17).

Fig. 3. Plot of $b_{\max}(\eta_0, \lambda)$ from Eq. (20).

Fig. 4. Various limiting parameters of the secondary emission parameter λ .

If

$$\begin{aligned}
R = & \left(2N_i / bN_p \right) \left[\left(1 + b\eta_o \right)^{1/2} - 1 \right] - \left(1 - e^{-\eta_o} \right) \\
& + (1 - \lambda) \left(\eta_o / \pi \right)^{1/2} e^{-\eta_o} - (1/2) \left[1 - T(\eta_o) \right] e^{-\eta_o} \\
& + (1/2) \lambda \theta^{1/2} \left[1 - T(\eta_o / \theta) \right] e^{-\eta_o} ,
\end{aligned} \tag{22}$$

then $R = 0$ is the condition in which a minimum of potential appears between the wall and the sheath edge, as in curve II of Fig. 1. Figure 4 shows η_o at $R = 0$ as a function of the secondary emission parameter λ . Notice that solutions consistent with the conditions for asymptotic matching ($b \leq b_{\max}$) only occur for $\eta_o \lesssim 0.75$ (and $\lambda \lesssim 0.91$). Because θ is small, minute variations of potential strongly affect the distribution of secondary electron charge density. It is only when the sheath itself is nearly abolished that an actual potential minimum can occur.

Taking $s = 0$ at $\eta = \eta_o$ and $s > 0$ on the right,

$$\begin{aligned}
s = & \int_{\eta}^{\eta_o} dy \left\{ \left(4N_i / bN_p \right) \left[(1 + by)^{1/2} - 1 \right] - 2(1 - e^{-y}) \right. \\
& + 2\pi^{-1/2} (1 - \lambda) e^{-\eta_o} \left[\eta_o^{1/2} - (\eta_o - y)^{1/2} \right] \\
& - e^{-\eta_o} \left[T(\eta_o - y) - T(\eta_o) \right] \\
& \left. + \lambda \theta e^{-\eta_o} \left[T(\theta^{-1} \eta_o - \theta^{-1} y) - T(\theta^{-1} \eta_o) \right] \right\}^{-1/2} .
\end{aligned} \tag{23}$$

Solutions of $s < 0$ are applicable only when $R = 0$, in which case from Eq. (16) we find

$$\begin{aligned}
(1/2) (d\eta/ds)^2 = & - \left(2N_i / bN_p \right) \left[\left(1 + b\eta_o \right)^{1/2} - (1 + b\eta)^{1/2} \right] \\
& + \lambda \theta^{1/2} e^{-\eta_o} \left[\exp(\theta^{-1} \eta_o - \theta^{-1} \eta) - 1 \right] \\
& + e^{-\eta_o} \left\{ (1 - \lambda) \pi^{-1/2} - (1/2) \left[1 - T(\eta_o - \eta) \right] \right. \\
& \left. + (1/2) \lambda \theta^{1/2} \left[1 - T(\theta^{-1} \eta_o - \theta^{-1} \eta) \right] \right\} ,
\end{aligned} \tag{24}$$

and

$$\begin{aligned}
s = & - \int_{\eta}^{\eta_0} dy \left\{ - \left(4N_i / bN_p \right) \left[\left(1 + b\eta_0 \right)^{1/2} + \left(1 + by \right)^{1/2} \right] \right. \\
& + 2\lambda\theta^{1/2} e^{-\eta_0} \left[\exp\left(\theta^{-1}\eta_0 - \theta^{-1}y\right) - 1 \right] \\
& + 2\pi^{-1/2} (1 - \lambda) e^{-\eta_0} \left(\eta_0 - y \right)^{1/2} - \left[1 - T\left(\eta_0 - y\right) \right] \\
& \left. + \lambda\theta^{1/2} \left[1 - T\left(\theta^{-1}\eta_0 - \theta^{-1}y\right) \right] \right\}^{-1/2} . \quad (25)
\end{aligned}$$

The character of the solution for $s < 0$ is such that the potential difference, $\eta_0 - \eta$, rises rapidly on the spatial scale of the Debye length of the cold secondary electrons until $\eta = \eta_w$ just satisfies the condition $\lambda = \delta_t \exp[\theta^{-1}(\eta_w - \eta_0)]$. For $s > 0$, the space potential rises monotonically and vanishes as $s \rightarrow \infty$.

We show several typical cases in Fig. 5 for the condition $b = b_{\max}$ including $\eta_0 = \eta_0(b_{\max}, \lambda)$ as shown in Fig. 4. Curve IV applies to the condition corresponding to the intersection of this curve and the curve of $\eta_0(R = 0)$.

When $\eta_0 = 0$, η is always negative and

$$\begin{aligned}
s = & - \int_0^{|\eta|} dy \left\{ - \left(4N_i / bN_p \right) \left[1 - \left(1 - by \right)^{1/2} \right] + 2\lambda\theta^{1/2} \left(e^{y/\theta} - 1 \right) \right. \\
& + 2\pi^{-1/2} (1 - \lambda)y^{1/2} - [1 - T(y)] + \lambda\theta^{1/2} [1 - T(y/\theta)]^{-1/2} \left. \right\} . \quad (26)
\end{aligned}$$

The potential rises monotonically from the sheath edge to the wall.

The net heat flow can be measured at $s = 0$;

$$Q = (1 - r)Q_{po} - Q_{so} + Q_{io} , \quad (27)$$

where

$$\begin{aligned}
Q_{po} &= (1/2)N_p \int_0^w dE m_e^{-1} (E - e\Phi_0 + E_{\perp p}) f_p(E - e\Phi_0) , \\
Q_{so} &= (1/2)N_s \int_0^\infty dE m_e^{-1} (E + e\Phi_w - e\Phi_0 + E_{\perp s}) f_s(E + e\Phi_w - e\Phi_0) , \\
Q_{io} &= J_i (E_i + e\Phi_0) . \quad (28)
\end{aligned}$$

Fig. 5. Plot of typical variation of space potential in the sheath. Only Curve IV has an appreciable region of negative space charge: at $s = 0$, $d\eta/ds = 0$. For Curve I; $\lambda = 0$, $\eta_0 = 3.14$, $b = 2.0$: for Curve II; $\lambda = 0.5$, $\eta_0 = 2.52$, $b = 2.0$: for Curve III; $\lambda = 0.8$, $\eta_0 = 1.60$, $b = 1.99$: for Curve IV; $\lambda = 0.908$, $\eta_0 = 0.75$, $b = 1.74$.

The energy $E_{\perp}$ represents the kinetic energy of the electrons in the two degrees of freedom tangent to the wall. For Maxwellian electrons,

$$Q = Q_o \left[2(1 - r - \lambda\theta) + (1 - r - \lambda)(b^{-1} + \eta_o) \right] \left(N_p / N_i \right) e^{-\eta_o}, \quad (29)$$

where

$$Q_o = \left(2\pi m_e \right)^{-1/2} \left(kT_e \right)^{3/2} N_i. \quad (30)$$

The factor 2 in the bracketed first term in Eq. (29) represents $(1/2)kT$ in each of the two transverse degrees of freedom as well as kT for the weighted electron energy normal to the wall per incident electron. Figure 6 shows Q/Q_o vs λ along the boundary curve $\eta_o = \eta_o(b_{\max}, \lambda)$ when $r = 0$. It is clear that the effects of secondary emission can significantly alter the flow of heat to the ends.

Fig. 6. Plot of total heat flow Q/Q_0 vs the secondary emission parameter λ .

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