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ANOMALOUS LENGTH OF ELECTRON BUNCHES AS AN INSTABILITY THRESHOLD

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ABSTRACT

A mechanism for the anomalous length of electron bunches, based on the existence of a "fast" longitudinal instability, is proposed. The equilibrium length is obtained by requiring that the growth rate be sufficiently larger than the rate of synchrotron oscillations. The theory is used to describe the bunch length data for SPEAR at 1.5 GeV. The low voltage and/or high current regime is dominated by a set of "low" frequency, low Q resonators [e.g., $f = 320$ MHz, $\Delta f(\text{fwhm}) = 130$ MHz]. To fit the observations in the high voltage and/or low current regime, a high frequency, low Q impedance is required (e.g., $f = 3.8$ GHz, $\Delta f = 1.0$ GHz). The mechanism is mediated by the resistive component of the impedance. Thus, there is qualitative agreement with the observed distortion of the bunch tail. This is in contrast to the predictions of the potential well models based on a reactive impedance source. These latter theories yield large distortions of the head of the bunch. The calculated power dissipated in the assumed sources by the given electron bunch is not inconsistent with estimates made for SPEAR.

I. Introduction

We propose a mechanism for the anomalous electron bunch length based on the existence of a "fast" longitudinal instability.¹

We presume the presence of three time scales for electrons:

τ_r = radiation damping time

τ_s = synchrotron oscillation period

τ_g = characteristic time for the "fast" longitudinal instability ($\alpha_g = 1/\tau_g$ is the instability growth rate).

The relationship between these times is

$$\tau_r \gg \tau_s \gg \tau_g \quad . \quad (1.1)$$

Let σ_ℓ be the rms bunch length as determined from the instability threshold and let σ_o denote the rms equilibrium bunch length resulting from the balance of radiation damping and quantum excitations.² Then, it follows that the actual steady state bunch length σ_A will be given by

$$\sigma_A = \text{Larger} \{ \sigma_\ell, \sigma_o \} \quad . \quad (1.2)$$

The fast instability characteristically develops as a bunch widening with unstable particles quickly losing energy. The role of the synchrotron motion is to mediate the instability overshoot and convert bunch widening into bunch lengthening.

If the initial bunch length is below threshold, instability causes growth and overshoot. Eventually the slower radiation damping will reduce the bunch area close to the threshold. At this point, further reduction is inhibited by incipient fast instability

Since σ_ℓ depends on the bunch current, this theory predicts a

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1. E. Messerschmid and M. Month, "Theory for the Fast Blowup of Particle Bunches in Accelerators," submitted to Nucl. Instrum. Methods (1976).
  2. M. Sands, SLAC Report No. 121, SLAC (1970).

current threshold. For sufficiently small but nonzero current, we must have $\sigma_\ell < \sigma_0$, leaving the natural bunch length σ_0 as the observed one. If the current is increased sufficiently, then at some value I_{th} , we have $\sigma_\ell = \sigma_0$. Above this current, $I > I_{th}$, then $\sigma_\ell > \sigma_0$ and then the observed bunch length will be that corresponding to the instability threshold.

II. The "Fast" Longitudinal Instability³

From observations of proton bunches and computer simulations, single charged particle bunches have been shown to develop fast instabilities in the presence of sufficiently large longitudinal electric fields. Since growth rates are proportional to the frequency of the beam induced fields, these instabilities tend to be characterized by high frequencies, that is, high azimuthal mode numbers.

Fast Instability Threshold for a Single Bunch

It can be shown that for these fast instabilities, where $\tau_g \ll \tau_s \ll \tau_r$, the threshold criterion for an unstable mode, n_0 , can be expressed in terms of a threshold impedance, Z_{th} , and a threshold rms energy spread, σ_E , by

$$\left| \frac{Z_{th}}{n_0} \right| = \frac{2\pi n E}{eI} \times \frac{2}{E}, \quad (2.1)$$

where E is the particle energy, I is the dc current

$$I = N e f_0, \quad (2.2)$$

with N the number of particles in the bunch, f_0 the revolution

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3. Much of this section follows ref. 1. Further references to the fast instability for protons can also be found in ref. 1

frequency, x_E is the relative rms energy spread,

$$x_E = \sigma_E / E ,$$

and η is the frequency slip factor,

$$\eta = \frac{pdf}{fdp} = \left| \frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2} \right| ,$$

with γ and γ_{tr} the energy and transition energy in electron mass units.

Since $\gamma \gg \gamma_{tr}$, η can be expressed as

$$\eta = \frac{1}{\gamma_{tr}^2} . \quad (2.3)$$

In solving the dispersion relation, we have assumed Gaussian distributions in both energy and phase of the bunch. If f_0 is interpreted as corresponding to a full particle revolution, this implies we are considering the case of a single bunch.

The Effective Impedance Determining the Threshold for a Bunched Beam

The basic result for the fast instability of a single bunch is that a single azimuthal beam mode, say, n_0 , can excite fields at all modes, n . The limit of this coupling is determined by the "bunch spectrum," i.e., the bunch length. The factor coupling a field of mode n to a beam mode n_0 is found to be the usual coupling impedance of mode n . Writing

$$m = n - n_0 , \quad (2.4)$$

and denoting the coupling impedance of mode n by Z_m , it can be shown that the effective coupling impedance for Gaussian bunches is given by

$$Z_{eff} = \sum_{m=-\infty}^{\infty} Z_m e^{-m^2 \theta_{rms}^2} , \quad (2.5)$$

where the exponential term represents the "bunch spectrum" limitation

for a Gaussian bunch. The bunch length is represented in units of azimuthal angle (radians) by θ_{rms} :

$$\theta_{\text{rms}} = 2\pi B_{\text{rms}} = \frac{2\pi\sigma_{\ell}}{C}, \quad (2.6)$$

where σ_{ℓ} is the rms bunch length, C is the machine circumference, and B_{rms} is the "rms bunching factor."

The Threshold Bunch Length

The threshold bunch length is determined by replacing Z_{th} in Eq. (2.1) with Z_{eff} from Eq. (2.5):

$$\left| \frac{Z_{\text{eff}}}{n_0} \right| = \left| \frac{Z_{\text{th}}}{n_0} \right| = \frac{2\pi\eta E}{eI} x_E^2, \quad (2.7)$$

where Z_{eff} and x_E are both functions of the bunch length, which, at threshold, we denote σ_{ℓ} , or in terms of its angular length, θ_{rms} .

The bunch length is related to the relative energy spread, x_E , through the synchrotron wave number, ν_s , by²

$$\theta_{\text{rms}} = \frac{\pi}{\nu_s} x_E. \quad (2.8)$$

The current independent synchrotron wave number is given by

$$\nu_s = \left[\frac{\eta h e V \cos \phi_s}{2\pi E} \right]^{\frac{1}{2}}, \quad (2.9)$$

where V is the peak rf voltage, h is the harmonic number, and ϕ_s is the stable phase angle, the latter determined by the rate at which energy must be provided to the beam by the rf system due to losses resulting from synchrotron radiation and the various dissipative forces in the ring. For large overvoltages, the effect on ϕ_s due to synchrotron radiation energy loss can usually be neglected.

Case of a Purely Resistive Effective Impedance

If we conjecture that the coupling impedance is dominated at high frequencies by resonators, it is not improbable that the sum of many such "low" Q elements can be simulated by a single strong "very low" Q resonator. In this case, it can be easily seen that the effective impedance is purely resistive since the reactive component is an odd function of mode number. The resistive component, on the other hand, is even and, from Eq. (2.5) many modes can be seen to add up, limited only by the Gaussian cutoff factor.

Because the reactive component is negligible, we can use Eq. (2.8) relating bunch length and width, taking ν_s as the "natural" synchrotron wave number, from Eq. (2.9), determined only by the applied rf system.

Effective Impedance for a Single Resonator

Neglecting the reactive component, the impedance can be expressed by

$$Z_m = Z_R \frac{a^2}{a^2 + m^2}, \quad (2.10)$$

where

$$a = \frac{n_o}{2Q}, \quad (2.11)$$

Q is resonator quality related to the resonance width, as in Eq. (2.10), n_o is the central resonator frequency, and

$$Z_R = C_o Q, \quad (2.12)$$

with C_o an effective strength constant related to the strength of each individual "real" resonator and the manner in which they add up.

For example, M resonators in the TM_{01} mode, all at the same frequency and with unit transit time factor would yield a strength constant,

$$C_o = 111.4 M \quad . \quad (2.13)$$

The effective impedance in this case can therefore be written,

$$\frac{Z_{eff}}{n_o} = \frac{1}{2} C_o a \sum_m \frac{1}{a^2 + m^2} e^{-m^2 \theta_{rms}^2} \quad . \quad (2.14)$$

Note that in the limit, $a > (1/\theta_{rms})$, or $Q < \frac{1}{2} n_o \theta_{rms}$, we can put $m = 0$ in the Lorentz factor. Then, replacing the sum by an integration, we arrive at the limiting case of "low" Q,

$$Z_{eff} \rightarrow \frac{\sqrt{2\pi} Z_R}{\theta_{rms}} \quad . \quad (2.15)$$

Defining the function

$$F(a, \theta_{rms}) = \sum_m \frac{a}{a^2 + m^2} e^{-m^2 \theta_{rms}^2} \quad , \quad (2.16)$$

we have

$$\frac{Z_{eff}}{n_o} = \frac{1}{2} C_o F(a, \theta_{rms}) \quad . \quad (2.17)$$

Equation for Equilibrium Bunch Length in the Presence of a

Resonator

Using Eqs. (2.8) and (2.9), then Eq. (2.7) becomes

$$\left| \frac{Z_{eff}}{n_o} \right| = \left| \frac{Z_{th}}{n_o} \right| = \frac{hV |\cos \varphi_s|}{I} \theta_{rms}^2 \quad . \quad (2.18)$$

If we assume that energy loss due to dissipative ring elements does not have too significant an impact on $\cos \varphi_s$, we can take $\cos \varphi_s$ to correspond

to that value needed to compensate the energy loss due to synchrotron radiation alone.

Therefore, substituting Eq. (2.17) into Eq. (2.18), we have for the equilibrium bunch length the equation

$$\frac{1}{2} C_0 F(a, \theta_{rms}) = \frac{hV |\cos \varphi_s|}{I} \theta_{rms}^2. \quad (2.19)$$

This means that except for normalization, the threshold bunch length depends only on the two variables, $a = n_0/2Q$, and

$$b = hV |\cos \varphi_s| / I. \quad (2.20)$$

If U_0 is the energy loss per turn due to synchrotron radiation, then we have for $\cos \varphi_s$,

$$|\cos \varphi_s| = [1 - (U_0/V)^2]^{\frac{1}{2}}, \quad (2.21)$$

where

$$U_0 = C_Y E^4 / \rho, \quad (2.22)$$

with

$$C_Y = \frac{4\pi}{3} \frac{r_e}{E_0^3} = 8.85 \times 10^{-5} \text{ m} \cdot \text{GeV}^3.$$

ρ is the magnetic bending radius, r_e is the classical electron radius and E_0 is the electron rest energy.

Growth Rate for "Fast" Longitudinal Instability

For a resistive impedance, it can be shown that the growth rate,

$\alpha_g = 1/\tau_g$, can be expressed as

$$\alpha_g = \sqrt{S} [S^2 + 4\Delta_R^2]^{\frac{1}{4}} \sin \varphi, \quad (2.23)$$

where

$$\varphi = \frac{1}{2} \tan^{-1} \frac{2\Delta_R}{S}, \quad (2.24)$$

$$S = \frac{x_E \eta n_0 2\pi f_0}{\beta^2}, \quad (2.25)$$

and

$$\Delta_R = \frac{eIf_0}{2x_E} (Z_{eff} - Z_{th}). \quad (2.26)$$

The quantity $(2\Delta_R/S)$ can be put in the form

$$(2\Delta_R/S) = \frac{eI}{2\pi\eta E_x^2} \frac{(Z_{eff} - Z_{th})}{n_o} \quad (2.27)$$

Using Eq. (2.1) this can be written in the simple form,

$$(2\Delta_R/S) = \frac{Z_{eff}}{Z_{th}} - 1 \quad (2.28)$$

Thus, the growth rate is given by

$$\alpha_g = \frac{f_o eI}{x_E E} Z_{th} \left[1 + \left(\frac{Z_{eff}}{Z_{th}} - 1 \right)^2 \right]^{\frac{1}{2}} \sin \varphi \quad (2.29)$$

with

$$\varphi = \frac{1}{2} \tan^{-1} \left(\frac{Z_{eff}}{Z_{th}} - 1 \right) \quad (2.30)$$

The rate of synchrotron oscillations is

$$\alpha_s = v_s f_o \quad (2.31)$$

Therefore, the growth rate in terms of the synchrotron oscillation rate is given by

$$\frac{\alpha_g}{\alpha_s} = \frac{eI}{v_s x_E E} Z_{th} \left[1 + (z - 1)^2 \right]^{\frac{1}{2}} \sin \varphi \quad ,$$

or, using Eqs. (2.7), (2.8) and (2.9),

$$\frac{\alpha_g}{\alpha_s} = 2\pi n_o \theta_{rms} \left[1 + (z - 1)^2 \right]^{\frac{1}{2}} \sin \varphi \quad (2.32)$$

with

$$\varphi = \frac{1}{2} \tan^{-1} (z - 1) \quad (2.33)$$

For z not too far from unity, we can write approximately,

$$\frac{\alpha_g}{\alpha_s} = \pi n_o \theta_{rms} (z - 1) \quad (2.34)$$

A consistency requirement for the "fast" longitudinal instability is that this ratio be sufficiently large, i.e.,

$$\alpha_g / \alpha_s > 1 \quad . \quad (2.35)$$

or roughly

$$z - 1 > \frac{1}{\pi n_o \theta_{rms}} \quad . \quad (2.36)$$

Power Dissipation in Resistive Ring Elements

The elements in the ring coupling to the beam through resistive impedances will cause the beam to lose energy. The power dissipation can be shown to be⁴

$$P = \frac{1}{2} I^2 \sum_n \text{Re}(Z_n) e^{-n^2 \theta_{rms}^2} \quad . \quad (2.37)$$

For the "single resonator," the resistive part is given in Eq. (2.10) and we have

$$P = \frac{1}{2} I^2 Z_R \sum_n \frac{a^2}{a^2 + (n - n_o)^2} e^{-n^2 \theta_{rms}^2} \quad . \quad (2.38)$$

III. Application to SPEAR Results at 1.5 GeV^(5,6)

Theory

Consider Eq. (2.34). It can be viewed as a correction term to the threshold expression Eq. (2.18) in determining the "equilibrium" bunch length. Let us suppose that the fast instability will be effective at some particular growth rate: for example, when the growth rate is 4 times the synchrotron oscillation rate. Then for a given effective impedance, $Z_{eff}(n_o)$, centered at the frequency, n_o , the stability criterion, including the frequency dependent correction

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4. See, for example, E. Keil, CERN Report, CERN/ISR-TH/74-15 (1974).

term, can be expressed as

$$\frac{Z_{\text{eff}}(n_o)}{n_o} = \frac{Z_{\text{th}}}{n_o} \left(1 + \frac{4}{\pi n_o \theta_{\text{rms}}} \right) .$$

Actually, we should include the observation of a reactive impedance in the form of a factor which takes account of the fact that the synchrotron frequency is a function of current. From the general observation on the current dependence of the synchrotron frequency, we should include a correction factor, say $k(I)$. The actual synchrotron frequency is then related to the unperturbed synchrotron frequency ν_{so} by

$$\nu_s(I) = k(I)\nu_{so} . \quad (3.1)$$

The stability criterion can then be written

$$\frac{Z_{\text{eff}}(n_o)}{n_o} = \frac{hV |\cos \varphi_s|}{I} \theta_{\text{rms}}^2 k^2(I) \left(1 + \frac{4}{\pi n_o \theta_{\text{rms}}} \right) , \quad (3.2)$$

where we have used Eq. (2.18) for Z_{th}/n_o .

From the observations of the synchrotron frequency at SPEAR,⁵ we can choose k of the form

$$k(I) = e^{-45 I_o^2} , \quad (3.3)$$

with I taken in units of A .

Recall that,

$$Z_{\text{eff}} = Z_R \sum_m \frac{a^2}{a^2 + m^2} e^{-m^2 \theta_{\text{rms}}^2} .$$

Therefore, we interpret Eq. (3.2) as follows: given a set of experimental conditions, i.e., V , I and E ; and further given a particular

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5. M. A. Allen, G. E. Fischer, M. Matera, A. P. Sabersky, and P. B. Wilson, Proc. IXth Inter. Conf. on High Energy Accel., Stanford University, May 1974, p. 352.

set of resonant structures described by Z_R , Q and n_o , then an equilibrium θ_{rms} is predicted by the stability criterion Eq. (3.2). If there is more than a single set of resonant structures, for example, 2 resonances at widely different frequencies, say n_1 and n_2 , then the equilibrium θ_{rms} is the larger of the two, each being determined separately from Eq. (3.2). In other words,

$$\theta_{rms} = \max \{ \theta_{rms}(n_1), \theta_{rms}(n_2), \theta_{rms}(\text{natural}) \} . \quad (3.4)$$

To compare the theory with the experimental observations at SPEAR, we write Eq. (3.2) as

$$G(\theta_{rms}, a, n_o) = g , \quad (3.5)$$

where

$$G(\theta_{rms}, a, n_o) = \left(\frac{Z_{eff}}{n_o} \right) / \left[\theta_{rms}^2 \left(1 + \frac{4}{n_o \theta_{rms}} \right) \right] , \quad (3.6)$$

and

$$g = \frac{hV |\cos \phi_s| k^2 (I)}{I} . \quad (3.7)$$

Equation (3.5) can be inverted giving θ_{rms} as a function of g for a particular resonant impedance function.

We plot in Fig. 1 the function $\theta_{rms}^*(g)$ for two resonances, one centered at 320 MHz, the other at 3.8 GHz. There are two significant qualitative features demonstrated in these curves. First, θ_{rms} shows an extremely strong dependence on g for low g and secondly, θ_{rms} becomes rather insensitive to g at larger values. If the two curves cross as shown, it can then be seen how a low frequency resonance could dominate in the low V , high current regime (that is, relatively large bunch lengths) while under conditions of high voltage and/or low current, the dominant perturbing field is at much higher frequencies.

Fig. 1 Comparison of SPEAR data at $E = 1.5 \text{ GeV}$ with 2-Resonance Theory. The rms bunch length, θ_{rms} , versus the scaling parameter, g .

The reason behind the choice of these frequencies is related to the implied power dissipation. In our theory, it is the resistive part of the impedance that is responsible for the bunch length. This means that the bunch length is directly related to the energy dissipated in the ring elements comprising the source of the perturbing impedance. We, therefore, have chosen parameters for the resonant impedances that give power dissipation in the range that is apparently observed at SPEAR.

SPEAR Parameters, Observed Data and Comparison with 2-Resonance Theory

In Table I, we give the SPEAR parameters required for our analysis. The 1.5 GeV data^{5,6} are listed - in particular, the stability parameter g and the corresponding bunch length observed.

To compute g , we use for the function $k(I)$, describing the current dependence of the synchrotron frequency,⁵

$$k^2(I) = e^{-90I^2},$$

with I in amps. We also require the stable phase angle φ_s . Neglecting the energy loss due to resistive ring elements, $\cos \varphi_s$ can be obtained from Eq. (2.21).

The observed bunch lengths are plotted as a function of the g parameter in Fig. 1. Keeping in mind that the equilibrium bunch length is determined from Eq. (3.4), the 2-resonance model is in reasonably good agreement with the observed data. In fact, it might be argued that the single high frequency resonance is adequate.

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6. A. M. Sessler, PEP Note 28, LBL (1973).

TABLE I SPEAR I Data and Parameters

Peak Voltage (kV)	Phase Angle ($\cos \phi_s$)	Current (mA)	$g(\times 10^6)$ (Ω)	θ_{rms} (mrad)
80.0	0.897	8.3	343.9	6.9
		20.0	138.5	8.6
		33.8	76.7	10.0
		45.0	53.2	10.7
150.0	0.972	10.8	534.4	5.3
		21.0	266.9	6.6
		35.0	149.2	8.0
		60.0	70.3	9.4
440.0	0.997	15.0	1146.4	3.7
		30.0	539.4	5.0
		50.0	280.2	6.4
		80.0	123.3	7.6
		92.5	87.8	8.3

$f_0 = 1.275$ MHz, h (rf harmonic number) = 40, $\rho = 12.7$ m, and $E = 1.5$ GeV.

This is, of course, related to the scatter of the observed points in the $g - \theta_{\text{rms}}$ plane. This is presumably due to the uncertainty in the experimental parameters, although it is not completely inconceivable that the 2 variable picture is incomplete and more subtle effects underly this scatter. In any case, scaling with g appears to be highly significant.

If we neglect the scatter, then the 2-resonance theory explains the general observation that low frequency resonators in SPEAR (the ferrite kickers) are indeed a source of bunch lengthening in the low voltage - high current (i.e., large θ_{rms}) regime. The crossover reflects the fact that removal of these low frequency resonators will not affect the equilibrium bunch length for short bunches (i.e., in the high voltage and/or low current regime).

In Table II, we describe the parameters characterizing the two resonant impedance functions. The predicted power dissipation in the two resonant impedance sources, P_A and P_B , are listed in Table III for the different experimental conditions.

IV. Conclusions

We have described a mechanism for the anomalous electron bunch length in SPEAR based on the theory for a "fast" longitudinal instability in individual bunches. Attempting to fit the SPEAR data on bunch lengthening at $E = 1.5$ GeV, we require a high frequency, low Q impedance. With an impedance peaking at 3.8 GHz with a full width at half maximum of 1.0 GHz, we obtain agreement with the observations in the high voltage and/or low current regime. For larger bunch

TABLE II Characteristics of Resonant Impedances

	Symbol	Units	Res A	Res B
Peak Impedance	Z_R	Ω	3.23×10^4	1.24×10^5
Quality Factor	Q	-	2.5	3.75
Mode Number	n_o	-	250	3000
FWHM	Δn	-	100	800
Central Frequency	f	GHz	0.32	3.8
Spread (FWHM)	Δf	GHz	0.13	1.0
Peak Impedance	Z_R/n_o	Ω	129.2	41.3

TABLE III Power Dissipation into A and B Impedance Sources

Bunch Length $\theta_{rms}(mrad)$	Current $I(mA)$	Power A Watts	Power B Watts
6.9	8.3	22.2	19.1
8.6	20.0	81.4	89.8
10.0	33.8	169.4	219.6
10.7	45.0	268.0	364.0
5.3	10.8	66.3	42.3
6.6	21.0	156.5	128.5
8.0	35.0	282.8	294.7
9.4	60.0	605.4	740.9
3.7	15.0	239.6	119.0
5.0	30.0	581.6	346.5
6.4	50.0	972.4	758.5
7.6	80.0	1676.7	1630.4
8.3	92.5	1817.4	1977.8

Res A: $Z_R = 3.23 \times 10^4 \Omega$, $n_o = 250$ ($f_A = 0.32$ GHz, $\Delta f_A = 0.13$ GHz)

Res B: $Z_R = 1.24 \times 10^5 \Omega$, $n_o = 3000$ ($f_B = 3.8$ GHz, $\Delta f_B = 1.0$ GHz)

lengths (i.e., $\theta_{\text{rms}} \geq 8 \text{ mrad}$), almost any resonant impedance function gives a satisfactory fit. The presence of low frequency, low Q resonators (e.g., with $f = 320 \text{ MHz}$, $\Delta f = 130 \text{ MHz}$) would be suitable. There are three qualitative features of our model which are consistent with the experimental facts. First, there is general agreement with the effect of the ferrite kickers.^{5,7} If we reduce the low frequency impedance, the theory does predict a reduction in the bunch length in the low voltage - high current regime. For high voltages and/or low currents, the impact of the ferrite kicker removal actually diminishes. This is also predicted in our model. Secondly, the mechanism presented here is predicated on the resistive component of the impedance.⁸ The small but noticeable elongation of the tail would tend to suggest just such a resistive impedance^{1,9} mechanism rather than one mediated by a reactive impedance,^{7,10} in which case we would expect a large distortion of the head of the bunch,^{5,11} this latter effect not being observed. Finally, the power dissipated in the resistive impedances required in the theory is not inconsistent with observation.⁵

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7. F. J. Sacherer, PEP Note 45 and PEP Summer Study (1973).
  8. A different approach, but also conjecturing that the source of bunch lengthening arises from a resistive impedance source is due to A. N. Lebedev, Proc. of the Inter. School of Physics, Varenna, Italy, Academic Press, 1971, p. 184. A resistive impedance has also recently been suggested by A. G. Ruggiero (DESY notes, private communication, 1976). See also, the "Strong Turbulence" approach, P. J. Channel and A. M. Sessler, LBL 4613 (1975) submitted to Nucl. Instrum. Methods.
  9. H. G. Hereward, PEP Note 56 and PEP Summer Study (1973).
  10. C. Pellegrini and A. M. Sessler, Nuovo Cimento **3A**, 116 (1971).
  11. A. Papiernik, M. Chatard-Moulin and B. Jecko, Proc. IXth Inter. Conf. on High Energy Accel., Stanford, Calif., 1974, p. 375; and E. Keil, SLAC Report, PEP 126 (1975).