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THE EFFECT OF FINITE β ON STELLARATOR TRANSPORT

By

H.E. Mynick

APRIL 1984

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THE EFFECT OF FINITE β ON STELLARATOR TRANSPORT

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ABSTRACT

A theory of the modification of stellarator transport due to the presence of finite plasma pressure is developed, and applied to a range of stellarator configurations. For many configurations of interest, plasma transport can change by more than an order of magnitude in the progression from zero pressure to the equilibrium β limit of the device. Thus, a stellarator with transport-optimized vacuum fields can have poor confinement at the desired operating β . Without an external compensating field, increasing β tends to degrade confinement, unless the initial field structure is very carefully chosen. The theory permits one to correctly determine this vacuum structure, in terms of the desired structure of the field at a prescribed operating β . With a compensating external field, the deleterious effect of finite β on transport can be partially eliminated.

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I. INTRODUCTION

Attaining acceptably long confinement times at the conditions of low collisionality relevant to a fusion reactor is one of the issues of principal concern for stellarator design. Recent work¹⁻⁶ has shown, both through Monte-Carlo simulations and analytic theory, that stellarator transport is quite sensitive to the structure of the magnetic field strength $B(x) \equiv |B|(x)$. For example, it has been shown that transport rates can vary by more than an order of magnitude, for the same average ripple strength over a flux surface, by redistributing the ripple.^{2,3}

Efforts at transport optimization of realistic stellarator fields (i.e., fields computed from actual coil sets),^{7,8} or of model fields^{2,3} approximately representing such real fields,⁹ have focused on vacuum fields. However in a reactor, as in the coming generation of stellarator experiments, the operating plasma pressure will be near the equilibrium β limit for that device. Thus, the magnetic surfaces will be shifted an amount comparable to the minor radius of the plasma, and the structure of B will correspondingly be substantially modified. Accordingly, transport optimization should be carried out not for the vacuum fields, but for the fields at the desired operating β .

In this paper, we employ previously developed results from analytic theories of stellarator equilibrium and transport to develop an understanding of the effect of finite β on stellarator transport. An analytic theory for "1/v" transport in stellarators with more than one helical component in B has been developed,^{2,5,6} which requires as input only a description of B in flux coordinates,^{10,11} independent of the physical origin of that B . Analytic equilibrium results^{12,13} are used to relate the finite- β field strength $B^P(x) = B(x|\beta)$ to the corresponding vacuum field $B^V(x) = B(x|\beta = 0)$. The analysis of Refs. 12 and 13 is done in the convenient framework of flux coordinates,

and is accordingly well adapted to use in the transport results, which are also expressed in those terms.

We shall use the $1/\nu$ transport coefficient D_{-1} as the measure for the confinement properties of any specified $B(x)$. Under high-temperature operating conditions, the electrons may be expected to lie principally in this $1/\nu$ regime,^{14,15} but the ions will more likely lie in a still less collisional regime, scaling as $\nu^{1/2}$ or ν ,^{14,16,17} where the poloidal precession due to the radial electric field has a beneficial effect on confinement. Of the two roots $\phi_{\pm}$ of the ambipolarity constraint for the electrostatic potential $\phi(r)$ at which a stellarator can operate,^{18,19} it is the negative root $\phi(r=0) = \phi_- < 0$ for which the electrons are the rate-controlling species, and so it is at this root where D_{-1} gives a good measure of the confinement time. At the positive root ϕ_+ , for precise statements about the β dependence of transport, one would need expressions for the transport coefficients $D_{1/2}$, D_1 in the $\nu^{1/2}$ and ν^1 regimes relevant to ions. A transport theory in these regimes, valid for magnetic fields containing more than a single helical component in their harmonic composition, has not yet been developed. However, drawing from understanding derived from study of the β independence of D_{-1} , in Sec. V we shall be able to make some qualitative statements about the β dependence of $D_{1/2,1}$ as well.

The paper is organized as follows. In Sec. II the results needed from equilibrium theory are introduced, and used to obtain expressions describing the β dependence of the field structure. Section III describes the required transport results, discussing some of the limitations of that theory, and introduces a subclass of magnetic configurations to which the theory applies, whose study is useful in understanding the tendencies which emerge in the more general discussion which follows. In Sec. IV, the theory is applied to a

series of specific configurations of increasing complexity, from which an understanding of the physical origin of the transport modification is developed. Finally, in Sec. V, we summarize our findings, and discuss some of the limitations of the theory.

II. EQUILIBRIUM MAGNETICS

We work in flux coordinates^{10,11} (ρ, θ, ζ) , with θ and ζ the poloidal and toroidal angles, respectively, in the flux coordinate system, and ρ the minor radial variable, constant on a flux surface, but as yet otherwise unspecified. The magnetic field $\underline{B}$ then has the contravariant (Clebsch) representation

$$\underline{B} = \nabla\psi \times \nabla(\theta - i\zeta) \quad . \quad (1)$$

Here $2\pi\psi(\rho)$ is the toroidal flux contained within flux surface ρ , and $i(\rho)$ is the rotational transform.

It will be convenient at times to designate the triplet (ρ, θ, ζ) by $\underline{q} \equiv (q^1, q^2, q^3)$. The magnetic field strength $B \equiv |\underline{B}|$ may in general be written as

$$B(\underline{q}) = \sum_{n,m} B_{nm}(\rho) \cos \eta_{nm} \quad , \quad (2)$$

where $\eta_{nm} \equiv n\zeta - m\theta$. [For most stellarators of interest the series in $\sin \eta_{nm}$, absent in Eq. (2), can be eliminated by an appropriate choice of origin. This has been assumed in Eq. (2).] In Refs. 12 and 13, the difference $\tilde{B} \equiv B^P - B^V$ between the magnetic field at finite β and in the vacuum is calculated to first order in β , and the corresponding change

$$\tilde{q}^i \equiv q_v^i - q_p^i \quad (3)$$

in the flux coordinate variables is then determined. Specializing the radial variable ρ to $r \equiv (2\psi/B_0)^{1/2}$, with $B_0 \equiv B_{00}(\rho=0)$, $\tilde{B}$ has the covariant representation

$$\tilde{B} = \tilde{\nabla} r \tilde{B}_r + \tilde{\nabla} \theta \tilde{B}_\theta + \tilde{\nabla} \zeta \tilde{B}_\zeta, \quad (4)$$

with

$$\tilde{B}_r = -\left(\frac{\beta_0 B_0 R}{4r}\right) \sum' \frac{|m|\Delta r}{n-1m} \sin \eta_{nm}, \quad (5)$$

$$\tilde{B}_\theta = \left(\frac{\beta_0 B_0 R}{4}\right) \sum' \frac{n\Delta_\theta}{n-1m} \cos \eta_{nm}, \quad (6)$$

and

$$\tilde{B}_\zeta = -\left(\frac{\beta_0 B_0 R}{4}\right) \left\{ 2\left[1 - \left(\frac{r}{a}\right)^2\right] + \sum' \frac{n\Delta_\theta}{n-1m} \cos \eta_{nm} \right\}. \quad (7)$$

Here,

$$\Delta_r(r, n, m) \equiv \delta_{nm} \left[1 - \frac{|m|}{|m|+1} \left(\frac{r}{a}\right)^2 \right],$$

$$\Delta_\theta(r, n, m) \equiv \delta_{nm} \left[1 - \frac{|m|+2}{|m|+1} \left(\frac{r}{a}\right)^2 \right],$$

a is the value of r at the plasma edge, $\beta_0 \equiv 8\pi p_0/B_0^2$ is the value of β at $r = 0$, and the symbol $\sum'$ denotes a sum over all m and n , except for the

$(n,m) = (0,0)$ - term. The amplitudes $\delta_{nm}(\rho)$ are defined by

$$B^{-2}(q) = B_0^{-2} \left(1 + \sum' \delta_{nm} \cos \eta_{nm} \right), \quad (3)$$

and so, for small δ_{nm} , are related to the B_{nm} of Eq. (2) by

$$B_{nm} \approx -B_0 \delta_{nm}/2. \quad (9)$$

In deriving these expressions for the $\tilde{B}_1$, the simplifying assumptions were made that $\delta_{nm} \propto r^{|m|}$, that (nr/mR) for appreciable δ_{nm} is small, and that the pressure profile is given by $p(r) = p_0[1-(r/a)^2]$. The first term in braces in Eq. (7) thus represents the diamagnetic well.

From Eqs. (4-7), the displacements $\tilde{q}_1$ in going from the vacuum to finite- β flux coordinates are obtained. The finite- β ζ -coordinate is chosen as unchanged from the vacuum values i.e.,

$$\tilde{\zeta} \equiv 0. \quad (10)$$

The rest of the transformation is then specified by^{12,13}

$$\tilde{r} = \sum' r_{nm} \cos \eta_{nm}, \quad \tilde{\theta} = \sum' \theta_{nm} \sin \eta_{nm}, \quad (11)$$

$$r_{nm} = \left(\frac{r_0^2}{4} \right) \frac{|m| \Delta_z}{\varepsilon_t^{2(n-1)m} 2}, \quad (12)$$

$$\theta_{nm} = \left(\frac{\beta_0}{4} \right) \frac{m \theta}{\varepsilon_t^{2(n-1)m} 2} \quad (|m| \neq 1), \quad (13)$$

and

$$\theta_{nm} = \left(\frac{3\beta}{8} \right) \frac{\delta_{nm}}{\epsilon_t^2 (n-1m)^2} \left(\frac{r}{a} \right)^2 \quad (|m| = 1) \quad (14)$$

Here, $\epsilon_t \equiv r/R$. Expression (14) for θ_{nm} is valid relative to the shifted magnetic axis. Terms in $t' \equiv dt/dr$ present in the same expression in Ref. 13 are of the same order as terms coming from the r variation of t in the resonant denominator, which were neglected,²⁰ and so are not given in Eq. (12) through (14) here.

We now use these equilibrium results to compute the field strength $B(g|\beta)$ in flux coordinates, as a function of β . As in the introduction, we denote by the function $B^P(g)$ the finite-pressure value of B , and by $B^V(g) \equiv B(g|\beta = 0)$ its vacuum value. The difference between these two,

$$\tilde{B}(g) \equiv B^P(g) - B^V(g) \quad (15)$$

is the sum of two contributions, denoted $\tilde{B}_Q$ and $\tilde{B}_B$. $\tilde{B}_Q$ is the change in $B(g)$ due to the shift with β of the flux coordinate system, described by Eqs. (3) and (10) through (14). Keeping up to first order in β (as throughout this paper), this contribution is

$$\tilde{B}_Q = \tilde{g} \cdot \left(\frac{\partial}{\partial g} \right) B^V = \sum_1 \sum_2 \left[\left(\frac{\partial}{\partial r} B_1^V \right) r_2 \cos \eta_1 \cos \eta_2 + B_1^V m_1 \theta_2 \sin \eta_1 \sin \eta_2 \right]. \quad (16)$$

The subscripts 1,2 on $\sum$, B^V , r , η , and θ here are shorthand for (n_1, m_1) and (n_2, m_2) , and $\partial_r \equiv \partial/\partial r$ is the partial derivation with respect to r .

The contribution $\tilde{B}_B$ comes from the modification of B at a fixed spatial position x :

$$\tilde{B}_B = \frac{\tilde{B} \cdot \tilde{B}^V}{B^V} = - \left(\frac{\beta_0 B_0}{4} \right) \left\{ 2 \left[1 - \left(\frac{r_1}{a} \right)^2 \right] + \sum \Delta_0 \cos \eta_{nm} \right\} \quad (17)$$

Using the trigonometric identities for products of sines and cosines, the double sum in Eq. (16) may be regrouped into a single sum over (n, m) ,

$$\tilde{B}_Q = \sum_{n, m} \tilde{B}_{qnm} \cos \eta_{nm} \quad (18)$$

where

$$\begin{aligned} \tilde{B}_{qnm} = \frac{1}{2} \sum_{1,2} \{ & \delta(n_2 - n_1 - n) \delta(m_2 - m_1 - m) [(\partial_r B_1^V) x_2 + B_1^V m_1 \theta_2] \\ & + \delta(n_2 + n_1 - n) \delta(m_2 + m_1 - m) [(\partial_r B_1^V) x_2 - B_1^V m_1 \theta_2] \} \quad (19) \end{aligned}$$

with $\delta(k) = 1$ for $k = 0$ and $\delta(k) = 0$ otherwise.

$\tilde{B}_B$ is typically small compared with $\tilde{B}_Q$. To see this, we compare the scalings of their Fourier amplitudes $\tilde{B}_{qnm}$ and $\tilde{B}_{Bnm}$:

$$\begin{aligned} \tilde{B}_{qnm} &\sim \beta_0 B_0 \delta_{n_1 m_1} \delta_{n_2 m_2} m_1 m_2 / \epsilon_t^2 (n_2 - m_2)^2 \\ \tilde{B}_{Bnm} &\sim \beta_0 B_0 \delta_{nm} \quad (20) \end{aligned}$$

Taking the δ_{nm} to be of order a parameter δ and m to be of order 1 for those (n, m) which significantly contribute, one has

$$\tilde{B}_{qnm} / \tilde{B}_{Bnm} \sim \delta / \epsilon_t^2 (n_2 - m_2)^2 \quad (21)$$

The denominator in Eq. (21) comes from the q_{nm} factors (r_{nm} , θ_{nm}) in $\tilde{B}_q$. Taking $\epsilon_t \sim 5$, and $i \sim 1$, one sees that the $n_2 = 0$ contributions to $\tilde{B}_q$ cause it to dominate $\tilde{B}_B$. This corresponds to the fact that the $(n,m) = (0,-1)$ amplitudes q_{0-1} in Eq. (12) and (13), which produce the Shafranov shift, dominate the other q_{nm} 's for most stellarators. Thus, for analytic purposes, considerable simplification is achieved by neglecting $\tilde{B}_B$ in $\tilde{B}$, and keeping only the $(0,-1)$ component in the sum over (n_2, m_2) in Eq. (16) or (19):

$$\tilde{B} = \tilde{B}_q = \sum_1 [(\partial_r B_1^V) r_{0-1} \cos n_1 \cos \theta + B_1^V n_1 \theta_{0-1} \sin n_1 \sin \theta] \quad (22)$$

For the parameters considered in this paper, Eq. (22) is a good approximation to the full $\tilde{B}$, except for the heliac reference design,²¹ for which $\tilde{B}_B$ is again negligible, and only a single additional q_{nm} need be kept to provide a reasonable representation of the field.

III. TRANSPORT

For most stellarators, most of the B_{nm} in Eq. (2) are negligible. For a system with N field periods, it is often found²² that only those B_{nm} with $n = N$ or 0 are appreciable. In such cases, Eq. (2) simplifies to a form more amenable to analytic efforts^{5,23}:

$$B(q) = B_0 [1 - e_t - e_h \cos(n_{NM} - \alpha)] \quad (23)$$

Here

$$e_t(r, \theta) \equiv -\sum_m \left(\frac{B_{0m}}{B_0} \right) \cos m\theta, \quad e_h(r, \theta) \equiv (e_{hc}^2 + e_{hs}^2)^{1/2},$$

and

$$\alpha(r, \theta) \equiv \arctan \left(\frac{e_{hs}}{e_{hc}} \right),$$

with

$$e_{hc}(r, \theta) \equiv -\sum_m \left(\frac{B_{Nm}}{B_0} \right) \cos[(m-M)\theta],$$

and

$$e_{hs}(r, \theta) \equiv -\sum_m \left(\frac{B_{Nm}}{B_0} \right) \sin[(m-M)\theta].$$

M is chosen as the m value of the dominant ripple perturbation, so that e_{hc} and e_{hs} vary as slowly as possible with θ .

A subclass of the fields described by Eq. (23), with relevance to our present study of the β dependence of transport, is the family of "σ-configurations," whose transport was studied numerically and analytically in Refs. 2 and 3. On a specified surface $r = r_0$, these configurations have $\alpha = 0$, and

$$e_h(r_0, \theta | \sigma) \equiv e_h(r_0) (1 - \sigma \cos \theta). \quad (24)$$

Assuming, as before, that the individual amplitudes B_{nm} vary as $r^{|m|}$, Eq. (24) specifies the B_{nm} at all r . Taking the usual $\beta = 0$ form

$$e_t = e_t(r) \cos \theta = e_t(r) \cos n_{0-1} \theta, \quad (25)$$

the nonzero amplitudes of the σ configuration are given by

$$B_{00} = B_0, \quad B_{0-1} = -B_0 e_t, \quad B_{NM} = -B_0 e_h(r),$$

and

$$B_{N,M\pm 1} = \frac{\sigma}{2} B_0 \epsilon_h(r_0) (r/r_0)^{M\pm 1} \quad (26)$$

where $\epsilon_h(r) \propto r^M$, and M is assumed positive.

From Eq. (24) or (26), one sees that $\sigma = 0$ yields the simple "single-helicity" model traditionally adopted for analytic study of stellarator transport. For $\sigma < 0$, the ripple becomes localized toward the outside ($\theta = 0$) of the torus, while for $\sigma > 0$, the ripple becomes localized toward the inside ($\theta = \pi$) of the torus. Numerical³ and analytic^{2,5} studies have shown that positive σ has lower transport and negative σ greater transport (for constant θ -averaged ripple strength ϵ_h) than $\sigma = 0$, with an optimum reached around $\sigma \epsilon_h / \epsilon_t = 1.5$.

The theory for $1/v$ transport in σ configurations developed in Ref. 2 is extended in Refs. 5 and 6 to a theory valid for all configurations described by Eq. (23). The radial particle and energy fluxes Γ_n and Γ_E decompose into the product of two factors,

$$\Gamma = I_0 \gamma_v \quad (27)$$

where γ_v is an integral over velocity space, independent of the features of the magnetic geometry, and I_0 is a θ integral containing this geometric information:

$$\gamma_v \equiv \frac{8}{M_s^{2/2} \Omega_s^2 r_s^2} \int_0^\infty dE \frac{E^q}{v} \partial_r f_{Ms} \quad (28)$$

$$I_0 \equiv \oint \frac{d\theta}{2\pi} \epsilon_h^{3/2} [G_1(\partial_\theta e_t)^2 + 2G_2(\partial_\theta e_t)(\partial_\theta e_h) + G_3(\partial_\theta e_h)^2] \quad (29)$$

Here, M_s is the particle mass for species s , Ω_s its gyrofrequency, $q = 5/2$ for Γ_n and $7/2$ for Γ_p , E is the particle kinetic energy, f_{Ms} is the Maxwellian distribution function describing species s , and the G_i are numerical coefficients, arising from a pitch-angle integration over x^2 , the square of the radial drift velocity. Their values are⁵ $G_1 = 16/9$, $G_2 = 16/15$, $G_3 = 0.684$.

We may write $I_0 = I_{01} + 2I_{02} + I_{03}$, where I_{0i} ($i = 1, 2, 3$) is the contribution arising from the term in G_i in Eq. (29). I_{01} and I_{03} are positive definite. It is the cross term I_{02} which permits a reduction in transport by appropriate θ modulation of the ripple amplitude e_h . Using the family of σ configurations as an example, from Eqs. (24) and (25) one has $\partial_\theta e_h = -(\sigma \epsilon_h / \epsilon_t) \partial_\theta e_t$, and hence

$$I_0(\sigma) = [1 - 2(\sigma \epsilon_h / \epsilon_t) G_2 / G_1 + (\sigma \epsilon_h / \epsilon_t)^2 G_3 / G_1] I_{01}, \quad (30)$$

from which the beneficial effect on transport from positive σ through the negative second term is more readily apparent. It should be remembered that some residual σ dependence remains in I_{01} here, through the factor $e_h^{3/2}$ in the integrand. Most of the σ dependence, however, is held in the expression in brackets. This expression is a minimum, for $\sigma \epsilon_h / \epsilon_t = G_2 / G_3 \approx 1.56$, in approximate agreement with the result found numerically in Ref. 5 to minimize I_0 .

IV. APPLICATIONS

The dependence of the factor γ_v in Eq. (28) on temperature T , density n , field strength B , and so on β , contains the full dependence of the Γ of previous transport theory on these parameters. Thus, studying the variation

of I_0 with β succinctly describes the additional impact of the effect considered in this paper. Accordingly, in the remaining sections we shall be examining the function $I_0(\beta)$ for a variety of configurations.

Our approach, then, is to put the expressions for $B_{nm}(r|\beta)$ from Sec. II into Eq. (29) for I_0 , and to study this as a function of β . For physical insight, it will also be useful to plot the profiles $B(s|\beta)$ along a field line, and to consider their variation with β . (Here s measures the distance along a field line.) Before proceeding, we extend the range of applicability of the theory. Until now, we have assumed, along with Refs. 12 and 13, that one is given the vacuum amplitudes B_{nm}^V , and from this one computes the finite- β amplitudes B_{nm}^P , to first order in β . However, if instead one is given the amplitudes $B_{nm}^0 \equiv B_{nm}(\beta_0 = \beta_{00})$ at some initial central β value $\beta_0 = \beta_{00}$, and one wants to know the amplitudes $B_{nm}^1 \equiv B_{nm}(\beta_0 = \beta_{01})$ at $\beta_0 = \beta_{01} \equiv \beta_{00} + \Delta\beta_0$, precisely the same expressions given in Sec. II for the change in field components, shift in flux coordinates, and change in the field amplitudes B_{nm} are valid, but with β_0 there replaced by $\Delta\beta_0$. This is true even for $\Delta\beta_0 < 0$. Thus, if one knows the amplitudes B_{nm}^0 desired (for, e.g., transport optimization) at the operating $\beta_0 = \beta_{00}$, the vacuum amplitudes $B_{nm}^V = B_{nm}^1$ one should design for may be computed from the theory presented here, simply by taking $\beta_{01} = 0$, i.e., $\Delta\beta_0 = -\beta_{00}$. Thus, in the following, we will be plotting $I_0(\Delta\beta_0)$ for $\Delta\beta_0$ both positive and negative.

A. Tokamak

We now consider $B(s|\beta_0)$ and $I_0(\beta_0)$ for a series of configurations of increasing complexity. We begin by considering $B(s|\beta_0)$ for a tokamak, whose nonzero vacuum amplitudes are $B_{00} = B_0$, and $B_{0-1} = -B_0 \varepsilon_t$. Using Eq. (19) or Eq. (22), and $B_{nm} = r^{|m|}$, one easily finds

$$\tilde{B}_q = \frac{1}{2} B_{0-1} \left[\left(\frac{r_{0-1}}{r} - \theta_{0-1} \right) + \left(\frac{r_{0-1}}{4} + \theta_{0-1} \right) \cos 2\theta \right] \quad (31)$$

Henceforth, we shall ignore contributions to the $(0,0)$ component $\tilde{B}_{00}$ of $\tilde{B}$. These terms contribute only to the poloidal precession frequency $\dot{\theta}$ of ripple-trapped particles, and not to their radial drift $\dot{r}$. Since $\dot{\theta}$ does not enter at all in the $1/\nu$ -regime, $\tilde{B}_{00}$ is irrelevant. Moreover, even in the $\nu^{1/2}$ and ν regimes where $\dot{\theta}$ plays an important role, the ∇B -contribution to $\dot{\theta}$ coming from B_{00} is overwhelmed by the $E \times B$ contribution, unless β_0 is of order unity. The only remaining contribution to $\tilde{B} = \tilde{B}_q$ for a tokamak is thus $\tilde{B}_{0-2}$, which modifies the shape $e_t(\theta)$ of the toroidal well. In Fig. 1, we plot $B(s|\beta_0)$, for one full poloidal period, taking $e_t = 0.1$, $B_0 = 1$, $r/a = 0.5$, and for $\beta_0 = 0, 0.005$, and 0.01 . For these parameters, $r_{0-1}/r \approx (48.6)\beta_0$ and $\theta_{0-1} \approx (20.8)\beta_0$. Thus at $\beta_0 = 0.01$, $r_{0-1}/r \approx 1/2$, near the plasma equilibrium β limit. The positive sign of r_{0-1} corresponds to an outward shift of the plasma, and the positive sign of θ_{0-1} corresponds to the local rotational transform at finite β being enhanced on the outside of the torus ($\theta = 0$), and reduced on the inside. Both signs thus confirm what one expects physically.

B. σ Configurations

We now proceed to stellarators for which I_0 is a meaningful quantity. We begin with the σ configurations. We choose parameters characteristic of conventional circular-axis devices; $i \sim 0.5$, $m = 2$ or 3 , and N large, so that $1/N$ is small. Thus, the general features of the results presented here should be applicable to the magnetic structure of most of the experimental devices currently in operation or being planned (though most of these operate in a more collisional regime than the $1/\nu$ regime studied here). Specifically, we

take $l = 0.3$, $M = 2$, $N = 8$, $r/a = 0.5$, $\epsilon_e(r/a = 0.5) = 0.1 = \epsilon_h(0.5)$, and $B_0 = 1$. The surface $r = a/2$ is assumed to be the surface r_0 at which Eq. (24) holds precisely, and so the amplitudes B_{nm}^0 are given by Eq. (26). From Eq. (19) or (22), one readily obtains approximate expressions for the nonzero perturbation amplitudes $\tilde{B}_{nm} = \tilde{B}_{qnm}$:

$$\tilde{B}_{0-2} = \frac{1}{2} B_{0-1}^0 \left(\frac{r_{0-1}}{r} + \theta_{0-1} \right) = -\frac{\epsilon_e}{2} \Delta_+ \quad , \quad (32)$$

$$\tilde{B}_{N,M\pm 1} = \frac{1}{2} B_{NM}^0 \left(|M| \frac{r_{0-1}}{r} \pm M \theta_{0-1} \right) = -\epsilon_h \Delta_{\pm} \quad (33)$$

$$\tilde{B}_{N,M\pm 2} = \frac{1}{2} B_{NM\pm 1}^0 \left[|M \pm 1| \frac{r_{0-1}}{r} \pm (M \pm 1) \theta_{0-1} \right] = \left(\frac{\sigma \epsilon_h}{4} \right) (2\pm 1) \Delta_{\pm} \quad , \quad (34)$$

and

$$\begin{aligned} \tilde{B}_{NM} &= \frac{1}{2} B_{NM-1}^0 \left[|M+1| \frac{r_{0-1}}{r} - (M+1) \theta_{0-1} \right] + \frac{1}{2} B_{NM+1}^0 \left[|M-1| \frac{r_{0-1}}{r} + (M-1) \theta_{0-1} \right] \\ &= \sigma \epsilon_h \left(\frac{r_{0-1}}{r} - \frac{1}{2} \theta_{0-1} \right) \quad . \end{aligned} \quad (35)$$

Here, $\Delta_{\pm} \equiv (r_{0-1}/r \pm \theta_{0-1})$, and in the last form of each of Eqs. (32) through (35), use has been made of the choices $B_0 = 1$ and $M = 2$, and of Eqs. (26).

Specializing to $\sigma = 0$, only Eqs. (32) and (33) give nonzero $\tilde{B}_{nm}$. The amplitude $\tilde{B}_{0-2}$ is the same as in the tokamak case, already considered. From Eq. (33), one finds for the perturbation $\tilde{B}_h \equiv (\tilde{B} - \tilde{B}_{0-2} \cos 2\theta)$ of the helical ripple amplitude

$$\tilde{B}_h = -2\epsilon_h \left[\left(\frac{r_{0-1}}{r} \right) \cos \theta \cos n_{N2} + \theta_{0-1} \sin \theta \sin n_{N2} \right] \quad . \quad (36)$$

Since for $\Delta\beta_0 > 0$, r_{0-1} is positive, one sees from Eq. (36) that $\tilde{B}_h$ enhances the ripple amplitude toward $\theta = 0$ and reduces it toward $\theta = \pi$, thus making a system with $\sigma = 0$ at $\Delta\beta_0 = 0$ more like one with $\sigma < 0$ for $\Delta\beta_0 > 0$, and like $\sigma > 0$ for $\Delta\beta_0 < 0$. Physically, this enhancement of the ripple on the outside of the torus with increasing β occurs because the Shafranov shift (r_{0-1}) moves a given flux surface closer to the helical windings (which produce B_{NM}^0) toward $\theta = 0$, and further away toward $\theta = \pi$.

In Figs. 2 are shown $B(s|\Delta\beta_0)$, again for one poloidal period, and for $\Delta\beta_0 = -0.01$ (a), 0.0 (b), and 0.01 (c). As in the tokamak case, for $|\Delta\beta_0| = 0.01$, one is near the equilibrium β limit, and so at the limits of the range within which the linear approximation (in $\Delta\beta_0$) for $\tilde{B}$ is valid.

The plots of $B(s)$ in Fig. 2 were made keeping all the harmonics present in Eqs. (16) and (17), i.e., without the neglect of all q_{nm} except for q_{0-1} , which led to the approximate form in Eq. (22), and without neglecting all B_{nm} except those with $n = N$ or 0, as was done to obtain the form Eq. (23) which is suitable for use in expressions for transport. Since $q_{NM}/q_{0-1} \sim (1/N)^2 = 0.0014$, the former of these approximations is a quite good one for present parameters. The validity of the latter approximation is shown in Fig. 3, where the envelope of the ripple amplitude in Eq. (23), given by the two curves

$$B_{\pm}(\theta) \approx B_0 \{1 - e_t(\theta) \pm e_h(\theta)\} \quad (37)$$

is plotted, for the same parameters as in Fig. 2. One notes that the envelopes in Fig. 3 delineate very well the outer boundary of the curves in Fig. 2.

Using the forms for $e_t(\theta)$ and $e_h(\theta)$ shown in Fig. 3, I_0 is computed.

This is plotted in Fig. 4. The horizontal line is the reference value $I_0 = I_{00}$ which a $\sigma = 0$ configuration with the same values of ϵ_t and ϵ_h would have,

$$I_{00} \equiv \frac{1}{2} G_1 \epsilon_h^{3/2} \epsilon_t^2 \quad (38)$$

Thus, for the present case, which is a $\sigma = 0$ configuration at $\Delta\beta_0 = 0$, one finds $I_0 = I_{00}$ at $\Delta\beta_0 = 0$.

From $\Delta\beta_0 = 0$, one sees that transport increases toward $\Delta\beta_0 > 0$ and decreases toward $\Delta\beta_0 < 0$. This trend is consistent with the expectation, already mentioned, that increasing the outward shift by increasing β_0 results in a decrease in the σ value attributable to a given configuration. Closer consideration of Fig. 4, however, shows that this rule of thumb is by itself insufficient. While transport deteriorates rapidly for $\Delta\beta_0 > 0$ (as expected), the improvement for $\Delta\beta_0 < 0$ is far less than the order of magnitude reduction found for the model configurations studied in Refs. 2 and 3.

To help in understanding this, we refer to Fig. 5, in which is plotted the same $I_0(\Delta\beta_0)$ as in Fig. 4, along with its constituent parts $I_{0i}(\Delta\beta_0)$ ($i = 1, 2, 3$). The two positive definite terms $I_{01,3}$ are roughly quadratic about $\Delta\beta_0 = 0$, while the cross-term I_{02} , which (as has already been noted) allows for transport reduction, is roughly linear in $\Delta\beta_0$, reducing transport for $\Delta\beta_0 > 0$ ($\sigma < 0$ -like) and enhancing it for $\Delta\beta_0 < 0$ ($\sigma > 0$ -like). Thus, in the vicinity of $\Delta\beta_0 = 0$, it is the variation in I_{02} which dominates the variation in I_0 . Away from $\Delta\beta_0 = 0$, however, the quadratic variation of I_{01} (which arises from the $\tilde{E}_{0-2}$ contribution to e_t) dominates, so that I_0 increases for increasing $|\Delta\beta_0|$, for $\Delta\beta_0$ both positive and negative. In contrast, in Refs. 2 and 3, σ alone was varied, keeping ϵ_t and ϵ_h fixed, and also keeping the harmonic content of e_t unchanged. When varying σ alone, I_{01} is constant, and

so has no deleterious effect, while the beneficial effect of $I_{\theta 2}$ is still present.

As has been mentioned, transport is optimized for the σ configurations for $\sigma_H/\epsilon_t \approx 1.5$. In Figs. 6-8 are shown plots analogous to Figs. 2-5 just discussed, but for a configuration with $\sigma_H/\epsilon_t = 1.5$ at $\Delta\beta_0 = 0$. As in Fig. 2, the solid curves in Fig. 6 show the profiles $B(s|\Delta\beta_0)$ for one poloidal period, again for $\Delta\beta_0 = -0.01, 0.0$, and 0.01 . The dashed lines show the envelope profile of Eq. (37) (analogous to Fig. 3). In Fig. 7 $I_\theta(\Delta\beta_0)$ is plotted. One sees, as expected, that a minimum is reached close to $\Delta\beta_0 = 0$, with a value about a factor of 10 below the $\sigma = 0$ reference value $I_{\theta 0}$. Thus, at the operating β (which for present parameters should be near $\beta_0 = 0.01$), transport considerations prescribe a profile $B(s)$ like that shown in Fig. 6b. This implies that the vacuum field should be designed to have a profile like that in Fig. 6a, which one notes from Fig. 7 has much poorer transport characteristics.

From Fig. 8, one sees that the degraded confinement at $\Delta\beta_0 = -0.01$ comes principally from the increase in $I_{\theta 3}$, arising because of excessively large ripple toward $\theta = \pi$. One also notes that, in contrast to the $\sigma = 0$ case (Fig. 5), I_θ is smaller in magnitude than its constituent contributions $I_{\theta 1}$, reflecting the fact that the factor of 10 reduction in I_θ over $I_{\theta 0}$ at $\Delta\beta_0 = 0$ is achieved by a rather delicate cancellation of terms.

C. Vertical Field

Up to now, we have assumed that no external vertical field E_{ext} is applied as the plasma β is raised. Since the modification of transport discussed here arises principally from shifts in the flux surfaces with increasing β , one possible way to counteract unwanted effects of these shifts

is by cancelling them using external fields.

However, this cancellation cannot be complete. An applied uniform B_{ext} produces a shift r_{0-1}^{ext} independent of r , while the shift r_{0-1}^p [r_{0-1} in Eq. (12)] induced by the plasma itself is r dependent. The resultant total (0,-1) shift $r_{0-1} = r_{0-1}^p + r_{0-1}^{ext}$ therefore has the same r dependence as r_{0-1}^p , so that one may choose $r_{0-1} = 0$ on only one surface.

As an example, we choose B_{ext} so as to keep the magnetic axis fixed, $r_{0-1}(r=0) = 0$. Then r_{0-1} is given by Eq. (12), but with the factor 1 in Δ_r [defined following Eq. (7)] eliminated. The resultant shift is negative for $r > 0$, reflecting the fact that the axis shift in the absence of B_{ext} is larger than the shift at any other r .

In Figs. 9 and 10 are shown the $B(s)$ profiles and $I_0(\Delta\beta_0)$ for a configuration having $\sigma = 0$ at $\Delta\beta_0 = 0$ (just as considered in Figs. 2-5), and with the magnetic axis held fixed by B_{ext} . Since the radial shift at $r/a = 0.5$ is now negative, the qualitative effect on $B(s)$ and $I_0(\Delta\beta_0)$ is opposite that shown in Figs. 2-5, as might be expected. The precise form of $\tilde{B}$ is not the same as for the shifted axis case, however, since Eq. (14) for θ_{0-1} is unaffected by the presence of B_{ext} , as already indicated following Eq. (14). From Fig. 10, one sees that increasing β slightly improves transport, in contrast to the two shifting-axis examples already considered.

D. Heliac

In contrast to the configurations studied thus far, the heliac design²¹ seriously strains a number of the assumptions going into much of stellarator transport theory as it presently exists, including the theory summarized by Eqs. (27) and (28). Because of its helical axis, heliac has $1/N \sim 1$. (For the Princeton reference design, whose parameters are used here, one has $1 \approx$

0.5, and $N = 3$, hence $1/N = 0.5$.) This large transform eliminates the separation of length scales which exists in circular axis devices. There, the length $L_h = 2\pi R/N$ along a field line between ripple wells is small compared to the length $L_t = 2\pi R/1$ along a field line for one poloidal period. The fact that $L_h/L_t = 1/N \ll 1$ for those configurations is what makes meaningful the factorization of the ripple term in Eq. (23) into a rapidly oscillating term $\cos n_{NM}$ times a slowly varying modulating amplitude e_h . Moreover, the assumption in Eq. (23) that only B_{nm} 's having $n = 0$ or N are significant also fails for heliac; in addition to the fundamental $n = N = 3$ terms, there are also $n = 2N = 6$ components of comparable size. This feature, along with the fact that $L_h \sim L_t$, makes conservation of the bounce action J for ripple-trapped particles much less uniformly reliable, and so makes a believable collisionless orbit theory much more difficult.

However, $1/v$ transport does not depend on conservation of J , but only on the bounce averaged radial drift $\bar{r}$. In a complete theory, the full well shape should be taken into account, but restricting ourselves to only the $n = 0$ and $n = N$ components still provides a rough description of the well shape. Thus, we shall apply the transport theory of Eqs. (27) and (28), remembering the much more approximate character of the results, in comparison with the cases already considered.

The validity of the equilibrium theory given in Sec. II, in contrast, is not affected by the differences just noted for heliac parameters. Thus, the β scaling of the $B(s)$ profiles is still valid. However, the resonant denominator $(n - 1m)$ which previously permitted neglect of all q_{nm} except for $(n,m) = (0,-1)$ is now comparably small for the $(n,m) = (N,1)$ harmonic: $[0 - 1(-1)] = 1.5 = [N - 1(1)]$. Thus the approximation leading to Eq. (22) must be generalized for heliac, to include the effects of this helical shift q_{N1} .

For the parameters taken here [cf. Eq. (40)], one finds

$$r_{0-1}/r = (1.44)\delta_0, \quad r_{N1}/r = (1.11)\delta_0. \quad (39)$$

Neglect of $\tilde{B}_3$ in comparison with $\tilde{B}_q$ is still valid, however. From Eq. (21), one has

$$\frac{\tilde{B}_{qnm}}{\tilde{B}_{Bnm}} \sim \frac{\delta}{\epsilon^2} \gg 1.$$

A reasonable representation of the fields for the heliac reference design is given by taking six nonzero B_{nm} 's. At $r/a = 0.5$, these are

$$B_{00}^0 = 1.2, \quad B_{0-1}^0 = -0.162, \quad B_{30}^0 = -0.157, \quad B_{61}^0 = 0.129, \quad (40)$$

$$B_{31}^0 = -0.125, \quad \text{and} \quad B_{3-1}^0 = 0.0401.$$

(Although B_{30}^0 and B_{61}^0 are larger than B_{31}^0 , it is the (3,1) shifts which dominate. $q_{30} = 0$ because $m = 0$, and q_{61}/q_{31} is small because of the resonant denominators of these two components.)

In Figs. 11 and 12 are shown the $B(s)$ profiles and $I_0(\Delta\beta_0)$ for these heliac parameters. The envelope [Eq. (37)] used in computing I_0 is indicated by the dashed lines, as in Fig. 6. One sees that, though the full $B(s)$ is roughly bounded by the envelope, much of the detail of the well structure is lost, owing to the limitations on the transport theory already discussed.

Because the shifts in Eq. (39) are an order of magnitude smaller than their counterparts from the previous applications, an order of magnitude larger range in $\Delta\beta_0$ was taken. Even so, one sees that the magnetic structure,

and consequently the transport, is only weakly affected by β over this range. Therefore, it is probable that the principle danger of degrading heliac transport when finite β is introduced arises not from the effect studied here, but from the destruction of the magnetic surfaces by magnetic islands.¹³

V. DISCUSSION

This section summarizes the findings and discusses some of the limitations of the work presented here.

We have presented a theory of the effects on stellarator transport of varying the operating β . The theory can be used to project from vacuum fields to the fields at a desired operating β , or from the operating β back to the vacuum fields. The latter capability is necessary for efforts at transport optimization.

Analytic formulae for the β dependence of the Fourier amplitudes B_{nm} which determine transport have been developed. For many cases of interest, the general formulae simplify considerably [cf., e.g., Eq. (22)], permitting an analytic understanding of the β dependence of the transport integral I_0 to be extracted.

Such an understanding has been developed by considering a sequence of configurations of increasing complexity. For circular axis devices, the effect of changing β can be quite strong, changing transport by over an order of magnitude in moving from $\beta = 0$ to the equilibrium β limit. Without an external vertical field applied, increasing β tends to degrade confinement, unless the vacuum amplitudes are very carefully chosen. This is consistent with the results reported in Ref. 5 of applying Eq. (29) to amplitudes B_{nm} numerically generated from a 3D equilibrium code. The degradation arises from

both a shift of the ripple strength toward $\theta = 0$ as the plasma shifts outward, as well as modification in the toroidal well shape $e_{\theta}(\theta)$.

Using an applied vertical field to cancel the Shafranov shift, the degradation of confinement can be partially eliminated. It is difficult, however, to substantially reduce transport by this means. Transport optimized configurations require a rather precise mixture of Fourier amplitudes to achieve the required cancellation of terms in $\dot{r}$ or I_{θ} , for which the single additional "knob" of B_{ext} is insufficient.

Helical axis devices, by virtue of their large transform and so high equilibrium β limit β_{eq} , have transport less sensitive to β . This should also be true of circular axis configurations designed²⁴ to have large β_{eq} through elimination of δ_{0-1} .

The theory presented here suffers from the limitations of the equilibrium and transport theories it draws on. These are noted in Secs. II and III, as well as those discussed in Sec. IV in connection with heliac. In particular, we mention again the simple form for the pressure profile $p(r)$ and for the radial dependence $\delta_{nm} = r^{|m|}$ of the ripple amplitudes assumed by the equilibrium theory, as well as the fact that theory is only valid to first order in β . For the design of an actual machine, of course, the analytically tractable expressions used here would have to be replaced by numerically generated results. However, the general tendencies just described which the analytic theory exhibits should also be reflected by the more precise computations.

As mentioned in Sec. I, we have restricted ourselves to the calculation of the effect of finite β on the $1/v$ transport coefficient D_{-1} . The structure of D_{-1} is simple; it is essentially an average over all ripple-trapped particles of $\dot{r}^2$, $D_{-1} \sim \langle \dot{r}^2 \rangle$, where the bounce-averaged radial drift is given

by

$$\dot{r} = -\left(\frac{\mu B_0}{M \Omega_s r}\right) (\partial_\theta e_t + \partial_\theta e_h \overline{\cos \eta_{NM}}) \quad (41)$$

The overbar on $\cos \eta_{NM}$ means an average over one ripple-bounce period. Thus the term $\overline{\cos \eta_{NM}}$ is pitch-angle dependent, equal to 1 for deeply trapped particles, and -1 for particles barely ripple-trapped. As discussed in Refs. 2 and 3, for $\sigma > 0$ the term in $\overline{\cos \eta_{NM}}$ tends to cancel the term in $\partial_\theta e_t$ for all but the most shallowly trapped particles, permitting the reduction of D_{-1} .

In the less collisional $v^{1/2}$ and v regimes, expressions for the transport coefficients have thus far been developed only for the $\sigma = 0$ configuration. This is a somewhat singular case; as opposed to configurations having $\partial_\theta e_h \neq 0$, for the $\sigma = 0$ configuration only ripple particles barely trapped contribute to transport, because of the pitch-angle independence of the orbits of more deeply trapped particles. The presence of a nonzero $\partial_\theta e_h$ breaks this degeneracy in the orbit shape, causing the more deeply trapped bulk of the ripple particles to again dominate the transport. For these, Eq. (41) governs the same reduction or enhancement of $\langle \dot{r}^2 \rangle$ as is the case for D_{-1} . Thus, one may expect a given change in $e_h(\theta)$ to have an effect on $D_{1/2,1}$ qualitatively similar to that on D_{-1} .

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FIGURE CAPTIONS

- FIG. 1. Plot of the profiles of the magnetic field strength $B(s)$ along a field line for one complete poloidal period for a tokamak, for the three indicated values of the central β . The distance s along a field line is here parameterized by poloidal angle θ . The other parameters taken are $B_0 = 1$, $i = 0.3$, and $\epsilon_t = 0.1$ at $r/a = 0.5$.
- FIG. 2. Profiles $B(s)$ for the three indicated values of $\Delta\beta_0$, for a stellarator which is a $\sigma = 0$ configuration at $\Delta\beta_0 = 0$. Parameters chosen are $N = 8$, $M = 2$, $\epsilon_h(r/a = 0.5) = 0.1$, and otherwise the same parameters as in Fig. 1.
- FIG. 3. Envelopes of the ripple amplitude [Eq. (37)] for the same cases as in Fig. 2, used to compute I_0 in Fig. 4.
- FIG. 4. The portion I_0 of the radial flux dependent on magnetic geometry [Eq. (29)] versus $\Delta\beta_0$, for the stellarator of Figs. 2 and 3. The reference line I_{00} is the value of I_0 for a $\sigma = 0$ configuration with the same ϵ_t and ϵ_h .
- FIG. 5. Plot of the same $I_0(\Delta\beta_0)$ as in Fig. 4, along with its constituent parts I_{0i} ($i = 1, 2, 3$).
- FIG. 6. $B(s)$ profiles for a stellarator having $\sigma = 1.5$ at $\Delta\beta_0 = 0$, and otherwise the same parameters as the case in Fig. 2.

FIG. 7. Plot of $I_0(\Delta\beta_0)$ for the stellarator of Fig. 6.

FIG. 8. Plot of I_0 and the I_{0i} ($i = 1, 2, 3$) versus $\Delta\beta_0$, for the stellarator of Figs. 6 and 7.

FIG. 9. $B(s)$ profile for a stellarator with $\sigma = 0$ at $\Delta\beta_0 = 0$ (as in Fig. 2), but with a vertical field applied to keep the magnetic axis fixed as $\Delta\beta_0$ is varied.

FIG. 10. Plot of $I_0(\Delta\beta_0)$ for the configuration of Fig. 9.

FIG. 11. $B(s)$ profiles for the heliac reference design at $\Delta\beta_0 = -0.1, 0.0$, and 0.1 . Note that the distance L_h across a single ripple well is now comparable with the length L_t for variation of the ripple envelope (dashed lines).

FIG. 12. Plot of $I_0(\Delta\beta_0)$ for the heliac reference design.

Fig. 1

#84T0033

Fig. 2

#84T0035

Fig. 3

Fig. 4

#84T0029

Fig. 5

#84T0031

Fig. 6

Fig. 7

Fig. 8

#84T0032

Fig. 9

Fig. 10

#84T0037

Fig. 11

Fig. 12

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