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APPROPRIATE B INDICES
FOR
EVALUATING LOAD-CONTROLLING STRESSES
IN
PIPING PRODUCTS AT ELEVATED TEMPERATURES

ERDA Research and Development Report

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Rockwell International

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EVALUATING LOAD-CONTROLLING STRESSES
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PIPING PRODUCTS AT ELEVATED TEMPERATURES

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ABSTRACT

At the present time, rules for design of piping for nuclear power plant elevated temperature service are contained in Code Case 1592-8. A Task Group under the Code Working Group on Pipe Design has prepared a revised portion on piping for a future revision of Code Case 1592. This revised portion contains explicit equations for calculating stresses; those stresses are limited by the general stress limits of the Code Case.

The stress equations involve the use of stress indices which, except for the B_2' indices, are given in the present Code, NB-3600. This report started as an effort to develop appropriate B_2' indices for inclusion on the Code Case. However, the report shows that the B_1 and B_2 indices are based on limit load concepts hence there is no need for the B_2' indices.

The general concepts and motivation behind the stress index approach is described. The background of the B_1 and B_2 indices is given for the several types of piping products covered by the indices. Recommendations are given for the text of -3650, "Analysis of Piping Products", for inclusion in a future revision of Code Case 1592.

This report is concerned with stress indices in equations as used for checking the equivalent of "Primary Stress Intensity Limits". It does not address the use of stress indices and equations as used for "Limits on Deformation-Controlled Quantities".

NOMENCLATURE

- B_1 = primary stress index for pressure loading
- B_2 = primary stress index for moment loading
- C = coefficient in material-descriptive equation, $\sigma = C\dot{\epsilon}^n$ or $\sigma = C\dot{\epsilon}^n$
- C_1 = secondary stress index for pressure loading
- C_2 = secondary stress index for moment loading
- D_o = outside diameter of pipe
- D = diameter of run pipe of branch connection or tee
- d = diameter of branch pipe of branch connection or tee
- E = modulus of elasticity
- e = plastic strain
- $\dot{\epsilon}$ = creep strain rate
- F = peak stress category
- h = tR/r^2
- I = moment of inertia of pipe cross section
- $k_s = K_t - 1$, for the purpose of this report
- K_t = factor indicating plastic stress distribution over a cross section
- M = moment (see footnote 5 of Table 1* for subscripts)
- M_b = bending moment
- M_e = first yield moment
- M_i = vector moment
- M_L = limit moment
- M_p = limit moment for straight pipe = $4r^2ts_y$
- M_t = torsional moment
- M_{max} = maximum moment, test data
- M' = maximum moment, theory

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* Tables and Figures are included in numerical order at the end of the text.

NOMENCLATURE (Cont.)

$$m_b = M_b / (4r^2 t S_y)$$

$$m_t = M_t / (2\pi r^2 t S_y)$$

n = strain hardening or steady state creep exponent

P = internal pressure

P_o = range of internal pressure

P_m = general primary membrane stress category

P_L = local primary membrane stress category

P_b = primary membrane stress category

P_e = restraint of thermal expansion stress category

$$p = Pr / t S_y$$

Q = secondary stress category

R = bend radius of curved pipe

r = cross section radius of thin-wall pipe

r_o = outside radius of cross section

r_i = inside radius of cross section

S_m = time-independent allowable stress intensity per Code Case 1592.

S_o = allowable general primary membrane stress intensity per Code Case 1592.

S_t = time-dependent allowable stress intensity per Code Case 1592.

S_y = yield strength

T = wall thickness of run pipe of branch connection or tee

t = wall thickness of pipe (branch pipe for branch connections or tees)

T_a = range of average temperature on side a of a gross discontinuity.

T_b = range of average temperature on side b of a gross discontinuity.

Z = section modulus of pipe cross section

Z_r = section modulus of run pipe, branch connection or tee

NOMENCLATURE (Cont.)

Z_b = section modulus of branch pipe, branch connection or tee.

ΔT = thru-wall temperature gradient

σ = calculated stress

σ_o = outer fiber stress

ρ = radius of curvature or time rate of change of radius of curvature.

INTRODUCTION

At the present time, rules for design of components for nuclear power plant elevated temperature service are contained in ASME Boiler Code* Case 1592-8. This Code Case includes a subarticle -3600, "Piping Design". Under -3600 there is a sub-subarticle -3650, which reads as follows:

-3650 Analysis of Piping Components

-3651 General Requirements

Until special rules for piping components are developed for elevated temperature service, the analysis requirements are given by (a) and (b) below.

(a) The structural analysis shall demonstrate (by analysis or experiment or both) that the component fully complies with the requirements of -3200.

(b) The stress indices and stress equations of NB-3600 may be used to satisfy the limits on load-controlled stresses; may be used when creep effects are insignificant (-3211(c)); and may be used whenever linearly-elastic material models are appropriate.

A Task Group under the Code Working Group on Piping Design has prepared a revised -3600 which is intended to be incorporated in Code Case 1572-X; X being some future revision number. A draft of the proposed -3600 contents was attached to the minutes of the November, 1976 meeting of the Working Group on Piping Design. The part of -3600 pertinent to this report reads as follows:

-3650 ANALYSIS OF PIPING COMPONENTS

-3651 General Requirements

(a) Piping components which satisfy the requirements of -3640 and comply with the rules of -3650 satisfy the requirements of -3611.

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* The term "Code", as used in this report, refers to Reference (1); The ASME Boiler and Pressure Vessel Code, Section III, Division 1, "Nuclear Power Plant Components".

(b) The rules of -3200 and Appendix T may be used in lieu of the rules of -3650.

(c) The rules of -3650 apply only to linearly elastic material models of piping components. Where inelastic analysis of piping components is required, the rules of -3200 and Appendix T may be used.

(d) The forces and moments applied to piping components shall be determined using flexibility analyses as specified in -3672.

(e) The stress indices used in the analysis of -3650 shall comply with the rules of -3680.

-3652 Design Conditions

Piping components which comply with the rules of -3640 for pressure design and the rules of -3650 for Operating Conditions also satisfy the requirements for Design Conditions.

-3653 Consideration of Normal Conditions

-3653.1 Primary Stress Intensity Limits

The primary stress intensity limits of -3223 shall be satisfied. The primary stress intensities and associated limits shall be calculated using (a) through (h) below.

(a) The general primary membrane stress intensity, derived from P_m , shall be calculated using Equation (P-1).

$$P_m = \left[\frac{D_o}{2(t_s - a)} \right] P_o \quad (P-1)$$

where

P_o = operating pressure, psi

D_o = outside diameter of pipe, in. (-3683)

t_s = minimum specified wall thickness (nominal wall

thickness less fabrication tolerances). in.
(-3685)

a = thickness included in the minimum specified
wall thickness to allow for expected corrosion
and erosion in service. (3641)

(b) The primary membrane-plus bending stress
intensity, derived from $(P_L + P_b)$, shall be calculated
using Equation (P-2).

$$(P_L + P_b) = B_1 \left(\frac{D_o}{2t} \right) P + B_2 \left(\frac{D_o}{2I} \right) M_1 \quad (P-2)$$

where the notation is as defined for Equation (1) and,

B_1, B_2 = primary stress indices for the specific product
under investigation (-3680)

t = nominal thickness, in.

I = moment of inertia, in.⁴ (-3683)

M_1 = load-controlled moment resultant (-3680)

(c) The local primary membrane stress intensity,
derived from P_L , shall be calculated using Equation (P-3).

$$P_L = B_1 \left(\frac{D_o}{2t} \right) P_o + B_2' \left(\frac{D_o}{2I} \right) M_1 \quad (P-3)$$

where

B_2' = local primary membrane stress indices
for the specific product under investi-
gation (-3680).

(d) The primary bending stress intensity, derived
from P_b , shall be calculated using Equation (P-4).

$$P_b = (B_2 - B_2') \left(\frac{D_o}{2I} \right) (M_1) \quad (P-4)$$

(e) The load-controlled moment resultant, M_1 , induced by the specified loading conditions shall account for significant elastic follow-up under deformation-controlled loadings such as thermal expansion and anchor movement (see -3138).

(f) Moments for which only the relative algebraic signs of the components are known shall be included in M_1 as illustrated in the following example.

	<u>Sum of Moments with known sign</u>	<u>Moment with Indefinite sign</u>	<u>Combined Moment</u>	
			(1)	(2)
M_1	+4	± 1	+5	+3
M_2	-6	∓ 10	-16	+4
M_3	+3	∓ 7	-4	+10

The signs of the moment components for which only the relative algebraic signs are known shall be used as calculated to obtain the result of Column (1). The opposite signs shall be used to calculate the sum of moments for Column (2). The value of M_1 is the larger of the resultant moments of Columns (1) and (2).

(g) Moments for which only the magnitudes of the components are known shall be included in M_1 as illustrated in the following example.

	<u>Sum of Moments with known sign</u>	<u>Moments with unknown sign</u>	<u>Combined Moment</u>
M_1	+4	1	+5
M_2	-6	10	-16
M_3	+3	7	+10

The signs of the moment components with unknown sign are taken to be the same as the signs of the moment components with known sign in calculating the combined moment components.

(h) The quantity k_s in Equation (7) of -3223 shall be taken as given in Table -3653.1 for evaluation of piping

products using the stress index method.

Table -3653.1 K_s Values for Piping Products

<u>Product</u>	<u>$\frac{k}{s}$</u>
Straight pipe (with or without welds), tapered transition joints, curved pipe or butt welding elbows (bend radius $< 5D$) and reducers.	0.15
Other piping products	0.25

-3653.2 Limits on Deformation-Controlled Quantities
The rules of -3250 apply.

-3654 Upset Conditions
The procedures for evaluating Upset conditions are the same as those given for Normal conditions.

-3655 Emergency Conditions
-3655.1 Primary Stress Intensity Limits
The primary stress intensity limits of -3224 shall be satisfied. The procedures are as given in -3653.1.

-3655.2 Limits on Deformation-Controlled Quantities
The rules of -3250 apply.

-3656 Consideration of Faulted Conditions
The primary stress intensity limits of -3225 shall be satisfied. The procedures are as given in -3653.1.

-3657 Pressure Testing Conditions
The limits of -3226 shall be satisfied. The procedures for evaluating internal pressure as as given in -3653.1.

Equations (P-3) and (P-4) use B_2' stress indices; such indices are not given in the Code. The objective of this report started as an effort to develop appropriate B_2' indices for inclusion in the Code and for use in Equations (P-3) and (P-4). However, it soon became apparent that the B_2' indices should be taken as zero provided that B_1 and B_2 indices, when used in Equation (P-2), establishes an appropriate limit to combinations of pressure and moment loading. Accordingly, the major thrust of this report is concerned with Equation (P-2) and its associated B_1 and B_2 indices. We eventually reach the recommendation that -3653.1(c) and (d) be deleted and that the present -3653.1(h) be revised to simply say that $k_s = 0$.

The chapter on "Simplified Piping Analysis" briefly describes the background and general basis for the simplified methods of piping design and analysis contained in NB-3600. The generic background of the entire table in stress indices is discussed; in part, because the B_2 indices are related to the C_2 indices. The Chapter includes the present Code Table of stress indices which serves as a list of the various piping products* which will be evaluated in subsequent chapters of this report. The motivation behind the development of simplified analysis procedures is pointed out; presumably this same motivation will exist for high temperature piping systems in future nuclear power plants. Finally, the chapter on Simplified Piping Analysis points out the fundamental concept behind the B indices and its implications with respect to attempts to directly relate the B indices to the Code definitions of primary stress categories.

The subsequent six chapters of the report discuss the background and basis for various types of piping products, starting with straight pipe.

The "Summary" chapter is followed by "Recommendations", which contains recommendations for the text of -3650 "Analysis of Piping Products", for inclusion in a future revision of Code Case 1592.

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* Piping products are anything in a piping system; e.g., an elbow, a branch connection, a girth butt weld, a length of straight pipe.

SIMPLIFIED PIPING ANALYSIS

Subsection NB-3200 of the Code, "Design by Analysis", contains the basic engineering concepts of the Code rules for "no-creep" design for Class 1 components. Code Case 1592, -3200 (also entitled "Design by Analysis") contains similar basic engineering concepts for elevated temperature design. These basic concepts are more-or-less directly applicable to pressure vessels and, indeed, the basic design philosophy of the Code was developed with pressure vessels in mind.

The first edition of the Code (1963) and the 1968 edition covered only pressure vessels. Starting around 1965, work was initiated to expand the Code to cover other components such as pumps, valves and piping systems. For piping, this work resulted in publication of USAS B31.7 - 1969, "Nuclear Power Piping". The design portions of USAS B31.7 were, in large part transferred to the 1971 edition of the Code as NB-3600, "Piping Design".

For Class 1 piping, a strenuous and, on the whole, successful attempt was made to establish a set of rules which were equivalent to the basic Code design philosophy. However, the rules* for piping are given in terms of explicit requirements and rules rather than in the form of general concepts. This was accomplished by including NB-3640 on "Pressure Design" and NB-3650 on "Analysis of Piping Products".

NB-3640 establishes the starting point for the design and does so on the basis of the design pressure at the design temperature. These design conditions must be included in the Code-required Design Specification. The required minimum wall thickness of straight pipe is calculated by Equation (1) in NB-3641.1. Other piping products are selected (e.g. from ANSI standards) or designed to be at least equivalent in pressure capacity to straight pipe with the required minimum wall thickness. Explicit rules are given in NB-3643 for the pressure-design of fabricated branch connections.

Having met the rules of NB-3640, the Code user must now check that the requirements of NB-3650 are satisfied. To do this he must conduct

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* We are discussing here the simplified rules. Both NB-3600 and -3600 permit the use of NB-3200 or -3200 for design and/or analysis of piping systems and products.

two analyses:

- (1) An analysis of the piping system, from which the moments, M , used in NB-3650 are obtained.
- (2) A heat transfer analysis, starting with the fluid temperature transients and giving the ΔT 's, T_a and T_b used in NB-3650.

The requirements of NB-3650 are expressed by Code equations (9), (10), (11), (12), (13) and (14). These equations involve stress indices which, for many commonly used piping products, are given in Table NB-3683.2-1; included herein as Table 1. The task of checking compliance with NB-3650, having available the moments, ΔT 's, T_a and T_b , and stress indices, is relatively simple.

As shown in Table 1, stress indices are identified by letters B, C and K and by subscripts 1, 2 and 3. In the following, we will discuss the concepts behind the stress indices; first the C and K indices and then the B indices.

C and K Stress Indices

The C and K indices are based on either tests run at room temperature, on elastic theory* or combinations thereof. The C indices, when used in Code Equation (10), give a conservative estimate of the primary plus secondary stress range. In NB-3200 terminology, it gives $P_L + P_b + P_e + Q$. Equation (10) is deemed to be conservative because it simply adds together the stresses due to the various loads. The C indices give the maximum stresses due to each load but, in general, the maximum stresses due to the various loads do not occur at the same point and direction on the product. Adding them together, as done in Equation (10), assumes that all maximum stresses occur at the same point and in the same direction at that point.

The K indices are, in large part, based on fatigue tests run at room temperature. The K indices, when used in Code Equation (11), give a

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* On linear (small deformation) elastic theory, except for out-of-round piping products; see footnote (1) of Table 1.

conservative estimate of the primary plus secondary plus peak stress range. In NB-3200 terminology, it gives the equivalent of $P_L + P_b + P_e + Q + F$.

Code Equation (12) shows a limit of $3S_m$ but this limit may be exceeded provided certain restrictions are met and an appropriate modification of the fatigue evaluation is made. The additional restrictions are contained in Code Equations (12) and (13); the modification to the fatigue evaluation is contained in Code Equation (14) by use of the K_e factor. All three of these equations tie back to NB-3228.3, "Simplified Elastic Plastic Analysis". Conceptually the $3S_m$ limit on S_n [Code Equation (10)] is equivalent to a $2S_y$ (twice yield strength) limit which is the shakedown limit. If $S_n < 3S_m$, then the design fatigue curves given in the Code can be used directly. However, even if $S_n > 3S_m$, the fatigue life may still be ample for the anticipated loading history. Code Equation (12), by itself, is intended to prevent formation of plastic hinges in the piping system due to thermal expansion by itself. Equation (12) also restricts S_e stresses to about the maximum level permitted by ANSI B31.1, "Power Piping". Code Equation (13) expresses the equivalent of NB-3228.3(a). The K_e factor is taken from NB-3228.3(b).

B Stress Indices

The B stress indices are fundamentally different than the C and K indices in that the conceptual basis is limit loads of the piping product rather than stresses at a point on the piping product. The B indices, when used in Code Equation (9) and the $1.5 S_m$ limit, are intended to restrict combinations of loadings to those that are less than two-thirds of the limit load combination. In considering the basis of Code Equation (9), the requirements of NB-3640 must be kept in mind.

It should be noted that Code Equation (9) involves only sustained loads; i.e. pressure and moments caused by sustained loads such as weight. Loads due to anchor movements or equivalently, those due to restraint of thermal expansion, are not included. Analogously, thru-the-wall thermal gradient terms are not included because they are internally self-equilibrating.

It should also be noted that Code Equation (9) is used for

emergency or faulted conditions as well as normal and upset conditions.

The development and background of the B indices are discussed in some detail in subsequent portions of this report.

Motivation of the Simplified Analysis Approach

The Code user (or reader of this report) may ask the question: "In view of the generally applicable rules of NB-3200, why is there a need for the simplified analysis methods contained in NB-3600?" The answer is obviously a matter of economics. Piping systems contain many products, each of which pose an analytical problem equivalent to or more complex than the reactor pressure vessel. For example, an ANSI B16.9 tee, with its loadings of pressure, six independent moments, thru-the-wall temperature gradients and axial gradients, is perhaps more complex than the pressure vessel analysis because of the generally non-neglectable six independent moments. A piping system may contain a number of complex shaped and loaded components and there generally are several piping systems for each reactor vessel.

It is pertinent to note that the vessel rules of NB-3300 also make use of stress indices; see NB-3338. This indeed, was one of the precedent uses for stress indices in piping. In piping, however, the concept has been extended to cover various types of piping products (one of which is equivalent to the nozzle in a pressure vessel with pressure loading) and to moment and thermal gradient loadings as well as pressure.

Stress Indices for Elevated Temperature Piping

The proposed -3600 portion of Code Case 1592-X uses stress indices taken from Table 1 herein. An implied assumption is that sometime in the not too distant future, a number of nuclear power plants involving the use of elevated temperature piping will be designed. Granting the validity of this assumption, then we have the same motivation for development and use of stress indices in -3600 as we do for NB-3600.

The remainder of this report* is devoted to an assessment of the use of B indices for high temperature piping. As remarked previously, the the B indices are related to the limit loads of the piping product as a structure; not to stresses at a point on the structure. Accordingly, the Code definitions of P_m , P_L and P_b and Code limitations thereof are of little significance to evaluation of the B indices. In effect, the B indices reflect a higher level of sophistication and the basis is related to NB-3228.2, "Limit Analysis".

An aspect of Code interpretation of essential significance to this report is illustrated in Figure 1. Looking at a thru-the-wall piece of the straight pipe at A, in Figure 1, a stress exists which fits the definition of a general primary membrane stress and it is so classified on the vessel side of the reinforcement zone boundary (see NB-3227.5). Looking at the entire cross section, a stress exists which varies linearly across the cross section and has zero net axial force resultant. This fits the definition of a primary bending stress. This illustrates the difference in concept behind the B indices and the Code limits on stresses at a point. The B indices are related to the product (in this case a length of straight pipe) and not to the local state of stress at some point on the product.

The proposed -3650 contains portions in Appendix T of Code Case 1592-X which make use of B, C and K indices. This report does not assess the validity of those uses of the stress indices.

STRAIGHT PIPE

As shown in footnote (5) of Table NB-3683.2-1 (Table 1 herein), a straight pipe is, in general, subjected to a set of three orthogonal moments. The magnitude of these moments is determined by an analysis of the piping system.

Bending Moments

The moments M_1 and M_2 may be combined to a single bending moment, $M_b = [M_1^2 + M_2^2]^{1/2}$. Assuming that the cross sections remain circular, and assuming a material descriptive equation of $\sigma = C\epsilon^n$ for plasticity, or $\sigma = C\dot{\epsilon}^n$ for steady-state creep, the moment-radius-of-curvature, relationship is:

$$M = \frac{2\sqrt{\pi}}{n+3} \frac{C}{\rho^n} \left[\frac{\Gamma\left(\frac{n}{2} + 1\right)}{\Gamma\left(\frac{n+3}{2}\right)} \right] (r_o^{n+3} - r_i^{n+3}) \quad (1)$$

where: $\Gamma ( )$ = gamma function of ( )

r_o = outside radius of pipe

r_i = inside radius of pipe

ρ = radius of curvature or time rate of change of radius of curvature.

Equation (1) is based on the assumption that the neutral axis passes through the center of the cross section and that plane sections remain plane, hence the strain ϵ is given by y/ρ , where y is the distance from the neutral axis. Accordingly, $\sigma = C(y/\rho)^n$, and σ_o , the outer fibre stress is

$$\sigma_o = C(r_o/\rho)^n \quad (2)$$

The M versus σ_o relationship is:

$$M = \frac{2\sqrt{\pi}}{n+3} \frac{\sigma_o}{r_o^n} \left[\frac{\Gamma\left(\frac{n}{2} + 1\right)}{\Gamma\left(\frac{n+3}{2}\right)} \right] (r_o^{n+3} - r_i^{n+3}) \quad (3)$$

Both equations (1) and (3) contain the elastic and rigid-perfectly-plastic solutions as special cases. In the elastic solution, $\sigma = Ee$ ($C = E$, $n = 1$) and for $n = 1.0$, $\Gamma(3/2) = \sqrt{\pi}/2$ and $\Gamma(2) = 1.0$ so that:

$$M = E (\pi/4) (r_o^4 - r_i^4)/\rho, \quad \text{or} \quad (4)$$

$$M = \sigma_o (\pi/4) (r_o^4 - r_i^4)/r_o. \quad (5)$$

For the rigid-perfectly-plastic solution, $C = S_y$ (yield strength) and $n = 0$ and $\Gamma(1) = 0$, $\Gamma(3/2) = \sqrt{\pi}/2$ so that:

$$M_p = \frac{4}{3} S_y (r_o^3 - r_i^3) \quad (6)$$

For thin wall pipe, equations (3), (5) and (6) become

$$M = 2 \sqrt{\pi} r^2 t \left[\frac{\Gamma\left(\frac{n}{2} + 1\right)}{\Gamma\left(\frac{n+3}{2}\right)} \right] \sigma_o \quad (7)$$

$$M = \pi r^2 t \sigma_o \quad (8)$$

$$M_p = 4r^2 t S_y \quad (9)$$

where $r_o \cong r_i = r$.

Setting $\sigma_o = S_y$ in equation (8) to obtain $M_e = \pi r^2 t S_y$ (first yield moment), the ratio of M_p/M_e is $4/\pi = 1.273$; the "shape factor" for thin wall pipe. Table 2 gives analogous "shape factors" for various values of n and D_o/t (Pipe outside diameter to wall thickness ratio).

The use of $k_s = 0.15$ corresponds to a shape factor $K_t = 1.15$. As can be seen in Table 1, $K_t = 1.15$ is conservative for any $n < 1/3$. It is quite conservative for small D/t combined with small n . However, it should be recalled that the theory is based on the assumption that

the pipe cross section remains circular. For thin-wall pipe, this may not be a valid assumption.

The author is not aware of any test data on thin-wall pipe subjected to a bending moment at elevated temperature. However, some tests have been run at room temperature, investigating the analogous problem of time-independent plasticity. References (2) and (3), in particular, include results of tests on pipe with large D/t ratios; up to 116 in Reference (2) and up to 90 in Reference (3). Table 3 shows values of $M_{\max}/M_e$ derived from correlation equations of References (2) and (3), where $M_{\max}$ is the maximum moment capacity and M_e is the "first yield" moment. The important aspect of these results, with respect to this report, is that for thin-wall pipe ($D/t > \sim 50$), a shape factor of 1.15 may be unconservative.

The reduction in moment capacity from that predicted by the elementary theory as exemplified by equation (1) is apparently due to two factors:

- (1) The pipe cross section tends to ovalize when subjected to a bending moment, and
- (2) For large D/t , buckling may occur on the compressive side.

To include the ovalization effect, a finite-deformation theory must be used. Ades⁽⁴⁾ developed such a theory but leaves it in the form of integrals. Afendik⁽⁵⁾ developed an analogous theory using the material-descriptive equation: $\sigma = Ce^n$. For n between 0.15 and 0.25, Afendik gives relatively simple equations for calculating the moment-curvature relationship. Unlike the elementary theory, in which the moment always increases with increasing curvature, the finite-deformation theory leads to a maximum moment at some specific curvature; for greater curvatures the moment decreases. A plot of $M/(\epsilon \pi r^2 t)$ vs r/ρ is shown in Figure 2 for $D_o/t = 100$, $E = 3 \times 10^7$ and $\sigma = (40,000/.002^n)e^n$, with $n = 0.25$. The stress-strain relationship corresponds to a material with yield strength of 40,000 psi determined by 0.2 percent offset.

It can be seen in Figure 2 that the maximum moment occurs at r/ρ of about 1 percent. The nominal axial strain is equal to r/ρ so it follows that the maximum moment occurs at 1 percent total strain or about 0.8 percent plastic strain. Table 4 shows values of $M'/\epsilon \pi r^2 t$ and corresponding values of r/ρ' , where M' is the maximum moment and r/ρ' is

the nominal plastic strain at the maximum moment. The bottom-right set of parameters corresponds to Figure 2. Table 4 indicates that:

- (1) The maximum moment M' is substantially higher than the limit moment, $M = 4r^2tS_y$, and
- (2) The maximum moment occurs at high nominal strain values; usually above 1 percent total strain.

Accordingly, it appears that ovalization of the pipe cross section is not a problem, even for $D/t = 100$. However, the theories do not include buckling and the experimental data shown in Table 3 indicate that buckling may control and a shape factor of 1.15 may be unconservative for thin-wall ($D/t > \sim 50$) pipe.

It would be possible to develop a k_s for straight pipe which would be a function of D/t and perhaps n . However, in the interest of simplicity, it is suggested that k_s be made zero - giving a shape factor of 1.0. As discussed in the following for torsional moments, k_s is essentially zero for torsion.

Torsional Moments

The third of the three moments that may be applied to a pipe is the torsional moment, M_t in footnote (5) of Table 1 herein. Assuming a stress-strain relationship for the material of $\sigma = Ce^n$, the ratio of M_t/M_e turns out to be:

$$\frac{M_t}{M_e} = \frac{4}{3+n} \frac{1 - (r_1/r_o)^{3+n}}{1 - (r_1/r_o)^4} \quad (10)$$

This ratio gives the shape factor for torsional bending. Some examples are tabulated below.

n	M_t/M_e for D_o/t of:		
	10	40	∞
1.0	1.000	1.000	1.000
0.5	1.049	1.012	1.000
0.0	1.102	1.025	1.000

It is apparent from the above tabulation that, if we are to simply, yet conservatively, cover all materials and combinations of M_1 , M_2 , and M_3 , then the shape factor must be taken as 1.00 and, equivalently, $k_s = 0$.

The author is not aware of any test data in which maximum torsional moments of thin-wall pipe were determined; nor any theories beyond the elementary theory reflected by Equation (10). Presumably ovalization does not occur and, hopefully, buckling is not a limiting aspect for D/t up to 100 (maximum coverage of the stress indices).

Combined Pressure and Moments

With k_s taken as zero, for reasons discussed in the preceding, we can now look at Equation (P-2). For thin-wall straight pipe, Equation (P-2) can be written:

$$\frac{1}{2} \frac{Pr}{t} + \frac{(M_b^2 + M_t^2)^{1/2}}{\pi r^2 t} \leq \begin{cases} 1.5 S_m, & \text{or} \\ S_t \end{cases} \quad (11)$$

The limit on the right of Equation (11) is given in -3223; note that with $k_s = 0$, $K_t = 1.0$. It may be noted that conceptually the $1.5 S_m$ limit is equal to the yield strength of the material. For comparison with limit load theory, it is convenient to write Equation (11) in the shorthand form:

$$\frac{1}{2} p + \left\{ \left[(4/\pi) m_b \right]^2 + [2m_t]^2 \right\}^{1/2} = 1.0 \quad (12)$$

where: $p = Pr/tS_y$

$$m_b = M_b / (4r^2 t S_y)$$

$$m_t = M_t / (2\pi r^2 t S_y)$$

The theory for combinations of internal pressure, bending moment and torsional moment which produce limit loading is developed by Larson, et.al. (6). The theory is based on rigid-perfectly plastic material and

Mises yield criterion. Their limit load equation is:

$$(3/4) p^2 + m_b^2 + m_t^2 = 1.0 \quad (13)$$

In the following, we will present comparisons of equations (12) and (13) for (1) normal and upset conditions, (2) emergency conditions and (3) faulted conditions. We will do so first in terms of the S_m limits, with the interpretation that $S_m = (2/3) S_y$ and assessing the relationships in terms of time-independent plasticity. We will then do so in terms of the S_t limit, with the interpretation that S_t corresponds to a tolerable creep rate.

At this point, it is pertinent to look at whether S_m or S_t will control. The coefficients of S_m and S_t limits are tabulated below.

Condition	P_m		$(P_L + P_b)$	
	S_m	S_t	S_m	S_t
Normal/Upset	1.0	1.0	1.5	1.0
Emergency	1.2	1.0	1.8	1.0
Faulted	2.0	1.2	3.0	1.2

The Code requires that the smaller of the two limits be used in evaluations of P_m or $(P_L + P_b)$. Table 5 shows representative values of S_m and S_t , taken from Code Case 1592-8. The Code Case shows S_t values for 1 hour or more for materials other than 2-1/4 Cr-1 Mo; for which it shows S_t values for 10 hours or more. Presumably, the intent of the Code Case is that S_t shall not be greater than the highest value shown, even though the condition might exist for a shorter period of time. Proceeding on this assumption, it can be deduced from Table 5 that for the $(P_L + P_b)$ stress, the S_t limit almost always controls. The exceptions are for 2-1/4 Cr-1Mo at 700F, all times, and at 800F, times less than 10^4 hours. For the P_m stress, the main point we wish to establish is that under faulted conditions, the S_t limit always controls.

Interpretation Using S_m Limits

Design Conditions. For design conditions, -3222.1 requires that $P_m < S_o$ and $(P_L + P_b) < 1.5 S_o$. For design conditions, -3652 states that -3640 and -3650 for operating conditions must be met. Because -3640 uses S_o as the allowable stress in determining the required minimum pipe wall thickness (and other products are proportioned thereto), the limit $P_m < S_o$ is essentially met. The second limit is also met provided the Design Specification does not specify any design load other than internal pressure. In-so-far as the author is aware, internal pressure is the only design load included as such in Design Specifications for piping systems.

Normal and Upset Conditions. For normal and upset conditions, -3223 requires that $P_m < S_m$ and $(P_L + P_b) < 1.5 S_m$. Taking $1.5 S_m$ to be equivalent to S_y , Equation (12) represents the Code limits for combinations of pressure and moment. Equations (12) and (13) are plotted in Figure 3 for a value of $p = 2/3$; i.e. the hoop stress in the pipe is at its maximum permissible value. It is apparent in Figure 3 that Equation (P-2) provides a bound such that the combination of pressure and moment is less than 2/3 of the limit load combination.

Figure 4 compares Equations (12) and (13) on the assumption that P in Equation (P-2) may be taken as zero. It can be seen that there are combinations of m_b and m_t which, while well below the limit load, are somewhat above 2/3 of the limit load. However, we note that P as used in Equation (P-2) is not defined. In the analogous Equation (9) of NB-3600, P is defined as the design pressure and hence cannot be zero. It can be shown that if $p > 0.42$, then Equation (P-2) gives moment bounds that are less than 2/3 of the limit load. For light water reactors, this would usually be the case. That is, the pipe wall thickness would not be more than 2.38 times that required for the design pressure. For LMFBR piping, this might not always be the case. However, considering that we are at most 18% over the "magic" 2/3 of limit load criterion and do not exceed 79% of the limit load, the use of $P = P_o$ is considered to be adequately conservative.

Emergency Conditions

For emergency conditions, -3224 requires that $P_m < 1.2S_m$. Again interpreting $S_m = (2/3) S_y$, the maximum value of p is 0.8. Also, -3224 requires that $(P_L + P_b) < 1.8 S_m$. The right hand side of Equation (12) becomes $1.8 \times (2/3) = 1.2$. Comparisons of the Code limit and the theoretical limit load are shown in Figure 5. It can be seen that the Code equations restrict the moments to less than the limit load; this is deemed to be appropriate for emergency conditions.

Faulted Conditions

For faulted conditions, -3225 requires that P_m shall not exceed "one of the Faulted Condition limits in Appendix F". Now, for piping, one of the faulted condition limits is that the pressure shall not exceed 2 times the design pressure. This leads to an upper bound on p of $2 \times 2/3 = 4/3$. For piping, Appendix F limits the equivalent of $(P_L + P_b)$ to $3.0 S_m$. Equation (13), for $p = 4/3$, gives imaginary values of m_b and/or m_t because $p = 4/3$ is itself above the yield pressure. Figure 6 shows the moment combinations permitted by Equation (P-2).

It is apparent that we cannot establish from Figure 6 that straight pipe will not grossly deform under faulted conditions based on the S_m limits of Code Case 1592-8. This is not a peculiarity of Equation (P-2) but rather is inherent in the Code Case faulted condition limits. However, for piping systems in particular, it cannot be said that gross plastic deformation will occur, even at the extreme condition permitted of $p = 0$, $m_t = 0$, $m_b = 1.571$. What we have been discussing are the conditions under which a plastic hinge may form at one cross section of a piping system. If a piping system consisted of a cantilevered length of straight pipe, then the formation of one hinge would lead to gross plastic deformation. However, most piping systems include at least two anchors and usually three hinges are necessary to obtain gross plastic deformation.

The problem of adequate design rules for faulted conditions needs further study for both low temperature and high temperature applications. It is beyond the scope of this report to solve that problem.

Interpretation Using S_t Limits

The significance of the equation $\sigma = C\dot{\epsilon}^n$ is illustrated in Figure 7. The family of curves have slopes $\dot{\epsilon} = d\epsilon/dt$ that are proportional to the $1/n$ power of σ . If, for example, $n = 1/4$, then increasing the stress by 10% increases the creep rate by a factor of 1.464. In limit load evaluations, n is assumed to be zero. In terms of creep rates, the equivalent is a postulated stress level, S_t , at and below which the creep rate is considered sufficiently slow so as to be tolerable, above which the creep rate is so fast as to be intolerable. The magnitude of S_t is, of course, a function of time and the implication is that the integral of the creep rate over the length of time corresponding to the S_t value is a tolerable amount of creep strain.

Normal and Upset Conditions. For normal and upset conditions, -3223 requires that $P_m < S_t$ and $(P_L + P_b) < K_t S_t$. With $K_t = 1.0$, both limits are simply S_t . Equation (P-2) then can be written just like Equation (12), except that in the definitions of p , m_b and m_t we substitute S_t for S_y . The limit load of equation of Reference (6), Equation (13) herein, becomes applicable to steady state creep rate by the change in definitions of p , m_b and m_t . With $P_m = 0$, $p = 0$ we obtain the comparisons shown in Figure 4. It can be seen in Figure 4 that the Code equation restricts moments to those combinations below the "creep limit load". The least margin of safety occurs at $m_t = 0$; for both $p = 1.0$ and $p = 0$ the margin is $4/\pi = 1.27$. For intermediate pressures, the margin of safety is higher; for example, for $p = 0.625$ the least margin of safety is 1.56.

Emergency Conditions. For emergency conditions, -3224 requires that $P_m < S_t$ and $(P_L + P_b) < S_t$ (with $K_t = 1.0$). These are the same limits as for normal and upset conditions.

Faulted Conditions. For faulted conditions, - 3225 requires that $P_m < 1.2S_t$ and $(P_b + P_d) < 1.2S_t$ (with $K_t = 1.0$). With $P_m = 1.2S_t$, $p = 1.2$ and equation (13) gives imaginary values of m_b and/or m_t because $p = 1.2$ is itself above the creep limit. Figure 8 shows the moment combinations permitted by Equation (P-2). For $p = 0$, the factor of safety is 1.06. As p

increases up to 0.5, the factor of safety increases to 1.21; as p increases further, the factor of safety decreases to 1.0 at $p = 0.94$. Accordingly, Equation (P-2) appears adequate except for the range of p between 0.94 and its permissible maximum of 1.2.

As remarked previously, we are not trying to solve the problem of faulted condition design rules in this report but it is apparent that the S_t limits of Code Case give more conservative rules than the analogous low-temperature design rules.

PRODUCTS TREATED LIKE STRAIGHT PIPE

Table 1 includes a number of products which, like straight pipe, are assigned a B_1 index of 0.5 and a B_2 index of 1.0. The basic concept is that this group of piping products are at least equal to straight pipe in resistance to gross plastic deformation. For example, a girth butt weld between two pieces of straight pipe is assumed to be as strong as the pipe itself. Implicit in this assumption is that the weld metal is at least equal to the pipe material in strength and resistance to creep. This concept is deemed to be appropriate for high temperature piping in-so-far as Equation (P-2) and the associated B_1 and B_2 indices are concerned.

An exception occurs for girth fillet welds for which $B_1 = 0.75$ and $B_2 = 1.5$. It could be contended that a full fillet weld (one with throat dimension not less than the pipe wall thickness) would also be as strong as the pipe. However, one of the uses of fillet welds is in conjunction with socket weld fittings and, for some combinations of pipe wall thickness and fittings, there may not be sufficient space to really get the equivalent of a full fillet weld. Accordingly, the B_1 and B_2 indices were increased by 50% so as to cover this possible condition.

CURVED PIPE/WELDING ELBOWS

Moment Loading

Analysis⁽⁷⁾ of a curved pipe subjected to in-plane or out-of-plane bending moments shows that the single parameter $h = tR/r^2$ describes the curved pipe completely; t = wall thickness, R = bend radius and r = cross section radius. For large values of h , the stresses are simply those in a straight pipe subjected to a moment M ; i.e. the axial membrane stress is $(M/Z) \sin\phi$, the hoop membrane stress is zero and thru-the-wall bending stress is zero. For small values of h , the maximum stresses are:

$$\sigma_{\max} = \frac{1.86}{h^{2/3}} \text{ for in-plane moments} \quad (14)$$

$$= \frac{1.59}{h^{2/3}} \text{ for out-of-plane moments} \quad (15)$$

For in plane bending, the maximum stress occurs at about $\phi = 0$ and 180° in the hoop direction and is essentially a pure thru-the-wall bending stress. The hoop membrane stress is zero at $\phi = 0$ and 180° and is relatively small compared at all ϕ compared to the maximum axial stress.

Figure 9 shows the normalized axial membrane stress, $\sigma_a / (M/\pi t^2)$, plotted against the circumferential location angle, ϕ . Stresses are shown for three geometries which pretty well span the range of interest; $h = 2.0$, 0.12 and 0.04 . These h values represent elbows with $D/t = 10$, $R/r = 10$; $D/t = 50$, $R/r = 3$; and $D/t = 100$, $R/r = 2$; respectively.

It can be seen in Figure 9 that the axial membrane stress for $h = 2.0$ is almost the same as for straight pipe. Limit moments on such curved pipe would, of course, be the same as for straight pipe. For $h = 0.12$ and $h = 0.04$, it is apparent that there is a high axial membrane stress and the pattern of axial stress around the circumference is quite different than for straight pipe. However, classifying these stresses as P_m or P_L or $(P_L + P_b)$, using the stress limits for these various classes of stresses given in the Code Case, might or might not

provide limits on moments such that gross plastic deformation would not occur. As we will discuss in the following, a more rational approach requires that we also consider the bending stress in the hoop direction. Comparisons of maximum axial membrane stresses and maximum hoop bending stresses per unit value of $M/\pi r^2 t$ are tabulated below.

h	In-Plane, M_2		Out-of-Plane, M_1	
	Axial	Hoop	Axial	Hoop
0.12	3.74	7.84	4.80	6.95
0.04	7.86	16.14	9.90	14.90

Torsional moments on curved pipe produce a shear stress of $\sigma_s = M_t/(2\pi r^2 t)$. However, it should be noted that a pure torsional moment on one end of a 90° arc length elbow translates into an out-of-plane moment on the other end. Accordingly, advantage of the relatively low shearing stress due to torsional moments could only be taken for short arc lengths; e.g. a 30° elbow.

The B_2 index for curved pipe is given by the equation:

$$B_2 = 0.75 \times \frac{1.95}{h^{2/3}} \quad (16)$$

The C_2 index reflects the maximum elastic stress. It may be noted that $1.95/h^{2/3}$ is a higher stress index than the numerical values cited earlier herein; e.g., $\sigma_{\max} = 1.86/h^{2/3}$ for in-plane moment. The reason is that

$M_i = (M_1^2 + M_2^2 + M_3^2)^{1/2}$ and the C_2 index is meant to be conservative for any combination of M_1 , M_2 and M_3 . Dodge and Moore⁽⁸⁾ describe their study of $\sigma_{\max}$ as a function of the moment vector M_i and found that $h = 1.95/h^{2/3}$ would give the maximum stress intensity for any moment vector.

The B_2 index is based on the simplistic concept that, for small h , the C_2 index represents a thru-the-wall bending stress and to develop a hinge, the load would have to be increased by a factor of 1.5; the shape

factor for a solid rectangular thru-the-wall section. This implies that B_2 could be $(1/1.5) C_2 = 0.667C_2$. However, in partial recognition of buckling, it was decided to use the more conservative $B_2 = 0.75C_2$.

Bolt and Greenstreet⁽⁹⁾ give results of limit moment tests on 6-inch welding elbows. These results are shown in Table 6. The following observations can be made.

- (1) The column headed M_L/ZS_y shows that the elbows had a significantly lower limit moment capacity than equivalent straight pipe. For straight pipe we would expect this ratio to be about 1.2.
- (2) The column head M_L/M_{cL} shows that the Code equation

$$\frac{PD_o}{2t} + B_2 \frac{M}{Z} < S_y \quad (17)$$

is conservative. Except for test 15 (the only test on austenitic stainless steel material) the Code B_2 index is perhaps overconservative.

- (3) Tests 13 and 14 were run with an internal pressure of 1500 psi; $PD_o/2t = 17,750$ psi. The material yield strength was 39,600 psi. This pressure did not reduce the moment capacity as compared to tests 10 and 12 with no internal pressure. Including the internal pressure in Equation (17) gives values of M_L/M_{cL} of 4.28 and 4.37 for tests 13 and 14, respectively. Accordingly, Equation (17) can be highly overconservative for combined pressure and moment loading.

Spence⁽¹⁰⁾ developed a theory for curved pipe which is applicable both to time-independent plasticity with $\sigma = Ce^n$ and to steady-state creep with $\sigma = C\dot{\epsilon}^n$. Figure 10 shows the moment-curvature relationship for an elbow with $R/r = 3$, $r/t = 25$, $h = 0.12$ and for a material described by $\sigma = (40,000/.002^4)e^{.25}$. The limit moment implied by the Code equation $B_2 M/Z < S_y$ is also shown.

The theoretical "limit moment" obviously depends upon what criterion is used to define "limit moment". If defined as that moment

at which the displacement is three times the elastic displacement, then the ratio of the limit moment to the Code limit moment is 2.43. The Code limit is then highly conservative. For more conservative definitions of limit moment we obtain:

<u>Total Displacement</u> <u>Elastic Displacement</u>	<u>Theoretical M</u> <u>Code M</u>
3.0	2.43
2.5	2.16
1.5	1.40

Table 7 summarizes the results a parametric study made using Spence's theory. It covers a range of strain hardening exponents, n , for two elbows; one with $h = 0.12$ (applicable, for example, to an elbow with $D/t = 50$, $R/r = 3$) and with $h = 0.04$ (applicable, for example, to an elbow with $D/t = 100$, $R/r = 2$). The axial membrane elastic stresses for these elbows are shown in Figure 9.

It can be seen in Table 7 that the limit moment derived from Spence's theory, M_s , is almost always substantially higher than the Code limit, M_{CL} . The exception is for relatively high values of n combined with a rather conservative definition of limit moment.

Pressure Loading

The C_1 index for curved pipe is $2R-r/[2(R-r)]$. It is based on membrane theory of toroidal shells and, when multiplied by Pr/t , gives the maximum membrane stress. The stress is located at $\phi = -90^\circ$ (see Fig. 9) and is in the hoop direction. The magnitude of C_1 depends only on the ratio of R/r . Some values are:

<u>R/r</u>	<u>C₁</u>
20	1.026
10	1.056
3	1.250
2	1.500

The B_1 index for curved pipe is 1.0. This is deemed satisfactory for bent pipe because C_1 is not much greater than 1.0 and the bending process increases the wall thickness in the region of $\phi = -90^\circ$. For ANSI B16.9 elbows ($R/r \approx 3$), the standard requires that the burst pressure of the elbow be at least as high as the equivalent straight pipe. This may or may not require increased wall thickness in the $\phi = -90^\circ$ region; the elementary theory ignores "end effects" of the attached pipe and these end effects may be sufficient to make the elbow as strong as equivalent straight pipe. For ANSI B16.28 elbows ($R/r \approx 2$), -3642.2 requires 20% increased thickness in the region of $\phi = -90^\circ$.

The B_1 index for straight pipe is 0.5 whereas for curved pipe the index is 1.0. The reason for this is that, for small values of h , the maximum stress due to moment loading is in the hoop direction hence it adds to the hoop pressure stress, not the axial pressure stress. It may be noted that for curved pipe with large h (which behaves like straight pipe) a substantial conservatism is introduced by the use of $B_1 = 1.0$ rather than $B_1 = 0.5$.

Combined Pressure and Moments

Elastic analysis⁽⁷⁾ of curved pipe subjected to an in-plane or out-of-plane bending moment shows that internal pressure may significantly decrease the stress due to the moment loading. Some examples of ratios of maximum stresses with $Pr/t = 17,000$ psi to maximum stress with $P = 0$ are shown below.

$\frac{R}{r}$	$\frac{r}{t}$	Stress Ratio
3	25	0.676
2	50	0.492
1.891	11.33	0.903

The last entry in the above tabulation represents the elbows shown in Table 6 tested with an internal pressure of 1500 psi.

A simple theory for the moment capacity of a curved pipe subjected to pressure and moment loadings is not known to the author. The elastic

theory and the only available test data suggest that Equation (P-2) is probably very conservative when pressure is significant. Equation (P-2) indicates that the limit moment of the test elbows with $P = 1500$ psi would be only 57 percent of the limit moment with $P = 0$, whereas the test data (Table 6) indicated essentially no reduction in the limit moment.

FABRICATED BRANCH CONNECTIONS

In the previous chapters on Straight Pipe and Curved Pipe, we started with a discussion of moment loadings because the theory for pressure loading by itself is relatively simple. However, for branch connections, while internal pressure is the simplest of loadings that we will consider, it is none-the-less quite complex and we start with a discussion of that aspect.

Pressure Loading

The C_1 and K_1 indices are given in Table 1 as 2.0 and 1.7, respectively. The $C_1 K_1$ product is $2 \times 1.7 = 3.4$. This is essentially the same as the maximum stress index shown in Table NB-3338.2(c)-1 of 3.3. The value of the $C_1 K_1$ product (and the indices shown in Table NB-3338.2(c)-1) were based on test data⁽¹¹⁾. That test data, if evaluated in terms of a nozzle reinforced as shown in Figure 11(a), indicates that the maximum stress intensity will not exceed:

$$S_{\max} \leq 3.3 \frac{P_o D}{2T} \quad (18)$$

where T is the thickness of the run pipe.

However, a branch connection may look like Figure 11(b) and still meet the Code reinforcing rules requirements. Equation (18) is not valid for the entire range of parameters covered by the stress indices; i.e. D/T up to 100 and d/D up to 0.5. Indeed, Reference (12) shows that Equation (18) can be invalid if $(d/D)\sqrt{D/T}$ is greater than about 0.6. Code actions to correct this aspect have been taken in NB-3300 (Vessel Design) by the simple expedient of restricting the use of the indices to $(d/D)\sqrt{D/T} \leq 0.8$. The 0.8 parameter limit was used rather than the 0.6 limit because Reference (12) remarks that "This (limit of 0.6) could, of course, be stretched up to 0.7 or 0.8 with no great amount of unconservatism". With $(d/D)\sqrt{D/T} \leq 0.8$, the maximum values of d/D , for various values of D/T , are:

D/T =	5	10	20	40	60	80	100
Max d/D=	0.358	0.253	0.179	0.126	0.103	0.089	0.08

These d/D limits are sufficiently large so that almost all nozzles in pressure vessels are covered. However, for branch connections in piping, the intent is to cover d/D up to 0.5 for D/T up to 100.

The B_1 index for branch connections is given in Table 1 as 1.0. This index is based on the concept that a branch connection reinforced as shown in Figure 11(a) has a burst pressure essentially equal to that of the unperforated and unreinforced run pipe. There are considerable test data that support this concept; see, for example, Reference (13). However, there are no data which indicate that a branch connection as shown in Figure 11(b) would have a burst pressure equal to that of the run pipe.

Appropriate revisions to the B_1 , C_1 and K_1 indices are under consideration by the Code Section III Working Group on Piping. Development of appropriate B_1 indices is anticipated to entail the following concepts:

- (1) If the branch connection is reinforced like Figure 11(a), then $B_1 = 1.0$.
- (2) If the branch connection is not reinforced; i.e., like Figure 11(b), then B_1 will be based on assessment of limit load test data and theories for such constructions. Preliminary evaluations suggest that an appropriate equation for B_1 may be:

$$B_1 = \frac{1}{1 - 0.35\sqrt{\rho}} \quad (19)$$

where $\rho = (d/D)\sqrt{D/T}$. Some values of B_1 based on Equation (19) are tabulated below.

D/T	B_1 for d/D of:				
	0.1	0.2	0.3	0.4	0.5
10	1.25	1.39	1.52	1.65	1.79
50	1.42	1.71	2.04	2.43	2.92
100	1.54	1.98	2.54	3.33	4.60

- (3) If the branch connection is intermediate between Figures 11(a) and 11(b), then an appropriate interpolation formula will be given. This may consist of the linear interpolation formula:

$$B_1 = \left(1 - \frac{A_a}{A_r}\right) B_1^* \quad , \text{ but not less than } 1.0 \quad (20)$$

where B_1^* is obtained by Equation (19) and:

A_a = available reinforcing area using Code rules except with required run pipe thickness replaced by the run pipe thickness.

A_r = required reinforcing area using Code rules except with required run pipe thickness replaced by the run pipe thickness.

The B_1 indices so developed are deemed to be appropriate for use in Equation (P-2).

Moment Loading

Schroeder⁽¹⁴⁾ has developed an upper bound theory for limit moments of branch connections. Ellyin⁽¹⁵⁾ has developed a lower bound theory. These theories are rather complex and a computer program is required to calculate the limit moment for a specific set of branch connection dimensions. In-so-far as the author is aware, there has been no attempt to develop simplified design rules based on these theories. This, of course, would entail running parametric studies using the computer programs.

Schoeder^(16,17,18) and Ellyin^(19,20) have reported results of limit load tests on branch connections. These are summarized in Table 8. We will not make any cross comparisons between limit load theory and test data in this report. We will, in the following, show how the test data compare with the Code formulations involved in Equation (P-2).

As indicated in footnote 5 of Table 1, the Code method of evaluation entails a separation of moments into those which traverse

through the branch pipe and those which traverse through the run pipe. Footnote 7 of Table 1 gives B_{2b} and B_{2r} indices applicable to moments which traverse through the branch or run pipe, respectively. Equation (P-2), as applied to branch connections, becomes:

$$(P_L + P_b) = B_1 \frac{P_o D}{2T} + B_{2b} \frac{M_b}{Z_b} + B_{2r} \frac{M_r}{Z_r} \quad (21)$$

Moments Through Branch, B_{2b} Index

The B_{2b} index is given in footnote (7) of Table 1 as equal to $0.75 C_{2b}$, but not less than 1.0, where:

$$C_{2b} = 3 \left(\frac{R_m}{T_r} \right)^{2/3} \left(\frac{r'_m}{R_m} \right)^{1/2} \left(\frac{T'_b}{T_r} \right) \left(\frac{r'_m}{r_p} \right) \quad (22)$$

The nomenclature used in Equation (22) is defined in Figure NB-3686.1-1, included herein as Figure 12. The background of the development of both the C_{2b} and B_{2b} indices is given in Reference (21). Briefly, the C_{2b} index, which represents the maximum value of primary-plus-secondary stress, was based on an empirical evaluation of some 19 test models in which the maximum stress was determined by use of strain gages. The form of the equation was guided by both Markl's⁽²²⁾ work and Bijlaard's⁽²³⁾ work. The B_{2b} index was set as $0.75 C_{2b}$ using the same simple concept as used for curved pipe; i.e. the C_{2b} index represents a thru-the-wall bending stress and to develop a hinge, the load would have to be increased by a factor of 1.5; the shape factor for a solid rectangular thru-the-wall section. As for elbows, this implies that B_{2b} could be $0.667C_{2b}$. Again, in partial recognition of buckling, it was decided to use the more conservative $B_{2b} = 0.75 C_{2b}$.

Equation (P-2), for zero pressure and for a moment load on the branch, can be written as:

$$B_{2b} \frac{M}{(\pi/4)d^2 t} \leq 1.5S_m \quad (23)$$

Replacing $1.5S_m$ by S_y , Equation (23) can be written:

$$\frac{M_L}{d^2 t S_y} = \frac{\pi/4}{B_{2b}} \quad (24)$$

Figures 13 and 14 compare test data with Equation (24) for M_{x3} (out-of-plane) and M_{z3} (in-plane), respectively. Some comments concerning this comparison are as follows:

- (1) The ratio (r/r_p) , for the test models, was essentially equal to unity, hence it does not appear in the parameter used in correlating the results.
- (2) The $(1/B_{2b})$ parameter does a reasonably good job of correlating the test results (Table 9).
- (3) The comparisons indicate that Equation (P-2), used with the $1.5S_m$ limit for normal/upset conditions, is highly conservative. The test data limit loads are much more than 1.5 times the Code-allowable moment; the implied requirement of NB-3228.2.

It may be noted that the available data are for M_{x3} or M_{z3} applied by itself. Equation (P-2) applies to the vector moment $[M_{x3}^2 + M_{y3}^2 + M_{z3}^2]^{1/2}$. The test data suggest that Equation (P-2) is conservative for any vector moment.

Moments Thru Run, B_{2r} Index

The B_{2r} index is given in footnote (7) of Table 1 as equal to $0.75 C_{2r}$ (but not less than 1.0), where

$$C_{2r} = 0.8 \left(\frac{R_m}{T_r} \right)^{2/3} \left(\frac{r'_m}{R_m} \right) \quad (25)$$

The nomenclature used in Equation (25) is defined in Figure 12 herein. The background of the development of both the C_{2r} and B_{2r} indices is given in Reference (21). The factor of 0.75 arises from the same concept as

previously discussed for the B_2 index for curve pipe and the B_{2b} index for branch connections.

In-so-far as the author is aware, there are no test data or theoretical calculations that indicate the moment capacity or limit moment for branch connections with the moment traversing the run pipe.

Combined Pressure and Moment

Table 8 includes a few tests where internal pressure was applied and held constant during the limit moment tests. Comparisons of the limit moments with and without internal pressure are shown in Table 9. The Code-implied relationship can be written as:

$$\frac{M_p}{M_o} = 1 - B_1 \frac{PD}{2TS_y} \quad (26)$$

where M_p = limit moment with internal pressure, M_o = limit moment with zero internal pressure and $B_1 = 1.0$. It can be seen in Table 9 that Equation (26) is conservative (predicts a greater decrease in limit moment with pressure than the test data) for all except the T11/T15 comparison. It is of interest that in two of the comparisons, internal pressure increased the limit moment. This is somewhat similar to tests on elbows; see Table 6 and compare tests 13 with 10 and 14 with 12.

ANSI B16.9 TEES

Branch connections in piping systems, where the branch-to-run diameter ratio is around one-half or larger, usually consist of a tee purchased to ANSI standard B16.9⁽²⁴⁾. These tees, from an analysis standpoint, are very complex structures. In addition to the basic complexity of two intersecting cylindrical shells, such tees contain intersection radii and complex variations in wall thickness, particularly in the intersection area.

Internal Pressure

In-so-far as the author is aware, there are no "limit pressure" theoretical analysis methods applicable to B16.9 tees. An elastic-plastic analysis, in principle, could be conducted using a general purpose (including plasticity), 3-D finite-element computer program. Several such programs are in use, but the author has not seen an elastic-plastic analysis of a B16.9 tee.

Experimentally, the only limit pressure test known to the author which covers branch connections identified as B16.9 tees is given in Reference (25). However, from the dimensions shown in Reference (25), these tees are not representative of B16.9 tees and, indeed, may not have been capable of withstanding the burst pressure requirement of ANSI B16.9.

The important pressure-strength requirement for ANSI B16.9 tees is the requirement that the tee (like other products under B16.9) must be capable of withstanding a pressure calculated by the equation:

$$p = \frac{2ST}{D} \quad (27)$$

where S = minimum specified tensile strength of the equivalent pipe

T = minimum wall thickness of the equivalent pipe

D = outside diameter of the equivalent pipe.

In conducting burst tests, pipe legs of at least two diameter lengths must be welded to the welding ends of the tee. Quite often, the burst failure occurs in the pipe legs, and special reinforcement must be placed

on the pipe legs to force failure into the tee.

In general, the shape of the tee is not altered very much by any pressure up to the burst pressure (at which pressure, failure usually consists of an axial crack through the crotch of the tee). This, of course, depends to some extent on the tee material and D/T - ratio. For carbon steel, schedule 80, the deformations are hardly noticeable by eye, for austenitic steel, schedule 10, bulging of the sides is quite noticeable.

The B_1 index for ANSI B16.9 tees is given in Table 1 as 1.0. The index is based on the ANSI B16.9 requirement that the burst strength of fittings must be at least equal to that of equivalent straight pipe.

Moment Loading

In-so-far as the author is aware, there are no "limit moment" theoretical analysis methods available for B16.9 tees and the author has not seen any elastic-plastic analyses of such tees with moment loading.

The only limit-moment tests on B16.9 tees known to the author consist of tests on 6 x 6 x 6 Schedule 40 carbon steel tees run at Oak Ridge National Laboratory. The results of these tests have not been completely evaluated nor have the results been published. It is the author's understanding that tests were conducted by welding 2-diameter lengths of Schedule 40 straight pipe to each end of the tee. The tee was then rigidly held at both ends of the run pipe and an out-of-plane moment was applied to the branch. The result was that the branch pipe yielded; the tee did not. The pipe legs were then cut off and the tee was annealed to a "dead soft" condition. The pipe legs were welded back on and the test was repeated. This time, the tee did yield.

If we assume that in the first test the tee material and pipe material had the same yield strength, then the limit moment of the 6 x 6 x 6 Schedule 40 tee, held on the run ends and subjected to an out-of-plane moment, is greater than the limit moment of equivalent straight pipe. The second test suggests that the limit moment of the tee is not necessarily very much greater than that of equivalent straight pipe.

Publication of these test results would be a valuable contribution to knowledge of the moment capacities of piping products.

The B_{2b} and B_{2r} indices for ANSI B16.9 tees are given in footnote

(9) to Table 1 as $B_{2b} = B_{2r} = 0.75C_{2b}$, where

$$C_{2b} = 0.67 \left(\frac{D}{2T} \right)^{2/3} \quad (28)$$

The equation for C_{2b} is based on the stress intensification factor developed by Markl⁽²²⁾ for ANSI B16.9 tees. The stress intensification is:

$$i = 0.9 \left(\frac{D}{8.8T} \right)^{2/3} \quad (29)$$

Recognizing that the stress intensification factor, i , represents about one-half of the maximum elastic stress, we obtain C_{2b} as shown in Equation (28). The factor of 0.75 relating the B_{2b} and B_{2r} indices to the C_{2b} index arises from the same concept as previously discussed for curved pipe and branch connections.

Equation (P-2), with the limit of $1.5S_m = S_y$, can be written for ANSI B16.9 tees as:

$$\frac{PD}{2T} + 0.5025 \left(\frac{D}{2T} \right)^{2/3} \left(\frac{M_b}{Z_b} + \frac{M_r}{Z_r} \right) \leq S_y \quad (30)$$

Equation (30), as applied to the tee tested at ORNL, implies that

$$0.5025 \left(\frac{3.1725}{.280} \right)^{2/3} \frac{M_L}{Z_b S_y} = 1.0 \quad (31)$$

where M_L is the limit moment. Equation (31) indicates that the limit moment is 0.39 times the yield moment of the equivalent pipe whereas the test data suggest that the limit moment is about the same as the equivalent pipe. Accordingly, in this only available check, the Code relationship is highly conservative.

ANSI B16.9 REDUCERS

Pressure Loading

The B_1 index is given in Table 1 as equal to 1.0. This index has the same basis as the $B_1 = 1.0$ for ANSI B16.9 tees; i.e. the requirement in ANSI B16.9 that fittings have a burst strength at least equal to that of equivalent straight pipe.

Moment Loading

The B_2 index is given in Table 1 as equal to 1.0. It should be noted that ANSI B16.9 covers both eccentric and concentric reducers. The cone angles are not controlled nor are the transition radii between tangents and the conical portion of the reducer. Reference (26) presents the results as an extensive parametric study of calculated elastic stresses in concentric reducers with moment loading. Footnotes (13) and (14) of Table 1 give the empirical equations derived in Reference (26). The indices are restricted to concentric reducers with cone angle not over 60° ; there are no C_2 or K_2 indices for eccentric reducers.

Reference (26) includes a discussion of the B_2 index and concludes that a theoretical limit load (or elastic plastic) analysis plus some test data are needed before reasonable values of B_2 indices for concentric reducers can be established. Accordingly, Reference (26) simply recognizes that the B_2 index of 1.0 may be unconservative but makes no recommendations for changing the B_2 index.

The inadequacy of $B_2 = 1.0$, if it exists, will be for a reducer with large cone angle, large D/t and very small transition sections. An extreme example, from Reference (26), is a reducer with $\alpha = 60^\circ$, $D/t = 100$, transition radii of zero. The calculated maximum stress intensity is $12 M/Z_2$, where Z_2 is the section modulus of the small-end attached pipe. This is an axial stress at the juncture of the conical section with the straight small-end pipe. The stress exists only at two points around the circumference. The stress is mainly a thru-the-wall bending stress; the membrane portion is about $1.0 M/Z_2$.

B_2 indices could be established using the same simple concept

used for curved pipe, branch connections and B16.9 tees; i.e. $B_2 = 0.75C_2$. However, considering that the calculated maximum elastic stress occurs at only two isolated points, making $B_2 = (2/3)C_2$ would be amply conservative. Table 10 shows B_2 indices that were obtained from the equation: $B_2 = (2/3)C_2$. While these probably are excessively conservative, it is recommended that they be used for Code Case 1592-X. Hopefully, in the next few years some theoretical and/or test data can be generated which will permit development of more accurate stress indices. It should be recalled in this regard that the Code user is not forced to use the stress indices; he can always make an NB-3200 (or-3200) analysis.

SUMMARY

The objective of this report started out as an effort to develop appropriate B_2' indices for use in Code Case 1592-X, Equations (P-3) and (P-4). However, in considering the limit load concepts behind the present B_1 and B_2 indices, it became apparent that Equations (P-3) and (P-4) and the B_2' indices used therewith are irrelevant to the basic purpose of those portions* of -3650 concerned with primary stress limits. That purpose is to insure that sustained load combinations are restricted to sufficiently low levels so that gross plastic deformation does not occur, either as a result of short-time yielding or creep.

Accordingly, the major thrust of this report is concerned with Equation (P-2) and its associated B_1 and B_2 indices.

In the following, we will summarize the report with respect to each of the products covered by the Code (Table 1 herein). However, there are two observations which are applicable to all products.

- (1) Buckling. Most of the background data behind the B_2 indices is for products with relatively small D/t ; i.e. 30 and less. For straight pipe, this report includes data which indicate that for $D/t > 50$, buckling may occur and may limit the moment capacity. For more complex piping products such as curved pipe, branch connections, tees and reducers, there are no available data to indicate if buckling will limit the moment capacity and, if so, at what D/t .
- (2) Faulted Conditions. In the chapter on straight pipe, it is pointed out that we cannot show that the B_1 and B_2 indices, when used in Equation (P-2) and the associated faulted condition limits, are necessarily conservative. The same concern exists for all piping products.

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* This report does not address -3653.2 or -3655.2, "Limits on Deformation-Controlled Quantities"

Straight Pipe

A rather extensive review of the elastic-plastic-creep characteristic of straight pipe is given. It is recognized that straight pipe seldom controls the design of piping systems; the more complex structures such as curved pipe branch connections or tees do. However, as we get into the more complex structures we establish relationships between straight pipe characteristics and those of the more complex structures. Hence straight pipe provides a significant reference level.

The evaluation of straight pipe leads to the conclusion that the present B_1 and B_2 indices, as used in Equation (P-2), are conservative for normal, upset or emergency conditions.

Curved Pipe

The elastic analysis of curved pipe shows that, for some values of the elbow parameter $h = tR/r^2$, a high axial membrane stress does exist. However, considering the curved pipe as a structural whole, these high axial membrane stresses have no particular significance. The complex set of bending and membrane stresses existing throughout the structure are appropriately evaluated by tests and/or by limit load analysis. Review of the test data and analysis indicate that the present B_1 and B_2 indices, as used in Equation (P-2), are conservative for normal, upset or emergency conditions.

Fabricated Branch Connections

The present B_1 indices are not necessarily conservative where Code-required reinforcement is obtained by excess thickness in the run pipe and with B_1 used as part of Equation (P-2); $(P_L + P_b) = B_1 DP/2T$ where T is the thickness of the run pipe. A potential solution to the problem with respect to B_1 indices is given. However, the entire problem is under review by the Code Section III Working Group on Piping hence no specific action is deemed necessary for Code Case 1592-X.

Considerable data exists on limit moments applied to the branch of branch connections. These data indicate that the B_1 and B_{2b} indices

are highly conservative. No data are available for limit moments applied to the run of branch connections but the author deems that the present B_{2r} indices are sufficiently conservative.

ANSI B16.9 Tees

The B_1 index of 1.0 is based on the burst strength requirements of ANSI B16.9.

The B_2 indices are based on the presumably highly conservative assumption that a thru-the-wall plastic hinge must form before gross yielding can occur. The one available limit moment test of a B16.9 tee indicates that the B_{2b} index is highly conservative.

ANSI B16.9 Reducers

The B_1 index of 1.0 is based on the burst strength requirements of ANSI B16.9.

The B_2 -index of 1.0 is deemed to be potentially unconservative for reducers with large cone angle and large D/t . It is recommended that the B_2 indices be related to the C_2 indices by: $B_2 = (2/3)C_2$.

Recommended Revisions to -3650

The recommended revised wording of -3650 is included herein in the next chapter. This revised wording incorporates the technical suggestions developed in this report. In terms of the Nov. 1976 draft (see pp 1-5):

- (1) -3651(e): Add special B_2 requirement for reducers.
- (2) Equation (P-2): Define P as P_o
- (3) -3653.1: Delete (c) and (d)
- (4) -3653.1: Change (h) so that $k_s = 0$

In addition, several more-or-less editorial changes are included:

- (1) The word "component" is changed to "product".
"Component", in Code terminology, is an entire

piping system

- (2) -3653.1: Words "and associated limits" deleted because the limits are in -3223, not calculated under -3653.1.
- (3) -3653.1(a) and (b): Words "derived from" deleted because explicit equations are given for calculating P_m and $(P_L + P_b)$.
- (4) -3651(c): Delete. The first sentence conveys the wrong impression of the basis for the B_1 and B_2 indices; they are not based on linearly elastic models. The second sentence doesn't seem to be needed because, when -3653.2 and -3655.2 refer to -3250 and -3250 refers to Appendix T, we find that the piping additions to Appendix T are under T-1320 "Satisfaction of Strain Limits Using Elastic Analysis" and under T-1430 "Limits Using Elastic Analysis".

Finally, -3653.1(f) and (g) are deleted. This reflects a consensus recommendation of the ASME Section III Working Group on Piping, Meeting on March 1, 1977.

RECOMMENDATIONS

It is recommended that the -3650 portion of Code Case 1592-X be revised (from the Nov. 1976 draft) to read as follows:

-3650 ANALYSIS OF PIPING PRODUCTS

-3651 General Requirements

(a) Piping products which satisfy the requirements of -3640 and comply with the rules of -3650 satisfy the requirements of -3611.

(b) The rules of -3200 and Appendix T may be used in lieu of the rules of -3650.

(c) The forces and moments applied to piping products shall be determined using flexibility analyses as specified in -3672.

(d) The stress indices used in the analysis of -3650 shall comply with the rules of -3680, except that B_2 for reducers shall be taken as $(2/3)C_2$ for reducers instead of 1.0.

-3652 Design Conditions

Piping products which comply with the rules of -3640 for pressure design and the rules of -3650 for Operating Conditions also satisfy the requirements for Design Conditions.

-3653 Consideration of Normal Conditions

-3653.1 Primary Stress Intensity Limits

The primary stress intensity limits of -3223 shall be satisfied. The primary stress intensities shall be calculated using (a) through (d) below.

(a) The general primary membrane stress intensity, P_m , shall be calculated using Equation (P-1).

$$P_m = \left[\frac{D_o}{2(t_s - a)} \right] P_o \quad (P-1)$$

where

P_o = operating internal pressure, psi

D_o = outside diameter of pipe, in. (-3683)

t_s = minimum specified wall thickness (nominal wall thickness less fabrication tolerances). in. (-3685)

a = thickness included in the minimum specified wall thickness to allow for expected corrosion and erosion in service. (3641).

(b) The primary membrane-plus bending stress intensity, $(P_L + P_b)$, shall be calculated using Equation (P-2).

$$(P_L + P_b) = B_1 \left(\frac{D_o}{2t} \right) P_o + B_2 \left(\frac{D_o}{2I} \right) M_i \quad (P-2)$$

where the notation is as defined for Equation (1) and,

B_1, B_2 = primary stress indices for the specific product under investigation (-3680)

t = nominal thickness, in.

I = moment of inertia, in.⁴ (-3683)

M_i = load-controlled moment resultant (-3680)

(c) The load-controlled moment resultant, M_i induced by the specified loading conditions shall account for significant elastic follow-up under deformation-controlled loadings such as thermal expansion and anchor movement (see -3138).

(d) The quantity k_s in Equation (7) of -3223 shall be taken as zero for evaluation of piping products using the stress index method.

-3653.2 Limits on Deformation-Controlled Quantities.

The rules of -3250 apply.

-3654 Upset Conditions

The procedures for evaluating Upset Conditions are the same as those given for Normal conditions

-3655 Emergency Conditions

-3655.1 Primary Stress Intensity Limits

The primary stress intensity limits of -3224 shall be satisfied. The procedures are as given in -3653.1.

-3655.2 Limits on Deformation-Controlled Quantities

The rules of -3250 apply.

-3656 Consideration of Faulted Conditions

The primary stress intensity limits of -3225 shall be satisfied. The procedures are as given in -3653.1.

-3657 Pressure Testing Conditions

The limits of -3226 shall be satisfied. The procedures for evaluating internal pressure are as given in -3653.1.

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TABLE 1: STRESS INDICES FOR PIPING COMPONENTS

TABLE NB-3683.2-1
STRESS INDICES FOR USE WITH EQUATIONS IN NB-3650

Piping Products and Joints	(Not Applicable for $D_o/t > 100$)								
	Internal Pressure			Moment Loading ⁵			Thermal Loading		
	B_1	C_1	K_1	B_2	C_2	K_2	C_3	C_4	K_3
Straight pipe, remote from welds or other discontinuities	0.5	1.0	1.0 ¹	1.0	1.0	1.0	1.0	...	1.0
Girth butt weld between straight pipe or between pipe and butt welding components ^{1,11}									
(a) flush	0.5	1.0	1.1 ¹	1.0	1.0	1.1	1.0	0.5	1.1
(b) as welded $r > 3/16"$ [and $\delta/r \leq 0.1$]	0.5	1.1	1.2 ¹	1.0	1.0	1.8	1.0	0.5	1.7
(c) as welded $r \leq 3/16"$ [or $\delta/r > 0.1$]	0.5	1.1	1.2 ¹	1.0	1.4	2.5	1.0	0.5	1.7
Girth fillet weld to socket weld fittings, slip on flanges, or socket welding flanges	0.75	2.0	3.0	1.5	2.1	2.0	1.8	1.0	3.0
Longitudinal butt welds in straight pipe ²									
(a) flush	0.5	1.0	1.1 ¹	1.0	1.0	1.1	1.0	...	1.1
(b) as welded $r > 3/16"$	0.5	1.1	1.2 ¹	1.0	1.2	1.3	1.0	...	1.2
(c) as welded $r \leq 3/16"$	0.5	1.4	2.5 ¹	1.0	1.2	1.2	1.0	...	1.2
Tapered transition joints per NB-4425 and Fig. NB-4233-1 ^{3,6,11}									
(a) flush or no girth weld closer than $\sqrt{rt}$	0.5	6	1.2	1.0	6	1.1	6	1.0	1.1
(b) as welded	0.5	6	1.2	1.0	6	1.8	6	1.0	1.7
Branch connections per NB-3643 ^{3,11}	1.0	2.0	1.7	7	7	7	1.8	1.0	1.7
Curved pipe or butt welding elbows per ANSI B16.9, ANSI B16.28 or MSS SP48 ^{1,4,10,11}	1.0	$\frac{2R-r}{2(R-r)}$	1.0	8	8	1.0	1.0	0.5	1.0
Butt-welding tees per ANSI B16.9 or MSS SP48 ^{1,9,11}	1.0	1.5	4.0	9	9	1.0	1.0	0.5	1.0
Butt-welding reducers per ANSI B16.9 or MSS SP48 ^{1,9,11}	1.0	13	14	1.0	13	14	1.0	0.5	1.0

1. (a) The values of K_1 shown for these components are applicable for components with out of roundness not greater than $0.08\ r$, where out of roundness is defined as $D_{max} - D_{min}$, and

D_{max} = maximum outside diameter of cross section, in.
 D_{min} = minimum outside diameter of cross section, in.
 t = nominal wall thickness, in.

- (b) If the cross section is out of round such that the cross section is approximately elliptical, an acceptable value of K_1 may be obtained by multiplying the tabulated values of K_1 by the factor F_{1a} :

$$F_{1a} = 1 + \frac{D_{max} - D_{min}}{t} \left[\frac{1.5}{1 + 0.455 \left(\frac{D_o}{t} \right)^3 \frac{P}{E}} \right]$$

where D_o = nominal outside diameter, in.

P = internal pressure, psi

(use maximum value of pressure in the load cycle under consideration.)

E = modulus of elasticity of material at room temperature, psi

Other symbols are defined in (a).

- (c) If $D_{max} - D_{min}$ is not greater than $0.08\ D_o$, and acceptable value of K_1 may be obtained by multiplying the tabulated values of K_1 by the factor F_{1b} :

$$F_{1b} = 1 + \frac{M S_y}{P D_o^2 / 2t}$$

where $M = 2$ for ferritic steels and nonferrous materials except nickel-chrome-iron alloys and nickel-iron-chrome alloys

$M = 2.7$ for austenitic steels, nickel-chromium-iron alloys, and nickel-iron-chromium alloys

S_y = yield strength at design temperature, psi (Tables 1-2.0)

P = design pressure, psi

D_o and t are defined in (a) and (b).

- (d) For flush welds as defined in Note 2, S may be taken as zero.

2. Welds in accordance with the requirements of this Subsection.

(a) *Flush welds* are defined as those welds which have been ground on both interior and exterior surface to remove weld irregularities and abrupt changes in contour due to misalignment. Thickness of weld reinforcement (total inside and outside) shall not exceed $0.1t$. No concavity on the root side is permitted. The finished contour shall nowhere have a slope (angle measured from tangent to

TABLE 1: STRESS INDICES FOR PIPING COMPONENTS (Cont.)

surface of pipe or, on tapered transition side of weld, to the nominal transition surface) greater than 7°, see sketch below.

- (b) *As welded* is defined as welds not meeting the special requirements for *flush welds*. At the intersection of a longitudinal butt weld in straight pipe with a girth butt weld or girth fillet weld.

$$B_1 = 0.5 \text{ and } B_2 = 1.0$$

The C_1 , K_1 , C_2 , K_2 , and K_3 indices shall be the product of the respective indices for the longitudinal weld and girth weld. For example, at the intersection of an *as welded* girth butt weld with an *as welded* longitudinal butt weld, C_1 is $1.1 \times 1.1 = 1.21$. C_2 for a girth fillet weld intersecting a longitudinal weld shall be taken as 2.0.

- The stress indices given are applicable only to branch connections in straight pipe with branch axis normal to the pipe surface and which meet the dimensional requirements and limitations of NB-3686 and Fig. NB-3686.1-1.
- R = curved pipe or elbow radius, in.
 r = mean radius of cross section, in.
 $= (D_o - t)/2$, where t = nominal wall thickness
- The values of moment, M_t , shall be obtained from an analysis of the piping system in accordance with NB-3672. M_t is defined as the range of moment loading applied during the specified operating cycle:

Straight Through Pipe

M_t = moment at Point A

$$M_t = \sqrt{M_1^2 + M_2^2 + M_3^2}$$

Curved Pipe or Welding Elbow

M_t = moment at Point A

$$M_t = \sqrt{M_1^2 + M_2^2 + M_3^2}$$

Branch Pipe

Moments calculated for point at intersection of run and branch centerlines

For M_b

$$M_b = \sqrt{M_{x3}^2 + M_{y3}^2 + M_{z3}^2} = \text{resultant moment on branch}$$

For M_r

$$M_r = \sqrt{M_{xr}^2 + M_{yr}^2 + M_{zr}^2} = \text{resultant moment on run}$$

where M_{xr} , M_{yr} and M_{zr} are determined as follows:

If M_{i1} and M_{i2} have the same algebraic sign, then $M_{ii} = 0$. If M_{i1} and M_{i2} have different algebraic signs, then M_{ir} is the smaller of M_{i1} or M_{i2} where $i = x, y, z$

For branch connections of tees, the M_i term of Equations (9), (10), (11), or (12) shall be replaced by the following pairs of terms:

Equation (9) $B_1 b \frac{M_b}{Z_b} + B_2 r \frac{M_r}{Z_r}$

Equations (10) & (12) $C_1 b \frac{M_b}{Z_b} + C_2 r \frac{M_r}{Z_r}$

Equation (11) $C_2 b K_2 b \frac{M_b}{Z_b} + C_2 r K_2 r \frac{M_r}{Z_r}$

where

$$Z_b = \pi (r'_m)^2 T_b$$

$$Z_r = \pi R_m^2 T_r$$

For branch connections per NB-3643 see Footnote 3 above

r'_m , T_b , R_m , and T_r are defined in Fig. NB-3686.1-1

For butt-welding tees per ANSI B16.9 or MS SP 48:

r'_m = mean radius of designated branch pipe

T_b = nominal wall thickness of designated branch pipe

R_m = mean radius of designated run pipe

T_r = nominal wall thickness of designated run pipe

- Indices are applicable to tapered transition joints with a girth butt weld at the thin end of the transition.

$$C_1 = 1.3 + 0.003 (D_o/t) + 1.5 (6/t)$$

$$\text{but not greater than } 2.0$$

$$C_2 = 1.4 + 0.004 (D_o/t) + 3.0 (6/t)$$

$$\text{but not greater than } 2.1$$

$$C_3 = 1.2 + 0.008 (D_o/t)$$

- $B_2 b = 0.75 C_2 b$ but not less than 1.0

- $B_3 r = 0.75 C_2 r$ but not less than 1.0

$$C_2 b = 3(R_m/T_r)^{1/3} (r'_m/R_m)^{1/2} (T_b/T_r) (r'_m/r_p), \text{ but not less than } 1.5$$

R_m , T_r , r'_m , T_b , and r_p are defined in Fig. NB-3686.1-1

$$K_2 b = 1.0$$

$$C_2 r = 0.8 (R_m/T_r)^{1/3} (r'_m/R_m), \text{ but not less than } 1.0$$

$$K_2 r = 2.0$$

The product of $C_2 r K_2 r$ shall be a minimum of 3.0

- $C_3 = \frac{1.95}{b_2^{1/3}}$, but not less than 1.5; $B_3 = 0.75 C_3$

$$b_2 = \frac{tR}{r^2}, \text{ where } t = \text{nominal pipe wall thickness}$$

R = bend radius of curved pipe or elbow

r = mean pipe radius

$$= (D_o - t)/2$$

TABLE 1: STRESS INDICES FOR PIPING COMPONENTS (Cont.)

9. $B_{2b} = D_{1r} = 0.75 C_{2b}$
 $C_{2b} = C_{2r} = 0.67 (R_m/T_r)^{1/2}$, but not less than 2.0
 R_m = mean radius of designated run pipe
 T_r = nominal wall thickness of designated run pipe

$$K_{2b} = K_{2r} = 1.0$$

10. The K indices given for fittings per ANSI B16.9 ANSI B16.28 or MSS SP48 apply only to seamless fittings with no connections, attachments, or other extraneous stress raisers on the bodies thereof. For fittings with longitudinal butt welds, the K indices shown shall be multiplied by the 1.1, for *flush welds* as defined in Note 2; by 1.3 for welds not meeting the requirements for *flush welds*.

11. The stress indices given predict stresses which occur in the body of a fitting. It is not required to take the product of stress indices for two piping products such as a tee, and a reducer, or a tee and a girth butt weld when welded together except for the case of curved pipe or butt welding elbows welded together or joined by a piece of straight pipe the length of which is less than 1 pipe diameter. For this specific case the stress index for the curved pipe or butt welding elbow must be multiplied by that for the girth butt weld. Excluded from this multiplication are the B_1 and C_1 indices. Their value is to be: $B_1 = 1.0$, $C_1 = 0.50$.

12. δ is defined as the maximum permissible mismatch as shown in Fig. NB-4233.1. A value of δ less than $1/32$ in. may be used provided the smaller mismatch is specified for fabrication.

13. (a) Nomenclature:

t_1 = nominal wall thickness, large end.
 t_2 = nominal wall thickness, small end
 D_1 = nominal outside diameter, large end
 D_2 = nominal outside diameter, small end
 α = cone angle, deg.

- (b) The indices given in (c) and (d) apply if the following conditions are met.

- (1) Cone angle, α , does not exceed 60 deg. and the reducer is concentric.

- (2) The wall thickness is not less than t_{1m} throughout the body of the reducer, except in and immediately adjacent to the cylindrical portion on the small end, where the thickness shall not be less than t_{2m} . Wall thicknesses t_{1m} and t_{2m} are to be obtained by Equation (1), NB-3641.1.

- (c) Reducers in which r_1 and $r_2 \geq 0.1 D_1$

$$C_1 = 1 + 0.0058 \alpha \sqrt{D_n/t_n}$$

$$C_2 = 1 + 0.36 \alpha^{0.4} (D_n/t_n)^{0.4} (D_2/D_1 - 0.5)$$

where D_n/t_n is the larger of D_1/t_1 and D_2/t_2 .

- (d) Reducers in which r_1 and/or $r_2 < 0.1 D_1$

$$C_1 = 1 + 0.00465 \alpha^{1.285} (D_n/t_n)^{0.19}$$

$$C_2 = 1 + 0.0185 \alpha \sqrt{D_n/t_n}$$

where D_n/t_n is the larger of D_1/t_1 and D_2/t_2 .

14. The K indices given in (a), (b), and (c) apply for reducers attached to the connecting pipe with "flush" or "as-welded" girth welds as defined in footnote (2). Note that the connecting girth weld must also be checked separately for compliance.

- (a) For reducers connected to pipe with "flush" girth butt welds:

$$K_1 = 1.1 - 0.1 \frac{L_m}{\sqrt{D_m t_m}}, \text{ but not less than } 1.0$$

$$K_2 = 1.1 - 0.1 \frac{L_m}{\sqrt{D_m t_m}}, \text{ but not less than } 1.0$$

where $L_m/\sqrt{D_m t_m}$ is the smaller of $L_1/\sqrt{D_1 t_1}$ and $L_2/\sqrt{D_2 t_2}$.

- (b) For reducers connected to pipe with "as-welded" girth butt welds where $t_1, t_2 \geq 3/16$ in. and $\delta_1/t_1, \delta_2/t_2 \leq 0.1$:

$$K_1 = 1.2 - 0.2 \frac{L_m}{\sqrt{D_m t_m}}, \text{ but not less than } 1.0$$

$$K_2 = 1.2 - 0.2 \frac{L_m}{\sqrt{D_m t_m}}, \text{ but not less than } 1.0$$

where $L_m/\sqrt{D_m t_m}$ is the smaller of $L_1/\sqrt{D_1 t_1}$ and $L_2/\sqrt{D_2 t_2}$.

- (c) For reducers connected to pipe with "as-welded" girth butt welds, where t_1 or $t_2 \leq 3/16$ in. or δ_1/t_1 or $\delta_2/t_2 > 0.1$:

$$K_1 = 1.2 - 0.2 \frac{L_m}{\sqrt{D_m t_m}}, \text{ but not less than } 1.0$$

$$K_2 = 2.5 - 1.5 \frac{L_m}{\sqrt{D_m t_m}}, \text{ but not less than } 1.0$$

where $L_m/\sqrt{D_m t_m}$ is the smaller of $L_1/\sqrt{D_1 t_1}$ and $L_2/\sqrt{D_2 t_2}$.

TABLE 2: SHAPE FACTORS, M_p/M_e , FOR STRAIGHT PIPE

n (a)	M_p/M_e for D_o/t of: (b)				
	10	20	40	60	∞
1.00	1.00	1.00	1.00	1.00	1.00
1/2	1.17	1.14	1.13	1.12	1.11
1/3	1.24	1.20	1.19	1.18	1.16
1/4	1.28	1.23	1.22	1.20	1.19
1/5	1.30	1.25	1.25	1.22	1.20
1/6	1.32	1.27	1.26	1.23	1.21
1/10	1.35	1.30	1.28	1.26	1.24
0	1.40	1.34	1.31	1.29	1.27

(a) n is the exponent in the material stress-strain or stress-creep-rate equation $\sigma = Ce^n$, σ = stress, e = strain or creep rate.

(b) M_p calculated by Equation (3). $M_e = S_y(\pi/4)(r_o^4 - r_i^4)/r_o$. Calculations based on $C = 40,000/.002^n$ psi and $\sigma_o = S_y = 40,000$ psi.

TABLE 3: TEST DATA ON SHAPE FACTORS,
 $M_{\max}/M_e$, FOR STRAIGHT PIPE

D_o/t	$M_{\max}/[S_y(\pi/4)(r_o^4 - r_i^4)/r_o]$	
	(a)	(b)
20	1.16	1.00
40	1.21	1.01
60	1.14	0.96
80	0.99	0.88
100	0.85*	0.80

(a) From Reference (2)

$$M_{\max} = q D_o^2 t S_y \left[\frac{500}{445 + D_o/t} \right]$$

S_y established by total strain = 0.005

For material like API X52 or X60

For D_o/t greater than about 50,
buckling occurs.

D_o/t	q
≤ 50	1.00
60	0.95
70	0.88
80	0.85

*Extrapolated to

$D_o/t = 100$,

$q = 0.75$

(b) From Reference (3)

$$M_{\max} = C_B D_o^2 t S_y$$

where $C_B = 1 - 0.0035 D_o/t$

Tests were on carbon steel material, specified minimum yield strengths ranging from 35,000 psi to 52,000 psi. In the evaluations, the specified minimum rather than actual may have been used.

TABLE 4: MAXIMUM MOMENT CAPACITY OF STRAIGHT
PIPE USING AFENDIK'S⁽⁵⁾ EQUATIONS

$\frac{D}{t}$	$n = 0.15^{(a)}$		$n = 0.25^{(a)}$	
	$\frac{M'}{E\pi r^2 t}$	$\frac{r}{\rho'}$	$\frac{M'}{E\pi r^2 t}$	$\frac{r}{\rho'}$
	(b)	(c)	(b)	(c)
10	0.0029	0.09	0.0038	0.09
20	0.0026	0.04	0.0031	0.04
40	0.0024	0.018	0.0026	0.02
60	0.0023	0.012	0.0024	0.014
80	0.0021	0.010	0.0023	0.012
100	0.0021	0.008	0.0021	0.008

(a) Plastic stress strain curve described by $\sigma = Ce^n$. The value of C was obtained by making $\sigma = 40,000$ at $e = 0.002$.

(b) M' is the maximum moment. The limit moment, by $M_L = 4r^2 t S_y$, with $S_y = 40,000$ psi, $E = 3 \times 10^7$ psi, is:

$$M_L / (E\pi r^2 t) = 0.00170$$

(c) ρ' is the radius of curvature at which M' occurs. Values of $M'/E\pi r^2 t$ were calculated for r/ρ in steps of 0.01 for r/ρ up to 0.1 and in steps of 0.002 for r/ρ up to 0.02.

TABLE 5: VALUES of S_o , S_m and S_t FROM CODE CASE 1592-8

Material	Temp, °F	S_o , ksi	S_m , ksi	S_t (ksi) for t (hours) of:				
				1	10^2	10^4	10^5	3×10^5
304SS	800	15.1	15.1	20.4	20.4	20.4	20.4	20.4
	1000	13.7	14.0	18.5	16.9	14.7	11.1	9.3
	1200	6.0	12.7	16.0	12.2	6.1	4.1	3.4
	1500	1.4	7.0/6.4	7.9	3.5	1.3	0.8	0.6
316	800	15.8	15.8	20.8	20.8	20.8	20.8	20.8
	1000	15.3	15.4	19.8	19.8	18.2	16.2	14.0
	1200	7.4	14.6	17.8	14.2	8.3	5.5	4.5
	1500	1.2	8/8.5	7.8	3.2	1.2	0.65	0.5
800H	800	15.3	15.3	20.2	20.2	20.2	20.2	20.2
	1000	14.4	14.4	19.4	19.4	18.9	18.5	17.0
	1200	8.4	13.8	18.2	16.6	9.4	7.2	6.4
	1400	3.6	11.0	12.6	6.8	4.0	3.0	2.6
				(10 hours)				
$2\frac{1}{4}$ Cr - 1 Mo	700	15.0	17.9	35.5	35.5	35.5	35.5	35.5
	800	15.0	17.9	33.2	30.4	25.0	18.0	16.1
	1000	7.8	15.9	17.1	14.2	8.7	6.3	5.2
	1200	1.6	9.5	6.2	5.9	2.7	1.7	1.4

TABLE 6: COMPARISON OF LIMIT LOAD TESTS⁽⁹⁾ WITH CODE LIMIT LOAD
EQUATION, $M_{cL} = ZS_y/B_2$. TESTS ON 6.625" NOMINAL OUTSIDE
DIAMETER WELDING ELBOWS, 90° ARC LENGTH

Test No.	Wall, in.,	Bend Radius, in	h	B ₂	Type of Moment	$\frac{M_L}{ZS_y}$	$\frac{M_L}{M_{cL}}$
(a)	t	R	(b)	(b)	(c)	d	(e)
1	0.280	9.0	0.2504	3.68	+M _z	0.45	1.66
2	↓	↓	↓	↓	-M _z	0.40	1.47
3	↓	↓	↓	↓	M _y	0.45	1.66
4	↓	↓	↓	↓	M _z	0.48	1.77
5	↓	↓	↓	↓	-M _z	0.51	1.88
6	↓	↓	↓	↓	M _y	0.49	1.80
7	0.432	9.0	0.4055	2.67	+M _z	0.75	2.76
8	↓	↓	↓	↓	-M _z	0.62	2.28
9	↓	↓	↓	↓	M _y	0.71	2.61
10	0.280	6.0	0.1669	4.82	+M _z	0.48	2.31
11	↓	↓	↓	↓	-M _z	0.46	2.22
12	↓	↓	↓	↓	M _y	0.50	2.41
13	↓	↓	↓	↓	+M _z +P	0.49	2.36
14	↓	↓	↓	↓	M _y +P	0.50	2.41
15	0.280	9.0	0.2504	3.68	-M _z	0.30	1.10

(a) Material: ASTM A106 Grade B, except Test No. 15: ASTM A312 Type 304L.

(b) $h = LR/l^2$, $l = (6.625-t)/3$, $B_2 = 0.75 \times 1.95/h^{2/3}$.

(c) P = 1500 psi internal pressure.

(d) Limit moment M_L determined by $\delta_t = (1 + a) \delta_e$, where δ_t = total deflection, δ_e = elastic deflection and $a = 0.25$ for test numbers 1, 2, 3, 10, 11, 12, 15; $a = 0.40$ for the other tests. Z = section modulus of elbow cross section.

(e) $M_{cL} = ZS_y/B_2$, $M_L/M_{cL} = (M_L/ZS_y)B_2$

TABLE 7: CURVED PIPE LIMIT MOMENTS AS CALCULATED BY SPENCE⁽¹⁰⁾ THEORY, M_s , AND RATIOS OF THEORETICAL TO CODE LIMIT MOMENTS, M_s/M_{cL} . CURVED PIPE WITH $h = 0.12$ and 0.04 .

n	h	K	K_s	B_2	a = 2.0 ^(d)		a = 1.5 ^(d)		a = 0.5 ^(d)	
					$\frac{M_s}{\pi r^2 t S_y}$	$\frac{M_s}{M_{cL}}$	$\frac{M_s}{\pi r^2 t S_y}$	$\frac{M_s}{M_{cL}}$	$\frac{M_s}{\pi r^2 t S_y}$	$\frac{M_s}{M_{cL}}$
(a)	(b)	(b)	(c)	(b)	(e)	(f)	(e)	(f)	(e)	(f)
1/2	0.12	13.75	41	6.011	0.554	3.33	0.277	1.66	0.138	0.832
	0.04	41.25	256	12.504	0.266	3.33	0.133	1.66	0.066	0.832
1/3	0.12	13.75	152	6.011	0.434	2.61	0.307	1.84	0.217	1.303
	0.04	41.25	1,960	12.504	0.209	2.62	0.148	1.85	0.105	1.308
1/5	0.12	13.75	2,050	6.011	0.387	2.32	0.325	1.96	0.274	1.64
	0.04	41.25	111,000	12.504	0.188	2.35	0.158	1.97	0.133	1.66
1/9	0.12	13.75	370,000	6.011	0.367	2.21	0.356	2.03	0.309	1.86
	0.04	41.25	3.5×10^8	12.504	0.179	2.24	0.164	2.05	0.151	1.88

(a) n in equation $\sigma = Ce^n$ or $\sigma = Ce^n$. Calculations based on $C = 40000/(\cdot 002)^n$.

(b) $h = tr/r^2$, $K = 1.65/h$, $B_2 = 0.75 \times 1.95/h^{2/3}$.

(c) K_s = plasticity or creep flexibility factor based on Spence⁽¹⁰⁾ theory.

(d) Limit moment M_s defined by $\theta_t = (1+a) \theta_e$, θ_t = total rotation, θ_e = elastic rotation.

(e) M_s = Spence limit moment, $S_y = 40,000$ psi.

(f) M_{cL} = Code limit = ZS_y/B_2 , $M_s/M_{cL} = [M_s/(\pi r^2 t S_y)] \times B_2$.

TABLE 8: SUMMARY OF LIMIT MOMENT TEST
DATA ON BRANCH CONNECTIONS

Moment (a)	Ref.	Test Iden.	$\frac{D}{T}$	$\frac{d}{D}$	$\frac{s}{S}$ (b)	$\frac{PD}{2TS_y}$	$\frac{M_L}{d^2 t S_y}$ (c)
M_{x3} ↓	(16)	T4	25	0.5	1.0	0	0.72
		T10	34.5	0.7	0.55	0	0.48
		T6	25	1.0	1.0	0	0.59
		T16	42	1.0	1.0	0	0.49
	↓	T13	34.5	0.7	0.55	0.56	0.32
	(17)	T18	34.5	0.4	0.5	0	0.57
		T20	34.5	0.6	0.5	0	0.46
		T21	34.5	0.3	0.5	0	0.75
		T22	34.5	0.8	0.5	0	0.33
	↓	T23	25.0	0.7	1.0	0	0.70
	(18)	T24	34.7	0.2	0.5	0	0.90
		T21	34.5	0.3	0.5	0	0.75
		T25	35.0	0.97	0.5	0	0.27
		T29	45.0	0.4	0.3	0	0.33
		T28	45.0	0.6	0.3	0	0.22
		T27	45.0	0.8	0.3	0	0.16
	↓	T26	45.0	0.96	0.3	0	0.17
	(19)	B2	31.0	0.65	0.98	0	0.50
	(19)	B4	22.9	0.65	1.65	0	0.84
	(20)	C2	30.0	0.6	1.02	0	0.69
		D2	29.7	0.79	1.02	0	0.52
		E2	30.0	1.0	1.00	0	0.41
		C3	30.0	0.6	0.99	0.5	0.56
		D3	30.0	0.8	1.01	0.5	0.58
		E3	30.0	1.0	1.00	0.5	0.45
M_{z3} ↓	(16)	T8	24.0	0.52	1.0	0	0.91
		T5	25.0	0.75	1.0	0	0.84
		T14	25.0	1.00	1.0	0	0.64
		T11	25.0	0.75	1.0	0.33	0.47
	↓	T12	27.0	1.00	1.0	0.27	0.49
	(17)	T15	56.5	1.00	1.0	0	0.50
	(17)	T19	34.5	0.40	0.5	0	0.70
	(19)	B1	31.0	0.65	0.98	0	0.66
	(20)	D1	30.1	0.79	1.00	0	0.63
	(20)	E1	30.0	1.00	1.00	0	0.46

(a) See Footnote (5) of Table 1.

(b) $s/S = (d/D)/(t/T)$. $s = Pd/2t$ and $S = PD/2T$

(c) M_L is the experimentally determined limit moment of the branch connection.

TABLE 9: COMPARISON OF LIMIT MOMENT TESTS WITH
AND WITHOUT INTERNAL PRESSURE

Moment (a)	Test Iden. (b)	$\frac{D}{T}$	$\frac{d}{D}$	$\frac{s}{S}$ (c)	$\frac{PD}{2TS_y}$	M^*		M_p/M_o	
						$P>0$ M_p^* (d)	$P=0$ M_o^* (d)	Test	Eq. (26)
M_{x3}	T13/T10	34.5	0.7	0.55	0.56	0.32	0.48	0.67	.44
	C3//C2	30.0	0.6	1.0	~0.5	0.56	0.69	0.81	.5
	D3//D2	30.0	0.8	1.0	~0.5	0.58	0.52	1.12	.5
	E3//E2	30.0	1.0	1.0	~0.5	0.45	0.41	1.10	.5
M_{z3}	T11/T15	25	0.75	1.0	0.33	0.47	0.84	0.56	.67
	T12/T14	~26	1.00	1.0	0.27	0.49	0.64	0.77	.73

(a) See footnote (5) of Table 1.

(b) See Table 8 for Reference.

(c) $s/S = (d/D)/(t/T)$. $s = Pd/2t$, $S = PD/2T$.

(d) $M^* = M_L/d^2tS_y$, where M_L is the experimentally determined limit moment of the branch connection.

TABLE 10: B_2 INDICES FOR CONCENTRIC REDUCERS^(a)
OBTAINED BY $B_2 = (2/3)C_2$

α	$\frac{D}{t}$	$\frac{D_2}{D_1}$	B_2	
			$r_1 \text{ and } r_2 > 0.1D_1$	$r_1 \text{ or } r_2 < 0.1D_1$
15	10	0.5	1.38	1.25
		0.9	1.69	1.25
	50	0.5	1.38	1.97
		0.9	1.99	1.97
	100	0.5	1.38	2.52
		0.9	2.15	2.52
30	10	0.5	1.60	1.75
		0.9	2.02	1.75
	50	0.5	1.60	3.28
		0.9	2.42	3.28
	100	0.5	1.60	4.37
		0.9	2.62	4.37
60	10	0.5	1.90	3.01
		0.9	2.45	3.01
	50	0.5	1.90	5.90
		0.9	2.98	5.90
	100	0.5	1.90	8.07
		0.9	3.25	8.07

(a) See footnote (13) of Table 1 for nomenclature.

FIGURE 1: ILLUSTRATION OF DIFFERENT CONCEPTS USED IN CLASSIFICATION OF STRESSES DUE TO A SUSTAINED MOMENT LOAD

FIGURE 2: COMPARISON OF THEORIES FOR MOMENT LOADING OF STRAIGHT PIPE WITH AND WITHOUT OVALIZATION OF CROSS SECTION

FIGURE 3: COMPARISON OF LIMIT LOAD THEORY WITH CODE EQUATION (P-2), STRAIGHT PIPE, $p = Pr / t S_y = 2/3$

FIGURE 4: COMPARISON OF LIMIT LOAD THEORY WITH CODE EQUATION (P-2), STRAIGHT PIPE, $Pr/tS_y = 0$.

$$\text{Code Eq. (P-2): } p/2 + \left\{ \left[(4/\pi)m_b \right]^2 + \left[2m_t \right]^2 \right\}^{1/2} = 1.2$$

$$p = Pr/tS_y$$

FIGURE 5. COMPARISON OF LIMIT LOAD THEORY⁽⁶⁾ WITH CODE EQUATION (P-2), STRAIGHT PIPE, EMERGENCY CONDITIONS

FIGURE 6: COMPARISON OF LIMIT LOAD THEORY⁽⁶⁾ WITH CODE EQUATION (P-2), STRAIGHT PIPE, FAULTED CONDITIONS

a. Plot of Creep Strain versus Time at Various Stress Levels

b. Plot of Creep Strain Rate versus Stress, Derived from (a)

FIGURE 7: IDEALIZED DATA TO GENERATE CREEP EQUATION OF THE FORM: $\sigma = C\dot{\epsilon}^n$

FIGURE 8: COMPARISON OF CREEP LIMIT THEORY WITH CODE EQUATION (P-2), STRAIGHT PIPE, FAULTED CONDITIONS

FIGURE 9: AXIAL MEMBRANE STRESSES IN CURVED PIPE WITH IN-PLANE MOMENT LOADING

FIGURE 10: EXAMPLE OF CALCULATED MOMENT VERSUS CURVATURE RELATIONSHIP FOR A CURVED PIPE.

T = run pipe thickness, T_r = required minimum run pipe thickness
 t_{br} = required minimum branch pipe thickness

FIGURE 11: EXTREMES OF CODE-PERMITTED REINFORCEMENT WITH RESPECT TO REQUIRED REINFORCEMENT AREA, $A_r = T_r d$.

FIG. NB-3686.1-1 NOZZLE DIMENSIONS

FIGURE 12: NOMENCLATURE USED IN EQUATIONS (22) and (25)

$$M^* = M_L / (d^2 + Sy)$$

M_L = experimental limit moment
or instability moment

FIGURE 13: CORRELATION OF TEST DATA FOR M_{x3} (OUT-OF-PLANE)
MOMENT, BRANCH CONNECTIONS

$$M^* = M_L / (d^2 t S_y)$$

M_L = experimental limit moment

FIGURE 14: CORRELATION OF TEST DATA FOR M_{z3} (IN-PLANE) MOMENT, BRANCH CONNECTIONS.