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PLANE STRAIN LARGE DEFORMATION ANALYSIS OF THIN CYLINDRICAL
SHELL SUBJECT TO EXTERNAL PRESSURE

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ABSTRACT

Included in this report are the analytical results of a plane strain large displacement analysis of a thin cylindrical shell subjected to non-uniform external pressure. The purpose of the analysis is to determine the ultimate load capacities and hence, the design margins of safety of the shell structure for both the static and the dynamic loading conditions.

The finite element method was used to solve the numerical problem. For dynamic analysis, the implicit Houbolt operator was utilized to perform the time integration. All numerical computations were accomplished using the nonlinear finite element program MARC.

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PLANE STRAIN LARGE DEFORMATION ANALYSIS OF THIN
CYLINDRICAL SHELL SUBJECT TO EXTERNAL PRESSURE

1. DESCRIPTION OF STRUCTURE

The structure under consideration is a long cylindrical shell having diameter and thickness of, respectively, 54 inches and .25 inches.

Material is SA516 carbon steel with the following properties: E (Young's Modulus) = 3×10^7 psi, ν (Poisson's Ratio) = 0.3, σ_{yd} (yield strength) = 35,000 ps, and ρ (mass density) = 0.000735 LB - sec²/in.⁴

2. LOADING CONDITIONS

The thin shell structure is subjected to non-uniform pressure shown in Figure 1. The pressure distribution is described by the following expression:

$$p = \begin{cases} p_o (1 - \theta^2/\alpha^2) & \text{For } \theta \leq \alpha \\ 0 & \text{For } \theta > \alpha \end{cases}$$

where p_o is a given value for static loading or a time-dependent function for a pressure pulse described by the following parabolic equation:

$$p_o = \begin{cases} 4 P_K t/\tau (1 - t/\tau) & \text{For } 0 \leq t \leq \tau \\ 0 & \text{For } t < 0 \text{ and } t > \tau \end{cases}$$

3. METHOD OF ANALYSIS

The large deformation finite element method is employed to perform the desired analysis. All numerical computations are conducted by utilization of the finite element computer code MARC. (Reference: MARC-CDC, Non-linear Finite Element Analysis Program, Control Data Corporation, Publication No. 17309500).

3. METHOD OF ANALYSIS (cont'd)

The finite element model (Figure 2) is formed by MARC type 15 axisymmetric thin shell elements.

A large number is added to all r nodal coordinates in order to simulate the plane strain effect. Only one-half of the structure is considered due to symmetry conditions. Twelve (12) shell elements and four (4) springs are presented in the model. The springs are used to take into account the beam stiffness which exists in the actual BWR Mark I Suppression pool vent header structure.

Due to the non-linear large deformation nature of the problem, the load has to be applied incrementally during a static buckling analysis. An increment of 0.5 psi of the maximum surface pressure was considered in the static analysis. A non-positive definite stiffness matrix was detected by the computer program as the maximum external surface pressure is increased from 17.5 psi to 18.0 psi during the static analysis.

In performing the dynamic analysis for each chosen peak pressure, a numerical time integration scheme is required to obtain the transient structural responses. Both implicit and explicit integration schemes are available in MARC. The implicit Houbolt operator is used in the dynamic analysis. For each chosen peak pressure not exceeding the critical pressure, a converged bounded solution can be achieved. In other words the displacement of the shell increases with time, reaches a maximum value, and then starts to decrease. If the chosen peak

pressure is greater than the critical value, the displacement tends to go unbounded and will never reach a finite maximum. The dynamic analyses were performed for peak pressure of 5, 10, 15, 20, 25, 30, 35, and 40 psi. Converged solutions were obtained up to 35 psi. At 40 psi, solution became unbounded. Another loading case of peak value of 37 psi was then added to the investigation. Unbounded displacement again was detected.

4. ANALYSIS RESULTS

Figure 3 shows the transient displacement at Node 1 of the structure under dynamic loading. Peak external pressure values in psi for these curves are at 5, 10, 15, 20, 25, 30, 35, 37, and 40, respectively. Maximum response of the structure was obtained except for the last two loading conditons of 37 and 40 psi.

Figure 4 shows the load-displacement relations at node 1 for both the static and the dynamic responses with Table 1 showing the numerical values.

Stress and strain states at the bottom exterior surface of the cylinder are presented by Tables 2 and 3, as well as, Figures 5, 6, 7, 8, 9 and 10.

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TABLE 1 DISPLACEMENT AT NODE 1

P _o (psi)	Static Displ. (inches)	Dynamic Displ. (inches)
0	0.	0.
2.5	0.344	-
5.0	0.716	0.740
7.5	1.117	-
10.0	1.572	1.532
12.5	2.288	-
15.0	3.648	2.468
17.5	5.806	-
20.0	∞	3.866
25.0	∞	6.252
30.0	∞	9.623
35.0	∞	15.570
37.0	∞	∞
40.0	∞	∞

TABLE 2 STRESSES & STRAINS AT BOTTOM SHELL SURFACE FOR STATIC ANALYSIS

MAXIMUM PRESSURE (PSI)	HOOP STRESS (PSI)	LONG. STRESS (PSI)	HOOP STRAIN (%)	EQUIV. STRESS (PSI)	EQUIV. STRAIN (%)	STRESS INTENSITY (PSI)
2.5	-11,680	-3,500	-0.035	10,380	0.035	11,680
5.0	-24,270	-7,280	-0.074	21,570	0.072	24,270
7.5	-37,740	-11,320	-0.115	33,540	0.112	37,740
10.0	-40,430	-15,140	-0.169	35,380	0.169	40,430
12.5	-42,040	-19,680	-0.307	36,440	0.320	42,040
15.0	-44,870	-22,210	-0.616	38,860	0.665	44,870
17.5	-48,570	-24,060	-1.022	42,060	1.003	48,570

TABLE 3 STRESSES & STRAINS AT BOTTOM SHELL SURFACE FOR DYNAMIC ANALYSIS

MAXIMUM PRESSURE (PSI)	HOOP STRESS (PSI)	LONG. STRESS (PSI)	HOOP STRAIN (%)	EQUIV. STRESS (PSI)	EQUIV. STRAIN (%)	STRESS INTENSITY (PSI)
5	-19,370	-5,810	-0.059	17,220	0.057	19,370
10	-39,410	-11,900	-0.121	35,010	0.118	39,410
15	-41,090	-17,190	-0.219	35,750	0.223	41,090
20	-43,460	-21,210	-0.462	37,640	0.492	43,460
25	-47,320	-23,440	-0.885	40,980	0.966	47,320
30	-48,500	-24,030	-1.015	42,000	1.111	48,500
35	-52,790	-26,150	-1.486	45,720	1.639	52,790

Figure 1 Loading Condition

Figure 2 Finite Element Model

Figure 3 Transient Structural Responses

Figure 4 Load vs. Displacement Curves

Figure 5 Equivalent Stress vs. Pressure

Figure 6 Stress Intensity vs. Pressure

Figure 7 Equivalent Strain vs. Pressure

Figure 8 Equivalent Stress vs. Equivalent Strain

Figure 9 Hoop Strain vs. Pressure

Figure 10 Hoop Stress vs. Hoop Strain

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