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DETERMINATION OF PULSE BROADENING IN OPTICAL FIBERS FROM NUMERICAL SOLUTIONS
OF A WAVE EQUATION

M. D. Feit and J. A. Fleck, Jr.

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ABSTRACT

Dispersion is one of the most important properties of optical fibers, since it determines their information carrying capacity. In the past estimates of fiber dispersion have been based on a highly approximate modal analysis. We describe herein a new method based on accurate numerical solutions of Maxwell's wave equation for calculating pulse dispersion in graded index fibers. The methods described should also be useful for studying many properties of fiber optic transmission.

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DETERMINATION OF PULSE BROADENING IN OPTICAL FIBERS
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INTRODUCTION

Temporal or pulse broadening is an important property of optical fibers, since it determines their ultimate information carrying capacity. Pulse dispersion has already been analyzed by a number of workers in terms of the group delay properties of individual propagating modes. In the case of multi-mode graded index fibers analysis has been based on an application of the WKB approximation.⁽¹⁻²⁾

In this paper, we wish to describe a new method for determining pulse broadening that is based on a direct numerical solution of a wave equation. Since the method is numerical, it can in principle be used to model realistic fibers by employing in the calculation experimental material dispersion data, frequency dependent loss data, and index profile data. The only restriction on the index profile is that the maximum refractive index change between core and cladding be small compared to 1.

The method can in principle be applied to both coherent (beam type) and incoherent sources, although our efforts so far have been restricted to beam type sources.

The assumed properties of the fiber need not be axially constant, but the length of the fiber over which calculations can be extended is obviously limited by practical considerations of computation time. For typical fibers practically calculable fiber lengths are of the order of centimeters. Since one is normally interested in dispersion resulting from propagation over kilometer distances, it is thus necessary to establish the scaling of dispersion accurately over centimeter distances. This places unusually exacting requirements on the accuracy of the numerical calculation method. These requirements for accuracy are fully met by the finite Fourier series method of solution which is described herein.

This report is designed as an interim progress report, since it describes results which were obtained primarily for the purpose of testing the method. In the future, dispersion calculations will be performed on fibers of practical interest such as the Corning fiber.

Dispersion is of course, the most difficult of calculations. The computer code to be described can be economically applied to many problems of practical interest such as characterizing the mode structure, splicing losses, the effects of illuminating the fiber in various ways, etc.

WAVE EQUATION AND SOLUTION METHOD

We begin with the usual assumption that the propagation of a single frequency component of light in a fiber wave-guide can be described by the scalar Helmholtz equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} + \frac{\omega^2}{c^2} n^2(\omega, x, y) E = 0, \quad (1)$$

where $E(x, y, z)$ is the transverse component of the electric field, ω is the circular frequency of the light, and the refractive index $n(\omega, x, y)$ is assumed to depend only on the transverse coordinates x and y . This dependence of n on coordinates will subsequently be relaxed, so that n can be assumed to depend "weakly" on z as well.

Let E be expressed in the form

$$E(x, y, z) = E(x, y, z) e^{-ikz}, \quad (2)$$

where

$$k = \frac{n_0 \omega}{c}, \quad (3)$$

and n_0 is some reference value of the refractive index lying somewhere between the value in the cladding and the maximum value in the core. If $E(x, y, z)$ is a slowly varying function of z in the sense that

$$\frac{\partial^2 E}{\partial z^2} \ll k^2 E, \quad (4)$$

one can neglect the second derivative of E with respect to z on substituting expression (2) into Eq. (1). The result is the familiar parabolic or paraxial form of the wave equation

$$2ik \frac{\partial E}{\partial z} = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + k^2 \left(\frac{n^2}{n_0^2} - 1 \right) E = 0 \quad (5)$$

The solution of Eq. (5) over a short axial distance Δz can be expressed in symbolic form as

$$E(x, y, \Delta z) = \exp \left\{ - \frac{i\Delta z}{2k} \left[\nabla_{\perp}^2 + k^2 \left(\frac{n^2}{n_0^2} - 1 \right) \right] \right\} E(x, y, 0) \quad , \quad (6)$$

where

$$\nabla_{\perp}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad (7)$$

Equation (6) can be replaced to second order accuracy in Δz by the symmetrized split operator form

$$E(x, y, \Delta z) = e^{-\frac{i\Delta z}{4k} \nabla_{\perp}^2} e^{-i\Delta z \chi} e^{-\frac{i\Delta z}{4k} \nabla_{\perp}^2} E(x, y, 0) \quad , \quad (8)$$

where

$$\chi = \frac{k}{2} \left(\frac{n^2}{n_0^2} - 1 \right) \quad (9)$$

Expression (8) is the basis for the computational algorithm represented in Fig. 1 and is interpretable as follows. After propagation through a distance $\Delta z/2$ in a homogeneous medium of refractive index n_0 , the beam enters a periodic lens wave-guide system with lenses separated by the distance Δz and with phases

$$\phi = -\Delta z \chi \quad (10)$$

imposed by the lenses. The homogeneous medium propagation between lenses is governed by the equation

$$2ik \frac{\partial E}{\partial z} = \nabla_{\perp}^2 E \quad (11)$$

FIG. 1 - The solution algorithm for wave equation is equivalent to periodic array of lenses. The incremental axial distance Δz is determined by the condition that further reduction in Δz does not affect solution. The individual lens properties are determined by fiber refractive index profile.

The combination of the phase and propagation operations (10) and (11) is an example of operator splitting. Equation (11) is most accurately solved by means of a finite Fourier series representation

$$E(x,y,z) = \sum_{m=-\frac{N}{2}+1}^{\frac{N}{2}} \sum_{n=-\frac{N}{2}+1}^{\frac{N}{2}} E_{mn}(\Delta z) e^{\frac{2\pi i(mx+ny)}{N}} \quad (12a)$$

with

$$E_{mn}(\Delta z) = E_{mn}(0) e^{\frac{i}{2k} \left(\frac{2\pi}{N}\right)^2 (m^2+n^2)\Delta z} \quad (12b)$$

The solution method embodied in Eqs. (12a) and (12b) is highly accurate, and, when implemented with the FFT algorithm, is efficient as well. If the solution and spectrum are negligible on the appropriate mesh boundaries, the computed solution $E(x,y)$ at the points of the computational grid is, practically speaking, free from truncation error. The only approximation in the calculation is thus the replacement of the optical fiber by the lens waveguide illustrated in Fig. 1. The expansion (12a) therefore has the character of a true modal expansion, although the basis functions are not the normal modes of the optical wave-guide.

We now wish to generalize the paraxial form of solution to Eq. (1) to one that includes the effects of second derivatives of the amplitude function $E(x,y,z)$. We wish at the same time to retain the finite Fourier series representation of the solution (12a) for reasons of numerical accuracy. This implies that some form of operator splitting will be required.

The solution to Eq. (1) may be written formally as

$$E(x,y,\Delta z) = \exp \left[\pm i\Delta z (\nabla_{\perp}^2 + \frac{\omega^2}{c^2} n^2)^{1/2} \right] E(x,y,0) \quad (13)$$

The square root in Eq. (13) may be written

$$(\nabla_{\perp}^2 + \frac{\omega^2}{c^2} n^2)^{1/2} = \frac{\nabla_{\perp}^2}{(\nabla_{\perp}^2 + \frac{\omega^2}{c^2} n^2)^{1/2} + \frac{\omega}{c} n} + \frac{\omega}{c} n \quad (14)$$

The phase and propagation operators can be split in Eq. (14) if n is replaced by n_0 in the first right hand member. Equation (14) then becomes

$$(\nabla_{\perp}^2 + \frac{\omega^2}{c^2} n^2)^{1/2} \approx \frac{\nabla_{\perp}^2}{(\nabla_{\perp}^2 + k^2)^{1/2} + k} + k + k \left(\frac{n}{n_0} - 1 \right) \quad (15)$$

The operation

$$\exp \left\{ i\Delta z \left[\frac{\nabla_{\perp}^2}{(\nabla_{\perp}^2 + k^2)^{1/2} + k} + k \right] \right\} E(x,y,0) \quad (16)$$

is equivalent to solving the wave equation

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2} n_0^2 E = 0 \quad (17)$$

with $E(x,y,0)$ as an initial condition, while discarding the backward moving wave solution.

Making use of the approximation (15) in Eq. (13) and expressing the result in symmetrized split operator form, one obtains the following generalization of Eq. (8)

$$E(x,y,\Delta z) = \exp \left\{ \frac{-i\Delta z}{2} \left[\frac{\nabla_{\perp}^2}{(\nabla_{\perp}^2 + k^2)^{1/2} + k} \right] \right\} e^{-i\Delta z \chi} \left\{ \frac{-i\Delta z}{2} \left[\frac{\nabla_{\perp}^2}{(\nabla_{\perp}^2 + k^2)^{1/2} + k} \right] \right\} E(x,y,0) \quad (18)$$

where

$$\chi = k \frac{n}{n_0} - 1 \quad (19)$$

Equation (18) may also be interpreted in terms of Fig. 1, but the beam propagation between lenses is governed by Eq. (17). Thus Eq. (12b) must be replaced by

$$E_{mn}(\Delta z) = E_{mn}(0) \exp \left\{ i\Delta z \left[\frac{\kappa_x^2 + \kappa_y^2}{(-\kappa_x^2 - \kappa_y^2 + k^2)^{1/2} + k} \right] \right\}, \quad (20)$$

where κ_x and κ_y are the transverse wave numbers

$$\kappa_x = \frac{2\pi x}{L}, \quad \kappa_y = \frac{2\pi y}{L}, \quad (21)$$

and L is the transverse period of the solution.

The algorithm represented by Eqs. (15) - (19) should be accurate for

$$\Delta/n_0 \ll 1, \quad (22)$$

where Δ is the maximum variation in refractive index between the core and cladding of the fiber. It also represents an improvement on the parabolic or paraxial wave equation (5), which gives vanishing dispersion for a beam propagating in a medium with a refractive index varying quadratically with radius.

COMPUTATION OF PULSE DISPERSION

The time variation of the electric field is obtained from the Fourier superposition

$$E(x,y,z,t) = \int_{-\infty}^{\infty} f(\omega) E(x,y,z,\omega) e^{i(\omega t - k z)} d\omega, \quad (23)$$

where $E(x,y,z,\omega)$ is the frequency response function of the fiber obtained by solving Eqs. (18) to (20) for frequency ω , and $f(\omega)$ is the Fourier transform of the input pulse waveform $f(t)$. The numerical evaluation of the integral in Eq. (23) requires the computation of $E(x,y,z,\omega_i)$ for a select set of frequencies ω_i , which for practical reasons must be few in number. Typical numbers range from 5 to 8.

The variance

$$\sigma^2 = \langle t^2 \rangle - \langle t \rangle^2 \quad (24)$$

computed for the pulse time history at an axial position z is a measure of the pulse broadening at that location. Ideally, one would like to compute the averages in Eq. (24) using the beam power

$$P(t) = \iint dx dy |E(x,y,z,t)|^2 \quad (25)$$

as a weighting function, but this procedure will require processing an impractically large quantity of data. As an alternative, one can determine σ^2 as a function of z by averaging over the on-axis intensity $I(t) = |E(0,0,z,t)|^2$. This result should differ little from the power weighted result, since it can be shown that the variance (24) averaged over intensity is quite insensitive to transverse position.

For evaluating the integral in Eq. (23) the functional form

$$\log E(z, \omega_i) = \sum_n a_n (\omega_i - \omega_0)^n \quad (26)$$

is used to fit the computed values of the on-axis response function

$E(z, \omega_i)$. (Transverse coordinates have been omitted for simplicity.)

Convergence of the integral (23) is guaranteed by the presence of the function $f(\omega)$. Obviously, only values of ω_i within the pass band of $f(\omega)$ need be included.

The scaling of $\sigma(z)$ to large distances is determined by assuming the form

$$\sigma^2(z) = a + bz^2, \quad (27)$$

where the quantity $b^{1/2}$ represents the pulse dispersion per unit propagation distance, the values of a and b are determined from a least-squares fit of the quadratic form (26) to calculated values of $\sigma^2(z)$. Expression (27), which represents a linear scaling of dispersion with distance for large z -values can be derived from a modal analysis. The form (27) can also be justified by the fact that it provides a very consistent fit to calculated values of $\sigma^2(z)$.

ACCURACY OF NUMERICALLY COMPUTED SECOND DERIVATIVES

OF PHASE WITH RESPECT TO FREQUENCY

Dispersion depends on the second as well as higher order derivatives of the phase of $E(x,y,z,\omega)$ with respect to ω . In many situations of interest it depends predominately on the second derivative. It is thus crucial that the numerical solution of a propagation equation give accurate second phase derivatives with respect to frequency, if it is to be relied upon for an accurate description of dispersive phenomena. In the present application, accuracy requirements are particularly stringent since one needs to scale the small changes in $\partial^2 \phi / \partial \omega^2$ that occur in propagation paths of the order of a few centimeters upward to propagation paths of the order of kilometers.

A numerical solution to Eq. (5) based on Eq. (8) for a Gaussian beam propagating in a medium with refractive index that depends quadratically on radius has been compared to the analytic solution that is available for these conditions. If $n(r)$ is assumed to have the form

$$\frac{n(r)}{n_0} = 1 - (\beta r)^2, \quad (28)$$

where $x^2 + y^2 = r^2$, then the on-axis electric field can be written

$$E = \frac{1}{\cos \beta z + i D \sin \beta z}, \quad (29a)$$

$$D = \frac{1}{k\sigma^2\beta}, \quad (29b)$$

where σ is the initial $1/e$ intensity radius of the Gaussian beam. From Eq. (29a)

$$\phi = \tan^{-1} (D \tan \beta z) \quad , \quad (30a)$$

$$\frac{\partial^2 \phi}{\partial \omega^2} = \frac{2(D/\omega^2) \tan z}{(1 + D^2 \tan^2 \beta z)^2} \quad (30b)$$

Expression (30b) as a function of z is compared with the numerically computed derivative in Fig. 2 for $\beta = 8.88 \text{ cm}^{-1}$, $\lambda = 1 \text{ } \mu\text{m}$, $\sigma = 30 \text{ } \mu\text{m}$, and $\Delta z = 25 \text{ } \mu\text{m}$. The phase derivatives have been divided by z . The field E and the phase were calculated numerically for a center frequency and two symmetrically placed frequencies differing from the center value by 10%. A parabola as a function of frequency fit to the three phase values for each z value used in the computation. Differentiation of the parabola provided the second phase derivative exhibited in Fig. 2.

The numerical and analytic results are seen to be in very close agreement. This leads to confidence that the numerical method embodied in Eqs. (8) and (12b) and its generalization, contained in Eqs. (18) and (20), are of sufficient accuracy to be applied to the dispersion problem. The final justification for the numerical method, however, will lie in the consistency of the results.

The above graphs represent second derivatives of phase with respect to frequency for wave propagating in infinite medium with parabolic index variation. Analytic solution is available for this case.

FIG. 2

COMPARISON OF PROPAGATION IN FINITE AND
INFINITE QUADRATIC REFRACTING MEDIA

In the paraxial approximation a Gaussian beam launched in an infinite quadratic refracting medium undergoes periodic changes in size as it focuses and defocuses but always retains its Gaussian shape. This behavior is modified in some important ways if the refractive index is quadratic only up to a radius $r = a$ and constant beyond, even if the Gaussian beam is initially well contained in the core region.

Figure 3 shows a sequence of beam intensity distributions obtained from a numerical solution of Eq. (18) for a beam that is uniform in phase and Gaussian in shape at $z = 0$. The medium refractive index is described by

$$n = n_1 \left[1 - \Delta \left(\frac{r}{a} \right)^2 \right] , \quad r \leq a \quad (31)$$

$$n = n_1 (1 - \Delta) , \quad r > a ,$$

with $a = 60 \mu\text{m}$, $n_1 = 1.5$, $\Delta = 3.2 \times 10^{-3}$, and $\sigma = 30 \mu\text{m}$ is the $1/e$ intensity radius of the initial Gaussian beam. Beyond the first focus the intensity distribution departs markedly from a Gaussian shape, exhibiting a ring structure.

In Figure 4 the on-axis phase corresponding to Figure 3 is displayed as a function of z . For comparison purposes the on-axis phase of the same

FIG. 3 - Gaussian beam intensity distribution does not remain Gaussian after first focus if beam propagates in a finite quadratic refractive medium such as an optical fiber.

beam launched in what is effectively an infinite quadratic refracting medium is also displayed. The "infinite" medium was created by extending the core radius in Eq. (31) from 2 to 5 beam radii. Since the numerical solution of Eq. 5 for this case is in excellent agreement with Eqs. (29) and (30), we shall refer to this case as the infinite medium case. The refractive index curvature at the origin is the same for both media. Hence one would expect beams in the two media to display the same periodic focusing pattern. Evidence of the same periodicity is apparent in both phase plots, but the finite quadratic medium phase contains a scalloped structure that is absent in the "infinite" medium case. This scalloped appearance of the phase along with the appearance of rings in the intensity pattern is due to the diffraction of the beam edges at the core cladding interface. In quantum mechanical terms one can think of the beam in the finite medium case as exciting energy levels close to the top of the potential well. These levels will be more closely spaced than levels at the bottom of the well. Consequently, a beating between the various energy states included in the wave packet should be evident in the amplitude and phase.

One further difference between the two curves in Figure 4 is that $\phi(z)/z$ for the "infinite" medium case shows a stronger downward trend than for the finite medium case. Since $\phi(z)$ for the infinite medium is a periodic function, $\phi(z)$ for a finite medium exhibits an upward trend with increasing z .

FIG. 4 - Comparison of phase for initially Gaussian beams propagating in infinite and finite quadratic refracting media.

NUMERICAL EXAMPLE OF DISPERSION CALCULATIONS

Figure 5 shows the results of a series of dispersion calculations based on the method outlined in the previous sections. The optical fiber used has the same set of parameters as were used for Figure 4.

We display results for the rms (1 σ) pulse spreading in psec/km as a function of the refractive index. Curve a corresponds to pure modal dispersion, while curve b also includes profile dispersion. That is, the shape of the index profile remains fixed, but Δ is a function of frequency. For these calculations, we have taken $\partial \ln \Delta / \partial \ln \omega = 0.1$. The input pulse was assumed to be temporally Gaussian with an initial 1/e width of 1 psec. The central frequency contained in the input pulse corresponds to a wavelength of 1 μ m. For comparison, we also show the corresponding results of Olshansky and Keck², which were derived from a WKB treatment assuming equal excitation of all trapped modes. Curve A should be compared with a, and B with b. Both theories lead to the same optimal exponent for minimum dispersion. However, there are qualitative differences. The present results indicate a greater sensitivity to deviations from the optimal value, at least near the optimum, and are generally somewhat higher. Because the WKB solution assumes greater excitation of higher order modes than is true for our Gaussian beam, and these higher order modes would be expected to lead to a higher dispersion, the differences between the two theories cannot be attributed to difference in modal excitation.

The log amplitude and phase curves for the various frequencies involved for $\alpha=2$ and no dispersion are shown in Fig. 6.

FIG. 5 - Comparison between dispersion calculation based on WKB method (Keck Olshansky) and detailed wave equation calculation. The two sets of curves represent cases with and without profile dispersion.

FIG. 6 - The calculation of dispersion requires solution of wave equation for a number of frequencies. Shown here are on-axis phase and log amplitude vs. propagation distance. Note how close the amplitudes lie for different frequencies.

CONCLUSIONS

In summary, we have implemented a method for the calculation of pulse dispersion in realistic graded fibers based on a direct numerical solution of the Helmholtz wave equation. The accuracy of the numerical solution was tested by comparing analytical and numerical results for the second derivative of phase with respect to frequency in a case for which an analytical solution exists. This is a sensitive test, and the excellent agreement found gives confidence in the numerical method. Modal, waveguide, material and profile dispersion may all be treated. In addition, further effects such as those due to frequency dependent absorption and bends in the fiber could be treated by this method.

The results obtained for a well contained coherent Gaussian pulse at 1 μm wavelength indicate a greater sensitivity to deviations from the optimal refractive index profile exponent α than had been expected.

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