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RELATIVISTIC PARTICLE BEAM IN A SEMI-INFINITE, AXIALLY SYMMETRIC CONDUCTING CHANNEL EXTENDING FROM A PERFECTLY CONDUCTING PLANE

V. K. NEIL AND R. K. COOPER

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Relativistic Particle Beam in a Semi-infinite, Axially Symmetric
Conducting Channel Extending from a Perfectly Conducting Plane*

V. K. Neil and R. K. Cooper†

Lawrence Livermore Laboratory
Livermore, CA 94550

I. INTRODUCTION

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In some experiments with field emission diodes a pulse of relativistic electrons emerges through a metal anode into a gas. The pulse creates an ionized, highly conducting channel. If the gas is contained within a conducting cylinder, the return currents consist of surface currents in the cylinder wall as well as plasma currents in the channel. If, on the other hand, the gas is contained within an insulating cylinder, no wall currents are present and any return current must flow in the channel.

The purpose of this work is to investigate in part the electromagnetic field configuration generated if there is no wall surrounding the channel.

An azimuthally symmetric channel of uniform, finite, conductivity σ and radius "a" is the model we treat in this work. In cylindrical coordinates, a perfectly conducting surface (sometimes referred to as the ground plane) of infinite extent is located at the plane $z = 0$. The conducting channel is centered on the z axis. It is in contact with the conducting surface at $z = 0$, and extends to infinity along the z axis. The region $r > a$ is occupied by free space.

A pulse of charged particles crosses the perfectly conducting surface and moves along the channel. The pulse of particles is azimuthally symmetric

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† Permanent address: Los Alamos Scientific Laboratory, Los Alamos, NM.

and coaxial with the channel. Although the results of this work are independent of the radial distribution of particles in the pulse, for convenience we take this radial distribution to be uniform out to the channel radius "a" (i.e., the pulse fills the channel radially). All particles in the pulse have the same axial velocity v . This velocity is held constant so that the effects of the electromagnetic fields on the particles' motion is neglected.

A complete description of the electromagnetic fields is beyond the scope of this work. We shall demonstrate that if Q is the total charge in the pulse, then the total charge that passes through any plane $z > 0$ is exactly equal to Q . That is, the charge carried by the beam and only this charge, is transported through the channel. We shall also demonstrate that as the pulse moves through the channel and the distance from the ground plane to the pulse approaches infinity, the electromagnetic field configuration approaches that which results from the steady state with the same pulse moving in a channel of infinite length in the absence of a ground plane.

II. FOURIER TRANSFORMS OF THE ELECTRIC AND MAGNETIC FIELD

The presence of the ground plane may be taken into account by the method of images. We take the charge density $\rho(r, z, t)$ and the axial current density $j(r, z, t)$ of the pulse only to be of the form

$$\rho(r, z, t) = \frac{Q}{\pi a^2 v} \left[f\left(t - \frac{z}{v}\right) \Theta(z) - f\left(t + \frac{z}{v}\right) \Theta(-z) \right] \Theta(a - r), \quad (1)$$

$$j(r, z, t) = \frac{Q}{\pi a^2} \left[f\left(t - \frac{z}{v}\right) \Theta(z) + f\left(t + \frac{z}{v}\right) \Theta(-z) \right] \Theta(a - r), \quad (2)$$

in which Θ is the unit step function. The function f is normalized such that

$$\int_{-\infty}^{\infty} f(\tau) d\tau = 1. \quad (3)$$

The head of the pulse emerges from the ground plane at time $t = 0$. For $t < 0$ no charge or current is present and all electromagnetic fields are zero.

We now take the Fourier transform in both t and z . We indicate a transformed quantity with two tildas. We have

$$\tilde{\tilde{\rho}}(k, \omega) = \frac{Q}{\pi a^2 v} \int_{-\infty}^{\infty} dt e^{i\omega t} \left[\int_0^{\infty} f(t - \frac{z}{v}) e^{-ikz} dz - \int_{-\infty}^0 f(t + \frac{z}{v}) e^{-ikz} dz \right]. \quad (4)$$

In the integral over z from zero to infinity, we set $t - (z/v) = \tau$, and in the integral over z from minus infinity to zero we let $t + (z/v) = \tau$. Introducing the quantity $\tilde{f}(\omega)$ by the definition (ω has a positive imaginary part.)

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(\tau) e^{i\omega\tau} d\tau, \quad (5)$$

we find

$$\tilde{\tilde{\rho}}(k, \omega) = \frac{2kQ}{\pi i v a^2} \frac{\tilde{f}(\omega)}{[k^2 - (\omega/v)^2]}. \quad (6)$$

Similarly we find

$$\tilde{\tilde{j}}(k, \omega) = \frac{2\omega Q}{\pi i v a^2} \frac{\tilde{f}(\omega)}{[k^2 - (\omega/v)^2]}. \quad (7)$$

Expressions for the transforms of the electromagnetic field inside and outside the channel may be derived by standard techniques, such as given in Refs. (1) and (2). The details of this calculation will not be given here. It suffices to point out that the transform $\tilde{\tilde{F}}(r, k, \omega)$ of any field component $F(r, z, t)$ may be expressed in the form

$$\tilde{\tilde{F}}(r, k, \omega) = \tilde{\tilde{G}}_F(r, k, \omega) \tilde{j}(k, \omega) . \quad (8)$$

The only field components arising from our current and charge distributions are E_r , E_z , and B_θ . We will be particularly interested in $\tilde{\tilde{B}}_\theta(a, k, \omega)$, the transform of B_θ at the edge of the channel. For this component one can derive the expression for $\tilde{\tilde{G}}_B(a, k, \omega)$; namely,

$$\tilde{\tilde{G}}_B(a, k, \omega) = \frac{4\pi}{c\alpha} \frac{I_1(\alpha a)}{[I_0(\alpha a) - HI_1(\alpha a)]} , \quad (9)$$

in which I_n is the n^{th} order modified Bessel function of the first kind and the quantity α is defined by

$$\alpha^2 = k^2 - (\omega/c)^2 - (4\pi i\omega\sigma/c^2) . \quad (10)$$

Gaussian units are employed in this work. The quantity H in Eq. (9) is given by

$$H = \frac{(4\pi\sigma - i\omega)}{i\omega} \frac{\lambda}{\alpha} \frac{K_0(\lambda a)}{K_1(\lambda a)} , \quad (11)$$

in which K_n is the n^{th} order modified Bessel function of the second kind, and

$$\lambda^2 = k^2 - (\omega/c)^2 . \quad (12)$$

III. THE STEADY STATE PROBLEM

We now consider a pulse of charge that is moving in an infinite conducting channel. That is, we are treating the same problem as in the previous section without the initial and boundary conditions introduced by the presence of the ground plane. We take the current distribution to be

$$j = \frac{Q}{2\pi a} f\left(t - \frac{z}{v}\right) \Theta(a - r) . \quad (13)$$

Proceeding as above we find the double Fourier transform to be

$$\tilde{j}(k, \omega) = \frac{2Q}{a} \tilde{f}(\omega) \delta\left(k - \frac{\omega}{v}\right) . \quad (14)$$

We now demonstrate that for this steady state problem the total charge that passes a position z is just equal to Q . The total charge Q_t that passes a position z is

$$Q_t = \int_{-\infty}^{\infty} dt \int (j + \sigma E) dA , \quad (15)$$

in which the second integral extends over the cross-sectional area of the channel. We evaluate Q_t by noting that

$$2\pi a B_{\theta}(a, z, t) = \frac{4\pi}{c} \int \left(j + \sigma E_z + \frac{1}{4\pi} \frac{\partial E_z}{\partial t} \right) dA . \quad (16)$$

The displacement current term will contribute nothing when integrated over time from $-\infty$ to ∞ , since there is no electric field in the channel long before or long after the pulse has passed the position z . Combining Eqs. (15) and (16) we have

$$Q_t = \frac{ca}{2} \int_{-\infty}^{\infty} B_{\theta}(a, z, t) dt . \quad (17)$$

We employ Eq. (8) with $\tilde{F}(r, k, \omega) = \tilde{B}_\theta(a, k, \omega)$ along with Eq. (14) for $\tilde{j}(k, \omega)$ to perform the Fourier inversion and obtain $B_\theta(a, z, t)$. We have

$$B_\theta(a, z, t) = \frac{Q}{2(\pi a)^2} \int_{-\infty}^{\infty} \tilde{f}(\omega) \tilde{G}_B(a, \omega/v, \omega) e^{-i\omega(t - \frac{z}{v})} d\omega, \quad (18)$$

in which we have employed the $\delta(k - \frac{\omega}{v})$ in Eq. (14) to perform the integration over k . We wish to integrate Eq. (18) over t from $-\infty$ to ∞ . We interchange integration order and employ the definition

$$\int_{-\infty}^{\infty} e^{-i\omega t} dt = 2\pi\delta(\omega). \quad (19)$$

The integration over ω becomes trivial, and the result is

$$\int_{-\infty}^{\infty} B_\theta(a, z, t) dt = \frac{Q}{\pi a^2} f(0) \tilde{G}_B(a, 0, 0). \quad (20)$$

From Eqs. (3) and (5), we have $f(0) = 1$. Examination of Eqs. (9), (10), and (11) in the limit $k = \omega/v$ and $\omega \rightarrow 0$ we have

$$\tilde{G}_B(a, 0, 0) = 2\pi a/c. \quad (21)$$

Employing Eqs. (20) and (21) in Eq. (17), we obtain the result $Q_t \equiv Q$.

IV. TOTAL CHARGE PASSING THROUGH THE CHANNEL

We employ the convolution theorem to demonstrate that, even in the presence of the ground plane, the total charge Q_t that passes through the channel is exactly equal to the pulse charge, Q . We may write

$$\tilde{B}_\theta(a, z, \omega) = \int_{-\infty}^{\infty} \tilde{j}(z - x, \omega) \tilde{G}_B(a, x, \omega) dx. \quad (22)$$

In this expression the quantities with one tilde are defined by

$$\tilde{j}(z, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{j}(k, \omega) e^{ikz} dk, \quad (23)$$

and the analogous relation for $\tilde{B}_\theta$ and $\tilde{G}_B$. Employing Eq. (7), we find

$$\tilde{j}(z, \omega) = \frac{Q\tilde{f}(\omega)}{\pi a^2} \begin{cases} e^{i\omega z/v}, & z > 0 \\ e^{-i\omega z/v}, & z < 0. \end{cases} \quad (24)$$

On the other hand, for the steady state problem treated in the previous section, we have for all z ,

$$\tilde{j}(z, \omega) = \frac{Q\tilde{f}(\omega)}{\pi a^2} e^{i\omega z/v}. \quad (25)$$

The functions given by Eqs. (24) and (25) are identical in the limit $\omega = 0$.

We further emphasize that the quantity $\tilde{G}_B(z, \omega)$ is the same for both physical situations.

We now calculate the total charge passing through the channel, as given by Eq. (17). Following the procedure of the previous section, we have

$$\begin{aligned} \int_{-\infty}^{\infty} B_\theta(a, z, t) dt &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \tilde{B}_\theta(a, z, \omega) e^{-i\omega t} d\omega \\ &= \tilde{B}_\theta(a, z, 0). \end{aligned} \quad (26)$$

From Eqs. (17), (22), and (26) we obtain

$$Q_t = \frac{ca}{2} \int_{-\infty}^{\infty} \tilde{j}(z - x, 0) \tilde{G}_B(a, x, 0) dx. \quad (27)$$

Since $\tilde{j}(z - x, 0)$ and $\tilde{G}_B(a, x, 0)$ are given by the same expressions with and without the ground plane, we have already established $Q_t = Q$ in the absence of the ground plane, it follows from Eq. (27) that $Q_t = Q$ in the presence of the ground plane.

V. FIELD CONFIGURATION IN THE LIMIT $t \rightarrow \infty$

We wish to examine expressions for any field component $F(r, z, t)$ in the limit $t \rightarrow \infty$ but $z - vt$ remains finite. We have

$$F(r, z, t) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dk e^{ikz} \int_{-\infty}^{\infty} \tilde{j}(k, \omega) \tilde{G}_F(r, k, \omega) e^{-i\omega t} d\omega. \quad (28)$$

We make the change of variable $\Omega = \omega - kv$ in Eq. (28) to obtain

$$F(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{ik(z - vt)} W(r, k, \Omega), \quad (29)$$

in which

$$W(r, k, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{j}(k, \Omega + kv) \tilde{G}_F(r, k, \Omega + kv) e^{-i\Omega t} d\Omega. \quad (30)$$

To examine the behavior as $t \rightarrow \infty$ we employ the final value theorem, which for our notation takes the form

$$\lim_{t \rightarrow \infty} W(r, k, t) = - \lim_{\Omega \rightarrow 0} \left[i\Omega \tilde{j}(k, \Omega + kv) \tilde{G}_F(r, k, \Omega + kv) \right]. \quad (31)$$

In taking the limit $\Omega \rightarrow 0$ we simply set $\Omega = 0$ in $\tilde{G}_F$, and employ Eq. (7) with $\omega = \Omega + kv$ to obtain

$$- \lim_{\Omega \rightarrow 0} \left[i\Omega \tilde{j}(k, \Omega + kv) \tilde{G}_F(r, k, \Omega + kv) \right] = \frac{Qv}{\pi a^2} \tilde{f}(kv) \tilde{G}_F(r, k, kv). \quad (32)$$

Employing Eqs. (31) and (32) in Eq. (29), we have

$$\lim_{t \rightarrow \infty} F(r, z, t) = \frac{Qv}{2(\pi a)^2} \int_{-\infty}^{\infty} \tilde{f}(kv) \tilde{G}_F(r, k, kv) e^{ik(z-vt)} dk. \quad (33)$$

Equation (18) (which is written explicitly for $F = B_\theta$ but is valid also for E_r and E_z) gives an expression for a field component in the steady state situation. By replacing kv with ω in Eq. (33) we see that in the limit $t \rightarrow \infty$, the expression for a field component F with the ground plane is identical to the expression for the same field component in the steady state (no ground plane) situation.

As a final comment, we note that with the boundary and initial conditions imposed by the presence of the ground plane, all field components are identically zero for $z > ct$. For finite time the solutions to the two problems discussed here are indeed different. For $z > ct$ the solution with the ground plane consists of the steady state solution plus an additional contribution that is equal and opposite. For $z < ct$, the additional contribution represents the fields from the image charge and current. But in the limit $t \rightarrow \infty$, the additional contribution vanishes.

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