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BIAS REMOVAL FOR CONVENTIONAL MAGNETOTELLURIC DATA

by

John A. Stodt

September 1983

Work performed under Contract Number: DE-AC07-80ID12079

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**Prepared for
U.S. Department of Energy
Division of Geothermal Energy**

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ABSTRACT

An algorithm has been developed for the removal of autopower bias errors from conventional magnetotelluric (MT) data. Nonlinear equations involving autopowers as unknowns with crosspower coefficients are obtained by cross substitutions between the eight referenced impedance equations to eliminate the four impedances. Although exact solutions to the nonlinear equations can be computed, they are often sensitive to noise components in the crosspower coefficients, due to degeneracy of the equations as Z_{xx} and Z_{yy} approach zero or as the coherence between the magnetic field components H_x and H_y approaches unity. However, the ordinary coherence functions between various pairs of the horizontal field components provide additional information. This information is incorporated as constraints by developing a constrained iterative solution for the autopowers. The solution is outlined and its usefulness is explored with examples of its application to field data.

INTRODUCTION

Magnetotelluric (MT) data are obtained as sets of simultaneous measurements of orthogonal electric and magnetic field measurements at a given site on the earth's surface. Usually, the data sets are Fourier transformed and complex transfer functions between the field components at the air-earth interface are calculated. When the assumptions concerning the plane-wave nature of the source fields are satisfied (e.g. Madden and Nelson, 1964; Swift, 1967) the signal components (subscript s) of the measured fields are related to each other in the frequency domain by the linear relations

$$E_{si} = Z_{ix}H_{sx} + Z_{iy}H_{sy}, \quad i = x \text{ or } y \quad (1)$$

and

$$H_{sz} = T_{zx}H_{sx} + T_{zy}H_{sy} \quad (2)$$

Equations (1) and (2) are referred to as the MT impedance and tipper relations and can be rewritten in the general form

$$O_{si} = G_{ix}I_{sx} + G_{iy}I_{sy}, \quad i = x, y, \text{ or } z. \quad (3)$$

Equation (3) describes a dual input, single output linear system. The input signals I_{sx} and I_{sy} are linearly related to the output signal O_{si} through the transfer functions G_{ix} and G_{iy} . The transfer functions depend only on frequency and the conductivity structure of the earth.

The transfer functions are usually estimated from measured data using a least squares approach (e.g. Sims et al, 1971), which leads to the estimates (subscript m)

$$G_{mix} = W_{mix}/V_m \quad (4)$$

and

$$G_{miy} = W_{miy}/V_m, \quad (5)$$

where

$$W_{mix} = \overline{O_i A^*} \overline{I_y B^*} - \overline{I_y A^*} \overline{O_i B^*},$$

$$W_{miy} = \overline{I_x A^*} \overline{O_i B^*} - \overline{O_i A^*} \overline{I_x B^*},$$

and

$$V_m = \overline{I_x A^*} \overline{I_y B^*} - \overline{I_y A^*} \overline{I_x B^*}.$$

Stable estimates are obtained for any conductivity structure when the reference fields (A,B) are chosen as one of the pairs (E_x, E_y) , (H_x, H_y) , (E_x, H_x) , or (E_y, H_y) , provided the input signals H_{sx} and H_{sy} are not perfectly coherent. The individual terms, $\overline{XY^*} = N^{-1} \sum_{k=1}^N X_k Y_k^*$, in equations (4) and (5) are auto and cross spectral estimates (e.g. Oppenheim and Schaffer, 1975) of the measured field components. They are computed as an average of crossproducts $X_k Y_k^*$ of different harmonics within narrow bandwidths of the Fourier transforms of a single data set, or of harmonics obtained from different data sets at the frequency of interest. The symbol * in these expressions indicates complex conjugation, and the bar represents an average over N different crossproducts.

Generally, the measured fields used to compute the estimates G_{mix} and G_{miy} are comprised of both signal and noise terms. By definition, signal components satisfy the plane wave impedance relationships defined by equations (1-3), while noise components do not. Estimates of the transfer functions computed from noise-free data using equations (4) and (5) are accurate provided they are computed from properly sampled sequences of data having sufficient length to suppress truncation errors (e.g. Sims and Bostick, 1969). In this paper, aliasing and truncation effects are neglected and attention is restricted to field measurements which are sums of signal and additive random noise components, e.g.,

$$I_x = I_{sx} + i_x ,$$

where lower case letters identify noise terms which are uncorrelated with the signal components. Recent analyses of the effects of such noise on the estimates G_{mix} and G_{miy} are presented by Gamble et al (1979b), Pederson (1982), and Stodt (1983a,b,c).

The presence of additive random noises in the field measurements introduces random and bias errors into the estimates G_{mix} and G_{miy} . The exact effect is difficult to predict in the general case when both signal and noise components are nonstationary. One expects a reduction of random errors as the number of data sets averaged with similar noise to signal ratios increases. However, bias errors, introduced because of the nonlinearities of equations (4) and (5), are not necessarily reduced by averaging and present a serious problem.

The primary source of bias errors in G_{mix} and G_{miy} is the bias error in the spectral estimates introduced by noise components in the measured fields. It can be shown that

$$E[\overline{XY^*}] = \overline{X_S Y_S^*} + E[\overline{xy^*}] , \quad (6)$$

where $E[\cdot]$ denotes statistical expectation with respect to the conditional distribution of the noise terms (Stodt, 1983b). Persistent bias errors are introduced when the noise components x and y are correlated so that $E[\overline{xy^*}]$ is non-zero. In particular, autpowers of noisy field measurements are always biased because the noise is self-correlated.

The mean squared error (MSE) of the estimates G_{mij} measures the effects of both random and bias errors and is defined by

$$\text{MSE } G_m = E[|G_m - G|^2] = \text{Var } G_m + |\text{Bias } G_m|^2 ,$$

where

$$\text{Bias } G_m = E[G_m] - G$$

and

$$\text{Var } G_m = E[|G_m|^2] - |E[G_m]|^2 .$$

Bias errors caused by autpowers of noisy field measurements can be severe and will dominate the MSE of conventional estimates when a sufficient number of data sets are averaged to reduce the variances to negligible levels. In order to reduce bias errors, Goubau et al (1978) suggest using two additional field measurements, obtained simultaneously with the base field measurements but at a separate site, as the references (A,B) in equations (4) and (5). Gamble et al (1979a) demonstrate the effectiveness of this approach when the base and reference sites are separated far enough that the base field noises are uncorrelated with the reference field noises. However, additional equipment and a more complicated field procedure are required to obtain

remote reference measurements. If one is to avoid additional complications and expense, procedures for reducing bias errors in conventional estimates must be developed.

Attempts to remove autopower bias from conventional estimates have not been as successful as the remote reference technique. The first attempts to control random and bias errors simply involved identifying and rejecting noisy data sets, using predicted coherencies (e.g. Swift, 1967) and a suitable cutoff criteria. Unfortunately, it is often difficult to obtain a sufficient amount of data which satisfies a suitably strict rejection criteria over the entire frequency range of interest. It is sometimes possible to average the four stable estimators to reduce the effects of noise (Sims et al, 1971). However, the success of this approach is unpredictable because both the random and bias errors in each of the estimators are, in general, different. Kao and Rankin (1977) presented a cyclic averaging technique to reduce the autopower bias errors. However, Hernandez and Jacobs (1979) have questioned the validity of the procedure when the noise to signal ratios vary between channels.

Boehl et al (1977) noted that phase data were often biased significantly less than the corresponding amplitude data. They developed a technique to predict amplitude data from smoothed phase data using a Hilbert transform relation. The transfer functions were assumed to be minimum phase.

Goubau et al (1978) reported several attempts either to calculate and remove autopower bias errors or to develop unbiased estimators for conventional magnetotelluric data. They reported limited success in removing bias errors, but often at the expense of introducing

unacceptably large random errors.

Jupp (1978) presented a generalized least squares estimation procedure which accounts for noises in all components of the measured fields, but the theory requires the variance of the noises in all channels to be uniform. Unfortunately, Lienert et al (1980) report that in their experience the procedure is sensitive to this assumption. Thus, the effectiveness of the procedure is questionable when the noise is nonstationary or varies between channels.

Finally, Lienert et al (1980) attempted to estimate the noise to signal powers in the horizontal field components using relationships between the signal to noise power ratios and the multiple coherence functions derived by White (1973). However White's expressions were derived by assuming the signal components of all channels to be perfectly coherent. This assumption is appropriate for the seismic model White considered, but it does not apply to the magnetotelluric model.

This paper examines the conditions under which bias errors can be reduced in estimates calculated using local references. An algorithm is presented which reduces or removes bias errors under suitable conditions, which are specified. The usefulness of the algorithm is explored with examples of field data processed with and without the algorithm.

THEORY

Derivation of equations

We focus attention on the two impedance relations defined by equation (1). Multiplying them through in turn by E_x^* , E_y^* , H_x^* , and H_y^* and averaging a number of data sets generates two sets of referenced equations. Using matrix notation, we can write them as

$$\begin{bmatrix} \underline{A} \\ \underline{B} \end{bmatrix} \begin{bmatrix} \underline{Z} \end{bmatrix} = \begin{bmatrix} \underline{C} \\ \underline{A}^{*T} \end{bmatrix} \quad (7)$$

where $\underline{Z}$ is the 2 x 2 impedance tensor,

$$\begin{aligned} \underline{A} &= \begin{bmatrix} b^* & c^* \\ d^* & e^* \end{bmatrix}, \\ \underline{B} &= \begin{bmatrix} P_{H_x} & f^* \\ f & P_{H_y} \end{bmatrix}, \\ \underline{C} &= \begin{bmatrix} P_{E_x} & a^* \\ a & P_{E_y} \end{bmatrix}, \end{aligned}$$

and $\underline{A}^{*T}$ is the transposed conjugate of $\underline{A}$. The quantities P_{E_x} , P_{E_y} , P_{H_x} , and P_{H_y} represent autopowers of the indicated fields, while the lower case symbols $a = \overline{E_x E_y^*}$, $b = \overline{E_x H_x^*}$, $c = \overline{E_x H_y^*}$, $d = \overline{E_y H_x^*}$, $e = \overline{E_y H_y^*}$, and $f = \overline{H_x H_y^*}$ represent the specified crosspowers. With the

partitioning indicated in equation (7), we obtain the relations

$$\underline{A} \underline{Z} = \underline{C} \quad (8)$$

and

$$\underline{B} \underline{Z} = \underline{A}^{*T} \quad (9)$$

Solving for $\underline{Z}$ in equation (8) and substituting the result into equation (9) eliminates the impedances, giving

$$\underline{B} \underline{A}^{-1} \underline{C} = \underline{A}^{*T} \quad (10)$$

Equating the elements of the matrices and rearranging terms in a straightforward manner, we obtain

$$d^* e f^* P_{E_x} + a c^* e P_{H_x} - |e|^2 P_{E_x} P_{H_x} = a b^* e f^* - b e (\det \underline{A}), \quad (11)$$

$$d e^* f P_{E_x} + a b^* d P_{H_y} - |d|^2 P_{E_x} P_{H_y} = a c^* d f + c d (\det \underline{A}), \quad (12)$$

$$b^* c f^* P_{E_y} + a^* c e^* P_{H_x} - |c|^2 P_{E_y} P_{H_x} = a^* c d^* f^* + c d (\det \underline{A}), \quad (13)$$

and

$$b c^* f P_{E_y} + a^* b d^* P_{H_y} - |b|^2 P_{E_y} P_{H_y} = a^* b e^* f - b e (\det \underline{A}), \quad (14)$$

where $\det \underline{A}$ is the determinant of the matrix $\underline{A}$. The autopowers in equations (11) through (14) can be treated as unknowns to be solved for in terms of coefficients containing only crosspowers.

Other equations can be generated by partitioning the matrix equation (7) differently, by different cross substitution in equations (8) and (9), or by starting with the admittance formulation of the MT

relations. The different approaches produce the following additional equations:

$$de^*f P_{E_x} + bc^*f P_{E_y} - |f|^2 P_{E_x} P_{E_y} = a^*be^*f + af[c^*d - a^*f^*], \quad (15)$$

$$ac^*e P_{H_x} + ab^*d P_{H_y} - |a|^2 P_{H_x} P_{H_y} = ab^*ef^* + af[c^*d - a^*f^*], \quad (16)$$

$$|f|^2 P_{E_y} + |e|^2 P_{H_x} + |d|^2 P_{H_y} - P_{E_y} P_{H_x} P_{H_y} = 2\text{Re}(de^*f), \quad (17)$$

$$|f|^2 P_{E_x} + |c|^2 P_{H_x} + |b|^2 P_{H_y} - P_{E_x} P_{H_x} P_{H_y} = 2\text{Re}(bc^*f), \quad (18)$$

$$|a|^2 P_{H_y} + |e|^2 P_{E_x} + |c|^2 P_{E_y} - P_{H_y} P_{E_x} P_{E_y} = 2\text{Re}(ac^*e), \quad (19)$$

and

$$|a|^2 P_{H_x} + |d|^2 P_{E_x} + |b|^2 P_{E_y} - P_{H_x} P_{E_x} P_{E_y} = 2\text{Re}(ab^*d). \quad (20)$$

It is important to note that equations (15) through (20) were obtained merely by rearrangement. They provide no additional information for the solution of the autopowers. However, they do provide alternatives to equations (11) through (14) when seeking a solution for the autopowers.

The signal autopower estimates $P_{E_{sx}}$, $P_{E_{sy}}$, $P_{H_{sx}}$, and $P_{H_{sy}}$ satisfy equations (11) through (20) provided the calculated crosspowers are the crosspower estimates of the signal components of the measured fields. However, multiple solutions to the equations exist because of their nonlinearity and the physically meaningful solution must be identified. In addition, the solution will be corrupted if noise components of the measured fields correlate between channels so that the

crosspowers are biased, or if the random errors in the crosspowers have not been suppressed by sufficient averaging.

Direct solutions for the autopowers

Equations (11) through (16) can each be separated into real and imaginary parts, generating two equations having the forms

$$MX + NY - OXY = P \quad (21)$$

and

$$mX + nY = p, \quad (22)$$

where X and Y represent a pair of autopowers, M , N , O , and P represent the real parts of the coefficients, and m , n , and p represent the corresponding imaginary parts. When the coefficients are noise free, one of the intersections of the hyperbola defined by equation (21) and the line defined by equation (22) gives the signal autopowers. It is straightforward to solve equation (22) for X and substitute the result into equation (21) to generate a quadratic equation in Y . Two solutions for Y are computed from the quadratic formula and the corresponding solutions for X are obtained by back substitution.

Goubau et al (1978) used this approach with equation (16) to calculate $P_{H_{sx}}$ and $P_{H_{sy}}$. The corresponding electric field autopowers were then computed from equations (17) and (18) using both pairs of calculated magnetic field autopowers, and criteria were established to select the physically meaningful set of solutions. Theoretically, solutions can be obtained by this approach provided the diagonal impedance elements Z_{xx} and Z_{yy} are nonzero.

Unfortunately, the method is not stable enough to produce reliable

estimates when significant noise is present. Gamble et al (1979a) provide an example of the application of the technique with field data. The random errors introduced by the method are unacceptably large. The reason for the instability becomes apparent upon examination of the behavior of the coefficients of equations (21) and (22). It can be shown that the signal components of the imaginary coefficients m , n , and p approach zero as Z_{xx} and Z_{yy} approach zero or as the coherence between H_{sx} and H_{sy} approaches unity. In these cases, even small amounts of noise in the field measurements can produce noise components in m , n , and p which overshadow their signal components. Consequently, the solutions to equations (21) and (22) will be seriously corrupted.

Instability of the direct solutions

The instability of the direct solutions as the coherence between H_{sx} and H_{sy} approaches unity is expected; the impedance calculations themselves become indeterminate in this case. However, the instability when noise is present and Z_{xx} and Z_{yy} are small must be overcome. We require additional information. It appears, upon casual examination, that the required additional information is available in equations (11) through (14). Unfortunately, these equations are not independent. If E_x is eliminated from equations (11) and (12) by cross substitution, equation (16) is generated. Similarly, equation (16) is also generated when E_y is eliminated from equations (13) and (14). Further evidence of the dependence of equations (11) through (16) is obtained when they each are used individually to solve for the autopowers. Exactly the same pair of solutions for $P_{E_{sx}}$, $P_{E_{sy}}$, $P_{H_{sx}}$, and $P_{H_{sy}}$ are obtained regardless of the noise levels in the crosspower coefficients.

Further degeneracy occurs when Z_{xx} and Z_{yy} are zero and in addition the coherence between H_{sx} and H_{sy} is zero, indicating the magnetic fields are unpolarized. In this case, the expected values of the crosspowers a , b , e , and f are zero and equations (11), (14), (15), and (16) degenerate. In addition, equations (12), (18), and (19) reduce to

$$P_{E_x} P_{H_y} = |c|^2, \quad (23)$$

while equations (13), (17), and (20) reduce to

$$P_{E_y} P_{H_x} = |d|^2. \quad (24)$$

Obviously, there is insufficient information available in equations (23) and (24) to solve for the four autopowers.

In the presence of noise, the point at which solutions for the autopowers become seriously corrupted is a complicated function of the noise levels in the crosspower coefficients, the coherence between H_{sx} and H_{sy} , and the relative magnitudes of Z_{xx} and Z_{yy} compared to the magnitudes of Z_{xy} and Z_{yx} . Our goal is to develop a solution for the autopowers which is robust in the sense that the instability of the direct solutions is controlled as the equations degenerate.

Fortunately, the ordinary coherence functions defined for various pairs of the field measurements E_x , E_y , H_x , and H_y provide additional information which can be used to constrain solutions for the autopowers, using an iterative approach.

Constraints

Before an iterative solution is attempted, it is useful to constrain the region over which a solution is sought as tightly as possible. The signal autopowers satisfy the inequalities

$$P_{X_s} \geq 0$$

and

$$P_X - P_{X_s} \geq 0, \quad (25)$$

where X represents a measured field component, E_x , E_y , H_x , or H_y . Equation (25) provides the sharpest upper bound available. However, sharper lower bounds are provided by the ordinary and multiple coherence functions associated with the dual input, single output linear systems used to model the MT process (e.g. Stodt, 1983a).

The ordinary coherence estimate between two measured fields X and Y is given by

$$C_{XY}^2 = |\overline{XY^*}|^2 / P_X P_Y \quad (26)$$

and satisfies the inequality,

$$0 \leq C_{XY}^2 \leq 1. \quad (27)$$

If X and Y are any pair of the field measurements E_x , E_y , H_x , or H_y , six ordinary coherence functions are obtained. From the upper bound of equation (27) we obtain the constraint

$$P_X P_Y \geq |\overline{XY^*}|^2.$$

Since the expected value of $\overline{XY^*}$ is $\overline{X_s Y_s^*}$ when the noise components of X and Y are uncorrelated, we define lower bounds

$$P_{X_s} P_{Y_s} - |\overline{X_s Y_s^*}|^2 \geq 0, \quad (28)$$

which the calculated signal autopowers are required to satisfy. The region bounded by equations (25) and (28) is shown in Figure 1. The

sharpest of the constraints given by equation (28) are usually obtained when (X,Y) are (E_x, H_y) and (E_y, H_x) since these field pairs are typically the most strongly correlated. In fact, when Z_{xx} and Z_{yy} are zero, the signal components of these field pairs are perfectly correlated and two equality constraints are obtained.

The multiple coherence function defined by (e.g. Bendat and Piersol, 1971)

$$c_{I_x I_y - O_i}^2 = \frac{|\overline{I_y O_i^*}|^2 P_{I_x} + |\overline{I_x O_i^*}|^2 P_{I_y} - 2 \operatorname{Re}[\overline{I_x I_y^*} \overline{I_x^* O_i} \overline{I_y O_i^*}]}{P_{O_i} [P_{I_x} P_{I_y} - |\overline{I_x I_y^*}|^2]} \quad (29)$$

gives an estimate of the correlation of two measured inputs I_x and I_y to the output O_i of a dual input, single output linear system. Two multiple coherence functions are defined for the MT impedance formulation, where I_x and I_y are chosen as H_x and H_y . Two additional multiple coherence functions are defined for the MT admittance formulation, where I_x and I_y are chosen as E_x and E_y . The multiple coherence functions satisfy the inequality

$$0 \leq c_{I_x I_y - O_i}^2 \leq 1.$$

The upper bound provides the constraints,

$$\begin{aligned} P_{I_x} P_{I_y} P_{O_i} - |\overline{I_x I_y^*}|^2 P_{O_i} - |\overline{I_y O_i^*}|^2 P_{I_x} - |\overline{I_x O_i^*}|^2 P_{I_y} \\ + 2 \operatorname{Re}[\overline{I_x I_y^*} \overline{I_x^* O_i} \overline{I_y O_i^*}] \geq 0. \end{aligned} \quad (30)$$

If the field measurements I_x , I_y , and O_i are noise free, $c_{I_x I_y - O_i}^2$ is unity and equation (30) becomes an equality constraint. Inspection

shows that the four equations generated in this manner are identical to equations (17) through (20).

Constrained iterative solution for the autopowers

The constraints described in the previous section can be incorporated mathematically in an iterative solution for the autopowers to control instability. Various approaches to the solution are possible, depending upon the equations chosen for the solution, the iterative technique employed, and how the constraints are incorporated. Appropriate choices can affect greatly the efficiency of the solution. The approach described below has proven useful with field data.

Equations (17) through (20) are solved iteratively for the autopowers. These equations can be generated by appropriate cross substitutions in equations (11) through (16); no information is lost by restricting our attention to them. Equations (17) through (20) can each be rewritten in the form

$$f_m(P_{sj}, P_{sk}, P_{sl}) = 0, m = 1, 2, 3, \text{ or } 4 \quad (31)$$

where P_{sj} , P_{sk} , P_{sl} are the appropriate signal autopowers. Simultaneous solutions to the four equations are absolute minima of the objective function

$$g = \sum_{m=1}^4 W_m^2 e_m^2, \quad (32)$$

where the e_m are the residuals,

$$e_m = f_m(P_j, P_k, P_l), \quad (33)$$

obtained when P_j , P_k , P_l are not solutions to equation (31). The

quantity W_m is a positive weighting factor chosen to insure that each residual contributes appropriately to the objective function g . Useful weighting is achieved when W_m is chosen as the measured autopower P_i , $i \neq j, k$, or l .

A constrained iterative gradient method is employed to search for minima of the objective function g . Successive iterates are obtained using the iteration formula

$$\underline{P}_{i+1} = \underline{P}_i - t_i \underline{R}_i \underline{q}_i, \quad (34)$$

where $\underline{P}_i$ is a column vector of the parameters $P_{E_x}, P_{E_y}, P_{H_x}$, and P_{H_y} at the i th iteration. The column vector $\underline{q}_i$ is the gradient of the objective function g . It is given by

$$\underline{q}_i = \underline{J}_i^T \underline{e}_i, \quad (35)$$

where $\underline{e}_i$ is the column vector of residuals and the matrix $\underline{J}_i$ is the matrix of derivatives,

$$\underline{J}_i = [j_{mn}] = [W_m \frac{\partial e_m}{\partial p_n}] ; m, n = 1, 2, 3, 4. \quad (36)$$

Both $\underline{J}_i$ and $\underline{e}_i$ are evaluated at the point $\underline{P}_i$ in parameter space. The choice of the positive scaling factor t_i and the operational matrix $\underline{R}_i$ depend on the particular gradient method used. For example, if $\underline{R}_i$ is the identity matrix iterations proceed in the direction of the negative of the gradient of the objective function g , and equation (34) represents the method of steepest descent. If $\underline{R}_i$ is the inverse of the Hessian matrix $\underline{H}_i$ (matrix of second derivatives) of g and t_i is unity, equation (34) represents Newton's method. Newton's method finds the minimum of the quadratic function

$$f(\underline{v}) = \underline{g}_i^T \underline{v} + \frac{1}{2} \underline{v}^T \underline{R}_i \underline{v}, \quad (37)$$

where $\underline{v}$ is the parameter change vector $\underline{v} = \underline{P}_{i+1} - \underline{P}_i$ and $\underline{R}_i$ is a positive definite matrix. Thus, if the objective function g is quadratic and $\underline{R}_i$ is chosen as the inverse of the Hessian matrix $\underline{H}_i$, the minimum is obtained in a single iteration. When g is not quadratic, a solution is not obtained in a single step, and progress toward a solution depends on how well the function $g'(\underline{v}) = g(\underline{P}_i) + f(\underline{v})$ approximates g in the vicinity of $\underline{P}_i$. The scale factor t_i is used to control the magnitude of $\underline{v}$ at each iteration. If the gradient $\underline{g}_i$ is nonzero and $\underline{R}_i$ is a positive definite matrix, a factor t_i always exists such that $g(\underline{P}_{i+1}) < g(\underline{P}_i)$, and progress toward a minimum of the objective function is made (see e.g. Bard, 1974, p. 86).

The particular choice of the gradient method used affects the efficiency of the solution. The Gauss approximation to Newton's method with Marquardt's modification (see e.g. Bard, 1974, pp. 88-106) has provided good results. With the Gauss approximation, the matrix $\underline{R}_i$ is chosen as

$$\underline{R}_i = [\underline{J}_i^T \underline{J}_i]^{-1}. \quad (38)$$

The matrix $\underline{J}_i^T \underline{J}_i$ is always positive definite and is a first order approximation to the Hessian matrix. In practice it is useful to scale the parameters at each iteration so that the individual elements in each column of the matrix $\underline{J}_i$ are weighted by the reciprocal of the magnitude of the column vector. This scaling, which is assumed in subsequent discussions, reduces the diagonal elements of the matrix $\underline{J}_i^T \underline{J}_i$ to unity.

Marquardt's modification provides an additional means of

controlling both the magnitude and direction of the parameter change vector $\underline{v}$, by adding a constant term to the diagonal elements of $\underline{J}_i^T \underline{J}_i$. The modified parameter change vector becomes

$$\underline{v} = -t_i [\underline{J}_i^T \underline{J}_i + r_i \underline{I}]^{-1} \underline{q}_i, \quad (39)$$

where r_i is the Marquardt factor and $\underline{I}$ is the identity matrix. Successive iterations using equation (39) are invariant to the scaling of the parameters when the matrix $\underline{J}_i$ is normalized as discussed in the previous paragraph. When r_i is small, the parameter change vector $\underline{v}$ approximates a Newton step. Alternatively, when r_i is large, $\underline{v}$ is approximately

$$\underline{v} = \frac{-t_i}{r_i} \underline{q}_i$$

and represents a step in the steepest descent direction. Thus, as r_i varies from small to large values, $\underline{v}$ is rotated from the Newton step direction towards the steepest descent direction and is shortened in length. Proper selection of r_i and t_i at each iteration is important to obtain an efficient solution. The choice of the Marquardt factor r_i is discussed subsequently. The scale factor t_i is adjusted at each iteration using an interpolation-extrapolation technique discussed by Bard (1974, p. 111) (see Appendix A).

The incorporation of constraints requires additional care. It is usual practice to terminate iterations when the percentage change $[g(\underline{p}_i) - g(\underline{p}_i + 1)]/g(\underline{p}_i)$ is small, or the magnitude of $\underline{v}$ is small compared to the magnitude of $\underline{p}_i$. When a parameter step $\underline{v}$ violates a constraint boundary, the simplest approach is to adjust the scale factor t_i so that the point $\underline{p}_i + 1$ lies on the boundary of the feasible

region. On the next iteration however, it is possible for the parameter step $\underline{v}$ to point out of the feasible region, even though the objective function g could be reduced by proceeding along the boundary. In this case, iterations will terminate prematurely unless special provision is made to modify the direction of $\underline{v}$ as well as its magnitude. Bard (1974, p. 146-153) presents a projection algorithm for use with linear constraints which provides an appropriate modification of $\underline{v}$ in this situation. The technique is summarized in Appendix B.

The available constraint information is given by equations (25), (28), and (30). If we work with log parameters, the constraints represented by equations (25) and (28) are both linear, i.e.

$$\log P_X - \log P_{X_S} \geq 0 \quad (40)$$

and

$$\log P_{X_S} + \log P_{Y_S} - \log |\overline{XY^*}|^2 \geq 0. \quad (41)$$

The computations presented in this paper were obtained by reparameterizing to log parameters and minimizing the objective function g subject to the constraint equations (40) and (41), using Bard's linear constraint algorithm. Note that although the constraints represented by equation (30) are not forced to be satisfied at each iteration, they are treated implicitly as equality constraints because the objective function g is synthesized from them.

It is interesting to examine the form of the matrix $\underline{J}_1$. Assigning indices 1, 2, 3, 4 to the field components E_x , E_y , H_x , H_y and accounting for the change to log parameters, equation (36) becomes

$$\underline{J}_1 = [W_m P_n \frac{\partial e_m}{\partial P_n}], \quad m, n = 1, 2, 3, 4 \quad (42)$$

where

$$\frac{\partial e_m}{\partial p_m} = 0$$

and

$$\frac{\partial e_m}{\partial p_n} = p_r p_s - |\overline{rs^*}|^2 \quad r, s \neq m, n.$$

Note that the constraint equation (41) provides a bound on the elements of $\underline{J}_i$ in addition to providing bounds for the parameters $\underline{P}_i$.

The iteration strategy used to generate the examples in this paper is summarized in the following steps. It is assumed the logarithmic parameters are rescaled at each iteration and the equations from which the objective function g is formed are each weighted by the appropriate measured autopower, as discussed previously.

1. Use the measured autopower as the initial guess; set the Marquardt factor r_1 to 0.1; initialize the matrix of active constraints (see Appendix B)
2. Compute the objective function $g(\underline{P}_i)$ and the Jacobian $\underline{J}_i$ for the i th iteration.
3. If $r_i > 100$, go to step 14.
4. Compute the parameter change vector $\underline{v}_i$ from equation (39) with $t_i = 1$.
5. If $g(\underline{P}_i + \underline{v}_i) > g(\underline{P}_i)$; multiply r_i by 10 and go to step 3.
6. If $\underline{P}_i + \underline{v}_i$ violates any constraints which were active at $\underline{P}_i$, compute the projection of $\underline{v}_i$ on the active constraints which are violated (see Appendix B).
7. Adjust the parameter t_i to achieve a further reduction of g , if possible (see Appendix A).

8. If $\underline{p}_i + \underline{v}_i$ violates any constraints which are inactive at $\underline{p}_i$, adjust t_i so that $\underline{p}_i + \underline{v}_i$ lies on those constraints; if $g(\underline{p}_i + \underline{v}_i) > g(\underline{p}_i)$, increase r_i by 10 and go to step 2.
9. If no constraints which are inactive at $\underline{p}_i$ become active at $\underline{p}_i + \underline{v}_i$, set $r_{i+1} = \max(.1r_i, 10^{-7})$.
10. If the maximum parameter change is less than .01 percent, go to step 14.
11. Update the identification of active and inactive constraints (see Appendix B).
12. If no constraints are active at $\underline{p}_i + \underline{v}_i$, set $r_{i+1} = \max(.1r_i, 10^{-7})$.
13. Go to step 2 for the next iteration.
14. Terminate the iterations and save the result.

EXAMPLES OF BIAS REMOVAL

In this section, examples of the removal of bias from conventional MT data are presented. The data were collected in southwestern Utah by personnel from the University of Texas at Austin in the summer of 1977. The data were recorded in three bands. The low frequency band (< 0.5 Hz) and the mid band (0.5 Hz - 5 Hz) were obtained digitally with 12 bit resolution and analyzed using the decimation technique discussed by Wight et al (1977). The data above 5 Hz were obtained one frequency at a time, using a phase sensitive heterodyne detection system, also described by Wight et al.

The figures to be presented share the following features. The plotting symbols U and D represent quantities obtained from impedances calculated from equations (4) and (5) using different reference field pairs. The symbol U corresponds to quantities calculated using E_x and E_y as the reference pair. Impedances obtained using these references are biased high when E_x or E_y are noisy. The symbol D corresponds to quantities calculated using H_x and H_y as the reference pair. Impedances obtained using these references are usually biased low when H_x or H_y are noisy. The symbol X has been used to plot all quantities computed from impedances obtained using autopowers calculated with the bias removal algorithm. Ideally, these quantities are unbiased. The tipper estimate computed using E-fields as references is also plotted with an X since it contains no autopowers and will not be biased by

noise correlations in the crosspowers if E-field noises are uncorrelated with H-field noises. Plots of phase data are presented in the range 0° to 180° to retain graphical resolution. The absolute value of phase data in the range 0° to -180° has been plotted and is identified with a + sign at the center of the plotting symbol.

Figures (2) and (3) present apparent resistivities computed from the off-diagonal impedances obtained in the measuring directions for two stations identified as U-12 and U-34 respectively. The bias errors in the estimates from both sites are evident from the systematic separation of the two different estimators plotted in each figure. From the behavior of the upward and downward biased estimators, it appears the magnetic field measurements are noisier than the electric field measurements at both sites, and that the measurements at station U-12 are noisier than those at station U-34. Figures (4) and (5) present the phase data corresponding to the amplitude data in Figures (2) and (3). In each case, the phase data appears less affected by autopower bias errors than the amplitude data, as noted by Boehl et al (1977).

The different noise contents in the data from stations U-12 and U-34 allow us to explore the usefulness of the bias removal algorithm. The exact solutions to equations (11) through (16) are listed in Tables 1 and 2 for data from stations U-12 and U-34 respectively. The solution which satisfies the constraints most closely is listed first. Solutions which satisfy all the constraints are identified by a ">", while the entry "....." identifies complex solutions. In these examples, an exact solution satisfies all the constraints only about 20 percent of the time, and it is not uncommon

to observe complex solutions, i.e., equations (21) and (22) do not intersect. Examination of both solutions in Tables 1 and 2 shows that with few exceptions it is straightforward to choose the "most reasonable" solution. Theoretically, both solutions could satisfy all the constraints to complicate identification of the correct solution, but this did not occur with the data from stations U-12 and U-34.

The constrained iterative solutions for stations U-12 and U-34 are listed in Table 3. They were obtained using the iteration strategy described in the previous section. Ideally, the constrained iterative solution should be identical to the exact solution in those cases where the exact solution satisfies all the constraints. Comparison of the appropriate solutions in Tables 1, 2, and 3 shows that the worst disagreement in these cases is approximately 10 percent and that usually the disagreement is less than 2 percent. These differences arise because the convergence criteria were met before the absolute minimum of the objective function g was reached. The iterative solutions can be improved through "fine tuning", i.e., restarting the iterations using a different Marquardt factor or a different convergence criterion. Such fine tuning has not been done in order to give an idea of the performance of the algorithm with a minimum of operator intervention.

Figures 6 and 7 present apparent resistivities for stations U-12 and U-34 which were computed using the constrained iterative solutions for the autopowers presented in Table 3. Both estimates are plotted with an X in these figures; in general they very nearly superimpose. Making an intuitive judgement based on the smoothness of the resulting curves and the indications in Figures 2 and 3 that the magnetic field

measurements contained more noise than the electric field measurements, the bias removal is deemed successful in both cases, although the higher noise contents of the data from station U-12 has produced more scatter in those estimates than is present in the estimates from station U-34. The estimates in Figures 6 and 7 should be compared to the logarithmic average of the upward and downward biased estimates plotted in Figures 2 and 3, which is commonly used. Figure 8 presents the phase data for station U-12 corresponding to the amplitude data of Figure 6. The phase data is essentially unchanged, lending support to the notion that the original phase data in Figure 3 was essentially unbiased. The phase data for station U-34 which corresponds to the amplitude data of Figure 7 is very nearly a repeat of Figure 5 and has not been plotted.

Figures 9 and 10 are plots of tipper magnitudes from stations U-12 and U-34 respectively. The upper plot in each figure presents tipper estimates computed without bias removal. The symbol X indicates references (E_x, E_y) were used; these estimates contain no autopowers and the crosspowers from which they are computed are unbiased if electric field measurement noise is uncorrelated with magnetic field measurement noise. The symbol D indicates references (H_x, H_y) were used; equations (4) and (5) show these estimates are biased downward by bias errors in the magnetic field autopowers. Such bias errors are evident in the top plot of both Figures 9 and 10. Assuming the tippers calculated with the references (E_x, E_y) are unbiased, we can check the effectiveness of bias removal by comparing these estimates to estimates obtained by substituting the calculated magnetic field autopowers from Table 3 for the measured autopowers in

the downward biased estimates. This comparison is made in the lower plot of Figures 9 and 10, where the symbol D now represents the tipper estimate computed using references (H_x, H_y), but with the calculated autopowers from Table 3 substituted for the measured autopowers. The general agreement between the two estimates in the lower plot of Figure 10 is reasonable evidence that autopower bias errors are being removed at station U-34. From examination of Figure 9, this conclusion appears warranted at station U-12 as well, although the apparently larger random errors exhibited by the tipper estimates at this site make the conclusion more tenuous.

Finally, an example is presented where bias removal has failed. Figure 11 presents apparent resistivities computed from off-diagonal impedances obtained in the measuring directions for a station identified as U-6. Figure 12 presents the corresponding apparent resistivities obtained at station U-6 using autopowers calculated with the constrained iterative algorithm. Examination of the original upward and downward biased estimates in Figure 11 indicates that the magnetic fields are noisier than the electric fields, yet the "bias removed" estimates in Figure 12 tend to be an average of the upward and downward biased estimates and are obviously incorrect in the mid band between 0.1 and 10 Hz. The on-diagonal impedances were small at this station, as was the coherency between H_x and H_y in general. This example appears to be a case where equations (17) through (20) have degenerated to equations (23) and (24); there is not enough information available to solve unambiguously for the signal autopowers. Note however, that the method fails gracefully, giving approximately the logarithmic average of the upward and downward

biased estimates. This tendency is due to the form of equations (23) and (24) and the fact that the iteration strategy used is invariant to the scale of the parameters. In the degenerate case, the algorithm tends to assign equal noise to signal ratios to the autopowers as a minimum of the objective function g is sought within the constraints.

CONCLUSIONS

A constrained iterative algorithm has been presented which solves for the signal autopowers from a nonlinear set of equations generated from conventional MT data. It is assumed fundamentally that the noise components of the measured fields are uncorrelated so that the crosspowers are unbiased.

The examples presented demonstrate the range of effectiveness of the algorithm. Examination of Table 3 and Figures 3 and 6 show that noise to signal ratios as high as 1.5 have been dealt with effectively at station U-12. Theoretically, even higher noise to signal ratios could be handled, but in practice sufficient averaging is necessary to reduce the noise in the crosspower estimates to useful levels. Limited experience so far indicates that spectral estimates comprised of an average of 50 to 100 harmonics are usually sufficient to handle noise to signal ratios up to roughly 1.

The algorithm is stable in the sense that it extracts useful information from the nonlinear equations when it is available and defaults in a predictable fashion when the equations degenerate. When a solution to the nonlinear equations lies within the constrained region, the iterative algorithm generally finds it. When noise in the crosspower coefficients causes solutions to the nonlinear equations to lie outside the constraints, the constrained minimum of g often provides a good estimate of the signal autopowers. In cases of

degeneracy where the constrained minimum of g is nonunique, the algorithm tends to a solution which assigns approximately equal noise to signal ratios to the measured autopowers.

The constrained solutions for the autopowers generally produce estimates of apparent resistivity which lie between the upward and downward biased estimators computed with the measured autopowers. By plotting the upward-biased, downward-biased, and "unbiased" estimates together, the geophysicist can usually assess readily the success of the bias removal based on the behavior of the curves. Plotting the upward-biased and downward-biased estimates separately gives greater insight into the noise contents of the data than does plotting an average. This practice is recommended even if bias removal is not performed.

The algorithm for bias removal presented in this paper is not expected to work well if the measured fields contain significant correlated noise components. The only recourse in this situation is to obtain remote reference measurements, separating the reference site far enough that noise components at the base and remote sites are uncorrelated.

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REFERENCES

- Bard, Y., 1974, Nonlinear parameter estimation: New York, N.Y., Academic Press, Inc., 341 p.
- Bendat, J. S., and Piersol, A. G., 1971, Random data: analysis and measurement procedures: New York, John Wiley & Sons, Inc., 407 p.
- Boehl, J. E., Bostick, F. X., Jr., and Smith, H. W., 1977, An application of the Hilbert transform to the magnetotelluric method: tech. rep., Electr. Geophys. Res. Lab., Univ. of Texas, Austin.
- Gamble, T. D., Goubau, W. M., and Clarke, J., 1979a, Magnetotellurics with a remote reference: Geophysics, v. 44, p. 53-68.
- , 1979b, Error analysis for remote reference magnetotellurics: Geophysics, v. 44, p. 959-968.
- Goubau, W. M., Gamble, T. D., and Clarke, J., Magnetotelluric data analysis: Removal of bias: Geophysics, v. 43, p. 1157-1166.
- Hernandez, W., and Jacobs, J., 1979, On: "Enhancement of signal-to-noise ratio in magnetotelluric data" (Dominic W. Kao and David Rankin): Geophysics, v. 44, p. 1594-1597.
- Jupp, D. L. B., 1978, Estimation of the magnetotelluric impedance functions: Phys. Earth Planet. Inter., v. 17, p. 75-82.
- Kao, D. W., and Rankin, D., 1977, Enhancement of signal to noise ratio in magnetotelluric data: Geophysics, v. 42, p. 103-110.
- Lienert, B. R., Whitcomb, J. H., Phillips, R. J., Reddy, I. K., and Taylor, R. A., 1980, Long term variations in magnetotelluric apparent resistivities observed near the San Andreas fault in northern California: J. Geomag. Geoelectr., v. 32, p. 757-775.
- Madden, T. R., and Nelson, P. H., 1964, A defense of Cagniard's magnetotelluric method: ONR report, project NR-371-401, Geophysics Lab., MIT.
- Oppenheim, A. V., and Schaffer, R. W., 1975, Digital signal processing: Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 585 p.

- Pedersen, L. B., 1982, The magnetotelluric impedance tensor - its random and bias errors: Geophysical Prospecting, v. 30, p. 188-210.
- Sims, W. E., and Bostick, F. X., Jr., 1969, Methods of magnetotelluric analysis: tech. rep. no. 58, Electr. Geophys. Res. Lab., Univ. of Texas, Austin.
- Sims, W. E., Bostick, F. X., Jr., and Smith, H. W., 1971, The estimation of magnetotelluric impedance tensor elements from measured data: Geophysics, v. 36, p. 938-942.
- Stodt, J. A., 1983, Weighted least squares estimates of the magnetotelluric transfer functions from nonstationary data: in Ph.D. thesis, University of Utah (submitted to Geophysics).
- _____, 1983, Generalized error analysis for conventional and remote reference magnetotellurics: in Ph.D. thesis, University of Utah (submitted to Geophysics).
- _____, 1983, Probability distributions for magnetotellurics: in Ph.D. Thesis, University of Utah (submitted to Geophysics).
- Swift, C. M., Jr., 1967, A magnetotelluric investigation of an electric conductivity anomaly in the Southwestern United States: Ph.D. Thesis, MIT.
- White, R. E., 1973, The estimation of signal spectra and related quantities by means of the multiple coherence function: Geophysical Prospecting, v. 21, p. 660-703.
- Wight, D. F., Bostick, F. X., Jr., and Smith, H. W., 1977, Real time transformation of magnetotelluric data: tech. rep., Electr. Geophys. Res. Lab., Univ. of Texas, Austin.

LIST OF TABLES

- Table 1. Exact autopower solutions to equations (11) through (16) for station U-12. The solution which most nearly satisfies the constraints is listed first. A ">" indicates a solution satisfied all the constraints. The entries ". . ." indicate the solutions are complex. The solutions have been normalized by the measured autopowers.
- Table 2. Exact autopower solutions to equations (11) through (16) for station U-34. The solution which most nearly satisfies the constraints is listed first. A ">" indicates a solution satisfied all the constraints. The entries ". . ." indicate the solutions are complex. The solutions have been normalized by the measured autopowers.
- Table 3. Constrained iterative autopower solutions to equations (17) through (20) for stations U-12 and U-34. The solutions have been normalized by the measured autopowers.

TABLE 1.

FREQUENCY	EXEX	EYEX	HXHX	HYHY	EXEX	EYEX	HXHX	HYHY
2. 930E-03	.961	.725	1.002	.399	.966	.945	.997	-3.918
3. 906E-03	.886	.936	1.030	1.658	1.179	1.542	.835	.365
5. 859E-03	.983	1.023	.983	.921	1.091	.756	.672	.698
7. 813E-03	.981	1.036	.971	.840	1.044	.614	.843	.727
1. 172E-02 >	.982	1.000	.991	.982	1.126	.760	.384	.656
1. 563E-02	.995	1.009	.985	.946	1.380	.686	1.962	.562
2. 344E-02 >	.983	.999	.985	.974	8.081	.260	4.674	.050
3. 125E-02	.998	1.002	.994	.974	-.926	-1.337	-6.405	-.086
4. 688E-02	.997	1.001	.996	.970	2.429	.585	2.133	.140
6. 250E-02 >	.992	.995	.980	.641	-1.061	1.539	.560	.007
9. 375E-02 >	.900	.996	.777	.427	-2.266	-6.271	-.276	-.081
1. 250E-01	.631	1.164	.660	.644	-4.052	.307	2.037	-.009
1. 875E-01	.965	1.027	.752	.386	4.182	.698	1.106	.089
2. 500E-01	.762	1.021	.806	.524	3.761	.727	1.127	.122
3. 750E-01	1.115	.956	.899	.406	2.594	.763	1.131	.160
4. 688E-01	1.111	.956	.940	.449	.303	1.252	.687	-2.909
5. 000E-01	.801	1.024	.867	.682	4.543	.848	1.039	.212
6. 250E-01	.917	1.055	.864	.545	-.089	1.781	.427	-.420
9. 375E-01	.845	1.013	.905	.833	.635	1.067	.854	1.717
1. 250E+00	1.272	.921	.999	.398	-.579	1.631	.489	-.213
1. 875E+01	1.135	.907	.969	.492	-1.570	-3.709	-.637	-.139
2. 500E+00 >	.884	.907	.897	.620	6.780	.455	1.844	.051
3. 750E+00 >	.668	.891	.765	.902	-5.167	.389	1.651	-.024
5. 000E+00	17.27	.351	1.317	.025	-.020	.748	.602	-.557
1. 000E+01	.834	.809	1.129	.953	1.818	-6.230	-.031	.411
1. 700E+01	.954	1.041	.927	.964	1.160	1.438	.683	.787
2. 600E+01 >	.965	.989	.988	.993	1.110	-.891	-.706	.861
4. 000E+01	...	...	...	...	...	...	...	...
7. 600E+01	.939	.976	1.014	.994	.894	1.031	.960	1.044
1. 000E+02	.936	.979	1.008	1.036	.909	1.059	.932	1.067
1. 560E+02	.927	.897	1.103	1.051	.458	.998	.986	3.086
2. 300E+02	.918	.909	1.070	1.017	1.456	1.032	.933	.710
5. 300E+02	.7373	1.025	.895	1.220	1.333	1.055	.869	.662

TABLE 2.

FREQUENCY	EXEX	EYEX	HXHX	HYHY	EXEX	EYEX	HXHX	HYHY
2. 930E-03	.968	.962	1.026	.991	.704	.679	1.382	1.445
3. 906E-03	...	...	...	...	...	...	...	...
5. 859E-03	.952	.992	1.045	1.056	-4.242	.444	1.957	.033
7. 813E-03	1.004	1.043	.962	.978	.499	.423	1.995	2.303
1. 172E-02	1.002	1.020	.976	.974	1.038	1.233	.834	.958
1. 563E-02	...	...	...	...	...	...	...	...
2. 344E-02	...	...	...	...	...	...	...	...
3. 125E-02	.953	.985	1.013	1.050	1.043	1.015	.985	.941
4. 688E-02	.993	.997	.999	1.002	-2.287	-18.63	-.072	-.403
6. 250E-02	1.003	1.002	.993	.994	2.019	1.372	.689	.586
9. 375E-02 >	.963	.958	.966	.968	.409	-1.063	-.476	2.065
1. 250E-01 >	.883	.978	.932	.815	.016	.261	3.261	-45.54
1. 875E-01 >	.863	.971	.949	.800	-.315	.486	1.789	-1.223
2. 500E-01	.971	1.050	.914	.688	-1.753	.688	1.356	-.164
3. 750E-01 >	.959	.990	.969	.725	11.82	.763	1.251	.112
4. 688E-01	1.021	.994	.977	.895	12.79	.673	1.432	.107
5. 000E-01	1.212	.969	.988	.563	2.778	.838	1.143	.242
6. 250E-01	1.078	.973	.997	.868	-2.216	.711	1.348	-.217
9. 375E-01 >	.939	.991	.988	.985	2.653	.826	1.187	.338
1. 250E+00	.757	1.024	.961	1.261	1.582	.941	1.047	.574
1. 875E+01	1.070	.978	1.005	.869	-.692	1.327	.735	-.947
2. 500E+00	1.055	.976	1.004	.879	-85.76	.553	1.765	-.004
3. 750E+00	1.243	.926	1.052	.716	3.402	.600	1.641	.244
5. 000E+00	1.142	.884	1.079	.735	2.986	-.077	-6.607	.263
1. 000E+01	.755	.739	1.213	1.188	7.388	-8.570	-.096	.116
1. 700E+01	...	...	...	...	...	...	...	...
2. 600E+01 >	.966	.979	.999	.978	6.841	-1.494	-.646	.136
4. 000E+01	2.781	.801	1.233	.343	.202	1.101	.897	4.766
7. 600E+01	...	...	...	...	...	...	...	...
1. 000E+02	.865	1.018	.971	1.135	.643	.464	2.142	1.526
1. 560E+02	...	...	...	...	...	...	...	...
2. 300E+02	.951	.946	1.049	1.023	.278	1.074	.922	4.411
5. 300E+02	.714	1.004	.950	1.384	1.185	1.378	.703	.796

TABLE 3.

FREQUENCY	STATION U-12				STATION U-34			
	EXEX	EYEX	HXHX	HYHY	EXEX	EYEX	HXHX	HYHY
2. 930E-03	.966	1.000	.997	.981	.989	.994	1.000	.968
3. 906E-03	.943	1.000	.996	.919	.995	.996	1.000	.999
5. 859E-03	.977	1.000	.991	1.000	.990	.998	.998	.995
7. 813E-03	.963	1.000	.821	1.000	.989	1.000	.992	.995
1. 172E-02	.982	1.000	.939	1.000	.998	1.000	.995	.999
1. 563E-02	.988	1.000	.989	.981	.999	.998	.998	.999
2. 344E-02	.983	.999	.985	.974	.999	.997	1.000	.998
3. 125E-02	.992	1.000	.985	.974	.996	1.000	.999	.995
4. 688E-02	.994	1.000	.996	1.000	.995	1.000	.996	1.000
6. 250E-02	.991	.995	.980	.642	1.000	1.000	.996	.997
9. 375E-02	.888	.999	.776	.436	.964	.961	.962	.967
1. 250E-01	.936	1.000	.764	.364	.878	.973	.936	.820
1. 875E-01	1.000	1.000	.772	.368	.861	.970	.950	.802
2. 500E-01	.821	1.000	.824	.465	.873	1.000	.954	.780
3. 750E-01	.939	1.000	.856	.521	.951	.991	.968	.732
4. 688E-01	.898	1.000	.889	.528	1.000	.998	.973	.914
5. 000E-01	1.000	.981	.913	.522	1.000	.993	.964	.705
6. 250E-01	1.000	1.000	.903	.494	1.000	.985	.985	.928
9. 375E-01	.904	1.000	.918	.734	.960	.988	.992	.961
1. 250E+00	1.000	.961	.954	.627	.965	.998	.987	.962
1. 875E+01	1.000	.929	.945	.603	1.000	.986	.997	.938
2. 500E+00	.825	.918	.885	.713	1.000	.981	.998	.932
3. 750E+00	.693	.889	.767	.832	.965	.973	1.000	.954
5. 000E+00	.691	.696	.667	.556	.876	.952	1.000	1.000
1. 000E+01	.914	.912	1.000	.858	.921	.896	1.000	.967
1. 700E+01	.921	1.000	.963	1.000	.972	1.000	.966	1.000
2. 600E+01	.966	.989	.988	.992	.971	.981	.997	.973
4. 000E+01	.898	1.000	.988	.992	.973	.987	1.000	.980
7. 600E+01	.935	.990	.979	1.000	.993	.994	.995	.993
1. 000E+02	.970	.993	.994	1.000	.991	.994	.994	.992
1. 560E+02	.982	.987	.997	.993	.995	.998	.998	.995
2. 300E+02	.959	.985	.979	.975	.981	.991	1.000	.992
5. 300E+02	.929	.967	.951	.912	.942	1.000	.957	1.000

LIST OF FIGURES

1. Graphical representation of the region over which a solution for the autopowers is sought. The region is bounded from above by the constraints $P_X - P_{X_S} \geq 0$ and $P_Y - P_{Y_S} \geq 0$. The doubly hatched area indicates the region bounded from below by the signal autopowers. The combination of singly and doubly hatched areas is bounded from below by constraints obtained from the ordinary coherence functions and represents the region over which an iterative solution is sought. Two examples of lower bounds are illustrated.
2. Apparent resistivities computed from off-diagonal impedances obtained in the measuring directions at station U-12. The symbol "U" indicates the impedances were computed with reference pair (E_x, E_y) ; the apparent resistivities are biased high by noises in E_x and E_y . The symbol "D" indicates the impedances were computed with reference pair (H_x, H_y) ; the apparent resistivities are biased down by noises in H_x and H_y .
3. Apparent resistivities computed from off-diagonal impedances obtained in the measuring directions at station U-34. Plotting conventions are the same as for Figure 2.

4. Phase data for station U-12 corresponding to the apparent resistivity data of Figure 2. The data has been plotted in the range 0° to 180° . The absolute value of the phase data in the range 0° to -180° has been plotted and is identified by a "+" sign at the center of the plotting symbol. The symbols "U" and "D" have the same meaning as in Figure 2.
5. Phase data for station U-34 corresponding to the apparent resistivity data of Figure 3. The plotting conventions are the same as for Figure 4.
6. Apparent resistivities computed from off-diagonal impedances obtained in the measuring directions at station U-12, using the constrained iterative solutions for the autopowers given in Table 3. The same estimators used to compute the data of Figure 2 were used. Both estimators are plotted with the symbol "X"; in general they superpose.
7. Apparent resistivities computed from off-diagonal impedances obtained in the measuring directions at station U-34, using the constrained iterative solutions for the autopowers given in Table 3. Plotting conventions are the same as for Figure 6.
8. Phase data from station U-12 corresponding to the apparent resistivity data of Figure 6. The symbol "X" has the same meaning as for Figure 6. Other plotting conventions are the same as for Figure 4.
9. Tipper magnitude, $T = (|T_{zx}|^2 + |T_{zy}|^2)^{1/2}$, from station U-12. The symbol "X" represents the estimate computed using the

reference pair (E_x, E_y) , which is unbiased by noise correlations in the crosspowers if noises in E_x and E_y are uncorrelated with noises in the magnetic field measurements. In the upper plot, the symbol "D" represents the estimate computed using the reference pair (H_x, H_y) , which is biased down by bias errors in the magnetic field autopowers. In the lower plot, the symbol "D" represents the estimate computed using the reference pair (H_x, H_y) , but using the constrained iterative solutions for the magnetic field autopowers given in Table 3.

10. Tipper magnitude, $T = (|T_{zx}|^2 + |T_{zy}|^2)^{1/2}$, from station U-34. Plotting conventions are the same as for Figure 9.
11. Apparent resistivities computed from off-diagonal impedances obtained in the measuring directions at station U-6. Plotting conventions are the same as for Figures 2 and 3.
12. Apparent resistivities computed from off-diagonal impedances obtained in the measuring directions at station U-6, using constrained iterative solutions for the autopowers. Plotting conventions are the same as for Figures 6 and 7. The apparent failure of the bias removal may be due to degeneracy of the nonlinear equations from which the iterative solution is sought, as discussed in the text.

Figure 1.

Figure 2.

Figure 3.

Figure 4.

Figure 5.

Figure 6.

Figure 7.

Figure 8.

Figure 9.

Figure 10.

Figure 11.

Figure 12.

APPENDIX A
INTERPOLATION - EXTRAPOLATION TO OPTIMIZE t_i

Our goal is to choose a scale factor t_i so that the inequality

$$h_i(t_i) \equiv g(\underline{p}_i - t_i \underline{R}_i \underline{q}_i) < g(\underline{p}_i)$$

is made as large as possible, i.e., the objective function is reduced as much as possible at the i th iteration by a step in the direction of $-\underline{R}_i \underline{q}_i$. One approach (e.g. Bard, 1974 p. 111) attempts to improve an initial choice, t_i' , by finding the stationary point of a quadratic function $a + bt_i + ct_i^2$ whose values match $h_i(t_i)$ at $t_i = 0$ and $t_i = t_i'$, and whose slope is that of $h_i(t_i)$ at $t_i = 0$, i.e.

$$a = h_i(0) = g(\underline{p}_i),$$

$$b = \left. \frac{dh_i}{dt_i} \right|_{t_i = 0} = -\underline{q}_i^T \underline{R}_i \underline{q}_i,$$

and

$$a + bt_i' + ct_i'^2 = h_i(t_i').$$

The last equation can obviously be solved for c in terms of known quantities. The quadratic $a + bt_i + ct_i^2$ has a stationary point at

$$t_i'' = -b/2c, \tag{A-1}$$

and t_i' is replaced by t_i'' if $h(t_i'') < h(t_i')$.

The choice of the initial value t_1^i and the interpolation-extrapolation strategy used in the algorithm presented in this paper are summarized in the following steps:

1. Set $t_1^i = t_1'' = 1$; set $\max t_1'' = 2t_1^i$; set $H = g(\underline{p}_1 + \underline{v}_1^i)$ where $\underline{v}_1^i = t_1^i R_1 q_1$.
2. If $H \leq g(\underline{p}_1)$, compute t_1'' from equation (A-1); if $t_1'' < 0$, set $t_1'' = 2t_1^i$.
3. Set $t_1'' = \min(t_1'', \max t_1'')$.
4. Compute $\underline{v}_1'' = -t_1'' R_1 q_1$.
5. If $H > g(\underline{p}_1)$, set $H = g(\underline{p}_1 + \underline{v}_1'')$; if $H > g(\underline{p}_1)$, compute t_1'' from equation (A-1) and go to step 4.
6. If $g(\underline{p}_1 + \underline{v}_1'') > H$; use $\underline{v}_1^i$ instead of $\underline{v}_1''$ as the parameter step.

APPENDIX B

INCORPORATION OF LINEAR CONSTRAINTS

Bard (1974 p. 146) provides a useful algorithm for the incorporation of linear constraints and its application in the algorithm for bias removal is summarized in this appendix.

We wish to find the minimum of the objective function $g(\underline{p})$ given by equation (20), within the feasible region bounded by equations (25) and (28) as shown in Figure 1. If we work with log parameters, the feasible region equivalent to that of Figure 1 is bounded by the linear constraints given by equations (40) and (41). Linear constraints can be written in the form

$$\underline{a}_j^T \underline{p} \geq b_j, \quad j = 1, 2, \dots, \quad (\text{B-1})$$

where we choose the sign convention so the constraints evaluate positive when $\underline{p}$ is within the feasible region. The j th constraint is defined active at the i th iteration of $\underline{a}_{j-1}^T \underline{p}_i = b_j$, and we define a matrix $\underline{A}_i$ at the i th iteration whose columns consist of the vectors $\underline{a}_j$ of the coefficients of the active constraints, i.e.,

$$\underline{A}_i^T \underline{p}_i = \underline{b}. \quad (\text{B-2})$$

From equations (B-1) and (B-2) we deduce that any feasible parameter step, $\underline{v} = \underline{p}_{i+1} - \underline{p}_i$, must satisfy the condition $\underline{A}_i^T \underline{v} \geq 0$.

In practice, the objective function $g(\underline{p})$ is minimized by solving

successively a sequence of linearized problems. At each iteration we replace $g(\underline{P})$ by the approximation

$$g'(\underline{v}) = g(\underline{P}_i) + \underline{v}^T \underline{q}_i + \frac{1}{2} \underline{v}^T \underline{R}_i^{-1} \underline{v} ,$$

where $\underline{R}_i$ is adjusted so that $g(\underline{P}_i + 1) < g(\underline{P}_i)$. The normal parameter step

$$\underline{v} = -\underline{R}_i \underline{q}_i ,$$

which is computed ignoring all constraints, minimizes $g'(\underline{v})$.

However, the minimum of $g'(\underline{v})$ may or may not lie in the feasible region.

One of two approaches is taken depending on whether $\underline{P}_i$ is in the interior of the feasible region or lies on the boundary. If $\underline{P}_i$ lies in the interior of the feasible region, we calculate and apply the normal parameter step. If the normal step violates any constraints it is truncated so that $\underline{P}_i + 1$ lies on the boundary of the feasible region. In the situation where $\underline{P}_i$ lies on one or more constraint boundaries the normal step, or some function of it, is taken if the step points into the feasible region. Otherwise, the direction of the normal step is modified by treating the active constraints which are violated as equality constraints and projecting the normal step along these boundaries. In this case, we must determine which of the active constraints at $\underline{P}_i$ to project the normal step onto. By definition, minimizing the quadratic function $g'(\underline{v})$ subject to the active constraints at $\underline{P}_i$ is a quadratic programming problem.

Suppose $\underline{v}_i$ is the solution to the quadratic programming problem at the i th iteration. If $\underline{v}_i$ is a constrained minimum, then according

to the Kuhn-Tucker conditions (e.g. Bard, 1974 p. 52) the gradient of $g'(\underline{v}_i)$ can be written as a linear combination of the gradients of the active constraints at $\underline{P}_i + \underline{1} = \underline{P}_i + \underline{v}_i$, i.e.,

$$\underline{q}_i + \underline{R}_i \underline{v}_i = \underline{A}_B \underline{l}_B, \quad (B-3)$$

where $\underline{A}_B$ is a matrix of constraints which are active at both $\underline{P}_i$ and $\underline{P}_i + \underline{1}$, and $\underline{l}_B$ is a vector of Lagrange multipliers. Bard (1974, p. 146) provides an efficient algorithm to determine the matrix $\underline{A}_B$ and the vector $\underline{l}_B$. The reader is referred to his work for details of implementation. The modified normal parameter step is obtained from equation (B-3) as

$$\underline{v}_i = \underline{R}_i^{-1} (\underline{A}_B \underline{l}_B - \underline{q}_i).$$

The actual parameter step $\underline{v}_i' = t_i \underline{v}_i$ is computed by interpolation-extrapolation as described in Appendix A, with the additional requirement that $\underline{P}_i + t_i \underline{v}_i$ must satisfy all the constraints which were inactive at $\underline{P}_i$. Writing these constraints as

$$\underline{c}_{j-1}^T \underline{P}_i + \underline{1} - b_j \geq 0, \quad (B-4)$$

we must have

$$\underline{c}_{j-1}^T \underline{P}_i + t_i \underline{c}_{j-1}^T \underline{v}_i - b_j \geq 0.$$

Constraints for which $\underline{c}_{j-1}^T \underline{v}_i < 0$ are violated. If J is the set of indices identifying these constraints, we require

$$t_i \leq t_{\max} = \min [(b_j - \underline{c}_{j-1}^T \underline{P}_i) / \underline{c}_{j-1}^T \underline{v}_i]$$

to ensure that $\underline{c}_{j-1}^T \underline{v}_i' \geq 0$. If t_i is chosen so that one or more of the

constraints given by equation (B-4) become active, the associated column vectors $\underline{c}_j^T$ are appended to the matrix $\underline{A}_B$ to form the matrix of active constraints $\underline{A}_{i+1}$ for the next iteration.