AN IMPACT PARAMETER STUDY OF ELASTIC HADRON-PROTON
SCATTERING AT 50 AND 175 GeV/c

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An Impact Parameter Study of Elastic Hadron-Proton
Scattering at 50 and 175 GeV/c

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In hadron-proton collisions at high energies, the impact profile function measures the range and strength of the strong interaction. It determines the effective size and shape of the hadrons involved, for purposes of both elastic scattering and particle production, but depends solely upon the behavior in t of the elastic differential cross-section. This paper presents such an analysis of the reactions πp , $K p$, and $p p$ at incident momenta from 50 to 175 GeV/c, using the elastic scattering data recently taken at the National Accelerator Laboratory by the Fermilab Single Arm Spectrometer Group[1].

Two derivations of the profile function are given: the classic optical model and the partial wave treatment. The first stresses the 'grey disc' interpretations of hadrons; both allow the decomposition of the profile into the elastic and inelastic overlap functions, which represent the probabilities of each of the two processes. A more rigorous procedure yielding essentially the same result involves the imposition of unitarity on the relativistic S-matrix and then requiring angular momentum conservation[2,3].

A computer program transforms quadratic exponential fits to the cross-section data to obtain the amplitude in b -space, rather than transforming the data points themselves. This method certainly biases the data, most noticeably at small t , but the fits do represent the data quite well, and smooth any erroneous fluctuations. Cross-section measurements extend to

$t = -.8(\text{GeV}/c)^2$; this calculation assumes pure exponential behavior thereafter, and accounts for the possibility of a sharp decrease in its slope at larger t . The real part of the scattering amplitude is neglected.

The results display good qualitative agreement with the shadow-diffraction scattering model, and with previous analyses[3,4a,4b]. They indicate distinct differences between meson-baryon and baryon-baryon interactions, and imply that dependence of the cross-section on energy derives primarily from a net change in peripheral processes. A test of the quark model and the Lipkin sum rules[5] shows substantial deviation from theoretical expectations.

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The Optical Model

The pronounced wave nature of elementary particles involved in high energy collisions suggests the use of an optical analogy. Reflection, refraction, and absorption-diffraction, the three processes, occurring in the scattering of light[6], correspond quantum mechanically to an infinite repulsive potential, a finite real potential, and an imaginary potential, respectively. The attractive nature of hadron-hadron interactions rules out reflection, and recent experimental results[7] indicate that the real part of the potential at these energies is small, at least at small t (this point is discussed below). Hence the almost entirely imaginary hadronic potential in some sense either 'absorbs' or 'diffracts' incident hadrons.

Since the wavelength (0.01 fermi) of the incoming particles of momentum about 100 GeV/c is short compared to the region of interaction (1 fermi), and because the source-scatterer, scatterer-detector distances are large compared with this region, Fraunhofer diffraction applies. This semi-classical approximation is valid since the mean trajectory followed by the particle approaches the classical one[8].

Consider a plane wave $\vec{k}_0$ incident on an azimuthally symmetric potential of finite radius R , such that $\lambda_0 = (2\pi/k_0) \ll R$. The scattered waves $\vec{k}$ ($|\vec{k}_0| = |\vec{k}|$) viewed far away from the target will be spherical plane waves with some dependence on the scattering angle θ . With the origin at the scattering center, the total wave function becomes

$$\Psi_{\text{total}} = \exp(i\vec{k}_0 \cdot \vec{r}) + f(\theta) (1/r) \exp(i\vec{k} \cdot \vec{r})$$

In order to determine the scattering amplitude $f(\theta)$, the scattered wave function $\Psi_S = f(\theta) (1/r) \exp(i\vec{k} \cdot \vec{r})$ will be calculated.

Defining the Green's function

$$G(\vec{r}, \vec{r}') = -(1/4\pi|\vec{r} - \vec{r}'|) \exp(ik|\vec{r} - \vec{r}'|),$$

it follows that

$$(\nabla'^2 + k^2) G(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}').$$

Outside the scatterer, Ψ_S satisfies the free particle Schroedinger equation,

$$(\nabla'^2 + k^2) \Psi_S(\vec{r}') = 0$$

Multiplying the first of these equations by $\Psi(\vec{r}')$, subtracting the second, and integrating over some volume V outside the scatterer one obtains

$$\Psi_S(\vec{r}) = \int_V d^3\vec{r}' \left\{ \Psi_S(\vec{r}') \nabla'^2 G(\vec{r}, \vec{r}') - G(\vec{r}, \vec{r}') \nabla'^2 \Psi_S(\vec{r}') \right\}$$

which reqs for an application of Green's Theorem, yielding

$$\Psi_S(\vec{r}) = \oint_S dS' \left\{ \Psi_S(\vec{r}') \hat{n}' \cdot \vec{\nabla}' G(\vec{r}, \vec{r}') - G(\vec{r}, \vec{r}') \hat{n}' \cdot \vec{\nabla}' \Psi_S(\vec{r}') \right\}$$

Thus $\Psi_S(r)$ is completely determined by its value and gradient on a closed surface outside the scatterer. Let S be the infinite hemisphere behind the target. Since the integrand vanishes on

the infinite sphere, the only values of Ψ_s that contribute are those on the plane perpendicular to $\vec{k}_0$ immediately behind the target.

In the Fraunhofer approximation, $r \gg r'$, the Green's function reduces to $G(\vec{r}, \vec{r}') = -(1/4\pi r) \exp(ikr) \exp(-i\vec{k} \cdot \vec{r}')$. Putting this and $\Psi_s(\vec{r}') = f(\theta) (1/r') \exp(ikr')$ into the integral equation above yields

$$f(\theta) = -\frac{1}{4\pi} \int_{\substack{z=0 \\ \text{PLANE}}} dS' \left\{ \Psi_s(\vec{r}') \hat{n} \cdot \vec{\nabla}' e^{-i\vec{k} \cdot \vec{r}'} - e^{-i\vec{k} \cdot \vec{r}'} \hat{n}' \cdot \vec{\nabla}' \Psi_s(\vec{r}') \right\}$$

On the plane just behind the scatterer and beyond its range, Ψ_s must be exactly zero. Setting the impact parameter $b = r \sin \theta$,

$$\Psi_s(\vec{r}) = \begin{cases} 0 & r > R \\ -W(b) e^{i\vec{k}_0 \cdot \vec{r}} & r < R \end{cases}$$

where $0 \leq W(b) \leq 1$ is a measure of the 'blackness' profile presented by the scatterer, and hence bears the name 'profile function'. $|W(b)|^2$ is the probability of elastic scattering at b , and since $|1 - W(b)|^2$ is the probability of no interaction, $2(|W(b)| - |W(b)|^2)$ is the probability of absorption.

To evaluate $f(\theta)$, note that

$$(i) \quad \hat{n}' \cdot \vec{\nabla}' W(b) = 0, \quad \hat{n}' \cdot \hat{z}' = -1, \quad \text{and} \quad \hat{n}' \cdot \vec{k} = -\cos \theta$$

$$(ii) \quad \vec{k}_0 \cdot \vec{r}' = 0, \quad \vec{k} \cdot \vec{r}' = k \sin \theta b \cos \phi$$

where $\vec{r}'$ is in the plane of integration and ϕ is the azimuthal angle.

$$\text{So} \quad f(\theta) = (ik/4\pi) (1 + \cos \theta) \int_0^R W(b) b db \int_0^{2\pi} \exp(ikb \sin \theta \cos \phi) d\phi$$

But the zero order Bessel function is defined as

$$J_0(x) = (1/2\pi) \int_0^{2\pi} \exp(ix \cos z) dz$$

so that $f(\theta) = (ik/2) (1 + \cos\theta) \int_0^R W(b) J_0(kb \sin\theta) b db$

For large k , $\theta \approx \sin\theta$, $\cos\theta \approx 1$, so

$$f(\theta) = ik \int_0^R W(b) J_0(kb\theta) b db$$

Transforming this to a function of q ($q^2 = -t$), where t is the four momentum transfer squared, use $t = -2p^2 (1 - \cos\theta) \approx -p^2 \theta^2$ to get $q \approx p\theta$. Thus

$$f(q) = ik \int_0^\infty W(b) J_0(qb/p) b db$$

where it is now assumed that $W(b)$ is zero for $b > R$. Applying the Fourier-Bessel Integral Theorem, we obtain

$$W(b) = (i/\pi p) \int_0^\infty f(q) J_0(bq/p) q dq$$

Thus, with the assumption that $f(\theta)$ is purely imaginary, and the relation $d\sigma(\theta) = |f(\theta)|^2 d\Omega$, the profile function of the scatterer can be calculated from the elastic differential cross-section.

In the case of a black disk of radius R , all incoming waves with $b < R$ are absorbed: $W(b) = 1$. Thus $f(\theta) = (iR/\theta) J_1(kR\theta)$, where $J_1(z) = (1/z) \int_0^z J_0(z) dz$, and hence $d\sigma/d\Omega(\theta) = [J_1(kR\theta) R/\theta]^2$. The cross-section is characterized by a sharp forward peak with a pronounced diffraction extrema: a classic diffraction pattern.

If, however, $W(b)$ is allowed to vary, the oscillations can disappear. Giving the target a halfwidth b_0 , $W(b) = A \exp(-(b/b_0)^2)$, and

$$f(\theta) = .5 i k A b_0^2 \exp(-(k\theta b_0/2)^2)$$

which gives

$$d\sigma/d\Omega(\theta) = C \exp(-(k\theta b_0)^2/2) = C \exp(-(b_0/\hbar)^2 t)$$

The cross-section dies as a Gaussian in θ , and thus as a simple exponential in t . This agrees qualitatively with experimental hadron-proton elastic differential cross-section data, which gives $d\sigma/dt(t) = A \exp(Et)$ where $E \approx 10$ (GeV/c)². So a typical halfwidth for the Gaussian $W(b)$ should be $b \approx 1$ fermi.

The Partial Wave Treatment

A quantum mechanical approach decomposes Ψ into partial waves, where the l -th wave is assumed to suffer only a phase shift δ_l in the scattering process (this is equivalent to the Fraunhofer approximation in the optical model). One obtains

$$f(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin \delta_l P_l(\cos \theta)$$

The quantity $a = \exp(i\delta_l) \sin \delta_l$ will be shown to be exactly $(1/2i)W(b)$.

Multiplying both sides of $f(\theta)$ by $P_{l'}(\cos \theta) \sin \theta d\theta$ and integrating, and using the orthogonality relation

$$\int_{-1}^1 P_l(x) P_{l'}(x) dx = (2/2l+1) \delta_{ll'}$$

one obtains

$$a_l = (p/2\hbar) \int_0^{2\pi} f(\theta) P_l(\cos \theta) \sin \theta d\theta$$

Since $f(\theta)$ is appreciable only for small θ , increasing the upper limit of integration to ∞ alters nothing. Using the approximation

$$P_l(\cos \theta) = J_0(l\theta) \text{ for } l \gg 1, \theta \ll 1 \text{ [9]}$$

and $q = p\theta$, $l = p\hbar/q$ to get

$$a(b) = (1/2\hbar p) \int_0^{\infty} f(q) J_0(bq/\hbar) q dq$$

which is, as promised, exactly $(1/2i)W(b)$.

The partial wave treatment permits calculation of the cross-sections in terms of the function $a(b)$:

$$\begin{aligned}\sigma_{\text{elastic}} &= \int |f(\theta)|^2 d\Omega = \frac{\hbar^2}{p^2} \int d\Omega \left| \sum_{l=0}^{\infty} (2l+1) a_l P_l(\cos\theta) \right|^2 \\ &= \frac{\hbar^2}{p^2} \sum_{l,l'} (2l+1)(2l'+1) a_l a_{l'}^* \int_0^{2\pi} d\phi \int_0^{\pi} P_l(\cos\theta) P_{l'}(\cos\theta) \sin\theta d\theta\end{aligned}$$

But
$$\int_0^{\pi} P_l(\cos\theta) P_{l'}(\cos\theta) \sin\theta d\theta = \frac{2}{2l+1} \delta_{ll'}$$

so

$$\sigma_{\text{elastic}} = 4\pi \frac{\hbar^2}{p^2} \int_0^{\infty} (2l+1) |a_l|^2 dl$$

Letting $l = pb/\hbar$, $dl = (p/\hbar) db$, then $\sigma_{\text{elas}} = 8\pi \int_0^{\infty} |a(b)|^2 b db$.

Hence $d\sigma_{\text{elas}} = 8\pi b |a(b)|^2 db$.

The optical theorem ($\text{Im}[f(\theta=0)] = (k/4\pi)\sigma_{\text{total}}$) now allows calculation of the total and inelastic cross-sections in terms of $a(b)$:

$$\begin{aligned}\sigma_{\text{total}} &= \text{Im} \left(\frac{4\pi\hbar}{p} \sum (2l+1) a_l P_l(\cos\theta) \Big|_{\theta=0} \right) \\ &\approx \text{Im} \left(8\pi \int_0^{\infty} a(b) J_0\left(\frac{pb\theta}{\hbar}\right) b db \Big|_{\theta=0} \right) \\ &= \text{Im} \left(8\pi \int_0^{\infty} a(b) b db \right)\end{aligned}$$

$$\frac{d\sigma_{\text{total}}}{db} = 8\pi b \text{Im}[a(b)]$$

$$\frac{d\sigma_{\text{inelastic}}}{db} = \frac{d\sigma_{\text{tot}}}{db} - \frac{d\sigma_{\text{elas}}}{db} = 8\pi b (\text{Im}[a(b)] - |a(b)|^2)$$

The next step entails providing the interpretation for $a(b)$, independent of the optical analogy, as is done in the following brief aside.

The classical differential cross-section can be written

$$d\sigma/d\Omega(\theta) = (b(\theta)/\sin\theta) |db/d\theta|$$

where azimuthal symmetry of the scatterer has been assumed. The nature of the potential determines the dependence of the impact parameter on the scattering angle θ . The profile presented by the target to the incoming particle can be obtained by finding the rate of change of effective scattering area as a function of radius as

$$d\sigma/db = (d\sigma/d\Omega) (d\Omega/db) = 2\pi b$$

As expected, the classical scattering area increases linearly to the radius of the scatterer and then drops to zero, independent of the potential. This arises directly from the deterministic nature of classical scattering;

i.e. that a particle of given energy on a potential at impact parameter b is always scattered the same amount into an angle $\theta(b)$. Note that $d\sigma(b)$ contains no information about where the particle scatters, i.e. about $\theta(b)$, but just whether or not it scatters at all.

Quantum mechanics obviously alters the above analysis. Now one expects that associated with each impact parameter b there is a probability amplitude $f(\theta, b)$ for scattering at any angle θ such that $\int_0^\pi f(\theta, b) d\theta = P(b)$ is the total probability that the incoming particle of impact parameter b scatters anywhere. (The classical limit for a target of radius R is $P(b) = \begin{cases} 1 & b < R \\ 0 & b > R \end{cases}$).

But $\frac{d\sigma}{db}$ measures the sum over all scattering angles θ and azimuthal angles ϕ of the probability amplitude $f(\theta, b)$:

$$\frac{d\sigma}{db} = \int p(\theta, b) b d\theta d\phi = 2\pi b P(b),$$

yielding

$$P(b) = (1/2\pi b) d\sigma/db(b)$$

Knowledge of $d\sigma(b)$ affords calculation of the probability of scattering or transmission at any b . If the scattering potential were a finite constant for $b < R$ and zero outside, the target would have a well defined edge, and a non-zero probability for transmission at all b . A potential with more character, a 'hard' core surrounded by a 'soft' cloud diminishing quickly in density yields the pictured situation. All incoming particles at R_0 are scattered, whereas only half of those at R_1 are.

It was shown above that for a particle with impact parameter b the probability that it will be scattered is $P(b) = (1/2\pi b) (d\sigma/db)$. Therefore, substituting the notation 'G' for the probability 'P' (by convention)

$$G_{\text{elastic}}(b) = 4 |a(b)|^2$$

$$G_{\text{inelastic}}(b) = 4 (\text{Im}[a(b)] - |a(b)|^2)$$

$$G_{\text{total}}(b) = 4 \text{Im}[a(b)]$$

In the black disc limit, $a(b) = i/2$ (see below), so that these expressions must be treated as relative probabilities: division by a factor of two renormalizes them. This comes as a natural consequence of the fact that two distinct scattering processes can and do occur. So $a(b)$ is a measure of the impact profile, and therefore directly related to $W(b)$ in the optical analogy. The elastic and inelastic 'overlap' functions, defined by the probabilities above, measure the effective overlap (for elastic and inelastic purposes, respectively) of the profile areas of the two particles in the collision. Intuition expects $G=1$ for 'head-on' collisions, and $G \ll 1$ for 'grazing' (peripheral) collisions.

What are the properties of $a(t)$? Recall that $a = \exp(i\delta_l) \sin\delta_l$, where δ_l is the phase shift for each partial wave. So

$$a_l = e^{i\delta_l} \left(\frac{e^{i\delta_l} - e^{-i\delta_l}}{2i} \right) = (e^{2i\delta_l} - 1)/2i$$

If δ_l is real (which implies the potential is real), then

$$a_l = (1/2) \sin 2\delta_l + i \sin^2 \delta_l \quad (\delta_l \text{ real})$$

In the elastic scattering limit a_l has comparable real and imaginary parts.

Now allow the phase shift to have an imaginary part (this corresponds to letting the potential have an imaginary (absorptive) part): $\delta_l = \beta + i\eta$. Then

$$a_l = \frac{e^{-2\eta}}{2} \sin 2\beta + i \left\{ \frac{1 - e^{-2\eta} \cos 2\beta}{2} \right\}$$

η need not be very large for the real part of a_l to be negligible, especially for small β .

The ratio Q of real to imaginary parts of the forward scattering amplitude $f(t=0)$ for hadron-proton collisions has magnitude less than about .15 in this energy range [7] (in fact, it crosses zero). The error in the approximation $\text{Im}[f(\theta)] = [d\sigma/dt]^{1/2}$ is on the order of Q^2 , since $[d\sigma/dt]^{1/2} = \text{Im}(f) (1 + Q^2/2)$, reducing the effect at $t=0$ to less than 2%. At larger $|t|$, the assumption that Q is small becomes less tenable. Calculation of the real part for $t \neq 0$, however, unfortunately becomes model dependent [2]. Contributions to $a(t)$ diminish with increasing $|t|$, somewhat reducing the effect of a possibly growing Q .

To a very good approximation, then, a is purely imaginary, with $Q < \text{Im}(a) < .5$. (From this point on, we shall write $a(t)$ and

mean $\text{Im}[a(b)]$). In the black disc limit of full absorption,
 $\sigma \rightarrow \infty$ and $a \rightarrow .5$. Summarizing,

$$\begin{aligned} d\sigma_{el} &= 8\pi b a^2 db, & d\sigma_{in} &= 8\pi t(a-a^2) dt, & d\sigma_{tot} &= 8\pi b a db, \\ G_{el} &= 4a^2, & G_{in} &= 4(a-a^2), & \text{and} & G_{tot} &= 4a. \end{aligned}$$

Now we have a precise expression for and interpretation of the impact profile function in high energy hadron-hadron collisions. The optical analogy has been valuable, providing the semi-classical explanation for results in impact parameter space, as well as yielding the black disc limit. This correspondence implies that diffraction of the incident beam causes elastic scattering; that elastic processes occur only as a by-product of absorptive ones. The function $a(t)$ determines both the elastic and inelastic scattering properties of the target as viewed by the incoming particle. This quantity could never be measured directly, and depends only on the elastic differential cross-section's behavior in t . We can gain considerable microscopic information from macroscopic measurements.

One result already evident is that the elastic impact profile is always less than or equal to the inelastic profile. And, if it decreases with b as our intuition suggests, it does so more than twice as quickly. The first conclusion follows directly from the fact that diffraction is the 'shadow' of absorption; the second explains how this diffraction depends on the relative greyness of the profile. Indeed, if we plot $G_{\text{elas}}(b)$ for some fixed b as a function of $z=1-G_{\text{inel}}$, we notice that it decreases infinitely sharply ($dG/dz(z=0)=-\infty$) from 1 at $z=0$, and continues to decrease until $z=1$, where $a=G_{\text{el}}=G_{\text{in}}=0$. Thus, even a small deviation from the black disc limit of total absorption dramatically suppresses diffraction effects. These points become clear upon examination of the other curves below: the relative probabilities of elastic and inelastic processes when there is

scattering (which diverge symmetrically from the black disc limit), and the overlap function of each process. Both assume a roughly Gaussian impact parameter dependence in the profile $a(b)$.

Calculation of the Impact Profile

Recall $a(b) = (1/2\pi p) \int_0^\infty f(q) J_0(bq/\hbar) q dq$. The forward elastic scattering amplitude $f(q)$ is related to the elastic differential cross-section by $d\sigma/d\Omega = |f(q)|^2$, which, in the small angle scattering limit, becomes $d\sigma/dt = (\pi/p^2) |f(q)|^2$. With the assumption that $f(q)$ is purely imaginary, $f(q) = i(p/\pi^{1/2}) (d\sigma/dt)^{1/2}$, and hence

$$a(b) = (1/2\pi\hbar^{1/2}) \int_0^\infty [d\sigma/dt(q)]^{1/2} J_0(bq/\hbar) q dq, \quad q = \sqrt{-t}.$$

As mentioned earlier, this paper shall analyze the impact parameter space representation of the six elastic reactions π^+p , K^+p , and p^+p at incident momenta 50, 70, 100, 140, and 175 GeV/c, using data taken at the Fermilab[1]. This differential cross-section data, taken in the range $.03 < |t| < 0.8$ (GeV/c)², may be represented well by $d\sigma/dt = A \exp(Bt + Ct^2)$. The parameters resulting from the final fits[10] to this data were used.

The quantitative behavior of the differential cross-sections at large t is largely unknown at these energies. What effect upon $a(b)$ will such a large t contribution have, and how can we account for it?

The first of these questions can be answered by analyzing the effect of the zero order Bessel function on the integral expression for $a(b)$. Since its argument depends on both b and q , its effect at $b=0$ is always the largest. For large q , however, $|J_0(bq/\hbar)|$ rapidly becomes small as a function of b . The lack of information at large t , then, will be reflected in a

corresponding uncertainty in $a(b)$ at small impact parameter. More specifically, appreciable cross-sections at large t will always tend to increase the impact profile in the vicinity of $b=0$. Since, however, $J_0(z)$ does oscillate about zero, the larger t contribution does also, becoming negative at about $b=.5$ fermi, though to much less an extent than at $b=0$. All of this is merely a restatement of the fact that collisions at high momentum transfer contain information about the 'cores' of the participating particles.

Correspondingly, the behavior of the cross-section at very small t has its largest effect, percentage-wise, at large b (since the dominant contributions to the integral at large b occur in the small t range). Use of the optical point in fits to the data helps reduce this problem.

Since the deviation in the cross-section from the pure exponential is relatively small, one reasonable way of extending the data is as a pure exponential, requiring continuity and smoothness at the real data cutoff. Another characteristic of the elastic cross-section that we might expect, a sharp decrease in the exponential slope at large t , is sometimes accompanied by a dip and shoulder at the position of the break. After the break, the cross-section decreases much more slowly, yielding an even larger contribution to $a(b)$ near $b=0$. Clearly these problems must be dealt with, though the solution will have to reflect our ignorance of the specific behavior at large t .

The impact profile was calculated for $d\sigma/dt$ in the three regions shown below. Region I consists of real data, and the

quadratic exponential fits were used. Region II is a continuous and smooth pure exponential extension of the real data. The error assigned to $a(b)$ for $d\sigma/dt$ in this region was taken to be of the same percentage as region I. Region III accounts for the possible break in slope.

At what value of t does the break occur for a given reaction at a given energy? J. Tran Thanh Van[11] offers a phenomenologically valid if theoretically incomplete argument for calculation of this t_{break} . When many elastic differential cross sections are plotted as a function of the variable $X = -q_{t,t}^2$, the break occurs for all reactions and energies at the value $X = 2080 (\text{mbGeV}/c)^2$. The relatively small dependence of $\sigma_{t,t}$ in the given range manifests itself in a correspondingly weak dependence of t_{break} . Consequently, the average values[12] shown below represent good approximations for the given purpose at all energies:

pp	38.5 mb	π^+p	23.4 mb	K^+p	19.0 mb
$\bar{p}p$	42.3 mb	π^-p	24.1 mb	K^-p	20.5 mb

Modulo the assumption of an exponentially decreasing cross-section, the effect of the break on $a(0)$ is negligible ($<1\%$) for all reactions except pp and $\bar{p}p$, where it occurs soon enough for the decrease in slope to matter. The experiment by Akerlof, et al.[13], which measured $d\sigma/dt$ at 100 and 200 GeV/c, suggests a dependence beyond t_{break} of $\exp(Dt)$, with $D = 1.1 \pm 0.3 (\text{GeV}/c)^2$. So, for

the reactions pp and $\bar{p}p$, $d\sigma/dt$ was extended into region III continuously as a pure exponential in t with slope $D=1.1$ $(\text{GeV}/c)^2$ (the possible presence of a dip was ignored, as was the probable increase in the contribution from the real part of $f(t)$ in this region). An arbitrary error of 30% was assigned to the contribution to $a(b)$ in this region. Region III contributed from 1.4% of the total $a(0)$ for pp at 50 GeV/c to 6% for $\bar{p}p$ at 175 GeV/c .

Below appears a plot of the contribution to $a(b)$ from regions II and III for pp at 175 GeV/c . Note its oscillatory nature and that at $b=0$ it attains 4% of the total $a(b)$.

Results

Tables 1 and 2 list the results of the above described calculation. For all reactions at 50 and 175 GeV/c the impact profile function and its associated error are shown at intervals of .1 fermi out to 2.0 fermi. The results at 70, 100, and 140 GeV/c lie more or less successively between these two values. The overlap functions and differential cross-sections can be obtained directly from these numbers.

Errors in the profile range from 3% for π^+p to 10% for $\bar{p}p$, and grow with t . The single most significant contribution to this uncertainty was that propagated by the 'A' term in the differential cross section fits $A \exp(Bt + Ct^2)$. The value of ΔA was very strongly influenced by the optical point uncertainty (i.e. $\Delta d\sigma/dt(t=0)$), set at 3% to reflect the overall scale normalization uncertainty in the spectrometer[1]. For example, a 1% optical point decreases ΔA by a factor of two, yielding approximately a factor of two decrease in the profile also. Thus roughly one-half of the profile error is due to the systematic uncertainty in the cross-section, independent of t .

Table 3 shows the 'opacity' (defined as the inelastic overlap function, in reference to the optical model) and elastic overlap function at $t=0$ for all reactions at all energies. Table 4 displays $\langle b \rangle$ and $\langle b^2 \rangle^{1/2}$ at 50 and 175 GeV/c obtained by the formulae

$$\langle b \rangle = \left(\int b d\sigma(b) \right) / \left(\int d\sigma(b) \right)$$

$$\langle b^2 \rangle = \left(\int b^2 d\sigma(b) \right) / \left(\int d\sigma(b) \right)$$

with the appropriate $d\sigma$ (elastic, inelastic, total) in each case.

Figures 1-18 display the results graphically. Data points are connected by straight lines for easier examination.

Figure 1: The impact profile for all reactions at 50 GeV/c, on normal and $\log$ vs b^2 scales.

2: Elastic and inelastic overlap functions for all reactions at 50 and 175 GeV/c on $\log$ vs b^2 scales.

3-8: Each displays the elastic and inelastic overlap functions for a reaction at 50 and 175 GeV/c on a $\log$ vs b^2 scale, as well as both overlap function energy differences (i.e. $G(175) - G(50)$) on a normal scale.

9-14: Each displays the elastic and inelastic differential cross sections on a normal scale, as well as both cross section energy differences (i.e. $d\sigma(175) - d\sigma(50)$).

15-16: Particle-antiparticle differential cross-section differences (i.e. $d\sigma(XY) - d\sigma(\bar{X}\bar{Y})$) for all reactions on a normal scale.

17-18: These display a test of Lipkin's sum rule[5] and the simple quark model. According to these theories, the profile combinations plotted should be identically zero for all b.

Lipkin rule 1:
$$\frac{\frac{2}{3} a(\pi^- p) + \frac{1}{3} a(\pi^+ p)}{\frac{1}{2} a(K^+ p) + \frac{1}{3} a(\bar{p} p)} - 1 = 0$$

Lipkin rule 2:
$$\frac{a(K^- p) - \frac{1}{2} a(K^+ p) + \frac{1}{3} a(pp)}{\frac{1}{2} a(\pi^- p) + \frac{1}{2} a(\pi^+ p)} - 1 = 0$$

Simple quark rule 1:
$$\frac{a(K^- p)}{a(\pi^+ p)} - 1 = 0$$

Simple quark rule 2:
$$\frac{\frac{2}{3} a(\pi^- p) + \frac{1}{3} a(\pi^+ p)}{\frac{1}{3} a(pp) + \frac{1}{3} a(\bar{p} p)} - 1 = 0$$

Simple quark rule 3:
$$\frac{a(K^+ p)}{\frac{2}{3} a(pp)} - 1 = 0$$

Conclusions

The following statements summarize the characteristics of the b-space results:

1) Table 3 reveals a few important features of the data. (We recall that the opacity at zero represents the relative probability of particle production (absorption) in a head-on collision, and hence has a black disc limit of 1.0).

(a) Pions are slightly blacker than kaons, and both mesons present a much more transparent profile than the proton and anti-proton. Since the absolute probability of any interaction (elastic or inelastic) is $2a(b)$, we see, for example, that a kaon at these energies has a 48% probability of missing a proton altogether in a 'head-on' approach, whereas a proton misses a proton only 16% of the time in such an approach.

(b) For each reaction throughout this energy range, the opacity at zero is moderately constant (within errors). The K^+p reaction could be getting blacker with increasing energy, however, and $p\bar{p}$ more transparent.

(c) The proton opacities fall within errors of Miettinen's [3] 'broad minimum' model for energy dependence. A falling proton opacity in this range is also consistent with his model.

(d) Particle and antiparticle look very similar at $b=0$.

2) Root mean-square interaction radii vary from .77 to .96 fermi.

in the order K^+p , π^+p , $K-p$, $\pi-p$, pp , $\bar{p}p$. Absorptive contributions to this total 'reaction size' dominate: the rms interaction distance for inelastic processes is 5% greater; for elastic scattering it is 30% smaller.

3) Wherever the 'size' of a reaction increases significantly with energy (as clearly occurs for K^+p), inelastic processes dominate this increase. This is consistent with the thesis that elastic processes are the shadow of absorption: elastic processes contribute much less than inelastic ones, especially at large impact parameter.

4) The $\log$ vs b^2 plots allow easy comparison with a pure Gaussian curve, which would be linear. The slight upward concavity of $a(b)$ at small impact parameters and the appearance of a tail at larger ones reflects the small t --large b , large t --small b correspondence: $d\sigma/dt$ departs positively from a single linear exponential fit at both small and large t .

5) With one possible exception, at all impact parameters the profiles and overlap functions for the six reactions increase from smallest to largest in the same order (K^+p , $K-p$, π^+p , $\pi-p$, pp , $\bar{p}p$), which is, of course, the order of increasing cross-sections. The possible exception occurs in the range $.1 < b < .5$, where the $\bar{p}p$ profile briefly rises by a small amount (well within errors) above the pp .

6) The inelastic overlap function exhibits a suppression (which decreases with increasing b) from a pure Gaussian (i.e. is 'concave down' on the $\log$ vs b^2 scale) until about 1 fermi. At this point it begins to level off, developing a pronounced tail which persists to 2 fermi. Conversely, the elastic overlap exhibits an upward deviation from the Gaussian at small b ('concave up') and develops a similar tail. The elastic overlap begins at a value 30-60% smaller than the opacity, and diverges from it quickly: by 1.3 fermi it is already at least 2 orders of magnitude smaller. This behavior vividly illustrates the absorption-diffraction nature of high energy scattering.

The π^+p , K^+p , K^-p , and pp reactions have increasing total cross-sections in this energy range (π^-p 's decreases until ~100 GeV/c, then increases; $\bar{p}p$'s decreases throughout the range [12]). At what impact parameters does this increase (or decrease) occur? Since $4d\sigma(b) = G(b) b db$, changes in the opacity near $b=0$ will have a smaller effect in $d\sigma(b)$ than such an increase for b near 1 fermi, and increases or decreases in the opacity at a given impact parameter manifest themselves in $d\sigma(b)$ at larger b .

For all reactions except K^+p , sizeable errors and a relatively narrow energy range ($\sqrt{s}=9.4$ to 18.1 GeV) continue to cloud the quantitative energy dependence of the t -space functions. The qualitative behavior of this dependence, however, does exhibit some interesting characteristics:

7) The elastic overlap change varies more or less monotonically from an extremum at $b=0$ to essentially zero at larger b . The opacity (inelastic overlap) change, however, reveals more structure:

(a) The K^+p opacity exhibits a definite increase with energy for $b>1$ (peaking at 1.2 fermi and falling by 2 fermi) and very well may be increasing for $0<b<.5$.

(b) The opacities of π^+p , $K-p$, and pp seem to show an increase centered in the region $.8<b<1.5$, reaching a maximum at about 1.2 fermi. This is in rough agreement with Miettinen's[3] result for pp at ISR energies.

(c) The $\bar{p}p$ reaction looks to be losing opacity in a 1 fermi wide region; the maximum change occurs at .8 fermi.

(d) $\pi-p$ displays little s dependence.

8) Increases in the opacity do indeed show up at larger b in the cross-sections. The increase are spread over a broad region 1 fermi wide, peaking at about $b=1.3$ fermi for π^+p , K^+p , $K-p$, and pp . Once again, $\pi-p$ is relatively flat, and $\bar{p}p$ exhibits a bell-shaped decrease centered at $b=.9$ fermi.

Miettinen[3] makes the point that the 'real' source(s) of change in G need not be located in the same t -range as the net charge. He proposes that it is a superposition of two effects:

(i) Increases at large b and decrease at small t , maintaining a constant σ_{tot} .

(ii) The opening up of new central channels.

9) The Pomeranchuk Theorem predicts that as $s \rightarrow \infty$,

$$|\sigma_t(XY) - \sigma_t(\bar{X}Y)| \rightarrow 0.$$

Figures 15 and 16 shows how close 175 GeV/c ($s=328 \text{ GeV}^2$) is to this asymptotia in the range $0 < t < 2$ fermi. The particle-antiparticle differential cross-sections differ in a 1 fermi wide region, reaching a maximum at $t=1.2$ fermi. That this is the same region in which some of the cross sections are growing holds well for theories of asymptotia, since it indicates that the region of change with energy is precisely the region where particle and antiparticle differ (and hence that the differential cross-sections may be approaching each other).

10) The simple quark model predicts that the protons and pions consist of 3 quarks apiece, and kaons 2 quarks apiece, yielding a prediction of $a(Kp) = (2/3)a(\pi p) = (2/3)a(pp)$. To account for observed deviations, Lipkin[5] has proposed that $a(Kp) = (1/2)a(\pi p) = (2/9)a(pp)$. Combination of this with the simple model yields the weaker relation $a(\pi p) = (1/2)a(Kp) + (1/3)a(pp)$. Choosing charge combinations satisfied by the Regge component, the following sum rules result:

$$(1) \quad (2/3)a(\pi^-p) + (1/3)a(\pi^+p) = (1/2)a(K^+p) + (1/3)a(\bar{p}p)$$

(2) $(1/2)a(\pi^-p) + (1/2)a(\pi^+p) = a(K^-p) - (1/2)a(K^+p) + (1/3)a(pp)$. The simple quark model predicts

$$(1) \quad a(K^-p) = a(\pi^+p)$$

$$(2) \quad (2/3)a(\pi^-p) + (1/3)a(\pi^+p) = (1/3)a(\bar{p}p) + (1/3)a(pp)$$

$$(3) \quad a(K^+p) = (2/3)a(pp)$$

Most rules seem to hold better at small b (about 20% deviation) than large b (about 50% deviation) (the π^+p and K^-p amplitudes, however, show excellent agreement). This seems to contradict the simple picture of 'valence quarks', which would predict large b agreement and small b deviation.

TABLE 1

IMPACT PROFILE FUNCTION & ERROR AT 50 GEV/C

B	PI+P		PI-P		K+P		K-P		PP		PBARP	
0.0	.30E 00	.96E-02	.32E 00	.10E-01	.26E 00	.12E-01	.27E 00	.16E-01	.41E 00	.23E-01	.41E 00	.39E-01
0.1	.30E 00	.94E-02	.31E 00	.99E-02	.26E 00	.11E-01	.27E 00	.15E-01	.41E 00	.22E-01	.40E 00	.37E-01
0.2	.28E 00	.89E-02	.29E 00	.93E-02	.24E 00	.11E-01	.25E 00	.15E-01	.39E 00	.20E-01	.38E 00	.33E-01
0.3	.26E 00	.82E-02	.26E 00	.85E-02	.22E 00	.97E-02	.23E 00	.13E-01	.36E 00	.18E-01	.35E 00	.27E-01
0.4	.23E 00	.73E-02	.23E 00	.75E-02	.19E 00	.84E-02	.20E 00	.12E-01	.32E 00	.17E-01	.32E 00	.27E-01
0.5	.19E 00	.64E-02	.20E 00	.67E-02	.16E 00	.72E-02	.17E 00	.10E-01	.29E 00	.15E-01	.28E 00	.25E-01
0.6	.16E 00	.57E-02	.16E 00	.59E-02	.13E 00	.64E-02	.14E 00	.90E-02	.25E 00	.13E-01	.25E 00	.22E-01
0.7	.13E 00	.49E-02	.13E 00	.51E-02	.10E 00	.54E-02	.11E 00	.77E-02	.21E 00	.11E-01	.22E 00	.18E-01
0.8	.99E-01	.40E-02	.10E 00	.42E-02	.78E-01	.44E-02	.85E-01	.63E-02	.17E 00	.92E-02	.18E 00	.15E-01
0.9	.75E-01	.31E-02	.79E-01	.33E-02	.57E-01	.35E-02	.65E-01	.49E-02	.13E 00	.75E-02	.15E 00	.13E-01
1.0	.56E-01	.23E-02	.60E-01	.25E-02	.41E-01	.26E-02	.48E-01	.37E-02	.10E 00	.59E-02	.12E 00	.11E-01
1.1	.41E-01	.18E-02	.46E-01	.19E-02	.29E-01	.18E-02	.36E-01	.27E-02	.80E-01	.44E-02	.98E-01	.88E-02
1.2	.30E-01	.14E-02	.35E-01	.15E-02	.20E-01	.15E-02	.27E-01	.23E-02	.60E-01	.33E-02	.77E-01	.69E-02
1.3	.21E-01	.12E-02	.26E-01	.13E-02	.13E-01	.16E-02	.19E-01	.20E-02	.44E-01	.27E-02	.59E-01	.53E-02
1.4	.15E-01	.10E-02	.19E-01	.11E-02	.89E-02	.21E-02	.14E-01	.18E-02	.31E-01	.22E-02	.45E-01	.44E-02
1.5	.10E-01	.84E-03	.14E-01	.90E-03	.59E-02	.45E-02	.98E-02	.16E-02	.22E-01	.18E-02	.34E-01	.38E-02
1.6	.70E-02	.64E-03	.96E-02	.68E-03	.38E-02	.11E-01	.68E-02	.13E-02	.16E-01	.14E-02	.25E-01	.31E-02
1.7	.46E-02	.43E-03	.66E-02	.46E-03	.24E-02	.22E-02	.45E-02	.89E-03	.11E-01	.10E-02	.19E-01	.25E-02
1.8	.29E-02	.50E-03	.44E-02	.48E-03	.15E-02	.72E-03	.29E-02	.10E-02	.74E-02	.91E-03	.14E-01	.19E-02
1.9	.18E-02	.49E-03	.28E-02	.49E-03	.84E-03	.10E-02	.18E-02	.11E-02	.49E-02	.93E-03	.98E-02	.15E-02
2.0	.10E-02	.42E-03	.17E-02	.44E-03	.44E-03	.99E-03	.10E-02	.98E-03	.31E-02	.87E-03	.70E-02	.15E-02

TABLE 2

IMPACT PROFILE FUNCTION & ERROR AT 175 GEV/C

B	PI+P		PI-P		K+P		K-P		PP		PBARP	
0.0	.31E-00	.12E-01	.31E-00	.12E-01	.28E-00	.91E-02	.28E-00	.14E-01	.38E-00	.36E-01	.40E-00	.58E-01
0.1	.30E-00	.12E-01	.30E-00	.12E-01	.27E-00	.90E-02	.27E-00	.14E-01	.38E-00	.35E-01	.39E-00	.55E-01
0.2	.28E-00	.11E-01	.28E-00	.12E-01	.26E-00	.84E-02	.26E-00	.13E-01	.36E-00	.33E-01	.36E-00	.49E-01
0.3	.26E-00	.11E-01	.26E-00	.11E-01	.23E-00	.76E-02	.23E-00	.12E-01	.34E-00	.30E-01	.33E-00	.41E-01
0.4	.23E-00	.93E-02	.23E-00	.93E-02	.20E-00	.67E-02	.20E-00	.10E-01	.31E-00	.28E-01	.30E-00	.40E-01
0.5	.19E-00	.82E-02	.19E-00	.82E-02	.17E-00	.58E-02	.17E-00	.89E-02	.27E-00	.25E-01	.26E-00	.36E-01
0.6	.16E-00	.72E-02	.16E-00	.73E-02	.14E-00	.52E-02	.14E-00	.79E-02	.24E-00	.22E-01	.23E-00	.32E-01
0.7	.13E-00	.61E-02	.13E-00	.62E-02	.11E-00	.44E-02	.11E-00	.67E-02	.20E-00	.19E-01	.20E-00	.27E-01
0.8	.10E-00	.50E-02	.10E-00	.51E-02	.82E-01	.36E-02	.86E-01	.55E-02	.17E-00	.16E-01	.17E-00	.23E-01
0.9	.77E-01	.39E-02	.78E-01	.40E-02	.61E-01	.28E-02	.66E-01	.43E-02	.14E-00	.13E-01	.14E-00	.19E-01
1.0	.58E-01	.29E-02	.50E-01	.30E-02	.45E-01	.21E-02	.50E-01	.32E-02	.11E-00	.11E-01	.11E-00	.16E-01
1.1	.43E-01	.22E-02	.45E-01	.23E-02	.33E-01	.15E-02	.37E-01	.24E-02	.83E-01	.81E-02	.90E-01	.13E-01
1.2	.32E-01	.18E-02	.34E-01	.18E-02	.24E-01	.13E-02	.28E-01	.20E-02	.64E-01	.63E-02	.71E-01	.97E-02
1.3	.23E-01	.15E-02	.25E-01	.15E-02	.17E-01	.12E-02	.21E-01	.17E-02	.48E-01	.50E-02	.55E-01	.75E-02
1.4	.16E-01	.12E-02	.18E-01	.12E-02	.12E-01	.11E-02	.15E-01	.15E-02	.35E-01	.39E-02	.42E-01	.63E-02
1.5	.11E-01	.99E-03	.13E-01	.99E-03	.84E-02	.10E-02	.11E-01	.14E-02	.26E-01	.30E-02	.32E-01	.55E-02
1.6	.79E-02	.73E-03	.91E-02	.73E-03	.57E-02	.78E-03	.74E-02	.12E-02	.18E-01	.23E-02	.25E-01	.46E-02
1.7	.52E-02	.52E-03	.62E-02	.54E-03	.37E-02	.47E-03	.50E-02	.82E-03	.13E-01	.17E-02	.19E-01	.38E-02
1.8	.34E-02	.59E-03	.41E-02	.59E-03	.23E-02	.56E-03	.33E-02	.98E-03	.91E-02	.14E-02	.14E-01	.29E-02
1.9	.21E-02	.59E-03	.26E-02	.57E-03	.14E-02	.60E-03	.20E-02	.11E-02	.62E-02	.13E-02	.10E-01	.22E-02
2.0	.13E-02	.52E-03	.16E-02	.49E-03	.77E-03	.55E-03	.12E-02	.95E-03	.41E-02	.11E-02	.74E-02	.25E-02

Table 3

Opacity at Zero

Reaction	50	70	100	140	175
π^+p	$.85 \pm .01$	$.86 \pm .02$	$.85 \pm .03$	$.86 \pm .02$	$.85 \pm .02$
π^-p	$.86 \pm .01$	$.85 \pm .02$	$.85 \pm .02$	$.85 \pm .02$	$.85 \pm .02$
K^+p	$.78 \pm .02$	$.80 \pm .02$	$.82 \pm .03$	$.80 \pm .02$	$.80 \pm .02$
K^-p	$.80 \pm .03$	$.78 \pm .02$	$.80 \pm .02$	$.78 \pm .03$	$.80 \pm .02$
pp	$.97 \pm .02$	$.97 \pm .02$	$.95 \pm .03$	$.95 \pm .03$	$.95 \pm .03$
$\bar{p}p$	$.97 \pm .03$	$.94 \pm .04$	$.95 \pm .05$	$.96 \pm .04$	$.96 \pm .05$

Elastic Overlap at Zero

π^+p	$.37 \pm .02$	$.38 \pm .03$	$.38 \pm .04$	$.39 \pm .03$	$.37 \pm .03$
π^-p	$.40 \pm .03$	$.37 \pm .03$	$.37 \pm .03$	$.37 \pm .03$	$.38 \pm .03$
K^+p	$.28 \pm .02$	$.30 \pm .02$	$.33 \pm .04$	$.31 \pm .03$	$.31 \pm .02$
K^-p	$.30 \pm .03$	$.28 \pm .03$	$.31 \pm .04$	$.28 \pm .03$	$.31 \pm .03$
pp	$.69 \pm .08$	$.66 \pm .09$	$.60 \pm .11$	$.61 \pm .10$	$.59 \pm .11$
$\bar{p}p$	$.67 \pm .13$	$.57 \pm .11$	$.60 \pm .16$	$.65 \pm .19$	$.63 \pm .18$

Table 4

Reaction	$\langle b^2 \rangle^{1/2}$			$\langle b \rangle$		
	Inel	Elas	Total	Inel	Elas	Total
π^+p at 50	.85 $\pm$.01	.55 $\pm$.01	.82 $\pm$.01	.76 $\pm$.01	.49 $\pm$.02	.72 $\pm$.01
175	.86 $\pm$.01	.56 $\pm$.01	.83 $\pm$.01	.77 $\pm$.01	.49 $\pm$.02	.73 $\pm$.01
π^-p at 50	.89 $\pm$.01	.57 $\pm$.01	.85 $\pm$.01	.79 $\pm$.01	.50 $\pm$.02	.75 $\pm$.01
175	.88 $\pm$.01	.57 $\pm$.01	.85 $\pm$.01	.78 $\pm$.01	.50 $\pm$.02	.74 $\pm$.01
K^+p at 50	.79 $\pm$.01	.52 $\pm$.01	.77 $\pm$.01	.71 $\pm$.01	.46 $\pm$.01	.67 $\pm$.01
175	.83 $\pm$.01	.53 $\pm$.01	.80 $\pm$.01	.74 $\pm$.01	.47 $\pm$.02	.70 $\pm$.01
K^-p at 50	.85 $\pm$.02	.55 $\pm$.02	.82 $\pm$.02	.76 $\pm$.02	.48 $\pm$.03	.72 $\pm$.02
175	.87 $\pm$.02	.55 $\pm$.01	.83 $\pm$.01	.77 $\pm$.02	.48 $\pm$.02	.73 $\pm$.02
pp at 50	.93 $\pm$.01	.61 $\pm$.01	.88 $\pm$.01	.84 $\pm$.02	.54 $\pm$.03	.78 $\pm$.02
175	.96 $\pm$.02	.64 $\pm$.03	.91 $\pm$.02	.86 $\pm$.03	.57 $\pm$.05	.81 $\pm$.03
$\bar{p}p$ at 50	1.00 $\pm$.02	.67 $\pm$.02	.95 $\pm$.02	.90 $\pm$.03	.59 $\pm$.04	.84 $\pm$.03
175	1.01 $\pm$.03	.67 $\pm$.04	.96 $\pm$.03	.91 $\pm$.04	.58 $\pm$.07	.85 $\pm$.04

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18

Notes

- 1) Fermilab Single Arm Spectrometer Group, Phys. Rev. Letters 35, 1195 (1975).
- 2) H.I. Miettinen and E.H. De Groot, 'Shadow Approach to Diffraction Scattering', Proceedings of the Eighth Rencontre de Moriond, Vol. 2, 193 (1973).
- 3) H.I. Miettinen, 'Impact Structure of Diffraction Scattering', Proceedings of the Ninth Rencontre de Moriond, Vol. 1, 363 (1974).
- 4a) R. Henzi and P. Valin, Physics Letters 48E, 119 (1974).
- 4b) P.S. Henyey et al., Nuclear Physics B70, 445 (1974).
- 5) H.J. Lipkin, Physics Letters 56B, 76 (1975).
- 6) The optical model is discussed in W.C. Lock, High Energy Nuclear Physics, 110-118 (1960)
- 7) R.E. Hendrick and E. Laurup, Phys. Rev. D11, 529 (1975).
- 8) This discussion closely follows the development in H. Frauenfelder and E.H. Henley, Subatomic Physics, 133-145 (1974).
- 9) Two compatible representations are

$$P_l(\cos \theta) = \sum_{k=0}^{\infty} \frac{(-1)^k (l+k)!}{(k!)^2 (l-k)!} \sin^{2k} \left(\frac{\theta}{2} \right)$$

$$J_0(l\theta) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2} \left(\frac{l\theta}{2} \right)^{2k}$$

Use of the small angle approximation ($\theta \approx \sin \theta$) and Stirling's formula ($N! = \exp(-N) + N \ln(N)$) for large l reduces the first expression to the second.

- 10) J. Butler, private communication.
- 11) J. Tran Than Van, 'Exploratory Dip Systematics in Elastic Scattering', Proceedings of the Eighth Rencontre de Moriond, Vol. 2, 235 (1973).
- 12) A.S. Carroll et al., Phys. Rev. Letters 33, 928 & 932 (1974).
- 13) C.W. Akerlof et al., Physics Letters 52E, 197 (1975).