Three-Dimensional Analysis of Voids in AM60B Magnesium Tensile Bars Using Computed Tomography Imagery

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Abstract

In an effort to increase automobile fuel efficiency as well as decrease the output of harmful greenhouse gases, the automotive industry has recently shown increased interest in cast light metals such as magnesium alloys in an effort to increase weight savings. Currently several magnesium alloys such as AZ91 and AM60B are being used in structural applications for automobiles. However, these magnesium alloys are not as well characterized as other commonly used structural metals such as aluminum.

This dissertation presents a methodology to nondestructively quantify damage accumulation due to void behavior in three dimensions in die-cast magnesium AM60B tensile bars as a function of mechanical load. Computed tomography data was acquired after tensile bars were loaded up to and including failure, and analyzed to characterize void behavior as it relates to damage accumulation. Signal and image processing techniques were used along with a cluster labeling routine to nondestructively quantify damage parameters in three dimensions. Void analyses were performed including void volume distribution characterization, nearest neighbor distance calculations, shape parameters, and volumetric renderings of voids in the alloy. The processed CT data was used to generate input files for use in finite element simulations, both two- and three-dimensional.

The void analyses revealed that the overwhelming source of failure in each tensile bar was a ring of porosity within each bar, possibly due to a solidification front inherent to the casting process. The measured damage parameters related to void nucleation, growth, and coalescence were shown to contribute significantly to total damage accumulation. Void volume distributions were characterized using a Weibull function,
and the spatial distributions of voids were shown to be clustered. Two-dimensional finite element analyses of the tensile bars were used to fine-tune material damage models and a three-dimensional mesh of an extracted portion of one tensile bar including voids was generated from CT data and used as input to a finite element analysis.
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Chapter 1

Background and Motivation

Research and development of cast light metals has become an important priority for U.S. and European car manufacturers in recent years. Cast light metals are being developed for automotive components to increase weight savings in an effort to meet federal and state emissions standards, increase fuel economy, reduce output of greenhouse gases such as CO₂, decrease dependence on foreign oil, and to increase driving affordability in general [Osborne, 2000]. Currently, nearly one-third of all castings shipped each year are to the car and light truck industry [Lessiter, 2000].

Although casting is generally more expensive than other material forming methods, cast parts offer many advantages to wrought or machined parts. In addition to near net shape, some additional advantages include part consolidation, three-dimensional design flexibility, and improved dimensional accuracy [Osborne, 2000]. In addition, parts cast without joints are stronger and more durable than parts with welded joints.

1.1 Cast Magnesium

Cast light metals currently being emphasized in the automotive industry include aluminum alloys, magnesium alloys, and cast light steels [Lessiter, 2000; Van Fleteren,
Of these cast light metals, cast magnesium alloys have huge potential for weight reduction in automobiles. Magnesium is the lightest of all the structural engineering metals, offering a potential 50 to 60% weight savings over commonly used materials. For example, an equal volume part made of magnesium is only 23% the weight of the same part made of steel, and only 64% the weight of the same part made of aluminum. Since 1990 the growth rate for magnesium casting has averaged roughly 10% per year [Cole, 1995]. According to the forecast report by the American Foundrymens Society, cast magnesium is expected to see a large annual growth rate over the next 10 years due to significant growth in automotive applications (see Figure 1.1).

Figure 1.1. American Foundrymens Society (AFS) forecasts for shipments of die-cast magnesium through 2009.
The current 2000 model year automobile design contains an average of 3.6 kg of magnesium [Osborne, 2000]. However, over 100 kg of magnesium applications in an average automobile have been identified. Some of the automotive applications that use magnesium include steering wheels and columns, seat frames, suspension arms, instrument panel substrates, transmission cases, shift cams, intake manifolds, valve covers and wheels. An example of one magnesium automotive application is shown in Figure 1.2. Additional development and use of cast magnesium structures have the potential to reduce approximately 100 kg of vehicle mass, which could reduce emissions by 5%, and reduce fuel consumption by about 2.32 kilometer/liter (1 mile/gallon) [Dolan, 2000]. The relative ease of recycling of magnesium in addition to the weight savings, makes the recent interest in the development of magnesium easy to understand.

Figure 1.2. Magnesium alloy cross car beam used in automobiles (courtesy of R. Osborne, General Motors).
Although interest in cast magnesium alloys is increasing, there remains a lack of fundamental knowledge regarding the behavior of magnesium alloys compared to aluminum alloys as well as a lack of data for accurate simulations and behavior predictions [Froes, et. al, 1998]. An effort is currently underway to improve scientific understanding of magnesium alloys in order to improve casting processes and optimize component designs. For example, it is of utmost importance to understand and eventually refine casting processes in order to decrease porosity inherent to the casting process [Pan, et al., 1991]. Quite simply, as porosity increases, mechanical properties worsen; e.g. the ultimate tensile strength and the percent tensile elongation decrease with increasing porosity [Osborne, 2000]. Figure 1.3 shows the effects of porosity on mechanical properties for a die-cast magnesium AM60 alloy. Thus it is important to not only understand the effect of porosity on mechanical properties, but to determine optimum processing techniques to minimize porosity in cast components. In addition, in order to develop in-use performance simulation models of new cast magnesium components, it is necessary to develop a linkage of the macroscopic mechanical properties to the cast microstructural features (e.g. porosity, cracks, etc.) to accurately characterize the materials.
Figure 1.3. Graphs demonstrating the effect of porosity on mechanical properties of a high pressure die-cast (HPDC) magnesium AM60 alloy (courtesy of R. Osborne, GM). Increasing porosity results in decreased ultimate tensile strength (UTS) and elongation.

While the response of a material to an applied load is inherently three-dimensional in nature, most engineering measurements are taken in only one or two-dimensions. For example, a load-displacement, or stress-strain curve is a one-dimensional measurement of three-dimensional mechanical behavior, while scanning electron microscopy (SEM) and transmission electron microscopy (TEM) images obtained by destructively slicing and polishing a surface of a sample are examples of two-dimensional measurements.

Radiography is a nondestructive technique that compresses three-dimensional information into two-dimensional images. Ultrasound is another nondestructive technique that is often used to determine total volume fraction of porosity while neglecting spatial and morphological information. In order to truly understand the three-
dimensional mechanical response of a material, it is necessary to quantify and understand the three-dimensional microstructure and resulting behavior in three-dimensions. Doing this with a nondestructive evaluation technique has another advantage. Because nondestructive evaluation techniques do not alter the object in any way while evaluating or characterizing it, as their name implies, it is possible to track void behavior and evolving microstructures with time or applied loads.

1.1.1 Optimization of design

One way to characterize or describe a material in three-dimensions is to use mathematical models to describe behavior of the material in simulated in-service conditions. Predictions from these simulations can ultimately lead to optimized processing parameters such as porosity minimizing casting techniques and improved designs of existing components. The predictions can also be used to aid in the development of new improved materials by identifying weaknesses in existing materials, and identifying properties that would improve the behavior of a material. For accuracy, simulations of cast light metals such as magnesium must include damage due to void, or pore, behavior in three dimensions. Damage accumulation in ductile metals such as magnesium subjected to monotonic loading is due to void nucleation, growth and coalescence [Cocks and Ashby, 1982; Garrison and Moody, 1987]. The size or volume,
Shape and spatial distribution of the voids in the material will have a strong influence on the damage evolution, localization and overall mechanical properties of the castings [Pan, et al., 1991; Samuel and Samuel, 1995; Horstemeyer, et al., 2000]. Thus it is critical to understand the three-dimensional behavior of voids in the cast material in order to develop accurate mathematical models that can be used to predict overall material behavior [Garrison and Moody, 1987].

Specifically it is vital to understand void behavior as a function of mechanical load, and its contribution to overall damage in the material. Studying void behavior as a function of mechanical loading can provide the linkage between the microstructural features (such as voids, cracks) and the macroscopic mechanical behavior that is necessary to develop more accurate mathematical models, and thus more reliable predictions of material behaviors.

1.2 Review of Literature

Generally, experiments to determine damage due to void behavior are performed "post-mortem"; in other words, experiments and analysis are performed at some point after damage has already begun, and usually after the material has been failed or otherwise tampered with or destroyed. While examining voids (porosity) in a material after failure may help to confirm or otherwise validate model predictions, this technique
does not provide additional information on void behavior. Specifically, this approach does not provide information on how voids change as a function of stress or load, such as nucleation of new voids, growth of existing voids, or the interactions between voids. Many of these void analyses are performed in two-dimensions, and results are extrapolated to three dimensions using often times broad assumptions. A brief summary of several commonly used void analysis techniques, destructive and nondestructive, is presented below.

1.2.1 Destructive Evaluation

Destructive evaluation is the term given to evaluation or characterization techniques that physically alter the object being evaluated. While these techniques are invaluable as research tools, many of these techniques result not only in physically altered, but destroyed objects. Several commonly used and important destructive evaluation techniques are briefly introduced below.

1.2.1.1 Mechanical loading

One of the most common destructive evaluation techniques used to study damage and determine mechanical properties of cast metals is by mechanically loading the sample, often to failure, and recording behavior, e.g. stress strain curves, force displacement curves, etc. A representative load displacement curve is presented in Figure
1.4 for three magnesium AM60B tensile bars loaded to 60% of their average failure load. Samples can be loaded in a variety of different ways including monotonic uniaxial tension, compression and torsion, cyclic loading to failure, and loading at varying temperatures. This technique is primarily useful in confirming overall macroscopic predictions of failure, but does not contribute significantly to an understanding of void behavior as it relates to internal damage accumulation and ultimate failure. However, in terms of generating information about materials and failure, and in confirming model predictions, this experimental technique is invaluable to material scientists.

Figure 1.4. Load-displacement curves for three magnesium AM60B tensile bars (Series F23 to 25) loaded to 60% of its average failure load.
1.2.1.2 Metallography

Because metals are opaque to light, by far the most common approach to study voids or porosity within cast metals is by analyzing two-dimensional planes (metallographic sections) through the material. The planar sections are obtained by carefully destructively cutting or slicing the sample at specific planes of interest using a thin wafering saw and then finely polishing the exposed surfaces of the sample to reveal microstructural details. The polished planes are then analyzed using an optical microscope. These techniques are commonly referred to as metallography. Figure 1.5 is a representative metallographic image of a failed magnesium AM60B tensile bar that has been cut and polished for analysis.

Figure 1.5. Optical light microscopy image of a cut and polished section of a magnesium alloy AM60B tensile bar H25 after failure at 10X magnification.
Quantitative stereology uses the principles of geometric probabilities to quantify cast microstructures through the analysis of metallographic sections [Underwood, 1981]. Metallography often uses statistical sampling of several planar sections to obtain quantitative information regarding the entire material. These statistics include counting objects either manually or in a semi-automated fashion using a computer, as well as measuring object sizes, spatial distributions and morphologies. However, for the statistics to be reliable, the planar section being analyzed must be representative of the entire volume of material.

Metallography is considered a “post-mortem” analysis technique. In other words, the analysis destroys the sample by slicing it into multiple planar sections, and after the analysis is complete no further information can be obtained. The techniques can be very useful to confirm material model predictions, or to generate information about a material, however it is not possible to say what effect the slicing and polishing has on the properties of the material. Additionally it is impossible to obtain further quantitative dynamic void behavior information.

In addition to being destructive, the two-dimensional nature of the analysis results in the loss of much information about the three-dimensional characteristics of the microstructure. For example, three-dimensional connectivity is difficult to determine, as
is the three-dimensional morphology of structures within the material such as voids and inclusions. However, many important quantitative analyses have been developed using metallography combined with stereological principles. Crepeau, et al. in 1989 provides an excellent overview regarding the use of quantitative stereology to determine various characteristics of cast microstructures such as volume fraction of voids or other objects of interest, surface area per unit volume, and integral mean curvature. In 1994 Gokhale and Drury presented a method for measurement of microstructural surface area using only three appropriately chosen metallographic sections. Gokhale in 1996 presented a method to determine the bivariate size and orientation distribution of microcracks using a limited number of planar sections, and Tewari, et al. in 1998 characterized spatial parameters of micropores in an aluminum alloy, including nearest, second nearest and third nearest neighbor distances.

While most of the techniques above focused on using a limited number of planar sections to statistically determine some characteristics of a material, Tewari and Gokhale in 2000 presented a technique to reconstruct the three dimensional microstructure of a W-Ni-Fe alloy using ninety metallographic sections. The technique consisted of using digital image processing on each of the planar sections to reconstruct the volume of the microstructure. The three-dimensional reconstruction provided information on connectivity and morphology of structures that would otherwise not be seen due to the
opaqueness of the alloy. However, because the sample was destroyed by the analysis, it is not possible to observe further microstructural changes within the material as a function of load or other environmental factors. Additionally, it is impossible to know how the slicing and polishing applied to a material affects the existing microstructure.

1.2.1.3 Fractography

Another common “post-mortem” technique used to study the behavior of voids and their effect on overall damage in a material is through examination of the failure surface [Hull, 1999]. This relatively straightforward technique involves using a microscope (optical, Scanning Electron Microscope, Tunneling Electron Microscope, etc.) to examine the fracture surface. A representative image of the fracture surface of a magnesium AM60B tensile bar acquired using an SEM are presented in Figure 1.6. Certain physical trends observed on the fracture surface can help to understand void behavior and material properties. For example, “dimpling”, or “taffy-like” pulls on the failure surface indicate a ductile type fracture, whereas smooth cleavage planes indicate a more brittle type of fracture. In most cases, fractographic analysis of the fracture features can only be used to suggest the influence of the microstructure on the fracture mechanism, given the size, shape and distributions of the features [Van Stone and Cox, 1976]. Usually a second technique, such as metallography, is used to confirm and directly identify which features affect the fracture process. Attempts have been made to
quantify the fracture features on failure surfaces and Bouchaud (1997) presents an excellent review of some quantitative fractography techniques.

Figure 1.6. SEM image of the fracture surface of AM60B tensile bar H25.

1.2.2 Nondestructive Evaluation

While destructive evaluation techniques are very useful to characterize and evaluate materials, other evaluation techniques exist that are nondestructive. Nondestructive evaluation (NDE) techniques do not physically alter the sample in any way while generating information about the material. Often NDE techniques are non-contact as well as non-destructive. NDE techniques can be useful to characterize the
evolving microstructure in a material, since these techniques can be used before, often
times during, and after loading without affecting material characteristics. Some examples
of commonly used NDE techniques are briefly described in the following sections.

1.2.2.1 Ultrasonic flaw detection

Ultrasonic techniques are commonly used to determine material properties and to
characterize the effects of anomalies such as voids, cracks or inclusions in materials
[Krautkrämer and Krautkrämer, 1990; McIntire, et al., 1991]. In addition, ultrasonic
techniques can provide valuable information regarding the presence of distributed
porosity in a parent matrix. The most common methods for ultrasonic characterization of
porosity involve monitoring, in a pulse-echo configuration, the back-scattered signal from
pores. This technique can provide a relative measure of the amount of porosity present,
and with the proper analytical support can provide an estimate of the average size of the
pores. One of the principal limitations to bulk ultrasonic characterization of porosity is
that the scattering cross section of a given discontinuity (i.e. pore) can be highly sensitive
to the shape. This means that the amplitude of the ultrasonic signal scattered, or
reflected, from a given pore may not give an accurate measure of the defect size [Martin,
et al., 2001]. In addition, in the presence of multiple scattering sites, such as in a porous
solid, the sound waves used for ultrasonic testing may be subject to multiple scattering
events. Thus, an ultrasonic signal that has passed through a porous solid may exhibit the
combined influence of any and all scatterers along the path of the sound waves. The net result is that the use of traditional ultrasonic techniques for acquisition of quantitative data characterizing the size and shape of individual scatterers (pores) in a solid containing a large number of distributed scatterers represents a difficult scenario.

While traditional ultrasonic techniques are somewhat limited in their ability to provide quantitative size, shape and other microstructural information such as orientation for distributed porosity, UT remains a useful tool for analyzing materials and for detection of discontinuities. Numerous investigations have correlated the effect of pore volume fraction on such parameters as the elastic moduli [Rice, 1993A, 1993B; Roth, et al., 1991]. The relations derived from these investigations often incorporate a 'shape factor' to account for the apparent, average pore shape.

Numerous empirical and semi-empirical relations have been applied, with varying degrees of success, to the evaluation of porosity levels in various ceramic and metallic material systems [Sayers and Smith, 1982; Spriggs, 1961; Nagarajan, 1971]. In general, however, these relations provide only average pore characteristics, via some 'fitting parameter', over the entire volume interrogated by the sound waves. Analytical modeling has shown limited use in bridging the gap between these empirical relations and the details of the pore characteristics such as size and shape. Analytical models generally are
forced to make broad assumptions as to the shape, size, and relative distributions of these pore characteristics.

Ultrasonic techniques are often used in combination with metallography or other evaluation techniques to confirm or correlate predicted or observed behaviors. In 2000, Taleff, et al., used pulse-echo ultrasound as a means to distinguish aluminum-magnesium alloys with porosity from alloys without porosity. These results were confirmed using metallography. However, detailed quantitative information about the microstructure of the material was not obtained using this technique. Sophisticated ultrasonic techniques based on phased array and time reversal algorithms are being developed for enhanced detection and characterization capabilities in inhomogeneous media [Fink and Prada, 2000; Chakroun, et al., 1995]. In addition, ultrasonic tomographic techniques are being developed to enhance large volume imaging capabilities, however at this time these techniques are not sufficiently developed to quantitatively characterize voids or other microstructures.

1.2.2.2 Acoustic microscopy

Another nondestructive technique based on ultrasonic principles that is used to characterize materials is acoustic microscopy [Briggs and Kolosov, 1996]. Acoustic microscopy can be used to characterize microstructures of material at high spatial resolutions and to determine elastic properties of materials. This technique works by
generating a short pulse of acoustic waves with a transducer that is brought to a focus using a spherical lens surface. The waves propagate through a coupling medium to the sample, where they are reflected either by features in the material or at its front and back surfaces. The reflected signal is then detected using the same transducer and lens as above. The lens is scanned in a raster pattern over an area of interest in order to generate an image and the reflected signal is used to modulate the brightness at each scanned spot. The frequency of the acoustic wave determines the resolution of the resulting image.

Accurate measurements can be made of the interface between coatings and substrate with a high frequency acoustic microscope with nanosecond pulses [Crossen, et al., 1994]. Acoustic microscopes have been used to characterize electronic components [Pfannschmidt, 1996; Crean, et al., 1995;], to detect small cracks [Knauss, et al., 1995], to characterize surface damage [Warren, et al., 1996], and to determine elastic properties of thin films [Achenbach, et al., 1995], to mention only a few applications. Depth resolution can be obtained by setting a time gate so that only signals from a certain range of specified depths are displayed in an image, which allows for a plane to be selected for inspection. The main disadvantage of this technique is that it is depth limited, dependent on the ultrasonic wavelength used for inspection. This technique can provide relatively high spatial resolution planar (two-dimensional) images, but again does not readily provide volumetric microstructural information.
1.2.2.3 Radiography

X-ray film radiography is an extremely valuable tool used to examine objects. Generally, film radiography is used to look for flaws and other anomalies in objects, such as voids, cracks and inclusions. Because the resolution of film can be between 40 to 80 line pairs per millimeter (lp/mm) for a fine grain film, it is possible to see defects in materials on the order of micrometers. In addition to film radiography, digital radiography is becoming increasingly popular as the technology improves and becomes less expensive. Digital radiography requires a digital X-ray detector, or film radiographs that are digitized using a film digitizer. A digitized film radiograph of three magnesium AM60B tensile bars is presented in Figure 1.7.

![Figure 1.7. Digitized film radiographs of three AM60B tensile bars (Series G23 to 25, from top to bottom, respectively). The darker regions represent less dense material. Note the crack clearly visible in the middle tensile bar, right threaded end.](image-url)
Results can be downloaded immediately to a computer for analysis, and the user is able to change image display parameters such as contrast and magnification in order to more easily pick out objects of interest in an image. These digital detectors and digitized film do not have the excellent spatial-resolution of film, however their ease of use and digital analysis are helping to increase their popularity. The main drawback with radiography is that radiographs are projections, or images of three-dimensional objects compressed into a two-dimensional shadow, and microstructural spatial information cannot be easily retrieved. Baaklini and Bhatt (1994) have developed a laboratory system for testing specimens of metal materials that incorporates a tensile-testing system with a radiography system. This system obtains radiographic images of tensile bars in situ and can note changes in microstructure. However, the information derived from these types of radiographic techniques does not include three-dimensional spatial changes in microstructure that occur with load.

1.2.2.4 X-ray computed tomography

X-ray computed tomography (CT) was developed as a way to extract three-dimensional spatial information from two-dimensional projections (radiographs). Computed tomography involves acquiring multiple radiographic projection images of an object at different angles using digital detectors. These projections are processed and
mathematically reconstructed using a computer to obtain a three-dimensional image or representation of the object. For X-ray CT, the final three-dimensional image is a map of the linear attenuation coefficient that can be examined through specific slice planes of interest. Figure 1.8 presents two slices from a reconstructed CT volume of one magnesium AM60B tensile bar after loading to 60% of its average failure load. A more detailed theory of CT will be presented in Chapter 4.

Figure 1.8. Examples of two slices from a reconstructed CT volume of AM60B tensile bar loaded to 60% of its average failure load. Top: slice through the y-axis; bottom: slice through the z-, or longitudinal axis. The darker regions indicate pores, and regions of lower density material.
CT allows for the dimensional spatial information to be readily obtained, including diameters, spatial locations, and thicknesses. Microstructure can be quantified and analyzed using CT, as shown by Horstemeyer and Gokhale in 1999, Waters, et al., in 1999 and 2000, Martz, et al., in 1999 and many others [Kinney, et al., 1994; 1995]. Taleff, et al., used ultra-high resolution CT to obtain volumetric rendered images of the porosity in an Al-Mg alloy. Because CT data is often volumetric, quantitative 3-D data can be readily obtained and used as input to finite element analyses.

Horstemeyer, et al. obtained statistical parameters related to void behavior based on a volumetric data set for use in two-dimensional finite element analyses, while others such as Bossart, et al. (1996) used a 3-D CT data set to segment out regions of interest to be meshed and used as input to a finite element code for further analysis. Hollister, et al. (1999) also used volumetric CT data of an engine part to create a mesh that was used as input to a finite element code for analysis. Tatum (1999) also used CT data of a pipe joint to create a mesh used as input to a finite element code for analysis.

Most analyses of microstructures using CT are performed on a slice by slice basis despite the volumetric nature of the CT information. While useful in a semi-quantitative sense to search for flaws or anomalies in an object, analyzing in two dimensions does not provide information on the three-dimensional connectivity or morphology of microstructures. Volumetric analyses of CT data have been developed and have recently
received more attention. For example, Kinney, et al., (1992) have used synchrotron radiation as an energy source for X-ray tomographic microscopy (XTM), a high-resolution variant of CT, on biological and other materials. The XTM has been used to look at damage accumulation with loading in metal matrix composites (MMCs) in three dimensions [Kinney, et al., 1992]. Connectivity studies have also been performed on three-dimensional CT volumes, and percolation studies have been performed using CT volumes [Kinney, et al., 1994]. Despite the amount of volumetric information derived from CT data, only recently has there been a larger effort to obtain quantitative three-dimensional engineering information from CT data [Bossart, et al. 1996; Waters, et al. 1999, 2000; Hollister, et al., 1999; Tatum, 1999, Martz, et al. 1999].

1.2.2.5 Other NDE techniques

Various other NDE techniques are available to characterize materials, such as eddy current and thermal imaging. However, as of now, none of these techniques can provide the wealth of three-dimensional data that CT can. Therefore this project uses CT to characterize damage evolution in a material loaded to failure by quantitatively characterizing void behavior in all three dimensions in three magnesium AM60B tensile bars.
1.2.3 Finite element modeling

In addition to experimental techniques available to characterize materials, there are also computational approaches such as finite element analyses commonly used to predict behavior of materials [Chapra and Canale, 1989; Cook, et al., 1989]. In this case, a model of the object of interest is created and meshed to contain many individual connected pieces, called elements. These elements behave independently of each other except at the boundaries. Depending on the geometry of the object, and the resolution of the mesh, the mesh of the object can contain hundreds to millions of elements. An example of a meshed cylinder extracted from a complete CT data set including voids from magnesium AM60B tensile bar H24 loaded to 60% of its average failure load is presented in Figure 1.9.

Elements can be different shapes such as hexahedral, tetrahedral, etc. Simulation studies are then performed on the meshed object, and results can be analyzed. Most finite element models are based on many assumptions such as homogeneous, isotropic material properties and microstructure. This may be due to a lack of detailed, three-dimensional spatial information about the object and its microstructure, as well as the need to make the computations reasonable in terms of computing resources and expense. For example, an analysis of a mesh containing 100 elements will run significantly faster than a mesh
Figure 1.9. Three-dimensional mesh of an extracted cylinder from tensile bar H24 loaded to 60% of its average failure load created from segmented, binary CT data. The mesh contains 808,000 elements, created from an extracted cylinder with a diameter of 100 pixels and length of 102 pixels for a total volume of 42.5 mm$^3$. 
containing 1,000,000 elements. The microstructure of a material dictates the mechanical properties and performance of the material, although typically an average value of some parameter (e.g. average yield strength) is applied to the entire object in a finite element simulation. Thus, it is critical to accurately model the microstructure of the material of interest in three dimensions to ensure accurate simulations. Gall, et al. (2000) presented a finite element analysis of an aluminum alloy, including damage evolution due to silicon particles. Their results showed good agreement with metallographic results, and great potential with respect to accurately modeling damage in cast materials. Some assumptions commonly used in finite element analyses are presented below.

1.2.3.1 Axisymmetric two-dimensional quarter plate approximation

One common way to decrease the amount of information in the model, and therefore the computational expense, is to assume symmetry in the object. For tensile bars, one assumption that is nearly always made is axial symmetry [Lo and Nakamura, 1994; Daoud and Cartwright, 1985]. In addition, often there is an assumed longitudinal symmetry. A two-dimensional axisymmetric quarter plate mesh created to represent one magnesium AM60B tensile bar is presented in Figure 1.10. While these assumptions are often necessary to compensate for limited computer resources, they are generally not reflective of the true nature of the materials. In particular, the microstructure of die-cast metals such as distributions of voids, void sizes, etc., is rarely symmetric, and when
studying damage evolution due to voids, this assumption can lead to large errors, as shown in this project.

![Diagram of a two-dimensional axisymmetric quarter-plate mesh for series H magnesium AM60B tensile bars created to match the notch geometry. Void volume fractions are supplied from the experimental CT data for each hexahedral element.]

**Figure 1.10.** Two-dimensional axisymmetric quarter-plate mesh for series H magnesium AM60B tensile bars created to match the notch geometry. Void volume fractions are supplied from the experimental CT data for each hexahedral element.

**1.2.3.2 Void assumptions**

Another common assumption made when modeling void behavior with finite element analysis is that of spherical void shapes. This assumption is not always valid, and may lead to large errors. But again, spherical voids are much easier to model and less computationally expensive than long, crack-like voids, or nonsymmetric voids. We
have found voids to be far from spherical in the cast magnesium alloy tensile bars under investigation (see Figure 1.11).

![Image of a void](image)

**Figure 1.11** Volumetric rendering of the largest void (0.178 mm³) found in tensile bar H24 loaded to 93% of its average failure load.

Frequently, finite element analyses use random spatial distributions of voids in their models. As will become apparent in this dissertation, this is often not a valid assumption. Spatial distributions of voids in many materials, particularly cast light metals, are often not random, due to the casting process which can result in solidification fronts and obvious spatial orientations of voids.

### 1.3 Thesis Outline and Methodology

Some of the steps required to characterize and eventually predict behavior of materials and to optimize development of new materials are presented in the flowchart in Figure 1.12.
This thesis presents a methodology incorporating CT and digital image processing techniques to quantify void behavior in all three dimensions as a function of uniaxial monotonic loading in die-cast AM60B magnesium alloy tensile bars. Void statistics were obtained and trends were analyzed with respect to void behavior with increasing load. The void statistics are being used to refine and calibrate existing material damage models. CT data was used to quantitatively characterize void distributions in the magnesium alloy tensile bars, and subsequently used to create two- and three-dimensional meshes used as input to finite element analyses. Due to the volumetric nature of the data, regions of interest were easily parsed out and analyzed for individual contributions to overall
damage in the material. The main contribution of this work is that the entire analysis was performed on a three-dimensional volume, as opposed to a slice-by-slice approach, and quantitative numbers descriptive of dynamic void behavior were nondestructively obtained as a function of monotonic load.

Overall damage accumulation in the die-cast AM60B tensile bars was found to be fueled by void growth and coalescence as well as by void nucleation. The void volume distributions were best fit with a Weibull distribution function. In addition, the spatial distribution of voids tended to be more clustered than random. One specific area within each tensile bar contributed more significantly to overall damage than other areas. Two-dimensional finite element analyses were performed and parameters were calibrated to fit with experimental damage results. The two-dimensional analysis showed damage growth in expected areas of the tensile bar. A three-dimensional mesh was created from the segmented CT data and used as input to a finite element analysis code, and simulations revealed that internal voids affected the macroscopic mechanical properties of the object.

The experimental and analytical approach presented in this dissertation provides dynamic and quantitative engineering information about void behavior in a cast magnesium alloy. Quantitative data describing total damage accumulation is being used to refine and calibrate existing material models, to confirm predictions of behavior, and to optimize the development of new materials. The above steps and the methodology
they represent may be used with any number of materials to obtain dynamic information
about evolving microstructures in materials, such as damage accumulation due to voids.
Eventually optimized materials and processing conditions may be determined based
partly on the damage accumulation information presented here.
Chapter 2

Material Models

Cast light metal components are frequently used as structural parts in an attempt to decrease the total weight of a structure, however adequate information does not currently exist about damage progression in cast light metals. Damage here is defined as the total void volume fraction in a material, with increasing damage indicating increasing total void volume in the material and failure occurring at some critical level. Existing material models are frequently used to explain damage accumulation data after damage has occurred. However models of material behavior are not as accurate as they must be in order to predict and explain damage accumulation as a function of loading in cast components. The majority of theories of damage evolution are purely phenomenological with little or no emphasis on the underlying physical mechanisms that bring about damage accumulation, or increasing void volume fraction, and ultimate failure. These physical mechanisms include void nucleation, void growth and interactions between voids such as coalescence. This dissertation attempts to make the connection between experiments and material models. Experimental results from CT data can help to refine
and calibrate material model parameters, resulting in improved, more accurate predictions of material behavior.

2.1 Damage accumulation in ductile metals

Damage accumulation in ductile metals is almost always due to the nucleation, growth and coalescence of voids [Horstemeyer and Gokhale, 1999; Garrison and Moody, 1987]. In order to develop comprehensive models of damage in cast metals several things must occur according to Horstemeyer, et al. (2000). First, the initial distribution of pores and defects must be quantified (note: throughout this dissertation, the terms void and pore will be used synonymously). Next, void nucleation, growth and coalescence behavior must be quantified as a function of deformation. Most voids nucleate at inclusions, precipitates, and other second phases of ductile materials [Palmer and Smith, 1967; Gurland, 1972; Cox and Low, 1974; Hahn and Rosenfeld, 1975]. These nucleated voids, along with existing voids, can then grow, and may interact and coalesce, or grow together with other nearby voids. Figure 2.1 shows a schematic representation of a fictitious ductile metal experiencing void nucleation, growth, and coalescence with increasing strain. Although these three events progress independently or simultaneously, with complex interactions between each, they are often treated in material damage models as separate entities for simplicity.
Finally, the quantified microstructural void information must be correlated to mechanical properties by way of internal state variable evolution equations for plasticity (permanent deformation) and damage (total void volume fraction). When damage accumulation parameters related to void nucleation, growth and coalescence are used in the internal state variable modeling framework, the term continuum damage mechanics is often used. Continuum mechanics is that branch of mechanics dealing with the stresses in solids, liquids and gases and the deformation or flow of these materials. Continuum mechanics describes relationships between gross phenomena, neglecting the structure of material on
a smaller scale and regarding the matter as indefinitely divisible. In other words, a continuum description disregards the molecular structure of matter, assuming no gaps or empty spaces, and furthermore assumes that all mathematical functions used are continuous functions, as well as all derivatives of functions. A continuum mechanics approach is generally adequate for the analysis of stress and deformation in most engineering problems, such as the one presented here [Malvern, 1969].

Internal state variables relate to the internal structure rearrangement in a material, i.e. the composition, morphology, damage accumulation due to void behavior, and internal stresses, and are those variables used in constitutive material models that are observable and may be measured. Internal state variables are useful to model collective effects of changing material structure involving multiple mechanisms at multiple length scales. For our purposes, the length scale of 30 micrometers to several millimeters is of most interest and can be considered "mesoscale". This length scale includes the effects of initial porosity due to casting as well as void-void interactions such as coalescence, void volume, shape and spatial distributions. The spatial scale length may be considered arbitrary when identifying a void nucleation event. The phenomena of void nucleation is defined as the creation of a void where previously there was none. However, a void cannot be identified until it is measured, and the ability of an experimental instrument to identify a void is directly related to the contrast sensitivity and spatial resolution of the
system. Therefore, a void nucleation event is defined by the contrast sensitivity and spatial resolution of the system used to measure voids. Any small void that is not selected as a void by the methodology described in this dissertation is not measured by the system, and is then, by the definition used here, not a void. A void that is just large enough to be measured by the system is by definition a void; if it was not measured in a previous state and is measured in a new state, it can be defined as a nucleation event. When defining nucleation events, it is critical to be consistent in the definition and implementation in the statistical void analysis.

2.2 Continuum damage mechanics

A brief development of the kinematics in the continuum damage mechanics framework is presented below and follows that of Davison, et al. (1977) and Bammann and Aifantis (1989). This development assumes the reader is familiar with continuum mechanics, and the tensors presented below are defined in traditional continuum mechanics textbooks (a letter with an underscore represents a second-order tensor, while a letter without an underscore is a scalar) [Lai, et al., 1993; Malvern, 1969; Gurtin, 1981]. In order to use a continuum damage mechanics approach to model damage accumulation in a material, it is necessary to relate damage in terms of some parameter to the kinematics of the material. In particular, it is necessary to directly relate damage to the
volumetric rate of deformation tensor. After the damage is related to the volumetric rate
of deformation, a further description of damage can be defined. It is useful to again make
clear the definition of damage used here. The measure of damage used here is the total
volume or area of the voids to the total volume or area of dense material including voids,
known as the void volume fraction, and is represented by the symbol \( \phi \).

Material point motion is described by elastic straining, inelastic flow, and
formation and growth of damage or void volume fraction. The deformation gradient, \( E \),
is a tensor that can be used to describe deformation of an object including rotation and
stretching. For this continuum damage model development, \( E \) can be described by three
separate deformation gradients; the elastic deformation gradient, \( E^e \); a set of continuously
distributed dislocations whose motion produces permanent shape changes or deformation
without a change in total volume, measured by \( E^p \) (isochoric inelastic or plastic); and a
continuous distribution of voids contributing to permanent total volume changes
described by \( E^d \) (dilatational inelastic or plastic). The multiplicative decomposition of
the deformation gradient is then given by

$$ E = E^e E^p E^d, $$

(2.1)

and is represented schematically in Figure 2.2. From Figure 2.2, the initial state, or state
0, goes to state 1 which represents shape changes without a change in volume, or
deformation due to dislocations described by $E_d$. Thus, the volume of state 0 equals the volume in state 1. The next intermediate configuration, or state 2, includes deformations that include volume changes due to damage accumulation caused by void nucleation, growth and coalescence, $E_v^p$. Finally, state 3, or the final configuration occurs after the final elastic deformation, $F^e$. This decomposition assumes that the motion of the body is described by a smooth displacement function, which precludes the initiation of discrete failure surfaces yet still allows a continuum description of damage accumulation.

![Figure 2.2 Schematic drawing of the multiplicative decomposition of the deformation gradient, $E$.](image)

To relate the damage accumulation due to voids to the kinematics, it is necessary to express $E_v^p$, the deformation gradient associated with dilatational plastic deformation due to voids, as a function of void nucleation and growth. The total damage, or the
volume of voids per total volume (volume of material including voids), is represented by the term $\phi$. Damage, $\phi$, can be defined as the ratio of the change in volume of an object in the elastically unloaded state (State 2) from its volume in the initial reference state, or the ratio of the total volume due to voids to its total volume in the elastically unloaded state. This can be expressed as

$$\phi = \frac{V_v}{V_2}. \quad (2.2)$$

Assume the volume and density of the material specimen in a reference, or initial configuration, state 0, are given as $V_0$ and $\rho_0$, respectively, and the volume and density of the material in some intermediate configuration, state 2, are given by $V_2$ and $\rho_2$. Then the change in total volume due to void nucleation, growth and coalescence from the reference configuration to the intermediate configuration is described by

$$V_2 = V_0 + V_v, \quad (2.3)$$

where $V_v$ is defined as the added volume due to voids. When transforming the configuration from State 0 to State 2, an added volume of voids, $V_v$, is introduced to the total volume $V_2$, however the volume of the solid matter remains unchanged due to inelastic incompressibility (conservation of mass).

The Jacobian of the deformation gradient, $J$, is related to the change in volume or density for a constant mass and is defined as
and must be positive. Given the definition of damage presented in equation 2.2, the volume in the reference state, \( V_0 \), can be expressed as \( V_0 = (1 - \phi) V_2 \), and then the Jacobian, \( J \), is determined by the damage, \( \phi \), as

\[
J = \det \mathbf{E}_v^p = \frac{1}{(1 - \phi)}.
\]  

We assume damage is restricted to produce isotropic dilatation, or volumetric changes only due to void nucleation and growth, which, when combined with equation 2.5, results in the volumetric part of the deformation gradient expressed as

\[
\mathbf{E}_v^p = \frac{1}{(1 - \phi)^{1/3}} I, \text{ or } \mathbf{E}_v^p = \Phi I,
\]  

where \( \Phi = (1 - \phi)^{-1/3} \), and \( I \) is the identity matrix. The corresponding velocity gradient that is associated with the deformation gradient becomes (note dots over some terms denote derivatives):

\[
\mathbf{L}_v^p = \dot{\mathbf{E}}_v^p \cdot I = \frac{\phi}{3(1 - \phi)} I.
\]  

The velocity gradient \( \mathbf{L} \) can be decomposed into a symmetric part commonly known as the stretching tensor, or plastic volumetric rate of deformation tensor, \( \mathbf{D} \), and an antisymmetric part known as the spin tensor, \( \mathbf{W} \). From equation (2.7), the stretching tensor, i.e., the plastic volumetric rate of deformation, is then defined as

\[
J = \det \mathbf{E}_v^p = \frac{V_2}{V_0} = \frac{\rho_0}{\rho_2}, \quad (2.4)
\]
\[
D^p = \frac{\phi}{3(1-\phi)} I. \quad (2.8)
\]

The spin tensor, \( W^p \), related to the rate of rotation, vanishes, indicating there is no volumetric component to the spin tensor. The trace of the stretching tensor given above is

\[
\text{tr}(D^p) = \frac{\phi}{(1-\phi)}, \quad (2.9)
\]

and now directly relates the damage, \( \phi \), to the plastic volumetric rate of deformation, the necessary goal in order to use continuum mechanics to describe damage accumulation.

Additionally, the elastic rate of deformation, \( D^e \), relates to the volumetric rate of deformation, \( D^v \), and the isochoric plastic rate of deformation, \( D^\rho \), by additive decomposition as \( D^e = D - D^v - D^\rho \). Analogous formulas hold for the elastic and deviatoric parts of the velocity gradients, expressed as \( L^e = \dot{F}^e F^{-1} \), and \( L^\rho = F^e \dot{F}^\rho F^{-1} F^e F^{-1} \) (note dots or derivatives for some terms) when there is no plastic spin and the elastic spin equals the total spin. The total velocity gradient, \( L = \dot{F} F^{-1} \), can be expressed as \( L = L^e + L^v + L^\rho \).

### 2.3 Damage, \( \phi \)

The above development directly relates the plastic volumetric rate of deformation to the damage, \( \phi \), a step that is necessary to use continuum damage mechanics. At this
point, it is possible to describe damage accumulation in terms of void nucleation, void
growth and void coalescence in the unstressed intermediate configuration, state 2. Let $N$
be the total number of voids in an initial representative continuum volume of material,
$V_0$, and let $\eta^*$ be the number of voids per unit volume in the reference configuration.

Then the void density is defined as

$$\eta^* = \frac{N}{V_0}. \quad (2.10)$$

The average void volume is defined as

$$v_v = \frac{1}{N} \sum_{i=1}^{N} v_i, \quad (2.11)$$

with $v_i$ defined as the void volume of each void that has been nucleated. Thus the total
volume of voids is given by $V_v = \eta^* V_0 v_v$. Inserting this into the definition of damage,

$$\phi = \frac{V_v}{V_2},$$

gives

$$\phi = \frac{\eta^* V_0 v_v}{V_0 + \eta^* V_0 v_v} = \frac{\eta^* v_v}{1 + \eta^* v_v}. \quad (2.12)$$

This definition of damage refers to the reference configuration, state 0. If the number of
voids per unit volume is defined in the intermediate configuration, state 2, then

$$\phi = \frac{V_v}{V_2} = \frac{V_v N}{N V_2} = v_v \eta, \quad (2.13)$$
where $\eta = \frac{N}{V_2} = \frac{N}{V_0 \cdot N} = \eta \cdot \frac{V_0}{V_2}$.

Most existing damage models contain nucleation and growth terms in some combination. The damage model used in this project adds a coalescence term to the other two terms. In a phenomenological manner, this can be included in the damage term as $\phi = \eta \cdot c$, with $c$ designating the coalescence behavior. It is a multiplicative model, meaning the void growth affects the void nucleation, which in turn has effects on void coalescence. The model has been developed in greater detail elsewhere [Horstemeyer and Gokhale, 1999; Horstemeyer, et al., 2000], but a brief description of each of the three components used to describe damage in this model is presented below.

### 2.3.1 Void nucleation

In ductile metals, voids nucleate at sites of local microscale stress raisers such as inclusions, precipitates, and other second phases, or at intersections of slip bands, grain boundaries, twin boundaries, and vacancy clusters. This nucleation model is related to void nucleation from second phase particles, which is assumed to be the dominant nucleation site. Nucleation of voids is assumed to occur by decohesion of the particle/matrix interface or by particle fracture, and more than one void can be nucleated at a given particle. The void nucleation model below is a function of a length scale parameter (in this case the mean size of inclusions in the magnesium, $d$), volume fraction
of second phase materials \( f \), stress state (invariants of stress, \( I \) and \( J \)), strain rate \( \varepsilon(t) \) and fracture toughness \( K_{IC} \).

The nucleation term developed by Horstemeyer and Gokhale (1999) can be expressed as

\[
\eta(t) = c_{\text{coeff}} \exp \left[ \frac{\varepsilon(t)d^{4/3}}{K_{IC} f^{4/3}} \left( a \left( \frac{4}{27} - \frac{J_3}{J_2^3} \right) + b \frac{2J_3}{J_2^{3/2}} + c \left( \frac{I_{1}}{\sqrt{J_2}} \right)^3 \right) \right] \exp \left( \frac{-T}{C_{\text{const}}} \right),
\]

where \( \eta(t) \) is the void nucleation density, \( \varepsilon(t) \) is the strain rate at time \( t \), and \( C_{\text{coeff}} \) is a material constant related to the initial void nucleation level. The material parameters \( a \), \( b \), and \( c \) relate to the volume fraction of nucleation events arising from local microstresses in the material and are determined experimentally from tension, compression and torsion tests in which the number density of void sites is measured at different strain levels. The stress invariants, denoted by \( I_1 \), \( J_2 \), and \( J_3 \), capture the stress state dependence on damage evolution. The volume fraction of the second phase material is \( f \), the average particle size is \( d \), and the bulk fracture toughness is \( K_{IC} \).

Horstemeyer and Gokhale (1999) have shown that the rate of nucleation decreases in a nonlinear fashion as initial volume fraction of second phase particles increases. Void nucleation occurs first at larger particles for a number of different materials, resulting in increased accumulation of total damage with increases in particle size as a function of strain. In addition, they have shown the differing effects on damage accumulation as a
function of stress state, e.g. tension, torsion and compression. This nucleation model is the only one to distinguish various stress states in its analysis. In addition it allows for nucleation to be a function of stress triaxiality.

The data acquired using CT in this project can be used to calibrate and refine the nucleation model by supplying void density data (number of voids per unit volume) as a function of increasing stress.

2.3.2 Void growth

Many void growth rules have been developed and studied [Tvergaard, 1990; Cocks and Ashby, 1980] however none of these can comprehensively deal with different levels of stress triaxialities, different hardening rates, different strain rates, and different temperature regimes. The void growth rule used in this project accounts for triaxiality and was developed by McClintock (1968). It is expressed as

$$v_v = \frac{4}{3} \left( R_0 \exp \left[ \frac{\varepsilon(t)}{2(1-n)} \sinh \left( \frac{\sqrt{3}}{3} \left[ \frac{\sqrt{2 J_2}}{\sqrt{3}} \right] \right) \right] \right)^3,$$

(2.15)

where the material constant $n^*$ is related to the strain hardening exponent and is determined from tension tests. $R_0$ is the initial radius of the voids, and all other variables are the same as defined above. According to the McClintock model, void volume grows as the strain and/or stress triaxiality (hydrostatic stress divided by deviatoric stress) increases. The void growth rate dependency on stress triaxiality is important to note here.
For the notched Bridgman tensile bars examined in this project, the maximum stress triaxiality occurs at the center of the necked region and hence, the largest rates of void growth would be expected there. The model also allows for voids to grow in tension but not in compression or torsion, which complies with experimental observations [Horstemeyer, et al., 2000].

The CT data can be used to refine the void growth rule by supplying initial void volumes as well as average void volumes as a function of stress and strain states.

2.3.3 Void coalescence

Coalescence is the joining of voids either at the microscale or at the macroscale, and can be a significant contributor to damage accumulation. Generally, voids coalesce by two main mechanisms. The first, referred to as void impingement, occurs when two neighboring voids grow together until they join as one, larger, void. The area between them gradually necks down to nothing. The second mechanism, called “void sheet” coalescence, occurs when a localized shear band occurs between two neighboring voids. The void sheet mechanism is related to particles nucleating small voids in between two larger voids as the larger voids impose their influence on the surrounding region. Therefore, coalescence is a function of both nucleation and void growth. The two different void coalescence mechanisms are presented schematically in Figure 2.3.
voids are dilute in a material, coalescence does not occur. When coalescence does occur, it causes a discontinuous jump in the nucleation evolution and growth evolution, as two voids become one larger void. However, these discontinuities are in the discrete regions within nucleation and growth rules and do not effect the damage rate equations, which evolve as internal state variables at a higher length scale and thus are continuous functions.

![Diagram of void coalescence mechanisms](image)

**Figure 2.3** Two different void coalescence mechanisms that have been observed in various materials. Left: natural coalescence, where two voids grow into one larger void. Right: void sheet mechanism, where a line of smaller voids form between two larger voids, eventually connecting the two larger voids as one.

Coalescence is generally defined as some function of the void growth and nucleation, $c = I + f(\eta, v)$, which in the limiting case of natural or simple coalescence is
equal to one, and can be changed to suit the material of interest. An example, of a more complicated coalescence function is

\[ c = (c_{\text{simple}} + c_{\text{coal}} \eta \nu) \exp\left(\frac{c_{\text{temp}}}{T}\right), \quad (2.16) \]

with coalescence terms due to temperature and other factors [Horstemeyer, 2000]. The coalescence parameter \( c_{\text{temp}} \) is determined experimentally, while the remaining two coalescence terms, \( c_{\text{simple}} \) and \( c_{\text{coal}} \) are determined numerically.

The CT data can be used to refine coalescence models by supplying nearest neighbor distances, and by providing information on discontinuous trends in the data.

2.4 Finite element applications

The advantage to using CT data to provide parameters for each of these three damage components lies in the three-dimensional nature of the analysis as well as the ability of CT to provide these parameters as a function of stress state. These data can be used to calibrate and “fine-tune” the models to better predict behavior of materials such as magnesium AM60B tensile bars.

The damage model presented above can be used in conjunction with material models such as the plasticity model developed by Bammann, et.al (1996), by modifying the plasticity model to account for stress state dependent damage evolution (see Horstemeyer et al., 2000). When implementing the plasticity-damage model into finite
element codes, one assumption that is commonly used is that as damage, \( \phi \), approaches unity, failure is assumed to occur within an element. While materials engineers designate failure at less than unity, it has been shown that after a few percent void volume fraction the damage goes rapidly to unity. While more complicated functions for the damaged elastic modulus are available, they are computationally expensive.

This damage model has previously been used in finite element analyses by Horstemeyer, et al., (2000) and results were compared with experimental data from notched Bridgman tensile bars. Quarter "plate" axisymmetric analyses using the finite element code ABAQUS were performed to study the effects of void nucleation, growth and coalescence. Model parameters were determined for cast A356 aluminum tensile bars by experimental observations. The models were shown to agree well with the experimental observations of damage accumulation from metallography, optical microscopy and mechanical tests in tension, compression and torsion. This indicates that the damage model presented here can accurately model damage accumulation in ductile metals such as magnesium AM60B due to void nucleation, growth, and coalescence.
Chapter 3

Die-cast Magnesium AM60B

Magnesium is a commonly used metal, both structurally and nonstructurally. Nonstructurally, magnesium is commonly used as an alloying agent with aluminum, zinc, lead, and other nonferrous metals. Some other nonstructural uses include some limited use in pyrotechnics, cathodic protection of other metals from corrosion, as an oxygen scavenger and a desulfurizer in the manufacture of nickel and copper alloys, and as a desulfurizer in the iron and steel industry [Polmear, 1999]. However, because of its good strength and stiffness at both room and elevated temperatures, and its extremely light weight compared to other structural metals, magnesium is increasingly being used in structural applications such as aircraft landing wheels, helicopter components, and in appliances and sporting goods, to name a few [Froes, et al., 1998; Eliezer, et al., 1998].

3.1 Magnesium production

Magnesium is produced from seawater, brines and magnesium-bearing minerals such as dolomite. Seawater contains about 0.13% magnesium by weight, which means there is a virtually unlimited supply [Shigley, 1951]. For example, at 1998 world use levels it was estimated that from the Dead Sea alone, there is enough magnesium to
satisfy demand for at least 22,000 years [Froes, et al., 1998; Eliezer, et al., 1998]. There are two principal extraction processes used today. These are the electrolyses of molten magnesium chloride and the thermal reduction of magnesium oxide [Cameron, et al., 1987; Desikan, 1985]. The majority of magnesium production is by the electrolytic method.

3.2 General properties of magnesium and its alloys

Magnesium is the world's lightest structural metal, with only two-thirds the density of aluminum [VanFleteren, 1996]. Magnesium has a hexagonal close-packed (HCP) crystal structure (see Figure 3.1). When coupled with large and variable grain sizes, the HCP structure has in the past lead to less than optimum mechanical properties. For example, the hexagonal close-packed structure limits its suitability for cold working, it can have poor corrosion resistance, and a low elastic modulus, as well as upper service temperature limits. On the other hand, magnesium alloys can be wrought, forged, extruded, and sheet and plate rolled. Magnesium has sufficient hardness for structural application, and the machinability of magnesium and its alloys is outstanding, requiring approximately 50% less power than machining aluminum [Avedesian and Baker, 1999]. Generally, as temperature decreases, the tensile strength and yield strength of magnesium and its alloys increases, while the ductility decreases. Conversely, increasing temperature
has an adverse affect on mechanical properties [Aune, 1995]. While the weight savings with magnesium has the potential to be enormous, in comparison with aluminum the tensile strength of magnesium is approximately 25% less, fatigue behavior (number of cycles to failure) is 35% less, hardness 30%, modulus 40%, and thermal expansion 15% less. Many magnesium alloys have been developed and continue to be researched and developed in an effort to improve mechanical properties.

Figure 3.1 Example of Hexagonal Close-Packed (HCP) crystal structure. The hexagonal planes are stacked such that atoms in successive planes nestle in triangular "grooves" of the preceding plane. There are six of these "grooves" surrounding each atom in the hexagonal plane, but only three of them can be covered by atoms in the adjacent plane. The first plane is labeled "A" and the second plane is labeled "B", and the ratio of c/a = 1.633. If the third plane is again in the "A" orientation and succeeding planes are stacked in the repeating pattern ABABA..., the resulting close-packed structure is HCP.

3.3 Magnesium alloys

Unalloyed magnesium is not commonly used as a structural material due to its tremendous susceptibility to galvanic corrosion and combustibility. There exists a
multitude of magnesium alloys. The naming convention is as follows. The first two letters of the alloy represent the two most abundant alloying elements used. For example, AM represents aluminum manganese. The numbers that follow the letters are the weight percentages of the alloying elements rounded off to the nearest whole number. For example, AM60, which is the alloy studied in this project, designates by weight percent an aluminum content of approximately 6% (5.67%), and a manganese content of close to 0% (0.377%). The letter following the numbers is used to distinguish between different alloys as compositions become standard. For example, AM60B would indicate that it is the second alloy composition developed containing approximately 6% aluminum and 0% manganese. Often a fourth identifier is included to indicate condition such as temper.

While magnesium alloys can be produced by forging, extruding, and sheet and plate rolling, most commonly used magnesium alloys are produced by casting. Some magnesium alloys commonly used for casting include: aluminum-manganese, with and without silicon or with zinc (abbreviated AS, AM, and AZ, respectively); zirconium (abbreviated by K), zinc-zirconium with and without rare earths (ZK, ZE, and EZ, respectively), thorium-zirconium with and without zinc (HK, HZ, and ZH, respectively) and silver-zirconium with rare earths or thorium (QE and QH, respectively). In these alloys, aluminum is added to improve strength, hardness and castability. Zinc also improves castability as well as increases corrosion resistance and with aluminum,
improves mechanical properties. Manganese is generally added up to its solubility limit, usually around 0.3% in the presence of aluminum, to improve corrosion resistance.

3.4 Die-casting magnesium alloys

The commonly used die-casting technique, often referred to as high pressure die-casting, consists of forcing a liquid metal into a mold under pressure and at a relatively high velocity. The liquid metal is allowed to solidify with the pressure maintained. The mold usually consists of two-pieces, made of steel and referred to as a die. When the two pieces are clamped together, they form the desired shape. After the liquid metal has solidified, the die is opened and the cast piece is removed. The technique allows for rapid casting rates, making it an inexpensive method. However, the metal used must have a low melting temperature, and the cast pieces produced must be relatively small [Callister, 1994].

Magnesium alloys are particularly well suited for casting for a variety of reasons. The alloys show high fluidity, which allows for thinner-walled cast sections, as well as complex and intricate three-dimensional designs [Osborne, 2000]. In addition, magnesium has a low specific heat per unit volume compared to other metals which allows for faster cycle times and reduced die wear. Finally, iron from the dies has low solubility in magnesium alloys, which reduces sticking. While castings are the preferred
method for producing magnesium products, there are several drawbacks. First, molten magnesium is combustible and therefore certain precautions must be taken while casting magnesium alloys. Second, and the most troublesome, is the inherent porosity that cast alloys contain. This porosity can be due to many factors from the casting process, including local feeding obstructions through dendritic solidification fronts, trapped gases, or temperature gradient driven solidification shrinkage [Campbell, 1997; Pan, 1995]. These pores or voids can exist at length scales ranging from sub-micrometer to several hundred micrometers, depending on the solidification process.

3.4.1 Casting porosity

As mentioned above, the casting process results in porosity due to the nature of the technique. The primary causes of porosity in castings are due to trapped gases and solidification shrinkage. Gas porosity can be caused by nucleation of gas bubbles caused by random, thermal atomic fluctuations within the liquid. In addition, pre-existing suspensions of bubbles may exist in the liquid resulting in porosity upon solidification [Campbell, 1997]. Gas bubbles can also form in crevices of a mold in contact with the liquid metal and then bubble into the metal, where they become trapped during the early stages of solidification. Hydrogen gas is the only gas with more than negligible solubility in magnesium and the only gas associated with porosity. When the hydrogen
concentration in the liquid alloy exceeds a critical value it nucleates gas bubbles and the bubbles within the liquid then grow, being controlled mainly by the rate of diffusion of gases through the metal [Bakke, et. al, 1990]. The resulting volume of porosity is inversely proportional to the pressure applied to it during its growth.

For the vast majority of cast materials, shrinkage porosity is the most common and most important defect in castings. In the casting process, there are three different contractions of material, or shrinkages observed. The first is liquid contraction, where the volume of the liquid metal decreases almost linearly with decreasing temperature. This contraction is generally not an issue, since the casting process compensates for the reduced volume by providing more liquid metal through a feed. The second contraction, solidification contraction, occurs at the freezing point and causes many problems in die-cast materials. Generally, liquids contract on freezing due to the rearrangement of atoms from a more random arrangement to an ordered crystalline array. The densest solids are those having cubic close-packed symmetry, such as magnesium which has a hexagonal close-packed structure. For example, the density of liquid magnesium is 1500 kg/m³, but the density of solid magnesium is 1655 kg/m³ which results in a total volume shrinkage of 4.2% [Avedesian, 1999]. Therefore, as the liquid metal solidifies, if no additional liquid metal is supplied to compensate for the shrinkage, the resulting casting volume is smaller than the mold volume by 4.2%.
The third and final shrinkage is solid contraction. As cooling progresses and the casting attempts to reduce its volume as a result of the solidification contraction, it is constrained to some extent by the mold through sticking. This constraint results in a larger than expected casting volume than that due to solidification shrinkage, which is expected to be 4.2% less volume. Consider a spherical mold injected with liquid metal. As the outermost shell of liquid metal solidifies, the innermost part of the metal in the spherical mold is still in liquid state. Assuming the outer solidified shell is attached to the die, as successive inner shells begin to solidify the reduced volume the solid metal occupies with respect to the liquid metal means that either a pore has to form or the solid metal has to expand a little and the liquid metal has to contract a little. While shrinkage porosity can be reduced by improving the cleanliness of the metal (minimizing void nucleation sites such as pieces of refractory, etc.), and applying sufficient pressure, the other approach to solving this problem is through adequate feeding mechanisms.

3.4.2 Effects of porosity

Porosity resulting from the casting process can have large effects on the behavior and performance of castings. The size or volume of the voids or pores is often not as important as their form (including shape and size) and position. For example, a large pore located in a low stressed region of a casting will most likely have negligible effects
on the properties of the casting, while a group of many smaller pores located in a sharp corner or other region subjected to high stresses may have catastrophic effects [Campbell, 1997]. Therefore, it is not necessary or even reasonable to attempt to remove all defects from a casting. It is more important to specify the regions that must be required to contain no, or minimal porosity in order to preserve performance of the casting. Thus the art of casting is to select the casting parameters in such a way as to prevent porosity from forming in regions that will be subjected to high stress.

The mechanical properties of a casting can be affected in many ways by porosity. Yield stress is relatively unaffected by porosity because by definition, no substantial yield has taken place yet in the material. Pores lead to a reduction in the total material volume of the casting, however porosity in most castings is usually only a few percent, and the reduction is minimal. The fracture toughness of a material, $K_{IC}$, loosely speaking, is a measure of the ability of a material to absorb energy up to fracture and has dimensions of stress times the square root of length. The fracture toughness of a casting is a material property, theoretically independent of defects, and therefore may be used to predict shapes and critical sizes of defects that will affect the fracture toughness of the object.

To determine the critical defect size that can be tolerated using the fracture toughness, the fracture toughness and the applied stress must be known. The largest defect size that can be tolerated, $d$, can be calculated from the equation $d = 2K_{IC}^2 / \pi\sigma^2$, where $\sigma$ is the
applied stress. Any defects smaller than $d$ can then be considered to have no effect on fracture toughness [Campbell, 1997].

The fatigue performance, or behavior with cyclic loading to failure, of cast alloys is relatively poor due to the fast initiation of fatigue cracks. However, the growth is slow compared to wrought alloys, as the crack must negotiate its way through the defects (voids, cracks, inclusions, etc.). Ductility decreases drastically with defects of any kind. Particle defects such as inclusions can quickly lead to voids due to decohesion from the matrix, and eventual failure. Several simple ductility models have been developed that help to explain the effect of porosity on ductility. Figure 3.2 presents a simple model of ductility illustrating the effects of porosity on elongation to failure.
Figure 3.2 A schematic of a simple ductile failure model, representing a solid homogeneous specimen with width \( l \), pore width \( d \), pore spacing \( s \), and elongation to failure \( e \). (a) solid necks down to 100\% reduction in area; (b) a single large pore leads to a "cup and cone" fracture; and (c) an array of smaller pores effectively serves to "perforate" the sample. From above it is clear that the elongation to failure \( e \) is directly related to the area or volume, quantity and spatial distribution of pores.

The ultimate tensile strength of a casting, which is a complex combination of ductility, work hardening and yield stress, is more difficult to understand as a function of increased porosity. The mechanical properties of a die cast material depend strongly on the specific casting technique and casting parameters (e.g. pressure, temperature, feed rate, etc.) used. The effect of the casting parameters may be greater than the effect of the alloying element concentration [Aune, 1995], and thus it is important to specify the casting process and parameters used when characterizing a die cast object.
3.5 Magnesium AM60B tensile bars

The magnesium alloy studied in this project is die-cast AM60B. The exact composition of the tensile bars obtained, expressed in weight percent, is 5.67% Al, 0.377% Mn, 0.15% Zn, 0.013% Si, 0.0011% Fe, <0.0035% Cu, <0.0001% Ni, <0.0033% Ca, and the remainder Mg. Die-cast AM60B magnesium alloy is commonly used for production of automotive components and other parts requiring good elongation and toughness combined with reasonable yield and tensile properties.

Notched Bridgman tensile bars were obtained from the Institute of Magnesium Technology in Quebec, Canada. Nine AM60B magnesium tensile bars with three different notch geometries were obtained, three bars for each notch geometry. Each set of three tensile bars was part of a larger group of more than 25 tensile bars for each notch geometry. The notch radii were 0.635 cm, 0.794 cm and 1.27 cm. The different notch geometries were designated for this study as F23-25, G23-25, and H23-25, in decreasing order of notch radius. The tensile bars were 11.4 cm in length, with an outer diameter of 1.27 cm. A picture of one tensile bar from each of the three notch geometries is shown in Figure 3.3.

The samples were cast using a 600-ton cold chamber die-casting machine with a 600-ton locking force and an injection temperature between 675 and 690 degrees Celsius.
The metal temperature was 750 degrees Celsius, and the die temperature was 300 degrees Celsius. The shot weight was 1.63 kg, the shot sleeve diameter was 3.81 cm, with a shot stroke of 29.21 cm. The average gate velocity was 41.15 m/s, with an average cycle time of 45 seconds.

Figure 3.3 Three magnesium AM60B notched Bridgman tensile bars, one each from three notch geometries. Top: Series H, notch radius is 0.635 cm. Middle: Series G, notch radius is 0.794 cm. Bottom: Series F, notch radius is 1.27 cm. Total length of tensile bars is 11.4 cm, with an outer diameter (excluding threads) of 1.265 cm.

3.5.1 Mechanical properties of AM60B

The magnesium alloy AM60B is being used more frequently for structural automotive applications in place of the more common alloy AZ91. The reason for this is better elongation and toughness than AZ91. However AM60B has lower tensile and yield strengths than AZ91. A table with several mechanical properties of AM60B as compared with AZ91 is presented in table 3.1.
Table 3.1 Typical mechanical properties of separately die-cast test bars of AM60 and AZ91 magnesium alloys (source: Magnesium Handbook, 1999).

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Tensile strength</th>
<th>Yield strength</th>
<th>Elongation*</th>
<th>Impact strength†</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPa</td>
<td>ksi†</td>
<td>MPa</td>
<td>ksi†</td>
</tr>
<tr>
<td>AM60</td>
<td>240</td>
<td>35</td>
<td>130</td>
<td>19</td>
</tr>
<tr>
<td>AZ91</td>
<td>250</td>
<td>36</td>
<td>160</td>
<td>23</td>
</tr>
</tbody>
</table>

*Elongation is in 50.8 mm (2 in) †Unnotched Charpy test bars ‡thousand pounds per square inch

AM60B is currently used in the automotive industry for automotive panel supports, seat frames, and steering wheel armatures. At the present time, AZ91 is more commonly used than AM60B, for accessory drive brackets, clutch housings, door mirror brackets, headlamp retainers, upper and lower inlet manifolds, oil filter adapter housings, power window regulator housings, and many other components. Essentially all of the magnesium components currently being used in automobiles, and those under development, contain aluminum as the principal alloying element in an effort to improve castability as well as mechanical properties.

3.5.2 Loading of tensile bars

Prior to obtaining the tensile bars, Westmoreland Mechanical Testing and Research Laboratory in Youngstown, Pennsylvania experimentally determined average failure loads, and stress strain behavior for each of the three notch geometries using tensile bars from the larger set of 25. The average ultimate tensile strengths, \( \sigma_{UTS} \), for the notch geometries were determined and are presented along with calculated percentages of average failure loads in table 3.2. From the average values of ultimate tensile strength,
percentages of failure loads were determined and selected for analysis. The percentages selected for analysis were 60, 87, 93, and 95-97% of the predetermined average failure load.

**Table 3.2** Average ultimate tensile strengths and percentages of average failure load for each notch geometry as determined experimentally by Westmoreland Mechanical Testing and Research Laboratory.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\sigma_{UTS}$ (MPa)</th>
<th>60% load (N)</th>
<th>87% load (N)</th>
<th>93% load (N)</th>
<th>95% load (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>241</td>
<td>9879</td>
<td>14327</td>
<td>15314</td>
<td>15642</td>
</tr>
<tr>
<td>G</td>
<td>221</td>
<td>9283</td>
<td>13460</td>
<td>14389</td>
<td>14698</td>
</tr>
<tr>
<td>H</td>
<td>207</td>
<td>9176</td>
<td>13304</td>
<td>14225</td>
<td>14529</td>
</tr>
</tbody>
</table>

After the tensile bars were initially scanned in their unloaded states using CT, tensile bars were uniaxially loaded at Sandia National Laboratories, Livermore, California using a 50 kip MTS machine. The samples were load controlled with a strain rate of $5 \times 10^{-4}$ cm/sec, and loaded up to the percentages of the previously determined average failure load. Selected samples were removed from the MTS after each loading and scanned using CT, then reloaded to the next selected percentage. Finally, the samples were loaded to failure and rescanned. A representative load-displacement curve for the series H tensile bars loaded to 60% of average failure load is presented in Figure 3.4.
3.5.3 Radiography of tensile bars

Upon receiving the tensile bars, all 25 were nondestructively examined using film radiography. The radiographs were taken using a polychromatic X-ray source at an energy of 40 kV with Kodak M-Readypack film. The source-to-film distance was 122 cm. A digitized film radiograph is show in Figure 3.5 for series H tensile bars.

Analysis of the film radiographs revealed many variations in film density which are due to cracks, porosity and inclusions within the tensile bars. Film digitization was performed with a DuPont NDT Scan II™ Model 35 film digitizing system. The NDT Scan II™ is a flat bed scanner that provides a 17.8 by 43 square centimeter scan area, 35-
micrometer pixel size, 12-bit pixel depth, and uses a 5000 element linear array CCD camera. The digitized image allows the user to employ a computer to vary the brightness and contrast for further analysis. This analysis reveals variations in film density even more clearly. However, to accurately characterize the behavior of voids in the magnesium alloy as a function of loading, it is necessary to acquire three-dimensional information not available from two-dimensional projection images, or radiographs. The three-dimensional information is acquired using computed tomography (CT).

Figure 3.5 Digitized film radiograph of series H tensile bars. Radiograph was acquired with an X-ray energy of 40kV, and digitized using a DuPont Scan II Model 35 film digitizing system. The color scale at the bottom relates color to digitized film density, darker areas indicating less dense material. The dark spots within the bars represent porosity and voids.
Chapter 4.

CT Background and Theory

Computed tomography was developed in order to obtain information about an object in all three dimensions. The linear X-ray attenuation coefficient is a sensitive measure of atomic composition and density [Dyson, 1990]. CT allows for volumetric measurement of the X-ray attenuation coefficient. Subtle microstructural differences in three dimensions appear as differences in attenuation coefficients. X-ray CT involves acquiring multiple radiographic projection images of an object at different angles using digital or electronic imaging detectors. These projections are processed and mathematically reconstructed using a computer to produce a three-dimensional data set containing information about the full volume of the object. These data can be viewed by selecting a plane anywhere in the volume and displaying it as a two-dimensional image. The data can also be rendered and displayed as a three-dimensional representation in a two-dimensional plane. The final three-dimensional data set is a map of the linear attenuation coefficient, which is a function of the energy of the X-rays and the composition (e.g. elemental and density) of the object inspected.

Cormack and Hounsfield developed CT as a tool for diagnostic medical imaging in the early 1970’s. While CT is most commonly recognized as a diagnostic medical
tool, in the mid-1980's CT was adapted for industrial nondestructive inspection and analysis of parts, and with the development of faster computers and new reconstruction methods [Smith, 1990; Feldkamp, et al., 1984], has proved superior to techniques such as radiography. A CT image reveals locations and dimensions of features, such as voids within an object as well as external dimensions, such as thickness or radius [Kak and Slaney, 1988; Herman, 1980]. The basic theory of CT is provided in this chapter and examples of some CT systems are presented in chapter 5. For a more detailed development and history of CT, please refer to the excellent texts by Kak and Slaney (1988), Herman (1980), Barrett and Swindell (1981), or Macovski (1983).

4.1 Basic mathematics of monochromatic X-ray CT

Tomography can be defined as the two-dimensional reconstruction of a parameter of interest within an object from measurements of the integrated values of the parameter through the object. The original mathematics for reconstructing the two-dimensional function from its one-dimensional projection data were developed by Radon in 1917, however CT researchers only “re”-discovered the techniques in the 1960s when computers, and their ability to perform complex and rigorous calculations made CT a viable option in diagnostic medicine [Hounsfield, 1973; Azevedo, 1991, Martz and Schneberk, 2001]. Neutrons [Hall, et al. 1999], protons [Pontau, et al., 1989], and
electrons can also be used in tomography, and can be complementary to X-rays. With X-ray tomography, the measurement one obtains from an X-ray detector is a measure of the transmitted intensity of the X-ray beam along a straight line through the object. The transmitted intensity relative to the incident intensity yields information about the object inspected. It is important to select an appropriate X-ray wavelength to minimize refraction and diffraction and to ensure that only effects due to absorption are measured [Azevedo, 1991].

The integrations along straight lines through the object are referred to as “line integrals.” For X-rays, the line integral represents the total attenuation due to absorption suffered by a beam of X-rays as it travels through an object. A projection is defined as a set of measurements of the integrated parameter of interest, in this case X-ray attenuation. The Fourier Slice Theorem relates one-dimensional measured projection data to the two-dimensional Fourier transform of the object cross-section and is the fundamental basis of CT [Kak, 1988; Herman, 1990].

The line integral approximation has been found to be a good match to the physics of X-ray photon transport through materials. The following development assumes that radiation travels in straight lines through materials and disregards the effects of scatter and secondary processes. It also assumes a monochromatic energy source, unless
otherwise noted. For monochromatic X-ray radiation the measured intensity at the X-ray
detector is described by

\[ I = I_0 \exp \left( - \int_L f(x, y) du \right), \]  

(4.1)

where \( I_0 \) is the intensity of the incident beam, \( L \) is the beam path through the object, and
\( du \) is the incremental distance along \( L \). Then \( f(x, y) \) is the X-ray absorption or linear
attenuation coefficient of the point \((x, y)\) in the object at some plane \( z \) and is directly
proportional to the energy of the X-rays, the volumetric density and the elemental
composition at that point. If the X-ray energy is fixed, a reconstruction of \( f(x, y) \) will give
an estimate of the relative density distribution coupled with the elemental composition
within the object. If we define the observed signal on a line \( L \) as the natural logarithm of
the ratio of monochromatic X-ray photons that enter an object to those that leave, then

\[ g_L = \ln \left( \frac{I_0}{I} \right). \]  

(4.2)

Clearly, a measurement of incident intensity, or a measurement of intensity with no
object present is necessary for CT. It follows that

\[ g_L = \int_L f(x, y) du. \]  

(4.3)

Equation 4.3 is defined as the line integral, also referred to as a ray-sum. In reality, due
to the finite-width of detectors, line integrals are really strip-integrals of some width,
however the line integral derivation is acceptable.
4.1.1 The Radon Transform

From the coordinate system presented in Figure 4.1, the line, $L$ can be more explicitly expressed in terms of $s$ and angle $\theta$.

Thus, the expression for the line integral can now be written more explicitly as

$$g(s, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - s) \, dx \, dy.$$  \hspace{1cm} (4.4)

Here, $s_0 = x \cos \theta + y \sin \theta$ for a line $L_0$ and the delta function selects only those points lying on $L$ for integration. This expression for the line integral is known as the Radon transform of $f(x, y)$. Note that the line integral is now an expression of $s$ and $\theta$, i.e. two-
One important property of the radon transform is that it is the same forward and reverse, giving symmetry. In other words, for reconstruction purposes, for parallel X-ray beam geometry, 180 degrees equals 360 degrees. The radon transform of an object function is usually displayed as a “sinogram” (see Figure 4.2).

![Figure 4.2](image)

**Figure 4.2** Left: Projection or magnesium AM60B tensile bar with fiducial wire (right hand side of this image). Right: Sinogram created from slice indicated in the projection. Note the half sinusoidal path the fiducial wire traces over 180 degrees. Over 360 degrees a full sine wave (or sine wave) is obtained.

A sinogram is a representation of the projection data with $s$ along the horizontal axis and $\theta$ along the vertical axis. It is called a sinogram because a distinct point $(x,y)$ not lying on the center axis of rotation traces out a sinusoidal path in this space. The goal of tomography is to find an estimate of $f(x,y)$ from a set of measurements of $g(s, \theta)$.

A set of line integrals, $g(s, \theta)$, or radon transforms is called a projection, and when a set of line integrals for all $s$ are obtained at a fixed angle $\theta$, with a parallel beam geometry, the set is called a parallel projection. A parallel projection can be expressed as

$$g_{\theta_0}(s) \equiv g(s, \theta)|_{\theta=\theta_0}.$$  \hspace{2cm} (4.5)
Fan- and cone-beam projections resulting from a pinhole or point source are also common X-ray source and detector geometries. For the CT data obtained in this project, we employed parallel and fan-beam data acquisition and image reconstruction.

4.1.2 The Fourier Slice Theorem

The Fourier slice theorem states that a one-dimensional Fourier transform of a parallel projection is equal to a line in the two-dimensional Fourier transform of the object. Therefore, using a complete set of parallel projection data it is possible to estimate the object function by performing a two-dimensional inverse Fourier transform as follows. Start by defining the two-dimensional Fourier transform of the object as

\[ F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(u+vy)} dx dy, \quad (4.6) \]

where \( u \) and \( v \) are coordinates in Fourier space. The Fourier transform of a projection at an angle \( \theta \) is defined as

\[ S_\theta(w) = \int_{-\infty}^{\infty} s_\theta(s) e^{-j2\pi ys} ds. \quad (4.7) \]

If we consider the simple case where \( \theta = 0 \), we start with the Fourier transform of the object along the line in the frequency domain given by \( v = 0 \). The Fourier transform integral now simplifies to

\[ F(u,0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi ux} dx dy. \quad (4.8) \]
Now that the phase factor is not dependent on $y$, the integral can be separated into two separate parts as

$$F(u,0) = \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} f(x,y)dy \right] e^{-j2\pi xu} dx. \quad (4.9)$$

Remembering the definition of the projection (equation 4.3) and substituting this into equation 4.9 results in

$$F(u,0) = \int_{-\infty}^{\infty} g_{\theta=0}(x)e^{-j2\pi xu} dx. \quad (4.10)$$

The right hand side of the equation represents the one-dimensional Fourier transform of the projection $g_{\theta=0}$, and gives the following relationship between a projection and the two-dimensional transform of the object

$$F(u,0) = S_{\theta=0}(u). \quad (4.11)$$

The Fourier Slice Theorem can be more explicitly stated as [Kak, 85]:

*The Fourier transform of a parallel projection of an image $f(x,y)$ taken at an angle $\theta$ gives a slice of the two-dimensional transform, $F(u,v)$, subtending an angle $\theta$ with the $u$-axis.*

Or put more simply:

$$S_{\theta}(w) = F(w,\theta) = F(w\cos\theta, w\sin\theta). \quad (4.12)$$

This equation is the essence of straight ray tomography. The overall idea of the Fourier Slice Theorem can be shown graphically, as in Figure 4.3.
Figure 4.3 The projection-slice theorem: the Fourier transform relationship of a 2-D function \( f(x,y) \) to its 1-D projection \( g_\theta(s) \) at angle \( \theta \).

By taking projections of an object at many angles from \( \theta_1, \ldots, \theta_n \) and Fourier transforming each of these, the values of \( F(u,v) \) can be determined. If an infinite number of projections are taken, then \( F(u,v) \) is known at all points in the uv-plane. And knowing \( F(u,v) \), the object function, \( f(x,y) \) can be recovered using the inverse Fourier transform which is written as

\[
f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} \, du \, dv.
\] (4.13)

For computational purposes, if \( x \) and \( y \) are finite, and \(-A/2 < x < A/2\) and \(-A/2 < y < A/2\), the inverse Fourier transform can be expressed discretely as

\[
f(x, y) = \frac{1}{A^2} \sum_{m} \sum_{n} F\left(\frac{m}{A}, \frac{n}{A}\right) e^{j2\pi\left[\frac{m}{A}x + \frac{n}{A}y\right]},
\] (4.14)
where \( m \) and \( n \) are summing indexes. For a finite number of Fourier components, or projections,

\[
f(x, y) = \frac{1}{A^2} \sum_{m=-N/2}^{N/2} \sum_{n=-N/2}^{N/2} F\left(\frac{m}{A}, \frac{n}{A}\right) e^{j2\pi[(m/A)x+(n/A)y]},
\]

(4.15)

where \(-N/2\) to \(N/2\) are the limits of the sums, and \( N \) is arbitrarily assumed to be an even integer. It is evident from above that the spatial resolution in a reconstruction is determined by \( N \). In other words, \( N \) is directly related to the minimum number of radiation detectors necessary in the \( x \) and \( y \) directions. The inverse transformation can be rapidly implemented by using the Fast Fourier Transform (FFT) algorithm, as long as the \( N^2 \) Fourier coefficients, \( F(m/A, n/A) \) are known. Because the function \( F(u, v) \) is only known along a finite number of radial lines about an origin and these must be interpolated to a square grid, the density of radial points gets sparser as one moves away from the origin. Thus, there is greater error in higher frequency components in an image than in low frequency components, due to the divergence of the radial lines, which results in some image degradation (see Figure 4.4). This image degradation can be minimized by acquiring more projection data sets to “fill in” the missing data by adding radial lines. A common rule loosely followed by CT researchers in determining the number of angular projections to acquire is known as the “\( \pi \) over 2” rule. This rule states that to obtain an appropriately sampled CT data set, the number of angular projections acquired must be at
least equal to $\pi/2$ times the number of ray sums (or line integrals) within the object to minimize the error in high frequency components [Wells, et al., 1997].

Figure 4.4 Collecting projections of the object at a number of angles gives estimates of the Fourier transform, $F(u,v)$, of the object at points along radial lines. This information is then interpolated back to a square grid to obtain an estimate of the object function, $f(x,y)$. Note the density of radial points becomes sparser as one gets farther away from the center. Hence, the interpolation error also becomes larger.

4.2 Polychromatic X-rays

While the above has been developed based on the assumption of monochromatic X-ray sources, most industrial X-ray sources are polychromatic, with broad energy spectra. Since the intensity of the photons exiting an object is dependent on energy, for
polychromatic sources the transmitted intensity must be integrated over all energies and
can be expressed as

$$I = \int_0^E I_0(E) \exp\{-\int_L f(x, y) du\} dE,$$

(4.16)

where $E$ is the energy of the photons. Thus the resulting intensity contains an integration
over a range of energies.

Polychromatic sources can be described by an average, or a characteristic X-ray
energy and therefore the above monochromatic development holds true to first order for
polychromatic X-ray sources as well. Efforts are currently underway to better
characterize and understand the dependency of transmitted intensity on X-ray energies
[Martz, et al., 2000]. For energies below 1.022 MeV, the resulting intensity is a result of
both photoelectric absorption and Compton scattering. At energies above 1.022 MeV,
pair production can occur in addition to photoelectric absorption and Compton scattering.
In order for pair production to occur, the incident X-ray must have an energy equal to or
greater than the rest mass of an electron-positron pair. The rest mass of each is 0.511
MeV, and therefore the incident X-ray must have an energy of at least 1.022 MeV in
order for pair production to occur. The work presented here was performed at energies
significantly below 1.022 MeV (25 keV, 75 keV, and 200 keV), and therefore pair
production was not a concern.
4.3 CT image reconstruction

After the projection data have been acquired, there are many different reconstruction techniques and algorithms available to reconstruct the data into two-dimensional slices or three-dimensional volumes. These techniques include Direct Fourier Inversion (DFI), backprojection techniques, series expansion techniques, and many others [Azevedo, et al., 1989; Bates and McDonnell, 1986; Gordon, 1974; Azevedo, 1991; Feldkamp, et al., 1984; Smith, 1990, Goodman, et al., 1993; Martz, et al. 2000]. The image reconstruction method depends upon the data acquisition geometry employed. Parallel-beam projections provide simple and fast reconstructions, but other CT geometries exist, such as fan- and cone-beams. The data presented here was acquired with parallel- and fan-beam geometries only and therefore a brief discussion of parallel- and fan-beam reconstruction algorithms is presented here.

4.3.1 Filtered backprojection (FBP)

For parallel projection geometries, by far the most common algorithm used for CT image reconstruction is called filtered backprojection (FBP). FBP is a fast, computationally efficient and easy to implement algorithm that provides high-quality reconstructions and is used in almost all applications of straight-ray, parallel-beam tomography. The theory behind FBP can be derived using the Fourier Slice Theorem.
More detailed derivations and descriptions of the algorithm are available elsewhere [Kak
and Slaney, 1988, Barrett and Swindel, 1981; Herman, 1980], but a brief qualitative
description is presented below. To start, the inverse Fourier transform is rewritten using
polar coordinates and the limits of integration are rearranged. FBP intuitively consists of
two steps. First, each projection is filtered using a simple weighting of each projection in
the frequency domain. Next, the filtered projections are “backprojected”, or smeared
over the image plane along the lines from which they came. A simplified description of
the FBP algorithm and the steps involved is presented below for projection data acquired
over \( K \) angles, \( \theta \), between 0 and 180 degrees:

1. Measure the projection data, \( g_d(s) \);

2. Fourier transform the projections to get \( \tilde{g}_d(w) \);

3. Multiply by the weighting function \( \frac{2\pi|w|}{K} \), where \( w \) is a given frequency;

4. Sum the inverse Fourier transforms of the filtered projections over the image
   plane.

Expressed another way

\[
 f(x, y) = \int_0^\pi \tilde{g}_\theta(x \cos \theta + y \sin \theta) d\theta ,
\]

(4.17)

where the 1-D filtered projections are expressed as

\[
 \tilde{g}_\theta(s) = \mathcal{F}^{-1} \left[ G_\phi(\xi) Q(\xi) \right](s),
\]

(4.18)
and $G_\theta$ is the Fourier transform of the projection, $Q$ is the filter function in frequency space and $\mathcal{F}'$ is the one-dimensional inverse Fourier transform operator. While many computational FBP algorithms exist, the Donner method has been used for this project [Huesman, et al., 1977]. The FBP technique, while very fast and easy to use, does not work well with incomplete (e.g. not enough line integrals or projection data) data sets. In addition, although it is designed to detect edges and features in an object, the filter actually amplifies noise in the projection data [Chang and Herman, 1980; Schneberk, et al. 1990]. Therefore it is important to properly select the filter to be used when using FBP to reconstruct CT images in order to minimize amplified noise.

4.3.2 Convolution backprojection (CBP)

Fan-beam geometry is a result of a small or pinhole source where the X-ray beam is divergent in two dimensions like a fan, as the name implies. Another backprojection algorithm similar to FBP called convolution backprojection (CBP) can be used to reconstruct projections acquired with fan-beam source and detector geometries. CBP uses a spatial domain convolution instead of the frequency domain filtering as used in FBP. Expressed another way

$$f(x, y) = B\{g_\theta(s) \ast \rho(s)\}.) \tag{4.19}$$

81
where \( p(s) \) is a discrete approximation to the spatial domain FBP equivalent filters, and \( B \) represents the backprojection operation. While the FBP algorithm performs the filtering operation on each projection in the frequency domain before backprojecting them in the spatial domain, the CBP algorithm uses a spatial domain convolution on the projections instead and then backprojects them in the spatial domain. For parallel-beam geometry, CBP is the same as FBP. However, because line integrals do not need to be uniformly spaced as is required for most FBP algorithms, CBP can be used in equi-angular fan-beam configurations where the detectors are not evenly spaced, and also with cone-beam source and detector geometries with areal detector arrays, provided the cone angle is small [Kak, 1979]. Although not as fast as FBP, the CBP algorithm is quite fast and easy to implement.

For both the FBP and CBP image reconstruction algorithms, two-dimensional slices or planes are reconstructed from one-dimensional projections. A three-dimensional volumetric reconstruction can be easily obtained from the slices by “stacking” successive planes. However, algorithms do exist that reconstruct entire three-dimensional volumes at once. An example of this is cone-beam reconstruction algorithms, where the divergent source projections are captured with an areal-detector array, and the two-dimensional projections are reconstructed as a group into a three-dimensional volume. These
reconstruction algorithms tend to be extremely computationally expensive and difficult to implement. Smith (1990) gives a good summary of several cone-beam algorithms.
Chapter 5

CT Data Acquisition Systems: Hardware

In general, an X-ray CT system consists of an X-ray source, an X-ray detector, and staging for mechanical manipulation of the source, detector, or object being scanned. The source, detector, and object staging are connected to a computer for mechanical control and for data storage, processing, reconstruction and analysis. While all CT systems must contain each of these components, there are many different hardware and geometric configurations available for acquiring CT data. A brief description of several types of X-ray sources and detectors is presented below, followed by a brief description of several commonly used CT scanning configurations.

5.1 X-Ray Sources

Radiation is generated when high velocity charged particles impinge on a target material, solid, liquid or gaseous. Radiation is also emitted by various processes during radioactive decay. Here it is useful to point out that the radiation is called either X- or $\gamma$ (gamma)-rays. They differ only in how their radiation is generated. X-rays are created from electronic interactions while $\gamma$-rays are generated by nuclear transitions. For more detailed descriptions of the physics of radiation, see texts by Brown (1971), Knoll
(1979) or Dyson (1990). The charged particles used in X-ray generators are usually electrons, but protons or alpha particles can also be used to generate X-rays. X-ray energy is defined with the electron-Volt (eV). An electron gains 1 eV of kinetic energy as it accelerates through a potential difference of 1 Volt. Energy sources can be categorized as polychromatic or monochromatic X-ray generators. Monochromatic sources produce X-rays with a single characteristic energy, while polychromatic sources produce X-rays with a continuous spectrum of energies with characteristic energy peaks. Polychromatic sources are more commonly used in both industrial and medical CT.

5.1.1 Polychromatic X-ray sources

X-ray tubes are one of the most commonly used sources of polychromatic X-rays. An X-ray tube consists of a heated filament housed in an evacuated volume with a solid metallic target. A heated filament is a copious source of electrons and is commonly made of a coiled tungsten wire. The filament forms the cathode, or negative electrode of the tube and the metallic target forms the anode, or positive electrode. Electrons produced by the filament are accelerated across the potential difference until they strike the target. A focusing cup serves to concentrate the stream of electrons onto a small area of the target, called the focal spot. The higher the temperature of the filament, the greater its emission of electrons and the larger the resulting tube current.
X-ray tubes emit only a small fraction, approximately 1%, of the dissipated energy as X-rays, with the rest dissipated as heat [Brown, 1971]. Metals are commonly used as target materials, in part due to their large thermal conductivities, since most of the energy applied to the tube is transformed into heat at the focal spot. The X-rays emitted have a continuous energy spectrum (called Bremsstrahlung, or “braking”, radiation) with characteristic energy peaks due to the target material. The efficiency of the target material in producing X-rays is proportional to its atomic number. Tungsten, with an atomic number of 74, is used in practically all high-output industrial X-ray tubes [Eastman Kodak Co., 1980]. Focal spot sizes of X-ray tubes can range anywhere from a few micrometers (microfocus sources) to several millimeters, with operating voltages of 160 to 450 kV.

When greater energies are desired (1-20MeV), linear accelerators, or Linacs, can be used to generate polychromatic X-rays. In Linacs, electrons are produced by thermionic emission and then accelerated by the electric field associated with a travelling or standing wave generated at microwave frequencies. Electrons are injected into an accelerator tube associated with a wave guide and the velocity of the electrons gradually increase along the tube until they collide with a target generating high energy X-rays. Maximum energies obtained for Linac sources range from 1 MeV to 20 MeV, and focal spot sizes range from less than 1-mm up to 2-mm. Because of the high energy
output, Linacs require more X-ray shielding and generally a dedicated facility. Linacs are not as common as X-ray tubes in industrial CT because of the higher investment required for equipment and facilities.

5.1.2 Monochromatic X- and γ-ray sources

In addition to the polychromatic X-ray sources mentioned above, monochromatic X- and γ-ray sources are also used in industrial CT. Radioisotope sources generate discrete energy X- and γ-rays through emission from a radioactive element with some characteristic energy dependent on the product element. The source size depends on the physical size of the encapsulated radioactive material, but is generally less than a few millimeters. Some examples of radioactive isotopic sources include Am-141 with an energy of 60 kV, Ir-192, with an average energy of 400 keV, Cs-137, with an energy of 662 keV, and Co-60, with an average energy of 1225 keV. Because the X- and γ-rays generated have discrete energies, radio isotopic sources can be very useful for quantitative CT, where spectral analysis is used to record only the discrete energies of interest.

Other sources of monochromatic X-rays include cyclotrons, betatrons, and synchrotrons, which all generate X-rays by accelerating electrons to relativistic velocities along a curved trajectory with a large radius of curvature. Any accelerated
charged particle that is bent can be used to generate X-rays but electron acceleration is most common. A typical electron accelerator emits radiation in a very broad range of photon energies, from microwaves to high energy (hard) X-rays. Synchrotron sources emit radiation into a small solid angle directed tangentially to the electron orbit, resulting in dramatically increased brightness as compared to more conventional X-ray sources. The electrons in the storage ring are accelerated by a magnetic field, either with a dipole magnet or with insertion devices, known as “wigglers” [Margaritondo, 1988]. The X-ray energy depends on the electron energy and the magnetic field. Electrons are continuously distributed in the electron storage ring; the X-ray output can be tuned using a monochromator to provide monochromatic radiation with a very high intensity.

5.2 X- and Γ-Ray Detectors

In addition to the many X- and Γ-ray sources available, some of which are mentioned above, there are also many different X-ray detector systems available for use in CT. An excellent text that develops the theory of radiation detection and presents examples of many different radiation detectors is that by Knoll (1979). Radiation detectors can be coarsely categorized as energy discriminating, and non-energy discriminating. Energy discriminating radiation detectors provide a response to individual X- or Γ-ray interactions that is proportional to the energy deposited in the
interaction. These types of detectors are used in radiation spectroscopy. Non-energy
discriminating radiation detectors simply record the accumulated deposition of X-ray
energies that is collected by the detector. The work presented here is performed with
non-energy discriminating radiation detectors, and therefore the descriptions presented
below are for commonly used non-energy discriminating detector systems. Essentially,
a radiation detector used in CT is any component or system of components that produces
a detectable electrical signal in response to detection of radiation, where the signal is
proportional to the deposited energy. This can be accomplished in many ways. Several
commonly used detector systems are described below.

A common non-energy discriminating detector system widely used in the
medical world is the image intensifier tube (IIT) [Hasegawa, 1991]. The IIT consists of
an input phosphor that converts X-ray energy to light, adjacent to a photocathode that
converts the light to electrons. The IIT then accelerates the electrons across a high
voltage potential in a way that preserves the geometrical configuration of the image.
The acceleration of the electrons coupled with focusing the electrons onto a much
smaller area output phosphor results in a large amplification of the input signal. This
output image is 30 to 10,000 times brighter than the image produced at the input
phosphor. The images are then recorded with a camera and converted to digital format
for reconstruction.
The detector systems used in this project and commonly used in industrial CT employ a scintillator that is sensitive to X-rays. These non-energy discriminating detector systems consist of a scintillating material such as sodium iodide (NaI), cadmium tungstate (CdWO₄), gadolinium oxide (Gd₂O₃), or terbium oxide (Tb₂O₃) that emits light when exposed to radiation. The brightness is proportional to the amount of radiation absorbed. A thin scintillator can be coupled either directly or with a fiber optic plate to a diode array or CCD, and a thicker scintillator can be coupled with an optical lens to a CCD camera, where the light signal is digitized. The detector systems used in each set of CT experiments presented here consist of a scintillating material, either a terbium-doped monolithic glass or fiber-optic plate, or a scintillating CdWO₄ single crystal, optically coupled with a lens to a CCD camera that records the light images. This type of detector system provides good spatial-resolution with moderate acquisition times [Martz, et al. 1992; Dolan, et al., 1994; Martz, et al., 1991; Savona, et al., 1996].

5.3 Stages and Computers

Beyond the assortment of X- and γ-ray sources and detectors available, mechanical staging and computers offer additional choices for assembling a CT system. Mechanical staging used in CT is usually capable of x, y and z translation, and θ rotation, and is controlled by a computer. Often, the mechanical staging is manipulated and
images are captured on a PC, and then transferred to more powerful computers for processing, reconstruction and analysis. Figure 5.1 shows a summary of the steps required to acquire, process, display and analyze CT data.

![Diagram of the steps required in CT imaging from acquisition of data to display and analysis.](image)

**Figure 5.1** A representative flow diagram of the steps required in CT imaging from acquisition of data to display and analysis.

With the development of more efficient reconstruction algorithms, it is now possible to acquire and process all data using a PC. The CT data presented here were acquired with a PC or a Sun workstation and then transferred to a Silicon Graphics Workstation for reconstruction and analysis.

### 5.4 Geometric configuration of CT systems

In addition to the X- and Γ-ray source, detector, staging and computing choices, there are several ways to configure CT system components to acquire data. CT systems are commonly classified according to geometric set-up and mode of data acquisition [Hasegawa, 1991; Martz and Schneberk, 2001]. Brief descriptions of these
classifications are presented below and some schematic drawings of three of the most common configurations used for industrial CT are shown in Figure 5.2.

(a) Single detector

(b) Linear-array (1D) of detectors

(c) Area-array (2D) of detectors

Figure 5.2 Three of the most common data acquisition geometries used in industrial CT imaging. (a) First-generation, discrete-beam "translate-rotate" scanner configuration using a single detector; (b) well-collimated, fan-beam configuration using a linear detector array; (c) cone-beam configuration using an area-array detector.
5.4.1 First-generation CT systems

First generation CT systems, also known as “translate–rotate” systems, consist of a single X- or \( \Gamma \)-ray source and an associated single detector placed on opposite sides of an object and controlled by a computer. The source–detector pair are moved in concert perpendicular to the X-ray beam along a straight-line path in one plane, or equivalently, the object is moved while the source–detector pair remain stationary (see Figure 5.2(a)). Each position along the straight line, or ray-sum, is stored in the computer and a complete set of ray-sums at positions along the line is a parallel projection. The object is then rotated by some angular increment from 0 to 180 degrees and scanned along a line at each angle on the same plane to obtain the full set of projection data needed to reconstruct one plane. Hence the name “translate–rotate” CT system. The object can then be vertically translated to collect projection data for other planes in a similar manner. This type of CT system can provide low-scatter, high contrast and artifact free CT slices and the spatial resolution of the scan is technically limited only by the size of the X- or \( \Gamma \)-ray beam and accompanying detector. However the time required for acquisition can be enormous, making the CT data acquisition time–prohibitive. For this reason, arrays of detectors are more commonly used in industrial CT to collect multiple ray-sums at once from a fan– or cone–beam X-ray source geometry.
5.4.2 Second-generation CT systems

Many commonly used industrial CT systems consist of a well-collimated fan-beam X-ray source geometry, coupled with a linear detector array on the opposite side of the object (see Figure 5.2(b)). The object is rotated at angular increments equal to the fan-angle to collect the complete set of projection data needed for reconstruction of one slice or plane. If the object is larger than the width of the fan-beam, then the source and detector must be translated as well to capture the full object projection data and hence this is known as a “translate-rotate” CT system. The object can then be vertically translated to acquire CT data for multiple planes.

5.4.3 Third-generation CT systems

Another commonly used CT system, known as a third-generation CT scanner, uses either a linear array or an areal array, located opposite a cone-beam X-ray source geometry (see Figure 5.2(c)). If the object is small with respect to the length of the detector and is able to fit in the field-of-view of the detector, then this CT geometry is known as third-generation. This configuration allows for the collection of CT data for one entire slice plane with just one angular rotation and hence is commonly referred to as a “rotate only” CT system. Areal array detectors may allow for the acquisition of CT data for the entire object with one rotation. While these CT systems are significantly
faster than the first- or second-generation systems, there is increased scatter from the part into the multiple detectors, as well as difficulty in multiple detector calibration, and finite gaps separating detectors. Both second- and third-generation scanners can acquire data using either fan or cone beams. They differ in that third-generation scanners acquire all data necessary to reconstruct one slice plane by rotating the object or source-detector pair only. This results in faster CT data acquisition times than first- or second-generation scanners, however the increase in noise and other artifacts that result from third-generation systems requires more time spent in pre- and post-processing of the CT images. Generally, while first-generation scanners take longer to acquire the CT projection data, less time is spent in pre- and post-processing of the CT images because the data acquired has fewer artifacts than the CT data acquired with second- or third-generation scanners. In other words, there is a trade off between acquisition time and time spent processing data to minimize artifacts in the final CT images.

5.4.4 Fourth-generation and helical CT systems

Medical CT commonly uses third or fourth-generation CT systems [Hasegawa, 1991]. A third-generation CT system uses a fan-beam X-ray source with a circular arc detector array that moves in concert with the source around the patient. Fourth-generation CT systems consist of a fixed circular detector array with a rotating X-ray
tube. Another recently developed medical CT system is known as “helical CT”, which consists of a row or several rows of radiation detectors arranged along a planar circular arc centered in the source [Turbell and Danielsson, 2000; Armato, et al., 1999; Hu, et al., 1999]. The patient is then moved through the gantry at a constant rate of speed while CT data is continuously acquired with a helical rotating source and detector pair. These systems are beginning to be used in industrial applications as well. In addition to the strong guidelines and limits on radiation dosage required for medical CT, it generally employs a spatial-resolution of approximately 0.5-mm in plane with a 1.5-mm slice plane thickness, significantly larger than the resolutions of interest in this project.

5.5 Scan parameter selection

In addition to selecting the proper hardware and CT system geometry, scan parameters must be carefully determined with respect to the project at hand. Selecting an appropriate X-ray energy for CT is crucial. The selected energy must be high enough so that absorption of the photons is not excessive, but low enough that the attenuation signal can be distinguished from the incident intensity signal, $I_0$, in the presence of system noise [Martz, et al., 1989]. If the path length of one ray-sum is denoted as $L$, then the optimum X-ray energy can be derived from the well known relationship of $\mu L = 2$, where $\mu$ is the linear attenuation coefficient [Grodzins, 1983]. This is equivalent to
an effective attenuation of 86%. This relationship optimizes the signal-to-noise ratio in
the data and was derived by calculating the necessary time to obtain a scan in a vacuum
at a given sensitivity or resolution, which is related to the photon statistics, and then
setting the derivative of the time with respect to the attenuation coefficient equal to zero.
By calculating optimum thickness versus energy, one can select an appropriate energy
for the particular geometry of the object of interest. However, the incident intensity, $I_0$,
is generally not measured in a vacuum, and in addition the chord length, $L$, through an
object varies and hence the true optimum energy condition is $1 \leq \mu L \leq 2$ [Bryant and
McIntire, 1985]. Other scan parameters that should be optimized for a particular CT
problem include selecting adequate scanning times in order to ensure adequate photon
statistics, and determining the appropriate number of ray sums and angular projections to
acquire to ensure adequate spatial resolution [Wells, et al. 1997].

5.6 Experimental CT systems used

For the purpose of obtaining CT data for magnesium AM60B tensile bars, several
CT systems were investigated. Lawrence Livermore National Laboratory (LLNL) has
several CT systems available for acquiring CT data. The systems used at LLNL are built
in a modular fashion, meaning that the components of each CT system can be changed or
modified to suit the object being scanned. In addition to the LLNL systems, the X-ray
Tomographic Microscope (XTM) located at the Stanford Synchrotron Radiation Laboratory (SSRL) also became available for use in this project. A total of three CT systems (PCAT, KCAT and XTM) were used to acquire AM60B tensile bar CT data. These systems are described below.

5.6.1 PCAT

The first CT system used to obtain CT data of the tensile bars is located at LLNL and called PCAT for Photometric Computed Axial Tomography. A photo of the PCAT system is shown in Figure 5.3.

Figure 5.3 Photograph of the PCAT scanner located at LLNL.
PCAT consists of a 450-kV polychromatic X-ray tube source with a divergent small-angle cone-beam geometry, located 1482 mm in front of the tensile bars to be scanned. The tensile bar is located on a mechanical stage, capable of rotation. Located 23-mm behind the tensile bar is a scintillating glass plate. Two thicknesses and configurations of scintillating glass were used for this project; a 6-mm thick monolithic plate and a 12-mm thick fiber-optic bundle. The glass is a silicate host matrix containing Gd₂O₃ doped with terbium oxide [Placious, et al., 1991]. Behind the glass at a distance of 387.4-mm is a mirror that is turned at an angle of 45 degrees to redirect the detector image perpendicular to the X-ray beam. The image is redirected with the turning mirror in order to shield the electronics of the camera. The redirected image is captured with a 200-mm Micro-Nikor lens CCD camera, located 292.1-mm from the mirror. The field of view of the detector is approximately 6.45 x 6.45 square centimeters and a complete CT data set can be acquired with rotation only; therefore, this CT system is a third-generation scanner. The source-to-object distance (s.o.d.), as well as the object-to-detector distance (o.d.d) can be varied with this system, and in all of the PCAT experiments to be presented here, the s.o.d. was 1482 mm, and the o.d.d. was 23 mm, for a total source-to-detector distance (s.d.d.) of 1505 mm. This results in a very small geometric magnification of approximately 1.02 (the geometric magnification for fan-beam geometries can be defined as s.d.d./s.o.d).
Two X-ray energies were initially used, in order to determine an optimum energy for scanning the tensile bars. One CT scan was acquired using an X-ray energy of 200 kV, and an additional scan was acquired with an energy of 75 kV. CT scans were acquired over an angular range of 180 and 360 degrees, with angular increments of either 1 or 0.5 degrees respectively, for a total of 360 projection sets. Spatial-resolution for PCAT with the above mentioned setup was approximately 70 to 100 micrometers. A summary of the parameters varied for PCAT is presented in Table 5.1. Initial CT scans for several tensile bars were acquired using PCAT.

Table 5.1 Parameters investigated to optimize two LLNL CT systems for data acquisition of the Mg tensile bars.

<table>
<thead>
<tr>
<th>CT Sys.</th>
<th>kV</th>
<th>mA</th>
<th>Spot Size</th>
<th>sod</th>
<th>odd</th>
<th>Thick/Scint.</th>
<th>CCD</th>
<th>Filter</th>
<th>Acq. Time</th>
<th>MTF 10 lp/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCAT</td>
<td>200</td>
<td>10.0</td>
<td>1 mm (o.d.)</td>
<td>1482</td>
<td>23</td>
<td>6-mm plate</td>
<td>1024 x 1024 pixels</td>
<td>none</td>
<td>~25 hours</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>12-mm f.o.³</td>
<td>14 bits</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KCAT</td>
<td>75</td>
<td>0.12</td>
<td>10 μm (o.d.)</td>
<td>77</td>
<td>15</td>
<td>2-mm plate</td>
<td>768 X 512 pixels</td>
<td>none</td>
<td>~5 hours</td>
<td>~0.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3-mm f.o.³</td>
<td>.076 mm Al</td>
<td></td>
<td></td>
<td>~0.52</td>
</tr>
</tbody>
</table>

¹ source-to-object distance, ² object-to-detector distance, ³ fiber optic, ⁴ line-pairs/mm

5.6.2 KCAT

The second modular style CT system located at LLNL that was used to examine the tensile bars is known as KCAT – Knolls Computed Axial Tomography – and is shown in Figure 5.4. KCAT is a higher resolution CT system that utilizes a 160-kV polychromatic X-ray tube with a small-angle cone-beam geometry. The tensile bar was
located 77-mm directly in front of the source, and a scintillating glass detector was located 15-mm behind the object. The geometric magnification for this setup was approximately 1.2 (92 mm/77 mm). A CCD camera is located directly behind the detector to record projection images. The field-of-view for KCAT was approximately one square centimeter.

An X-ray source energy of 75 kV was used to examine the tensile bars, with and without a filter of 0.076-mm thick aluminum. The filter was used in an attempt to minimize beam-hardening artifacts. In addition to varying the source energy, several gadolinium oxide terbium-doped scintillating glasses were investigated, including LKH5, LKH6, and a fiber optic glass plate [Placious, et al., 1991]. Table 5.1 presents a summary of the parameters varied for KCAT. CT data sets were collected over 180 and 360 degrees, in increments of 0.5 or 1 degrees respectively, for a total of 360 projection sets, with a resulting spatial-resolution of around 40-60 micrometers. Initial CT scans for all nine tensile bars were acquired using KCAT with a filtered energy of 75 kV, using the fiber-optic scintillating glass bundle. Data were also acquired after the nine tensile bars were loaded to 60% of their average failure loads.
5.6.3 X-Ray Tomographic Microscope (XTM)

The final CT system used to study the tensile bars is the X-ray Tomographic Microscope (XTM) located at Stanford Synchrotron Radiation Laboratory in Palo Alto, California, and shown in Figure 5.5.

![Photograph of the X-ray Tomographic Microscope (XTM) located on beamline 10-2, at the Stanford Synchrotron Radiation Laboratory (SSRL), Palo Alto, CA.](image)
The XTM is a high-spatial resolution CT system described in detail elsewhere [Kinney and Nichols, 1992]. The radiation source is the 31-pole, X-ray wiggler beamline 10-2 with beamline energy tuned to 25 keV. Each tensile bar was mechanically rotated on a stage in front of an X-ray scintillating CdWO4 single crystal of dimensions 25.0 mm x 25.0 mm by 0.7-mm thick. The scintillator was optically coupled to a 12-bit CCD camera, with a resulting pixel size of 0.02368 mm. 360 projections were acquired over 180 degrees for each CT scan, which was approximately 1000 x 80 pixels (or approximately 23.7-mm wide by 1.9-mm high). Due to the limited height of the field of view, each tensile bar was scanned several times (five or six), translated vertically between each scan, in order to obtain CT data for the entire notched region of the bar.

The XTM uses a parallel-beam geometry X-ray source, with a rectangular field-of-view and hence there is no geometric magnification. Unfortunately, the XTM was not available for use until after all tensile bars had been loaded to 60% of their average failure loads. However, one series of tensile bars with the smallest notch geometry (Series H, with a notch radius of 0.635-cm) was scanned using the XTM after 60%, 87%, 93%, 95-97% of their average failure load, and failure.
Chapter 6

CT Data Processing and Image Reconstruction

The basic steps necessary to acquire and reconstruct CT data are presented in Figure 6.1.

First, transmitted intensity images must be acquired, along with incident intensity and dark current images. The intensity images must then be converted to ray sums. Next, sinograms may or may not be created from the ray sums. Finally the ray sums can be reconstructed into CT slices. While reconstructed CT images can and do provide
extremely useful information, it is subject to several unique problems, often referred to as CT artifacts. Artifacts can result from many different things; for example data-acquisition errors such as hardware misalignment, improper scan parameters, or erroneous analog to digital conversions. Additional sources of artifacts include noise, improper choice of the reconstruction algorithm, and X-ray scatter, to name a few.

Several sources of CT artifacts are presented below.

As previously discussed, at energies less than 1.022 MeV attenuation of photons in a material can occur by either absorption or scatter. Scatter is in all angles, from forward- to back-scatter; yet for CT the only scatter that matters is the forward-scatter that is recorded with the X-ray detector. Detected scatter can lead to artifacts in reconstructed CT images because the effects of scatter can and often changes with the rotation of the object and is different in each projection. Scattered energy has greater effects when the transmitted intensity through the object is low, than when the intensity is high [Kak and Slaney, 1988]. Detected scatter can lead to streaks in reconstructed images. One way to prevent scatter is to have perfectly collimated X-ray detectors. Another way to minimize the effects of scatter is to use energy discriminating detectors. This assumes that the energy of the incident photon is less after a scattering incident than when it enters the object, which is generally a reasonable assumption.
Another common CT artifact is known as aliasing. Aliasing is a result of insufficient data, for example undersampling, not acquiring enough projections, or geometric constraints. Artifacts due to aliasing can appear as Gibbs phenomenon, streaks, or Moire patterns. Another inherent source of error in CT is noise. Generally, there is a trade off between image noise reduction and spatial resolution [Martz and Schneberk, 2001].

Many other CT artifacts exist, such as partial volume effects due to a large detector size with respect to objects of interest, accidental object motion while CT scanning, hollow projections resulting from a completely opaque object surrounded by less dense material, and limited data sets. It is important to select an appropriate CT system, considering hardware, X-ray energies, reconstruction algorithms and the object geometry, in order to extract the information of interest while minimizing CT artifacts.

The CT data presented here was subject to several CT artifacts and steps were taken to minimize their effects on the reconstructed CT images. Image processing steps for artifact reduction can be performed at various stages along the way. Some of these artifact reduction techniques include outlying pixel removal, ring reduction, smoothing, and beam-hardening corrections and will be discussed below. Table 6.1 shows which artifact reduction techniques were used on each data set.
Three different CT systems were used to acquire data for the AM60B tensile bars, as discussed in Chapter 5. The data processing steps required for each of three different CT data sets are similar, but not exactly the same, depending on which CT system was used to acquire the data. The data acquired using PCAT and KCAT, located at LLNL, require data processing and reconstruction steps that are identical. The data acquired using the XTM required similar, but not identical processing steps. The text below presents a chronological discussion of the processing steps used on the PCAT and KCAT data, followed by the processing steps used on the XTM data.

### 6.1 LLNL CT systems – data processing and reconstruction

All CT data acquired with systems located at LLNL (PCAT, KCAT) are saved in an identical file format. Each CT data set acquired is assigned a file name with the extension “.sct”, hereafter referred to as the SCT file [Brand, et al., 1995]. The SCT file contains all relevant scan parameters (such as geometry of source, source energy, number of projections acquired, etc.), some of which are necessary to process and correctly

<table>
<thead>
<tr>
<th>CT System</th>
<th>Artifact Reduction Techniques Applied</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Location)</td>
<td>Outlier Removal</td>
</tr>
<tr>
<td>PCAT (LLNL)</td>
<td>X</td>
</tr>
<tr>
<td>KCAT (LLNL)</td>
<td>X</td>
</tr>
<tr>
<td>XTM (SSRL)</td>
<td>X</td>
</tr>
</tbody>
</table>
reconstruct an image from a set of projections. All of the programs used to reconstruct
the CT data read from the SCT file and write a new updated SCT file describing the
output data files and most processing steps that have been performed on the data set. The
SCT files essentially keep track and record all operations and processing that has been
performed on a set of projections. The projection images are saved in a format
compatible with a signal-processing program called VIEW [Brase, et al. 1988], along
with the SCT file describing the entire set of projections. These files are acquired with
and saved onto a PC. The files are then transferred to a Silicon Graphics, Inc.
workstation for processing and reconstruction. The VIEW files transferred from the PC
to the SGI for further processing must be converted to account for the different platforms.
In VIEW, a “swap” command is applied to each file that swaps the signal data from IEEE
to DEC byte format, to make it compatible with the SGI computing platform.

As mentioned above, the projection data acquired using PCAT and KCAT is
saved in a format compatible with a data- and image-processing software package called
VIEW. VIEW was developed at LLNL and provides a set of general-purpose filtering,
analysis, and display options for multidimensional signal processing [Brase, et al., 1993;
Azevedo, et al. 1990]. The main emphasis of the program is on the processing of time
sequences and multidimensional images. It contains a database made up of signals and
sequences, a set of commands that act on the signals and sequences in the database, and a user interface that utilizes the database and the set of user-selected commands.

When a CT file is saved in VIEW format, it produces two files. The first is an ASCII parameter file that describes the signal and tells the program how to access it, and the second file contains only binary data. VIEW signals can be many data types; for example, byte, word, longword, real and complex. Most commands in VIEW will work with any of the data types. In addition, VIEW signals can be from one to four-dimensional, however most signal processing in VIEW is done in two dimensions.

6.1.1 Creating ray sum images

To reconstruct CT images, first the ray sums must be created from the intensity projection images. To do this, several things are needed in addition to the intensity projection data set. As described in Chapter 4, assuming a monochromatic X-ray source, the linear attenuation coefficient is calculated from the transmitted intensity, \( I \), as well as the incident intensity, \( I_0 \), of the X-ray beam. Hence, for each CT scan it is necessary to obtain an image of the incident intensity, \( I_0 \), or a projection without the object present. The incident intensity is often erroneously referred to as the background intensity and is usually acquired before and after each CT scan. In addition to an incident intensity projection, a "dark-current" image is also obtained. The dark current is the charge that
accumulates within the CCD camera due to the thermal generation of electrons. The dark-current image is captured when the camera and X-rays are turned off to measure the dark current and to account for any bad pixels or nonlinearities in the captured image.

For each set of projection data, after swapping the file formats, the incident intensity and the dark current images are read into VIEW, converted to real data, and a threshold is applied to “clip” any negative values. The dark current is then subtracted from the incident intensity image. Once this is done, the rest of the projection data set, the transmitted intensity projections, is read into VIEW, swapped, converted to real data and clipped, and then processed using the following steps. The dark-current image is subtracted from each projection image. The attenuation image is then created by calculating the natural log of the incident intensity minus the dark current divided by the incident intensity minus the dark current, represented mathematically as

$$\mu L = \ln \frac{I_0 - D}{I - D}, \tag{6.1}$$

where $D$ represents the dark current image, $\mu$ is the linear attenuation coefficient, $L$ is the chord or path length, and $I$ and $I_0$ are the transmitted and incident intensity images, respectively. This is also shown graphically in Figure 6.2. The resulting images are now dimensionless ray sums and can be reconstructed into cross-sectional slices. Once data acquired with any CT system is converted to ray sums, the CT codes developed at LLNL
and employed here can be used to reconstruct the ray-sum data into CT slices or volumes. However, before the ray-sum data are reconstructed into CT images, a few additional processing steps are required to minimize artifacts.

\[ \text{In (Incident intensity)} - \text{Dark current} = \text{Ray sum image} \]

\[ \text{ln (Transmitted intensity)} - \text{Dark current} = \text{Ray sum image} \]

**Figure 6.2** Image demonstrating how ray-sum projection images are created from intensity projections. The images on the left are all in units of intensity, while the resulting ray sum image is dimensionless. The color bars under each image relate the colors to values.

### 6.1.2 Outlier removal

Once the ray-sums have been calculated as above, additional processing can be done to eliminate artifacts and to improve the reconstructed images. The first preprocessing technique applied is to remove any outlying pixels to eliminate singularly bad ray-sums. A bad ray-sum can be the result of one bad detector, a single bad measurement of \( I_0 \), the incident intensity, erroneous analog to digital conversion, or many other
communication errors. Singularly bad ray sums are not easily noticed in the radon
transform or sinogram since they do not occur for every projection. However, bad
isolated points always emerge noticeably as aberrant straight lines in the reconstructed
image, as shown in Figure 6.3.

![Figure 6.3 Example of CT streak artifact resulting from a singularly bad ray sum.](image)

Some solutions to this problem include smoothing (e.g. median filter), or using an outlier
removal technique (i.e., finding and replacing singularly bad ray-sums with some local
average) [Schneberk, et al. 1990]. The latter technique is generally preferable to
smoothing since any smoothing can result in decreased spatial resolution for the entire
image, as opposed to correcting only the singularly bad ray-sums as in the outlier removal routine.

The outlier removal processing technique consists of creating a median filtered image of the projection and comparing it to the unfiltered image. Any pixels with a difference outside a user-specified range are then removed and replaced by the median value from the local neighborhood as shown in Figure 6.4.

Figure 6.4 Outlying pixels are removed from attenuation projection images to minimize artifacts. Left: Ray-sum image containing outlying pixels. Middle: Ray-sum image with outlying pixels removed. Right: Difference between the images, showing pixels that were removed.

This helps to eliminate streaks that are reconstructed in the cross-section image due to bad pixels (see Figure 6.3). While this processing can also be performed on the intensity images, it is generally best to apply it to only the ray sum images [Brand, et al. 1995]. After outlying pixels have been removed, the ray sum images are converted into sinograms using the RECON software package.

The software system for CT simulation and reconstruction used in this project is called RECON and was created at LLNL [Brand, et al. 1995; Azevedo, et al. 1990]. It
was designed to be compatible with the VIEW software package, though neither VIEW or VIEW format files are required in order to use the RECON package. RECON consists of a number of interconnected codes that run a variety of processing codes and reconstruction codes (parallel-, fan-, and cone-beam X-ray source geometries, emission tomography, etc.), as well as simulation codes. Data in two or three dimensions is handled equivalently. The bulk of the code is written in the C programming language, with some algorithms written in FORTRAN. RECON operates on UNIX workstations and VAX computers. Recently RECON has been ported to PCs and Macintosh computers [Goodman, et al., 1999]. RECON requires a SCT file as described above in addition to the data files in order to process data.

RECON contains a number of reconstruction algorithms that perform reconstruction from CT projection data. The algorithms include backprojection methods such as FBP and CBP described in Chapter 4, as well as Feldkamp cone-beam backprojection. In addition RECON includes Direct Fourier Inversion (DFI), Forward Projection Methods, Constrained Conjugate Gradient (CCG) methods, Algebraic Reconstruction Techniques (ART), and many others. In addition to the reconstruction algorithms, RECON contains a CT simulation program that computes projection, image or Fourier transform information from an input image or object description file that can be used to optimize CT data acquisition and reconstruction.
6.1.3 Creation of sinograms

As discussed in Chapter 4, sinograms are a representation of the data, with the ray sums on the horizontal axis, and the angle it was acquired on the vertical axis. Projections of an object not lying on the center of rotation trace out a sinusoidal path through the angular range. A sinogram is calculated for each slice-plane using RECON, and there is one sinogram associated with every reconstructed CT slice. While it is possible to reconstruct CT slices using ray sums only, the two-dimensional ray-sum projections were converted to sinograms for further processing and to remove ring artifacts.

6.1.4 Ring removal

In addition to the outlier removal routine performed on the ray-sum images to minimize CT artifacts, additional processing steps can be applied to the sinograms, resulting in reduced artifacts in the reconstructed slice. One extremely common artifact found in CT reconstructions is known as the ring artifact. Ring artifacts appear in CT reconstructions as concentric circles surrounding the center axis of rotation of the CT image. These rings are caused by detector-to-detector imbalances and appear as straight vertical lines in a sinogram. These lines are mapped into annular rings (except for the center axis of rotation which maps into a point) in the reconstructed CT image with backprojection reconstruction algorithms. Differences in detector offsets or gains can
cause these imbalances. One effective technique for removing ring artifacts was
developed by Kowalski (1978). In this approach, the offset for each detector is removed
by averaging together several projections in the sinogram and then performing a high
pass filter on the result. This technique can be used repeatedly until rings are removed
with minimal distortion of the image.

For every CT data set acquired, it was necessary to perform ring-removal
processing on the sinograms. Several different ring removal filters were selected
manually and optimal values were determined by what looked best in the corrected CT
image. The sinograms processed here were filtered 3 times, varying the amplitude and
order of the filter each time. The first filter used an amplitude of 0.001 and an order of 3,
the second filter had an amplitude of 0.57 with an order of 7, and the final filter had an
amplitude of 0.83 with an order of 45. It was determined that a combination of the three
filters mentioned above, with different amplitudes and orders, provided the best results
[Schneberk, 2000]. Images of a sinogram showing the effects of the ring removal
operation are shown in Figure 6.5.
Figure 6.5 Figure demonstrating the effect of the ring removal filtering applied to sinograms. The vertical lines shown in the top right image were removed from the top left sinogram. Vertical lines reconstruct into rings, as shown by the CT reconstructions.

6.1.5 Cupping Artifact

One extremely common CT artifact is a result of the physical nature of the polychromatic X-rays and non-energy discriminating detectors used in a CT scan and is known as the cupping artifact. Both photoelectric absorption and Compton scattering are dependent on energy, however absorption much more so than scattering. X-rays are attenuated differently by materials based on the energy of the incident photons, with a preferential absorption of lower energy photons as the beam passes through the object resulting in a spectral shift in the photons. In other words, the mean energy of the beam
passing through and eventually exiting the object is higher in energy than the beam entering the object, since lower energy photons were absorbed more readily than the higher energy photons. This "beam-hardening" effect, coupled with scatter, can result in severe errors in the measurement of attenuation. Beam-hardening and scatter generally appear as a "cupping" artifact in the reconstructed CT image. This results in a perceived density gradient, with higher density material appearing at edges and low density material appearing toward the center of objects, as shown in Figure 6.6.

There are several ways to deal with this artifact. First, and the most obvious, would be to either choose a monochromatic energy source or an energy discriminating detector to eliminate the problem, or to filter a polychromatic source in such a way as to
result in a more monoenergetic beam that would help minimize beam hardening artifacts. However, scatter would still be a problem. Second, preprocessing as well as postprocessing approaches exist that use image processing techniques to minimize the effects [Hammersberg and Mångård, 1998]. Another method used to eliminate beam-hardening artifacts is by using a dual-energy technique described elsewhere [Alvarez and Macovski, 1976; Duerinckx and Macovski, 1978].

To help reduce beam-hardening artifacts, and to help optimize the CT data collection and processing, a 99% pure magnesium specimen was obtained and is shown in Figure 6.7.

Figure 6.7 Photograph of 99% pure magnesium phantom created to help optimize CT data acquisition, processing, and reconstruction.
This magnesium phantom was machined to be a tiered cylinder with a length of 1.27 cm for each tier. The diameters of each cylindrical tier are 0.33, 1.02, 1.91, and 2.54 cm. A CT scan was acquired for the magnesium phantom using KCAT, and the results were analyzed to help reduce the cupping artifact. First, the projection data from the magnesium phantom was reconstructed into cross-sectional CT images at the 1.02-cm tier only. This tier was selected for reconstruction because it most closely matched the diameter of the notched region of the tensile bars being studied. Then, using VIEW, one reconstructed slice of the phantom was projected along the y-axis. The projection command collapses a two-dimensional signal into a one-dimensional signal by summing along an axis; in this case, the y-axis. This projection was then divided by the total number of pixels along the axis, and multiplied by the total chord length. This gave $\mu L$ as a function of $L$. The results were plotted and a third order polynomial was fitted to the data, as shown in Figure 6.8. This polynomial is the beam-hardening correction applied to each sinogram. Every sinogram is multiplied by the polynomial before being reconstructed into a slice, resulting in a corrected CT image with significantly less cupping artifact. The beam-hardening correction used for PCAT and KCAT data was

$$-0.41854x + 1.8088x^2 - 0.07129x^3. \quad (6.2)$$
While this technique results in a decreased density gradient or cupping artifact in the CT image, the noise in the image is increased significantly because every point in the sinogram, including noise, is multiplied by the third order polynomial. However, despite the increased noise in the corrected CT images, they still provide data that is more easily segmented and analyzed than the CT images without the correction. An example of one reconstructed CT image from the magnesium phantom with and without the beam-hardening correction is shown in Figure 6.9.
Figure 6.9 Images demonstrating the effects of the beam-hardening correction on a reconstructed CT slice from the magnesium phantom shown in Figure 6.7. Left: CT image without beam-hardening correction. Middle: CT image after the beam-hardening correction has been applied to the sinogram. Right: Lineouts from both CT images; black line is from uncorrected image, red line is from corrected image.

6.1.6 Data reconstruction using CBP

Once all the projections were processed to remove the outlying pixels and the sinograms were processed for ring removal and beam hardening, the sinograms were reconstructed into cross-sectional CT images. The projection data acquired using PCAT or KCAT were reconstructed using a Convolution Backprojection (CBP) algorithm applied slice-by-slice, due to the small-angle X-ray source cone-beam configuration. The SCT files for each data set contained the parameters necessary for reconstruction, including the geometric setup (s.o.d, s.d.d., and o.d.d.), source energy and current, number of rays (horizontal detector elements), angular range (0 to 180, or 0 to 360 degrees), and number of slices (vertical detector elements).
Each CT scan has an implicit center of rotation, about which the object rotates. All reconstruction algorithms require accurate knowledge of this center to produce high-quality reconstructions. These algorithms usually assume the central detector is the center axis of rotation. This is not always the case. If the center provided to the algorithm is incorrect, the reconstruction process will distort the final reconstructed images. If the projected center is incorrect, the sinusoidal paths selected from the sinograms will not use the correct ray-sums and are then incorrectly included in the backprojection sum for a certain location. The centering error appears as a “tuning fork” artifact for CT scans acquired over 180 degrees, and as a blurred halo or edge effect for CT scans acquired over 360 degrees (see Figure 6.10). When the correct center is used the centering artifacts are eliminated. Centering errors can be fixed by either physically measuring the center, which is very difficult to do accurately, or by calculating the correct center from sinograms using a least squares technique described elsewhere [Azevedo, et al., 1990]. In this study, the centers of rotation for each CT scan were selected by hand to minimize centering artifacts in the reconstructed CT slices. A representative reconstructed CT image from both PCAT and KCAT are presented in Figure 6.11.
Figure 6.10 Example of centering error artifacts in CT images. Top: Left image acquired with XTM over 180 degrees with the incorrect center. Note the "tuning fork" artifact. Right image has the correct center.
Bottom: Left image acquired using KCAT over 360 degree. Note the "halo" effect and the blurring of features within the image. Right image has the correct center.
Figure 6.11 Examples of reconstructed CT images from the LLNL CT systems. Left: reconstructed image from data acquired with PCAT at 75 kV. Right: reconstructed image from data acquired with KCAT at 75 kV, filtered with 0.003" of aluminum.

6.2 XTM data processing and reconstruction

Data acquired with the XTM is acquired and saved in a slightly different format than KCAT and PCAT data. All XTM projection images were acquired with a SUN workstation and saved as raw data. Dark current and incident intensity images were also acquired several times during the CT scan. The raw XTM data was transferred to an SGI and converted into VIEW format using a conversion code developed at LLNL [Roberson, 2000]. To convert the raw data into VIEW format, the images were initially read into a commercially available signal and image processing software package called IDL as raw data. The files were then saved as VIEW compatible files. A SCT file was also created.
for each XTM data scan that included all relevant XTM scan parameters. Once the XTM
data files are in VIEW format, the processing steps are very similar for the three CT
systems, as shown in Table 6.1.

The projection images were processed into ray-sum images \( \ln \frac{I_0}{I} \) in the same
manner as the PCAT and KCAT data. Outlying pixels were removed to minimize streaks
in the final CT image, and the ring removal routine was also applied to the sinograms
with the same amplitudes and orders as used for the PCAT and KCAT data. The beam-
hardening correction was not applied to data acquired with the XTM, due to the
monochromatic nature of the X-ray energy used. The XTM employs a parallel-beam X-
ray source geometry, and therefore all sinograms were reconstructed using the FBP
algorithm described in Chapter 4. Again, the rotational centers of each scan were
selected by hand to minimize centering artifacts in the reconstructed images.

Due to the limited height of the field-of-view of the XTM system, it was
necessary to acquire several vertical scans with a few overlapping slices for each tensile
bar in order to acquire data over the entire notched region of interest. Each scan was
processed and reconstructed separately, and then the resulting sets of reconstructions
were stacked to create one larger CT set that included the entire region of interest. An
example of two reconstructed CT images acquired using the XTM are shown in Figure 6.12.

Figure 6.12 Examples of reconstructed CT images from data acquired with the XTM. Bottom slice is image from the plane indicated in top image.

6.3 Comparison of Reconstructed Images

The reconstructed slices from all three CT systems were compared to determine which system produced the highest contrast and artifact-free slices for further analysis.

First, the CT data sets acquired with PCAT at two energies, 75 kV and 200 kV, were
compared, and two representative slices from tensile bar G24 are shown in Figure 6.13.

By inspection, the data acquired with the lower energy (75 kV) showed more contrast, as well as less noise than the data acquired with the higher energy (200 kV), confirmed by one-dimensional lineouts from the images.

Figure 6.13 Comparison of CT slices from tensile bar G24 acquired with PCAT at 200 and 75 kV, with lineouts from indicated regions.
Next, the 75 kV PCAT data was compared to the 75 kV KCAT data, shown in Figure 6.14. The KCAT data acquired with the filtered energy appeared to show more contrast and less noise than the PCAT data acquired with an unfiltered X-ray energy source, again confirmed by lineouts from the images.

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**Figure 6.14** CT slice from tensile bar H24 acquired with KCAT at 75 kV, with lineout from indicated region.

**Figure 6.15** CT slice from tensile bar H24 acquired with KCAT at 25 kV, with lineout from indicated region.
Finally, the data acquired with KCAT was compared to data acquired with the XTM at a monochromatic X-ray energy of 25 kV (see Figure 6.15).

By inspection, the XTM data appeared to show more detail, and have higher contrast and less noise than the KCAT data. The signal to noise ratios (SNR) were calculated for CT images acquired with the PCAT, KCAT and XTM CT systems to confirm that XTM data contained the least amount of noise in the reconstructed images [ASTM, 1995, Hammersberg and Mångård, 1999]. The SNR used here is defined as

\[
SNR = \frac{\Delta S}{\sigma_{\Delta S}} = \frac{|m_2 - m_1|}{\sqrt{\sigma_1^2 + \sigma_2^2}},
\]

where \(\Delta S\) is the difference between two signals, \(S_1\) and \(S_2\), and \(m\) and \(\sigma\) are the mean values and the standard deviations of the signals, respectively, as indicated in Figure 6.16. Two SNRs were calculated for each CT image; one for the magnesium alloy, and one for a void within the magnesium alloy. The calculated values for CT slices from one tensile bar, H24, loaded to 60% of its average failure load are presented in Table 6.2.

<table>
<thead>
<tr>
<th>CT System</th>
<th>Signal to Noise Ratio (SNR)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material</td>
</tr>
<tr>
<td>PCAT*</td>
<td>6.712</td>
</tr>
<tr>
<td>KCAT</td>
<td>7.889</td>
</tr>
<tr>
<td>XTM</td>
<td>20.135</td>
</tr>
</tbody>
</table>

*SNR for PCAT at an energy of 75 kV was calculated from tensile bar G24 loaded to 60%.
Figure 6.16 Example of a lineout showing the calculation of the mean and standard deviation needed to
calculate the SNR.

The XTM data has by far the best magnesium alloy SNR, at more than twice that of the
other CT systems, and more than 17% greater void SNR as well. In addition, because
there is no beam hardening artifact in the data acquired with the monotonic energy XTM,
there is less preprocessing necessary. While there appears to be a vertical gradient in the
XTM images (see Figure 6.12), which could be due to fluctuations in the X-ray beam or
energy harmonics, it does not affect subsequent processing and segmentation routines.
Because of the more visible details, the quantitatively higher SNR, and the minimal image processing used, it was decided to use the XTM data for further void analysis.

The XTM became available for use only after all tensile bars had been loaded to 60% of their average failure loads. For this reason, the analysis performed on the XTM data begins with 60% instead of 0%. Efforts to extrapolate back to an initial state using initial data acquired with KCAT were not successful.
Chapter 7

Void Analysis and Results

This chapter presents the experimental techniques and methodology used to analyze void behavior in the magnesium AM60B tensile bars. Results are presented here and will be discussed in depth in Chapter 8. In order to perform a void analysis on the reconstructed CT image data, it is helpful to segment out the voids from the dense material, in this case AM60B magnesium, in order to correctly identify and label voids in the CT image. The most critical step in any quantitative image analysis application is the correct segmentation of the features to be measured, in this case voids [Fabbri, 1980]. Ideally, the result is a three-dimensional volume containing only voids with minimal image processing techniques used on the data. In other words, a volume in which voids have a value of one and everything else, including dense material, has a value of zero. The segmentation of the voids from the material in this manner makes it easier to characterize the voids and their behavior as a function of mechanical loading.

The steps used here for segmentation of voids in the CT image data are shown in the flowchart in Figure 7.1. The segmentation steps presented in the flowchart are described in greater detail in the sections to follow.
7.1 Segmentation of voids

The resulting pixel size in the final CT images differed depending on which CT system was used to acquire the data. For example, the smallest pixel size was 0.017 mm for data acquired with PCAT, and the largest pixel size was 0.023 mm for data acquired with the XTM. To be consistent in the void analysis, all data sets were resampled to the largest pixel size before further analysis. Working in the VIEW image-processing
program, each set of reconstructed data was resampled to create CT volumes with an identical voxel volume of 0.02368 mm x 0.02368 mm x 0.02368 mm (1.33×10⁻⁵ mm³).

To decrease the size of each data set, the empty space surrounding the tensile bar in each slice image was eliminated. Thus, each slice image was extracted to a size on the order of 560 x 560 pixels. To further decrease the size of each data set, approximately 260 slices were selected for analysis. The 260 selected slices contained the entire notched region of each tensile bar data set. The reduced data set resulted in increased data processing speeds while retaining the full CT information of interest. For each set of data, a threshold of zero was applied to each reconstructed slice to eliminate any negative values. Next, a binary mask was created of the outside surface of each tensile bar from each slice by searching for a user-specified pixel value representing the object, in this case the edge of the tensile bar. The mask images were then filtered using a 3 x 3 median filter to smooth rough edges of the mask that are due to small variations in pixel values on the edge of the part. The mask slices were then stacked to create a mask volume of binary data representing the dense material, excluding voids. Representative images of the mask volume are shown in Figure 7.2. The rest of the data processing was performed using the IDL software package [Research Systems, 1999].
Each resampled and extracted CT data set was read into IDL as a three-dimensional floating point array. The maximum value in the CT volume was determined and a new three-dimensional array of identical size as the CT volume was created, containing at every point the maximum value. The original CT volume was then subtracted from this new array, resulting in an inverted CT volume as shown in Figure 7.3, with fully dense material now appearing with attenuation values near zero and voids and low-density regions appearing with attenuation values near the maximum. Next, the binary mask volume is read into IDL and multiplied by the inverted image, resulting in an inverted image of the tensile bar CT data, with zeroes outside the part, and the inverted CT image inside, as shown in Figure 7.4.
Figure 7.3 Representative image of an inverted CT reconstruction from tensile bar H24 loaded to 60% of its average failure load. The solid magnesium alloy now appears dark while voids and regions of low-density alloy appear light.

Figure 7.4 Image of the inverted CT reconstruction shown in Figure 7.3 after the binary mask volume (Figure 7.2) has been applied. The mask serves to eliminate all pixels not lying within the tensile bar, leaving only pixels containing the magnesium alloy (low values) or voids (high values).
This inverted and masked volume can be used to segment voids from the material by creating a histogram and selecting an upper threshold value, with anything above that value indicating a void. In the ideal case the histogram is a bimodal distribution with two peaks, one representing the magnesium alloy and the other representing voids. A threshold in this case is selected midway between the two peaks. However, the histograms of these inverted and masked CT volumes are not bimodal. Instead, the data appears as one large peak with a tail on the higher end, see Figure 7.5.

Histogram of inverted and masked CT volume for tensile bar H24 loaded to 60% of average failure load

Figure 7.5 Histogram created from the inverted and masked CT volume, used to select a threshold for segmentation of voids from magnesium alloy. Note that there is no clear distinction between magnesium alloy (low values) and voids (higher values). A threshold value is selected midway in the shoulder region, as indicated, and as discussed in the text.
In this case, thresholds were selected midway on the “shoulder” of the tail. While this threshold selection technique is subjective, and the results can vary considerably with respect to the threshold selected, simple studies of threshold sensitivity have revealed that thresholds selected in this manner give consistent results.

To check the validity of the threshold selection, each data set was analyzed using seven different thresholds. A void analysis to be described later was performed on the volume at each threshold selected and the maximum void volume resulting from each of these thresholds was determined. The results were then plotted with the maximum void volume on the vertical axis and the selected threshold on the horizontal axis, as shown in Figure 7.6. Ideally the plots would show a flat plateau over a narrow range of threshold values, indicating that a threshold selected from the plateau region would give results that show little sensitivity to the selected threshold[Schwartz, et al., 1994]. This is not the case with the magnesium alloy CT data, however there is a well-defined inflection in the curve (see Figure 7.6) and thresholds selected from this region have a minimal effect on the maximum void volume results. The thresholds selected for the data presented here lie near the inflection in every case, and therefore were selected as consistently as possible, given the subjective nature of the threshold selection process.
Figure 7.6 Plot showing the sensitivity of the single maximum void volume versus the selected threshold value. Note that the curve flattens out around 0.1, indicating a decreased sensitivity. Threshold values for each data set were selected near this plateau region.

Once the void threshold value has been selected from the histogram, the threshold value is applied to the inverted and masked data set, resulting in a segmented binary volume. The remaining voxels define the voids selected for further analysis. Due to an abundance of "salt and pepper" noise (single voxels) remaining in the PCAT and KCAT data after the application of the threshold, the segmented volume was filtered using a 3 by 3 median filter to eliminate the single voxels. The filtering is not a desirable processing step since any filtering used to eliminate noise will affect the volume and shape of the segmented voids by smoothing them as well. The XTM data did not require filtering as
there was significantly less noise in the segmented image. A representative image of a segmented slice from XTM data is shown in Figure 7.7.

Figure 7.7 Final void segmented CT image from tensile bar H24 loaded to 60% of average failure load. A threshold was used to segment voids from dense material as discussed in the text.

Once the CT data has been segmented, voids must be distinguished from the noise, i.e., only the voids should be identified for further analysis. Using the IDL software package, a cluster analysis routine, written in the C programming language (see Appendix A), was applied to each data set and was used to identify and label clusters of voxels [Hoshen and Kopelman, 1976; Haupt, 2000]. Explained simply, the routine searches a three-dimensional volume of binary data for voxels with a value of one, and
then looks for any connected voxels with values of one. The routine looks for voxels connected in any of 26 ways, including connected corners, edges, and faces. As the routine finds two voxels that are connected in some way, it gives both of them the same unique numeric label. It proceeds through the volume labeling all subsequent connected voxels with the same unique label until it finds a voxel with a value of zero, which it ignores and does not label. The routine then proceeds along until it encounters another voxel with a value of one, which it assigns a new unique numeric label. The routine proceeds by rastering across a plane in one direction, followed by the perpendicular direction in the same plane, then moves to the next plane down where the process is repeated. As it goes, it checks for voxel connectivity both behind and above its current path to ensure that seemingly newly found unconnected voxels are not connected in any way with previously labeled voxels. The result of this routine is a three-dimensional array containing uniquely labeled clusters, some of which are voids, while some are noise or low-density magnesium alloy.

7.2 Void statistics

Voids can be identified from the labeled cluster array by defining a void to be a cluster of a certain minimum volume. To determine which clusters are voids, the labeled cluster arrays were analyzed as follows. A histogram was created from the labeled
volume. The resulting histogram is a plot of the cluster volume, in number of connected voxels, versus numeric cluster label, as seen in Figure 7.8. The list of cluster labels and their corresponding volumes were exported to a spreadsheet program for further analysis. Sorting this information and manipulating it in various ways revealed many statistical parameters of interest for describing voids in the magnesium AM60B tensile bars.

Figure 7.8 Cluster label histogram created from the cluster labeled segmented void volume for tensile bar H24 loaded to 60% of average failure load. The histogram is a graphical representation of the cluster volumes (in number of connected voxels) in the segmented void volume.
7.2.1 Minimum void volume

The experimental cluster volume distribution was easily obtained by creating a histogram from the previous cluster labeled volume histogram. This provides a graphical representation of the frequency of occurrence versus cluster volume. When graphed on a log-log plot (see Figure 7.9), the void volume histogram for tensile bar H24-60% has two distinct slopes.

![Cluster Volume versus Frequency](image)

**Figure 7.9** Cluster volume histogram created from the cluster label histogram in Figure 7.8. There are many more small clusters in the volume than large. Note the two distinct slopes as indicated with the solid black lines, with the change occurring around the 100-voxel size. Only clusters larger than 100 connected voxels were selected for void analysis.
At smaller volumes, there are a large number of occurrences, and the frequency of occurrence decreases at essentially the same steep rate until some critical cluster volume, where the slope flattens out dramatically. This is observed for all tensile bars and loading conditions. For the data presented in Figure 7.9, the slope seems to change at a volume of approximately 100 connected voxels. To be consistent, and to eliminate false void detection due to noise, it was decided to perform void analyses on clusters with a volume larger than the critical volume of 100 connected voxels only. One hundred connected voxels corresponds to a volume of $1.33 \times 10^{-3}$ mm$^3$. Although it is certain that many voids exist at smaller volumes, for the purposes of obtaining information on void growth and nucleation, it is acceptable to define a void as any cluster containing a minimum of 100 connected voxels. As long as the definition of the minimum volume of a void remains constant, then the void nucleation, growth and coalescence effects can be observed relative to this volume.

7.2.2 Damage statistics

Once the definition of a void has been established, in this case clusters containing a minimum of 100 connected voxels or 0.00133 mm$^3$, and all smaller clusters discarded, it is simple to calculate the void density, or the number of voids per unit volume, a value related to the nucleation parameter described in Chapter 2. The binary mask volume
created from the CT data is used to calculate the total volume of material including voids. The number of voids (clusters greater than 100 voxels) can easily be obtained from the histogram, and then the void density, related to the nucleation term \( \eta \), can be determined, recalling that \( \eta \) is defined as the number of voids per unit volume.

Other statistical trends related to total damage accumulation such as average, median, and maximum void volume were determined for each data set, as well as the total void volume fraction, calculated as the average void volume times the void density. The total void volume fraction is related to the total damage accumulation, \( \phi \), as discussed in Chapter 2. These data are presented in the 7.1 for Series H tensile bars, loaded from 60% of their average failure load up to and including failure. The subscript "m" indicates that the parameter is measured, as opposed to theoretical. The statistical parameters were calculated for each failed specimen, however, because the two failed surfaces were held together while being scanned, it is unclear how relevant the resulting statistics are. While they are included in Figure 7.1 (a-c), when discussing trends the failed data sets are not considered unless otherwise stated.

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Table 7.1a. Summary of the statistical void analysis results determined from the XTM data at 60%, 87%, 93% of average failure load and failure for tensile bar H23.

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Number of voids/ mm³ ηₚ</th>
<th>Avg. void vol. (mm³) νₚ</th>
<th>Median void vol. (mm³)</th>
<th>Max Void vol. (mm³)</th>
<th>Damage ( \phi_n (10^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H23-1 (60%)</td>
<td>0.51</td>
<td>0.0058 ±0.012</td>
<td>0.0029</td>
<td>0.13</td>
<td>3.0</td>
</tr>
<tr>
<td>H23-2 (87%)</td>
<td>0.42</td>
<td>0.0068 ±0.020</td>
<td>0.0029</td>
<td>0.28</td>
<td>2.9</td>
</tr>
<tr>
<td>H23-3 (93%)</td>
<td>0.58</td>
<td>0.0082 ±0.032</td>
<td>0.0027</td>
<td>0.52</td>
<td>4.8</td>
</tr>
<tr>
<td>H23-4* (failure)</td>
<td>0.37</td>
<td>0.0047 ±0.0060</td>
<td>0.0026</td>
<td>0.049</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 7.1b. Summary of the statistical void analysis results determined from the XTM data at 60%, 87%, 93%, 95% of average failure load and failure for tensile bar H24.

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Number of voids/ mm³ ηₚ</th>
<th>Avg. void vol. (mm³) νₚ</th>
<th>Median void vol. (mm³)</th>
<th>Max Void vol. (mm³)</th>
<th>Damage ( \phi_n (10^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H24-1 (60%)</td>
<td>0.52</td>
<td>0.0054 ±0.0085</td>
<td>0.0026</td>
<td>0.077</td>
<td>2.8</td>
</tr>
<tr>
<td>H24-2 (87%)</td>
<td>0.52</td>
<td>0.0065 ±0.015</td>
<td>0.0026</td>
<td>0.18</td>
<td>3.4</td>
</tr>
<tr>
<td>H24-3 (93%)</td>
<td>0.46</td>
<td>0.0069 ±0.015</td>
<td>0.0027</td>
<td>0.18</td>
<td>3.2</td>
</tr>
<tr>
<td>H24-4 (95%)</td>
<td>0.58</td>
<td>0.0075 ±0.028</td>
<td>0.0025</td>
<td>0.43</td>
<td>4.3</td>
</tr>
<tr>
<td>H24-5* (failure)</td>
<td>0.41</td>
<td>0.0070 ±0.022</td>
<td>0.0025</td>
<td>0.31</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Table 7.2c. Summary of the statistical void analysis results determined from the XTM data at 60%, 87%, 93%, 97% of average failure load and failure for tensile bar H25.

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Number of voids/ mm³ ηₚ</th>
<th>Avg. void vol. (mm³) νₚ</th>
<th>Median void vol. (mm³)</th>
<th>Max Void vol. (mm³)</th>
<th>Damage ( \phi_n (10^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H25-1 (60%)</td>
<td>0.75</td>
<td>0.0066 ±0.014</td>
<td>0.0027</td>
<td>0.16</td>
<td>5.0</td>
</tr>
<tr>
<td>H25-2 (87%)</td>
<td>0.65</td>
<td>0.0064 ±0.014</td>
<td>0.0026</td>
<td>0.16</td>
<td>4.1</td>
</tr>
<tr>
<td>H25-3 (93%)</td>
<td>0.47</td>
<td>0.0074 ±0.016</td>
<td>0.0029</td>
<td>0.18</td>
<td>3.5</td>
</tr>
<tr>
<td>H25-4 (97%)</td>
<td>0.50</td>
<td>0.0081 ±0.023</td>
<td>0.0027</td>
<td>0.26</td>
<td>4.0</td>
</tr>
<tr>
<td>H25-5* (failure)</td>
<td>0.34</td>
<td>0.0066 ±0.022</td>
<td>0.0026</td>
<td>0.29</td>
<td>2.2</td>
</tr>
</tbody>
</table>

*This analysis was performed on a failed sample with two failure surfaces.
7.2.3 Void volume distributions

Experimental void volume probability density curves were created from the labeled histograms, using a bin volume of 0.001 mm³ and normalizing by the total number of voids in the data set, making the area under the curve equal to one. The probability density is the probability of a void lying in a particular volume bin (0.001 mm³). When the void volume data is presented in this way, characteristic statistical parameters and well-known probability distribution functions can be used to describe and characterize the data, and also to detect trends in the data.

![Probability density curves](image-url)

(a)
Figure 7.10(a-c) Probability density curves for tensile bars H23-25, after loading to 60% of average failure load up to and including failure. Note the long tail to the right of each maximum, indicating a positive skewness.
The experimental probability density curves created for each tensile bar at each loading condition are shown in Figure 7.10(a-c). Each of the resulting probability density curves is unimodal (i.e., each has only one peak) and positively skewed. Skewness can be used as an indication of the degree of asymmetry that a distribution exhibits [Gu, et al., 1986]. A probability density curve is said to be skewed to the right, or has positive skewness, if it has a tail to the right of the central maximum, or mode. Conversely, if the probability density curve has a tail to the left of the central maximum, the distribution is said to be negatively skewed. The skewness can be calculated as

\[ \text{Skewness} = \frac{\bar{x} - \text{Mode}}{s}, \]  

(7.1)

where \( \bar{x} \) is the average value, in this case average void volume, the mode is the value that occurs most frequently, and \( s \) is the standard deviation. Thus, a positively skewed distribution with a tail to the right of the mode would have a positive value of skewness.

Some statistical parameters for each probability density curve are presented in Table 7.2, including the average void volume, standard deviation and mode.
Table 7.2 Statistical parameters for each data set.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Loading (% failure load)</th>
<th>Average void volume (mm³)</th>
<th>Standard Deviation</th>
<th>Mode (mm³)</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>H23</td>
<td>60</td>
<td>0.0058</td>
<td>0.0118</td>
<td>0.002</td>
<td>0.3235</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>0.0068</td>
<td>0.0199</td>
<td>0.002</td>
<td>0.2418</td>
</tr>
<tr>
<td></td>
<td>93</td>
<td>0.0082</td>
<td>0.0325</td>
<td>0.002</td>
<td>0.1909</td>
</tr>
<tr>
<td></td>
<td>100 (failure)</td>
<td>0.0047</td>
<td>0.0060</td>
<td>0.002</td>
<td>0.4493</td>
</tr>
<tr>
<td>H24</td>
<td>60</td>
<td>0.0054</td>
<td>0.0085</td>
<td>0.002</td>
<td>0.3958</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>0.0065</td>
<td>0.0151</td>
<td>0.002</td>
<td>0.2994</td>
</tr>
<tr>
<td></td>
<td>93</td>
<td>0.0069</td>
<td>0.0152</td>
<td>0.002</td>
<td>0.3215</td>
</tr>
<tr>
<td></td>
<td>95</td>
<td>0.0075</td>
<td>0.0284</td>
<td>0.002</td>
<td>0.1920</td>
</tr>
<tr>
<td></td>
<td>100 (failure)</td>
<td>0.0070</td>
<td>0.0224</td>
<td>0.002</td>
<td>0.2209</td>
</tr>
<tr>
<td>H25</td>
<td>60</td>
<td>0.0066</td>
<td>0.0145</td>
<td>0.002</td>
<td>0.3202</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>0.0064</td>
<td>0.0144</td>
<td>0.002</td>
<td>0.3058</td>
</tr>
<tr>
<td></td>
<td>93</td>
<td>0.0074</td>
<td>0.0162</td>
<td>0.002</td>
<td>0.3337</td>
</tr>
<tr>
<td></td>
<td>97</td>
<td>0.0081</td>
<td>0.0230</td>
<td>0.002</td>
<td>0.2660</td>
</tr>
<tr>
<td></td>
<td>100 (failure)</td>
<td>0.0066</td>
<td>0.0223</td>
<td>0.002</td>
<td>0.2043</td>
</tr>
</tbody>
</table>

7.2.3.1 Weibull Distribution

Another way to characterize an experimental void volume probability density curve is to fit it with a well-defined mathematical distribution function. While experimental size or volume distributions of particles such as voids are commonly modeled using lognormal, beta, or gamma distribution functions, the distribution function that fit these experimental void volume distributions best is the Weibull distribution function [Weibull, 1951; Hastings and Peacock, 1975]. While the Weibull function is simpler to use and has more versatility than other commonly used distribution functions, it is important to point out that the application of this distribution function is empirical.
rather than theoretical and it is not suggested that any fundamental physical relationship
exists between void volume distributions and the Weibull distribution function.

However, the Weibull distribution function was selected to describe the experimental
void volume distributions in the tensile bars in an effort to try to say something more
about the voids and their behavior and characteristics. The Weibull distribution is most
commonly used in analyzing fatigue strength data [Kip, et al., 1995; Kip, 1995; Yang, et
al. 1987]. However, it has also been used to describe particle size distributions in Al-Li
alloys [Gu, et al., 1986], to unfold two-dimensional size distribution data to three
dimensions [Fang, et al., 1993], and to characterize the type and severity of wear on
materials by characterizing the size distributions of wear particles [Roylance and Pocock,
1983]. Experimental cumulative void volume probability curves can be created from the
probability density curves, by summing the probability versus void volume. The
experimental cumulative probability data can then be plotted and the resulting curve can
be fit with a mathematical function or distribution, in this case the Weibull distribution
function, presented in Figure 7.11(a-c).
Cumulative Probabilities with Weibull Fit
Tensile bar H23

Cumulative probability

H23-60%: \( p(x) = 1 - \exp\left(-\frac{x}{0.0042}\right)^{0.88} \)
H23-87%: \( p(x) = 1 - \exp\left(-\frac{x}{0.0041}\right)^{0.778} \)
H23-93%: \( p(x) = 1 - \exp\left(-\frac{x}{0.0039}\right)^{0.669} \)
H23-Failure: \( p(x) = 1 - \exp\left(-\frac{x}{0.0037}\right)^{0.861} \)

(b) Cumulative Probabilities with Weibull Fit
Tensile bar H24

Cumulative probability

H24-60%: \( p(x) = 1 - \exp\left(-\frac{x}{0.0042}\right)^{0.778} \)
H24-87%: \( p(x) = 1 - \exp\left(-\frac{x}{0.0041}\right)^{0.678} \)
H24-93%: \( p(x) = 1 - \exp\left(-\frac{x}{0.0040}\right)^{0.659} \)
H24-Failure: \( p(x) = 1 - \exp\left(-\frac{x}{0.0037}\right)^{0.861} \)
The cumulative probability is the probability of a void having a volume smaller than a certain bin volume. The cumulative probability of the Weibull distribution function, $P(x)$, with two parameters is expressed as

$$P(x) = 1 - e^{-\left(\frac{x}{a}\right)^b}, \quad (7.2)$$

where $x$ is the parameter of interest (void volume in this case) and $a$ and $b$ are the Weibull parameters. When the experimental cumulative probability curves are fit with a distribution function of the form of equation 7.2, the resulting curves give reasonably good agreement with the experimental data, with an average value of Chi squared equal
to 0.077. The values of the Weibull parameters calculated for each data set along with the Chi squared values are presented in Table 7.3.

Table 7.3 Calculated Weibull parameters for each data set.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Loading (% failure load)</th>
<th>$a$</th>
<th>$a$ average</th>
<th>$b$</th>
<th>$b$ average</th>
<th>Chi Squared</th>
<th>Chi Squared average</th>
</tr>
</thead>
<tbody>
<tr>
<td>H23</td>
<td>60</td>
<td>0.0042</td>
<td>0.0040 $\pm 0.00024$</td>
<td>0.881</td>
<td>0.802 $\pm 0.088$</td>
<td>0.0299</td>
<td>0.0392 $\pm 0.017$</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>0.0041</td>
<td></td>
<td>0.778</td>
<td></td>
<td>0.0492</td>
<td></td>
</tr>
<tr>
<td></td>
<td>93</td>
<td>0.0039</td>
<td></td>
<td>0.689</td>
<td></td>
<td>0.0575</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100 (failure)</td>
<td>0.0037</td>
<td></td>
<td>0.860</td>
<td></td>
<td>0.0202</td>
<td></td>
</tr>
<tr>
<td>H24</td>
<td>60</td>
<td>0.0039</td>
<td>0.0037 $\pm 0.00028$</td>
<td>0.778</td>
<td>0.692 $\pm 0.054$</td>
<td>0.0243</td>
<td>0.0445 $\pm 0.014$</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>0.0038</td>
<td></td>
<td>0.678</td>
<td></td>
<td>0.0405</td>
<td></td>
</tr>
<tr>
<td></td>
<td>93</td>
<td>0.0040</td>
<td></td>
<td>0.659</td>
<td></td>
<td>0.0433</td>
<td></td>
</tr>
<tr>
<td></td>
<td>95</td>
<td>0.0036</td>
<td></td>
<td>0.706</td>
<td></td>
<td>0.0556</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100 (failure)</td>
<td>0.0033</td>
<td></td>
<td>0.639</td>
<td></td>
<td>0.0589</td>
<td></td>
</tr>
<tr>
<td>H25</td>
<td>60</td>
<td>0.0046</td>
<td>0.0044 $\pm 0.00036$</td>
<td>0.976</td>
<td>1.017 $\pm 0.130$</td>
<td>0.1137</td>
<td>0.1276 $\pm 0.027$</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>0.0044</td>
<td></td>
<td>1.034</td>
<td></td>
<td>0.1132</td>
<td></td>
</tr>
<tr>
<td></td>
<td>93</td>
<td>0.0046</td>
<td></td>
<td>0.937</td>
<td></td>
<td>0.1652</td>
<td></td>
</tr>
<tr>
<td></td>
<td>97</td>
<td>0.0048</td>
<td></td>
<td>0.905</td>
<td></td>
<td>0.1458</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100 (failure)</td>
<td>0.0038</td>
<td></td>
<td>1.234</td>
<td></td>
<td>0.0999</td>
<td></td>
</tr>
</tbody>
</table>

The probability density function, $p(x)$, of the Weibull distribution function is given by the derivative of the cumulative probability;

$$
\frac{dP(x)}{dx} = p(x) = \frac{b}{a} \left(\frac{x}{a}\right)^{b-1} e^{-\left(\frac{x}{a}\right)^b}.
$$

(7.3)

The median void volume and the average void volume can also be calculated from the Weibull parameters as [Roylance and Pocock, 1983]:

$$
median = a (\ln 2)^{1/b},
$$

(7.4)

$$
average = a \Gamma\left(1 + \frac{1}{b}\right).
$$

(7.5)
Table 7.4 Weibull parameters and the calculated median and average, compared to experimental median and average void volumes (mm$^3$) for each data set.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Loading (% failure load)</th>
<th>$a$</th>
<th>$b$</th>
<th>Weibull median (mm$^3$)</th>
<th>Experimental median (mm$^3$)</th>
<th>Weibull average (mm$^3$)</th>
<th>Experimental average (mm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H23</td>
<td>60</td>
<td>0.0042</td>
<td>0.88</td>
<td>0.0028</td>
<td>0.0029</td>
<td>0.0045</td>
<td>0.0058</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>0.0041</td>
<td>0.78</td>
<td>0.0026</td>
<td>0.0029</td>
<td>0.0047</td>
<td>0.0068</td>
</tr>
<tr>
<td></td>
<td>93</td>
<td>0.0039</td>
<td>0.69</td>
<td>0.0023</td>
<td>0.0027</td>
<td>0.0050</td>
<td>0.0082</td>
</tr>
<tr>
<td></td>
<td>100 (failure)</td>
<td>0.0037</td>
<td>0.86</td>
<td>0.0024</td>
<td>0.0026</td>
<td>0.0040</td>
<td>0.0047</td>
</tr>
<tr>
<td>H24</td>
<td>60</td>
<td>0.0039</td>
<td>0.78</td>
<td>0.0024</td>
<td>0.0026</td>
<td>0.0045</td>
<td>0.0054</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>0.0038</td>
<td>0.68</td>
<td>0.0022</td>
<td>0.0026</td>
<td>0.0050</td>
<td>0.0065</td>
</tr>
<tr>
<td></td>
<td>93</td>
<td>0.0040</td>
<td>0.66</td>
<td>0.0023</td>
<td>0.0027</td>
<td>0.0054</td>
<td>0.0069</td>
</tr>
<tr>
<td></td>
<td>97</td>
<td>0.0036</td>
<td>0.71</td>
<td>0.0021</td>
<td>0.0025</td>
<td>0.0045</td>
<td>0.0075</td>
</tr>
<tr>
<td></td>
<td>100 (failure)</td>
<td>0.0033</td>
<td>0.64</td>
<td>0.0019</td>
<td>0.0025</td>
<td>0.0046</td>
<td>0.0070</td>
</tr>
<tr>
<td>H25</td>
<td>60</td>
<td>0.0046</td>
<td>0.98</td>
<td>0.0032</td>
<td>0.0027</td>
<td>0.0046</td>
<td>0.0066</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>0.0044</td>
<td>1.03</td>
<td>0.0031</td>
<td>0.0026</td>
<td>0.0043</td>
<td>0.0064</td>
</tr>
<tr>
<td></td>
<td>93</td>
<td>0.0046</td>
<td>0.94</td>
<td>0.0031</td>
<td>0.0029</td>
<td>0.0047</td>
<td>0.0074</td>
</tr>
<tr>
<td></td>
<td>97</td>
<td>0.0048</td>
<td>0.91</td>
<td>0.0032</td>
<td>0.0027</td>
<td>0.0050</td>
<td>0.0081</td>
</tr>
<tr>
<td></td>
<td>100 (failure)</td>
<td>0.0038</td>
<td>1.23</td>
<td>0.0028</td>
<td>0.0026</td>
<td>0.0035</td>
<td>0.0066</td>
</tr>
</tbody>
</table>

These median and average void volume values were calculated using the Weibull parameters for each data set and are presented in Table 7.4, along with experimental values.

7.2.4 Nearest neighbor distance

The nearest neighbor distance for each void was determined using a program created in IDL (see Appendix B). The voids previously identified for analysis were relabeled using the aforementioned cluster labeling routine, with labels ranging from 1 through $n$. The coordinates of the centroid of each void were then calculated and input to an array of size $3 \times n$. Next, the three-dimensional distance between two points,

$$\text{dist} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}, \quad (7.6)$$

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was calculated between the centroid of each void \((i)\) and the centroid of every other void 
\((j)\) and input into a size \(n \times n\) array. The nearest neighbor distance for each void was
determined as the minimum distance for each void in the \(n \times n\) array and saved to a file.
Second and third nearest neighbor distances can be determined in the same way. The
nearest neighbor distributions can then be analyzed to detect spatial distribution changes
from one loading condition to another.

Nearest neighbor distance probability density curves were created for each data
set, and the average nearest neighbor distance and the standard deviation were calculated.
Clark and Evans in 1954 postulated an elegant way to determine characteristics of the
spatial distribution compared to a randomly distributed pattern for a two-dimensional
space. The derivation can be extrapolated to three dimensions and is presented in
Appendix C. In a random distribution of a set of points on a given area, or for a set of
randomly distributed voids in a given volume, it is assumed that any void has had the
same chance of occurring in any sub-volume as any other void, that every sub-volume
has had the same chance of receiving a void as any other sub-volume of the same size,
and that the spatial location of any void has not been influenced by any other void.
Therefore randomness, described here as a spatial concept, is intimately dependent upon
the boundaries of the volume chosen. In other words, a spatial distribution can be
classified as random in a certain specified volume, but decidedly non-random in a smaller specified volume within the original larger volume.

The degree of randomness of a spatial population with respect to a random distribution is determined as follows. First, the mean nearest neighbor distance is calculated from the data. Then an expected theoretical mean nearest neighbor distance is determined based on a perfectly random distribution of voids, and compared to the experimental mean distance. The degree of randomness, \( R \), is the ratio,

\[
R = \frac{r_A}{r_E}, \tag{7.7}
\]

where \( r_A \) is the actual, or experimental mean nearest neighbor distance and \( r_E \) is the theoretical expected mean nearest neighbor distance for a random distribution. This ratio can be used to determine the degree to which the data shows random or aggregated distributions. The expected void mean nearest neighbor distance can be calculated using void density and is defined as (see Appendix C)

\[
\bar{r}_E = 0.812(\eta \pi)^{-\frac{1}{3}}, \tag{7.8}
\]

where \( \eta \) is the density of voids per unit volume and is determined from the experimental data. If \( R \), as defined above, has a value close to unity, then the data exhibits random spatial distributions. If it is less than unity, the spatial distribution tends toward aggregation, or clustering. A value of \( R \) greater than unity would indicate a more ordered
distribution. The experimental nearest neighbor distributions were determined and mean nearest neighbor distance values were found for each data set. The ratio $R$ was calculated for each set of data and is presented in Table 7.5.

Table 7.5 The degree of randomness ratio, $R$, is calculated for each sample and provides an indication of randomness in the spatial distribution of the voids.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Loading (% of failure)</th>
<th>Density (number of voids/mm$^3$)</th>
<th>$\bar{r}_A$ (mm)</th>
<th>$\bar{r}_E$ (mm)</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H23</td>
<td>60%</td>
<td>0.51</td>
<td>0.60</td>
<td>0.69</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>87%</td>
<td>0.42</td>
<td>0.64</td>
<td>0.74</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>93%</td>
<td>0.58</td>
<td>0.56</td>
<td>0.66</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>Failure</td>
<td>0.37</td>
<td>0.63</td>
<td>0.77</td>
<td>0.81</td>
</tr>
<tr>
<td>H24</td>
<td>60%</td>
<td>0.52</td>
<td>0.65</td>
<td>0.69</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>87%</td>
<td>0.53</td>
<td>0.63</td>
<td>0.69</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>93%</td>
<td>0.46</td>
<td>0.65</td>
<td>0.72</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>0.58</td>
<td>0.62</td>
<td>0.66</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>Failure</td>
<td>0.41</td>
<td>0.67</td>
<td>0.75</td>
<td>0.90</td>
</tr>
<tr>
<td>H25</td>
<td>60%</td>
<td>0.75</td>
<td>0.55</td>
<td>0.61</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>87%</td>
<td>0.65</td>
<td>0.59</td>
<td>0.64</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>93%</td>
<td>0.47</td>
<td>0.63</td>
<td>0.71</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>97%</td>
<td>0.50</td>
<td>0.61</td>
<td>0.70</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>Failure</td>
<td>0.34</td>
<td>0.68</td>
<td>0.79</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Another way to determine the degree of randomness in the spatial distribution of voids is by comparing the experimental distribution of nearest neighbor distances to an ideally random Poisson distribution [Russ, 1991]. Comparing the distributions can reveal the presence of clustering by a reduction in the mean value of the experimental data compared to the ideal random distribution. An ideal random Poisson distribution was created and compared to one set of experimental nearest neighbor distances, and is shown
in Figure 7.12. The nearest neighbor distance was also plotted against void volume in Figure 7.13 to determine what effect void volume has on its spatial location relative to other voids.

![Diagram showing nearest neighbor distance distributions](image)

**Figure 7.12** Experimental nearest neighbor distance probability density curve for tensile bar H24 loaded to 60% of average failure load compared to a probability density curve generated from an ideal random Poisson distribution. The experimental distribution is shifted to the left, indicating a tendency for voids to cluster.
7.2.5 Void volume fraction distribution

The experimental void volume fraction (average void volume times void density) spatial distribution is another parameter that was explored. Using IDL, a routine was created that calculated the total void volume fraction in a series of increasing diameter annuli at one location on the z-axis (see Appendix D). The void volume fractions were calculated for annuli on other z-axis locations until the entire tensile bar notch region had been analyzed. While this routine provided a map of void volume fraction as a function of radius and the z-axis, the void volume fractions calculated are summed around $\theta$. In other words, this calculation assumes axial symmetry. The results can be presented as
Figure 7.14 Contour plots of void volume fraction as a function of \( r \) and \( z \), averaged over \( \theta \), for tensile bar H24 loaded to 60% and 95% of its average failure load and input to a two-dimensional quarter-plate axisymmetric finite element mesh.
contour graphs of void volume fraction, as shown in Figure 7.14 (a-b) for tensile bar H24 loaded to 60 and 95% of average failure load. This information is valuable input to two-dimensional finite element analyses, using quarter-plate axisymmetry assumptions, as described in section 7.3.1.

7.2.6 Void shape

Shape information about the voids is another parameter of interest, however most shape factors are based on two-dimensional analysis methods [Hawkins, 1993; and Bernhardt, 1994]. In keeping with the intent of utilizing the full three-dimensional information, a three-dimensional shape factor was chosen to analyze the morphology of each void. The volumetric shape factor used is defined as [Clift et al., 1978]

\[ K = \frac{V}{d^3}, \]  

(7.8)

where \( V \) is the total volume (in voxels) of the object of interest (one void in this case), and

\[ d = \sqrt[3]{\frac{4A_p}{\pi}} \]  

(7.9)

is the "projected area diameter", defined as the diameter of a sphere with the same projected area as the void. The projected area of the void is given by \( A_p \), and was calculated for each void in three dimensions (projected onto the \( yz \)-, \( zx \)-, and the \( xy \)-planes) using a routine written in IDL (see Appendix E). The projected area diameter, \( d_p \),
was then calculated for all three dimensions for each void. Three orthogonal shape factors, $K_x$, $K_y$ and $K_z$ were calculated for the maximum void after each loading condition for tensile bar H24 and the standard deviation between the three shape factors for each void was determined. Results are presented in Table 7.6. This volumetric shape parameter and the standard deviation between them may provide insight into three-dimensional morphological changes of a void with increasing load.

Table 7.6 Volumetric shape factors for the maximum void in tensile bar H24, after loading to 60, 87, 93, 95% of average failure load and failure.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$K_x$</th>
<th>$K_y$</th>
<th>$K_z$</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>H24-60%</td>
<td>0.075</td>
<td>0.18</td>
<td>0.37</td>
<td>0.15</td>
</tr>
<tr>
<td>H24-87%</td>
<td>0.065</td>
<td>0.087</td>
<td>0.082</td>
<td>0.012</td>
</tr>
<tr>
<td>H24-93%</td>
<td>0.091</td>
<td>0.082</td>
<td>0.082</td>
<td>0.0050</td>
</tr>
<tr>
<td>H24-95%</td>
<td>0.078</td>
<td>0.060</td>
<td>0.076</td>
<td>0.010</td>
</tr>
<tr>
<td>H24-Failure</td>
<td>0.073</td>
<td>0.081</td>
<td>0.075</td>
<td>0.0037</td>
</tr>
</tbody>
</table>

7.3 Destructive evaluation

Fractographic analysis was performed on tensile bar H25 after failure. The failure surface was examined using a scanning electron microscope (SEM) and images were captured with a computer. A SEM image of the failure surface of tensile bar H25 is shown in Figure 7.15. A metallographic analysis on tensile bar H25 was then performed. The sample was cut and identified as either “top”, or “bottom”, depending on the failure surface (see Figure 7.16).
Figure 7.15 SEM photograph of the fracture surface of tensile bar H25. The large round feature in the center of the picture is the damaged ring. Note the dimpling surrounding the ring, indicating ductile fracture.

Figure 7.16 Diagram illustrating nomenclature used in metallographic analysis. The image of the left shows the two fracture surfaces, the top containing the "button", and the bottom containing the resulting "divot". The image on the right shows how each fracture surface was quartered and labeled.
As the damaged ring pulled out of the material it created a divot on one side (referred to as the bottom) and a matching plug on the other (referred to as the top). The failed samples were quartered using a 0.05-cm thick wafering saw, and mounted in a two-part epoxy resin. Samples were ground with SiC and water at approximately 300 rpm, and then polished using a vibrating diamond polisher to a final polish of 0.05 μm. Samples were analyzed using an optical microscope at 10 to 100X magnification, and photographs were taken and digitized with a scanner. An image of the top part of tensile bar H25, section 3, is shown in Figure 7.17.

Figure 7.17 Optical microscope photograph of the top part of tensile bar H25, section 3, magnified at10X. Note that the damaged ring is clearly visible.
7.4 Other applications for segmented CT data

7.4.1 FEA – 2D

The segmented binary CT data can provide very useful engineering data as input to finite element analyses. Using a finite element code, the robustness of the damage model proposed in Chapter 2 can be further evaluated and improved if necessary. In order to use a finite element code, the object of interest must be meshed, or segmented into a series of small volumetric elements. In order to make the number of elements in the mesh small and hence easier to work with, several assumptions can be made. First, an assumption of symmetry, both axial and longitudinal, can reduce a three-dimensional problem into a two-dimensional mesh. This type of analysis is commonly referred to as a quarter-plate axisymmetric analysis [Horstemeyer, et al., 2000]. To use the experimental CT results as input to this mesh, the void volume fraction was determined from the data as a function of radius \( r \), and height \( z \), averaged over \( \theta \), as discussed in section 7.2.5.

A finite element model simulating the tensile loading of AM60B magnesium alloy notched tensile bar H24 was performed. In this analysis, the three dimensional specimen is modeled as a two dimensional axisymmetric quarter-plate, using the ABAQUS finite element code. The input to the model was the void fraction distribution,
as shown in Figure 7.14(a-b). Experimental void volume fraction values were input for each corresponding element in the model.

The material constitutive response is modeled using the modified BCJ plasticity-damage model that incorporates void nucleation, growth and coalescence [Horstemeyer and Gokhale, 1999]. The elastic properties of the magnesium alloy AM60B were taken from CenBase Materials database at Sandia National Laboratories, California, with a value for Young’s modulus of $E = 44.79$ GPa, and a Poisson ratio of $\nu = 0.291$. The material constants utilized by the plasticity and damage model were based on previously known values and/or values that fit the experimental data best. The experimentally observed void volume fractions measured at 60% of average failure were used to extrapolate the data back to an initial condition of no load.

The boundary conditions for the model include symmetry conditions on the left and bottom edges, and a prescribed uniform, normal displacement of 0.06773 mm on the top edge. The displacement is applied in approximately 50000 increments. The application of incremental displacements causes nonlinear evolution of the stress state due to deformation induced changes in the plasticity and damage state variables as predicted by the modified BCJ model. The two-dimensional simulation was performed using input from tensile bar H24 at 60% and 95% of average failure load. Results were
Figure 7.18 Results from the two-dimensional finite element simulation for tensile bar H24. Results are shown for simulations to 60% and 95% of average failure load.
obtained for the simulation at approximately 60% and 95% of average failure load (see Figure 7.18(a-b)) and compared with experimental results (see Figure 7.14(a-b)).

7.4.2 FEA – 3-D

The binary segmented void data can be used to volumetrically render the three-dimensional morphology of the tensile bars with voids, and to generate three-dimensional meshes for use in a finite element code. Representative slices from the volume of segmented binary data used for rendering and as input for finite element analyses is shown in Figure 7.19.

Figure 7.19 Representative binary slices from tensile bar H24 after loading to 60% of average failure load. This volume was used to render data in three dimensions, and to create input mesh files for finite element codes.
In finite element simulations, the numerical solution of a boundary value problem or an initial value problem defined by a set of differential equations is determined within each finite element or volume. The solutions in each element are then connected across the finite element mesh using piecewise polynomial interpolation to give a solution for the stress state and deformation of the integrated model, given a particular boundary condition and/or forcing function. The binary data created from the CT data can be used to generate a data file containing lists of all elements and the associated node locations within the CT volume, which can then be used to generate an input mesh for use in a three-dimensional finite element analysis. This technique is sometimes referred to as a “voxel-to-element” meshing technique. Because the voxels are cubic, with a volume of $1.33 \times 10^5$ mm$^3$, the resulting mesh elements are also cubic with the same volume, and there are eight-nodes associated with each element. Each element contains either solid magnesium alloy or air (voids). A brief description of the algorithm used to generate the input file for the finite element code used here, LS-DYNA, is given in Appendix F.

LS-DYNA is a commercial finite element code for nonlinear dynamic analysis of structures [LSTC, 1999]. The loading of the tensile bars is quasi-static and is well-suited for an implicit finite element code such as LLNL’s NIKE3D [Maker, 1995]. However the size of the mesh generated from the tensile bars is extremely large. Implicit solvers have a large memory requirement and the size of the meshes used here resulted in
memory storage requirements too large even for the supercomputers available at LLNL such as ASCI-Blue Pacific [Nowak, 1999]. Therefore the decision was made to use a code with an explicit solver, such as LS-DYNA that does not have such large memory requirements.

The original segmented volume for tensile bar H24 contains approximately 568 x 568 x 260 voxels, or a total of 8.4 x $10^7$ voxels. Even with an explicit solver, the enormity of the mesh resulted in extremely inefficient and unreasonably long processing times. Given this, a small portion of the three-dimensional segmented CT volume was used to generate a mesh to allow for faster and more efficient processing and analysis. The portion used is a cylinder with a diameter of 100 pixels and containing 101 slices for a total of $8.08 \times 10^5$ elements. It was extracted from the center of the notched region of the original segmented volume of tensile bar H24 loaded to 60% of its average failure load.

For the magnesium tensile bars, the material is modeled as purely elastic and the voids are considered a vacuum. The material properties used were a density of 1.81 g/cm$^3$, a Young’s modulus of $E = 44.8$ Gpa, and Poisson’s ratio of $\nu = 0.33$. The requirement for the explicit calculation is that the time-scale for the loading must be small enough to eliminate any dynamic response from the tensile bar. The boundary condition used was a strain (displacement) of $23.68 \times 10^{-3}$ mm applied to the top and
bottom faces of the cylinder in a time dependent fashion. The time used was selected to be longer than the inverse of the fundamental mode of the structure to eliminate any dynamic response.

Three-dimensional finite element results are presented in Figure 7.20 (a-c) as contours of z-axis, or axial displacements for the extracted portion of tensile bar H24. Figure 7.20 (a) is a slice through the center of the cylinder, and Figure 7.20 (b) is a slice through the y-axis of the cylinder showing contours of displacement. Figure 7.20 (c) shows the volumetric rendering of the mesh with contours of displacement.
LARGEST TEST GEOMETRY 808K ZONES

Test = 0.3055
Contours of Z-displacement
min = -0.307005, at node 1254
max = 0.307007, at node 1690005

(a)

LARGEST TEST GEOMETRY 808K ZONES

Test = 0.3055
Contours of Z-displacement
min = -0.307005, at node 1254
max = 0.307007, at node 1690005

(b)
Figure 7.20 Results from three-dimensional finite element simulation for an extracted cylinder from tensile bar H24 loaded to 60% of average failure load, displaying contours of displacement for (a) a slice through the $a$-axis, (b) a slice through the $y$-axis, and (c) image showing the outer surface (isometric view) of the meshed cylinder.
Chapter 8

Discussion of Results

This chapter presents a more detailed discussion of the results described and presented in Chapter 7. A discussion of the qualitative results is presented first, followed by a discussion of the quantitative void analysis results and finally results from finite element analyses. Most of the figures and tables referred to in this chapter can be found in Chapter 7. As with any research project, this discussion will lead to several ideas for research to be done in the future. Chapter 9 will present conclusions from this project as well as future work that may be done to resolve some of the issues defined here.

8.1 Qualitative/Semi-quantitative CT analysis

CT data can be used to qualitatively visualize the behavior and morphology of voids in volumes of optically opaque materials. One way to analyze the data qualitatively is to visually compare two-dimensional CT slices between data sets. CT data is most often analyzed in two dimensions. For example, three reconstructed CT slices from tensile bar H24 after three different loading conditions, 60, 87, and 93% of average failure load, are easy to visually compare, as shown in Figure 8.1.
Figure 8.1 Three CT slices from tensile bar H24 after loading to 60, 87, and 93% of average failure load. The circled regions indicate an area where coalescence is occurring. Note also the growth of surrounding voids with increased load.

The slices were located at approximately the same vertical plane in the tensile bar; each slice was selected by visually comparing it to the previously selected slices, since the data sets are not registered to each other. From the three slices it appears that certain voids are growing larger, while other voids located near each other seem to be coalescing into one larger void. The slices can be analyzed quantitatively, for example by determining the void area versus material area and comparing changes between slices. These types of qualitative or semi-quantitative two-dimensional analyses through inspection of slices can give the observer a sense of void behavior inside the material as load is applied. For example in the slices mentioned above, there is evidence of void growth and coalescence occurring with increasing load. Also, there appears to be a ring of low-attenuating material within each slice. Recalling from Chapter 4 that the X-ray attenuation...
I alloy, density or y-dimensional information only and the connectivity and morphology of the voids is not readily revealed when examining two-dimensional slices.

In addition to a qualitative two-dimensional slice comparison, it is possible to volumetrically render CT data sets for animation and display, shown in Figure 8.2. This rendered data provides even more information about the behavior of voids, as well as their shapes and spatial distributions. For example, many voids that are seen in the volumetric rendering do not appear to have spherical shapes, nor do they appear to be spatially distributed in an organized way. The spatial distribution of the voids in fact appears to be somewhat random, with more voids located or clustered toward the center of the cylinder than toward the outer edges. The shape of many of the voids appears highly axisymmetric and jagged and several voids appear to be oriented approximately 45 degrees to the vertical axis. Some voids seem to be oriented vertically and grouped around a ring centered in the middle of the tensile bar. This appears to be the same ring evident in Figure 8.1. While this information can help the observer to visualize the coefficient is a function of elemental composition and density for monochromatic radiation, and because we are relatively certain the material is AM60B magnesium alloy throughout the tensile bars, the low-attenuating ring is most certainly due to a low-density region, possibly due to porosity. This ring is also apparent in slices through the x- or y-axes, as shown in Figure 6.12. These types of qualitative analyses provide two-dimensional information only and the connectivity and morphology of the voids is not readily revealed when examining two-dimensional slices.
structure and locations of voids and to qualitatively understand failure mechanisms, it does not provide quantitative engineering information that can be utilized in the design and optimization of new materials and more accurate material models.

8.1.1 Void shape analysis

As described in Chapter 7.2.6, three orthogonal volumetric shape factors, $K_x$, $K_y$, and $K_z$, and the standard deviations between them were calculated for the maximum void at each loading condition for tensile bar H24 and are presented in Table 7.6. There seems to be no discernible trend in the volumetric shape factors with load. The standard
deviation between the three parameters varies from a maximum of 0.15 to a minimum of 0.0037. To determine what significance, if any, the shape parameters contain with respect to changing void symmetry or morphology, the maximum void for each loading condition for tensile bar H24 was rendered volumetrically and animated for analysis, and is shown in Figure 8.3. It is useful to point out that the location of the maximum void within this tensile bar is changing with loading. In other words, the voids rendered in Figure 8.3 are not the same void.

The volumetric rendering helps to visualize the void morphology and to sense any obvious changes in void shape. The shape of each void does not appear spherical or to any degree symmetric when the volumetric shape factors’ standard deviation is small, for example at 93% loading, as one might expect. In fact, perhaps the best description of the shapes of the maximum voids is that they are similar to the shape of a corn flake. The standard deviations do not appear to correlate in any way with the changing shapes of the voids. Therefore, the volumetric shape factor used does not seem to provide useful information in terms of predicting or evaluating changes in shape or morphology. The volumetric renderings of the maximum voids with their highly nonsymmetric and jagged shapes suggest strongly that assuming spherical void shapes in a modeling simulation will not accurately represent the microstructure of the AM60B tensile bars.
8.2 Void Volume Distributions

While the qualitative and semi-quantitative analyses mentioned above can help give the observer a sense of what is happening with the voids in the material (e.g., it appears voids are growing and coalescing with increasing loads, and void shapes do not appear spherical), quantitative information about void behavior in three dimensions is the
overall objective of this thesis. To obtain quantitative information about voids in the material, a three-dimensional statistical void analysis was performed on the segmented volumes, as described in Chapter 7.2.

The probability density curves created for each set of data are unimodal, with long tails sitting to the right of the central maxima [see Figure 7.10(a-c)]. The value of skewness for each tensile bar calculated for each probability density curve and presented in Table 7.2 decreased overall with loading, from 60% to the final loading before failure. When the probability density curves are plotted to show the peaked region in more detail [see Figure 8.4(a-c)], it becomes apparent that for each data set, the majority of the voids have a volume between 0.001 and 0.02 mm³. The mode, or the void volume with the highest probability, occurs at 0.002 mm³ for each data set, and the small median volume value indicates that the majority of the voids have a smaller volume. The median is defined as the void volume that equally divides the void volume distribution into equal parts and is presented in Table 7.1 (a-c). For every data set, the median values vary from 0.0025 mm³ to 0.0029 mm³, decreasing slightly with increasing load. The average value of the median for each data set was 0.0027 mm³ with a standard deviation of 0.0001, or less than 4%. The standard deviations of the distributions increase overall with increasing load, suggesting that the larger voids are growing even larger. Combined with a decreasing median, it seems that not only are smaller voids growing to the minimum
void volume, but existing larger voids are also growing with increasing load. The tail regions of each probability density curve do not seem to show any detectable pattern, as shown in Figure 8.5(a-c). The maximum void volume (see Table 7.1 (a-c)) increases with increasing load for each tensile bar, as expected, and the largest increase in maximum void volume occurs at the last loading condition before failure for each tensile bar.

![Void Volume Probability Density Curves](image)

**Void Volume Probability Density Curves**
**Peak Region**
**Tensile bar H23**

- H23-60%
- H23-87%
- H23-93%
- H23-Failure

Probability

Void volume (mm$^3$)

(a)
Figure 8.4(a-c) Peaked regions of the probability density curves for each of the three tensile bars at all loading conditions. Note that the probability decreases to zero around 0.02 mm$^3$ for each data set, and the value that occurs most frequently, referred to as the mode, is 0.002 mm$^3$. 
Void Volume Probability Density Curves
Tail Region
Tensile bar H23

(a)

Void Volume Probability Density Curves
Tail Region
Tensile bar H24

(b)

185
8.2.1 Weibull parameters

The cumulative probability curves of void volume for each set of data were fit with a two-parameter Weibull function, as described in Chapter 7.2.3.1. The Weibull parameters, $a$ and $b$, have been shown to have different effects on the shape of the Weibull probability distribution function curves. Gu, et al., have shown that different values of the parameter $b$, the exponential term sometimes referred to as the Weibull modulus or the shape parameter, affect the symmetry of the probability density function curve by shifting it to the left or right. The parameter $a$, also referred to as the scaling
parameter, has an effect on the location of the mean value. An examination of the
Weibull parameters, $a$ and $b$ (see Table 7.3), reveals several interesting trends in the data
relating to void behavior.

Within loading conditions for the same tensile bar, the values of $a$ deviate by only
6 to 8 percent and the values of $b$ deviate from less than 10% to around 13%. Precluding
the failure data for each tensile bar, for tensile bar H23, parameter $a$ decreases with
increasing load, from 0.0042 to 0.0039, a decrease of more than 7%. For tensile bar H24,
while initially the $a$ parameter increases slightly with load, the overall value of $a$
decreases with increasing load from 0.0039 to 0.0036, or by approximately 8%. Tensile
bar H25 does not follow the same trend, with $a$ increasing slightly with load from 0.0046
to 0.0048, or by nearly 4%.

The parameter $b$ decreases with load for two of the three tensile bars. With
increasing load, tensile bar H23 shows a decrease in the parameter $b$ of 28%, H24 shows
a decrease of 10%, and H25 shows a decrease of 8%. When the failure data sets are
considered in the analysis, $a$ decreases by 13.5%, and $b$ decreases by 2% for tensile bar
H23, $a$ decreases by 18%, and $b$ decreases by 21.7% for H24, and for H25, $a$ decreases
by 21% while $b$ increases by 26% with increasing load.

When every data set from the three tensile bars including all loading conditions
and failure are considered, the overall average value of $a$ is 0.004 with a standard
deviation of 0.0004, or approximately 10% deviation. The values for $b$ for every data set including all loading conditions and failure ranged from 0.659 to 1.23 with an overall average value of 0.840 and a standard deviation of 0.17, or approximately 20%.

Considering that the parameters being compared are for three different tensile bars, with up to 5 loading conditions each for a total of 14 data sets, the agreement seems good, and at each loading condition including failure the void distributions can be fit with very similar Weibull functions. This seems to imply that the void nucleation, growth and coalescence behaviors are interacting in such a way as to result in similar shaped void distributions at each loading condition. This is useful information for modeling behavior of the tensile bars.

Using the two parameters derived from the Weibull function fit, a Weibull probability density function curve can be plotted using the equation for $p(x)$, given in equation 7.3, and an example is presented in Figure 8.6. The results show good agreement with experimental data, confirming that the void volume distributions can be reasonably described by a Weibull function with two parameters. Referring to Table 7.4, the median and average void volumes calculated with the Weibull parameters are compared with the experimentally determined values. The Weibull calculated median void volume values are much closer to the experimental median values than the calculated Weibull average void volume values are to the experimental average values.
From the above discussion it is evident that the void volume distributions in the magnesium AM60B tensile bars can be adequately and correctly described with a Weibull distribution function. This information can be extremely useful and valuable as input to numerical models, resulting in more accurate predictions of behavior.

Weibull Probability Density Function
versus Experimental Probability Density Curve
Tensile bar H24 loaded to 93% of average failure load

Figure 8.6 Weibull probability density function calculated using the Weibull parameters compared to experimental probability density curve for tensile bar H24 loaded to 93% of average failure load.
8.3 Nearest neighbor distance distributions

The experimental nearest neighbor distance distributions revealed that the voids have a tendency to cluster, as compared to a random Poisson distribution. For each data set the value of $R$, defined as the ratio of the actual average nearest neighbor distance to the expected average nearest neighbor distance of a random distribution, was less than one. This indicates a clustered spatial distribution of voids compared to an ideal random spatial distribution. In addition, the experimental nearest neighbor distance probability density curve created from tensile bar H24 loaded to 60% of average failure load (shown in Figure 7.12) gave a mean nearest neighbor distance of 0.65 mm, significantly less than the mean nearest neighbor distance of 1.49 mm given by the ideal random Poisson distribution. In addition, the mode of the experimental probability density curve was shifted to the left of the ideal random Poisson probability density curve. This confirms that the spatial distribution of voids in the magnesium AM60B tensile bars tends to be clustered compared to a random or ordered spatial distribution of voids.

While there is no obvious trend in the plots of void volume versus nearest neighbor distance (shown in Figure 7.13), when the average nearest neighbor distance is calculated for void volumes divided above and below the median value, the smaller void volumes consistently have larger average nearest neighbor distances compared to the
larger voids. The values calculated for tensile bar H24 at all loading conditions up to and including failure are presented in Table 8.1

Table 8.1 Average nearest neighbor distances calculated for tensile bar H24. Void volumes were sorted and grouped as above and below the median values.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Loading (% of failure load)</th>
<th>Median value (mm³)</th>
<th>Avg. near. neighbor dist. (mm)</th>
<th>Avg. near. neighbor dist. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>&lt; median</td>
<td>&gt; median</td>
</tr>
<tr>
<td>H24</td>
<td>60</td>
<td>0.0026</td>
<td>0.69</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>0.0026</td>
<td>0.66</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>93</td>
<td>0.0027</td>
<td>0.67</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>95</td>
<td>0.0025</td>
<td>0.66</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.0025</td>
<td>0.71</td>
<td>0.64</td>
</tr>
</tbody>
</table>

The larger nearest neighbor distances for smaller voids indicates that a smaller void is more likely to be located farther away from other voids. Conversely, larger voids tend to be located closer to other larger voids. This is consistent with the clustering behavior discussed above. This spatial distribution information is important when attempting to model behaviors of materials including effects due to voids.

8.4 Damage Statistics

To understand damage accumulation and eventual failure in the magnesium AM60B tensile bars, several parameters related to void nucleation, growth and coalescence were measured. While each theoretical damage event or phenomena, i.e., void nucleation, growth and coalescence, is not independent of the others and is driven by a complex interaction between the three, each theoretical event can be closely associated
with a measurable parameter. Thus, the discussion below presents each of the three theoretical damage events presented in Chapter 2 in terms of its associated measurable parameter, while also including the effects of the other theoretical events in terms of their associated measurable parameters.

8.4.1 Nucleation

The "Number of voids/mm³", or the void density parameter is related to the void nucleation event, \( \eta \), presented in Chapter 2. This measured parameter is presented in the second column of Tables 7.1 (a-c). This parameter gives an indication of both new voids being created and voids being destroyed through coalescence. Because we have identified a minimum void volume for analysis, here nucleation is defined as the creation of a void with a volume larger than the minimum specified volume, \( 1.33 \times 10^{-3} \) mm³. As mentioned in Chapter 2, for measurement purposes a void is not considered to exist or have been nucleated if it has a volume less than \( 1.33 \times 10^{-3} \) mm³. It is important to once again distinguish measurable nucleation related parameters (for example the void density) from nucleation as a physical phenomena. The void density, expressed as \( \eta_m \) in Tables 7.1 (a-c), which is related to the void nucleation term, \( \eta \), is plotted against load in Figure 8.7.
At first glance, for all three tensile bars there seems to be no obvious trend in void density as a function of load. For tensile bar H23, $\eta_m$ decreases from 60% to 87% of average failure load. This is most likely due to larger voids coalescing at a faster rate than voids are growing to the minimum measurable volume, or nucleating. From 87% to 93% (just before failure) of average failure load $\eta_m$ increases, indicating that voids are nucleating at a faster rate than voids are coalescing. The final value of void density at the loading condition just before failure (93% of average failure load) is higher than the value of void density at the first loading condition (60% of average failure load).
Tensile bar H24 shows an initial increase in $\eta_m$ from 60% to 87% of average failure load. The increase is very small, which could indicate that either a small number of voids are nucleating, or within uncertainty the void density remains constant. From 87% to 93% of average failure load $\eta_m$ shows a large decrease, indicating that coalescence is occurring faster than voids are nucleating. From 93% to 95% of average failure load $\eta_m$ increases dramatically. This indicates that nucleation is occurring at a faster rate than existing voids are coalescing. As in H23, tensile bar H24 shows a final value of void density just before failure (95% of average failure load) that is higher than the value at the first loading condition (60% of average failure load).

Tensile bar H25 follows the trend of H23, with an initial decrease in $\eta_m$ at lower loading conditions, from 60% to 93% of average failure load. This again indicates that coalescence is more dominant or occurs at a faster rate than nucleation of voids. From 93% to 97% of failure load, $\eta_m$ increases significantly, indicating that nucleation occurs faster than coalescence. The final value of $\eta_m$ for H25 just before failure (97% of average failure load) is significantly less than the value at the first loading condition (60% of average failure load).

At the final loading condition before failure for each tensile bar, the void density parameter, $\eta_m$, increases significantly for each tensile bar. Thus one explanation for the
fluctuations in void density with load may be that at the lower loads (60 and 87%), in general larger voids are growing and coalescing with other voids faster than smaller voids are growing to the measurable minimum void volume, or "nucleating". At higher loads just before failure (93, 95, and 97%), void growth in terms of nucleation dominates at the expense of coalescence; and voids that were initially smaller than the minimum volume have grown to the minimum volume and are considered to have nucleated. Nucleation as defined here contributes to total damage accumulation in the tensile bars as evidenced by fluctuations in the void density parameter, $\eta_m$, with load. In addition, the decreasing median void volume [column 4, Table 7.1 (a-c)] indicates an increase in the total number of smaller voids in the tensile bar, confirming that nucleation of voids contributes to damage in these tensile bars.

8.4.2 Void growth

The average void volume is related to the theoretical void growth term presented in Chapter 2, $V_v$. The measurable average void volume is described by $V_m$. The subscript "m" again indicates that this parameter is measurable. The third column in Tables 7.1(a-c) presents the measured average void volume for each data set. A plot of $V_m$ versus load is presented in Figure 8.8 below.
As shown in Figure 8.8, in samples H23 and H24, $v_m$ increases as a function of load from 60% up to the final loading condition before failure, which is to be expected. The average void volume in sample H25 initially decreases by a small amount, and then increases. The apparent decrease may be due to uncertainty in the data, and to errors in signal and image processing techniques such as threshold selection. The initial decrease in $v_m$ with load for tensile bar H25 may also be explained by the growth of many smaller voids to the minimum void volume, or nucleation, which would decrease the average void volume. For each tensile bar there are two distinct slope regions in the graph shown in Figure 8.8. The first region, at lower loads, shows $v_m$ increasing at a much lower or
slightly decreasing rate than the second region, at higher loads, where \( v_m \) increases at a much greater rate. Intuitively, one might expect all three tensile bars to exhibit similar behavior since each of the tensile bars are the same material and were cast in the same process. Differences may be due to measurement uncertainty, signal processing errors, or to initial distributions of porosity that result in the different sample behavior.

Void growth contributes to total damage in the tensile bars as evidenced by the increase in average void volume with load. In addition the increase in the standard deviation of the void volume distribution (see column 3, Table 7.1 (a-c)) indicates that existing large voids are growing as well as small voids. Finally the decreasing median void volume indicates that smaller voids are growing to the minimum void volume, or nucleating. These trends indicate that void growth is contributing to damage in these tensile bars through the growth of large voids as well as through the growth of small voids to the minimum measurable volume.

8.4.3 Void coalescence

While there is not a coalescence term per se that can be calculated from the CT data, information about void coalescence can be inferred by analysis of both the measured void density parameter (related to nucleation), \( \eta_m \), and the measured average void volume (related to void growth), \( v_m \). For example, if only coalescence were
occurring, that is without physical void nucleation or growth, then the measured void density parameter, $\eta_m$, would decrease as two or more voids coalesce to form one void, and the average void volume, $v_m$, would increase as several small voids combine to form one larger void. Conversely, if no coalescence occurs and there is only physical void nucleation and growth, then the void density and average void volume terms, $\eta_m$ and $v_m$, would consistently increase as smaller voids grow to the defined minimum volume for nucleation and existing voids grow larger. For each tensile bar, the average void volume, $v_m$, increases with load, while the void density, $\eta_m$, increases as well as decreases with load. Thus it is evident that coalescence contributes to total damage in the AM60B tensile bars as evidenced by the fluctuations in the void density parameter coupled with the increasing average void volume with load.

8.4.4 Total damage

The total damage accumulation in the AM60B tensile bars is due to the effects and interactions of the three damage events described in Chapter 2; void nucleation, growth and coalescence. While the interactions between the three events are not explicitly derived from the measurable parameters discussed above, much can be inferred through an analysis of the parameters and their effects on total measured damage. The total damage accumulation in each tensile bar is determined from the definition of
damage presented in Chapter 2, $\phi = \eta v_c$. Because the coalescence term is embedded within the theoretical nucleation, $\eta$, and growth terms, $v$, for these data sets the total damage is calculated as $\phi_m = \eta_m v_m$, or the measured void density times the measured average void volume, and represents the total volume fraction of voids measured in the material. The total measured damage is presented in Figure 8.9 as a function of load.

![Total Measured Damage versus Load](image)

**Figure 8.9** Plot of total measured damage (void volume fraction) versus loading condition for tensile bars H23-25, loaded from 60 to 97% of average failure load. The total measured damage is calculated as void density times average void volume.

Intuitively one would expect the total damage to increase with increasing load up to failure. However, the damage accumulation trends with load are not obvious for this data. Tensile bar H24 shows an initial decrease in $\phi_m$ loading from 60% to 87% of
average failure load, followed by a decrease from 87% to 93% of average failure load.

At the final loading condition before failure (97% of average failure load) $\phi_m$ for tensile bar H24 shows a large increase. Tensile bars H23 and H25 show an initial decrease in $\phi_m$ with load from 60% to 93-95% of average failure load. Both tensile bars H23 and H25 show a large increase in total measured damage at the final loading condition before failure (93% and 97% of average failure load, respectively).

For each statistical parameter discussed above, measured void density, measured average void volume, and measured total damage in particular, the largest increases occur at the final loading conditions before failure. Thus it is reasonable to say that more damage occurs at higher loads than at lower loads in the AM60B tensile bars. Because the total measured damage is calculated by multiplying the measured void density and the measured average void volume, the effects of both parameters are apparent in the plot of $\phi_m$ versus load.

It appears that the void density parameter has the larger effect on the shape of the total measured damage trend than the measured average void volume. Thus, the shape and general trend observed in damage accumulation in these tensile bars is primarily driven by the measured parameter most closely associated with nucleation events and coalescence, the void density. However, void growth affects the measured void density parameter by way of growing small voids to the minimum measurable void volume.
Therefore it is certain that damage accumulation in these AM60B tensile bars is due to all three damage events described in Chapter 2; void nucleation, growth and coalescence.

Some of the apparent discontinuities or fluctuations in the damage parameter as a function of loading that are not readily explained by an examination of the measured damage parameters could be due to errors in the acquisition and subsequent processing of the CT data. The inconsistencies will be especially apparent in the damage parameter, $\phi_m$, since the errors are additive. In other words, the errors or uncertainty in the measured void density and the measured average void volume would result in a much greater uncertainty in the measured total damage.

One particularly large source of error can be attributed to registration of data sets. In these experiments, alignment of the same slices for comparison is nearly impossible from data set to data set due to the time that elapsed between CT experiments, when the XTM was disassembled and reassembled multiple times, resulting in changed detector configurations. In addition, the tensile bars were affixed to the mechanical stage for scanning with wax and it was nearly impossible to assure registration with this method. Contributing to the registration errors is the fact that during each loading condition, the sample bars were plastically deformed by a small amount. Other sources of error include the semi-subjective selection of a threshold for segmentation (see Chapter 7.1) that can affect the measured void density and measured average void volume parameters, which
in turn would result in cumulative errors in the calculation of measured total damage
accumulation. While it is difficult if not impossible to quantify the uncertainty in this
data, if we estimate the error to be a maximum of 20% for any given data set, which is a
reasonable estimate based on an evaluation of variations in the measured total damage
parameter between data sets, the trends would indeed reveal an increasing accumulation
of internal damage with increased loading, a result that is intuitive.

8.4.5 Effects of low-attenuating ring

Because each CT scan results in a three-dimensional volume of information, it is a
simple task to parse the data in order to reveal important spatial trends. For example,
throughout the length of each tensile bar in Series H, there is a circular region or ring of
low X-ray attenuating material, most likely a result of low-density region caused by
porosity (see Figure 6.12 and Figure 8.1). Initially, this low-attenuating ring was
believed to be a ring artifact caused by an imbalance between detectors, and a lot of time
and energy was spent in an attempt to eliminate or minimize this apparent “artifact”.

When the XTM CT data were examined more closely with other CT data and
compared to further analysis of the film radiographs of the tensile bars (see Figure 8.10),
it was determined that the low attenuating ring was in fact not an artifact, but a true
representation of the internal structure of the tensile bars. This ring is centered about the
longitudinal axis, with a diameter of approximately 1/3 that of the notch. The ring appears to contain significant areas of porosity and low-density material and is clearly seen in radiographic images, as well as CT images. This region contributes as will be demonstrated to the damage progression more than other regions due to its higher initial damaged state.

![Digitized radiograph and CT reconstruction](image)

Figure 8.10 Digitized radiograph (left) and reconstructed CT slice (right) from tensile bar H24-60% showing the low-attenuating ring. It appears as two straight vertical lines in the radiograph and as a ring centered about the middle of the CT slice.

To determine the effect of this ring on measured total damage accumulation in the tensile bars, two volumes of data were extracted from within the full CT volume. First, a
Failure are presented in Table 8.2(c).

Dimensional damage statistics for the extracted volumes for each tensile bar not including the extracted volume using the techniques developed for the full CT volumes. The three extracted annuli has a total volume of 16.4 mm³ and the extracted inner cylinder has a total volume of 18.3 mm³, and the extracted inner cylinder region. The inner cylinder is indicated as "I", and the damaged annulus as "2", in the images. Figure 8.11 (left) shows a cross-section through the z-axis, and through the y-axis (right) showing approximate extracted image showing the approximate locations of the extracted regions is given in Figure 8.11. An annulus containment the low-attenuation ring. An inner annulus containing the inner cylinder of 80 pixels, and a length of 130 slices was extracted from the low-attenuation ring. Next, a hollow cylinder, or annulus, with inner radius of 50 cm of the notched region for each tensile bar. This inner cylinder did not contain the cylinder with a radius of 50 pixels and a length of 130 slices was extracted from the
Table 8.2 (a) Statistics for extracted regions within tensile bar H23. Scanned at SSRL at 60%, 87%, 93% and failure.

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Number of voids/ mm$^3$ $n_m$</th>
<th>Avg. void volume (mm$^3$) $V_m$</th>
<th>Median void volume (mm$^3$)</th>
<th>Max Void Volume (mm$^3$)</th>
<th>Total Damage $\phi_m$ ($10^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H23-1 (60%)</td>
<td>2.378</td>
<td>0.00491 ±0.00430</td>
<td>0.00356</td>
<td>0.01925</td>
<td>11.676</td>
</tr>
<tr>
<td>H23-2 (87%)</td>
<td>2.012</td>
<td>0.00503 ±0.00416</td>
<td>0.00364</td>
<td>0.01753</td>
<td>10.120</td>
</tr>
<tr>
<td>H23-3 (93%)</td>
<td>2.805</td>
<td>0.00485 ±0.00424</td>
<td>0.00329</td>
<td>0.01988</td>
<td>13.604</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Number of voids/ mm$^3$ $n_m$</th>
<th>Avg. void volume (mm$^3$) $V_m$</th>
<th>Median void volume (mm$^3$)</th>
<th>Max Void Volume (mm$^3$)</th>
<th>Total Damage $\phi_m$ ($10^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H23-1 (60%)</td>
<td>1.912</td>
<td>0.00670 ±0.01252</td>
<td>0.00216</td>
<td>0.06814</td>
<td>12.810</td>
</tr>
<tr>
<td>H23-2 (87%)</td>
<td>1.748</td>
<td>0.00991 ±0.03449</td>
<td>0.00212</td>
<td>0.19640</td>
<td>17.323</td>
</tr>
<tr>
<td>H23-3 (93%)</td>
<td>2.130</td>
<td>0.01529 ±0.06011</td>
<td>0.00211</td>
<td>0.37696</td>
<td>32.568</td>
</tr>
</tbody>
</table>
Table 8.2 (b) Statistics for extracted regions within tensile bar H24. Scanned at SSRL at 60%, 87%, 93%, 95%, and failure.

### Inner Cylinder

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Number of voids/ mm$^3$ $\eta_m$</th>
<th>Avg. void volume (mm$^3$) $v_m$</th>
<th>Median void volume (mm$^3$)</th>
<th>Max Void Volume (mm$^3$)</th>
<th>Total Damage $\phi_m (10^{-3})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H24-1 (60%)</td>
<td>2.439</td>
<td>0.00323 ±0.00265</td>
<td>0.00234</td>
<td>0.01295</td>
<td>7.878</td>
</tr>
<tr>
<td>H24-2 (87%)</td>
<td>2.439</td>
<td>0.00323 ±0.00217</td>
<td>0.00255</td>
<td>0.01174</td>
<td>7.878</td>
</tr>
<tr>
<td>H24-3 (93%)</td>
<td>2.134</td>
<td>0.00335 ±0.00240</td>
<td>0.00250</td>
<td>0.01230</td>
<td>7.149</td>
</tr>
<tr>
<td>H24-4 (95%)</td>
<td>2.805</td>
<td>0.00375 ±0.00337</td>
<td>0.00256</td>
<td>0.02004</td>
<td>10.519</td>
</tr>
</tbody>
</table>

### Annulus

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Number of voids/ mm$^3$ $\eta_m$</th>
<th>Avg. void volume (mm$^3$) $v_m$</th>
<th>Median void volume (mm$^3$)</th>
<th>Max Void Volume (mm$^3$)</th>
<th>Total Damage $\phi_m (10^{-3})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H24-1 (60%)</td>
<td>1.530</td>
<td>0.00742 ±0.00922</td>
<td>0.00251</td>
<td>0.03561</td>
<td>11.353</td>
</tr>
<tr>
<td>H24-2 (87%)</td>
<td>1.585</td>
<td>0.00744 ±0.00966</td>
<td>0.00300</td>
<td>0.04128</td>
<td>11.792</td>
</tr>
<tr>
<td>H24-3 (93%)</td>
<td>1.421</td>
<td>0.00786 ±0.00903</td>
<td>0.00335</td>
<td>0.03253</td>
<td>11.169</td>
</tr>
<tr>
<td>H24-4 (95%)</td>
<td>1.475</td>
<td>0.01113 ±0.02508</td>
<td>0.00304</td>
<td>0.13055</td>
<td>16.417</td>
</tr>
</tbody>
</table>
Table 8.2 (c) Statistics for extracted regions within tensile bar H25. Scanned at SSRL at 60%, 87%, 93%, 97% and Failure.

### Inner Cylinder

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Number of voids/ mm³ ( \eta_m )</th>
<th>Avg. void volume (mm³) ( \nu_m ) ±</th>
<th>Median void volume (mm³)</th>
<th>Max Void Volume (mm³)</th>
<th>Total Damage ( \phi_m (10^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H25-1 (60%)</td>
<td>2.99</td>
<td>0.00583 ±0.00858</td>
<td>0.00270</td>
<td>0.04487</td>
<td>17.432</td>
</tr>
<tr>
<td>H25-2 (87%)</td>
<td>3.11</td>
<td>0.00466 ±0.00747</td>
<td>0.00242</td>
<td>0.04140</td>
<td>14.493</td>
</tr>
<tr>
<td>H25-3 (93%)</td>
<td>2.20</td>
<td>0.00585 ±0.00933</td>
<td>0.00349</td>
<td>0.04747</td>
<td>12.870</td>
</tr>
<tr>
<td>H25-4 (97%)</td>
<td>2.32</td>
<td>0.00520 ±0.00836</td>
<td>0.00235</td>
<td>0.04038</td>
<td>12.064</td>
</tr>
</tbody>
</table>

### Annulus

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Number of voids/ mm³ ( \eta_m )</th>
<th>Avg. void volume (mm³) ( \nu_m ) ±</th>
<th>Median void volume (mm³)</th>
<th>Max Void Volume (mm³)</th>
<th>Total Damage ( \phi_m (10^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H25-1 (60%)</td>
<td>2.08</td>
<td>0.00853 ±0.01073</td>
<td>0.00376</td>
<td>0.05451</td>
<td>17.742</td>
</tr>
<tr>
<td>H25-2 (87%)</td>
<td>2.02</td>
<td>0.00743 ±0.00843</td>
<td>0.00368</td>
<td>0.04126</td>
<td>15.009</td>
</tr>
<tr>
<td>H25-3 (93%)</td>
<td>1.53</td>
<td>0.01219 ±0.01679</td>
<td>0.00481</td>
<td>0.07168</td>
<td>18.651</td>
</tr>
<tr>
<td>H25-4 (97%)</td>
<td>1.64</td>
<td>0.01126 ±0.02372</td>
<td>0.00376</td>
<td>0.13066</td>
<td>18.466</td>
</tr>
</tbody>
</table>

Analysis and comparison of the results from the two extracted volumes for each tensile bar, makes it clear that the void statistics from the annulus containing the low-attenuating ring change in a much different fashion than the statistics from the inner cylinder. When examining the statistics we are mostly interested in the trends from 60% to the final loading conditions before failure.
For tensile bar H24, which reflects the basic trends in the other two tensile bars as well, both the extracted inner cylinder and annulus show expected trends such as increasing average void volume and increasing maximum void volume as the load increases from 60% to 95% of average failure load. However, the average void volume and the maximum void volume within the annulus containing the low-attenuating ring increase much more with load compared to the increases in average and maximum void volume for the inner cylinder. Specifically, for tensile bar H24 the maximum void volume increased by a factor of 1.54 for the extracted inner cylinder, while the maximum void volume increased by a factor of 2.6 for the extracted annulus containing the low-attenuating ring from 60% to 95% of average failure load. The average void volume for the annulus increased by almost 50%, compared to a 20% increase for the inner cylinder. The total damage within the inner cylinder increased from $7.88 \times 10^3$ to $10.52 \times 10^3$, or approximately 34%, when loaded from 60 to 95% of average failure load. The total damage within the annulus containing the low-attenuating ring increased from $11.35 \times 10^3$ to $16.42 \times 10^3$, or by approximately 45% for the same loading conditions. The measured void density parameter does not change significantly within the annulus as compared to the inner cylinder, and in fact decreases by approximately 3%. This may indicate that coalescence of existing voids is more dominant within the annulus compared to the inner cylinder, which shows an increase in the measured void density parameter of 208.
about 15%. In general, for each tensile bar the annulus containing the low-attenuating ring contains fewer voids as defined here (minimum volume is \(1.33 \times 10^{-3} \text{ mm}^3\)), however the voids that are measured within the annulus are larger than the voids measured within the inner cylinder.

For these tensile bars, the void growth rate, as reflected by the average void volume parameter as a function of load, within the annulus significantly exceeded that of the inner cylinder despite lower applied stress triaxiality. Triaxiality is defined as the hydrostatic stress divided by the deviatoric stress, where the hydrostatic stress relates to volume change and the deviatoric stress relates to shape change. This higher void growth rate in the annulus occurs despite the fact that based on the geometry of the notched tensile bars the inner cylinder experiences the maximum triaxial stress and is expected to exhibit the highest rates of void growth according to the void growth rule described in Chapter 2. This illustrates the importance of utilizing the correct initial void volume and spatial distributions, related to damage accumulation, when using material models to predict behavior.

From the above discussion it is suggested that the annulus containing the low-attenuating ring contributes to overall damage progression through the mechanisms of void growth and coalescence, while the inner cylinder contributes to damage accumulation more through void nucleation than through void growth or coalescence for about 15%. In general, for each tensile bar the annulus containing the low-attenuating ring contains fewer voids as defined here (minimum volume is \(1.33 \times 10^{-3} \text{ mm}^3\)), however the voids that are measured within the annulus are larger than the voids measured within the inner cylinder.

For these tensile bars, the void growth rate, as reflected by the average void volume parameter as a function of load, within the annulus significantly exceeded that of the inner cylinder despite lower applied stress triaxiality. Triaxiality is defined as the hydrostatic stress divided by the deviatoric stress, where the hydrostatic stress relates to volume change and the deviatoric stress relates to shape change. This higher void growth rate in the annulus occurs despite the fact that based on the geometry of the notched tensile bars the inner cylinder experiences the maximum triaxial stress and is expected to exhibit the highest rates of void growth according to the void growth rule described in Chapter 2. This illustrates the importance of utilizing the correct initial void volume and spatial distributions, related to damage accumulation, when using material models to predict behavior.

From the above discussion it is suggested that the annulus containing the low-attenuating ring contributes to overall damage progression through the mechanisms of void growth and coalescence, while the inner cylinder contributes to damage accumulation more through void nucleation than through void growth or coalescence for...
each tensile bar. The effect of the low-attenuating ring on the total damage evolution within each tensile bar seems to be confirmed by the failure surfaces of the three failed tensile bars. A photograph of the failed tensile bar H24 is shown in Figure 8.12.

![Photograph of failed tensile bar H24. Note the low-attenuating ring is clearly visible on both fracture surfaces.](image)

The low-attenuating ring is easily observed when examining the fracture surface of the failed tensile bars, and all three tensile bars in series H failed in a similar manner, with the circular ring pulling out of the material. This is also confirmed by examining the CT data from the failed tensile bars. The low-attenuating ring is clearly seen in the CT reconstruction of the failed tensile bars (see Figure 8.13) and the failure surface seems to initiate within this area. One cannot stress enough the importance of using the correct
initial conditions and spatial distributions of voids within a material when attempting to
model damage accumulation due to voids in an object.

Figure 8.13 Reconstructed CT image through the y-axis from failed tensile bar H24. Note how the low-
attenuating ring in the top half seems to have pulled out of the bottom half.

8.4.6 Metallography of damaged region

The metallographic images of the quartered sections of the failed tensile bars
revealed much finer detail than that visible in the CT images. However, the
metallographic analysis confirmed several important results from the CT analysis. For
example, the low-attenuating ring section was clearly visible and appears in the optical
microscope photograph in Figure 7.17 as a strip of very fine pores, confirming that the
low-attenuation is due to lower density resulting from porosity. At higher magnification
(see Figure 8.14) the strip of fine pores that causes the low-attenuating behavior shows
evidence of void coalescence, with several larger voids forming on the strip of fine pores. This behavior was suggested from the CT analysis (see Figure 8.13). The metallographic images further confirm that the low-attenuating or damaged ring within each tensile bar was the overwhelming source of damage accumulation and ultimately the cause of failure in the tensile bars.

Figure 8.14 Optical microscope photograph of the top part of tensile bar H25, section 3 (see Figure 7.16), magnified 50X. Note the coalescence of larger voids on the low-attenuating ring.

8.4.7 Fractography

Failed tensile bar H25 was also subjected to a fractographic analysis of its failure surfaces using a scanning electron microscope (SEM). This analysis of the failed surfaces of the tensile bars revealed several interesting characteristics. As expected, the
casting voids were readily observed, along with evidence of ductile coalescence in between, appearing as a dimpled pattern and shown in Figure 7.15. In addition, the material on the fracture surface located near and inside the low-attenuating ring appears different than material located near the outer edge of the tensile bar. Specifically, the material in the low-attenuating ring appears to consist of droplets of magnesium alloy (see Figure 8.15 (a)) and does not appear to be fully dense, as does material near the edge (see Figure 8.15 (b)). This difference in apparent density may be due to the solidification process involved in die-casting and the less-dense material near the low-attenuating or damaged ring may help to explain the failure of the tensile bars near the ring.
Figure 8.15 (a) SEM image of the magnesium alloy located in and around the low-attenuating ring. Note the material looks like droplets and does not appear to be fully dense. (b) SEM image of magnesium alloy located away from the low-attenuating ring.
8.4.8 Summary of CT, metallography, and fractography results

The CT damage analysis, coupled with the metallographic and fractographic analyses, convincingly implies that the low-attenuating ring due to porosity causes failure in the AM60B tensile bars through the growth and coalescence of large voids within the ring of smaller pores. Further, this coalescence and growth of voids on and within the low-attenuating ring leads to the initiation of the failure surface for each of the three tensile bars examined here. Given this data as the starting point, the next question to address is whether the material damage models used to predict failure in the AM60B tensile bars correctly predicts failure in the same mode as that experimentally observed.

8.5 Finite Element Analyses

The next step is to use the CT data as input to two different finite element analyses. One code uses a two-dimensional mesh, incorporating assumptions of symmetry, with the input obtained from the void volume fraction distribution analysis described in Chapter 7.2.5. The other code uses a three-dimensional mesh including voids generated from the binary CT volumes. A discussion of the results of these analyses is presented below.
8.5.1 Two-dimensional analysis

The two-dimensional quarter-plate axisymmetric finite element simulation described in Chapter 7.4.1 revealed increasing damage with increasing load for tensile bar H24. The simulated results shown in Figure 7.18 show increasing contours of damage (void volume fraction) along the low-attenuating ring region, confirmed by the experimental data (Figure 7.14). Comparing the experimental data at 95% to the simulation results at 95% of average failure load, the simulation shows a maximum void volume fraction of 0.06, while the experimental data shows a maximum of 0.0597. These numbers agree well. However, the simulation at 95% of average failure load gives higher void volume fraction values near the center of the tensile bar that were not observed experimentally (see Figure 7.14). This could be due to uncertainty regarding the initial condition of zero load. The initial condition was determined by extrapolating back to zero load using the 60% average failure load data. This was necessary as the XTM CT system was not available for use until after all tensile bars had been loaded to 60% of their average failure loads. The differences could also be due to a registration error in determining the experimental void volume fractions. For example, if the routine used to calculate the void volume fraction distribution was started on one sample at a certain loading condition at a slightly different location than another, then results would be
different from sample to sample, and loading condition to loading condition. The discrepancy could also be due to errors in the parameters chosen for the simulation that were subjectively selected based on previous knowledge. Finally, the differences in spatial distribution of damage accumulation could be due to errors in the mesh or in the simulation model, which would need to be addressed.

8.5.2 Three-dimensional analysis

If the extracted cylinder from the magnesium tensile bar contained no voids and was modeled as a purely elastic material, then the finite element simulation would give smooth contours of displacement throughout the volume. The presence of the voids in the simulation presented in Chapter 7 affected the axial displacement contours as shown in Figure 7.20 (a-c). Figure 7.20 (a) presents a slice through the z-axis. The large anomalous red “bulls-eye” in the middle of the circular planar image is evidence of the voids within the tensile bar affecting the overall macroscopic or continuum mechanical response of the object. Similarly, Figure 7.20 (b) shows several anomalies in the displacement contours that appear as jags in the contour lines, confirming the effects of the internal voids on the macroscopic behavior. The isometric external view of axial displacement contours shown in Figure 7.20 (c) does not contain the same types of anomalies seen in the two internal planar slices, and has smooth contours across the outer
surface. Additionally, as seen in Figure 7.20 (a) the outer edges of the planar slice shows anomalous red areas, which are most likely a result of the faceting or “voxelizing” of the mesh which generate large stress concentrations at corners.

The three-dimensional finite element simulations on a small cylindrical portion of tensile bar H24 revealed that the presence of voids within the mesh affected the overall response of the object. While this result may seem obvious, many finite element modelers and analysts do not seem to appreciate the value in using a mesh created from the actual geometry of the object under investigation. Tomographers have tried to get finite element modelers interested in the use of CT data to generate finite element meshes for years. In doing so, several finite element modelers have even expressed the idea that using CT data to generate a finite element mesh is a supreme waste of time, an idea that unfortunately is not uncommon [Hollister, 1999]. Encouragingly, more and more modelers are beginning to see the light with respect to modeling “as-built” geometries of the object being studied. While many issues remain with respect to efficiently converting voxel CT data to finite elements, such as decreasing the size of the mesh itself and dealing with the faceted structures created, the potential in developing this process is enormous.
Chapter 9

Conclusions and Future Work

In this chapter a summary of the most important conclusions drawn from the results and discussion presented in Chapter 7 and Chapter 8 are presented. Several ideas for future work as it applies to this research are then suggested.

9.1 Conclusions from void analysis

The research results described in this dissertation represent the most recent work in three-dimensional quantitative analysis of CT data as it applies to damage accumulation due to voids in magnesium alloy AM60B tensile bars. Analyses of the CT data acquired from three magnesium AM60B tensile bars at different loading conditions led to eight important conclusions regarding the behavior of voids in the alloy as a function of load.

1. Voids in the AM60B tensile bars are growing and coalescing with increasing load, as evidenced by a qualitative comparison of CT slices at different loading conditions, and by an examination of the three-dimensional volumetrically rendered CT data at different loading conditions.
2. Many voids within the tensile bars are not spherical or show any degree of symmetry, as shown by the volumetric renderings of the largest volume void after each loading condition in tensile bar H24. Thus, the often-used modeling assumption of spherical void shapes within these tensile bars is not accurate.

3. Void volume distributions for every data set were described by a Weibull distribution function with two parameters. Furthermore, the values of the two parameters did not vary significantly between loading conditions, indicating that voids within the magnesium alloy behaved in such a way as to produce similar shaped void volume distributions regardless of loading.

4. Voids were shown to be spatially distributed in a clustered orientation, as compared to an ideal random Poisson or ordered spatial distribution. This was confirmed by calculating the degree of randomness, R. It was further confirmed by generating an ideal random Poisson probability density curve and comparing it to an experimental probability density curve.

5. The total damage accumulation in the AM60B tensile bars was shown quantitatively to be due to the three void damage phenomena described in Chapter 2; void nucleation, growth and coalescence. Most of the total damage accumulates at the final loading condition before failure.
6. The low X-ray-attenuating ring consisting of very small pores within each tensile bar caused failure in each bar through the coalescence and growth of large voids within and on the ring. Quantified damage statistics from the low-attenuating ring region confirmed this, as well as metallographic and fractographic images which showed the coalescence of large voids within and on the low-attenuating ring.

7. CT data was used as input to a two-dimensional finite element code for analysis and model parameters were tuned to fit the experimental data. The damage model used showed reasonable agreement with experimental damage results.

8. CT data was used to create an “as-built” three-dimensional finite element mesh, which reflected the true geometry and internal structural features of one magnesium AM60B tensile bar. Simulations revealed that overall mechanical behavior was influenced by the presence of voids within the tensile bar.

9.2 Future Work

There are several limitations in the experimental method presented in this dissertation that should be addressed. First, registration between CT data sets was shown to be a problem. A technique should be developed to ensure that identical volumes are
used when comparing CT data for the same sample between loading conditions. The use of a fiducial marker within the tensile bars could help make registration easier by providing a consistent reference against which to align all data. However, even this approach may not be practical due to the nature of the experiments. Plastic displacements that occur as a result of loading can result in the fiducial becoming irrelevant. In addition, the nature of the CT scan requires movement of the part, and ensuring an object starts and ends a scan at a particular angular increment is not trivial. The registration error is a difficult problem to overcome. Precision staging and a more accurate method of attaching the tensile bars to the mechanical stage (i.e. not wax) may help minimize registration errors.

A data and image processing issue that must be addressed is the selection of threshold values used for segmentation of voids from magnesium alloy for further analysis. In this project thresholds were selected somewhat subjectively which leads to uncertainty in the data. The selection of a threshold value for segmentation of voids or other objects of interest within a CT volume is not a trivial problem, as attenuation values can vary by a large amount throughout the volume by slice. A more robust quantitative, consistent threshold selection or other segmentation technique such as using both thresholds and gradients is needed to increase certainty in void analysis results. Additional investigation should be performed regarding the selection of a minimum void
volume, and the significance, if any, of the two different slope regions observed in the cluster label histogram shown Figure 7.9 should be determined.

CT data should be acquired for initial (unloaded) tensile bars as well as for a greater number of loading conditions than the four or five loadings presented here. Also, the loading conditions should be interpreted based on percentages of actual failure of particular samples, as opposed to percentages of average failure loads, as presented here. Increased loading conditions would lead to a better understanding of the damage mechanisms and eventual failure in the magnesium alloy.

Clearly, more magnesium alloy tensile bars should be analyzed to obtain a better understanding of void behavior as it relates to damage accumulation in AM60B magnesium tensile bars. Not only would more samples contribute to the development of improved image processing techniques to decrease uncertainty in the data (e.g., a more quantifiable threshold selection), but a larger data set would also help to clarify and interpret void behavior trends as related to damage accumulation due to voids in magnesium AM60B.

The information presented in this dissertation should be used to optimize the casting process to minimize or eliminate the low-porosity ring that was the source of failure in each tensile bar studied. The initial region of low-porosity was shown to be the overwhelming source of damage accumulation and eventual failure in these tensile bars.
and therefore should be minimized if not eliminated through a thorough analysis and refinement of the die-casting process and parameters used to cast these tensile bars. Elimination of the ring of porosity would most likely increase the ultimate strengths of the tensile bars.

More effort should be given to using “as-built” geometries as opposed to assumptions about the object under study for validation and refinement of material models. To ensure robustness and accuracy in the models, developers of models must make an effort to use input data from the actual object of interest and its “as-built” geometry, including correct spatial and size distributions of internal features such as voids. For example, neglecting to include the ring of porosity that exists within each tensile bar studied here would undoubtedly lead to incorrect predictions of failure. In addition, methodologies should be developed or new material models should be developed to more efficiently use initial internal structural conditions derived from nondestructive as well as destructive data to accurately calibrate and refine model parameters. Most available material models do not have the capability to effectively use the wealth of NDE data that already exists.

More research needs to be applied for creating “as-built” three-dimensional meshes from nondestructive data such as that acquired with CT scans. Currently, there are very few codes available commercially to create “voxel-to-element” meshes. Codes
being developed by researchers were also very difficult to obtain. In addition, the meshes created are often too large for computer systems to handle. Several options exist to make the input mesh files to FEA programs smaller, such as adaptive mesh refinement programs, and using unstructured, versus structured meshes. Finally, the faceted surfaces that result from creating elements from cubic voxels can result in large stress concentrations. This voxelizing or faceting that results from the creation of the mesh from voxels must be addressed. This area is the most exciting in terms of potential development and future applications. Creating meshes from “as-built” geometries of real objects would reduce the number of assumptions currently necessary to perform finite element analyses, resulting in more accurate predictions of behaviors and optimized materials.
Appendix A: Cluster labeling

The IDL routine is used to call the external cluster labeling routine, written in the C programming language.

```idl
pro clusters, vol, n, all=all

; component labeling    (connectivity = 6 or 26)

; vol       : volume; all nonzero values will count as material
;           : vol is output with components labeled

; n         : number of components found

; keyword=all: for 26-connectivity. (Default is 6-connectivity)

t0=systime(1)

sz=size(vol)
if sz(4) ne 1 then begin
    if sz(4) eq 2 and max(vol) le 255 then
        vol=byte(vol) ; allow int array if max val <= 255
    else begin
        print,"Volume input must be a BYTE Array!"
        STOP
    end
end

x=sz(1)
y=sz(2)
z=sz(3)
n=0

v2=intarr(x,y,z)

if keyword_set(all) then connect = 26 else connect = 6

pval=0

result=call_external( value = [0B, 0B, 1B, 1B, 0B, 1B, 1B, 1B], $'
'/reine4/IDLwork/csrc/idl_c.a','getComponents', $vol, v2, x,y,z, n, pval, connect )

if sz(4) eq 1 then vol = v2
else vol = fix(v2) ; return same type that was input
```

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tf=systime(1)
delt=tf-t0
print,'Run time = ',delt,' seconds',' there are ',n,' clusters'
end
Appendix B: Nearest Neighbor Distance

The following routine was used in IDL to calculate the nearest neighbor distance for each selected void.

; this routine calculates the 3-D centroid (x,y,z) for voids determined from cluster analysis
; i.e. each void must have a different value
; it also calculates the 3-d euclidean distance between all voids and determines
; the nearest neighbor distance and writes it out to a text file
PRO CENT_DIST2, vol, cent=cent, dist=dist
his=histogram(vol); use a histogram to label voids
num=where(his gt 0, count); determine how many voids there are
list = count
cent=fltarr(3, count)
dist=fltarr(count,count)
s=size(vol)
tmpname='TEXT.txt'; widget to open an ascii file to write output to
filter_type= '*.txt'
fname=dialog_pickfile(/write,file=tmpname,filter=filter_type,TITLE=E='Select the file to write minimum nearest neighbor distance')
IF fname EQ '' THEN BEGIN
Print, 'No file name was specified'
stop
END
OPENW,2,fname
FOR i=2, list-1 DO $ ; loop to process each void
BEGIN
dex=where(vol eq i, count)
volume= count ;finding indices of void(i)

index_z = dex / [S[1] * S[2]]
index_y = (dex - (index_z * S[1] * S[2])) / S[1]
index_x = (dex - (index_z * S[1] * S[2])) - (index_y * S[1])

;find centroid of void
Cg_x = total(index_x)/count
Cg_y = total(index_y)/count
Cg_z = total(index_z)/count

;write results to a text file
Printf,2,'void number = ', i, ' volume = ', volume ;' centroid = ('',Cg_x, Cg_y, Cg_z,)'

; write each centroid into an array
; write results to screen

print, 'void number = ', i, ' volume = ', count, ' centroid = ('
  cent(0,i)=Cg_x
  cent(1,i)=Cg_y
  cent(2,i)=Cg_z
ENDDO

; now we have a list of void number, volume and it's centroid...calculate nearest neighbors now!
; Close, 2
; create array with distances between voids

FOR j=2, list - 1 DO $
  BEGIN
    xl=cent(0,j)
    yl=cent(1,j)
    zl=cent(2,j)
  ENDIFOR

FOR k=2, list - 1 DO $
  BEGIN
    x2=cent(0,k)
    y2=cent(1,k)
    z2=cent(2,k)

    distance= ((xl-x2)^2 +(yl-y2)^2 + (zl-z2)^2)^0.5
    print, 'distance between void', j, 'and void', k, '= ', distance
  ENDDO

ENDFOR

; write out minimum neighbor distance
; set all distances between same void to 1000

dex=where(dist eq 0)
dist(dex)=1000

; loop to determine and write out minimum distance for each void
FOR l=2, list - 1 DO $
  BEGIN
    ; write out to file selected above
    printf, 2, 'void number =', l, ' min dist = ', min(dist(l,*))
  ENDDO

END
Appendix C: Derivation of average nearest neighbor distance in an ideal random Poisson distribution

For particles or voids distributed in three-dimensional space with a Poisson process with a density of \( \eta \) (number of voids per unit volume), and if \( r \) is the distance between an individual void and its nearest neighbor, the distribution function for \( r \) is expressed as [Parzen, 1962]:

\[
p(r) = 4\pi \eta r^2 e^{-\frac{4}{3} \pi \eta r^3}.
\]

The expected mean distance, \( \bar{r}_E \), is obtained by multiplying the above expression by \( r \) and integrating over the interval from 0 to \( \infty \) [Clark and Evans, 1954].

\[
\bar{r}_E = \int_0^{\infty} 4\pi \eta r^3 e^{-\frac{4}{3} \pi \eta r^3} dr
\]

Solving the above integral using Mathematica gives

\[
\bar{r}_E = \frac{\Gamma(\frac{1}{3})}{6^{\frac{2}{3}} \pi^{\frac{1}{3}} \eta^{\frac{1}{3}}} = 0.812 (\pi \eta)^{\frac{1}{3}},
\]

where \( \eta \) is the void density.
Appendix D: Void Volume Fraction

The following IDL routine was used to calculate the void volume fraction averaged over 
0 at all r and z positions in a tensile bar. The routine is used on the segmented data 
containing only voids larger than the minimum size of 100 connected voxels.

PRO FRACTEST, VOLALL

; this procedure reads in a volume, extracts slices then extracts 
concentric rings, calculates total and prints. 
; Then each successive volume is totaled to get total for each ring. 
The slices, radius and the totals are printed to an ascii file.

tmpname='TEXT.txt'
filter_type='*.txt'
fname=dialog_pickfile(/write, file=tmpname, filter=filter_type,TITLE='Select the file to write')

IF fname EQ ' ' THEN BEGIN
Print, 'No file name was specified'
stop
ENDIF

OPENW,1,fname ;Open LUN1 for fname with write access 
 ; opened an ascii file to write information to

FOR j=0,144, 8 DO $
BEGIN
    vol=volall(*,*,j:(j+7))
 ; selected slices to extract rings from
    FOR i= 1, 35 DO $
    BEGIN
    ; creating loops to extract cylinders

        volex=cyl_ex(vol,center=[289,286],radius=8*(i),
background=0,/resize)

        zdist=(j+4)
radius=8*(i)
;radiusmm=10*(i)*(.02368) ; in mm
;volumemm=3.14*(.14)(radiusmm)^2 ; in cubic mm 
volume=(3.14)*4*(radius)^2 ; pi * radius^2 * length

dex=total(volex) ; total number of void voxels

Print, 'Total volume of cylinder in pixels is ', volume
Print, 'Total number of occupied sites is', dex
PRINTF, l, 'z dist (pix)= ',zdist,' radius (pix)= ',radius, ' void pix = ',dex ; 'total volume = ', volume
Appendix E: Shape Parameter

The following IDL routine was used to calculate the projected area, $A_p$, of each void in three dimensions (yz-, zx-, and xy-planes).

```idl
PRO proj_all, image

; select a file to write output to
tmpname='TEXT.txt'; widget to open an ascii file to write output to
filter_type= '*.txt'
fname=dialog_pickfile(/write,file=tmpname,filter=filter_type,$
TITLE='Select the file to write projected area')

IF fname EQ '' THEN BEGIN
Print, 'No file name was specified'
stop
END

OPENW, 2, fname

his=histogram(image); use a histogram to label voids
num=where(his gt 0,count); determine how many voids there are
list=count

dim=size(image)
IF dim[0] EQ 3 THEN BEGIN
xdim = dim[1]
ydim = dim[2]
zdin = dim[3]
FOR n=1, list-1 DO $
BEGIN

    vol=where(image eq n,count)
    volume=count ; determine total volume of void

    cimage=bytarr(ydim,zdim)
    pimage=bytarr(ydim,zdim)

    FOR i=0, xdim-1 DO BEGIN
        w=WHERE(image(i,*,*) EQ n, count)
        IF count GT 0 THEN BEGIN
            print, 'Found ',count,' pixels at slice # ',',i
cimage(w)=255
            pimage = (pimage + cimage)/2
        ENDIF
        dex=where(pimage gt 0, count)
    ENDFOR

    print, 'Total area for void number ',',n,' = ',',count
```

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printf, 2, 'Void Num = ', n, ' Volume = ', volume, ' Projected area in x = ', count

ENDFOR
close, 2

ENDIF ELSE BEGIN
print, 'Must be a 3D image'
ENDIF ELSE

END
Appendix F: Algorithm developed to write finite element input files

Algorithm used to generate input files for finite element code:

Dimensions = (X=2, Y=2, Z=2)
There are (X+1)*(Y+1)*(Z+1) nodes = 27

Program:

\[\text{yinc} = \text{xdim} + 1\]
\[\text{zinc} = (\text{xdim} + 1) * (\text{ydim} + 1)\]
\[\text{counter} = 1\]

For \(z = 0\) to \(z\text{dim}\)
  For \(y = 0\) to \(y\text{dim}\)
    For \(x = 0\) to \(x\text{dim}\)
      Element = counter, counter+1,
                 counter+yinc+1, counter+yinc,
                 counter+zinc, counter+zinc+1,
                 counter+zinc+yinc+1, counter+zinc+yinc
      counter = counter+1
    EndFor (x)
  EndFor (y)
  counter = counter+yinc
EndFor (z)

Example object – 2 x 2 x 2 cube
Example output file

*--------------------------------- NODE DEFINITIONS --------------------------------- *

1,0,0,0
2,1,0,0
3,2,0,0
4,0,1,0
5,1,1,0
6,2,1,0
7,0,2,0
8,1,2,0
9,2,2,0
10,0,0,1
11,1,0,1
12,2,0,1
13,0,1,1
14,1,1,1
15,2,1,1
16,0,2,1
17,1,2,1
18,2,2,1
19,0,0,2
20,1,0,2
21,2,0,2
22,0,1,2
23,1,1,2
24,2,1,2
25,0,2,2
26,1,2,2
27,2,2,2

*--------------------------------- ELEMENT CARDS FOR SOLID ELEMENTS --------------------------------- *

1,1,1,2,5,4,10,11,14,13
2,1,2,3,6,5,11,12,15,14
3,1,4,5,8,7,13,14,17,16
4,1,5,6,9,8,14,15,18,17
5,1,10,11,14,13,19,20,23,22
6,1,11,12,15,14,20,21,24,23
7,1,13,14,17,16,22,23,26,25
8,1,14,15,18,17,23,24,27,26
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