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SECOND LAW OF THERMODYNAMICS

BY

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**PLASMA PHYSICS
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Tokamak Microturbulence and the Second Law
of Thermodynamics

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ABSTRACT

The second law of thermodynamics is shown to place interesting limits on fluctuation amplitudes and on the form of interaction of the turbulence with the plasma in large tokamaks.

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The second law of thermodynamics places important restrictions on fluctuations in present day tokamaks. This arises from the entropy density increase due to classical collisions if the distribution function deviates from a local Maxwellian. The temperature T times the rate of entropy density increase $\dot{S}_C$ is the rate organized, high grade energy is converted to low grade thermal energy by classical collisions. Assuming microturbulent processes obey the second law, the deviations from the Maxwellian distribution produced by the turbulence must not cause a rate of entropy increase due to classical effects which is greater than the rate of high grade energy being fed into the system. This can be stated mathematically in terms of the often quoted energy confinement time τ_E

$$\int T \dot{S}_C d^3x \leq \frac{1}{\tau_E} \int \frac{1}{2} n T d^3x \quad (1)$$

with n the electron density.

The expression for $\dot{S}_C$ is well known. Let the exact electron or ion distribution function be $f = f_M(1 + \hat{f})$. The local Maxwellian f_M is defined so the appropriate moments give the particle, momentum, and energy densities. Then,

$$\dot{S}_C = - \int \hat{f} C(\hat{f}) d^3v \quad (2)$$

with $C(\hat{f})$ the linearized Fokker-Planck collision operator. A simple limit can be placed on $\dot{S}_C$, for

$$\dot{S}_C = - \int \hat{f} C(\hat{f}) d^3v \geq \int \nu_D(v) \hat{f}^2 f_M d^3v \quad (3)$$

with $1/v_D$ the time it takes classical collisions to deflect a particle 90° . Of course, if $\hat{f}$ is highly localized in velocity space, $\dot{S}_C$ will be much larger than the expression using v_D . Combining inequalities (1) and (3), we find that

$$\langle v_D^2 \hat{f}^2 \rangle \leq 1 \quad (4)$$

with the angular brackets implying both an average over the Maxwellian in velocity space and the appropriate spacial average. In present day tokamaks $\tau_E \sim 40$ msec., while the electron collision time $1/v_D \sim 100$ μ sec., which implies the root mean square value of $\hat{f}$ must be less than about 5%. The comparable number for ions is much larger due to their smaller collision rate. It is about 40%.

Fluctuations in electron density and temperature have been observed¹ in the central regions of tokamaks at levels of order 1%. The extent to which these fluctuations imply deviations from a local Maxwellian is model dependent.

In addition to placing constraints on fluctuation amplitudes, the second law places constraints on the interaction of the time averaged electron or ion distributions with the microturbulence. Since the microturbulent fluctuations are assumed to be responsible for enhanced transport coefficients in tokamaks, the scattering of the averaged electron distribution due to microturbulence must be larger than that due to classical effects. If the drag term due to fluctuations were not comparably enhanced, the electron distribution function would be very non-Maxwellian with energy plateaus. Since the second law implies this cannot

be the case in modern tokamaks, the turbulence must obey a fluctuation-dissipation theorem of the standard type.²

The interaction of the electron gas with the microturbulence takes the form of a collision operator. If we assume the microturbulence is only locally correlated, the second law implies the effective linearized collision operator must be of the form

$$C(\hat{f}) = \frac{\partial}{\partial \vec{v}} \cdot \left[\frac{T}{m} f_M^{\vec{v}} \cdot \frac{\partial \hat{f}}{\partial \vec{v}} - \hat{R} \right] \quad (5)$$

with $\hat{R}$ a flux in velocity space which restores the conservation laws like energy and momentum. To obtain a simple expression for $\hat{R}$, we will now let f_M be a stationary Maxwellian with any small motion of the Maxwellian accounted for in $\hat{f}$. We assume $\frac{\vec{v}}{v}$ can be chosen so

$$\left\langle \frac{\vec{v}}{v} \cdot \frac{\vec{v}}{v} \right\rangle = 0 \quad (6)$$

$$mn \left\langle \frac{\vec{v}}{v} \cdot \frac{\vec{v}}{v} \right\rangle = m_f n_f \left\langle \frac{\vec{v}}{v} \cdot \frac{\vec{v}}{v} \right\rangle \quad (7)$$

$$mn \left\langle \frac{\vec{v}}{v} \cdot \frac{\vec{v}}{v} \cdot \frac{\vec{v}}{v} \right\rangle = m_f n_f \left\langle \frac{\vec{v}}{v} \cdot \frac{\vec{v}}{v} \cdot \frac{\vec{v}}{v} \right\rangle \quad (8)$$

with the subscript "f" implying the field particles off which the species is scattering and the brackets $\langle \dots \rangle$ now implying an average over a Maxwellian. The three conditions (6) to (8) can be easily shown to hold for classical collisions and in part make the choice of $\hat{R}$ specific. If $\hat{R}$ is chosen so

$$\hat{R} = f_M^{\vec{v}} \cdot (\vec{E} + n\vec{v}) \quad (9)$$

with $\vec{\xi}$ and η independent of velocity, then momentum and energy conservation imply

$$\vec{\xi} = \frac{T}{m_f} \left\langle \frac{\vec{v}}{v_f} \right\rangle^{-1} \cdot \left\langle \frac{\vec{v}}{v_f} \cdot \frac{\partial \hat{f}_f}{\partial \vec{v}_f} \right\rangle \quad (10)$$

$$\eta = \frac{T}{m_f} \left\langle \frac{\vec{v}}{v_f} \cdot \frac{\vec{v}}{v_f} \cdot \vec{v}_f \right\rangle^{-1} \left\langle \frac{\vec{v}}{v_f} \cdot \frac{\partial \hat{f}_f}{\partial \vec{v}_f} \right\rangle \quad (11)$$

For classical collisions one would have

$$\langle v_{ij} \rangle = \frac{1}{3} \langle v_{kk} \rangle \delta_{ij} \quad (12)$$

but this need not be true more generally. With Eqs. (9) - (11) defining $\vec{R}$ the collision operator not only conserves energy and momentum but is Galilean invariant, Hermitian, and of course obeys an H theorem. Hermiticity implies the time reversability of the microturbulence and Onsager relations. This may or may not be correct and could be experimentally checked by comparing Onsager related quantities like the Ware pinch and the bootstrap current.

For classical collisions the form given above is a useful approximation to the exact collision operator For classical collisions

$$\frac{\vec{v}}{v} = v_E \frac{\hat{v} \hat{v}}{v} + v_\Omega (\hat{\delta} - \frac{\hat{v} \hat{v}}{v}) \quad (13)$$

$$v_E = v_0 \frac{\psi(x)}{x}, \quad v_\Omega = v_0 \frac{\phi(x) - \psi(x)}{x}, \quad v_0 = \frac{8\pi n_f e^2 e_f^2}{m m_f \alpha_f^2} \ell_n \Lambda$$

$$x = v/v_f, \quad \frac{2}{f} = 2T/m_f, \quad (x) = (-x')/2x^2$$

and (x) is the error function.

The second law of thermodynamics has been shown to place two important restrictions on microturbulence in tokamaks. First, any fluctuations in the electron distribution function must be small — less than a few percent. Second, the effective electron collision operator must have drag as well as scattering terms. Equivalently, one can say the effective collision operator must obey an H theorem with the Maxwellian as its equilibrium solution.

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